Sample Question Paper - 17
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

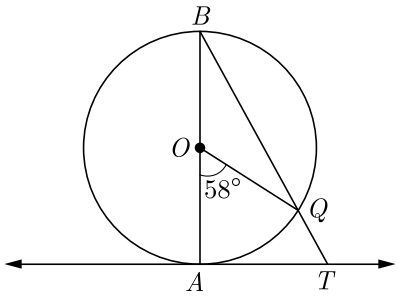
1. Solve for x : $\sqrt{6x+7}-(2x-7)=0$

OR

For what values of k, the equation $9x^2+6kx+4=0$ has equal roots?

2. The n^{th} term of an AP is given by $(-4n+15)$. Find the sum of first 20 terms of this AP.

3. In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ=58^\circ$, find $\angle ATQ$.



4. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs.25 per meter.

5. Find x and y from the following cumulative frequency distribution :

Classes	Frequency	c.f.
0-8	15	15
8-16	x	28
16-24	15	43
24-32	18	y
32-40	09	70

6. The frequency distribution of agricultural holdings in a village below :

Area of land (in hectare)	1-3	3-5	5-7	7-9	9-11	11-13
Number of families	20	45	80	55	40	12

Find the modal agricultural holding of the village.

OR

Find median of the data, using an empirical relation when it is given that Mode = 12.4 and Mean = 10.5.

Section B

7. Solve the following quadratic equation for x :

$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

8. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each AP is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

9. The angles of depression of the top and bottom of a building 50 meters high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower, and also the horizontal distance between the building and the tower.

10. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

OR

Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.

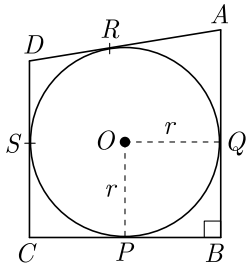
Section C

11. The angle of elevation of a cloud from a point 120 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud.

12. In figure, PQ is a chord of a circle O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.

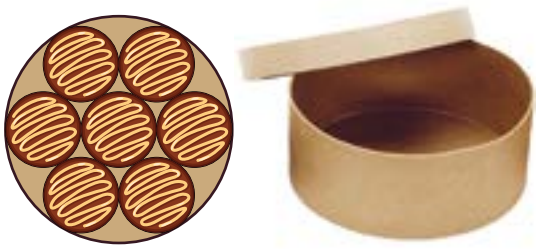
OR

In figure, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90^\circ$ and $DS = 5$ cm, then find the radius of the circle (in cm).



13. A bakery is an establishment that produces and sells flour-based food baked in an oven such as bread, cookies, cakes, pastries, and pies. Some retail bakeries are also categorized as cafés, serving coffee and tea to customers who wish to consume the baked goods on the premises.

Tania runs a bakery shop and her bakery is very famous for tasty biscuits. The amount of mixture required to make one biscuit is 18 cu cm. Before it is cooked, the mixture is rolled into a sphere. After the biscuit is cooked, the biscuit becomes a cylinder of radius 3 cm and height 0.7 cm. The increase in volume is due to air being trapped in the biscuit. Biscuits are packed in a cylindrical card box of height 14 cm. The arrangement of biscuits is shown below.



(i) What is the volume of the biscuits after it is cooked ? What is the volume of air trapped, while cooking the biscuit ?

(ii) How much space is vacant in box after biscuits are packed ?

14. The Kendriya Vidyalaya Sangathan is a system of premier central government schools in India that are instituted under the aegis of the Ministry of Education (MHRD), Government of India. As of October 2020, it has a total of 1239 schools. It is one of the world's largest chains of schools. The system came into being in 1963 under the name 'Central Schools'. Later, the name was changed to Kendriya Vidyalaya. Its schools are all affiliated to the Central Board of Secondary Education (CBSE). The objective of KVS is to cater to the educational needs of the children of transferable Central Government employees including Defence and Para-Military personnel by providing a common programme of education.

Commissioner of Regional office Jaipur prepare a table of the marks obtained of 100 students which is given below

Marks obtained	0-20	20-40	40-60	60-80	80-100
Number of students	15	18	21	29	p

He was told that mean marks of a student is 53.

(i) What is the value of model marks ?

(ii) What is the value of median marks ?

