Class - X Session 2022-23
Subject - Mathematics (Basic)
Sample Question Paper - 23

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:
1. This Question Paper has 5 Sections A, B, C, D, and E.
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. Section D has 4 Long Answer (LA) type questions carrying 5 marks each.
6. Section E has 3 Case Based integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

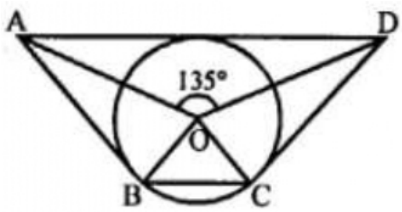
Section A

1. The centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by **[1]**

a) $\left(\frac{x_1+x_2+x_3}{6}, \frac{y_1+y_2+y_3}{6}\right)$ b) $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$

c) $\left(\frac{x_1+x_2+x_3}{4}, \frac{y_1+y_2+y_3}{4}\right)$ d) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

2. In the given figure, If ∠AOD = 135° then ∠BOC is equal to **[1]**



a) 45° b) 25°

c) 52.5° d) 62.5°

3. The probability of getting an even, number, when a die is thrown once is **[1]**

a) $\frac{5}{6}$ b) $\frac{1}{3}$

c) $\frac{1}{2}$ d) $\frac{1}{6}$

4. The abscissa of any point on the y-axis is **[1]**

a) 0
b) 1
c) y
d) -1

5. One equation of a pair of dependent linear equations is $-5x + 7y = 2$, then the second equation can be **[1]**

a) $-10x + 14y + 4 = 0$
b) $-10x - 14y + 4 = 0$
c) $10x - 14y + 4 = 0$
d) $10x + 14y + 4 = 0$

6. The distance of the point (4, 7) from the x-axis is **[1]**

a) 7
b) 4
c) 11
d) $\sqrt{65}$

7. If an event cannot occur then its probability is **[1]**

a) $\frac{3}{4}$
b) $\frac{1}{2}$
c) 0
d) 1

8. The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is **[1]**

a) 5 : 1
b) 4 : 1
c) 2 : 1
d) 3 : 1

9. A month is selected at random in a year. The probability that it is March or October, is **[1]**

a) $\frac{1}{6}$
b) $\frac{3}{4}$
c) None of these
d) $\frac{1}{12}$

10. If one root of the equation $2x^2 + ax + 6 = 0$ is 2 then a = ? **[1]**

a) -7
b) $\frac{7}{2}$
c) $\frac{-7}{2}$
d) 7

11. $(x + 1)^2 - x^2 = 0$ has **[1]**

a) no real roots
b) 1 real root
c) 2 real roots
d) 4 real roots

12. If $\tan \theta = \sqrt{3}$, then $\sec \theta =$ **[1]**

a) $\sqrt{\frac{3}{2}}$
b) 2

c) $\frac{2}{\sqrt{3}}$ d) $\frac{1}{\sqrt{3}}$

13. The exponent of 3 in the prime factorization of 864 is: **[1]**

a) 2 b) 3

c) 4 d) 8

14. The distance between the points (a cos 25°, 0) and (0, a cos 65°) is **[1]**

a) None of these b) 3a

c) a d) 2a

15. The angle of elevation and the angle of depression from an object on the ground to an object in the air are related as ________. **[1]**

a) greater than b) equal

c) less than d) None of these

16. Mode is: **[1]**

a) least frequent value b) None of these

c) middle most value d) most frequent value

17. The LCM of two numbers is 1200. Which of the following cannot be their HCF? **[1]**

a) 500 b) 200

c) 600 d) 400

18. The value of k so that the system of equations 3x - 4y - 7 = 0 and 6x - ky - 5 = 0 have a unique solution is **[1]**

a) $k \neq -8$ b) $k \neq 4$

c) $k \neq -4$ d) $k \neq 8$

19. **Assertion (A):** The H.C.F. of two numbers is 16 and their product is 3072. Then their L.C.M. = 162 **[1]**
Reason: If a, b are two positive integers, then H.C.F. × L.C.M. = a × b

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** Two similar triangles are always congruent. **[1]**
Reason (R): If the areas of two similar triangles are equal then the triangles are congruent.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Is the given statement correct or not correct? **[2]**
If a die is thrown, there are two possible outcomes- an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

22. Two rails are represented by the equations: $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$. Will the rails cross each other? **[2]**

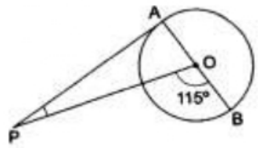
OR

Solve algebraically each of the following pair of linear equations for x and y
$x + y = a + b$, $ax + by = a^2 + b^2$.

