CBSE Class 11 Physics
Sample Paper 04 (2020-21)

Maximum Marks: 70
Time Allowed: 3 hours

General Instructions:

i. All questions are compulsory. There are 33 questions in all.
ii. This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
iii. Section A contains ten very short answer questions and four assertion reasoning MCQs of 1 mark each, Section B has two case based questions of 4 marks each, Section C contains nine short answer questions of 2 marks each, Section D contains five short answer questions of 3 marks each and Section E contains three long answer questions of 5 marks each.
iv. There is no overall choice. However internal choice is provided. You have to attempt only one of the choices in such questions.

Section A

1. A body is acted upon by a number of external forces. Can it remain at rest?
2. Why can speed of a particle not be negative?

OR

Can a tumbling beaker that has slipped off the edge of a table considered as a point object?

3. Can the couple acting on a rigid body produce translatory motion?
4. Would we have more sugar to the kilogram at the pole or at the equator?

OR

A thief with a box in his hand jumps from the top of a building. What will be the load experienced by him during the state of free fall?

5. Oil is sprinkled on sea waves to calm them. Why?
6. Under what condition the three vectors cannot give zero resultant?
7. Why has second been defined in terms of periods of radiations from cesium -133?

OR

The mass of a body is measured by two persons 10.2 kg and 10.23 kg. Which one is more accurate and why?

8. Sound waves of wavelength λ travelling in a medium with a speed of v m/s enter into another medium where, its speed is 2v m/s. What is the wavelength of the sound wave in the second medium?

OR

The speed of sound does not depend upon its frequency. Give an example in support of this statement.

9. How do you justify that when a body is being heated at melting point, the temperature remains Constant?
10. When is the sum of two vectors:
    1. maximum?
    2. minimum?
11. **Assertion:** For projection angle $\tan^{-1}(4)$, the horizontal range and the maximum height of a projectile are equal.

    Reason: The maximum range of projectile is directly proportional to square of velocity and inversely proportional to acceleration due to gravity.
    a. Assertion and reason both are correct statements and reason is correct explanation for assertion.
    b. Assertion and reason both are correct statements but reason is not correct explanation for assertion.
    c. Assertion is correct statement but reason is wrong statement.
    d. Assertion is wrong statement but reason is correct statement.
12. **Assertion:** Spring balances show correct readings even after they had been used for a long time interval.

    Reason: On using for long time, spring balances loose its elastic strength.
    a. Assertion and reason both are correct statements and reason is correct explanation

for assertion.

b. Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c. Assertion is correct statement but reason is wrong statement.

d. Assertion is wrong statement but reason is correct statement.

13. **Assertion:** If a gas container in motion is suddenly stopped, the temperature of the gas rises.

Reason: In given process, the kinetic energy of ordered mechanical motion is converted into the kinetic energy of random motion of gas molecules.

a. Assertion and reason both are correct statements and reason is correct explanation for assertion.

b. Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c. Assertion is correct statement but reason is wrong statement.

d. Assertion is wrong statement but reason is correct statement.

14. **Assertion:** A man who falls from a height on a cement floor receives more injury than when he falls from the same height on a heap of sand.

Reason: The impulse applied by a cement floor is more than the impulse by sand floor.

a. Assertion and reason both are correct statements and reason is correct explanation for assertion.

b. Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c. Assertion is correct statement but reason is wrong statement.

d. Assertion is wrong statement but reason is correct statement.

Section B

15. **Read the case study given below and answer any four subparts:**

A perfectly elastic collision is defined as one in which there is no loss of kinetic energy in the collision. An inelastic collision is one in which part of the kinetic energy is changed to some other form of energy in the collision.

i. During inelastic collision between two bodies, which of the following quantities always remain conserved?
   a. Total kinetic energy.
   b. Total mechanical energy.
   c. Total linear momentum.
   d. none of these
ii. A ball is dropped from a height 10 m. The ball is embedded in sand 1m and stops, then
   a. momentum is conserved
   b. kinetic energy is conserved
   c. both of them conserved
   d. none of these
iii. A particle of mass m moving with velocity v strikes a stationary particle of mass 2m and sticks to it. The speed of the system will be:
   a. v/2
   b. v/3
   c. v
   d. 3v
iv. The coefficient of restitution of e for a perfectly inelastic collision is:
   a. 0
   b. 1
   c. -1
   d. infinite
v. If the momentum of an object is doubled. How does its kinetic energy change?
   a. becomes half
   b. becomes double
   c. becomes four times
   d. none of these

16. **Read the case study given below and answer any four subparts:**
The figure shows the different modes of transfer of heat, heat transfer is defined as the movement of heat across the border of the system due to a difference in temperature between the system and its surroundings. The temperature difference exists between the two systems, heat will find a way to transfer from the higher to the lower system.

i. The sea breeze is caused by:
  a. conduction
  b. convection
  c. radiation
  d. none of these

ii. At what factor heat absorbed on radiation by the body depends on?
  a. distance between body
  b. source of heat
  c. its color
  d. all of the above

iii. When heat is transferred by molecular collision, it is referred to as heat transfer by:
  a. convection
  b. conduction
  c. radiation
  d. convection and radiation

iv. Thermal conductivity of air with rise in temperature:
  a. increase
  b. decrease
  c. constant
  d. none of these

v. Mass transfer does not take place in-
  a. conduction
  b. convection
  c. radiation
  d. none of these

Section C

17. Determine the speed with which the earth would have to rotate on its axis so that a Person on the equator would weigh $\frac{3}{5}$th as much as at present. Take the equatorial

radius as 6400 km.