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

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SECTION A

1. Solve for x : $\sqrt{6x+7}-(2x-7)=0$

Ans :

We have $\sqrt{6x+7}-(2x-7)=0$

or, $\sqrt{6x+7}=(2x-7)$

Squaring both sides we get

$$\begin{aligned} 6x+7 &= (2x-7)^2 \\ 6x+7 &= 4x^2-28x+49 \\ 4x^2-34x+42 &= 0 \\ 2x^2-17x+21 &= 0 \\ 2x^2-14x-3x+21 &= 0 \\ 2x(x-7)-3(x-7) &= 0 \\ (x-7)(2x-3) &= 0 \end{aligned}$$

Thus $x=7$ and $x=\frac{2}{3}$.

OR

For what values of k, the equation $9x^2+6kx+4=0$ has equal roots?

Ans :

We have, $9x^2+6kx+4=0$

For equal roots, $D=0$,

$$\begin{aligned} b^2-4ac &= 0 \\ (6k)^2-4(a)(4) &= 0 \\ 36k^2-144 &= 0 \\ 36k^2 &= 144 \\ k^2 &= 4 \\ k &= \pm 2 \end{aligned}$$

2. The n^{th} term of an AP is given by $(-4n+15)$. Find the sum of first 20 terms of this AP.

Ans :

Let the first term be a, common difference be d, n th term be a_n and sum of n term be S_n.

We have $a_n=-4n+15$

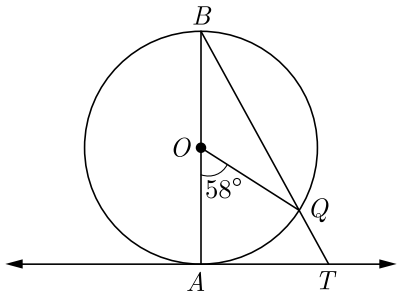
$$\begin{aligned} a_1 &= -4\times 1+15=11 \\ a_2 &= -4\times 2+15=7 \\ a_3 &= -4\times 3+15=3 \\ d &= a_2-a_1=7-11=-4 \end{aligned}$$

Now, we have $a=11$, $d=-4$

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a+(n-1)d\right] \\ S_{20} &= \frac{20}{2}\left[2\times 11+(20-1)\times(-4)\right] \\ &= 10\left[22-76\right]=10\times(-54)=-540 \end{aligned}$$

Thus $S_{20}=-540$.

3. In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ=58^\circ$, find $\angle ATQ$.



Ans :

We have $\angle AOQ=58^\circ$

Since angle $\angle ABQ$ and $\angle AOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2} \times 58^\circ = 29^\circ$$

Here OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus $\angle BAT = 90^\circ$

$\angle ABQ = \angle ABT$

Now in ΔBAT,

$$\angle ATB = 90^\circ - \angle ABT$$

$$= 90^\circ - 29^\circ = 61^\circ$$

Thus $\angle ATQ = \angle ATB = 61^\circ$

4. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs.25 per meter.

Ans :

We have radius $r = 7$ m and height $h = 24$ m

Slant height of tent,

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$$

$$= \sqrt{625} = 25 \text{ m.}$$

Curved surface area of cone,

$$\pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Curves surface area of tent will be required area of cloth. Let x meter of cloth is required

$$5x = 550 \text{ or, } x = \frac{550}{5} = 110 \text{ m.}$$

Thus 110 m of cloth is required.

$$\text{Cost of cloth} = 25 \times 110 = Rs.2750.$$

5. Find x and y from the following cumulative frequency distribution :

Classes	Frequency	c.f.
0-8	15	15
8-16	x	28
16-24	15	43
24-32	18	y
32-40	09	70

Ans :

From the cumulative frequency distribution,

$$15 + x = 28 \Rightarrow x = 28 - 15 = 13$$

and $43 + 18 = y \Rightarrow y = 61$

Hence, $x = 13$ and $y = 61$

6. The frequency distribution of agricultural holdings in a village below :

Area of land (in hectare)	1-3	3-5	5-7	7-9	9-11	11-13
Number of families	20	45	80	55	40	12

Find the modal agricultural holding of the village.

Ans :

Class 5-7 has the maximum frequency 80, therefore this is model class.

