Class XII Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If A is a square matrix of order 3 and |A| = 5, then the value of |2A'| is **[1]**

a) -10 b) -40

c) 40 d) 10

2. Since α = probability of Type-I error, then 1 - α **[1]**

a) Probability of not rejecting H_0 when H_0 is true.

b) Probability of rejecting H_0 when H_0 is true.

c) Probability of rejecting H_0 when H_a is true.

d) Probability of not rejecting H_0 when H_0 is true.

3. An 8 year annuity due has a present value of ₹ 1000. If the interest rate is 5% p.a., the amount of each annuity is closest to **[1]**

[Use $(1.05)^{-8} = 0.676$]

a) ₹ 154.73 b) ₹ 109.39

c) ₹ 104.72 d) ₹ 147.36

4. Which of the term is not used in a linear programming problem: **[1]**

a) Concave region
b) Objective function
c) Slack in equation
d) Feasible Region

5. If y = $e^x + e^{x + \dots \infty}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{y}{(1-y)}$
b) $\frac{1}{(y-1)}$
c) $\frac{1}{(1-y)}$
d) $\frac{y}{(y-1)}$

6. If X is normally distributed with mean 20 and standard deviation 4, then standard normal variable Z corresponding to X = 21 is **[1]**

a) -1.25
b) 1.25
c) 0.25
d) -0.25

7. Two dice are thrown n times in succession. The probability of obtaining a double six atleast once is **[1]**

a) $(\frac{1}{12})^n$
b) $(\frac{1}{36})^n$
c) 1 + $(\frac{35}{36})^n$
d) 1 - $(\frac{35}{36})^n$

8. y = $ae^{mx} + be^{-mx}$, where a, b are arbitrary constants satisfies which of the following differential equation? **[1]**

a) $\frac{dy}{dx}$ + my = 0
b) $\frac{dy}{dx}$ - my = 0
c) $\frac{d^2y}{dx^2}$ - m^2y = 0
d) $\frac{d^2y}{dx^2}$ + m^2y = 0

9. In a 1000 m race, A can beat B by 100 m. In a race of 400 m, B can beat C by 40 m. By how many meters will A beat C in a race of 500 m? **[1]**

a) 105 m
b) 85 m
c) 115 m
d) 95 m

10. Integrating factor of $x\frac{dy}{dx} - y$ = x^4 - 3x is **[1]**

a) log x
b) x
c) -x
d) $\frac{1}{x}$

11. A milkman mixed some water with milk to gain 25% by selling the mixture at the cost price. The ratio of water and milk respectively is: **[1]**

a) 1 : 5
b) 5 : 4
c) 4 : 5
d) 1 : 4

12. The solution of $\frac{x-3}{x+5}$ > 0, $\neq$ -5, x $\in$ R is **[1]**

a) x < -5
b) x < -5 or x > 3
c) no solution
d) x > 3

13. If a man goes 18 km downstream in 4 hours and returns against the stream in 12 hours, then the speed of the stream in km/hr is **[1]**

a) $\frac{3}{2}$
b) 3
c) 1
d) $\frac{7}{4}$

14. If the objective function for an L.P.P. is Z = 3x + 4y and the comer points for unbounded feasible region are (9, **[1]**

0), (4, 3), (2, 5) and (0, 8), then the minimum value of Z occurs at

a) (4, 3) b) (9, 0)

c) (2, 5) d) (0, 8)

15. The necessary condition for third quadrant region in xy plane is: **[1]**

a) $x > 0, y < 0$ b) $x < 0, y < 0$

c) $x < 0, y = 0$ d) $x < 0, y > 0$

16. A sample of 50 bulbs is taken at random. Out of 50 we found 15 bulbs are of Bajaj, 17 are of Surya and 18 are of Crompton. What is the point estimate of population proportion of Surya? **[1]**

a) 0.3 b) 0.34

c) 0.36 d) 0.4

17. $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to **[1]**

a) $-\frac{e^x}{1+x^2} + C$ b) $-\frac{e^x}{(1+x^2)^2} + C$

c) $\frac{e^x}{1+x^2} + C$ d) $\frac{e^x}{(1+x^2)^2} + C$

18. The most commonly use mathematical method for measuring the trend is: **[1]**

a) Semi average method b) Time series method

c) Method of least squares d) Moving average method

19. Let A be a non-singular matrix of order n. **[1]**

Assertion (A): adj(adj A) = $|A|^{n-2}$ A

Reason (R): |adj A| = $|A|^{n-1}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If the average cost (AC) of producing x units of an item is given by AC = $3x^2 - 7x + 5 - \frac{11}{x}$, then the marginal cost (MC) of producing 5 units is ₹160. **[1]**

Reason (R): MC = $\frac{d}{dx}$(C).

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Assuming a four yearly cycle, calculate the trend by the method of moving averages from the following data: **[2]**

Year	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993
Value	12	25	39	54	70	87	105	100	82	65

22. A machine has a scrap value of ₹ 22500 after 15 years of its purchase. If the annual depreciation charge is ₹ 8500, find its original cost using linear method. **[2]**

OR

Mrs Vandana invested ₹ 35000 in a shares of a company and reinvested the earnings every year in buying the shares of the same company. At the end of 5 years, the value of shares increased to ₹ 56000. Calculate the compound annual growth rate of her investment.

23. Evaluate: $\int_0^3 [x] dx$. **[2]**

24. The value of a car depreciates by 12.5 % every year. By what percent will the value of the car decrease after 3 years? **[2]**

OR

Find the effective rate which is equivalent to nominal rate of 10% p.a. compounded monthly. [Given that: $(1.00833)^{12} = 1.1047$]

25. A vessel contains a mixture of two liquids X and Y in the ratio 3 : 5. 8 litres of mixture are drawn off from the vessel and 8 liters of liquid X is filled in the vessel. If the ratio of liquids X and Y is now becomes 7: 10, how many litres of liquids X and Y were contained by the vessel initially? **[2]**

Section C

26. Solve the differential equation: $x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x$ **[3]**

OR

A radioactive substance disintegrates at a rate proportional to the amount of substance present. If 50% of the given amount disintegrates in 1600 years. What percentage of the substance disintegrates in 10 years? (Take $e^{\frac{-\log 2}{160}} = 0.9957$)

27. A loan of ₹ 250000 at the interest rate of 6 % p.a. compounded monthly is to be amortized by equal payments at the end of each month for 5 years, find **[3]**

i. the size of each monthly payment.
ii. the principal outstanding at beginning of 40th month.
iii. interest paid in 40th payment.
iv. principal contained in 40th payment and
v. total interest paid. (Given $(1.005)^{60} = 1.3489$, $(1.005)^{21} = 1.1104$)

28. The marginal revenue function of a commodity is given by $MR = 11 - 3x + 4x^2$, find the revenue function. Also, find the demand function. **[3]**

29. Find the mean and variance of the number of heads in the two tosses of a coin. **[3]**

OR

A traffic engineer records the number of bicycle riders that use a particular cycle track. He records that an average of 3.2 bicycle riders use the cycle track every hour. Given that the number of bicycles that use the cycle track follow a Poisson distribution, what is the probability that

a. 2 or less bicycle riders will use the cycle track within an hour?
b. 3 or more bicycle riders will use the cycle track within an hour?