23. Find the zeroes of the quadratic polynomial given as: $x^2 + 7x + 10$, and also verify the relationship between the zeroes and the coefficients. **[2]**

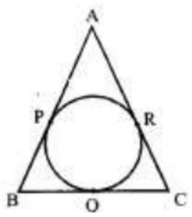
24. Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear. **[2]**

25. In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115°$, find $\angle APO$. **[2]**



OR

In the adjoining figure, sides AB, BC and CA of a triangle ABC, touch a circle at P, Q, and R respectively. If PA = 4 cm, BP = 3 cm and AC = 11 cm, then find the length of BC.



Section C

26. In $\triangle ABC$, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$. Find the value of cos A cos C - sin A sin C **[3]**

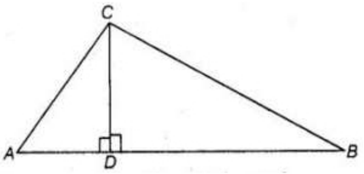
27. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 year. Find the ages of Ani and Biju. **[3]**

28. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers. **[3]**

OR

Express the HCF/GCD of 48 and 18 as a linear combination.

29. In figure, if $\angle ACB = \angle CDA$, AC = 8 cm and AD = 3 cm, find BD. **[3]**



30. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A. **[3]**

OR

From an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that.

i. $PA.PB = PN^2 - AN^2$

ii. $PN^2 - AN^2 = OP^2 - OT^2$

iii. $PA.PB = PT^2$

31. From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β. Show that the height in miles of aeroplane above the road is given by $\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$. **[3]**

Section D

32. Solve: $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2, x\neq-\frac{1}{2}, 1$ **[5]**

OR

Solve for y:

$\frac{y+3}{y-2}-\frac{1-y}{y}=\frac{17}{4}; y\neq 0\cdot 2$

33. In the figure, OB is the perpendicular bisector of the line segment DE, FA $\perp$ OB and F E intersect OB at point C. Prove that $\frac{1}{OA}+\frac{1}{OB}=\frac{2}{OC}$. **[5]**

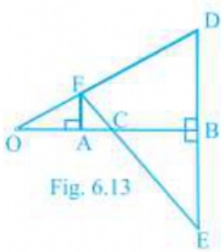


Fig. 6.13

34. Find the area of the segment of a circle of radius 12 cm whose corresponding sector central angle 60°. (Use $\pi = 3.14$). **[5]**

OR

Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending angle of 90° at the centre.

35. The following is the cummulative frequency distribution (of less than type) of 1000 persons each of age 20 years and above. Determine the mean age. **[5]**

Age below (in years)	30	40	50	60	70	80
Number of persons	100	220	350	750	950	1000

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Find the volume of the Hermika, if the side of cubical part is 10 m.

(ii) Find the volume of cylindrical base part whose diameter and height 48 m and 14 m.

(iii) If the volume of each brick used is 0.01 m^3, then find the number of bricks used to make the cylindrical base.

OR

If the diameter of the Anda is 42 m, then find the volume of the Anda.

37. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find an increase in the production of TV every year.

(ii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find in which year production of TV is 1000.

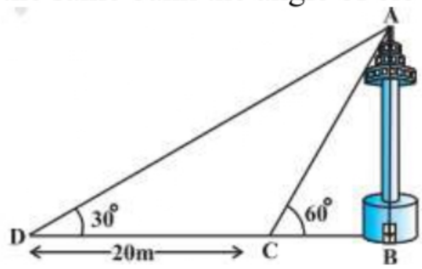
(iii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the 10th year.

OR

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the total production in first 7 years.

38. **Read the text carefully and answer the questions:** **[4]**

A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60^{o} from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30^{o}.



(i) Find the width of the canal.

(ii) Find the height of tower.

(iii) Find the distance between top of the tower and point D.

OR

Find the distance between top of tower and point C.

SOLUTION

Section A

1. **(d)** $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

   Explanation: The centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

   It is the point of intersection of the three medians in the triangle. it is also called the centre of gravity of the triangle.
2. **(a)** 45°

   Explanation: In the given figure, ∠AOD = 135°

   We know that if a circle is inscribed in a quadrilateral, the opposite sides subtend supplementary angles.

   ∠AOD + ∠BOC = 180°

   135° + ∠BOC = 180°

   ⇒∠BOC = 180° – 135° = 45°
3. **(c)** $\frac{1}{2}$

   Explanation: Even number on a die are 2,4,6.

   ∴ Probability P = $\frac{3}{6} = \frac{1}{2}$
4. **(a)** 0

   Explanation: Since coordinates of any point on y-axis is (0, y)

   Therefore, the abscissa is 0.
5. **(c)** 10x – 14y +4 = 0

   Explanation: If the equation of a pair of dependent linear equations, then

   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

   Given: a_1 = -5, b_1 = -5and c_1 = 2.

   For satisfying the condition of dependent linear equations, the values of a_2, b_2 and c_2 should be the multiples of the values of a_1, b_1 and c_1.

