In each of exercises 1 to 10 show that given differential eqn. is homogeneous and solve each of them.

Q No. 1 $(x^2+xy)\,dy = (x^2+y^2)\,dx$ ....(1)

Sol.

$$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$$

$$= \frac{1+y^2/x^2}{1+y/x} = \frac{1+(y/x)^2}{1+y/x} = F(y/x)$$

$\therefore$ (1) is homogenous differential equation.

Put $y = vx$.

$$\frac{dy}{dx} = v + x\frac{dv}{dx}.$$

$\therefore$ (1) becomes.

$$v + x\frac{dv}{dx} = \frac{1+v^2}{1+v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v = \frac{1+v^2-v-v^2}{1+v} = \frac{1-v}{1+v} = -\frac{v-1}{v+1}.$$

$$\Rightarrow \frac{1+v}{v-1}\,dv = -\frac{dx}{x}$$

$$\Rightarrow \frac{v-1+1+1}{v-1}\,dv = -\frac{dx}{x}$$

$$\Rightarrow \left(1+\frac{2}{v-1}\right)dv = -\frac{dx}{x}.$$

$$\Rightarrow \quad \int 1\,dv + \int \frac{2}{v-1}\,dv = -\int \frac{1}{x}\,dx$$

$$\Rightarrow \quad v + 2\log|v-1| = -\log|x| + c$$

$$\Rightarrow \quad v + \log(v-1)^2 + \log|x| = c$$

$$\Rightarrow \quad v + \log|x(v-1)^2| = c$$

$$\Rightarrow \quad \log|x(v-1)^2| = c - v$$

$$\Rightarrow \left| x\left(\frac{y}{x}-1\right)^2 \right| = e^{-v+c}$$

$$\Rightarrow x\frac{(y-x)^2}{x^2} = \pm e^{-v+c}$$

$$\Rightarrow (y-x)^2 = \pm x e^{-y/x}\cdot e^{c}$$

$$\Rightarrow (y-x)^2 = x e^{-y/x}\cdot c'$$ where $c' = \pm e^{c}$ is an arbitrary const.

Q No. 2 $y' = \frac{x+y}{x}$

Sol. Given differential eqn is $y' = \frac{x+y}{x}$ or $\frac{dy}{dx} = 1 + y/x = F(y/x)$ ...(1)

Which is homogeneous differential eqn.

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\therefore$ (1) becomes

$$v + x\frac{dv}{dx} = 1+v$$

$$\Rightarrow x\frac{dv}{dx} = 1+v-v$$

$$\Rightarrow dv = \frac{1}{x}dx$$

$$\Rightarrow \int dv = -\int \frac{1}{x}dx \Rightarrow v = \log|x| + c$$

or $\frac{y}{x} = \log|x| + c$

or $y = x(\log|x| + c)$

Q No. 3 $(x-y)\,dy - (x+y)\,dx = 0$

Sol. Given diff. eqn. is $(x-y)\,dy - (x+y)\,dx = 0$

or $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+y/x}{1-y/x} = F(y/x)$

Which is homogeneous differential eqn.

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}.$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{1+v}{1-v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v} = \frac{1+v^2}{1-v}$$

$$\Rightarrow \quad \frac{1-v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

$$\Rightarrow \quad \left[\frac{1}{1+v^2} - \frac{2v}{2(1+v^2)}\right]dv = \frac{1}{x}\,dx$$

$$\Rightarrow \quad \int\frac{1}{1+v^2}\,dv - \frac{1}{2}\int\frac{2v}{1+v^2}\,dv = \int\frac{1}{x}\,dx$$

$$\Rightarrow \quad \tan^{-1}v - \frac{1}{2}\log|1+v^2| = \log|x| + c$$

$$\Rightarrow \quad 2\tan^{-1}v - \log(1+v^2) - 2\log|x| = 2c$$

or $\quad 2\tan^{-1}v - \log\left((1+v^2)x^2\right) = 2c$

or $\quad 2\tan^{-1}\frac{y}{x} - \log\left(\left(1+\frac{y^2}{x^2}\right)x^2\right) = c'$ where $c' = 2c$ is arbit const.