Also, write the mean expectation and variance for the random variable X.

30. For the following data, use the weighted average of price relative method to construct the index number for the year 2010, taking the year 2005 as the base year. **[3]**

Commodity	**Weight (W)**	**Price in 2005 (P_0)**	**Price in 2007 (P_1)**
E	15	22	30
F	12	15	18

G	8	17	20
H	17	12	15
I	20	25	32

31. Consider the following hypothesis test: **[3]**

$H_0 : p \geq 0.75$

$H_a : p < 0.75$

A sample of 300 provided a sample proportion of 0.68.

i. Compute the value of the test statistic.
ii. What is the p-value?
iii. At α = 0.05, what is your conclusion?
iv. What is the rejection rule using critical value? What is your conclusion?

Section D

32. Solve the linear programming problem by graphical method: **[5]**

Maximize Z = 10x + 6y

Subject to

$3x + y \leq 12$

$2x + 5y \leq 34$

$x, y \geq 0$

OR

A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make an item of A and half an hour to make an item of B. The maximum time available per day is 16 hours. The profit on an item of A is ₹300 and on one item of B is ₹160. How many items of each type should be produced to maximize the profit? Solve the problem graphically.

33. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If there are 640 litres of the 8% solution, how many litres of 2% solution will have to be added? **[5]**

34. Find the probability distribution of number of doublets in three throws of a pair of dice. **[5]**

OR

If the diameters of ball bearings are normally distributed with mean 0.6140 inches and standard deviation of 0.0025 inches, determine the percentage of ball bearings with diameters

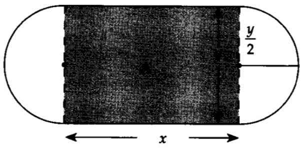
i. between 0.610 and 0.618 inches inclusive
ii. greater than 0.617 inches
iii. less than 0.608 inches
iv. equal to 0.615 inches.

35. A machine costing ₹50,000 has a useful life of 4 years. The estimated scrap value is ₹10,000. Using the straight-line method, find the annual depreciation and construct a schedule for depreciation. Also, find the depreciation rate percent. **[5]**

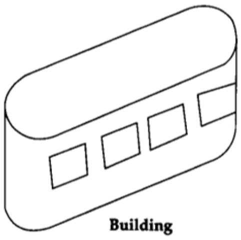
Section E

36. **Read the text carefully and answer the questions:** **[4]**

An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200 m as shown below:



Design of Floor


Building

(a) If x and y represents the length and breadth of the rectangular region, then find the relation between the variables.

(b) Find the area of the rectangular region A expressed as a function of x.

(c) Find the maximum value of area A.

OR

The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semicircular ends. Find the value of x for which the whole area is maximum.

37. **Read the text carefully and answer the questions:** **[4]**

In the year 2010, Mr. Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions:

(a) Determine the EMI.

(b) Find the principal paid by Mr. Aggarwal in the 150^{th} instalment.

(c) Find the total interest paid by Mr. Aggarwal.

OR

How much was paid by Mr. Aggarwal to repay the entire amount of home loan?

[Use $(1.00625)^{240} = 4.4608$; $(1.00625)^{91} = 1.7629$]

38. An amount of ₹5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is ₹358. If the combined income from the first two investments is ₹70 more than the income from the third, find the amount of each investment by matrix method. **[4]**

OR

Show that the matrix, $A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$ satisfies the equation, $A^3 - A^2 - 3A - I_3 = O$. Hence, find A^{-1}.

Solution

Section A

1.
(c) 40
Explanation: Given |A| = 5, order of A = 3.
So, |2A'| = 2^3 |A'| = 8|A| = 8 $\times$ 5 = 40

2. **(a)** Probability of not rejecting H_0 when H_0 is true.
Explanation: Probability of not rejecting H_0 when H_0 is true.

3.
(d) ₹ 147.36
Explanation: Present value = ₹ 1000, Let each annuity be ₹ x,
n = 8 years, r = 5% $\Rightarrow$ i $= \frac{5}{100}$ = 0.05
$\therefore$ 1000 $= \frac{x(1+0.05)}{0.05}$ [1 - (1 + 0.05)$^{-8}$]
$= \frac{x(1.05)}{0.05}$ [(1 - (1.05)$^{-8}$]
= 21x [1 - 0.676]
= 21x $\times$ 0.324
$\Rightarrow$ x $= \frac{1000}{21 \times 0.324}$ = ₹ 146.97
Closest to ₹ 147.36

4. **(a)** Concave region
Explanation: Concave region

5. **(a)** $\frac{y}{(1-y)}$
Explanation: We can write it as
$\Rightarrow y = e^{x+y}$
log y = (x + y) log e
Differentiating with respect to x,we get
$\Rightarrow \frac{1}{y} \frac{dy}{dx} = 1 + \frac{dy}{dx}$
$\Rightarrow \left(\frac{1}{y} - 1\right) \frac{dy}{dx} = 1$
$\Rightarrow \frac{dy}{dx} = 1 \left(\frac{y}{1-y}\right)$

6.
(c) 0.25
Explanation: Given μ = 20, σ = 4
$Z = \frac{X-\mu}{\sigma} \Rightarrow Z = \frac{21-20}{4} = 0.25$

7.
(d) 1 - $(\frac{35}{36})^n$
Explanation: 1 - $(\frac{35}{36})^n$

8.
(c) $\frac{d^2y}{dx^2}$ - m^2y = 0
Explanation: y = ae^{mx} + be^{-mx}
$\Rightarrow \frac{dy}{dx}$ = mae^{mx} - bme^{-mx}
and $\frac{d^2y}{dx^2}$ = m^2 ae^{mx} + m^2be^{-mx} = $m^2(ae^{mx} + be^{-mx})$
$\Rightarrow \frac{d^2y}{dx^2}$ = m^2y i.e. $\frac{d^2y}{dx^2}$ - m^2y = 0

9.

(d) 95 m

Explanation: When A runs 1000 m, B runs 900 m

Hence, when A runs 500 m, B runs 450 m

Again, when B runs 400 m, C runs 360 m

And, when B runs 450 m, C runs = 360 $\times$ $\frac{450}{400}$ = 405 m

Required distance = 500 - 405 = 95 meter

That means when A runs 500 meter then B can run 450 m then C runs 405 m

10.