   ∴ The values would be $a_2 = -5 \times (-2) = 10, b_2 = 7 \times (-2) = -14$ and

   $c_1 = 2 \times (-2) = -4$

   ∴ The second equation can be 10x - 14y = - 4
6. **(a)** 7

   Explanation: The distance of the point (4, 7) from x-axis = 7
7. **(c)** 0

   Explanation: The event which cannot occur is said to be impossible event and probability of impossible event is zero.
8. **(a)** 5 : 1

   Explanation: Ratio of the total surface area to the lateral surface area $=$

   $\frac{\text{Total surface area}}{\text{Lateral surfice area}}$

   $= \frac{2\pi r(h+r)}{2\pi rh}$

   $= \frac{h+r}{h}$

$= \frac{(20+80)}{20}$
$= \frac{100}{20}$
$= \frac{5}{1}$
= 5 : 1
Hence, the required ratio is 5:1

9. **(a)** $\frac{1}{6}$
Explanation: No. of months in a year = 12
Probability of being March or October = $\frac{2}{12} = \frac{1}{6}$

10. **(a)** -7
Explanation: One root of the equation $2x^2 + ax + 6 = 0$ is 2 i.e. it satisfies the equation
$2(2)^2 + 2a + 6 = 0$
$8 + 2a + 6 = 0$
$2a = -14$
$a = -7$

11. **(b)** 1 real root
Explanation: Given: $(x + 1)^2 - x^2 = 0$
$\Rightarrow x^2 + 1 + 2x - x^2 = 0$
$\Rightarrow 2x + 1 = 0$
$\Rightarrow x = \frac{-1}{2}$
Therefore, $(x + 1)^2 - x^2 = 0$ is a linear polynomial and has one real root.

12. **(b)** 2
Explanation: Since $\sec\theta = \sqrt{1+\tan^2\theta}$
$\therefore \sec\theta = \sqrt{1+\left(\sqrt{3}\right)^2}$
$= \sqrt{1+3} = \sqrt{4} = 2$

13. **(b)** 3
Explanation: Prime factorization of 864 = $32 \times 27 = 2^5 \times 3^3$
Therefore the exponent of 3 in the prime factorization of 864 is 3

14. **(c)** a
Explanation: Distance between ($a \cos 25^o$, 0) and (0, $a \cos 65^o$)
$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$= \sqrt{(0-a\cos 25^\circ)^2+(a\cos 65^\circ-0)^2}$
$= \sqrt{a^2\cos^2 25^\circ + a^2\cos^2 65^\circ}$
$= \sqrt{a^2\left[\cos^2 25^\circ+\cos^2 65^\circ\right]}$
$= a\sqrt{\cos^2(90^\circ-65^\circ)+\cos^2 65^\circ}$
$= a\sqrt{\sin^2 65^\circ+\cos^2 65^\circ}$
$= a(\sqrt{1}) = a$

15. **(b)** equal
Explanation: The angle of elevation and the angle of depression from an object on

the ground to an object in the air are related as equal if the height of objects is the same.

16. **(d)** most frequent value

    Explanation: Mode is the most frequent value of observation or a class.

17. **(a)** 500

    Explanation: It is given that the LCM of two numbers is 1200 .
    We know that the HCF of two numbers is always the factor of LCM.
    500 is not the factor of 1200.
    So this cannot be the HCF.

18. **(d)** $k \neq 8$

    Explanation: Given: $a_1 = 3$, $a_2 = 6$, $b_1 = -4$, $b_2 = -k$, $c_1 = -7$ and $c_2 = -5$
    If there is a unique solution, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
    $\Rightarrow \frac{3}{6} \neq \frac{-4}{-k}$
    $\Rightarrow -3k \neq -4 \times 6$
    $\Rightarrow k \neq 8$

19. **(d)** A is false but R is true.

    Explanation: $\frac{3072}{16} = 192 \neq 162$

20. **(d)** A is false but R is true.

    Explanation: Two similar triangles are not congruent generally. So, A is false but R is true.

Section B

21. Total outcomes that can occur are 1, 2, 3, 4, 5, 6
    Number of possible outcomes of a dice = 6
    Numbers which are odd = 1, 3, 5
    Total numbers which are odd = 3
    Numbers which are even = 2, 4, 6
    Total numbers which are even = 3
    Probability of getting an odd number $= \frac{\text{Number of outcomes where there is an odd number}}{\text{Total number of outcomes}}$
    $= \frac{3}{6} = \frac{1}{2}$
    Hence, the given statement is correct.