18. A ball is dropped on a floor from a height of 2m. After the collision, it rises up to a height of 1.5 m. Assuming that 40% of mechanical energy lost goes to thermal energy into the ball. Calculate the rise in temperature of the ball in the collision. The specific heat capacity of the ball is 800J/k. Take g = 10 m/s^2.

OR

A motorcyclist just loops a vertical loop of radius 8.0 m. What is his minimum speed at the highest and the lowest points of the loop? What is the reaction force due to the loop on the motorcyclist at the lowest point? Given that the combined mass of motorcyclist and the rider is 180 kg.

19. Particle executes S.H.M. of amplitude 25cm and time period 3s. What is the minimum time required for the particle to move between two points 12.5cm on either side of the mean position?

OR

A 40gm mass produces on extension of 4cm in a vertical spring. A mass of 200gm is suspended at its bottom and left pulling down. Calculate the frequency of its vibration.

20. Explain why a cricketer moves his hands backward while holding a catch.
21. A car travelling with a speed of 90 km/h on a straight road is ahead of a scooter travelling with a speed of 60 km/h. How would the relative velocity be altered, if scooter is ahead of the car?
22. What do you mean by interference of waves? What do you mean by constructive and destructive interference? State their conditions.
23. The vector sum of a system of non-collinear forces acting on a rigid body is given to be non-zero. If the vector sum of all the torques due to the system of forces about a certain point is found to be zero, does this mean that it is necessarily zero about any arbitrary point?
24. The resistance of a metallic wire is given by R = V/I, where V is the potential difference and I is the current. In the circuit, the potential difference across the resistance is V = (8 ± 0.5) V and current in the circuit I = (4 ± 0.2) A. What is the value of resistance with its percentage error?

OR

The radius of the atom is of the order of $2\overset{\circ}{A}$ and radius of a nucleus is of the order of a fermi. How many magnitudes higher is the volume of the atom as compared to the volume of the nucleus?

25. Derive:
    1. v = u + at
    2. $v^2 - u^2 = 2as$

    by calculus method.

Section D

26. A railway car of mass 20 tonnes moves with an initial speed of 54km/hr. On applying brakes, a constant negative acceleration of $0.3 m/s^2$ is produced.
    i. What is the breaking force acting on the car?
    ii. In what time it will stop?
    iii. What distance will be covered by the car before it finally stops?
27. The time of oscillation T of a small drop of a liquid under surface tension (whose dimensions are those of force per unit length) depends upon the density d, the radius r and the surface tension σ. Derive dimensionally the relationship T $\propto \sqrt{dr^3/\sigma}$.

OR

The frequency 'ν' of vibration of stretched string depends upon

i. its length l,
ii. its mass per unit length 'm' and
iii. the tension T in the string

Obtain dimensionally an expression for frequency ν.

28. a. Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

    $\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$

    b. Show that the projection angle θ_0 for a projectile launched from the origin is given by

    $\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$

where the symbols have their usual meaning.

OR

The greatest and the least resultant of two forces acting at a point are 29 N and 5 N, respectively. If each force is increased by 3 N. Find the resultant of two new forces acting at a right angle to each other.

29. Two bodies at different temperatures T_1 and T_2 are brought in contact. Under what condition, they settle to mean temperature? (after they attain equilibrium)

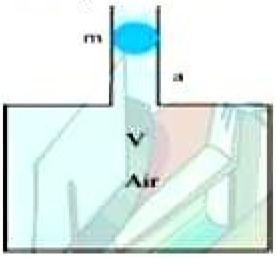
30. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^6 m; G = $6.67 \times 10^{-11} \mathrm{Nm^2kg^{-2}}$.

Section E

31. A cylindrical piece of cork of base area A, density ρ and height L floats in a liquid of density ρ_L. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically and find its time period of oscillations.

OR

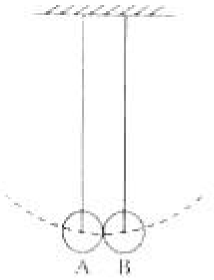
An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Figure). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal.



32. Show that total mechanical energy of a freely falling body remains constant throughout the fall.

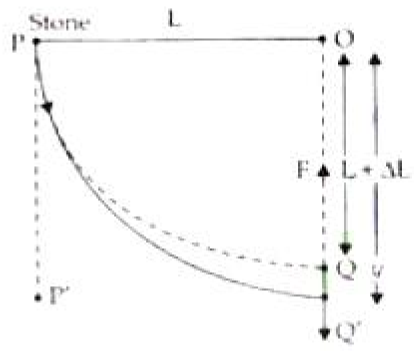
OR

Two pendulums with identical bobs and lengths are suspended from a common support such that in rest position the two bobs are in contact (figure). One of the bobs is released after being displaced by 10° so that it collides elastically head-on with the other bob.



i. Describe the motion of two bobs.
ii. Draw a graph showing variation in energy of either pendulum with time, for $0 \le 2T \le t$, where T is the period of each pendulum.

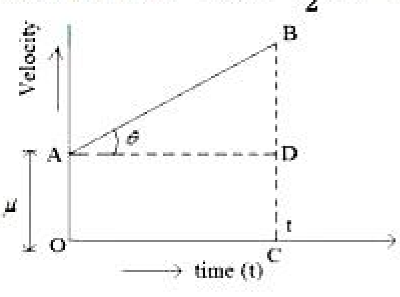
33. A stone of mass m is tied to an elastic string of negligible mass and spring constant k. The unstretched length of the string is L and has negligible mass. The other end of the string is fixed to a nail at a point P. Initially the stone is at the same level as the point P. The stone is dropped vertically from point P.



i. Find the distance y from the top when the mass comes to rest for an instant, for the first time.
ii. What is the maximum velocity attained by the stone in this drop?

