

## RECTILINEAR FIGURES

### 6.01 Triangle and its Angles :

We have studied about angles, made by straight lines at a point. In this chapter, we shall study about a plane figure formed by more than two lines. If we take three non-collinear points in a plane and join them by taking two points at one time, then we will get three line segments. Thus, the figure so formed bounded by three line segments is called a triangle.

*"A plane figure bounded by three line segments by joining three non-collinear points in a plane is called a triangle."*

Fig. 6.01 three non-collinear points  $A$ ,  $B$  and  $C$  are joined. Figure  $ABC$ , so formed by joining line segments  $AB$ ,  $BC$  and  $CA$ , is called triangle. Symbol ' $\Delta$ ' is used in place of the word 'triangle'. So, triangle  $ABC$  will be denoted by  $\Delta ABC$ . These three points which make a triangle are called vertices of triangle. Three line segments of triangle are called its **sides**. Angles formed at the vertices of a triangle by three line segments are called **angles** of triangle.

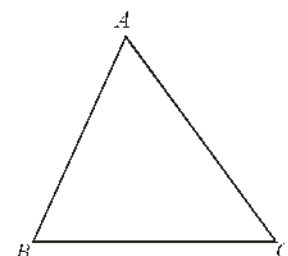


Fig. 6.01

From Fig. 6.02, it is clear that in  $\Delta ABC$  :

- (i) points  $A$ ,  $B$  and  $C$  are its vertices.
- (ii) line segments  $AB$ ,  $BC$  and  $CA$  are its sides.
- (iii)  $\angle CAB$ , (or  $\angle A$ ),  $\angle ABC$  (or  $\angle B$ )  $\angle BCA$  (or  $\angle C$ )

are the angles of the triangle. If the sides of  $\Delta ABC$  are extended in order, then angle between extended side and adjacent side is called an **exterior angle** of triangle.

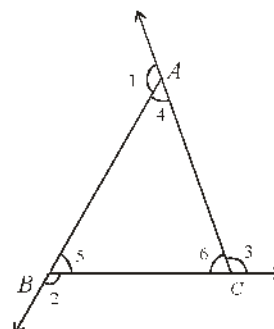


Fig. 6.02

In figure 6.2  $\angle 1, \angle 2, \angle 3$  are exterior angles of triangle ABC.  $\angle 4, \angle 5, \angle 6$  are interior angles of triangle ABC.

Triangles can be classified on the basis of their sides or angles.

### 6.02 Classification of Triangles on the Basis of Sides

**(i) Scalene Triangle :** A triangle having all the three sides of different measure is called scalene triangle. In Fig. 6.03,  $\triangle ABC$  is a scalene triangle.

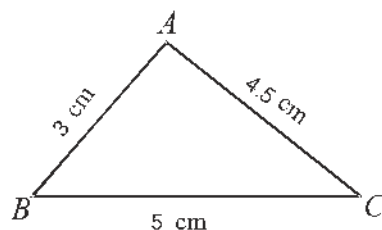


Fig. 6.03

**(ii) Isosceles Triangle :** If two sides of a triangle are of equal measure, then it is called an isosceles triangle. In Fig. 6.04,  $\triangle PQR$  is an isosceles triangle in which  $PQ = PR$ .

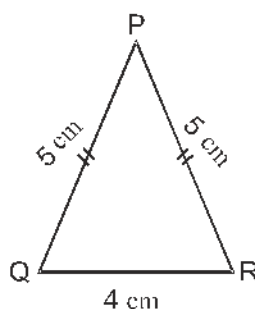


Fig. 6.04

**(iii) Equilateral Triangle :** A triangle, whose all sides are equal is called an equilateral triangle. In Fig. 6.05,  $\triangle ABC$  is an equilateral triangle in which  $AB = BC = CA$ .

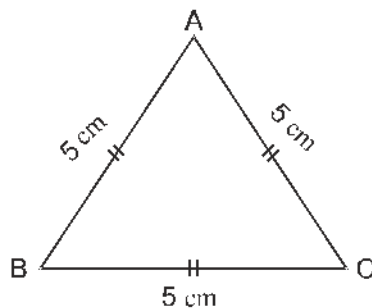


Fig. 6.05

### 6.03 Classification of Triangle on the basis of Angles :

**(i) Acute-angled Triangle :** A triangle, whose each angle is acute, is called an acute-angled triangle. In Fig. 6.06,  $\triangle PQR$  is an acute-angled triangle, since  $\angle P$ ,  $\angle Q$  and  $\angle R$  are acute angles.

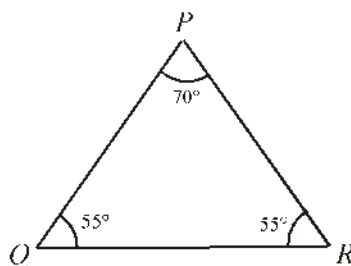


Fig. 6.06

**(ii) Right-angled Triangle :** A triangle with one angle a right angle is called a right-angled triangle. In Fig. 6.07,  $\triangle ABC$  is a right-angled triangle, since  $\angle B = 90^\circ$ .

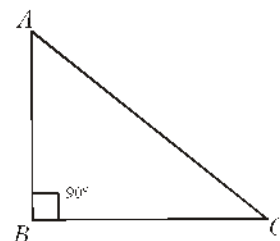


Fig. 6.07

**(iii) Obtuse-angled Triangle :** A triangle with one angle an obtuse angle, is known as an obtuse-angled triangle. In Fig. 6.08,  $\triangle PQR$  is an obtuse-angled triangle, since here  $\angle PQR = 120^\circ$ .

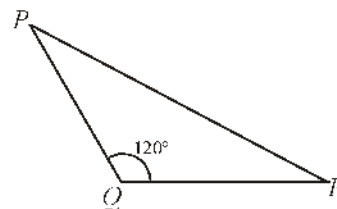


Fig. 6.08

The sum of three angles of a triangle is  $180^\circ$  proof of this geometrical fact is following.