(d) $\frac{1}{x}$

Explanation: $\frac{dy}{dx} - \frac{y}{x}$ = x^3 - 3

I.F. = $e^{\int -\frac{1}{x}dx}$ = $e^{-\log x} = e^{\log x^{-1}}$ = $\frac{1}{x}$

11.

(d) 1 : 4

Explanation: Cost price of 1 litres of milk = ₹ 100

$\therefore$ Mixture sold for ₹ 125

= $\frac{125}{100} = \frac{5}{4}$ litres

$\therefore$ Quantity of mixture = $\frac{5}{4}$ litres

$\therefore$ Quantity of milk = 1 litre

$\therefore$ Quantity of water = $\frac{5}{4} - 1 = \frac{1}{4}$ litre

$\therefore$ Required ratio = $\frac{1}{4}$: 1 = 1 : 4

12.

(b) x < -5 or x > 3

Explanation: $\frac{x-3}{x+5}$ > 0, x $\neq$ -5,

$\Rightarrow$ (x - 3) > 0 and (x + 5) > 0 or (x - 3) < 0 and (x + 5) < 0

$\Rightarrow$ x > 3 and x > -5 or x < 3 and x < -5

$\Rightarrow$ x > 3 or x < -5

13. **(a)** $\frac{3}{2}$

Explanation: Rate downstream $= \frac{18}{4} = \frac{9}{2}$ km/hr

Rate upstream $= \frac{18}{12} = \frac{3}{2}$ km/hr

Now, the speed of the stream $= \frac{\text{Rate Downstream} - \text{Rate Upstream}}{2}$

$\Rightarrow \frac{\frac{9}{2} - \frac{3}{2}}{2} = \frac{6}{4} = \frac{3}{2}$

14. **(a)** (4, 3)

Explanation: (4, 3)

15.

(b) x < 0, y < 0

Explanation: In 3^{rd} quadrant x < 0 and y < 0.

16.

(b) 0.34

Explanation: 0.34

17.

(c) $\frac{e^x}{1+x^2} + C$

Explanation: Given $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$

$\Rightarrow \int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ = $\int e^x \left(\frac{1+x^2-2x}{(1+x^2)^2}\right) dx$

$\Rightarrow \int e^x \left(\frac{1+x^2-2x}{(1+x^2)^2}\right) dx$ = $\int e^x \left\{\left(\frac{1+x^2}{(1+x^2)^2}\right) + \left(\frac{-2x}{(1+x^2)^2}\right)\right\} dx$

= $\int e^x \left\{\left(\frac{1}{(1+x^2)}\right) + \left(\frac{-2x}{(1+x^2)^2}\right)\right\} dx$

Now using the property: $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)$

Now in $\int e^x\left\{\left(\frac{1}{(1+x^2)}\right)+\left(\frac{-2x}{(1+x^2)^2}\right)\right\}dx$

$\Rightarrow$ f(x) = $\frac{1}{(1+x^2)}$

$\Rightarrow$ f'(x) = $\frac{-2x}{(1+x^2)^2}$

$\Rightarrow \int e^x\left\{\left(\frac{1}{(1+x^2)}\right)+\left(\frac{-2x}{(1+x^2)^2}\right)\right\}dx = \frac{e^x}{1+x^2}+C$

$\Rightarrow \int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx = \frac{e^x}{1+x^2}+C$.

18.

(c) Method of least squares

Explanation: Method of least squares

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: We know that for a non-singular matrix A of order n.

adj(adj A) = $|A|^{n-2}$ A

and |adj A| = $|A|^{n-1}$

$\therefore$ Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Given AC = $3x^2 - 7x + 5 - \frac{11}{x}$

Total cost C = AC $\times$ x

$\Rightarrow$ C = $3x^3 - 7x^2 + 5x - 11$

Now, Marginal Cost (MC) = $\frac{d}{dx}C = \frac{d}{dx}(3x^3 - 7x^2 + 5x - 11)$

= $9x^2 - 14x + 5$

$[MC]_{x=5}$ = ₹(9 $\times$ 52 -14 $\times$ 5 + 5) = ₹ 160

$\therefore$ Both Assertion and Reason true. Reason is the correct explanation of Assertion.

Section B

21. **Calculation of 4-year centred moving average:**

Year	Value	4-yearly moving total	4-yearly moving average	4-yearly centre moving average
1984	12			
1985	25	130	32.5	
1986	39			39.75
		188	47	
1987	54			54.75
		250	62.5	
1988	70			70.75
		316	79	
1989	87			84.75
		362	90.5	
1990	105			92
		374	93.5	
1991	100			90.75
		352	88	
1992	82			
1993	65			

22. Scrap value = ₹ 22500, useful life = 15 years, annual depreciation = ₹ 8500

$\therefore$ 8500 = $\frac{\text{original value} - 22500}{15}$

$\Rightarrow$ original value - 22500 = 127500

$\Rightarrow$ original value = ₹ 127500 + ₹ 22500 = ₹ 150000

OR

Given P.V. = ₹ 35000, F.V. = ₹ 56000, n = 5 years

So, CAGR = $\left(\frac{F.V.}{P.V.}\right)^{\frac{1}{n}} - 1$ = $\left(\frac{56000}{35000}\right)^{\frac{1}{5}} - 1$ = $(1.6)(1.6)^{\frac{1}{2}}$ - 1

Let x = $(1.6)^{\frac{1}{5}}$

$\Rightarrow$ log x = $\frac{1}{5}$ log 1.6

= $\frac{1}{5} \times$ 0.2041 = 0.04082

$\Rightarrow$ x = antilog 0.04082 = 1.098

So, CAGR = 1.098 - 1 = 0.098

Hence, CAGR = 0.098 $\times$100 % = 9.8 %

23. $\int_0^3 [x]dx = \int_0^1 [x]dx + \int_1^2 [x]dx + \int_2^3 [x]dx$

$= \int_0^1 0dx + \int_1^2 1dx + \int_2^3 2dx$

= 0 + $[x]_1^2$ + $[2x]_2^3$ = 1 + 6 - 4 = 3

24. Let the present value of the car be ₹P, then

value of the car after 3 years = P(1 - i)^3 = P$\left(1 - \frac{12.5}{100}\right)^3$

= P$\left(1 - \frac{1}{8}\right)^3$ = P$\left(\frac{7}{8}\right)^3$

Decrease in the value of car = P - P$\left(\frac{7}{8}\right)^3$ = P$\left[1 - \frac{343}{512}\right] = \text{P} \times \frac{169}{512}$

$\therefore$ Percentage decrease in the value of the car after 3 years

= $\left(\frac{\text{Decrease in value}}{\text{Present value}} \times 100\right)\% = \left(\frac{\text{P} \times \frac{169}{512}}{\text{P}} \times 100\right)\%$

= $\frac{169 \times 25\%}{128} = \frac{4225}{128}$ % = 33$\frac{1}{128}$%

OR

Here, r = 10% p.a., p = 12 months

So, effective rate (per rupee) = $\left(1 + \frac{10}{1200}\right)^{12}$ - 1 = (1.00833)^{12} - 1

= 1.1047 - 1 = 0.1047

Hence, the effective rate = 0.1047 $\times$ 100% = 10.47%

25. Let initially liquids X and Y be 3x litres and 5x litres respectively in the vessel.

After drawing off 8 litres of mixture:

Quantity of liquid X left in the mixture = 3x - $\frac{3}{8} \times$ 8 = (3x - 3) litres

Quantity of liquid Y left in the mixture = 5x - $\frac{5}{8} \times$ 8 = (5x - 5) litres

Further 8 litres of liquid X are mixed in the mixture.