OR

Establish $s = ut + \frac{1}{2}at^2$ from velocity-time graph for a uniform accelerated motion?



CBSE Class 11 Physics
Sample Paper 04 (2020-21)

Solution

Section A

1. Yes it can be at rest, only if the external forces acting on the body can be represented in magnitude and direction by the sides of a closed polygon taken in the same order. OR in simple words if the vector sum of all the forces acting on a body is zero.
2. Because speed is distance travelled per second and distance can never be negative.

   OR

   No, it cannot be. Because the size of the beaker cannot be neglected in comparison to the height of the table.

3. No. It can produce only rotatory motion.
4. $mg_p = m'g_e$ since, $g_p > g_e$, (p- pole and e- equator)
   $\therefore m' > m$. Therefore, we shall have a greater mass of sugar at the equator.

   OR

   The load experienced by him will be zero because during the state of free fall, the acceleration is equal to the acceleration due to gravity. So, the thief will be in the state of weightlessness.

5. The surface tension of sea-water without oil is greater than the oily water. Thus, on sprinkling oil on sea , the water without oil pulls the oily water against the direction of breeze, which calms down the sea waves.
6. If all the three vectors are not lying in the same plane, they cannot produce the resultant equal to zero.
7. Second has been defined in terms of periods of radiation from Cesium -133 because:
   i. this period is accurately defined.
   ii. this period is not affected by a change in physical conditions like pressure,temperature and volume etc.
   iii. it can be reproduced easily in a good laboratory.

OR

The value of mass of a body (m) = 10.23 kg is more accurate because the accuracy depends upon the number of digits after decimal(being correct to the 2nd place of decimal).

8. Frequency in the first medium, $\nu = \frac{v}{\lambda}$
We know that frequency will remains the same as before in the second medium too.
$\therefore \nu' = \nu \Rightarrow \frac{2v}{\lambda'} = \frac{v}{\lambda} \Rightarrow \lambda' = 2\lambda$ which means the wavelength will become double that of the previous one.

OR

If sounds are produced by different musical instruments simultaneously, then all these sounds are heard at the same time.

9. When a body is being heated below the melting point, the heat supplied increases the potential as well as the kinetic energy of the molecules. Due to the increase in the kinetic energy of the molecules, the temperature increases. But at melting point, heat goes, to increase only the potential energy of molecules and hence the temperature remains the same.
10. The sum of two vectors is maximum, when both the vectors are in the same direction (i.e. parallel, $\theta = 0°$) and is minimum when they act in opposite direction(when $\theta = 180°$).
We know that, $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$, from parallelogram law of vector addition.
    i. For R to be maximum,
    $\cos\theta = +1$
    i.e. $\theta = 0^\circ$
    $R_{max} = \sqrt{A^2 + B^2 + 2AB} = A + B$
    ii. For R to be minimum,
    $\cos\theta = -1$ or $\theta = 180^\circ$
    $R_{min} = \sqrt{A^2 + B^2 + 2AB(-1)} = A - B$
11. (b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
Explanation: Both assertion and reason are true but reason is not the correct explanation of assertion.

The horizontal range of projectile,

$R = \frac{u^2 \sin 2\theta}{g}$

$\therefore \quad R_{\text{max.}} = \frac{u^2}{g}$

The maximum height attained by projectile,

$H = \frac{u^2 \sin^2 \theta}{2g}$

If H = R, then $\frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin 2\theta}{g}$

or $\frac{\sin^2 \theta}{2} = 2 \sin \theta \cos \theta$

or tan θ or θ = $\tan^{-1}(4)$

12. (d) Assertion is wrong statement but reason is correct statement.

    Explanation: Assertion is false but reason is true.

    when a spring balance has been used for a long time, the spring in the balance gets fatigued and there is loss of strength of the spring. In such a case, the extension in the spring is more for a given load and hence the balance gives wrong readings.

13. (a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

    Explanation: Assertion and reason both are correct statements and reason is correct explanation for assertion.

14. (c) Assertion is correct statement but reason is wrong statement.

    Explanation: Cement floor being hard, stops the fall of them an immediately, i.e., At is very small. In case of heap of sand, momentum of all of the man is reduced to zero in a much longer time. Hence,

    $f\Delta t = F\Delta T = \Delta P$

    as $\Delta t << \Delta T$ so f >> F

    (where f and F are impulse applied by a cement floor and heap of sand.)

Section B

15. i. c
    ii. a
    iii. b
    iv. a
    v. c
16. i. (b) convection

ii. (d) all of the above
iii. (a) convection
iv. (a) increase
v. (c) radiation

Section C

17. Acceleration due to gravity at the equator is

$g_e = g - r\omega^2$

$mg_e = mg - mr\omega^2$

or $\frac{3}{5}\,mg = mg - mr\omega^2 \left[\because mg_e = \frac{3}{5}mg\right]$

$\therefore \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2\times 9.8}{5\times 6400\times 10^3}} = 7.8 \times 10^{-4} rad/s$

18. Initial height, h_1 = 2m

Final height, h_2 = 1.5m

for a body at rest, K.E = 0

Since, potential energy = mechanical energy

Then, Mechanical energy lost = $|mg(h_1 - h_2)|$

$= |1 \times 10(2 - 1.5)|$

$= |10 \times (0.5)|$

= 5 J

Now,

(mechanical energy lost) × 40% = heat gained by ball

$\frac{40}{100} \times 5 = \text{cm}\Delta T$

where c = specific heat of ball

m is the mass of ball = 1 kg

$\frac{40}{100} \times 5 = 800 \times 1 \times \Delta T$

$\Delta T = \frac{40\times 5}{100\times 800} = \frac{1}{400}$

$\frac{1}{400} = \Delta T$

$\Delta T = 2.5 \times 10^{-3}$ °C

Hence, the rise in the temperature of the ball in the collision is 2.5×10^{-3} °C.