**Theorem 6.1.** *The sum of the three angle of a triangle is equal to two right angles.*

**Given :** A triangle  $ABC$  and its angles namely  $\angle 1$ ,  $\angle 2$  and  $\angle 3$ .

**To prove**  $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

**Construction :** Through A, draw a line parallel to  $BC$ .

**Proof:**  $\because BC \parallel \ell$

$$\angle 2 = \angle 5 \quad (\text{Alternate angles}) \dots(1)$$

$$\text{and} \quad \angle 3 = \angle 4 \quad (\text{Alternate angles}) \dots(2)$$

On adding equations (1) and (2), we get

$$\angle 2 + \angle 3 = \angle 5 + \angle 4 \quad \dots(3)$$

Adding to both sides of equation (3), we get

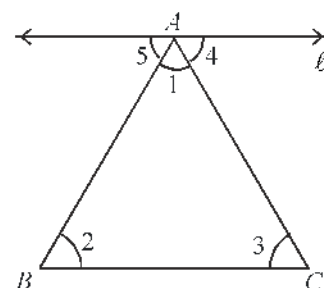


Fig. 6.09

$$\angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 5 + \angle 4 \quad \dots\dots(4) \quad \dots (iv)$$

(Sum of angles at a point on a line is  $180^\circ$  )

$$\angle 1 + \angle 5 + \angle 4 = 180^\circ \quad \dots\dots(5)$$

From (4) and (5),

$$\text{Thus} \quad \angle 1 + \angle 2 + \angle 3 = 180^\circ$$

Hence Proved

**Corollary 1.** If a side of a triangle is produced the exterior angle so formed is equal to the sum of the two interior opposite angles.

Sum of the three interior angles of a triangle is  $180^\circ$

$$\text{In Fig. 6.10, } \angle 1 + \angle 2 + \angle 3 = 180^\circ \quad \dots (i)$$

By linear pair property

$$\angle 3 + \angle 4 = 180^\circ \quad \dots (ii)$$

From equations (i) and (ii), we get

$$\angle 1 + \angle 2 + \angle 3 = \angle 3 + \angle 4$$

$$\text{Thus,} \quad \angle 1 + \angle 2 = \angle 4$$

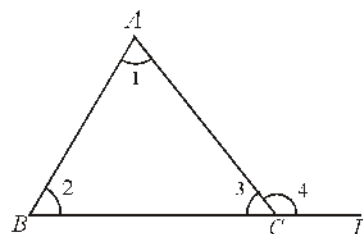


Fig. 6.10

**Corollary 2.** An exterior angle of a triangle is greater than either of the interior opposite angles.

In Fig. 6.10,  $\angle 4 = \angle 1 + \angle 2$  (From Corollary 1)

$$\Rightarrow \quad \angle 4 > \angle 1$$

$$\text{and} \quad \angle 4 > \angle 2$$

**Corollary 3.** In a right-angled triangle, right angle is the greatest angle,

$$\because \quad \text{Sum of the three angles of a triangle} = 180^\circ$$

$$\therefore \quad \text{Sum of one right angle} + \text{Sum of two other angles} = 180^\circ$$

$$\Rightarrow \quad 90^\circ + \text{Sum of other two angles} = 180^\circ$$

$$\therefore \quad \text{Sum of other two angles} = 90^\circ$$

$$\Rightarrow \quad \text{Remaining two angles are acute angles.}$$

Thus, Right angle is greater than remaining two acute angles.

Note: In each triangle, at least two angles are acute angles.

**Corollary 4.** Sum of the four angles of a quadrilateral is equal to four right angles.

In Fig. 6.11, ABCD is a quadrilateral having four angles  $\angle A, \angle B, \angle C$  and  $\angle D$ . Line AC divides quadrilateral into two triangles.

$$\text{In } \triangle ABC \quad \angle 1 + \angle 2 + \angle 3 = 180^\circ \quad \dots\dots(1)$$

$$\text{and in } \triangle ADC, \quad \angle 4 + \angle 5 + \angle 6 = 180^\circ \quad \dots\dots(2)$$

From equations (i) and (ii), we get

$$\begin{aligned} \Rightarrow \quad & \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ \\ \text{Thus,} \quad & (\angle 1 + \angle 4) + \angle 2 + (\angle 3 + \angle 5) + \angle 6 = 360^\circ \\ & \angle A + \angle B + \angle C + \angle D = 360^\circ \end{aligned}$$

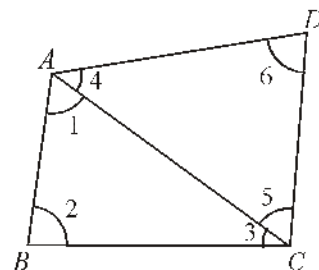


Fig. 6.11

### Illustrative Example

**Example 1 :** In Fig. 6.12, one angle of  $\triangle ABC$  is  $40^\circ$ . If the difference between remaining two angles is  $30^\circ$  then find them.

**Solution :** Let  $\angle x$  and  $\angle y$  are remaining two angles of  $\triangle ABC$ .

$$\begin{aligned} \therefore \quad & \angle x + \angle y + 40^\circ = 180^\circ \\ \Rightarrow \quad & \angle x + \angle y = 140^\circ \quad \text{.....(i)} \\ \Rightarrow \quad & \angle x - \angle y = 30^\circ \quad \text{(Given) \quad \quad \quad .....(ii)} \end{aligned}$$

On adding (i) and (ii), we get

$$\begin{aligned} & \angle x + \angle y + \angle x - \angle y = 140^\circ + 30^\circ \\ \Rightarrow \quad & 2\angle x = 170^\circ \\ \Rightarrow \quad & \angle x = 85^\circ \end{aligned}$$

$$\begin{aligned} \text{and from (i)} \quad & \angle y = 140^\circ - \angle x \\ & = 140^\circ - 85^\circ = 55^\circ \end{aligned}$$

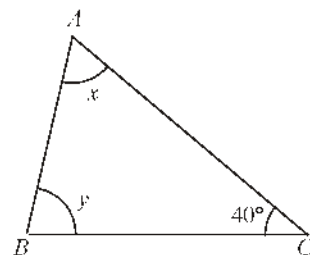


Fig. 6.12

Thus, the required angles are  $\angle x = 85^\circ$  and  $\angle y = 55^\circ$ .

