Chapter 2

Straight Line

Contents

Monge

A straight line is the simplest geometric curve. Monge (1781 A.D.) gave the modern 'Point Slope' form of equation of a line as y - y' = m (x - x') and condition of perpendicularity of two lines as mm' + 1 = 0

S.f. Lacroix (1765-1843 A.D.) was a prolific text book writer; but his contributions to analytic geometry are found scattered. He gave the 'two point' form of equation of a line as $y-\beta=\frac{\beta'-\beta}{\alpha'-\alpha}(x-\alpha)$. *He also gave the formula for finding angles between two lines.*

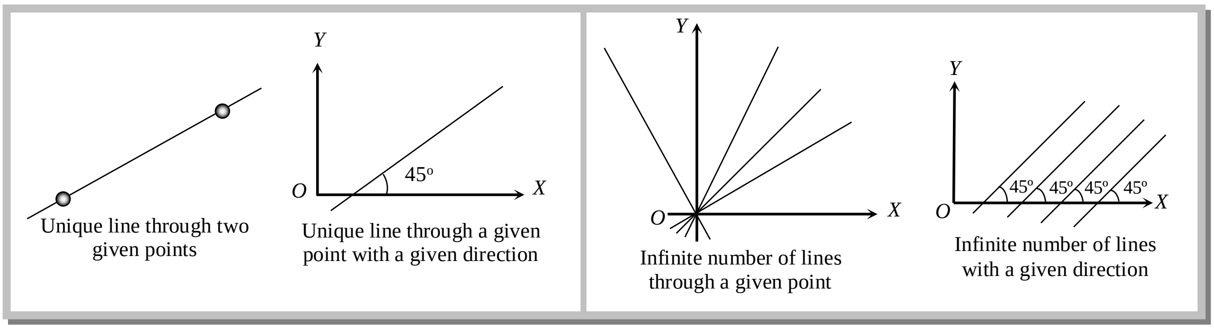
Straight Line

2.1 Definition

The straight line is a curve such that every point on the line segment joining any two points on it lies on it. The simplest locus of a point in a plane is a straight line. A line is determined uniquely by any one of the following:

(1) Two different points (because we know the axiom that one and only one straight line passes through two given points)

(2) A point and a given direction.



Unique line through two given points

Unique line through a given point with a given direction

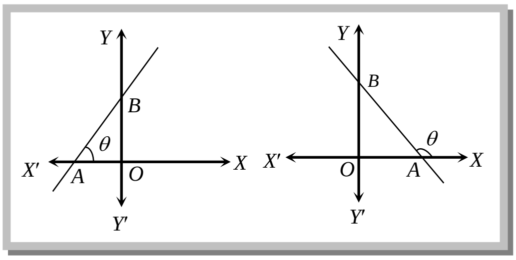
Infinite number of lines through a given point

Infinite number of lines with a given direction

Thus, to determine a line uniquely, two geometrical conditions are required.

2.2 Slope (Gradient) of a Line

The trigonometrical tangent of the angle that a line makes with the positive direction of the *x*-axis in anticlockwise sense is called the slope or gradient of the line.



The slope of a line is generally denoted by *m*. Thus, $m = \tan\theta$

(1) Slope of line parallel to x – axis is $m = \tan 0^o = 0$.

(2) Slope of line parallel to y – axis is $m = \tan 90^o = \infty$.

(3) Slope of the line equally inclined with the axes is 1 or –1.

(4) Slope of the line through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\frac{y_2 - y_1}{x_2 - x_1}$ taken in the same order.

(5) Slope of the line $ax + by + c = 0, b \neq 0$ is $-\frac{a}{b}$.

(6) Slope of two parallel lines are equal.

(7) If m_1 and m_2 be the slopes of two perpendicular lines, then $m_1.m_2 = -1$.

Note : ❑ m can be defined as $\tan\theta$ for $0 < \theta \leq \pi$ and $\theta \neq \frac{\pi}{2}$

❑ If three points *A*, *B*, *C* are collinear, then

Slope of *AB* = Slope of *BC* = Slope of *AC*

Example: 1 The gradient of the line joining the points on the curve $y = x^2 + 2x$, whose abscissae are 1 and 3, is **[MP PET 1997]**

(a) 6 (b) 5 (c) 4 (d) 3

Solution: (a) The points are (1, 3) and (3, 15)

Hence gradient is = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{12}{2} = 6$

Example: 2 Slope of a line which cuts intercepts of equal lengths on the axes is **[MP PET 1986]**

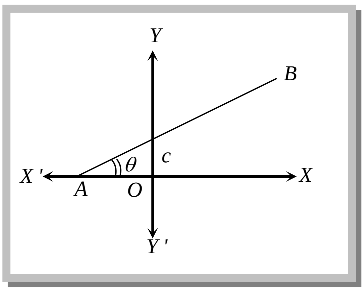
(a) – 1 (b) 0 (c) 2 (d) $\sqrt{3}$

Solution: (a) Equation of line is $\frac{x}{a} + \frac{y}{a} = 1$

$\Rightarrow \quad x + y = a \quad \Rightarrow \quad y = -x + a$. Hence slope of the line is – 1.

2.3 Equations of Straight line in Different forms

(1) **Slope form :** Equation of a line through the origin and having slope m is $y = mx$.