or $\quad 2\tan^{-1}\frac{y}{x} - \log(x^2+y^2) = c'$

QNo 4 $\quad (x^2-y^2)\,dx + 2xy\,dy = 0$

Soln. Given diff. eqn is $(x^2-y^2)\,dx + 2xy\,dy = 0$

or $\quad \frac{dy}{dx} = \frac{y^2-x^2}{2xy} = \frac{\left(\frac{y}{x}\right)^2 - 1}{2\cdot y/x}$ which is homo. ...(1)

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}.$$

$\therefore$ (1) becomes

$$v + x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v^2-1}{2v} - v = \frac{v^2-1-2v^2}{2v} = \frac{-1-v^2}{2v} = -\frac{v^2+1}{2v}$$

$$\Rightarrow \quad -\frac{dx}{x} = \frac{2v}{1+v^2}$$

$$\therefore \quad -\int\frac{1}{x}\,dx = \int\frac{2v}{1+v^2}\,dv$$

$$\Rightarrow -\log|x| = \log(1+v^2) + \log c$$

$$\Rightarrow \log[1+v^2] = -(\log|x| + \log c) = -\log c|x|$$

$$\Rightarrow 1+v^2 = c|x|^{-1}$$

$$\Rightarrow 1+\frac{y^2}{x^2} = \pm\frac{1}{x}\cdot c = \frac{1}{x}(\pm c) = \frac{1}{x}c'$$

$$\Rightarrow x^2+y^2 = c'x$$

Q No. 5 $\quad x^2\frac{dy}{dx} = x^2-2y^2+xy.$

Sol. Given diff. eqn is. $x^2\frac{dy}{dx} = x^2-2y^2+xy$

$$\frac{dy}{dx} = \frac{x^2-2y^2+xy}{x^2} = 1-2(y/x)^2+(y/x) \quad \cdots (1)$$

Which is homo. diff. eqn.

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}.$$

(1) becomes

$$v + x\frac{dv}{dx} = 1-2v^2+v$$

$$\Rightarrow x\frac{dv}{dx} = 1-2v^2.$$

$$\Rightarrow \frac{dv}{1-2v^2} = \frac{1}{x}dx$$

$$\Rightarrow \int\frac{1}{1-2v^2}\cdot dx = \int\frac{1}{x}dx.$$

$$\Rightarrow \frac{1}{2}\int\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2-v^2}+c = \log|x|$$

$$\Rightarrow \log|x| = \frac{1}{2}\cdot\frac{1}{2\cdot\left(\frac{1}{\sqrt{2}}\right)}\log\left|\frac{\frac{1}{\sqrt{2}}+v}{\frac{1}{\sqrt{2}}-v}\right|+c.$$

$$\Rightarrow \log|x| = \frac{1}{2\sqrt{2}}\log\left|\frac{1+\sqrt{2}v}{1-\sqrt{2}v}\right|+c$$

$$\Rightarrow \log|x| = \frac{1}{2\sqrt{2}} \log\left|\frac{1+\sqrt{2}\, y/x}{1-\sqrt{2}\, y/x}\right| + c$$

$$\Rightarrow \log|x| = \frac{1}{2\sqrt{2}} \log\left|\frac{x+\sqrt{2}y}{x-\sqrt{2}y}\right| + c$$

Q No 6. $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

Sol : Given diff. eqn is $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$.

or $\frac{dy}{dx} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2} = f(y/x)$ ....(1)

Which is homo. diff eqn.

Put $y = vx$.

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes.

$$v + x\frac{dv}{dx} = v + \sqrt{1+v^2}.$$

or $x\frac{dv}{dx} = \sqrt{1+v^2}.$

or $\frac{dv}{\sqrt{1+v^2}} = \frac{1}{x}\,dx.$

$$\Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{1}{x}\,dx.$$

$$\Rightarrow \log\left|v+\sqrt{1+v^2}\right| = \log|x| + C$$

$$\Rightarrow \log\left(\frac{y}{x} + \sqrt{1+y^2/x^2}\right) = \log x + \log A \;;\; A>0$$

$$\Rightarrow \frac{y}{x} + \sqrt{\frac{x^2+y^2}{x^2}} = Ax \Rightarrow y + \sqrt{x^2+y^2} = Ax^2.$$

where $A \neq 0$ is any arbitrary constant.