22. The pair of linear equations are given as:
    $x + 2y - 4 = 0$ ...(i)
    $2x + 4y - 12 = 0$ ...(ii)
    We express x in terms of y from equation (i), to get
    $x = 4 - 2y$
    Now, we substitute this value of x in equation (ii), to get
    $2(4 - 2y) + 4y - 12 = 0$
    i.e., $8 - 12 = 0$
    i.e., $-4 = 0$
    Which is a false statement. Therefore, the equations do not have a common solution.
    So, the two rails will not cross each other.

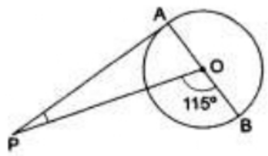
OR

$x + y = a + b$.......(i)
$ax + by = a^2 + b^2$.......(ii)

multiply (i) by b we have
bx + by = ab + bb......(iii)
subtracting (ii) and (iii)
⇒ (a - b)x = a(a - b)
⇒ x = a put x = a in (i) we get y =b.

23. We have,
$x^2 + 7x + 10 = (x + 2)(x + 5)$
So, the value of $x^2 + 7x + 10$ is zero when x + 2 = 0 or x + 5 = 0, i.e., when x = – 2 or x = –5. Therefore, the zeroes of $x^2 + 7x + 10$ are -2 and -5.
Now,
sum of zeroes = $-2 + (-5) = -(7) = \frac{-(7)}{1} = \frac{-(\text{ Coefficient of } x)}{\text{Coefficient of } x^2}$
product of zeroes = $(-2) \times (-5) = 10 = \frac{10}{1} = \frac{\text{Constant term}}{\text{coefficient of } x^2}$

24. If the given points are collinear, then the area of the triangle with these points as vertices will be zero.
$\therefore \frac{1}{2}$[x (2 – 0) + 1 (0 – y) + 7(y –2)] = 0
⇒ $\frac{1}{2}$[2x – y + 7y –14] = 0
⇒ $\frac{1}{2}$[2x + 6y –14] = 0
⇒ 2x + 6y – 14 = 0
⇒ x + 3y – 7 = 0 ....Dividing throughout by 2
⇒ x = -3y + 7
Which is the required relation between x and y.

25. 

We know that the tangent at a point to a circle is perpendicular to the radius passing through the point of contact.
$\therefore \quad \angle OAP = 90^\circ$
Now, $\angle AOP + \angle BOP = 180^\circ$
$\Rightarrow \angle AOP = 180^\circ - \angle BOP$
= 180° -115°
= 65°.
Now, $\angle OAP + \angle AOP + \angle APO = 180^\circ$ [sum of angles of a triangle is 180°]
$\Rightarrow \quad \angle APO = 180^\circ - (\angle OAP + \angle AOP)$
= 180° - (90° + 65°) = 25°.

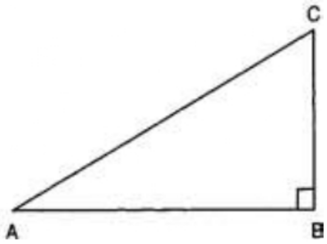
OR

In triangle ABC, we have
BP = BQ = 3cm
AP = AR = 4cm
(tangents drawn from an external point to the circle are equal).
So, RC = AC - AR
= 11 - 4
= 7 cm
Hence RC = CQ = 7 cm

Then, BC = BQ + QC
= 7 + 3
= 10 cm

Section C

26.



we have,
$\tan A = \frac{1}{\sqrt{3}} = tan 30^\circ$
$\therefore A = 30^\circ$
In $\triangle$ABC, we have
$\angle A + \angle B + \angle C = 180^\circ$
$\Rightarrow 30^\circ + 90^\circ + \angle C = 180^\circ$
$\Rightarrow 120^\circ + \angle C = 180^\circ$
$\Rightarrow \angle C = 180^\circ - 120^\circ = 60^\circ$
So,
cos A.cos C - sin A.sin C
$= \cos 30^\circ . \cos 60^\circ - \sin 30^\circ . \sin 60^\circ$
$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 0$

27. Let the ages of Ani and Biju be x yr. and y yr, respectively.
According to the given condition,
$x - y = \pm 3$ ..(i)
Also, age of Ani's father Dharam = 2x years
And age of Biju's sister = $\frac{y}{2}$ years
According to the given condition,
$2x - \frac{y}{2} = 30$
or, $4x - y = 60$ .....(ii)
Case I : When $x - y = 3$ ......(iii)
On subtracting eqn. (iii) from eqn. (ii),
$3x = 57$
$\therefore x = 19$years
On putting $x = 19$ in eqn. (iii),
$19 - y = 3$
$\therefore$y = 16 years
Case II : When $x - y = -3$...(iv)
On subtracting eqn. (iv) from eqn. (ii),
$3x = 60 + 3$
$3x = 63$
$\therefore x = 21$ years
On putting $x = 21$ in eqn. (iv), we get
$21 - y = -3$
$\therefore y = 24$ years
Hence, Ani's age = 19 years or 21 years.
Biju age = 16 years or 24 years

28. Numbers are of two types - prime and composite.
Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.
It can be observed that
$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1)$
$= 13 \times (77 + 1) = 13 \times 78 = 13 \times 13 \times 6$
The given expression has 6 and 13 as its factors.
Therefore, it is a composite number.
$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$
$= 5 \times (1008 + 1) = 5 \times 1009$
1009 cannot be factorized further
Therefore, the given expression has 5 and 1009 as its factors.
Hence, it is a composite number.

