9. TRIANGULATION

DEFINITION

The horizontal control in Geodetic survey is established either by triangulation or by precise traverse. In triangulation, the system consists of a number of inter-connected triangles in which the length of only one line is called the base line and the angles of the triangle are measured very precisely.

		First order or 1° Triangulation	Second order or 2° Triangulation	Third order or 3° Triangulation
1.	Average triangle closure →	< 1 seconds	3 seconds	6 seconds
2.	Maximum triangle closure →	≯ 3 seconds	8 seconds	12 seconds
3.	Length of base line →	5 to 15 kilometers	1.5 to 5 km	0.5 to 3 km
4.	Length of the sides of triangles →	30 to 150 kilometers	8 to 65 km	1.5 to 10 km
5.	Actual error of base →	1 in 300,000	1 in 150,000	1 in 750000
6.	Probable error of base →	1 in 1000000	1 in 500,000	1 in 250,000
7.	Discrepancy between two → measures of a section	$10\ mm\sqrt{kilometers}$	$20\ mm\sqrt{km}$	$25\ mm\sqrt{km}$
8.	Probable error of computed → distance	1 in 60000 to 1 in 250000	1 in 20000 to 1 in 50,000	1 in 5000 to 1 in 20,000
9.	Probable error in astronomic → azimuth	0.5 seconds	2.0 seconds	5 seconds

Remember

- When the shape of the triangle is such that any error in the measurement of angle has minimum effect upon the lengths of the calculated side, than such a triangle is called well conditioned triangle.
- The best shape of well condition triangle is Isosceles with base angle equal to 56°14′.
- The triangle having angle <30° and angle > 120° should be avoided.

- **Criterion of strength of figure**

  The strength of figure is a factor to be considered in establishing a triangulation system for which the computation can be maintained within a desired degree of precision.

  The square of a probable error (L^2) that would occur in the sixth place of the logarithm of any side,

$$L^2 = \frac{4}{3}d^2R$$

where, $R = \frac{D-C}{D}\Sigma\left[\delta_A^2 + \delta_A\,\delta_B + \delta_B^2\right]$

d = Probable error of an observed direction in seconds

D = Number of directions observed (forward and/or backward)

δ_A = Difference per second in the sixth place of a logarithms of the sine of the distance angle A of each triangle

δ_B = Same as δ_A but for the distance angle B

C = Number of angles and side conditions

$C = (n' - s' + 1) + (n - 2s + 3)$

n = Total number of lines

n′ = Number of lines observed in both directions

s = Total number of stations

s′ = Number of occupied stations

$(n' - s' + 1)$ = Number of angle conditions

$(n' - 2\delta + 3)$ = Number of side conditions

SIGNALS AND TOWERS

- **A signal is a device erected to define the exact position of an observed station.**

  A. **Non Luminous Signals:** Diameter of signal in cms = 1.3 D to 1.9 D
  Height of signal in cms = 13.3 D
  where, D = distance in kms (Length of sight) for non luminous signals

  B. **Luminous or Sun Signals:** Used when length of sight distance > 30 Kms.
  Phase of Signals: It is the error of bisection which arises, when the signal is partly in light and partly in shade.

- **Correction**

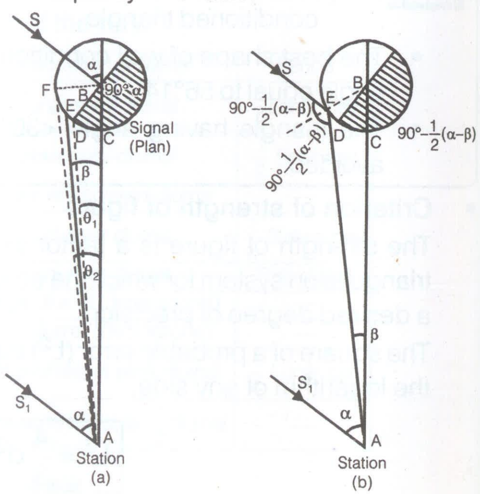
  (i) When observation is made on the bright portion.
  Phase correction,

$$\beta = \frac{r\cos^2\frac{\alpha}{2}}{D}\ \text{radians}$$

α = Angle which the direction of sun makes with line of sight.

r = radius of the signal.