Q No. 7. $\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\} y\,dx = \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\} x\,dy.$

Sol : On dividing both sides of given diff eqn by x^2, we get

$$\frac{dy}{dx} = \frac{\frac{y}{x}\left\{\cos\frac{y}{x} + \frac{y}{x}\sin\frac{y}{x}\right\}}{\frac{y}{x}\sin\frac{y}{x} - \cos\frac{y}{x}} = f(y/x) \quad \cdots\cdots (1)$$

Which is homo. diff. eqn.

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ (1) becomes

$$v + x\frac{dv}{dx} = \frac{v(\cos v + v\sin v)}{v\sin v - \cos v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v\cos v + v^2\sin v}{v\sin v - \cos v} - v = \frac{v\cos v + v^2\sin v - v^2\sin v + v\cos v}{v\sin v - \cos v}$$

$$x\frac{dv}{dx} = \frac{2v\cos v}{v\sin v - \cos v}$$

$$\Rightarrow \left(\frac{v\sin v - \cos v}{v\cos v}\right)dv = \frac{2}{x}\,dx$$

$$\Rightarrow \left(\tan v - \frac{1}{v}\right)dv = \frac{2}{x}\,dx$$

$$\Rightarrow \int\left(\tan v - \frac{1}{v}\right)dv = 2\int\frac{1}{x}\,dx$$

$$\Rightarrow -\log|\cos v| - \log|v| = 2\log|x| + c$$

$$\Rightarrow \log|v\cos v| + 2\log|x| = -c$$

$$\Rightarrow |v\cos v|\,|x|^2 = e^{-c}$$

$$\Rightarrow x^2 v\cos v = \pm e^{-c} = c' \text{ (say)}$$

$$\Rightarrow x^2 \cdot \frac{y}{x} \cdot \cos\frac{y}{x} = c'$$

$$\Rightarrow xy \cdot \cos y/x = c'$$

which is sequired general solution.

Q No. 8 $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$

Sol ∴ given diff eqn is $x\frac{dy}{dx} - y + x\sin\frac{y}{x} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sin\frac{y}{x} = F\left(\frac{y}{x}\right) \quad \ldots(1)$$ ∴ homogeneous diff. eqn.

Put $y = vx$.

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}.$$

∴ (1) becomes $v + x\frac{dv}{dx} = v - \sin v.$

$$\Rightarrow \operatorname{cosec} v \, dv = -\frac{dx}{x}$$

$$\Rightarrow \int \operatorname{cosec} v \, dv = -\int \frac{dx}{x}$$

$$\Rightarrow \log|\operatorname{cosec} v - \cot v| = -\log|x| + c$$

or $\log|x(\operatorname{cosec} v - \cot v)| = c$

or $|x(\operatorname{cosec} v - \cot v)| = e^c$

or $x(\operatorname{cosec} v - \cot v) = \pm e^c = c'$ (say)

$\Rightarrow x\left(\operatorname{cosec}\frac{y}{x} - \cot\frac{y}{x}\right) = c'$ which is reqd. general sol..

Q No. 9. $y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$

Sol. given diff. eqn can be written as.

$$\frac{y}{x} + \left[\log\left(\frac{y}{x}\right) - 2\right]\frac{dy}{dx} = 0 \qquad [\text{Dividing by } x\,dx]$$

or $$\frac{dy}{dx} = \frac{\frac{y}{x}}{2 - \log\frac{y}{x}} \quad \ldots(1)$$

Put $y = vx. \Rightarrow \frac{y}{x} = v.$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}.$$

∴ (1) becomes

$$v + x\frac{dv}{dx} = \frac{v}{2 - \log v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{2 - \log v} - v = \frac{v - 2v + v\log v}{2 - \log v} = \frac{v[\log v - 1]}{2 - \log v}.$$

$$\Rightarrow \frac{2-\log v}{v(\log v-1)}\,dv = \frac{dx}{x}$$

$$\Rightarrow \int \frac{2-\log v}{v(\log v-1)}\,dv = \int \frac{dx}{x} \quad \cdots\cdots (2)$$