**Example 2.** From Fig. 6.13, find  $\angle RPQ$ ,  $\angle QRP$  and  $\angle PQR$ .

**Solution :** From Fig. 6.13

$$\begin{aligned} & \angle x + \angle x = 126^\circ \\ \Rightarrow \quad & 2\angle x = 126^\circ \\ \Rightarrow \quad & \angle x = 63^\circ \\ \text{and} \quad & \angle RPQ = 63^\circ \\ \text{and} \quad & \angle PQR = 63^\circ \\ \text{now} \quad & \angle y + 126^\circ = 180^\circ \quad \text{(Linear pair)} \\ \therefore \quad & \angle y = 180^\circ - 126^\circ = 54^\circ \\ \text{or} \quad & \angle QRP = 54^\circ \end{aligned}$$

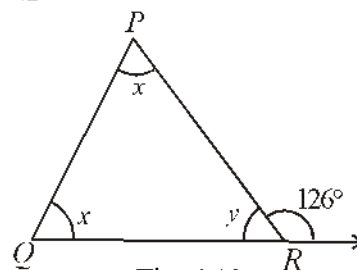


Fig. 6.13

**Example 3.** In Fig. 6.14, Find  $\angle x$ ,  $\angle y$  and  $\angle ACD$ . Here,  $BA \parallel CE$ .

**Solution :** Here,  $\angle x = 42^\circ$  (Alternate angles)

$$\begin{aligned}
 \text{Now } \angle ACD &= \angle x + 66^\circ \\
 &= 42^\circ + 66^\circ \\
 &= 108^\circ \\
 \angle y + \angle ACD &= 180^\circ \quad \dots (i) \\
 \text{or } \angle y + 108^\circ &= 180^\circ \\
 \Rightarrow \angle y &= 180^\circ - 108^\circ \\
 \Rightarrow \angle y &= 72^\circ
 \end{aligned}$$

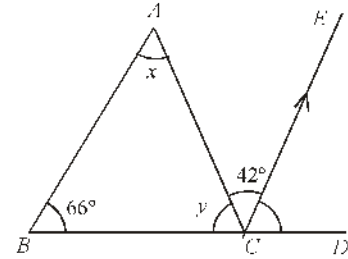


Fig. 6.14

**Example 4.** If in a  $\triangle ABC$ , bisectors of angle B and C intersect each other at point

'O', then show that  $\angle BOC = 90^\circ + \frac{1}{2}\angle A$ .

**Solution :** Draw  $\triangle ABC$  as shown in Fig. 6.15 and then draw  $BO$  and  $CO$ , the bisectors of  $\angle B$  and  $\angle C$ .

$$\begin{aligned}
 \angle A + \angle ABC + \angle ACB &= 180^\circ \\
 &\text{(Sum of three angles of a } \triangle \text{ is } 180^\circ) \\
 \text{or } \frac{1}{2}\angle A + \frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB &= \frac{1}{2} \times 180^\circ \\
 \text{or } \frac{1}{2}\angle A + \angle OBC + \angle OCB &= 90^\circ \quad \dots (1)
 \end{aligned}$$

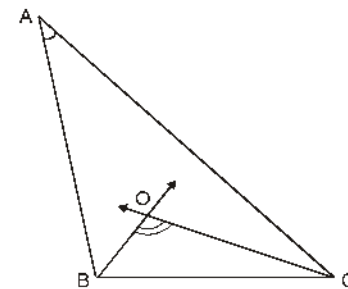


Fig. 6.15

Given  $BO$  and  $CO$  are the bisectors of  $\angle B$  and  $\angle C$  respectively.

$$\therefore \angle BOC + \angle OBC + \angle OCB = 180^\circ \quad \text{(the dangles of } \triangle OBC) \dots\dots (2)$$

Subtracting (1) from (2), we get

$$\begin{aligned}
 \angle BOC + \angle OBC + \angle OCB - \frac{1}{2}\angle A - \angle OBC - \angle OCB &= 180^\circ - 90^\circ \\
 \Rightarrow \angle BOC - \frac{1}{2}\angle A &= 90^\circ
 \end{aligned}$$

$$\text{Thus, } \angle BOC = 90^\circ + \frac{1}{2}\angle A.$$

**Example 5.** In Fig. 6.16, if  $BE \perp AC$ ,  $\angle EBC = 30^\circ$  and  $\angle FAC = 20^\circ$ , then find the values of  $\angle x$  and  $\angle y$ .

**Solution :** In  $\triangle BCE$ ,  
 $90^\circ + 30^\circ + x = 180^\circ$

or  $120^\circ + \angle x = 180^\circ$

or  $\angle x = 180^\circ - 120^\circ$

or  $\angle x = 60^\circ$

Now,  $\angle y = \angle FAC + \angle x$  (Exterior angle = Sum of interior opposite angles)

$$\angle y = 20^\circ + 60^\circ = 80^\circ$$

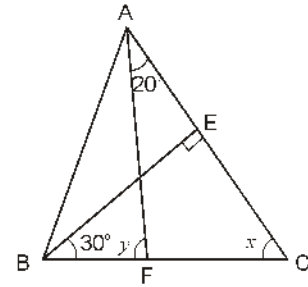


Fig. 6.16

### Exercise 6.1

1. In Fig. 6.17, find all the angles of  $\triangle ABC$

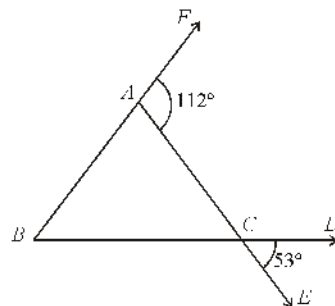


Fig. 6.17

2. In Fig. 6.18,  $\triangle ABC$  is an equilateral triangle. Find the values of  $\angle x$ ,  $\angle y$  and  $\angle z$  from the figure.

