(Olympiad Comprehensive Notes) (IMO)

NOTES

Fractions

Fraction is a number, which in used to represent the part of a whole. It is expressed in the form of $\frac{P}{Q}$ where P and Q are natural numbers. The upper part of the fraction is called **numerator** and the lower part is **denominator**. For $\frac{5}{9}$ is a fraction, where 5 is numeration and 9 is denominator.

> Example:

Represent the shaded part of the figure as a fraction.



Solution:

 $\frac{1}{4}$

Like Fraction

The fractions, which have the same denominators are called like fractions.

> Example:

$$\frac{5}{17}, \frac{8}{17}, \frac{12}{17}, \frac{19}{17}$$
 are like fractions

Unlike Fraction

The fractions, which do not have the same denominators, in other words, the fractions with different denominators are called unlike fractions.

> Example:

$$\frac{8}{9}, \frac{4}{13}, \frac{9}{8}, \frac{10}{12} \text{ are unlike fractions.}$$

Unit Fraction

The fractions which have the numerator 1 are called unit fractions.

> Example:

$$\frac{1}{3}$$
, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{1}{15}$ are unit fractions.

Proper Fraction

If the numerator of a fraction is smaller than the denominator, the fraction is called proper fraction.

> Example:

$$\frac{5}{7}$$
, $\frac{7}{9}$, $\frac{4}{15}$, $\frac{9}{16}$ are proper fractions.

Improper Fraction

If the numerator of a fraction is greater than the denominator, the fraction is called improper fraction.

> Example:

$$\frac{178}{128}$$
, $\frac{321}{65}$, $\frac{712}{100}$ are improve fractions.

Mixed Fraction

Mixed fraction is the sum of a whole number and a proper fraction. Both the whole number and the fraction are written together, but the sign of the addition (+) does not appear between them.

> Example:

$$4\frac{5}{7}, 9\frac{1}{3}, 7\frac{8}{12}, 8\frac{6}{13}$$
 are mixed fractions.

Equivalent Fractions

Two or more fractions are said to be equivalent fractions/ if they have the same value. In other words, when equivalent fractions are reduced into their simplest form, they give the same fraction.

> Example:

The equivalent fractions of
$$\frac{8}{13}$$
 are $\frac{16}{26}, \frac{24}{39}, \frac{32}{52}$.

Comparison of Unit Fractions

If the numerators of two fractions are same and their denominators are different, then the fractions having smaller denominator will be the greater one. To take an example, $\frac{1}{P}$ and $\frac{1}{Q}$ are unit fractions where P and Q are natural numbers.

If
$$Q < P$$
 then $\frac{1}{P} < \frac{1}{Q}$.

> Example:

Compare between $\frac{1}{13}$ and $\frac{1}{15}$. Which fraction is greater than other?

Solution:

$$\frac{1}{13} > \frac{1}{15}$$

Because 13 < 15, therefore $\frac{1}{13} > \frac{1}{15}$.

Comparison of Like Fractions

Like fractions have same denominator. The fraction which has bigger numerator is greater.

> Example:

Compare between $\frac{15}{74}$ and $\frac{19}{74}$. Which fraction is greater than other?

Solution:

$$\frac{19}{74} > \frac{15}{74}$$
.

Since denominators are same and 19 > 15

Therefore,
$$\frac{19}{74} > \frac{15}{74}$$
.

Comparison of Unlike Fractions

To compare unlike fractions, unlike fractions are converted into like fractions. Conversion is done by multiplying the numerator and denominator of both the fractions by a suitable numbers, such that, denominators become equal.

> Example:

Compare between $\frac{5}{8}$ and $\frac{3}{5}$. Which fraction is the greater?

Solution:

$$\frac{5}{8} > \frac{3}{5}$$

$$\frac{5}{8} \times \frac{5}{5} = \frac{25}{40}$$
 and $\frac{3}{5} \times \frac{8}{8} = \frac{24}{40}$

Since
$$\frac{25}{40} > \frac{24}{40}$$
, therefore, $\frac{5}{8} > \frac{3}{5}$.

Conversion of Improper Fraction into Mixed Fraction

If $\frac{a}{b}$ is an improper fraction, then divide a by b by long division method. The mixed fraction equivalent to improper fraction

is
$$Q\frac{R}{b}$$
, where

$$Q = Quotient of \frac{a}{b}$$

$$R = Remainder left after \frac{a}{b}.$$

> Example:

Convert $\frac{48}{5}$ into mixed fraction.

Solution:

$$\therefore \frac{48}{5} = 9\frac{3}{5}$$

Conversion of Mixed Fraction into Improper Fraction

If $a\frac{b}{c}$ is mixed fraction, then its equivalent fraction is $\frac{a \times c + b}{c}$.

> Example:

Convert $11\frac{1}{3}$ into improper fraction.

Solution:

$$11\frac{1}{3} = \frac{11 \times 3 + 1}{3} = \frac{33 + 1}{3} = \frac{34}{3}$$