Now for $\int \frac{2-\log v}{v(\log v-1)}\,dv \quad \cdots\cdots (3)$

Put $\log v = t$

$$\frac{1}{v}\,dv = dt$$

$$\therefore \int \frac{2-t}{t-1}\,.\,dt = \int \left(-1+\frac{1}{t-1}\right) dt = -t+\log|t-1|$$

$\therefore$ (2) becomes

$$-t+\log|t-1| = \log|x|+c$$

$$\Rightarrow -\log v + \log\left[|\log v|-1|\right] - \log|x| = c$$

$$\Rightarrow \log\left|\frac{|\log(v)-1|}{|x|v}\right| = c$$

$$\Rightarrow \frac{\log v-1}{x} = \pm v e^{c} = c'v \quad \text{where } c' = \pm e^{c} \neq 0$$

$$\Rightarrow \frac{\log(y/x)-1}{x} = c'\,y/x \qquad [\because v = y/x]$$

$$\Rightarrow \log|y/x)-1 = c'y$$

QNo 10. $(1+e^{x/y})\,dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$

Sol $\Rightarrow \frac{dx}{dy} = \frac{e^{x/y}\left(1-\frac{x}{y}\right)}{e^{x/y}+1} \quad \cdots\cdots (1)$

Since RHS of (1) is of form $\phi(x/y)$ which is homogeneous. diff. eqn.

Put $\frac{x}{y} = v$ i.e. $x = vy$

$$\therefore \frac{dx}{dy} = v + y\frac{dv}{dy}.$$

$\therefore$ (1) becomes $v + y\frac{dv}{dy} = \frac{e^v(v-1)}{e^v+1}$

$$\Rightarrow y\frac{dv}{dy} = \frac{e^v(v-1)}{e^v+1} - v = \frac{ve^v - e^v - ve^v - v}{e^v+1}$$

$$\Rightarrow \frac{e^v+1}{v+e^v}\,dv = -\frac{dy}{y}.$$

$$\Rightarrow \int\left(\frac{e^v+1}{v+e^v}\right)dv = -\int\frac{1}{y}\,dy.$$

$$\Rightarrow \log|v+e^v| = -\log|y| + c$$

$$\Rightarrow \log|y(v+e^v)| = c$$

$$\Rightarrow |y(v+e^v)| = e^c \Rightarrow y(e^v+v) = \pm e^c = c'(\text{say})$$

$$\Rightarrow y\left(e^{x/y} + \frac{x}{y}\right) = c'$$

or $ye^{x/y} + x = c'$

Which is required soln. of given diff. eqn.

For each of diff. eqn in excercise from 11 to 15 find the particular soln. satisfying the given condition:

11 Q.No 11. $(x+y)\,dy + (x-y)\,dx = 0$; $y = 1$ when $x = 1$

Sol. Given diff eqn is

$$(x+y)\,dy + (x-y)\,dx = 0$$

$$\text{or } (x+y)\,dy = -(x-y)\,dx$$

$$\text{or } \frac{dy}{dx} = -\frac{(x-y)}{(x+y)} = -\left(\frac{1-y/x}{1+y/x}\right) = f(y/x) \quad \text{--(1)}$$

which is homogeneous diff. eqn.

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}.$$

$\therefore$ (1) becomes $v + x\frac{dv}{dx} = -\left(\frac{1-v}{1+v}\right)$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1-v}{1+v} - v.$$

or $\frac{xdv}{dx} = \frac{-1+v-v-v^2}{1+v} = -\frac{1+v^2}{1+v}.$

$$\Rightarrow \frac{1+v}{1+v^2}dv = -\frac{1}{x}dx$$

Integrating both sides

$$\int \frac{1+v}{1+v^2}dv = -\int \frac{1}{x}dx$$

ie $\int \frac{1}{1+v^2}dv + \frac{1}{2}\int \frac{2v}{1+v^2}dv = -\log|x| + C$

$$\tan^{-1}v + \frac{1}{2}\log|1+v^2| = -\log|x| + C$$

or $2\tan^{-1}v + \log|1+v^2| + 2\log|x| = C$

or $2\tan^{-1}(y/x) + \log\left|1 + y^2/x^2\right| + \log|x|^2 = 2C = C'$ (say) $\left[\text{As } v = y/x\right]$

or $2\tan^{-1} y/x + \log\left(\frac{x^2+y^2}{x^2} \cdot x^2\right) = C'$ .....(2)

Now when $x = 1,\ y = 1$

$\therefore 2\tan^{-1}\left(\frac{1}{1}\right) + \log\left((1)^2+(1)^2\right) = C'$

$\Rightarrow 2 \times \pi/4 + \log 2 = C'$

$\therefore$ from (2) $2\tan^{-1}(y/x) + \log(x^2+y^2) = \log 2 + \pi/2$

Which is required particular soln.