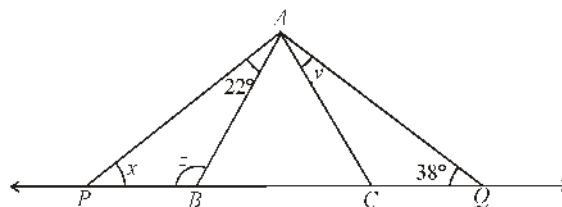


Fig. 6.18

3. In Fig. 6.19, sides  $AB$  and  $AC$  of  $\triangle ABC$  are produced to  $E$  and  $D$  respectively. If angle bisectors  $BO$  and  $CO$  of  $\angle CBE$  and  $\angle BCD$  meet each other at point  $O$ , then prove that;

$$\angle BOC = 90^\circ - \frac{\angle x}{2}$$

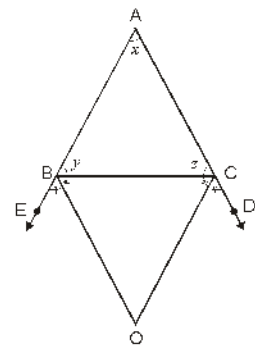


Fig. 6.19

4. In Fig. 6.20,  $\angle P = 52^\circ$  and  $\angle PQO = 64^\circ$ , if  $QO$  and  $RO$  are the angle bisectors of  $\angle PQR$  and  $\angle PRQ$  respectively, then find the values of  $\angle x$  and  $\angle y$ .

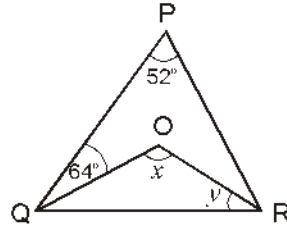


Fig. 6.20

5. In Fig. 6.21, if  $AB \parallel DE$ ,  $\angle BAC = 35^\circ$  and  $\angle CDE = 53^\circ$ , then find the value of  $\angle DCE$ .

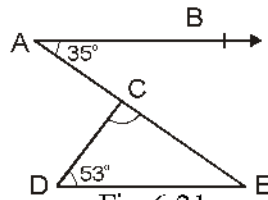


Fig. 6.21

6. In Fig. 6.22, if lines  $PQ$  and  $RS$  intersect each other at point  $T$ , such that  $\angle PRT = 40^\circ$ ,  $\angle RPT = 95^\circ$  and  $\angle TSQ = 75^\circ$ , then find  $\angle SQT$ .

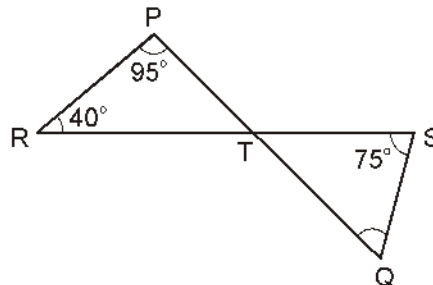


Fig. 6.22

7. In Fig. 6.23, sides  $QP$  and  $RQ$  of a triangle  $PQR$  are produced upto  $S$  and  $T$  respectively. If  $\angle SPR = 135^\circ$  and  $\angle PQT = 110^\circ$  then find  $\angle PRQ$ .

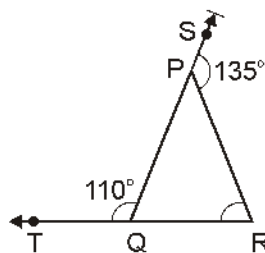


Fig. 6.23



8. In Fig. 6.24. If  $PQ \perp PS$ ,  $PQ \parallel SR$ ,  $\angle SQR = 28^\circ$  and  $\angle QRT = 65^\circ$ , then find the values of  $x$  and  $y$ .

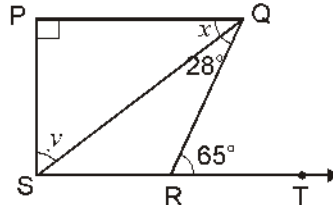


Fig. 6.24

9. In Fig. 6.25, side  $QR$  of  $\triangle PQR$  is produced to point  $S$ . If the bisectors of  $\angle PQR$  and  $\angle PRS$  meet at point  $T$ , then prove that :  $\angle QTR = \frac{1}{2} \angle QPR$

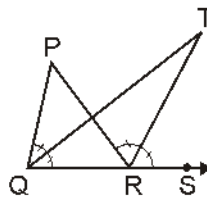


Fig. 6.25

10. In  $\triangle ABC$ ,  $A$  is right angle.  $L$  is any point on  $BC$  such that  $AL \perp BC$ . Prove that :  $\angle BAL = \angle ACB$
11. The angles of a triangle are in ratio 2:3:4. Find the all angles of the triangle.

### 6.04 Rectilinear Figures

A closed plane figure, bounded by at least three straight lines is called rectilinear figure. If we take  $n$  ( $n \geq 3$ ) different points on the plane such that :

- Line segments made by any two points out of  $n$  considered points, should not pass through any remaining point ( $n - 2$  points) except its own end points.
- Two line segments from a point are not collinear.

Then join these points in an order and figure so obtained is called polygon of  $n$  sides. These points are called vertices of the polygon and line segments which make polygon are called **sides of polygon**. Angles subtended by line segments at the vertices are called **interior angles** of polygon.

It is clear that in a  $n$  sided polygon :

- There are  $n$  vertices.
- There are  $n$  sides.
- There are  $n$  interior angles.

Polygons are classified on the basis of number of sides and the measure of angles.

**(i) Triangle :** When  $n=3$ , then rectilinear figure is called triangle.

**(ii) Quadrilateral :** When  $n=4$ , the rectilinear figure is called quadrilateral.

**(iii) Pentagon :** A polygon with 5 sides is called pentagon. In Fig. 6.26, ABCDE is a pentagon, which AB, BC, CD, DE and EA are its sides and  $\angle EAB$ ,  $\angle ABC$ ,  $\angle BCD$ ,  $\angle CDE$  and  $\angle DEA$  are its interior angles.

