MIND MAP : LEARNING MADE SIMPLE CHAPTER - 12

Introduction to Three Dimensional Geometry

Coordinates of the Centroid of a Triangle

The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

Eg: The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, –5, 7) and (–1, 7, –6), respectively, find the coordinates of the point C.

Sol: Let the coordinates of C be (x, y, z) and the coordinates of the centroid G be (1, 1, 1). Then $\frac{x+3-1}{3}$ =1, i.e., x=1;

$\frac{y-5+7}{3}$ = 1, i.e., y=1;

$\frac{z+7-6}{3}$ =1, i.e., z=2. So, C (x,y,z) = (1,1,2)

Coordinates of a Midpoint

The coordinates of the midpoint of the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

Eg: Find the midpoint of the line joining two points P(1, –3, 4) and Q(–4, 1, 2).

Sol: Coordinates of the midpoint of the line joining the points P & Q are

$\left(\frac{1-4}{2}, \frac{-3+1}{2}, \frac{4+2}{2}\right)$ i.e. $\left(\frac{-3}{2}, -1, 3\right)$

Introduction

- In three dimensions, the coordinate axes of a rectangular cartesian coordinate system are three mutually perpendicular lines. The axes are called x, y and z axes.
- The three planes determined by the pair of axes are the coordinate planes, called xy, yz and zx-planes.
- The three coordinate planes divide the space into eight parts known as octants.
- The coordinates of a point P in 3D Geometry is always written in the form of triplet like (x,y,z). Here, x, y and z are the distances from yz, zx and *yx* planes, respectively.

Eg:

- Any point on x-axis is : (x, 0, 0)
- Any point on y-axis is : (0, y, 0)
- Any point on z-axis is : (0, 0, z)

Distance between Two Points

Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Eg: Find the distance between the points P(1, –3, 4) and (–4, 1, 2).

Sol: The distance PQ between the points P & Q is given by

$$PQ = \sqrt{(-4-1)^2+(1+3)^2+(2-4)^2}$$

$$= \sqrt{25+16+4} = \sqrt{45} = 3\sqrt{5} \text{ units}$$

Section Formula

The coordinates of the point R which divides the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally and externally in the ratio m : n are given by

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right) \ \& \ \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$$

respectively.

Eg: Find the coordinates of the point which divides the line segment joining the points (1,–2, 3)and (3, 4, –5) in the ratio 2:3 internally.

Sol : Let P(x, y, z) be the point which divides line segment joining A (1,–2, 3) and B (3, 4, –5) internally in the ratio 2:3. Therefore,

$$x = \frac{2(3)+3(1)}{2+3} = \frac{9}{5} \quad y = \frac{2(4)+3(-2)}{2+3} = \frac{2}{5} \quad z = \frac{2(-5)+3(3)}{2+3} = \frac{-1}{5}$$

Thus, the required point is $\left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right)$.