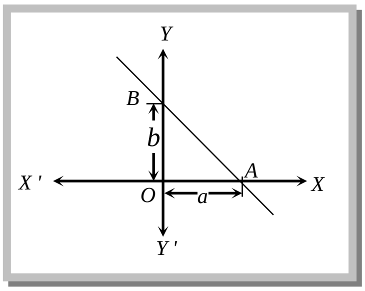


(2) **One point form or Point slope form :** Equation of a line through the point (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

(3) **Slope intercept form :** Equation of a line (non-vertical) with slope m and cutting off an intercept c on the y-axis is $y = mx + c$.

The equation of a line with slope m and the x-intercept d is $y = m(x - d)$

(4) **Intercept form :** If a straight line cuts x-axis at A and the y-axis at B then OA and OB are known as the intercepts of the line on x-axis and y-axis respectively.



The intercepts are positive or negative according as the line meets with positive or negative directions of the coordinate axes.

In the figure, OA = x-intercept, OB = y-intercept.

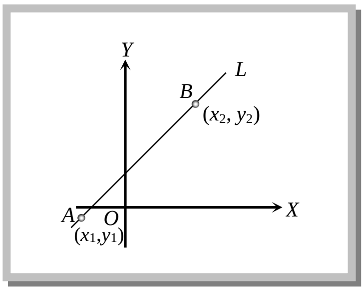
Equation of a straight line cutting off intercepts a and b on x–axis and y–axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$.

Note: ❑ If given line is parallel to X axis, then X-intercept is undefined.

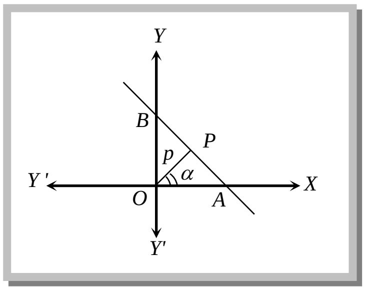
❑ If given line is parallel to Y axis, then Y-intercept is undefined.

(5) **Two point form:** Equation of the line through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$. In the determinant form it is gives as:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0 \text{ is the equation of line.}$$



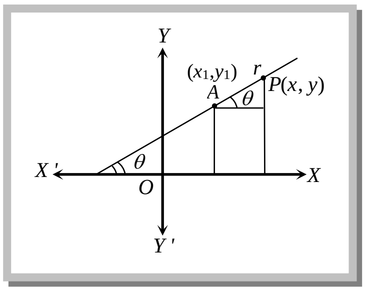
(6) **Normal or perpendicular form :** The equation of the straight line upon which the length of the perpendicular from the origin is p and this perpendicular makes an angle α with x-axis is $x\cos\alpha + y\sin\alpha = p$.



(7) **Symmetrical or parametric or distance form of the line :** Equation of a line passing through (x_1, y_1) and making an angle θ with the positive direction of x-axis is $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$,

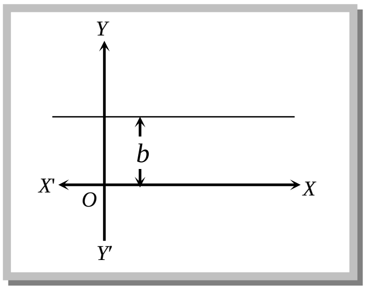
where r is the distance between the point $P(x, y)$ and $A(x_1, y_1)$.

The coordinates of any point on this line may be taken as $(x_1 + r\cos\theta, y_1 + r\sin\theta)$, known as parametric co-ordinates, 'r' is called the parameter.



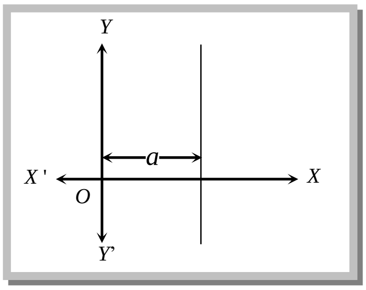
Note : ❑ Equation of x-axis $\Rightarrow y = 0$

Equation a line parallel to x-axis (or perpendicular to y-axis) at a distance 'b' from it $\Rightarrow y = b$



❑ Equation of y-axis $\Rightarrow x = 0$

Equation of a line parallel to y-axis (or perpendicular to x-axis) at a distance 'a' from it $\Rightarrow x = a$



Example: 3 Equation to the straight line cutting off an intercept 2 from the negative direction of the axis of y and inclined at 30° to the positive direction of x, is **[MP PET 2003]**

(a) $y + x - \sqrt{3} = 0$ (b) $y - x + 2 = 0$ (c) $y - \sqrt{3}x - 2 = 0$ (d) $\sqrt{3}y - x + 2\sqrt{3} = 0$

Solution: (d) Let the equation of the straight line is $y = mx + c$.

Here $m = \tan 30° = \dfrac{1}{\sqrt{3}}$ and $c = -2$

Hence, the required equation is $y = \dfrac{1}{\sqrt{3}}x - 2 \Rightarrow \sqrt{3}y - x + 2\sqrt{3} = 0$.

Example: 4 The equation of a straight line passing through (– 3, 2) and cutting an intercept equal in magnitude but opposite in sign from the axes is given by **[Rajasthan PET 1984; MP PET 1993]**

(a) $x-y+5=0$ (b) $x+y-5=0$ (c) $x-y-5=0$ (d) $x+y+5=0$

Solution: (a) Let the equation be $\frac{x}{a}+\frac{y}{-a}=1 \Rightarrow x-y=a$

But it passes through (–3, 2), hence $a=-3-2=-5$. Hence the equation of straight line is $x-y+5=0$.