OR

Let us find HCF of 48 and 18
$48 = 18 \times 2 + 12$
$18 = 12 \times 1 + 6$
$12 = 6 \times 2 + 0$
Hence HCF (48, 18) = 6
Now, $6 = 18 - 12 \times 1$
$6 = 18 - (48 - 18 \times 2)$
$6 = 18 - 48 \times 1 + 18 \times 2$
$6 = 18 \times 3 - 48 \times 1$
$6 = 18 \times 3 + 48 \times (-1)$
i.e., 6 = 18x + 48y .... (1)
where x= 3, y = -1
$\therefore 6 = 18 \times 3 + 48 \times (-1)$
$= 18 \times 3 + 48 \times (-1) + 18 \times 48 - 18 \times 48$
$= 18(3 + 48) + 48(-1 - 18)$
$= 18 \times 51 + 48 \times (-19)$
6 = 18x + 48y ...... (2)
where x = 51, y = -19
Hence, x and y are not unique.

29. In triangle ABC and CDA ∠ ACB = ∠ CDA. AC = 8 cm and AD = 3 cm we have to find BD
Let ∠ACB = x
∠CDA = x
CD intersect AB at D and Let
∠DCB = a then ∠ACD = x -a
∠CDA = ∠DCB + ∠CBD
⇒ x = a + ∠CBD
⇒ ∠CBD = x - a ...(∠CBD = ∠CBA as D is a point on line BA)
Now in Triangle Δ ABC
$\frac{AC}{Sin(\angle CBA)} = \frac{AB}{Sin(\angle ACB)}$
$\Rightarrow \frac{8}{Sin(x-a)} = \frac{AB}{Sinx}$

$\Rightarrow \frac{Sin(x)}{Sin(x-a)} = \frac{AB}{8}$ - eq 1

Now in Triangle Δ ACD

AC/Sin(∠CDA) = AD / Sin(∠ACD)

⇒ 8/Sin(x) = 3/Sin(x-a)

⇒ 8/3 = Sin(x) / Sin(x-a)

⇒ Sin(x) / Sin(x-a) = 8/3 - eq 2

equating eq 1 & eq 2

AB/8 = 8/3

=> AB = 64/3

AB = AD + BD

⇒ 64/3 = 3 + BD

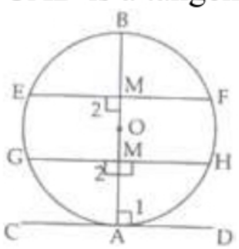
⇒ BD = 64/3 - 3

⇒ BD = 55/3

⇒ BD = 18.33 cm

30. Given: A circle with centre O and AOB is diameter.

CAD is a tangent at A. Chord EF || tangent CAD



To prove: AB bisects any chord EF || CAD.

Proof: OA radius is perpendicular to tangent CAD.

∴ ∠1 = 90°

CAD || EF [Given]

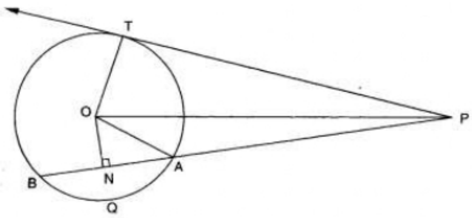
∴ ∠1 = ∠2 = 90° [alternate interior angles]

Point M is on diameter which passes through centre O.

∵ Perpendicular drawn from centre to chord bisect the chord.

Hence, AB bisect any chord EF || CAD.

OR



i. PA .PB = (PN - AN)(PN + BN)

= (PN - AN) (PN + AN) $\left[\begin{array}{c} \because ON \perp AB \\ \therefore N \text{ is the mid-point of } AB \\ \Rightarrow AN = BN \end{array}\right]$

= $PN^2 - AN^2$

ii. Applying Pythagoras theorem in right triangle PNO, we obtain

$OP^2 = ON^2 + PN^2$

$\Rightarrow PN^2 = OP^2 - ON^2$

$\therefore$ PN^2 - AN^2 = (OP^2 - ON^2) - AN^2

= OP^2 - (ON^2 + AN^2)

= Op^2 - OA ^{2} [Using Pythagoras theorem in ΔONA]