OR

Here it is given that, r = 8.0 m and m = 180 kg.

Now we know that the Minimum speed at highest point of loop = $\sqrt{gr} = \sqrt{8.0 \times 10}$ =8.9 ms^{-1}

Also, we have Minimum speed at lowest point of loop = $\sqrt{5gr} = \sqrt{5 \times 8.0 \times 10}$= 20 ms^{-1}

and Minimum reaction force at lowest point R = 6mg = $6 \times 180 \times 10$ = 1080 N

19. Given, r = 25 cm: T = 3s: y = 12.5 cm

The displacement $y = \frac{r \sin 2\pi t}{T}$

$12.5 = 25 \sin \frac{2\pi}{3} t$ or $\frac{2\pi}{3} t = \frac{\pi}{6}$ or t = 0.25s.

The minimum time taken by the particle 2t = 0.5 s

OR

Here mg' = 40g = 40 × 980 dyne: I = 4cm.

Say K is the force constant of spring, then

mg = kl or k = mg/l

$k = \frac{40 \times 980}{4} = 9800\ dyne\ cm^{-1}$

When the spring is loaded with mass m = 200 g

$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{9800}{200}} = 1.113 s^{-1}$

20. From 2nd law of motion, we have

F = ma = $m\frac{\Delta v}{\Delta t}$

Where

'F' is the force experienced by the cricketer as he catches the ball.

'm' is the mass of the ball

'Δt' is the short time of the impact with the hand of a cricketer.

We can thus see from the equation that impact force is inversely proportional to the impact time, Thus, if the impact is for a shorter period of time then the force will be large. It also shows that the force experienced by the cricketer decreases with the increase in the impact time.

Therefore, the cricketer moves his hand backwards while taking a catch to increase the impact time, and hence decrease the impact force on his hand and prevent it from getting hurt.

21. Let v_c and v_s be the velocities of the car and the scooter respectively.

v_c = 90 km/h and v_s = 60 km/h [given]

When the car is ahead of the scooter, then the relative velocity of the car with respect to the scooter is

$v_{cs} = v_c - v_s$ = 90 - 60 = 30 km/h

When the scooter is ahead of the car, then the relative velocity of the car with respect to the scooter is

$v_{cs} = v_c - v_s$ = 90 - 60 = 30 km/h

So there is no change in the relative velocity of the car.

22. **Interference** of waves is the phenomenon of superposition of two waves having same frequency, constant phase travelling in the same direction due to which amplitude and intensity becomes maximum at some points and minimum at some points.
Constructive interference occurs when waves superimpose in the same phase i.e., the crest of one wave (in transverse waves) coincides with crest of another wave and vice-versa. As a result, the resultant amplitude and hence intensity of the resultant wave becomes maximum. 'Thus, for constructive interference, the phase difference between the superposing waves is given by $\Delta\phi$ = 0 or $2n\pi$, where n is an integer i.e., n = 1, 2, 3...
Destructive interference occurs when waves superimpose in mutually opposite phase i.e., in a superposition of two transverse waves crest of one wave exactly coincides with trough of another wave. As a result, the resultant amplitude and hence intensity of the resultant wave becomes minimum. For destructive interference, the phase difference is given by $\Delta\phi$ = (2n - 1)π , where n =1, 2, 3...

23. The vector sum of all torques due all torques due to forces at a point is zero. It does not mean that the resultant of forces are zero. E.g., Torque on sea-saw of a boy and child can be equal (can be balance). If the point of support of sea-saw changes without changes their position, the torques will not balance the sea-saw. So it is not necessary that, if the sum of all torques due to different forces at a point is zero, it will may not be zero for other arbitrary point.
$G_i \sum_{i=1}^{n} F_i \neq 0$
τ about a point P(let)
$\tau = \tau_1 + \tau_2 + \ldots + \tau_n = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i = 0$ (given)
τ about any other point $Q1$ (say) $\vec{r}_i$ will be different forces
$\sum_{i=1}^{n} (\vec{r}_i - a) \times F_i = \sum_{i=1}^{n} \vec{r} \times F_i - a \sum_{i=1}^{n} F_1$
As a and $\sum F_i$ are not zero. So sum of all the torques about any arbitrary point need not

be zero necessarily.

24. It is given that, V = (8 ± 0.5) V and I = (4 ± 0.2) A

Now we know that Resistance, R = $\frac{V}{I}=\frac{8}{4}=2\Omega$

Now, relative errors in V and I are given by:

$\frac{\Delta V}{V}=\pm\frac{0.5}{8}=\pm\left(\frac{0.5}{8}\times 100\right)\%=\pm 6.25\%$

and $\frac{\Delta I}{I}=\pm\frac{0.2}{4}$

$=\pm\frac{0.2\times 100}{4}\%=5\%$

Also, $\left(\frac{\Delta R}{R}\right)_{\max}=\frac{\Delta V}{V}+\frac{\Delta I}{I}$

= 6.25 + 5 = 11.25%

Therefore, experimentally determined value of resistance is R = (2 +11.25%) Ω.