Example: 5 The equation of the straight line passing through the point (4, 3) and making intercept on the co-ordinates axes whose sum is – 1, is **[AIEEE 2004]**

(a) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{2}+\frac{y}{1}=1$ (b) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

(c) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{2}+\frac{y}{1}=1$ (d) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

Solution: (d) Let the equation of line is $\frac{x}{a}+\frac{y}{-1-a}=1$, which passes through (4, 3). Then $\frac{4}{a}+\frac{3}{-1-a}=1 \Rightarrow a=\pm 2$

Hence equation is $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$.

Example: 6 Let *PS* be the median of the triangle with vertices $P(2,2), Q(6,-1)$ and $R(7, 3)$. The equation of the line passing through (1, – 1) and parallel to *PS* is **[IIT Screening 2000]**

(a) $2x-9y-7=0$ (b) $2x-9y-11=0$ (c) $2x+9y-11=0$ (d) $2x+9y+7=0$

Solution: (d) S = mid point of $QR=\left(\frac{6+7}{2},\frac{-1+3}{2}\right)=\left(\frac{13}{2},1\right)$

$\therefore$ Slope (m) of $PS=\frac{2-1}{2-\frac{13}{2}}=\frac{-2}{9}$; $\therefore$ The required equation is $y+1=\frac{-2}{9}(x-1) \Rightarrow 2x+9y+7=0$

2.4 Equation of Parallel and Perpendicular lines to a given Line

(1) Equation of a line which is parallel to $ax+by+c=0$ is $ax+by+\lambda=0$

(2) Equation of a line which is perpendicular to $ax+by+c=0$ is $bx-ay+\lambda=0$

The value of λ in both cases is obtained with the help of additional information given in the problem.

Example: 7 The equation of the line passes through (a, b) and parallel to the line $\frac{x}{a}+\frac{y}{b}=1$, is **[Rajasthan PET 1986, 1995]**

(a) $\frac{x}{a}+\frac{y}{b}=3$ (b) $\frac{x}{a}+\frac{y}{b}=2$ (c) $\frac{x}{a}+\frac{y}{b}=0$ (d) $\frac{x}{a}+\frac{y}{b}+2=0$

Solution: (b) The equation of parallel line to given line is $\frac{x}{a}+\frac{y}{b}=\lambda$.

This line passes through point (a, b).

$\therefore \frac{a}{a}+\frac{b}{b}=\lambda \Rightarrow \lambda=2$

Hence, required line is $\frac{x}{a}+\frac{y}{b}=2$.

Example: 8 A line passes through (2, 2) and is perpendicular to the line $3x+y=3$. Its y-intercept is **[IIT 1992]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

Solution: (d) The equation of a line passing through (2, 2) and perpendicular to $3x+y=3$ is

$y-2=\frac{1}{3}(x-2)$ or $x-3y+4=0$. Putting $x=0$ in this equation, we obtain $y=\frac{4}{3}$.

So y-intercept = $\frac{4}{3}$.

Example: 9 The equation of line passing through $\left(-1,\frac{\pi}{2}\right)$ and perpendicular to $\sqrt{3}\sin\theta+2\cos\theta=\frac{4}{r}$ is **[EAMCET 2003]**

(a) $2=\sqrt{3}\,r\cos\theta-2r\sin\theta$ (b) $5=-2\sqrt{3}\,r\sin\theta+4r\cos\theta$

(c) $2=\sqrt{3}\,r\cos\theta+2r\sin\theta$ (d) $5=2\sqrt{3}\,r\sin\theta+4r\cos\theta$

Solution: (a) Equation of a line, perpendicular to $\sqrt{3}\sin\theta+2\cos\theta=\frac{4}{r}$ is $\sqrt{3}\sin\left(\frac{\pi}{2}+\theta\right)+2\cos\left(\frac{\pi}{2}+\theta\right)=\frac{k}{r}$

It is passing through $\left(-1,\frac{\pi}{2}\right)$. Hence, $\sqrt{3}\sin\pi+2\cos\pi=k/-1\Rightarrow k=2$

$\therefore\ \sqrt{3}\cos\theta-2\sin\theta=\frac{2}{r}\Rightarrow 2=\sqrt{3}\,r\cos\theta-2r\sin\theta$.

Example: 10 The equation of the line bisecting perpendicularly the segment joining the points (– 4, 6) and (8, 8) is **[Karnataka CET 2003]**

(a) $6x+y-19=0$ (b) $y=7$ (c) $6x+2y-19=0$ (d) $x+2y-7=0$

Solution: (a) Equation of the line passing through (–4, 6) and (8, 8) is

$y-6=\frac{8-6}{8+4}(x+4)\Rightarrow y-6=\frac{2}{12}(x+4)\Rightarrow 6y-x=40$ ......(i)

Now equation of any line $\perp$ to it is $6x+y+\lambda=0$ .....(ii)

This line passes through the midpoint of (–4, 6) and (8, 8) *i.e.,* (2, 7)

$\therefore$ From (ii) $12+7+\lambda=0\Rightarrow\lambda=-19$, $\therefore$ Equation of line is $6x+y-19=0$

2.5 General equation of a Straight line and its Transformation in Standard forms

General form of equation of a line is $ax+by+c=0$, its

(1) **Slope intercept form:** $y=-\frac{a}{b}x-\frac{c}{b}$, slope $m=-\frac{a}{b}$ and intercept on y-axis is, $C=-\frac{c}{b}$

(2) **Intercept form :** $\frac{x}{-c/a}+\frac{y}{-c/b}=1$, x intercept is $=\left(-\frac{c}{a}\right)$ and y intercept is $=\left(-\frac{c}{b}\right)$

(3) **Normal form :** To change the general form of a line into normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2+b^2}$ like

$-\frac{ax}{\sqrt{a^2+b^2}}-\frac{by}{\sqrt{a^2+b^2}}=\frac{c}{\sqrt{a^2+b^2}}$, where $\cos\alpha=-\frac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha=-\frac{b}{\sqrt{a^2+b^2}}$ and $p=\frac{c}{\sqrt{a^2+b^2}}$

2.6 Selection of Co-ordinate of a Point on a Straight line

(1) If the equation of the straight line be $ax+by+c=0$, in order to select a point on it, take the x co-ordinate according to your sweet will. Let $x=\lambda$; then $a\lambda+by+c=0$ or $y=-\frac{a\lambda+c}{b}$;

$\therefore\ \left(\lambda,-\frac{a\lambda+c}{b}\right)$ is a point on the line for any real value of λ. If $\lambda=0$ is taken then the point will be $\left(0,-\frac{c}{b}\right)$.