Here $l = 5, f_1 = 80, f_0 = 45, h = 2, f_2 = 55$

$$\text{Mode, } M_o = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$

$$= 5 + \frac{80 - 45}{160 - 45 - 55} \times 2$$

$$= 5 + \frac{35 \times 2}{60}$$

$$= 6.17$$

OR

Find median of the data, using an empirical relation when it is given that Mode = 12.4 and Mean = 10.5 .

Ans :

Mode, $M_o = 12.4$

Mean, $M = 10.5$

Median, M_d

$$= \frac{1}{3}M + \frac{2}{3}M_o$$

$$= \frac{1}{3}(12.4) + \frac{2}{3}(10.5) = \frac{12.4}{3} + \frac{21}{3}$$

$$= \frac{12.4 + 21}{3} = \frac{33.4}{3} = 11.13$$

Section B

7. Solve the following quadratic equation for x :

$$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$$

Ans :

We have $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

$$x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$$

$$x\left(x+\frac{a}{a+b}\right)+\frac{a+b}{a}\left(x+\frac{a}{a+b}\right)=0$$

$$\left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right)=0$$

Thus $x = \frac{-a}{a+b}, \frac{-(a+b)}{a}$

8. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each AP is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Ans :

Let the first term be a, common difference be d, n th term be a_n and sum of n term be S_n.

We have $S_1 = 1 + 2 + 3 + ... n$

$S_2 = 1 + 3 + 5 + ...$ up to n terms

$S_3 = 1 + 4 + 7 + ...$ upto n terms

Now $S_n = \frac{n(n+1)}{2}$

$S_2 = \frac{n}{2}[2+(n-1)2] = \frac{n}{2}[2n] = n^2$

and $S_3 = \frac{n}{2}[2+(n-1)3] = \frac{n(3n-1)}{2}$

Now, $S_1 + S_3 = \frac{n(n+1)}{2} + \frac{n(3n-1)}{2}$

$= \frac{n[n+1+3n-1]}{2} = \frac{n[4n]}{2}$

$= 2n^2 = 2s_2$ Hence Proved

9. The angles of depression of the top and bottom of a building 50 meters high as observed from the top of a tower are 30° and 60° respectively. Find the height of the tower, and also the horizontal distance between the building and the tower.

Ans :

Let AB be the building of height 50 m and CD be tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.

Let, $AE = BD = x$

Now, $DE = AB = 50$ m

$CE = CD - DE = (h - 50)$ m

Again, we have

$\angle CAE = 30°$ and $\angle CBD = 60°$

In $\Delta AEC, \tan 30° = \frac{CE}{AE}$

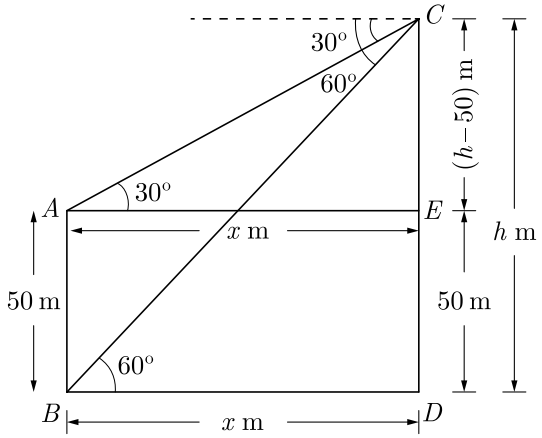
$$\frac{1}{\sqrt{3}} = \frac{h-50}{x}$$

$$x = \sqrt{3}(h-50) \quad ...(1)$$

In $\Delta BDC, \tan 60° = \frac{CD}{BD}$

$$\sqrt{3} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \quad ...(2)$$



From equation (1) and equation (2), we have

$$\sqrt{3}(h-50) = \frac{h}{\sqrt{3}}$$

$$3(h-50) = h$$

$$3h - 150 = h$$

$$2h = 150 \Rightarrow h = 75$$

Substituting $h = 75$ in equation (2), we get

$$x = \frac{h}{\sqrt{3}} = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

$$= 25 \times 1.73 = 43.2 \text{ m}$$

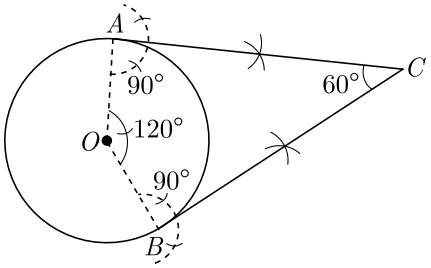
Hence, height of the tower is 75 m and distance between the building and the tower is 43.2 m.

10. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

Ans :

Steps of Construction :

1. Draw a circle of radius 5 cm with centre O.
2. Taking a point A on the circle draw $\angle AOB = 120°$.



3. Draw a perpendicular on OA at A.
4. Draw another perpendicular on OB at B.
5. Let the two perpendiculars meet at C.
   Thus, CA and CB are the two required tangents to the given circle which are inclined to each other at 60°.

Justification :

In a quadrilateral $OACB$, using angle sum property, we

$$\begin{aligned} 120^\circ + 90^\circ + 90^\circ + \angle ACB &= 360^\circ \\ 300^\circ + \angle ACB &= 360^\circ \\ \angle ACB &= 360^\circ - 300^\circ \\ &= 60^\circ. \end{aligned}$$

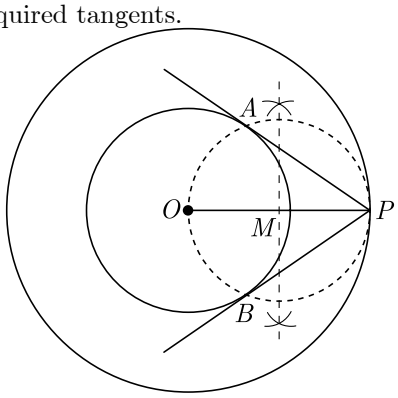
OR

Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.

Ans :

Steps of Construction :

1. Draw a circle with centre O and radius 4 cm.
2. Draw another circle with centre O and radius 6 cm.
3. Take a point P on outer circle and join OP.
4. Draw perpendicular bisector of OP which intersect OP at M.
5. Draw a circle with centre M which intersects inner circle at points A and B.
6. Join AP and BP. Thus AP and BP are required tangents.

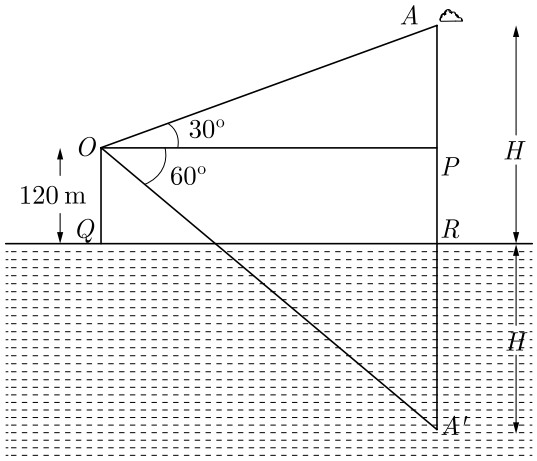


Section C

11. The angle of elevation of a cloud from a point 120 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud.

Ans :

As per given in question we have drawn figure below.



Here A is cloud and A' is refection of cloud.

In right ΔAOP we have

$$\begin{aligned} \tan 30^\circ &= \frac{PA}{OP} \\ \frac{1}{\sqrt{3}} &= \frac{H-120}{OP} \\ OP &= (H-120)\sqrt{3} \qquad \ldots(1) \end{aligned}$$

In right ΔOPA' we have

$$\begin{aligned} \tan 60^\circ &= \frac{PA'}{OP} \\ \sqrt{3} &= \frac{H+120}{OP} \\ OP &= \frac{H+120}{\sqrt{3}} \qquad \ldots(2) \end{aligned}$$

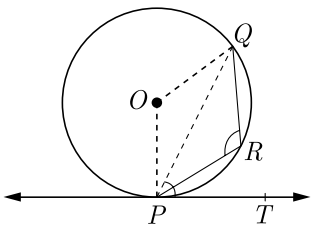
From (1) and (2), we get

$$\begin{aligned} \frac{H+120}{\sqrt{3}} &= \sqrt{3}(H-120) \\ H+120 &= 3(H-120) \\ H+120 &= 3H-360 \\ 2H &= 480 \Rightarrow H = 240 \end{aligned}$$

Thus height of cloud is 240 m.