So, quantity of liquid X in the mixture = (3x - 3 + 8) litres = (3x + 5) litres

According to given, $\frac{7}{10} = \frac{3x+5}{5x-5}$.

$\Rightarrow$ 35x - 35 = 30x + 50 $\Rightarrow$ 5x = 85

$\Rightarrow$ x = 17

Hence, the quantity of liquid X = 3 $\times$ 17 = 51 litres and the quantity of liquid Y = 5 $\times$ 17 = 85 litres initially.

Section C

26. The given differential equation is

x log x$\frac{dy}{dx}$ + y = $\frac{2}{x}$ log x

$\Rightarrow \frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x^2}$ ...(i)

This is a linear differential equation of the form

$\frac{dy}{dx}$ + Py = Q, where P = $\frac{1}{x \log x}$ and Q = $\frac{2}{x^2}$

$\therefore$ I.F. = $e^{\int Pdx} = e^{\int \frac{1}{x \log x} dx} = e^{\int \frac{1}{t} dt}$, where t = log x

$\Rightarrow$ I.F. = e$^{\log t}$ = t = log x

Multiplying both sides of (i) by I.F. = log x, we get

log x$\frac{dy}{dx} + \frac{1}{x}$y = $\frac{2}{x^2}$ log x

Integrating both sides with respect to x, we get

y log x = $\int \frac{2}{x^2}$ log x dx + C [Using: y(I.F.) = $\int$ Q (I.F.) dx + c]

$\Rightarrow$ y log x = $2 \int \underset{I}{\log x} \underset{II}{x^{-2}} dx + C$

$\Rightarrow$ y log x = $2 \left\{\log x \left(\frac{x^{-1}}{-1}\right) - \int \frac{1}{x} \left(\frac{x^{-1}}{-1}\right) dx\right\}$ + C

$\Rightarrow$ y log x = $2\left\{-\frac{\log x}{x}+\int x^{-2}dx\right\}$ + C

$\Rightarrow$ y log x = $2\left\{-\frac{\log x}{x}-\frac{1}{x}\right\}$ + C

$\Rightarrow$ y log x = $-\frac{2}{x}$ (1 + log x) + C, which gives the required solution.

OR

Let A denote the amount of the radioactive substance present at any instant t and

let A_0 be the initial amount of the substance.

It is given that

$\frac{dA}{dt}\alpha A\Rightarrow\frac{dA}{dt}=-\lambda A$ ...(i)

where λ is the constant of proportionality such that $\lambda > 0$. Here, the negative sign indicates that A decreases with the increase in t.

Now,

$\frac{dA}{dt}=-\lambda A$

$\Rightarrow\frac{1}{A}dA=-\lambda dt$

$\Rightarrow\int\frac{1}{A}dA=-\lambda\int 1.\,dt$

$\Rightarrow$ log A = $-\lambda$t + C ...(ii)

Initially i.e. at t = 0, we have A = A_0. Putting t = 0 and A = A_0 in (ii), we get

log A_0 = 0 + C $\Rightarrow$ C = log A_0

Putting C = log A_0 in (ii), we get

log A = $-\lambda$t + log A_0

$\Rightarrow\log\left(\frac{A}{A_0}\right)=-\lambda t$ ...(iii)

It is given that A = $\frac{A_0}{2}$ at t = 1600 years. Putting A = $\frac{A_0}{2}$ and t = 1600 in (iii), we get

log($\frac{1}{2}$) = -1600 $\lambda\Rightarrow\lambda=\frac{1}{1600}$ log 2

Substituting the value of λ in (iii), we get

$\log\left(\frac{A}{A_0}\right)=-\left(\frac{1}{1600}\log 2\right)t$

$\Rightarrow\frac{A}{A_0}=e^{-\frac{\log 2}{1600}t}$

$\Rightarrow A=A_0e^{-\frac{\log 2}{1600}t}$

Putting t = 10, we obtain the amount of the radioactive substance present after 10 years and is given by

A = A_0 (0.9957) [$\because e^{-\frac{\log 2}{160}}$ = 0.9957]

$\therefore$ Amount that disintegrates in 10 years = A_0 - A = A_0 - 0.9957 A_0 - 0.0043

percentage of the amount disintegrated in 10 years = $\frac{0.0043A_0}{A_0}\times$ 100 = 0.43

Hence, 0.43% of the original amount disintegrates in 10 years.

27. Given, P = ₹ 250000, i = $\frac{6}{12\times 100}$ = 0.005 and n = 5 $\times$ 12 = 60.

i. EMI = $\frac{250000\times 0.005\times(1.005)^{60}}{(1.005)^{60}-1}$

= $\frac{250000\times 0.005\times 1.3489}{0.3489}$ = ₹ 4832.69

ii. Principal outstanding at beginning of 40 th month

= $\frac{\text{EMI}\left[(1+i)^{60-40+1}-1\right]}{i(1+i)^{60-40+1}}=\frac{4832.69\times\left[(1.005)^{21}-1\right]}{0.005\times(1.005)^{21}}$

= $\frac{4832.69\times[1.1104-1]}{0.005\times 1.1104}=\frac{4832.69\times 0.1104}{0.005\times 1.1104}$ = ₹ 96096.72

iii. Interest paid in 40th payment = $\frac{\text{EMI}\left[(1+i)^{60-40+1}-1\right]}{(1+i)^{60-40+1}}$

= $\frac{4832.69\times\left[(1.005)^{21}-1\right]}{(1.005)^{21}}=\frac{4832.69\times 0.1104}{1.1104}$ = ₹ 480.48

iv. Principal paid in 40 th payment = EMI - Interest paid in 40 th payment

= 4832.69 - 480.48 = ₹ 4352.21

v. Total interest paid = n $\times$ EMI - P = 60 $\times$ 4832.69 - 250000

= 289961.40 - 250000 = ₹39961.40

28. Let R(x) be the revenue function of x units of the product and MR be the marginal revenue function, then

MR = 11 - 3x + 4x^2.