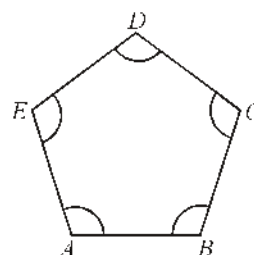


Fig. 6.26

**(iv) Hexagon :** A polygon with 6 sides is called hexagon. In Fig. 6.27 ABCDEF is a hexagon. Angles namely  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$ ,  $\angle 4$ ,  $\angle 5$  and  $\angle 6$  are its interior angles and AB, BC, CD, DE, EF and FA are its sides.

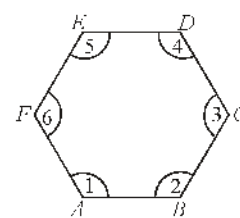


Fig. 6.27

**(v) Heptagon :** A polygon with 7 sides is called heptagon. In Fig. 6.28, ABCDEFG is a heptagon.

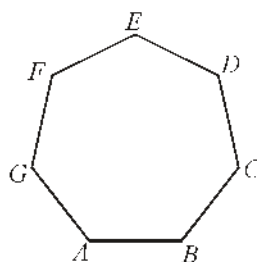


Fig. 6.28

**(vi) Octagon :** A polygon with 8 sides is called an octagon. In Fig. 6.29, ABCDEFGH is an octagon.

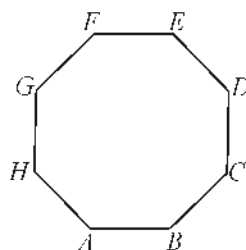


Fig. 6.29

**Convex Polygon :** If each angle of a polygon is less than two right angles, then the polygon is called a convex polygon. Unless otherwise stated, a polygon means a convex polygon.

**Concave Polygon :** If at least one angle of the polygon is greater than two right angles, it is called a concave polygon. In the given figure 6.30, ABCDEFGH is a concave polygon, since its interior angle  $\angle FED$  is greater than two right angles.

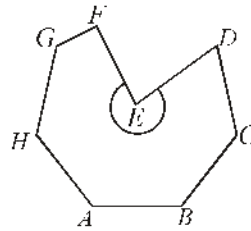


Fig. 6.30

**Equilateral Polygon :** A polygon with all sides equal is called an equilateral polygon. (Fig. 6.31)

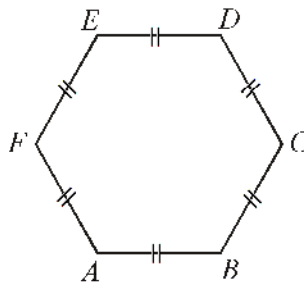


Fig. 6.31

**Equiangular Polygon :** A polygon with all interior angles equal is called an equiangular polygon. (Fig. 6.32)

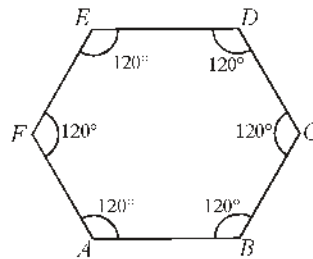


Fig. 6.32

**Regular Polygon :** A polygon which is equilateral and equiangular, is called regular polygon (Fig. 6.33).

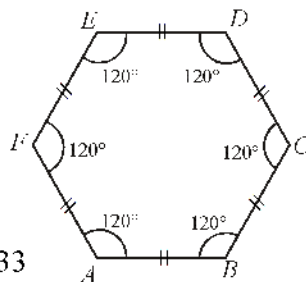


Fig. 6.33

**Exterior angles of a polygon :** if the sides of a polygon are produced in order (*i.e.* clockwise or anticlockwise direction) then the angles outside the polygon, which are supplementary angles of interior angles, and called exterior angles of the polygon.

In Fig. 6.34, the sides of polygon ABCDEF is produced in same order and we obtained exterior angles as

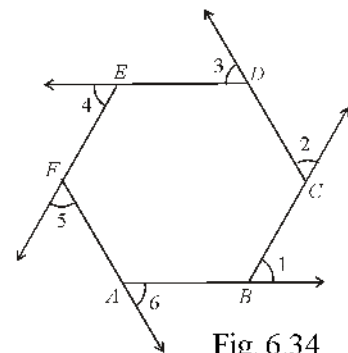


Fig. 6.34

$\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$  and  $\angle 6$ .

**Perimeter of a Polygon :** The sum of all sides of a polygon is called its **perimeter**.

**Diagonals of a Polygon :** By joining the vertices of a polygon we get the straight lines (that are not the sides) are called diagonals of polygon. In Fig. 6.35, lines AC, AD and AE are diagonals from point A. Here, Total 9 diagonals

Total number of diagonals in  $n$ -sided polygon  $= \frac{n(n-1)}{2} - n$

e.g. if  $n=6$ , then number of diagonals

$$= \frac{6(6-1)}{2} - 6 = \frac{6 \times 5}{2} - 6 = 15 - 6 = 9$$

In a  $n$ -sided polygon, the diagonals drawn from vertices make  $(n-2)$  triangles. In Fig. 6.36, diagonals AC and AD of pentagon ABCDE makes three triangles. On the basis of above facts we will establish the formula to find the sum of all the interior angles of any polygon, and on this basis we will find the sum of all exterior angles of polygon.

### Theorem 6.2

(i) *Sum of interior angles of an  $n$ -sided polygon is equal to  $(2n-4)$  right angle.*

(ii) *Sum of exterior angles of an  $n$ -sided polygon is equal to 4 right angles.*

(iii) *Each interior angles of a regular polygon is equal to  $\frac{1}{n}(2n-4)$  right angle.*

The sides AB, BC, CD, DE, EF, FA, ... of a  $n$  sided polygon are produced in same order. In This way  $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6 \dots$  are respectively exterior angles at the vertices A, B, C, D, E, F, ...

To prove

(i) Sum of all interior angles =  $(2n-4)$  right angles

(ii) Sum of all exterior angles = 4 right angles

(iii) Each interior angle of a regular polygon =  $\frac{(2n-4)}{n}$  right angle

Construction : Draw diagonals AC, AD, AE, ... from the vertex A. In the way we get  $(n-2)$  triangles in total.