Q No. 12. $x^2\,dy + (xy + y^2)\,dx = 0$; $y = 1$ when $x = 1$

Soln. Given differential Equation is

$$x^2\,dy + (xy + y^2)\,dx = 0$$

$$\Rightarrow x^2\,dy = -(xy + y^2)\,dx$$

$$\Rightarrow \frac{dy}{dx} = -\frac{xy + y^2}{x^2} = -\left(\frac{y}{x} + \frac{y^2}{x^2}\right) = f(y/x) \quad -(1)$$

Which is homogeneous diff. eqn.

Put $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\therefore$ (1) becomes

$$v + x\frac{dv}{dx} = -(v + v^2)$$

or $$x\frac{dv}{dx} = -v - v^2 - v = -(2v + v^2)$$

$$\Rightarrow \frac{dv}{v^2 + 2v} = -\frac{dx}{x}$$

or $$\frac{dv}{v^2 + 2v + 1 - 1} = -\frac{1}{x}\,dx$$

or $$\frac{dv}{(v+1)^2 - (1)^2} = -\frac{1}{x}\,dx$$

$$\therefore \int \frac{1}{(v+1)^2 - (1)^2}\,dv = -\int \frac{1}{x}\,dx$$

$$\therefore \frac{1}{2}\log\left|\frac{v+1-1}{v+1+1}\right| = -\log|x| + c$$

or $$\log\left|\frac{v}{v+2}\right| + 2\log|x| = 2c = c' \text{ (say)}$$

or $$\log\left|\frac{y/x}{\frac{y}{x} + 2} \times x^2\right| = c'$$

$$\Rightarrow \log\left|\frac{yx^2}{y+2x}\right| = c'$$

or $$\frac{yx^2}{y+2x} = \pm e' = A \text{ (say)} \quad \ldots\ldots (2)$$

Now when $x=1$, $y=1$

$$\frac{1\times1^2}{1+2\times1} = A \quad \text{ie} \quad A = \frac{1}{3}$$

$\therefore$ from (2)

$$\frac{x^2y}{2x+y} = \frac{1}{3} \quad \text{or} \quad 3x^2y = 2x+y$$

Which is required particular soln. of given diff. eqn.

Q No 13. $\left(x\sin^2\left(\frac{y}{x}\right) - y\right)dx + x\,dy = 0$; $y = \frac{\pi}{4}$ when $x=1$

Sol.

Given diff. eqn is $\left(x\sin^2(y/x) - y\right)dx + x\,dy = 0$

or $$\sin^2\left(\frac{y}{x}\right) - \frac{y}{x} + \frac{dy}{dx} = 0$$

or. $$\frac{dy}{dx} = \frac{y}{x} - \sin^2(y/x) = F(y/x)$$

which is homogeneous diff. eqn. $\quad$ - (1)

To Solve Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\therefore$ (1) becomes

$$v + x\frac{dv}{dx} = v - \sin^2 v.$$

or. $$\frac{dv}{\sin^2 v} = -\frac{1}{x}dx$$

ie $$\int \operatorname{cosec}^2 v\,dv = -\int \frac{1}{x}dx$$

$$\Rightarrow \quad -\cot v = -\log|x| + c$$

or. $$\log|x| - \cot v = c$$

or $\log|x| - \cot\frac{y}{x} = c$ ..... (2)

Now when $x=1$ $y=\pi/4$

$\therefore$ $\log|1| - \cot\left(\frac{\pi}{4}\right) = c$

or $0 - 1 = c$

$\therefore$ from (2) $\log|x| - \cot\left(\frac{y}{x}\right) = -1$

or $\log|x| + \log e = \cot(y/x)$ $[\because \log e = 1]$

or $\log|ex| = \cot(y/x)$ which is reqd. soln.