As MR = $\frac{d}{dx}$ (R(x)), so $\frac{d}{dx}$ (R(x)) = 11 - 3x + 4x^2

$\therefore$ R(x) = $\int$ (11 - 3x + 4x^2) dx

= 11x - $3 \cdot \frac{x^2}{2} + 4 \cdot \frac{x^3}{3}$ + k, where k is constant of integration.

When x = 0, R(x) = 0

$\Rightarrow$ 0 = 11 $\times$ 0 - $\frac{3}{2}$ $\times$ 0 + $\frac{4}{3}$ $\times$ 0 + k $\Rightarrow$ k = 0.

$\therefore$ R(x) = 11x - $\frac{3}{2}$x^2 + $\frac{4}{3}$x^3.

If p is the price per unit when x units of the product are sold, then

R(x) = p·x

$\Rightarrow$ px = 11x - $\frac{3}{2}$x^2 + $\frac{4}{3}$x^3

$\Rightarrow$ p = 11 - $\frac{3}{2}$x + $\frac{4}{3}$x^2, which is the corresponding demand function.

29. Let X be a random variable denoting the number of heads in the two tosses of a coin. Therefore, X can take values 0, 1 or 2 such that

P (X = 0) = (Probability of getting no head) = P(TT) = $\frac{1}{4}$

P (X = 1) = (Probability of getting one head) = P(HT or TH) = $\frac{2}{4} = \frac{1}{2}$

and, P(X = 2) = (Probability of getting both heads) = P(HH) = $\frac{1}{4}$

Thus, the probability distribution of X is as follows:

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Computation of mean and variance:

x_i	$p_i = P(X = x_i)$	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{4}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{4}$	$\frac{1}{2}$	1
		$\Sigma p_i x_i$ = 1	$\Sigma p_i x_i^2 = \frac{3}{2}$

Thus, we have,

$\Sigma p_i x_i$ = 1 and $\Sigma p_i x_i^2 = \frac{3}{2}$

$\therefore \overline{X}$ = Mean = $\Sigma p_i x_i$ = 1 and Var (X) = $\Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{3}{2} - 1 = \frac{1}{2}$

Hence, Mean = 1 and Variance = $\frac{1}{2}$

OR

Given mean = X = 3.2

Let X be the number of bicycle riders which use the cycle track.

a. Required probability = P(X $\leq$ 2) = P(X = 0) + P(X = 1) + P(X = 2)

   = $\frac{e^{-3.2}(3.2)^0}{0!} + \frac{e^{-3.2}(3.2)^1}{1!} + \frac{e^{-3.2}(3.2)^2}{2!}$

   = e$^{-3.2}$ [1 + 3.2 + 5.12]

   = 0.041 $\times$ 9.32 = 0.382

b. Required probability = P(X $\geq$ 3) = 1 - P(X $\leq$ 2)

   = 1 - 0.382 = 0.618

   Also, mean expectation and variance of X are λ = 3.2

30.

Commodity	**Weight (W)**	**Price in 2005 (P_0)**	**Price in 2010 (P_1)**	**R = $\frac{P_1}{P_0} \times 100$**	**RW**
E	15	22	30	136.36	2,045.4
F	12	15	18	120	1440
G	8	17	20	117.64	941.12
H	17	12	15	125	2,125
I	20	25	32	128	2,560

31. Given p_0 = 0.75, n = 300, $\bar{p}$ = 0.68

i. $Z = \frac{\bar{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.68-0.75}{\sqrt{\frac{0.75\times0.25}{300}}}$

$= \frac{-0.07\times10}{\sqrt{0.25\times0.25}} = \frac{-0.07}{0.25}$ = -2.8

ii. ∵ Z = -2.8 < 0

So, p-value of -2.8 = area under the standard normal curve to the left of Z

= 0.0026

∴ p-value = 0.0026

iii. Given α = 0.05

∵ p-value < 0.05

So, reject H_0.

iv. Rejection rule using critical value

Reject H_0 if Z $\leq$ - Z_α

Here, α = 0.05. So Z_α = $Z_{0.05}$ = 1.645

$\Rightarrow -Z_\alpha$ = -1.645

∵ -2.8 < -1.645

$\Rightarrow$ Z < -Z_α

So, reject H_0

Section D

32. Given,

Objective function is: Z = 10x + 6y

Constraints are:

2x + 5y $\leq$ 34

3x + y $\leq$ 12

x, y $\geq$ 0

First convert the given inequations into corresponding equations and plot them:

2x + 5y $\leq$ 34 $\Rightarrow$ 2x + 5y = 34 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 $\Rightarrow$ y = $\frac{34}{5}$ $\left(0, \frac{34}{5}\right)$ ...first coordinate.

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation.

If the given line does not pass through the origin then just put (0, 0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequations also and find the common region.

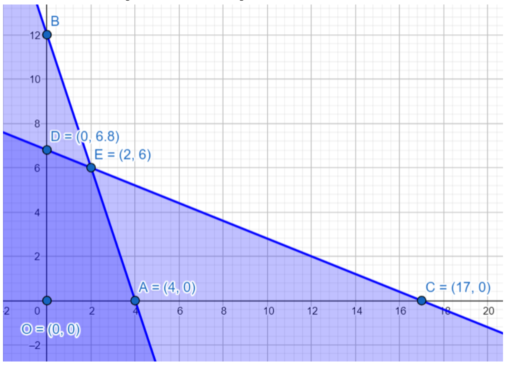
3x + y $\leq$ 12 $\Rightarrow$ 3x + y = 12 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 $\Rightarrow$ y = 12(0, 12) ...first coordinate.

Put, y = 0 $\Rightarrow$ x = $\frac{12}{3}$ = 4(4, 0) ...second coordinate

x = 0 is the y-axis and y = 0 is the x-axis



Hence, we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving 3x + y = 12 and 2x + 5y = 34 gives (2, 6)

Similarly solve other combinations by observing graph to get other coordinates.

From figure we have obtained coordinates of corners as:

$(0, 0), (4, 0), (0, \frac{34}{5}), (2, 6)$

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because z = 10x + 6y$

$\therefore$ Z at $(2, 6) = 10 \times (2) + 6 \times (6) = 56$

Z at $(0, \frac{34}{5}) = 10 \times (0) + 6 \times (\frac{34}{5}) = \frac{204}{5}$

Z at $(4, 0) = 10 \times (4) + 10 \times 0 = 40$

Z at (0, 0) = 0

We can see that Z is maximum at (2, 6) and max. value is 56

$\therefore$ Z is maximum at x = 2 and y = 16; and max. value is 56

OR

Let x and y be the number of items of A and B that should be produced each day to maximize the profit.

A number of items cannot be negative.

Therefore, $x \geq 0, y \geq 0$

It is also given that the firm can produce at most 24 items in a day.

$\therefore x + y \leq 24$

Also, the time required to make an item of A is one hour, and the time required to make an item of B is half an hour.