D = Distance of sight.



(ii) When the Observation is made on the bright line:

$$\beta = \frac{r\cos\frac{\alpha}{2}}{D}\ \text{radians}$$

ROUTINE OF TRIANGULATION SURVEY

The routine of triangulation survey generally consists of the following operations:

1. Reconnaissance
2. Erection of singles and towers
3. Measurement of base lines
4. Measurement of horizontal angles
5. Astronomical observations at Laplace stations, and
6. Computations

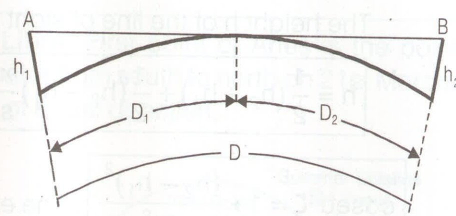
INTERVISIBILITY AND HEIGHT OF STATIONS

(a) **The distance between the stations:** If here is no obstruction due to intervening ground, the distance of the visible horizon from a station of known elevation above datum is given by

$$h = \frac{D^2}{2R}(1-2m)$$

where, h = height of the station above datum
D = distance to the visible horizon
R = mean radius of the earth

(b) **Relative elevation of stations:** If there is no obstruction due to intervening ground, the formula $h = \frac{D^2}{2R}(1-2m)$ may be used to get the necessary elevation of a station at distance, so that it may be visible from another station of known elevation



Let, h_1 = known elevation of station A above datum

h_2 = required elevation of B above datum

D_1 = distance from A to the point of tangency

D_2 = distance from B to the point of tangency

D = the known distance between A and B

then, $h_1 = 0.06728\,D_1^{\,2}$

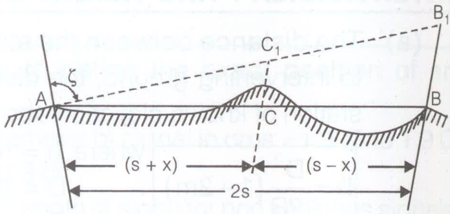
$$D_1 = \sqrt{\frac{h_1}{0.0728}} = 3.8553\sqrt{h_1}$$

where D_1 is in km and h_1 is in meters, $D_2 = D - D_1$

$$h_2 = 0.06728\, D_2^2 \text{ meters}$$

(c) Profile of the intervening ground: In the reconnaissance, the elevations and positions of peaks in the intervening ground between the proposed stations should be determined. A comparison of their elevations should be made to the elevation of the proposed line of sight to ascertain whether the line of sight is clear off the obstruction or not. The problem can be solved by using the principles discussed in the factors (1) and (2) above, or by a solution suggested by Captain G.T. McCaw. The former method will be clear from the worked out examples.

Captain GT McCaw's Method



Let, h_1 = height of station A above datum

h_2 = height of station B above datum

h = height of line of sight at the obstruction C

2s = distance between the two stations A and B

(s + x) = distance of obstruction C from A

(s − x) = distance of obstruction C from B

ζ = zenith distance from A to B

The height h of the line of sight at the obstruction is given by

$$h = \frac{1}{2}(h_2 + h_1) + \frac{1}{2}(h_2 - h_1)\frac{x}{s} - (s^2 - x^2)\operatorname{cosec}^2\zeta\left(\frac{1-2m}{2R}\right)$$

$\operatorname{cosec}^2\zeta = 1 + \dfrac{(h_2 - h_1)^2}{4s^2}$. The expression $\dfrac{1-2m}{2R} = 0.574$.

If x, s and R are substituted in miles, and h_1, h_2 and h are in feet.

and $\dfrac{1-2m}{2R} = 0.06728$

If x, s and R are in km and h_1, h_2 and h are in meters.

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