Similarly a suitable point can be taken as $\left(-\frac{c}{a},0\right)$.

(2) If the equation of the line be $x=c$ then a point on it can be taken as (c,λ) where λ has any real value.

In particular (c, 0) is a convenient point on it when $\lambda=0$.

(3) If the equation of the line be $y=c$ then a point on it can be taken as (λ,c) where λ has any real value.

In particular (0, c) is a convenient point on it when $\lambda=0$.

Example: 11 If we reduce $3x+3y+7=0$ to the form $x\cos\alpha+y\sin\alpha=p$, then the value of p is **[MP PET 2001]**

(a) $\frac{7}{2\sqrt{3}}$ (b) $\frac{7}{3}$ (c) $\frac{3\sqrt{7}}{2}$ (d) $\frac{7}{3\sqrt{2}}$

Solution: (d) Given equation is $3x+3y+7=0$, Dividing both sides by $\sqrt{3^2+3^2}$

$$\Rightarrow \frac{3x}{\sqrt{3^2+3^2}}+\frac{3y}{\sqrt{3^2+3^2}}+\frac{7}{\sqrt{3^2+3^2}}=0 \Rightarrow \frac{3}{3\sqrt{2}}x+\frac{3}{3\sqrt{2}}y=\frac{-7}{3\sqrt{2}}, \quad \therefore \quad p=\left|\frac{-7}{3\sqrt{2}}\right|=\frac{7}{3\sqrt{2}}.$$

2.7 Point of Intersection of Two lines

Let $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ be two non-parallel lines. If (x', y') be the co-ordinates of their point of intersection, then $a_1x'+b_1y'+c_1=0$ and $a_2x'+b_2y'+c_2=0$

Solving these equation, we get $(x',y')=\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}, \frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}\right) = \left(\frac{\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}, \frac{\begin{vmatrix} c_1 & c_2 \\ a_1 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}}\right)$

Note : ❑ Here lines are not parallel, they have unequal slopes, then $a_1b_2-a_2b_1 \neq 0$.

❑ In solving numerical questions, we should not remember the co-ordinates (x', y') given above, but we solve the equations directly.

2.8 General equation of Lines through the Intersection of Two given Lines

If equation of two lines $P=a_1x+b_1y+c_1=0$ and $Q=a_2x+b_2y+c_2=0$, then the equation of the lines passing through the point of intersection of these lines is $P+\lambda Q=0$ or $a_1x+b_1y+c+\lambda(a_2x+b_2y+c_2)=0$; Value of λ is obtained with the help of the additional information given in the problem.

Example: 12 Equation of a line passing through the point of intersection of lines $2x-3y+4=0$, $3x+4y-5=0$ and perpendicular to $6x-7y+3=0$, then its equation is **[Rajasthan PET 2000]**

(a) $119x+102y+125=0$ (b) $119x+102y=125$ (c) $119x-102y=125$ (d) None of these

Solution: (b) The point of intersection of the lines $2x-3y+4=0$ and $3x+4y-5=0$ is $\left(\frac{-1}{17}, \frac{22}{17}\right)$

The slope of required line = $\frac{-7}{6}$.

Hence, Equation of required line is, $y-\frac{22}{7}=\frac{-7}{6}\left(x+\frac{2}{34}\right) \Rightarrow 119x+102y=125$.

Example: 13 The equation of straight line passing through point of intersection of the straight lines $3x-y+2=0$ and $5x-2y+7=0$ and having infinite slope is **[UPSEAT 2001]**

(a) $x=2$ (b) $x+y=3$ (c) $x=3$ (d) $x=4$

Solution: (c) Required line should be, $(3x-y+2)+\lambda(5x-2y+7)=0$ .....(i)

$\Rightarrow (3+5\lambda)x-(2\lambda+1)y+(2+7\lambda)=0 \Rightarrow y=\left(\frac{3+5\lambda}{2\lambda+1}\right)x+\frac{2+7\lambda}{2\lambda+1}$ ......(ii)

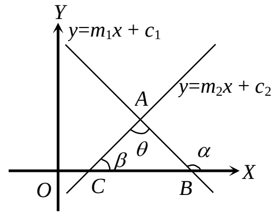
As the equation (ii) has infinite slope, $2\lambda+1=0 \Rightarrow \lambda=\frac{-1}{2}$

Putting $\lambda=\frac{-1}{2}$ in equation (i), We have $(3x-y+2)+\left(\frac{-1}{2}\right)(5x-2y+7)=0 \Rightarrow x=3$

2.9 Angle between Two non-parallel Lines

Let θ be the angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$.

and intersecting at A.



where, $m_1 = \tan\alpha$ and $m_2 = \tan\beta$

$\because$ $\alpha = \theta + \beta \Rightarrow \theta = \alpha - \beta$

$\Rightarrow$ $\tan\theta = \left|\dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}\right|$

$\therefore$ $\theta = \tan^{-1}\left|\dfrac{m_1 - m_2}{1 + m_1m_2}\right|$.