Therefore, the time required to produce x items of A and y items of B is $x + \frac{1}{2}y$ hours However, the maximum time available in a day is 16 hours. $x + \frac{1}{2}y \leq 16$

It is given that the profit on an item of A is ₹300 and on one item of B is ₹160. Therefore, the profit gained from x items of A and y items of B is ₹300x and ₹160y respectively.

Total profit Z = 300x + 160y

The mathematical form of the given LPP is:

Maximize Z = 300x + 160y

subject to constraints:

$x + y \leq 24$

$x + \frac{1}{2}y \leq 16$

$x \geq 0, y \geq 0$

First we will convert inequations into equations as follows:

$x + y = 24, x + \frac{1}{2}y = 16, x = 0$ and $y = 0$

Region represented by $x + y \leq 24$:

The line x + y = 24 meets the coordinate axes at A_1 (24, 0) and B_1 (0, 24) respectively.

By joining these points we obtain the line x + y = 24. Clearly, (0, 0) satisfies the x + y = 24. So, the region which contains the origin represents the solution set of the inequation $x + y \leq 24$

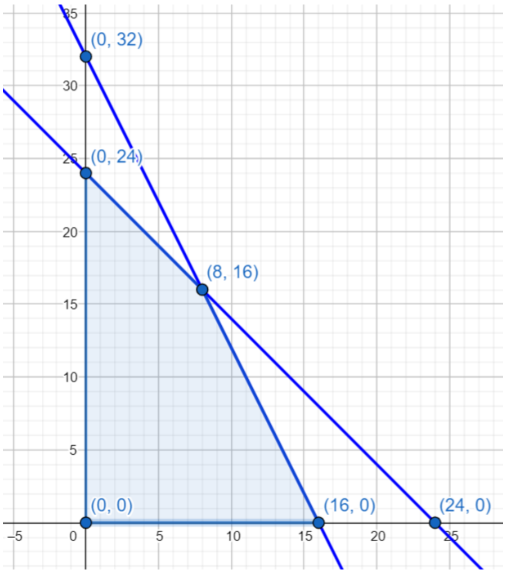
The region represented by $x + \frac{1}{2}y \leq 16$:

The line $x + \frac{1}{2}y = 16$ meets the coordinate axes at C_1 (16, 0) and D_1 (0, 32) respectively. By joining these points we obtain the line $x + m\frac{1}{2}y = 16$. Clearly, (0, 0) satisfies the inequation $x + \frac{1}{2}y \leq 16$. So, the region which contains the origin represents the solution set of the inequation $x + \frac{1}{2}y \leq 16$. Region represented by $x \geq 0$ and $y \geq 0$:

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$, and $y \geq 0$

The feasible region determined by the system of constraints $x + y \leq 24$, $x + \frac{1}{2}y \geq 16$, $x \geq 0$ and $y \geq 0$ are as follows.

The feasible region is shown in the figure:



In the above graph, the shaded region is the feasible region. The corner points are O(0, 0), C_1 (16, 0), E_1 (8, 16), and B_1 (0, 24)

The values of the objective function Z at corner points of the feasible region are given in the following table:

Corner Points	Z = 300x + 160y
O(0, 0)	0
C_1(16, 0)	4800
E_1(8, 16)	4960 $\rightarrow$ Maximum
B_1(0, 24)	3840

Clearly, Z is maximum at x = 8 and y = 16 and the maximum value of Z at this point is 4960

Thus, 8 items of A and 16 items of B should be produced in order to maximize the profit and the maximum profit is ₹4960.

33. Volume of the 8% solution = 640 litres

Boric acid present in the 8% solution = 8% of 640 ...(i)

And the rest 92% of 640 litres is water in the 8% solution.

Let volume of 2% solution added to 640 liters be x.

Boric acid present in 2% solution = 2% of x ...(ii)

New volume of 8% solution = 640 + x ...(iii)

Boric acid present in the new solution (that is, after adding x litres of 2% solution to 8% solution) = Boric acid present in the 8% solution + Boric acid present in the 2% solution [from (i) & (ii)]

$\Rightarrow$ Boric acid present in the new solution = 8% of 640 + 2% of x

$\Rightarrow$ Boric acid present in the new solution = $\left(\frac{8}{100} \times 640\right) + \left(\frac{2}{100} \times x\right)$

$\Rightarrow$ Boric acid present in the new solution = $\frac{2x}{100} + \left(\frac{8}{100} \times 640\right)$ ...(iv)

According to the question,

The resulting mixture is to be more than 4% but less than 6% boric acid.

That is, the boric acid content in the resulting mixture must be more than 4% but less than 6% boric acid.

So, first let us take boric acid content in the resulting mixture to be more than 4%.

$\Rightarrow$ Boric acid present in the new solution > 4% of the new volume of 8% solution

$\Rightarrow \frac{2x}{100} + \left(\frac{8}{100} \times 640\right) > \frac{4}{100} \times (640 + x)$

[from (iii) & (iv)]

$\Rightarrow \frac{2x}{100} + \frac{8 \times 640}{100} > \frac{4(640+x)}{100}$

$\Rightarrow \frac{2x+(8\times 640)}{100} > \frac{4(640+x)}{100}$

$\Rightarrow$ 2x + 5120 > 2560 + 4x

$\Rightarrow$ 5120 - 2560 > 4x - 2x

$\Rightarrow$ 2560 > 2x

$\Rightarrow$ 2x < 2560

$\Rightarrow x < \frac{2560}{2}$

$\Rightarrow$ x < 1280

Now, let us take boric acid in the resulting mixture to be less than 6%.

$\Rightarrow$ Boric acid present in the new solution < 6% of the new volume of 8% solution

$\Rightarrow \frac{2x}{100} + \left(\frac{8}{100} \times 640\right) < \frac{6}{100} \times (640 + x)$

[from (iii) & (iv)]

$\Rightarrow \frac{2x}{100} + \frac{8 \times 640}{100} < \frac{6(640+x)}{100}$

$\Rightarrow \frac{2x+(8\times640)}{100} < \frac{6(640+x)}{100}$

$\Rightarrow$ 2x + 5120 < 3840 + 6x

$\Rightarrow$ 5120 - 3840 < 6x - 2x

$\Rightarrow$ 1280 < 4x

$\Rightarrow$ 4x > 1280

$\Rightarrow x > \frac{1280}{4}$

$\Rightarrow$ x > 320

We have

x < 1280 & x > 320

$\Rightarrow$ 320 < x < 1280

Hence, the required liters of 2% solution to be added to 8% of the solution is between 320 liters and 1280 liters.