OR

Given that the Radius of an atom, (R_{Atom}) is 2$\mathring{A}$ = 2 $\times$ 10^{-10} m and the radius of a nucleus, ($R_{Nucleus}$) is 1 fermi = 10^{-15} m

Volume of an atom, V_{Atom} = $\frac{4}{3}\pi R^3_{Atom}$

Volume of a nucleus, $V_{Nucleus}$ = $\frac{4}{3}\pi R^3_{Nucleus}$

$\frac{V_{Atom}}{V_{Nuclues}}=\frac{\frac{4}{3}\pi R^3_{Atom}}{\frac{4}{3}\pi R^3_{Nucleus}}=\left[\frac{R_{Atom}}{R_{Nucleus}}\right]^3=\left[\frac{2\times 10^{-10}}{10^{-15}}\right]^3$ = 8 $\times 10^{15}$.

Volume of an atom is roughly 10^{16} times more than that of nucleus, therefore most of the atom is free (empty space) and nucleus is heavy and dense as per Rutherford's observation.

25. i. We know that, $a=\frac{dv}{dt}$

adt = dv

Integrating we get

$\int_0^t adt=\int_u^v dv$

at = v - u

v = u+at

ii. $a=\frac{dv}{dt}$

Multiply and Divide by dx, we get

$a = \frac{dv}{dt} \times \frac{dx}{dx}$
$a = \frac{dv}{dx} \times v$
$adx = vdv \left(\because \frac{dx}{dt} = v\right)$
Integrating within the limits, we have
$a\int_{0}^{s} dx = \int_{u}^{v} vdv$
$as = \frac{v^2}{2} - \frac{\nu^2}{2}$
$as = \frac{v^2 - \nu^2}{2}$
$v^2 - u^2 = 2as$

Section D

26. Mass of the railway car,m = 20 tonnes = 20 × 1000 kg=20×10^4 kg , Initial speed ,u = 54 km/hr = $54 \times \frac{5}{18}$=15 m/s

Negative acceleration,a = -0.3 m/s^2

a. Breaking force acting on the car ,F = -ma

$F = -(2 \times 10^4 kg) \times (-0.3 ms^{-2})$

$F = 6000N$

b. When the railway car stops, its final velocity is zero.

i.e. $v = 0$

Using the relation: $v - u = at$

$\Rightarrow\ 0 = 15 + (-0.3)t$

$\Rightarrow t = 50s$

c. Using the relation : $v^2 - u^2 = 2as$

$\Rightarrow\ 0 - (15)^2 = 2(-0.3)s$

$\Rightarrow\ s = 375m$

27. Let the time of oscillation of a small drop of a liquid is given by,

$T \propto d^a r^b d^c$

T = kd^{2}r^{2}d^2...........(i)

where, k is a dimensionless constant

Dimension of the given physical quantities are:

Surface tension, σ = [MT^{-2}], density, d = [ML^{-3}] and radius r = [L]

Substituting these dimensions in eq(i), we get

$[M^0L^0T^1] = k[ML^{-3}]^a[L]^b[MT^{-2}]^c$

Expanding the above equation we get

$[M^0L^0T^1] = [M^{a+c} L^{-3a + b}T^{-2c}]$

Equate the dimensions of M, L and T, we get

a + c = 0, -2c = 1, -3a + b = 0

Solving these, we get, c = -1/2, a = 1/2 and b = 3/2. Putting these values in equation (i) we get

$T \propto d^{1/2} r^{3/2} \sigma^{-1/2}$

i.e, $T \propto \sqrt{\frac{dr^3}{\sigma}}$

OR

Let the frequency of vibration of the string be given by

$\nu = Kl^a m^b T^c$ .... (i)

where K is a dimensionless constant

Dimensions of the given quantities are

$\nu = [T^{-1}]$, l = [L], T = [T], m = $[ML^{-1}$ and tension(T) = $[MLT^{-2}]$

Substituting these dimensions in equation (i), we get

$[T^{-1}] = [L]^a[ML^{-1}]^b[MLT^{-2}]^c$

Expanding the above equation we get

$[M^0L^0T^{-1}] = [M^{b+c}L^{a-b+c}T^{-2c}]$

Equate dimensions of M, L and T on both sides of equation (homogeneity rule), we get

b + c = 0 ,a -b + c =0 and - 2c = -1

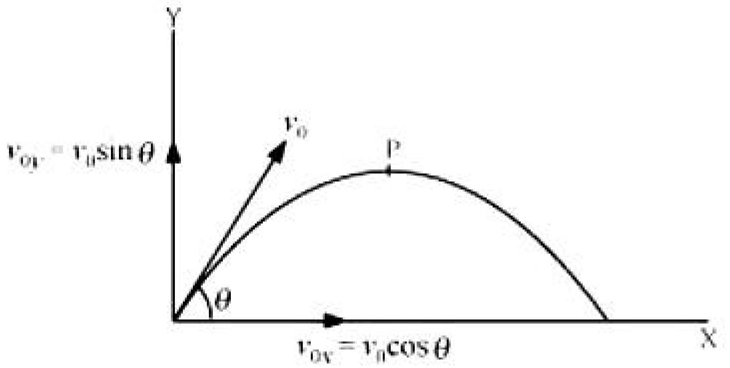
on solving, a = -1, $b = -\frac{1}{2}$ and $c = \frac{1}{2}$. Put these values in equation (i) we get

$\nu = Kl^{-1}m^{-1/2}T^{1/2}$

or, $\nu = \frac{K}{l}\sqrt{\frac{T}{m}}$

28. a. Let v_{Ox} and v_{Oy} respectively be the initial components of the velocity of the projectile along horizontal (x) and vertical (y) directions.

Let v_x and v_y respectively be the horizontal and vertical components of velocity at a point P.



Let 't' be the time taken by the projectile to reach point P.