(1) **Angle between two straight lines when their equations are given :** The angle θ between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by, $\tan\theta = \left|\dfrac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right|$.

(i) **Condition for the lines to be parallel :** If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel then, $m_1 = m_2 \Rightarrow \dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} \Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2}$.

(ii) **Condition for the lines to be perpendicular :** If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular then, $m_1m_2 = -1 \Rightarrow \dfrac{a_1}{b_1} \times \dfrac{a_2}{b_2} = -1 \Rightarrow a_1a_2 + b_1b_2 = 0$.

(iii) **Conditions for two lines to be coincident, parallel, perpendicular and intersecting :** Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are,

(a) Coincident, if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ (b) Parallel, if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ (c) Intersecting, if $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

(d) Perpendicular, if $a_1a_2 + b_1b_2 = 0$

Example: 14 Angle between the lines $2x - y - 15 = 0$ and $3x + y + 4 = 0$ is **[Rajasthan PET 2003]**

(a) $90°$ (b) $45°$ (c) $180°$ (d) $60°$

Solution: (b) $\tan\theta = \left|\dfrac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}\right| = \left|\dfrac{(3)(-1)-(2)(1)}{(3)(2)+(-1)(1)}\right| \Rightarrow \tan\theta = \left|\dfrac{-3-2}{6-1}\right| = \left|\dfrac{-5}{5}\right| = |-1|$

$\theta = \tan^{-1}|-1| = \tan^{-1}1 = 45^o$.

Example: 15 To which of the following types the straight lines represented by $2x + 3y - 7 = 0$ and $2x + 3y - 5 = 0$ belongs **[MP PET 1982]**

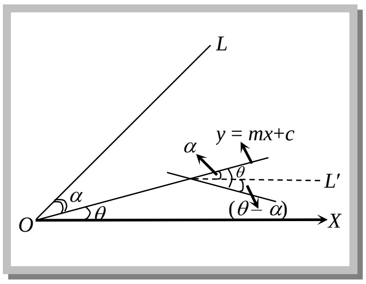
(a) Parallel to each other (b) Perpendicular to each other

(c) Inclined at 45° to each other (d) Coincident pair of straight lines

Solution: (a) Here, $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$; $\dfrac{2}{3} = \dfrac{2}{3} \neq \dfrac{7}{5}$. Hence, lines are parallel to each other.

2.10 Equation of Straight line through a given point making a given Angle with a given Line

Since straight line L makes an angle $(\theta+\alpha)$ with x-axis, then equation of line L is $y-y_1=\tan(\theta+\alpha)(x-x_1)$ and straight line L' makes an angle $(\theta-\alpha)$ with x-axis, then equation of line L' is



$\Rightarrow \qquad y-y_1=\tan(\theta-\alpha)(x-x_1)$

where $\qquad m=\tan\theta$

Hence, the equation of the straight lines which pass through a given point (x_1, y_1) and make a given angle α with given straight line $y=mx+c$ are

$$y-y_1=\frac{m\pm\tan\alpha}{1\mp m\tan\alpha}(x-x_1)$$

Example: 16 The equation of the lines which passes through the point (3, – 2) and are inclined at 60^o to the line $\sqrt{3}x+y=1$ **[IIT 1974; MP PET 1996]**

(a) $y+2=0,\ \sqrt{3}x-y-2-3\sqrt{3}=0$ (b) $x-2=0,\ \sqrt{3}x-y+2+3\sqrt{3}=0$

(c) $\sqrt{3}x-y-2-3\sqrt{3}=0$ (d) None of these

Solution: (a) The equation of lines passing through (3, –2) is $(y+2)=m(x-3)$ .....(i)

The slope of the given line is $-\sqrt{3}$.

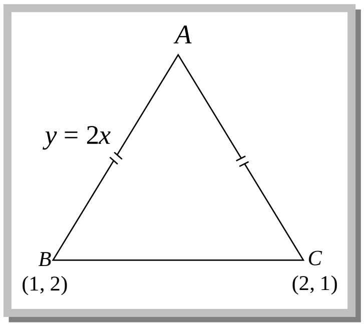
So, $\tan 60^o=\pm\frac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}$. On solving, we get $m=0$ or $\sqrt{3}$

Putting the values of m in (i), the required equation is $y+2=0$ and $\sqrt{3}x-y-2-3\sqrt{3}=0$.

Example: 17 In an isosceles triangle ABC, the coordinates of the point B and C on the base BC are respectively (1, 2) and (2, 1). If the equation of the line AB is $y=2x$, then the equation of the line AC is **[Roorkee 2000]**

(a) $y=\frac{1}{2}(x-1)$ (b) $y=\frac{x}{2}$ (c) $y=x-1$ (d) $2y=x+3$

Solution: (b) Slope of $BC=\frac{1-2}{2-1}=-1$



$\because\ AB=AC$, $\therefore\ \angle ABC=\angle ACB$

$\Rightarrow \left|\frac{2+1}{1+2(-1)}\right|=\frac{m+1}{1+m(-1)} \Rightarrow \frac{m+1}{1-m}=|-3| \Rightarrow \frac{m+1}{1-m}=\pm 3 \Rightarrow m=2,\frac{1}{2}$.