= OP^2 - OT^2 [$\because OA = OT =$ radius]

iii. From (i) and (ii), we obtain

PA.PB = PN^2 - AN^2 and PN^2 - AN^2 = OP^2 - OT^2

$\Rightarrow$ PA .PB = OP^2 - OT^2

Applying Pythagoras theorem in $\triangle OTP$, we obtain

OP^2 = OT^2 + PT^2

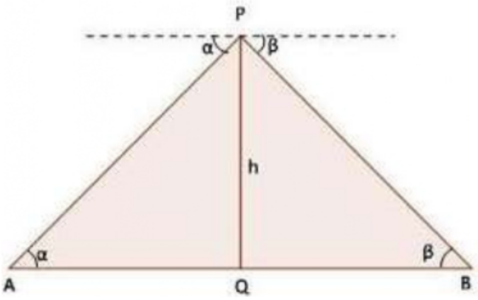
$\Rightarrow$ OP^2 - OT^2 = PT^2

Thus, we obtain

PA.PB = OP^2 - OT^2

and OP^2 - OT^2 = PT^2

Hence, PA.PB = PT^2.

31. 

Let h be the height of aeroplane above the road and A and B be two consecutive milestone.

In $\Delta\ AQP\ and\ \triangle BQP$,

$\tan\alpha = \frac{h}{AQ}$ and $\tan\beta = \frac{h}{BQ}$

$\Rightarrow AQ = h\ \cot\alpha$ and $BQ = h\ \cot\beta$

$\Rightarrow AQ + BQ = h(\cot\alpha + \cot\beta)$

$AB = h\left(\frac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}\right)$ OBJ OBJ OBJ

As, given that AB = 1 mile

$\Rightarrow h = \frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

Hence proved.

Section D

32. Given

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 0$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1} = \frac{1}{y}$

$\therefore$ Substituting this value

$y + \frac{1}{y} = 2$ or $\frac{y^2+1}{y} = 2$

or $y^2 + 1 = 2y$

or $y^2 - 2y + 1 = 0$

or $(y-1)^2 = 0$

Putting $y = \frac{x-1}{2x+1}$,

$\frac{x-1}{2x+1} = 1$ or $x - 1 = 2x + 1$

or x = -2

OR

Given equation, $\frac{y+3}{y-2} - \frac{1-y}{y} = \frac{17}{4}$

$\Rightarrow \quad \frac{y(y+3)-(1-y)(y-2)}{y(y-2)} = \frac{17}{4}$

$\Rightarrow \frac{(y^2+3y)-(-y^2+3y-2)}{y^2-2y} = \frac{17}{4}$

$\Rightarrow \frac{y^2+3y+y^2-3y+2}{y^2-2y} = \frac{17}{4}$

$\Rightarrow \quad \frac{2y^2+2}{y^2-2y} = \frac{17}{4}$

$\Rightarrow \quad 4(2y^2+2) = 17(y^2-2y)$

$\Rightarrow \quad 8y^2 + 8 = 17y^2 - 34y$

$\Rightarrow \quad 9y^2 - 34y - 8 = 0$

$\Rightarrow \quad 9y^2 - 36y + 2y - 8 = 0$

$\Rightarrow \quad 9y(y-4) + 2(y-4) = 0$

$\Rightarrow \quad (y-4)(9y+2) = 0$

$\Rightarrow \quad y - 4 = 0 \quad or \quad 9y + 2 = 0$

$\Rightarrow \quad y = 4 \quad or \quad y = -\frac{2}{9}$

$\therefore \quad y = 4, -\frac{2}{9}$

33. In $\triangle$AOF and $\triangle$BOD

$\angle$O = $\angle$O (Same angle) and $\angle$A = $\angle$B (each 90°)

Therefore, $\triangle$AOF $\sim$ $\triangle$BOD (AA similarity)

So, $\frac{\text{OA}}{\text{OB}} = \frac{\text{FA}}{\text{DB}}$

Also, in $\triangle$FAC and $\triangle$EBC, $\angle$A = $\angle$B (Each 90°)

and $\angle$FCA = $\angle$ECB (Vertically opposite angles).

Therefore, $\triangle$FAC $\sim$ $\triangle$EBC (AA similarity).