12. In figure, PQ is a chord of a circle O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.

Ans :



We have $\angle QPT = 60^\circ$

Here $\angle OPT = 90^\circ$ because of tangent at radius.

Now $\angle OPQ = \angle OQP$

$= \angle OPT - \angle QTP$

$= 90^\circ - 60^\circ = 30^\circ$

$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$

$= 180^\circ - (30^\circ + 30^\circ)$

$= 180^\circ - 60^\circ = 120^\circ$

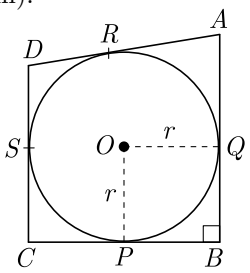
NowReflex $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$\angle PRQ = \frac{1}{2}$ Reflex $\angle POQ$

$= \frac{1}{2} \times 240^\circ = 120^\circ$

OR

In figure, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90^\circ$ and $DS = 5$ cm, then find the radius of the circle (in cm).



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Ans :

Since length of tangents from an external point to a circle are equal,

$DR = DS = 5$ cm

$AR = AQ$

$BQ = BP$

Now $AR = AD - DR$

$= 23 - 5 = 18$ cm

$AQ = AR = 18$ cm

$QB = AB - AQ = 29 - 18 = 11$ cm

$PB = QB = 11$

Now $\angle OQB = \angle OPB = 90^\circ$ because radius is always perpendicular to tangent.

Thus $OP = OQ = PB = BQ$

So, $POQB$ is a square. Hence, $r = OP = PB = 11$ cm

13. A bakery is an establishment that produces and sells flour-based food baked in an oven such as bread, cookies, cakes, pastries, and pies. Some retail bakeries are also categorized as cafés, serving coffee and tea to customers who wish to consume the baked goods on the premises.

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(i) What is the volume of the biscuits after it is cooked ? What is the volume of air trapped, while cooking the biscuit ?

(ii) How much space is vacant in box after biscuits are packed ?

Ans :

(i) Volume of the biscuit,

$= \pi r^2 h = \frac{22}{7} \times 3^2 \times 0.7 = 19.8$ cu cm

Volume of air trap

= Volume of biscuit − Volume of sphere

$= 19.8 - 18 = 1.8$ cu cm

(ii) From figure it is clear that radius of box is 3

times of biscuit radius i.e. $3 \times 3 = 9$.

$$\text{Volume of box} = \pi R^2 H$$
$$= \frac{22}{7} \times 9 \times 9 \times 14$$
$$= 22 \times 9 \times 9 \times 2$$
$$= 3564\,\text{cm}^3$$
$$\text{Volume of biscuits} = \pi r^2 h \times 140$$
$$= 19.8 \times 140$$
$$= 2772\,\text{cm}^3$$
$$\text{Vacant volume} = 3564 - 2772 = 792\,\text{cm}^3$$

14. The Kendriya Vidyalaya Sangathan is a system of premier central government schools in India that are instituted under the aegis of the Ministry of Education (MHRD), Government of India. As of October 2020, it has a total of 1239 schools. It is one of the world's largest chains of schools. The system came into being in 1963 under the name 'Central Schools'. Later, the name was changed to Kendriya Vidyalaya. Its schools are all affiliated to the Central Board of Secondary Education (CBSE). The objective of KVS is to cater to the educational needs of the children of transferable Central Government employees including Defence and Para-Military personnel by providing a common programme of education.

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Number of students	15	18	21	29	p

He was told that mean marks of a student is 53.

(i) What is the value of model marks ?

(ii) What is the value of median marks ?

Ans :

(i) Here, $l = 60$, $f_1 = 29$, $f_0 = 21$, $f_2 = 17$ and $h = 20$

$$\text{Mode, } M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$
$$= 60 + \frac{29 - 21}{2 \times 29 - 21 - 17} \times 20$$
$$= 60 + \frac{8}{58 - 38} \times 20 = 60 + 8 = 68$$

(ii) Now $3M_d = M_o + 2M = 68 + 2 \times 53$

$$M_d = \frac{174}{3} = 58$$

Hence median is 58.