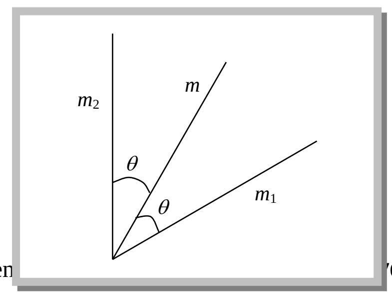
But slope of AB is 2; $\therefore\ m=\frac{1}{2}$ (Here m is the gradient of the line AC)

Equation of the line AC is $y-1=\frac{1}{2}(x-2) \Rightarrow x-2y=0$ or $y=\frac{x}{2}$.

2.11 A Line equally inclined with Two lines

Let the two lines with slopes m_1 and m_2 be equally inclined to a line with slope m

then, $\left(\frac{m_1-m}{1+m_1 m}\right)=-\left(\frac{m_2-m}{1+m_2 m}\right)$



Note : ❑ Sign of m in both brackets is same.

Example: 18 If the lines $y=3x+1$ and $2y=x+3$ are equally inclined to the line $y=mx+4$, then **6]**

(a) $\frac{1+3\sqrt{2}}{7}$ (b) $\frac{1-3\sqrt{2}}{7}$ (c) $\frac{1+3\sqrt{2}}{7}$ (d) $\frac{1\pm 5\sqrt{2}}{7}$

Solution: (d) If line $y = mx + 4$ are equally inclined to lines with slope $m_1 = 3$ and $m_2 = \frac{1}{2}$,then $\left(\frac{3-m}{1+3m}\right) = -\left(\frac{\frac{1}{2}-3}{1+\frac{1}{2}m}\right) \Rightarrow m = \frac{1\pm 5\sqrt{2}}{7}$

2.12 Equations of the bisectors of the Angles between two Straight lines

The equation of the bisectors of the angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given by, $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$ .....(i)

Algorithm to find the bisector of the angle containing the origin :

Let the equations of the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. To find the bisector of the angle containing the origin, we proceed as follows:

Step I : See whether the constant terms c_1 and c_2 in the equations of two lines positive or not. If not, then multiply both the sides of the equation by –1 to make the constant term positive.

Step II : Now obtain the bisector corresponding to the positive sign *i.e.*, $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$.

This is the required bisector of the angle containing the origin.

Note : ❑ The bisector of the angle containing the origin means the bisector of the angle between the lines which contains the origin within it.

(1) To find the acute and obtuse angle bisectors

Let θ be the angle between one of the lines and one of the bisectors given by (i). Find $\tan\theta$. If $|\tan\theta| < 1$, then this bisector is the bisector of acute angle and the other one is the bisector of the obtuse angle.

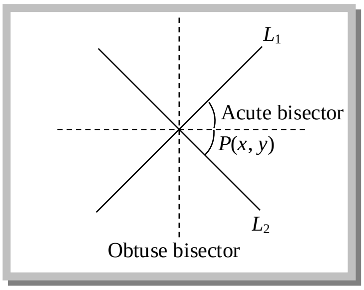
If $|\tan\theta| > 1$, then this bisector is the bisector of obtuse angle and other one is the bisector of the acute angle.

(2) Method to find acute angle bisector and obtuse angle bisector

(i) Make the constant term positive, if not. (ii) Now determine the sign of the expression $a_1a_2 + b_1b_2$.

(iii) If $a_1a_2 + b_1b_2 > 0$, then the bisector corresponding to "+" sign gives the obtuse angle bisector and the bisector corresponding to "–" sign is the bisector of acute angle between the lines.

(iv) If $a_1a_2 + b_1b_2 < 0$, then the bisector corresponding to "+" and "–" sign given the acute and obtuse angle bisectors respectively.



Note : ❑ Bisectors are perpendicular to each other.

❑ If $a_1a_2 + b_1b_2 > 0$, then the origin lies in obtuse angle and if $a_1a_2 + b_1b_2 < 0$, then the origin lies in acute angle.

Example: 19 The equation of the bisectors of the angles between the lines $|x| = |y|$ are **[Orissa JEE 2002]**

(a) $y = \pm x$ and $x = 0$ (b) $x = \frac{1}{2}$ and $y = \frac{1}{2}$ (c) $y = 0$ and $x = 0$ (d) None of these

Solution: (c) The equation of lines are $x + y = 0$ and $x - y = 0$.

$\therefore$ The equation of bisectors of the angles between these lines are $\frac{x+y}{\sqrt{1+1}} = \pm\frac{x-y}{\sqrt{1+1}} \Rightarrow x + y = \pm(x - y)$

Taking +*ve* sign, we get $y = 0$; Taking –*ve* sign, we get $x = 0$. Hence, the equation of bisectors are $x = 0,\ y = 0$.

Example: 20 The equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$ is

[IIT 1975, 1983; Rajasthan PET 2003]

(a) $21x + 77y - 101 = 0$ (b) $11x - 3y + 9 = 0$ (c) $31x + 77y + 101 = 0$ (d) $11x - 3y - 9 = 0$

Solution: (b) Bisector of the angles is given by $\frac{3x - 4y + 7}{5} = \pm \frac{12x + 5y - 2}{13}$

$\Rightarrow 11x - 3y + 9 = 0$ .....(i) and $21x + 77y - 101 = 0$ ......(ii)

Let the angle between the line $3x - 4y + 7 = 0$ and (i) is α, then $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{3}{4} - \frac{11}{3}}{1 + \frac{3}{4} \times \frac{11}{3}}\right| = \frac{35}{45} < 1 \Rightarrow \alpha < 45^o$

Hence $11x - 3y + 9 = 0$ is the bisector of the acute angle between the given lines.