So, $\frac{\text{FA}}{\text{EB}} = \frac{\text{AC}}{\text{BC}}$

But EB = DB (B is mid-point of DE)

So, $\frac{\text{FA}}{\text{DB}} = \frac{\text{AC}}{\text{BC}}$ (2)

Therefore, from (1) and (2), we have:

$\frac{\text{AC}}{\text{BC}} = \frac{\text{OA}}{\text{OB}}$

i.e. $\frac{\text{OC}-\text{OA}}{\text{OB}-\text{OC}} = \frac{\text{OA}}{\text{OB}}$

or OB . OC – OA . OB = OA . OB – OA . OC

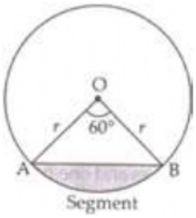
or OB . OC + OA . OC = 2 OA . OB

or (OB + OA). OC = 2 OA . OB

or $\frac{1}{\text{OA}} + \frac{1}{\text{OB}} = \frac{2}{\text{OC}}$ [Dividing both the sides by OA . OB . OC]

34. Area of minor segment = Area of sector – Area of $\triangle$OAB

In $\triangle$OAB,



$\theta = 60°$

OA = OB = r = 12 cm

$\angle B = \angle A = x$ [$\angle$s opp. to equal sides are equal]

$\Rightarrow \angle A + \angle B + \angle O = 180°$

$\Rightarrow x + x + 60° = 180°$

$\Rightarrow 2x = 180° - 60°$

$\Rightarrow x = \frac{120°}{2} = 60°$

$\therefore \triangle OAB$ is equilateral $\triangle$ with each side (a) = 12 cm

Area of the equilateral $\triangle = \frac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of $\triangle OAB$

$= \frac{\pi r^2\theta}{360°} - \frac{\sqrt{3}}{4}a^2$

$= \frac{3.14 \times 12 \times 12 \times 60°}{360°} - \frac{\sqrt{3}}{4} \times 12 \times 12$

$= 6.28 \times 12 - 36\sqrt{3}$

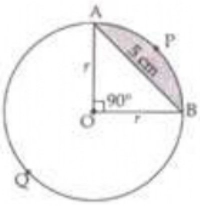
$\therefore$ Area of minor segment = $(75.36 - 36\sqrt{3})$ cm^2.

OR

Chord AB = 5 cm divides the circle into two segments minor segment APB and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, we are given that $\theta = 90°$

Area of $\triangle OAB = \frac{1}{2}$Base $\times$ Altitude = $\frac{1}{2}$r $\times$ r = $\frac{1}{2}r^2$



Area of minor segment APB

$= \frac{\pi r^2\theta}{360°}$– Area of $\triangle AOB$

$= \frac{\pi r^2 90°}{360°} - \frac{1}{2}r^2$

$\Rightarrow$ Area of minor segment = $\left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$ …(i)

Area of major segment AQB = Area of circle – Area of minor segment

$= \pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$

$\Rightarrow$ Area of major segment AQB $= \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right]$ …(ii)

Difference between areas of major and minor segment

$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$

$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$

$\Rightarrow$ Required area $= \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$

In right $\triangle$OAB,

$r^2 + r^2 = AB^2$

$\Rightarrow 2r^2 = 5^2$

$\Rightarrow r^2 = \frac{25}{2}$

Therefore, required area $= \left[\frac{1}{2}\pi \times \frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25}{4}\pi + \frac{25}{2}\right]$ cm^2

35.

Class interval	Frequency f_i	Mid-value X_i	$u_i = \frac{x_i - A}{h} = \frac{x_i - 45}{10}$	$f_i u_i$
20-30	100	25	-2	-200
30-40	120	35	-1	-120
40-50	130	45=A	0	0
50-60	400	55	1	400
60-70	200	65	2	400
70-80	50	75	3	150
	$\Sigma f_i = 1000$			$\sum f_i u_i = 630$

A = 45, h = 10,

$\Sigma f_i = 1000, \sum f_i u_i = 630$

Mean = A + $\left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$

= 45 + $\left\{10 \times \frac{630}{1000}\right\}$

= 45 + 6.3

= 51.3

Section E

36. **Read the text carefully and answer the questions:**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Volume of Hermika = side^3 = 10 × 10 × 10 = 1000 m^3

(ii) r = radius of cylinder = 24, h = height = 16

Volume of cylinder = $\pi r^2 h$

$\Rightarrow$ V = $\frac{22}{7} \times 24 \times 24 \times 14$ = 25344 m^3

(iii) Volume of brick = 0.01 m^3

$\Rightarrow$ n = Number of bricks used for making cylindrical base = $\frac{\text{Volume of cylinder}}{\text{Volume of one brick}}$

$\Rightarrow n = \frac{25344}{0.01}$ = 2534400

OR

Since Anda is hespherical in shape r = radius = 21

V = Volume of Anda = $\frac{2}{3} \times \pi \times r^3$

$\Rightarrow$ V = $\frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$\Rightarrow$ V = 44 $\times$ 21 $\times$ 21 = 19404 m^3

37. **Read the text carefully and answer the questions:**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We have, a_3 = 600 and

a_3 = 600

$\Rightarrow$ 600 = a + 2d

$\Rightarrow$ a = 600 - 2d ...(i)

$\Rightarrow$ a_7 = 700

$\Rightarrow$ a_7 = 700

$\Rightarrow$ 700 = a + 6d

$\Rightarrow$ a = 700 - 6d ...(ii)