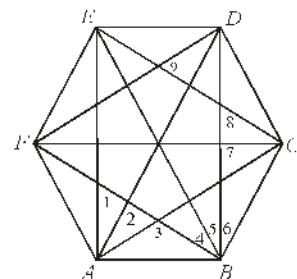


Fig. 6.35

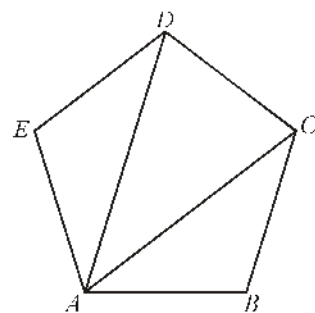


Fig. 6.36

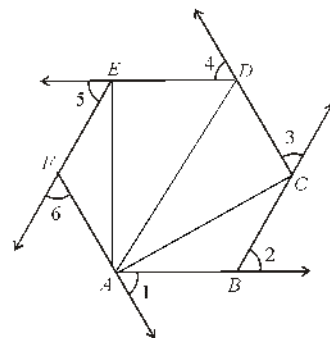


Fig. 6.37

**Proof(i) :** We know diagonals of polygon from its vertices divides it into  $(n-2)$  triangles.

$$\begin{aligned} & \text{Sum of all interior angles of } n \text{ sided polygon} \\ &= \text{Sum of interior angles of } (n-2) \text{ triangles made by the diagonals from the vertices} \\ &= (n-2) \times 2 \text{ right angle} \\ &= (2n-4) \text{ right angle} \end{aligned}$$

**Hence Proved**

**Proof(ii) :** We know that when the sides of polygon are produced, then sum of its pair of interior and exterior angles is equal to 2 right angles. Thus, there are  $n$  pairs of interior and exterior in a  $n$ -sided polygon and their sum  $= n \times 2$  right angles  $= 2n$  right angles.

Sum of  $n$  interior angles  $= (2n-4)$  right angle.

$$\begin{aligned} \text{Sum of } n \text{ exterior angles} &= 2n \text{ right angle} - (2n-4) \text{ right angle.} \\ &= (2n-2n+4) \text{ right angle} = 4 \text{ right angle.} \end{aligned}$$

**Proof(iii) :** If polygon is regular then its each interior angle  $= \frac{1}{n}(2n-4)$  right angle

**Proof :** We know that the sum of all the interior angles of  $n$ -sided polygon is equal to  $(2n-4)$  right angles. We know that, each angle of regular polygon is same. Let each angle of regular polygon is  $x^\circ$ , then  $nx = (2n-4)$  right angles (sum of all  $n$  angles)

$$x = \frac{1}{n}(2n-4) \text{ right angles} \quad \text{Hence Proved.}$$

### Illustrative Examples

**Example 6.** In a regular polygon, number of sides is 10. Find its each interior angle.

**Solution : First Method :**  $\therefore$  Each exterior angles or of  $n$ -sided regular polygon

$$\begin{aligned} &= \frac{4}{n} \quad (\text{right angles}) \\ &= \frac{4}{10} \times 90^\circ \quad (\text{Here } n = 10) \\ &= 36^\circ \end{aligned}$$

$$\begin{aligned} \therefore \text{Each interior angle} &= 180^\circ - 36^\circ \\ &= 144^\circ \end{aligned}$$

**Second Method :**

$$\begin{aligned} \text{Each interior angle of } n \text{ sided regular polygon} &= \frac{(2n-4) \text{ right angle}}{n} \\ &= \frac{(2 \times 10 - 4) \times 90^\circ}{10} \\ &= \frac{16 \times 90^\circ}{10} = 144^\circ \end{aligned}$$

**Example 7.** Find the sum of all interior angles of a heptagon.

**Solution :** Sum of interior angles of  $n$  sided regular polygon  $= (2n - 4)$  right angle.

$\therefore$  Sum of all the interior angles of 7 sided regular polygon.

$$= (2 \times 7 - 4) \times 90^\circ = 10 \times 90^\circ = 900^\circ$$

$$= 10 \times 90^\circ = 900^\circ$$

**Alternative method :**

Number of triangles in regular polygon made by all the diagonals from a vertex of a heptagon  $= 7 - 2 = 5$ .

$\therefore$  Sum of interior angles of a triangle  $= 180^\circ$

$\therefore$  Sum of interior angles of 5 triangles  $= 5 \times 180^\circ = 900^\circ$

**Example 8.** If each interior angle of a regular polygon is  $175^\circ$ , then find number of sides.

**Solution :**  $\therefore$  1 interior angle + 1 exterior angle  $= 180^\circ$

$$1 \text{ exterior angle} = 180^\circ - 1 \text{ interior angle}$$

$$= 180^\circ - 175^\circ = 5^\circ$$

$\therefore$  Sum of  $n$  exterior angles  $= 360^\circ$  (Let the number of sides are  $n$ .)

$$\therefore n \times 5^\circ = 360^\circ$$

$$\text{Thus, } n = \frac{360^\circ}{5^\circ} = 72$$

**Example 9.** Can a regular polygon be possible in which each interior angle is  $115^\circ$ ? Check it.

**Solution :**  $\therefore$  It is given that each interior angle of regular polygon  $= 115^\circ$  (If it possible.)

$$\text{Each exterior angle} = 180^\circ - 115^\circ = 65^\circ$$

let the number of sides be  $n$ .

$\therefore$  Sum of exterior angles of a polygon  $= 4$  right angles

$$\Rightarrow n \times 65^\circ = 360^\circ$$

$$\Rightarrow n = \frac{360^\circ}{65^\circ} = \frac{72}{13} = 5\frac{7}{13} \neq \text{a whole number}$$

Thus, a regular polygon cannot possible in which each interior angle is  $115^\circ$ .