2.13 Length of Perpendicular

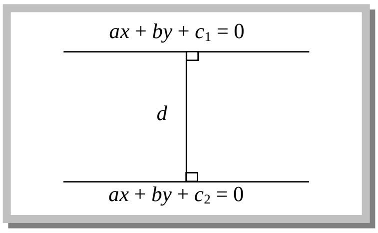
(1) **Distance of a point from a line :** The length p of the perpendicular from the point (x_1, y_1) to the line $ax + by + c = 0$ is given by $p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

Note : ❑ Length of perpendicular from origin to the line $ax + by + c = 0$ is $\frac{c}{\sqrt{a^2 + b^2}}$.

❑ Length of perpendicular from the point (x_1, y_1) to the line $x\cos\alpha + y\sin\alpha = p$ is $x_1\cos\alpha + y_1\sin\alpha - p$.

(2) **Distance between two parallel lines :** Let the two parallel lines be $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$.

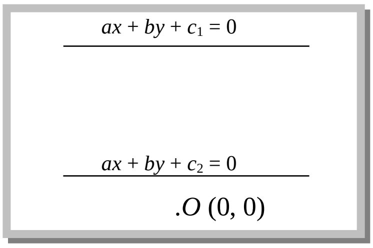
First Method : The distance between the lines is $d = \frac{|c_1 - c_2|}{\sqrt{(a^2 + b^2)}}$.



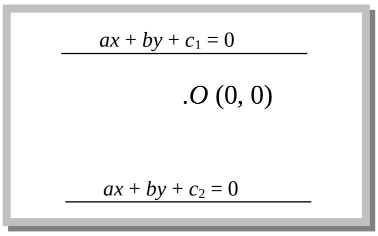
Second Method : The distance between the lines is $d = \frac{\lambda}{\sqrt{(a^2 + b^2)}}$, where

(i) $\lambda = |c_1 - c_2|$ if they be on the same side of origin.

(ii) $\lambda = |c_1| + |c_2|$ if the origin O lies between them.



Third method : Find the coordinates of any point on one of the given line, preferably putting $x = 0$ or $y = 0$. Then the perpendicular distance of this point from the other line is the required distance between the lines.



Note: ❑ Distance between two parallel lines $ax + by + c_1 = 0$ and $kax + kby + c_2 = 0$ is $\frac{\left|c_1 - \frac{c_2}{k}\right|}{\sqrt{a^2 + b^2}}$

❑ Distance between two non parallel lines is always zero.

2.14 Position of a Point with respect to a Line

Let the given line be $ax + by + c = 0$ and observing point is (x_1, y_1), then

(i) If the same sign is found by putting in equation of line $x = x_1, y = y_1$ and $x = 0$, $y = 0$ then the point (x_1, y_1) is situated on the side of origin.

(ii) If the opposite sign is found by putting in equation of line $x = x_1$, $y = y_1$ and $x = 0$, $y = 0$ then the point (x_1, y_1) is situated opposite side to origin.

2.15 Position of Two points with respect to a Line

Two points (x_1, y_1) and (x_2, y_2) are on the same side or on the opposite side of the straight line $ax + by + c = 0$ according as the values of $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign or opposite sign.

Example: 21 The distance of the point (–2, 3) from the line $x - y = 5$ is **[MP PET 2001]**

(a) $5\sqrt{2}$ (b) $2\sqrt{5}$ (c) $3\sqrt{5}$ (d) $5\sqrt{3}$

Solution: (a) $p = \left|\frac{x_1 - y_1 - 5}{\sqrt{1^2 + 1^2}}\right| = \left|\frac{-2 - 3 - 5}{\sqrt{1^2 + 1^2}}\right| = \left|\frac{-10}{\sqrt{2}}\right| = 5\sqrt{2}$

Example: 22 The distance between the lines $4x + 3y = 11$ and $8x + 6y = 15$ is **[AMU 1979; MNR 1987; UPSEAT 2000; DCE 1999]**

(a) $\frac{7}{2}$ (b) 4 (c) $\frac{7}{10}$ (d) None of these

Solution: (c) Given lines $4x + 3y = 11$ and $8x + 6y = 15$, distance from the origin to both the lines are $\left|\frac{-11}{\sqrt{25}}\right|$ and $\left|\frac{-15}{\sqrt{100}}\right| \Rightarrow \frac{11}{5}, \frac{15}{10}$

Clearly both lines are on the same side of the origin.

Hence, distance between both the lines are, $\frac{11}{5} - \frac{15}{10} = \frac{7}{10}$.

Example: 23 If the length of the perpendicular drawn from origin to the line whose intercepts on the axes are *a* and *b* be *p*, then **[Karnataka CET 2003]**

(a) $a^2 + b^2 = p^2$ (b) $a^2 + b^2 = \frac{1}{p^2}$ (c) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{p^2}$ (d) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

Solution: (d) Equation of line is $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow bx + ay - ab = 0$

Perpendicular distance from origin to given line is $p = \left|\frac{-ab}{\sqrt{a^2 + b^2}}\right| \Rightarrow \frac{\sqrt{a^2 + b^2}}{ab} = \frac{1}{p} \Rightarrow \frac{a^2 + b^2}{a^2b^2} = \frac{1}{p^2} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

Example: 24 The point on the *x*-axis whose perpendicular distance from the line $\frac{x}{a} + \frac{y}{b} = 1$ is *a*, is **[Rajasthan PET 2001; MP PET 2003]**