34. Let X denotes the number of doublets. X can take the value 0, 1, 2 or 3

Possible doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6)

Probability of getting a doublet = $\frac{6}{36} = \frac{1}{6}$

Probability of not getting a doublet = 1 - $\frac{1}{6} = \frac{5}{6}$

Now,

P(X = 0) = P(No doublet)

$= \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{125}{216}$

P(X = 1) = P(No doublet and two non-doublet)

$= \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$

$= \frac{25}{216} + \frac{25}{216} + \frac{25}{216}$

$= \frac{75}{216}$

P(X = 2) = P(Two doublet and one non-doublet)

$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{5}{216} + \frac{5}{216} + \frac{5}{216}$

$= \frac{15}{216}$

P(X = 3) = P(Three doublets)

$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{216}$

$\therefore$ The required probabilities are shown in the figure.

X	0	1	2	3
P(X)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

OR

Let X denote the diameters of ball bearings. Then X is a normal variate with mean μ = 0.6140 inches and standard deviation σ = 0.0025 inches.

Let Z be the standard normal variate. Then,

Z = $\frac{X-\mu}{\sigma} \Rightarrow Z = \frac{X-0.6140}{0.0025}$

i. When X = 0.610, we obtain: Z = $\frac{0.610-0.6140}{0.0025}$ = -1.6

When X = 0.618, we obtain: Z = $\frac{0.618-0.6140}{0.0025}$ = 1.6

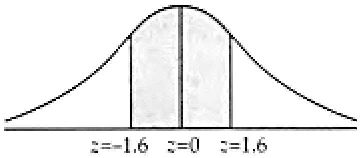
$\therefore$ P(0.610 $\leq$ X $\leq$ 0.618)

= P(-1.6 $\leq$ Z $\leq$ 1.6) [By symmetry]

= 2P(0 $\leq$ Z $\leq$ 1.6)

= 2 $\times$ 0.4452

= 0.8904 [See Table]



Thus, the percentage of ball bearings with diameters between 0.610 and 0.618 inches is 0.8904 $\times$ 100 = 89.04

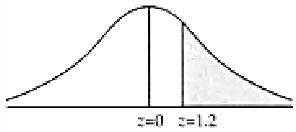
ii. When X = 0.617, we obtain: Z = $\frac{0.617-0.6140}{0.0025}$ = 1.2

$\therefore$ P(X $\geq$ 0.617)

= P(Z > 1.2)

= P(Z $\geq$ 0) - P(0 $\leq$ Z $\leq$ 1.2)

= 0.5 - 0.3849 = 0.1151



Thus, the percentage of ball bearings with diameters greater than 0.617 inches is 0.1151 $\times$ 100 = 11.51

iii. When X = 0.608, we obtain: Z = $\frac{0.608-0.6140}{0.0025}$ = -2.4.

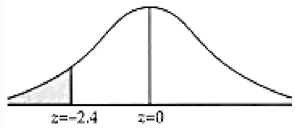
$\therefore$ P (X < 0.608)

= P (Z < - 2.4)

= P (Z > 2.4)

= P (Z $\geq$ 0) - P (0 $\leq$ Z $\leq$ 2.4)

= 05 - 0.4918 = 0.0082



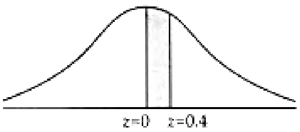
Thus, the percentage of ball bearings with diameters less than 0.608 inches = 0.0082 $\times$ 100 = 0.82

iv. When X = 0.615, we obtain: Z = $\frac{0.615-0.6140}{0.0025}$ = 0.4

$\therefore$ P(X = 0.615)

= P(Z = 0.4)

= 0.1554



Thus, the percentage of ball bearings with diameter equal to 0.615 inches = 0.1554 $\times$ 100 = 1554

35. We are given that:

C = Original value = ₹50,000, S = Salvage value = ₹10,000, n = Useful life = 4 years

The annual depreciation D is given by

D = $\frac{C-S}{n} \Rightarrow$ D = ₹$\left(\frac{50,000-10,000}{4}\right)$ = ₹10,000

At the beginning of the first year, the book value of the machine is ₹50,000. At the end of the first year, the accumulated depreciation is ₹10,000; hence the depreciation charge for the first year is ₹10,000. The book value at the end of the first year or in the beginning of the second year is ₹(50,000 - 10,000) = ₹40,000

At the end of the second year, we have:

Accumulated depreciation = ₹20,000

$\therefore$ Depreciation charge = ₹(20,000 - 10,000) = ₹10,000

$\Rightarrow$ Book value at the end of the second year = ₹(40,000 - 10,000) = ₹30,000

At the end of third year, we have:

Accumulated depreciation = ₹30,000

$\therefore$ Depreciation charge = ₹(30,000 - 20,000) = ₹10,000

$\Rightarrow$ Book value at the end of the third year = ₹(30,000 - 10,000) = ₹20,000

At the end of fourth year, we have:

Accumulated depreciation = ₹40,000

$\therefore$ Depreciation charge = ₹(40,000 - 30,000) = ₹10,000

⇒ Book value at the end of the third year = ₹(20,000 - 10,000) = ₹10,000

Clearly, this is the salvage value of the machine.

These values can be presented in the following tabular form which is known as the depreciation schedule of the machine:

Depreciation Schedule			
Year	Book value (Beginning of year)	Depreciation	Book value (End of year)
1	₹50,000	₹10,000	₹40,000
2	₹40,000	₹10,000	₹30,000
3	₹30,000	₹10,000	₹20,000
4	₹20,000	₹10,000	₹10,000

We find that

Annual depreciation amount = ₹10,000

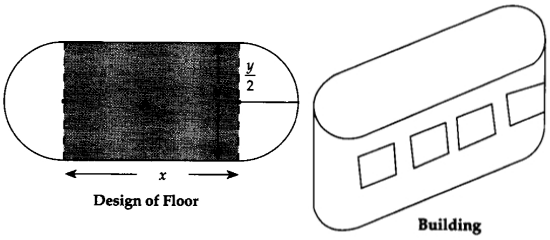
Cost of the machine - Salvage value of machine = ₹(50,000 - 10,000) = ₹40,000

∴ Depreciation rate percent = $\left(\frac{10,000}{40,000} \times 100\right)$ = 25%

Section E

36. **Read the text carefully and answer the questions:**

An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200 m as shown below:



(i) According to given information,

perimeter of floor of the building = 2x + $2.\left(\pi \cdot \frac{y}{2}\right)$ = 200

⇒ 2x + πy = 200.

(ii) Area of rectangular region of the floor = A = xy

= $x\left(\frac{200-2x}{\pi}\right) = \frac{2}{\pi}(100x - x^2)$

(iii) A = $\frac{2}{\pi}(100x - x^2)$ (from $\left(x\left(\frac{200-2x}{\pi}\right) = \frac{2}{\pi}(100x - x^2)\right)$

⇒ $\frac{dA}{dx} = \frac{2}{\pi}$ (100x - 2x) and $\frac{d^2A}{dx^2} = \frac{2}{\pi}(0 - 2) = -\frac{4}{\pi}$.