Applying the first equation of motion: v = u + at along the vertical and horizontal direction,

we get

$v_y = v_{oy} - gt$ and $v_x = v_{ox}$

$\therefore\ \tan\theta = \frac{v_y}{v_x} = \frac{v_{oy} - gt}{v_x}$

$\theta = \tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$

b. Maximum vertical height, $h_m = \frac{u_0^2 \sin^2(\theta)}{2g}$ .....(i)

Horizontal range, $R = \frac{u_0^2 \sin(2\theta)}{g}$ .....(ii)

Dividing equation (i) and (ii), we get

$\frac{h_m}{R} = \frac{\sin^2\theta}{2\sin 2\theta}$

$\frac{h_m}{R} = \frac{\sin\theta \times \sin\theta}{2 \times (2\ \sin\theta\ \cos\theta)}$

$\frac{h_m}{R} = \frac{1}{4}\frac{\sin\theta}{\cos\theta} = \frac{1}{4}\tan\theta$

$\tan\theta = \left(\frac{4h_m}{R}\right)$

$\theta = \tan^{-1}\left(\frac{4h_m}{R}\right)$

OR

Let P and Q be the two forces

We know that resultant of two vectors has maximum magnitude for the sum of vectors and minimum magnitude for the difference of vectors

Greatest resultant, R_1 = P + Q = 29 N

Least resultant, R_2 = P - Q = 5 N

P + Q = 29

P - Q = 5

Solving Eqs.(i) and (ii), we get

P = 17 N, Q = 12 N

When each force is increased by 3 N, then

P' = P + 3 = 17 + 3 = 20 N

Q' = 12 + 3 = 15 N

The resultant of new force,

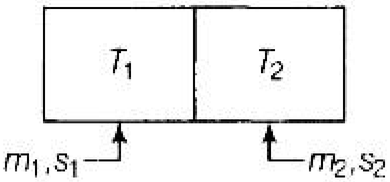
$R' = \sqrt{(20)^2 + (15)^2} = \sqrt{400 + 225}$ = 25 N

Let the resultant r, makes an angle θ with P', then,

$\tan\theta = \frac{15}{20} = 0.75$

$\Rightarrow \quad \theta = \tan^{-1}(0.75) = 36°52'$

Thus, the resultant of two new forces P' and Q' is 25N and angle made by the resultant with P' is 36°52'.

29. 

Let m_1 *and* m_2 are masses of bodies with specific heats s_1 *and* s_2, then if their temperature after they are in thermal equilibrium is T.

Then, if $T_1 > T > T_2$ and assuming no heat loss.

Heat lost by hot body = heat gained by cold body

$m_1 s_1 (T_1 - T) = m_2 s_2 (T - T_2)$

$\Rightarrow \frac{m_1 s_1 T_1 + m_2 s_2 T_2}{m_1 s_1 + m_2 s_2} = T$[equilibrium temperature]

So for, bodies to settle down to mean temperature,

$m_1 = m_2$ and $s_1 = s_2$ means bodies have same specific heat and have equal masses.

Then,

$T = \frac{T_1 + T_2}{2}$[mean temperature]

30. $Me = 6.0 \times 10^{24}\text{kg}$

Mass of the satellite, m = 200 kg

$Re = 6.4 \times 10^6 m$

$G = 6.67 \times 10^{-11}\text{Nm}^2\text{kg}^{-2}$

Height of the satellite, h = 400 km $= 4 \times 10^5 m = 0.4 \times 10^6 m$

Total energy of the satellite at height h =Kinectic energy + Potential energy

Total energy of the satellite at height h= $\frac{1}{2}mv^2 + \left(\frac{-GM_e m}{R_e + h}\right)$

Orbital velocity of the satellite, $v=\sqrt{\frac{GM_e}{R_x+h}}$

Total energy= $\frac{1}{2}m\left(\frac{GM_e}{R_e+h}\right)-\frac{GM_em}{R_e+h}=-\frac{1}{2}\left(\frac{GM_em}{R_e+h}\right)$

Energy required to send the satellite out of its orbit = -(Bound energy)

$=\frac{1}{2}\frac{GM_em}{(R_e+h)}$

$=\frac{1}{2}\times\frac{6.67\times10^{-11}\times6.0\times10^{24}\times200}{(6.4\times10^6+0.4\times10^6)}$

$=\frac{1}{2}\times\frac{6.67\times10^{-11}\times12\times10^{26}}{(6.8\times10^6)}$

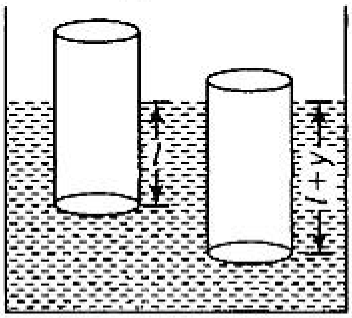
$=\frac{1}{2}\times\frac{6.67\times12\times10^9}{6.8\times10^6}=5.9\times10^9J$

Section E

31. Consider a cylinder of mass m, length L, density of material ρ and uniform area of cross-section A.

Therefore, mass of the cylinder(m) = A Lρ

Let the cylinder is floating in the liquid of density ρ_1



In equilibrium, let l be the length of cylinder dipping in liquid.