From (i) and (ii)

600 - 2d = 700 - 6d

$\Rightarrow$ 4d = 100

$\Rightarrow$ d = 25

(ii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We know that first term = a = 550 and common difference = d = 25

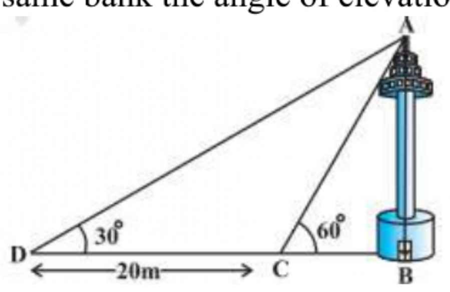
$a_n = 1000$
$\Rightarrow 1000 = a + (n - 1)d$
$\Rightarrow 1000 = 550 + 25n - 25$
$\Rightarrow 1000 - 550 + 25 = 25n$
$\Rightarrow 475 = 25n$
$\Rightarrow n = \frac{475}{25} = 19$

(iii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
The production in the 10th term is given by a_{10}. Therefore, production in the 10th year = $a_{10} = a + 9d = 550 + 9 \times 25 = 775$. So, production in 10th year is of 775 TV sets.

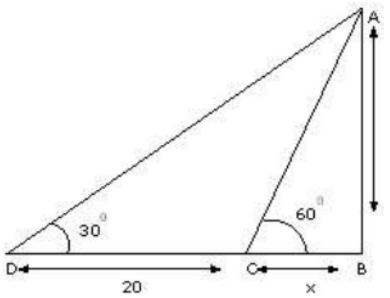
OR

Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).
$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$
$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$
$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$
$\Rightarrow S_7 = 4375$

38. **Read the text carefully and answer the questions:**
A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60^o from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30^o.



(i)



Let 'h' (AB) be the height of tower and x be the width of the river.

In $\triangle ABC, \frac{h}{x} = \tan 60^\circ$

$\Rightarrow h = \sqrt{3}x$ ...(i)

In $\Delta ABD, \frac{h}{x+20} = \tan 30^\circ$

$\Rightarrow h = \frac{x+20}{\sqrt{3}}$ ...(ii)

Equating (i) and (ii),

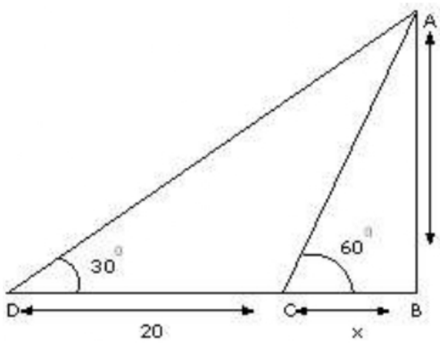
$\sqrt{3}x = \frac{x+20}{\sqrt{3}}$

$\Rightarrow$ 3x = x + 20

$\Rightarrow$ 2x = 20

$\Rightarrow$ x = 10 m

(ii)



Let 'h' (AB) be the height of tower and x be the width of the river.

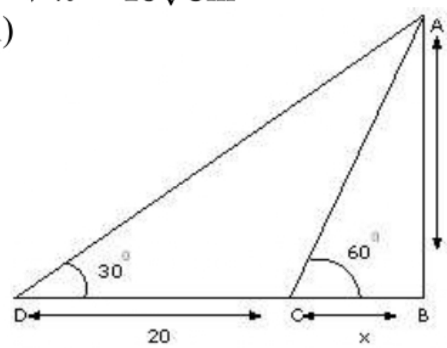
In $\triangle ABC, \frac{h}{x} = \tan 60^\circ$

$\Rightarrow h = \sqrt{3}x$ ...(i)

Put x = 10 in (i), $h = \sqrt{3}x$

$\Rightarrow h = 10\sqrt{3}$m

(iii)



In $\triangle$ABD

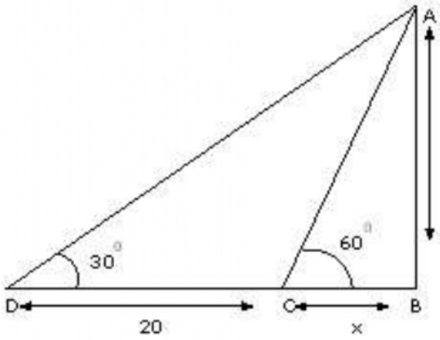
sin $30^o = \frac{AB}{AD}$

$\Rightarrow$ AD = $\frac{AB}{\sin 30^\circ}$

$\Rightarrow$ AD = $\frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}$ m

OR



In $\triangle$ABC

sin $60^o = \frac{AB}{AC}$

$\Rightarrow$ AC = $\frac{AB}{\sin 30^0}$

$\Rightarrow$ AC = $\frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AC = 20 m