Now, $\frac{dA}{dx} = 0 \Rightarrow \frac{2}{\pi}(100 - 2x) = 0 \Rightarrow x = 50$.

When x = 50, $\frac{d^2A}{dx^2} = \frac{4}{\pi} < 0$

⇒ A is maximum when x = 50.

Maximum value of A = $\frac{2}{\pi}(100 \times 50 - 50^2) = \frac{5000}{\pi} m^2$.

OR

Let Z be the area of the whole floor, then

$Z = xy + 2 \cdot \frac{1}{2}\pi\left(\frac{y}{2}\right)^2 = xy + \frac{\pi}{4} \cdot y^2$

$= x \cdot \frac{200-2x}{\pi} + \frac{\pi}{4} \cdot \left(\frac{200-2x}{\pi}\right)^2$

$= \frac{2}{\pi}(100x - x^2) + \frac{(100-x)^2}{\pi}$

$\Rightarrow \frac{dZ}{dx} = \frac{2}{\pi}(100 - 2x) + \frac{2}{\pi}(100 - x)(-1) = \frac{2x}{\pi}$ and $\frac{d^2Z}{dx^2} = -\frac{2}{\pi}$.

$\frac{dZ}{dx} = 0 \Rightarrow -\frac{2x}{\pi} = 0 \Rightarrow x = 0$.

When x = 0, $\frac{d^2Z}{dx^2} = -\frac{2}{\pi} < 0 \Rightarrow$ Z is maximum when x = 0.

37. **Read the text carefully and answer the questions:**

In the year 2010, Mr. Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions:

(i) Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ = 0.00625

and n = 12 $\times$ 20 = 240 months

EMI = $= \frac{Pi}{1-(1+i)^{-n}}$

$= \frac{30,00,000 \times 0.00625}{1-(1.00625)^{-240}-1}$

$= \frac{30,00,000 \times 0.00625 \times 4.4608}{3.4608}$

₹ 24167.82

(ii) Given P = ₹ 30,00,000, i = $\frac{7.5}{1200}$ = 0.00625

and n = 12 $\times$ 20 = 240 months

Interest paid on 150th instalment

$= \frac{EMI \times \left[(1+i)^{240-150+1} - 1\right]}{(1+i)^{240-150+1}}$

$= \frac{24167 \times [1.7629 - 1]}{1.7629}$

= ₹ 10458.70

$\Rightarrow$ Principal paid in 150th instalment = EMI - interest

= ₹ (24167.82 - 10458.70)

(iii) Total Interest paid = n $\times$ EMI - P

= ₹ (240 $\times$ 24167.82 - 30,00,000)

= ₹ 28,00,276.80

OR

Total amount paid = n $\times$ EMI

= 240 $\times$ 2416.81

= ₹ 5800276.8

38. Let x, y and z ₹ be the investments at the rates of interest of 6%, 7% and 8% per annum respectively. Then,

Total investment = ₹5000

$\Rightarrow$ x + y + z = 5000

Now, Income from first investment of ₹ x = ₹ $\frac{6x}{100}$

Income from second investment of ₹ y = ₹ $\frac{7y}{100}$

Income from third investment of ₹ z = ₹ $\frac{8z}{100}$

$\therefore$ Total annual income = ₹ $\left(\frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100}\right)$

$\Rightarrow \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100}$ = 358 [$\because$ Total annual income = ₹358]

It is given that the combined income from the first two investments is ₹70 more than the income from the third

$\therefore \frac{6x}{100} + \frac{7y}{100}$ = 70 + $\frac{8z}{100} \Rightarrow$ 6x + 7y - 8z = 7000

Thus, we obtain the following system of simultaneous linear equations:

x + y + z = 5000

6x + 7y + 8z = 35800

6x + 7y - 8z = 7000

This system of equations can be written in matrix form as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

or, AX = B, where A = $\begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$

Now, |A| = $\begin{vmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{vmatrix}$ = 1 (-56 - 56) - (-48 - 48) + (42 - 42) = -16 $\neq$ 0

So, A^{-1} exists and the solution of the given system of equations is given by X = A^{-1} B

Let C_{ij} be the cofactor of a_{ij} in A = $[a_{ij}]$. Then,

C_{11} = -112, C_{12} = 96, C_{13} = 0, C_{21} = 15, C_{22} = -14,

C_{23} = -1, C_{31} = 1, C_{32} = -2 and C_{33} = 1

$\therefore$ adj A = $\begin{bmatrix} -112 & 96 & 0 \\ 15 & -14 & -1 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

So, $A^{-1} = \frac{1}{|A|}$ (adj A) = $-\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

Hence, the solution is given by

$X = A^{-1} B = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix} = -\frac{1}{16}\begin{bmatrix} -560000 & +537000 & +7000 \\ 480000 & -501200 & -14000 \\ 0 & -35800 & +7000 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 2200 \\ 1800 \end{bmatrix}$

$\Rightarrow$ x = 1000, y = 2200 and z = 1800

Hence, three investments are of ₹1000, ₹2200 and ₹1800 respectively.

OR

Here, we have:

A = $\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$A^3 = A^2A$

$A^2 = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0-6 & 0+0-8 & -2+0-2 \\ -2+2+6 & 0+1+8 & 4-2+2 \\ 3-8+3 & 0-4+4 & -6+8+1 \end{bmatrix} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$

$A^2A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5^2+16-12 & 0-8+16 & 10-16-4 \\ 6-18+12 & 0-9+16 & -12+18+4 \\ -2-0+9 & 0-0-12 & 4+0+3 \end{bmatrix}$

$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix}$

Now, $A^3 - A^2 - 3A - I$

$= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1+5 & -8+8 & -10+4 \\ 0-6 & 7-9 & 10-4 \\ 7+2 & 12-0 & 7-3 \end{bmatrix} + \begin{bmatrix} -3-1 & -0-0 & 6-0 \\ 6-0 & +3-1 & -6-0 \\ -9-0 & -12+0 & -3-1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 & -6 \\ -6 & -2 & 6 \\ 9 & 12 & 4 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 6 \\ 6 & 2 & -6 \\ -9 & -12 & -4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Thus, $A^3 - A^2 - 3A - I = 0$

Multiply both sides by A^{-1}, we get

$A^{-1}A^3 - A^1A^2 - 3A^{-1}A - IA^{-1} = 0$

$A^2 - A - 3I = A^{-1}$ ...(since $A^{-1}A = I$)

$\Rightarrow A^{-1} = (A^2 - A - 3I)$

$$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5-1-3 & -8-0-0 & -4+2-0 \\ 6+2-0 & 7+1-3 & 4-2-0 \\ -2-3-0 & 0-4-0 & 3-1-3 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$