In equilibrium, weight of cylinder = Weight of liquid displaced

$\Rightarrow$ mg = A l ρ_1 g

$\Rightarrow$ m = A lρ_1 .... (ii)

Now say the cylinder is pushed down by y into the liquid, then

Total upward thrust, F_2 = A (l + y) ρ_1 g (since effective depth = l+y)

Restoring force, F = - (F_2 - mg)

$\Rightarrow$F = -[A(l + y)ρ_1g - Alρ_1 g] = -A$\rho_1$gy .... (iii)

We know that In SHM, F $\propto$ - y

$\Rightarrow$ F = - k y .... (iv)

Comparing equation (iii) with equation (iv) we get,

Spring factor, k = Aρ_1g

Inertia factor = mass of the cylinder(m) = ALρ

Now, we know the formula of time period, T = $2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$

Hence, T = $2\pi\sqrt{\frac{AL\rho}{A\rho_1 g}} = 2\pi\sqrt{\frac{L\rho}{\rho_1 g}}$ .... (v)

Using, m = Alρ_1 = ALρ

So, $l\rho_1 = L\rho$

Using the above value we get time period,

T = $2\pi\sqrt{\frac{l\rho_1}{g\rho_1}} = 2\pi\sqrt{\frac{l}{g}}$

OR

Given

⇒ Volume of the air chamber = V

⇒ Area of cross-section of the neck = a

⇒ Mass of the ball = m

The pressure inside the chamber is equal to the atmospheric pressure.

Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber, ΔV = ax

⇒ Volumetric strain $= \frac{\text{Change in volume}}{\text{Original valume}}$

⇒ $\frac{\Delta V}{V} = \frac{ax}{V}$

⇒ Bulk Modulus of air, $B = \frac{\text{Stress}}{\text{Strain}} = \frac{-p}{\frac{ax}{V}}$

In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.

⇒ $p = \frac{-Bax}{V}$

⇒ The restoring force acting on the ball, F = p × a

$= \frac{-Ba^2x}{V}$

In simple harmonic motion, the equation for restoring force is:

⇒ F = -kx ... (ii)

Where k is the spring constant

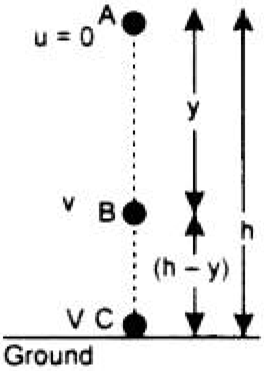
Comparing equations (i) and (ii), we get:

$= \frac{Ba^2}{V}$

⇒ Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{Vm}{Ba^2}}$$

32. As per principle of conservation of mechanical energy if there is no dissipation of mechanical energy, then total mechanical energy of a system must remain constant. Of course, potential energy may be converted into kinetic energy or vice versa but sum of the kinetic energy and potential energy of a system must remain unchanged.



Now consider a small sized object of mass m falling from a point A situated at a height h.

At point A: Since the object is at rest i.e., u = 0

∴K.E. of object at point A, $K_A = 0$

and P.E. of object $U_A = mgh$

$E_A = K_A + U_A = 0 + mgh = mgh$ .... (i)

At point B: Let the objects fall freely through a distance y so as to reach B, where its velocity v is given by

$v = \sqrt{2gy}$

∴ KE at point B

$K_B = \frac{1}{2}mv^2 = \frac{1}{2}m \cdot 2gy$ = mgy

and PE at point B, $U_B = mg(h - y)$

∴ $E_B = K_B + U_B = mgy + mg(h - y) = mgh$ ....` (ii)

At point C: Let the object just reaches the ground at point C, where its final velocity V is given by :

$V = \sqrt{2gh}$

∴ K.E at point C, $K_C = \frac{1}{2}mV^2 = \frac{1}{2}m \cdot 2gh = mgh$

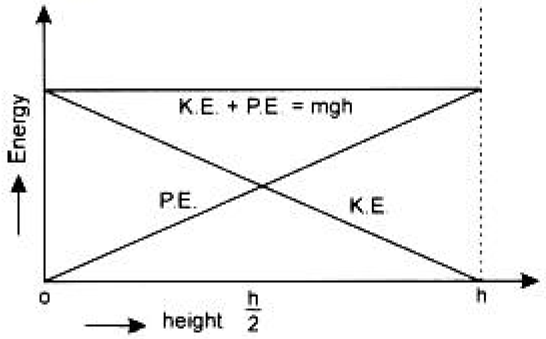
and P.E at point C, $U_C = 0$

∴ $E_C = K_C + U_C = mgh = 0 = mgh$

A simple comparison shows that

$E_A = E_B = E_C = mgh$ = a constant

A graph showing variation of K.E., P.E. and total energy of a freely falling body is shown below:



OR

i. Let at t = 0 A is at lowest position and B is at its height position at 10°.

$PE_A = 0 \quad PE_B = E, \quad KE_A = KE_B = 0$

K.E. of both are zero. Now bob is released. $t = \frac{T}{4} B$ reaches to A and collide elastically as both bobs are identical then

$KE_A = 0 \quad KE_B = E, \quad PE_A = 0$

$PE_B = 0$ At $t = \frac{2T}{4}$

A reaches at it's maximum height and B remains at it's lowest position.

$KE_A = 0 \quad KE_B = 0, \quad PE_A = E, \quad PE_B = 0$

At $t = \frac{3T}{4}$ Bob A hits the bob B which was at rest elastically and ball A becomes at rest, B moves upward.

$KE_A = 0$, $KE_B = E$, $PE_A = 0$, $PE_B = 0$ $E_A = 0 \quad E_B = E$ At $t = \frac{4T}{4}$

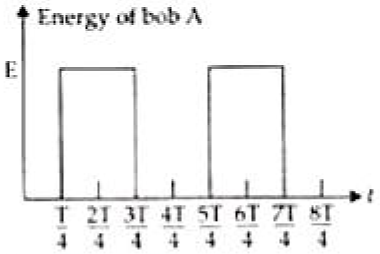
Bob B is at its maximum height and A is at lower height.