### Exercise 6.2

1. If a regular polygon has 8 sides, then:
  - (i) find the sum of its exterior angles.
  - (ii) find the measure of each exterior angle.
  - (iii) find the sum of its interior angle.
  - (iv) find the measure of each interior angle.
2. If the sum of interior angles of a regular polygon is  $2160^\circ$ , then what will be the number of sides of polygon? Find it.
3. Check, whether a regular polygon is possible with each interior angles  $137^\circ$ .
4. Find  $\angle CED$  and  $\angle BDE$  in the following Fig. 6.38.

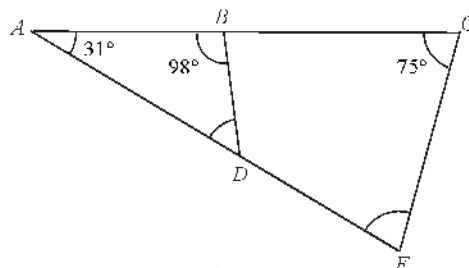


Fig. 6.38

#### Important Points

1. A plane figure bounded by  $n$  line segments is called  $n$ -sided polygon. In a  $n$ -sided polygon, there are  $n$  vertices,  $n$  interior angles and  $n$  sides.
2. Triangle, quadrilateral, pentagon, hexagon, heptagon and octagon are the name of the polygon according to  $n=3,4,5,6,7,8$  respectively.
3. Sum of three interior angles of a triangle is  $180^\circ$ .
4. Sum of all interior angles of  $n$ -sided regular polygon is equal to  $(2n-4)$  right angles.
5. A polygon with equal sides and equal angles is called regular polygon.
6. Each interior angles of  $n$ -sided regular polygon =  $\frac{(2n-4)}{n}$  right angles.
7. Sum of all exterior angles of a polygon =  $360^\circ$
8. Each exterior angle of  $n$ -sided regular polygon =  $\frac{4}{n}$  right angles.

### Miscellaneous Exercise - 6

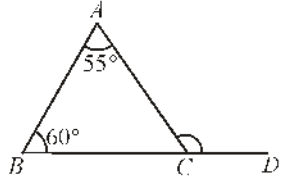
1. If two angles of a triangle are of measure  $90^\circ$  and  $30^\circ$  then the third angles is:  
 (A)  $90^\circ$                       (B)  $30^\circ$                       (C)  $60^\circ$                       (D)  $120^\circ$                       [   ]
  2. Three angles of triangle are in ratio 2:3:4, then the measure of its greatest angle is :  
 (A)  $80^\circ$                       (B)  $60^\circ$                       (C)  $40^\circ$                       (D)  $180^\circ$                       [   ]
  3. Measure of each angle of an equilateral triangle is :  
 (A)  $90^\circ$                       (B)  $30^\circ$                       (C)  $45^\circ$                       (D)  $60^\circ$                       [   ]
  4. Four angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4 ,then the measure of its smallest angle is :  
 (A)  $120^\circ$                       (B)  $36^\circ$                       (C)  $18^\circ$                       (D)  $10^\circ$                       [   ]
  5. In Fig.6.39, side  $BC$  of  $\triangle ABC$  is produced upto points  $D$ , If  $\angle A = 55^\circ$  and  $\angle B = 60^\circ$  ,then the measure of  $\angle ACD$  is :  
 (A)  $120^\circ$                       (B)  $110^\circ$   
 (C)  $115^\circ$                       (D)  $125^\circ$                       [   ]
- 

Fig.6.39
6. Sum of all interior angles of a hexagon is :  
 (A)  $720^\circ$                       (B)  $360^\circ$                       (C)  $540^\circ$                       (D)  $1080^\circ$                       [   ]
  7. The sum of exterior angles made by producing sides (in same order) of  $n$ -sided regular polygon :  
 (A)  $n$ -right angles                      (B)  $2n$  right angles  
 (C)  $(2n-4)$  right angles                      (D) 4 right angles                      [   ]
  8. In a regular polygon, number of sides in  $n$ , then the measure of each interior angle is :  
 (A)  $\frac{360^\circ}{n}$                       (B)  $\left(\frac{2n-4}{n}\right)$  right angles  
 (C)  $n$  right angles                      (D)  $2n$  right angles                      [   ]
  9. If one angles of a triangle is equal to the sum of other two angles, then the triangle is :  
 (A) Isosceles triangle                      (B) Obtuse angled triangle  
 (C) Equilateral triangle                      (D) Right angled triangle                      [   ]
  10. The exterior angle of a triangle is  $105^\circ$  and the interior opposite angles are same. Each of the equal angle is :  
 (A)  $37\frac{1}{2}^\circ$                       (B)  $52\frac{1}{2}^\circ$                       (C)  $72\frac{1}{2}^\circ$                       (D)  $75^\circ$                       [   ]



11. The angles of a triangle are in the ratio 5 : 3 : 7. This triangle is :  
 (A) Acute angled triangle (B) Obtuse angled triangle  
 (C) Right angled triangle (D) Isosceles triangle [ ]
12. If one angle of a triangle is  $130^\circ$ , then angle between the angular bisector of two angles of a triangle may be :  
 (A)  $50^\circ$  (B)  $65^\circ$  (C)  $145^\circ$  (D)  $155^\circ$  [ ]
13. In Fig. 6.40, find  $\angle A$ .