Q No 14. $\frac{dy}{dx} - \frac{y}{x} + \csc\left(\frac{y}{x}\right) = 0$; $y=0$ when $x=1$.

Soln : Given diff. eqn is $\frac{dy}{dx} - \frac{y}{x} + \csc(y/x) = 0$

or $\frac{dy}{dx} = \frac{y}{x} - \csc\left(\frac{y}{x}\right) = F(y/x)$ --- (1)

So this is a homogeneous diff. eqn.

To solve Put $y = vx$

so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

$\therefore$ (1) becomes

$v + x\frac{dv}{dx} = v - \csc v$

$\Rightarrow$ $\frac{dv}{\csc v} = -\frac{dx}{x}$.

$\Rightarrow$ $\int \sin v \, dv = -\int \frac{1}{x} dx$.

$\Rightarrow$ $-\cos v = -\log|x| + c$

$\therefore$ $-\cos(y/x) = -\log|x| + c$. —(2)

Now when $x=1$, $y=0$

$\therefore$ (2) becomes.

$-\cos 0 = -\log|1| + c$

$\Rightarrow$ $-1 = c$

$\therefore$ (2) becomes

$$-\cos(y/x) = -\log|x| - 1$$

or $\cos(y/x) = \log|x| + \log e$ $[\because \log e = 1]$

or $\cos(y/x) = \log[e|x|]$ or.

$\cos(y/x) = \log|ex|$ which is reqd particular solution.

QNo 15. $2xy + y^2 - 2x^2\frac{dy}{dx} = 0$; $y = 2$ when $x = 1$

Soln. given diff. eqn. is $2xy + y^2 = +2x^2\frac{dy}{dx}$

i.e $\frac{dy}{dx} = \frac{2xy+y^2}{2x^2} = \frac{y}{x} + \frac{1}{2}(y/x)^2 = f(y/x)$ —(1)

which is homogeneous diff. eqn.

To solve put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\therefore$ (1) becomes

$$v + x\frac{dv}{dx} = v + \frac{1}{2}v^2$$

$\Rightarrow$ $x\frac{dv}{dx} = \frac{1}{2}v^2$

$\Rightarrow$ $\frac{2}{v^2}dv = \frac{1}{x}dx$

$\Rightarrow$ $\int\frac{2}{v^2}dv = \int\frac{1}{x}dx$

$\Rightarrow$ $2\left(\frac{v^{-1}}{-1}\right) = \log|x| + c$

$\Rightarrow$ $-\frac{2}{v} = \log|x| + c$

or $-\frac{2x}{y} = \log|x| + c$ ---(2)

Now when $x = 1$, $y = 2$

from (2) $-\frac{2\times1}{2} = \log|1| + c \Rightarrow -1 = 0 + c$

$\therefore$ (2) becomes $-\frac{2x}{y} = \log|x| - 1$

or $\quad 1-\frac{2x}{y} = \log|x|$ which is reqd particular soln.

Q No.16. A homogeneous differential eqn of form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitutions.

(A) $y = vx$ (B) $v = yx$ (C) $x = vy$ (D) $x = v$

Soln: The eqn $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by substitution

$$\frac{x}{y} = v \quad \text{ie} \quad x = vy.$$

So. (C) is correct option. (See Question No 10)

Q No. 17. Which of following is a homogeneous diff. eqn.?

(A) $(4x+6y+5)dy - (3y+2x+4)dx = 0$

(B) $(xy)dx - (x^3+y^3)dy = 0$

(C) $(x^3+2y^2)dx + 2xy\,dy = 0$

(D) $y^2dx + (x^2-xy-y^2)dy = 0$

Sol. The eqn. $y^2dx + (x^2-xy-y^2)dy = 0$ which can be written as

$$\frac{dy}{dx} = -\frac{y^2}{x^2-xy-y^2} = -\frac{\frac{y^2}{x^2}}{1-\frac{y}{x}-\frac{y^2}{x^2}}$$

$$= -\frac{\left(\frac{y}{x}\right)^2}{1-\left(\frac{y}{x}\right)-\left(\frac{y}{x}\right)^2} = F(y/x)$$

$\therefore$ is homogeneous.

So (D) is correct option.

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