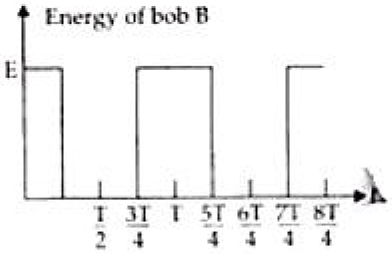
So, $KE_A = 0$, $KE_B = 0$, $PE_A = 0$, $PE_B = E$

The entire process is repeated.

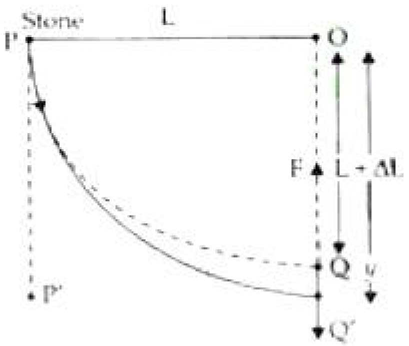
Time t	KEA	PEA	EA	KEB	PEB	EB
0	0	0	0	0	E	E
$\frac{T}{4}$	E	0	E	0	0	0
$\frac{2T}{4}$	0	0	0	0	E	E
$\frac{3T}{4}$	E	0	E	0	0	0
$\frac{4T}{4}$ =T	0	0	0	0	E	E

ii.





33. i. A stone is tied at P with string of length L. String is fixed with nail at 'O'. Stone is lifted upto height L, so that string is stretched as shown in given fig.



When stone fall under gravity. It tries to follow path PP' but due to elastic string it will go a part of circular path P to Q. This is like a centrifugal force that stretches the string outward and increases its length (ΔL). So the change in Potential energy of stone at Q' and p converts into mechanical energy in string of spring constant K.

So P.E of stone = mechanical Energy of string

$$mgy = \frac{1}{2}K(y-L)^2$$
$$mgy = \frac{1}{2}K\left(y^2 + L^2 - 2yL\right)$$

$2mgy = K\left[y^2 + L^2 - 2yL\right]$
$2mgh = Ky^2 - 2KyL + KL^2$
$Ky^2 - 2KyL - 2mgy + KL^2 = 0$
$Ky^2 - 2(KL + mg)y + KL^2 = 0$

Solving this equation by quadratic formula we get,

$D = b^2 - 4ac \quad (a = K \quad b = -2(KL + mg) \quad c = KL^2)$
$D = [-2(KL + mg)]^2 - 4(K)(KL)^2$
$D = +4\left[(KL)^2 + (mg)^2 + 2(KL)(mg)\right] - 4K^2L^2$
$D = 4\left[K^2L^2 + m^2g^2 + 2KLmg\right] - 4k^2L^2$
$= 4K^2L^2 + 4m^2g^2 + 8KLmg - 4K^2L^2$
$\sqrt{D} = \sqrt{4mg[mg + 2KI} = 2\sqrt{mg(mg + 2KL)}$
$y = \frac{-b \pm \sqrt{D}}{2a} = \frac{+2(KL + mg \pm 2\sqrt{mg(2KL + mg)}}{2K}$
$y = \frac{2[(KL + mg)] \pm \sqrt{mg(2KL + mg)]}}{2k} y = \frac{(KL + mg) \pm \sqrt{mg(2KL + mg)}}{K}$

ii. At maximum velocity at its lowest point acceleration is zero

$\therefore F = 0$

So the spring or string force $Kx$is balanced by gravitational force mg. So, these two forces will be equal and opposite

$\therefore mg = kx$ ... I where x is extension in string

Let v be the maximum velocity of stone at bottom of journey.

By law of conservation of energy

KE of stone + PE gain by string = P. E lost stone from p to Q'

$\frac{1}{2}mv^2 + \frac{1}{2}Kx^2 = mg(L + x)$
$mv^2 + Kx^2 = 2mg(L + x)$
$mv^2 = 2mgL + 2mgx - Kx^2$
$mg = Kx(fromI) \Rightarrow x = \frac{mg}{K}$
$\therefore mv^2 = 2mgL + 2mg \cdot \frac{mg}{k} - K\frac{m^2g^2}{K^2}$
$= 2mgL + \frac{2m^2g^2}{K} - \frac{m^2g^2}{K}$
$mv^2 = m\left[2gL + \frac{mg^2}{K}\right]$
$\therefore v = \left[2gL + \frac{mg^2}{K}\right]^{1/2}$

This will be the velocity.

OR

From the velocity-time graph, OA = u = initial velocity of the particle, acceleration, a = slope of the graph = tan θ = $\frac{DB}{AD}$, time taken = OC = AD = t, final velocity, v = BC

Displacement of the particle in time (t)

S = area under $v-t$ graph

S = area trapezium OABC

S = area of rectangle AODC + area of triangle ADB

$$\Rightarrow S=(OA\times OC)+\frac{1}{2}(AD\times BD)$$
$$\Rightarrow S=ut+\frac{1}{2}(AD)\times\left(\frac{AD\times DB}{AD}\right)$$
$$\Rightarrow S=ut+\frac{1}{2}(AD)^2\times\left(\frac{DB}{AD}\right)$$
$$\Rightarrow S=ut+\frac{1}{2}(t)^2\times\left(\frac{DB}{AD}\right)$$
$$\Rightarrow S=ut+\frac{1}{2}(t)^2\times(a)$$
$$\left[\because a=\tan\theta=\frac{BD}{AD}\right]$$
$$\therefore S=ut+\frac{1}{2}at^2$$