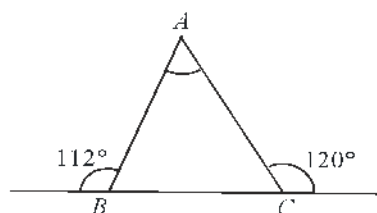


Fig. 6.40

14. In Fig. 6.41,  $\angle B = 60^\circ$  and  $\angle C = 40^\circ$ . Find the measure of  $\angle A$ .

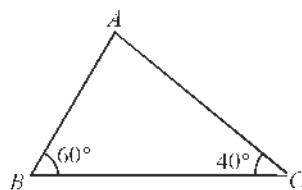


Fig. 6.41

15. In Fig. 6.42  $m \parallel QR$ , then find  $\angle QPR$

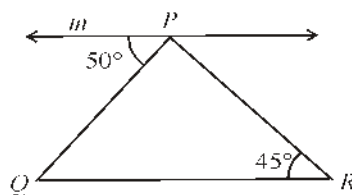


Fig. 6.42

16. In Fig. 6.43, find  $\angle A$

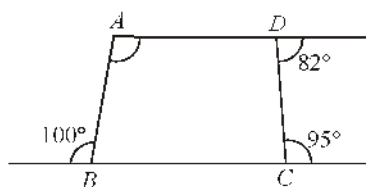


Fig. 6.43

17. Four interior angles of a pentagon are  $40^\circ$ ,  $75^\circ$ ,  $125^\circ$  and  $135^\circ$ , then find the measure of the fifth angle.
18. If each exterior angle of a regular polygon is  $45^\circ$ , then find the number of its sides.
19. The number of sides of a regular polygon is 12, then find the measure of each interior angle.
20. Sum of all interior angles of a polygon is 10 right angles. Find the number of sides.
21. Check, whether a polygon is possible if each of its interior angle is of measure  $110^\circ$ .
22. If in a  $\triangle ABC$ ,  $\angle A + \angle B = \angle C$ , then find the greatest angle of  $\triangle ABC$ .
23. Find the sum of interior angles of an octagon.
24. Find each interior angle of regular decagon.
25.  $110^\circ$ ,  $130^\circ$  and  $x^\circ$  are the exterior angles obtained by producing the sides of a triangle in same order. Find the value of  $x^\circ$ .
26. A hexagon has one interior angle of measure  $165^\circ$  and remaining each interior angle of measure  $x^\circ$ , then find measure of remaining angle.
27. In Fig. 6.44,  $AB \parallel DC$ , Find  $\angle x$ ,  $\angle y$  and  $\angle z$ .

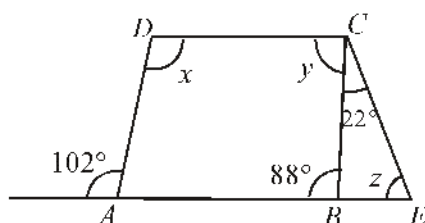


Fig. 6.44

28. In Fig. 6.45, find the value of  $\angle x$  and  $\angle y$ , where  $\angle x - \angle y = 10^\circ$ .

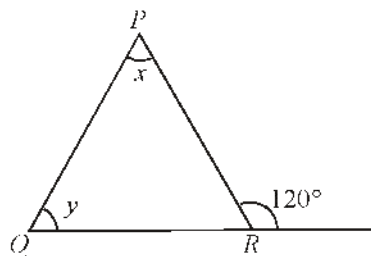


Fig. 6.45

29. In a polygon, two angles are of  $90^\circ$  each remaining each angle is of measure  $150^\circ$ . Find the number of sides of this polygon.

30. In Fig. 6.46, prove that :  $\angle x + \angle y = \angle A + \angle C$ .

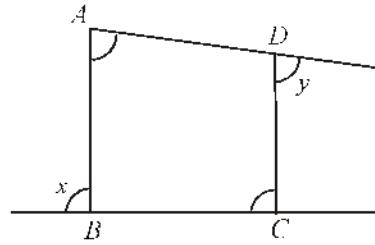


Fig. 6.46

31. In Fig. 6.47, lines  $BO$  and  $CO$  are the bisectors of  $\angle B$  and  $\angle C$ , Find angle  $x$

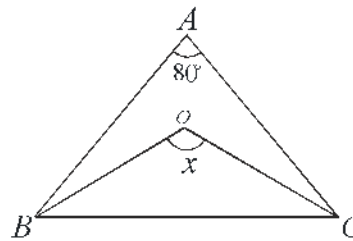


Fig. 6.47

32. In Fig 6.48,  $\angle Q > \angle R$ ,  $PA$  is the bisector of  $\angle QPR$  and  $PM \perp QR$ . Prove that :

$$\angle APM = \frac{1}{2}(\angle Q - \angle R)$$

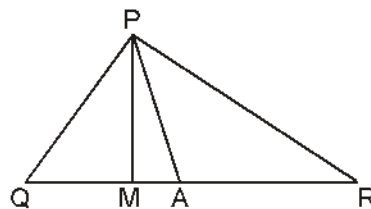


Fig. 6.48

## Answers

### Exercise 6.1

1.  $\angle A = 68^\circ$ ,  $\angle B = 59^\circ$ ,  $\angle C = 53^\circ$
2.  $\angle x = 38^\circ$ ,  $\angle y = 22^\circ$ ,  $\angle z = 120^\circ$
4.  $\angle x = 116^\circ$ ,  $\angle y = 32^\circ$
5.  $92^\circ$
6.  $60^\circ$
7.  $65^\circ$
9.  $\angle x = 37^\circ$  and  $\angle y = 53^\circ$
11.  $40^\circ$ ,  $60^\circ$ ,  $80^\circ$

### Exercise 6.2

1. (i)  $360^\circ$  (ii)  $45^\circ$  (iii)  $1080^\circ$  (iv)  $135^\circ$
2. 14      3. No      4.  $74^\circ$

### Miscellaneous Exercise 6

- |                                     |                 |                 |   |                |                  |
|-------------------------------------|-----------------|-----------------|---|----------------|------------------|
| 1. (C)                              | 2. (A)          | 3. (D)          | 4. (B)  | 5. (C)         | 6. (A)           |
| 7. (D)                              | 8. (B)          | 9. (D)          | 10. (B)   | 11. (A)        | 12. (D)          |
| 13. $\angle A = 52^\circ$           | 14. $80^\circ$  | 15. $85^\circ$  | 16. 97  | 17. 165        |                  |
| 18. 8                               | 19. $150^\circ$ | 20. 17          | 21. No  | 22. $\angle C$ | 23. $1080^\circ$ |
| 24. $144^\circ$                     | 25. $120^\circ$ | 26. $111^\circ$ | 27. $x = 102^\circ$ , $y = 92^\circ$ , $z = 66^\circ$ |                |                  |
| 28. $x = 65^\circ$ , $y = 55^\circ$ | 29. 6           | 30. (B)         | 31. $140^\circ$                                       |                |                  |

□