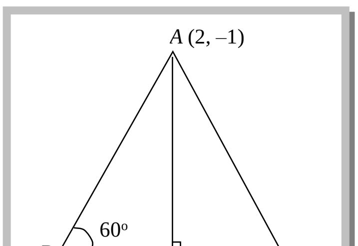
(a) $\left[\frac{a}{b}(b \pm \sqrt{a^2 + b^2}), 0\right]$ (b) $\left[\frac{b}{a}(b \pm \sqrt{a^2 + b^2}), 0\right]$ (c) $\left[\frac{a}{b}(a \pm \sqrt{a^2 + b^2}), 0\right]$ (d) None of these

Solution: (a) Let the point be $(h, 0)$ then $a = \pm\frac{bh + 0 - ab}{\sqrt{a^2 + b^2}} \Rightarrow bh = \pm a\sqrt{a^2 + b^2} + ab \Rightarrow h = \frac{a}{b}(b \pm \sqrt{a^2 + b^2})$

Hence the point is $\left[\frac{a}{b}(b \pm \sqrt{a^2 + b^2}), 0\right]$

Example: 25 The vertex of an equilateral triangle is (2, –1) and the equation of its base is $x + 2y = 1$. The length of its sides is **[UPSEAT 2003]**

(a) $\frac{4}{\sqrt{15}}$ (b) $\frac{2}{\sqrt{15}}$



(c) $\frac{4}{3\sqrt{3}}$ (d) None of these

Solution: (b) $|AD| = \left|\frac{2-2-1}{\sqrt{1^2+2^2}}\right| = \frac{1}{\sqrt{5}}$

$\because \tan 60^o = \frac{AD}{BD} \Rightarrow \sqrt{3} = \frac{1/\sqrt{5}}{BD} \Rightarrow BD = \frac{1}{\sqrt{15}} \Rightarrow BC = 2BD = \frac{2}{\sqrt{15}}$

2.16 Concurrent Lines

Three or more lines are said to be concurrent lines if they meet at a point.

First method : Find the point of intersection of any two lines by solving them simultaneously. If the point satisfies the third equation also, then the given lines are concurrent.

Second method : The three lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ are concurrent if,

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Third method : The condition for the lines $P=0$, $Q=0$ and $R=0$ to be concurrent is that three constants a, b, c (not all zero at the same time) can be obtained such that $aP+bQ+cR=0$.

Example: 26 If the lines $ax+by+c=0$, $bx+cy+a=0$ and $cx+ay+b=0$ be concurrent, then **[IIT 1985; DCE 2002]**

(a) $a^3+b^3+c^3+3abc=0$ (b) $a^3+b^3+c^3-abc=0$ (c) $a^3+b^3+c^3-3abc=0$ (d) None of these

Solution: (c) Here the given lines are, $ax+by+c=0$, $bx+cy+a=0$, $cx+ay+b=0$

The lines will be concurrent, *iff* $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow a^3+b^3+c^3-3abc=0$

Example: 27 If the lines $4x+3y=1$, $y=x+5$ and $5y+bx=3$ are concurrent, then b equals **[Rajasthan PET 1996; MP PET 1997; EAMCET 2003]**

(a) 1 (b) 3 (c) 6 (d) 0

Solution: (c) If these lines are concurrent then the intersection point of the lines $4x+3y=1$ and $y=x+5$, is (–2, 3), which lies on the third line.

Hence, $\Rightarrow 5\times 3-2b=3 \Rightarrow 15-2b=3 \Rightarrow 2b=12 \Rightarrow b=6$

Example: 28 The straight lines $4ax+3by+c=0$ where $a+b+c=0$, will be concurrent, if point is **[Rajasthan PET 2002]**

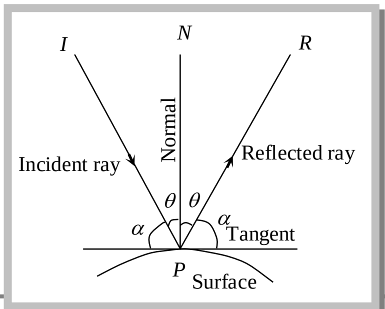
(a) (4, 3) (b) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (d) None of these

Solution: (b) The set of lines is $4ax+3by+c=0$, where $a+b+c=0$

Eliminating c, we get $4ax+3by-(a+b)=0 \Rightarrow a(4x-1)+b(3y-1)=0$

They pass through the intersection of the lines $4x-1=0$ and $3y-1=0$ *i.e.*, $x=\frac{1}{4}, y=\frac{1}{3}$ *i.e.*, $\left(\frac{1}{4}, \frac{1}{3}\right)$

2.17 Reflection on the Surface



Here IP = Incident Ray

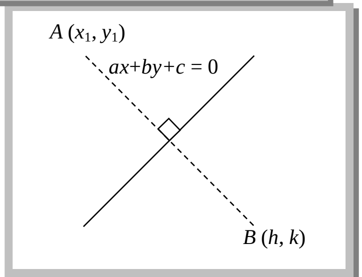
PN = Normal to the surface

PR = Reflected Ray

Then, $\angle IPN = \angle NPR$

Angle of incidence = Angle of reflection

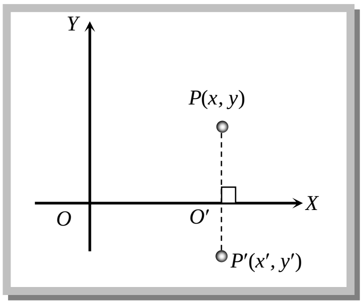
2.18 Image of a Point in Different cases



(1) **The image of a point with respect to the line mirror :** The image of $A(x_1, y_1)$ with respect to the line mirror $ax+by+c=0$ be $B(h, k)$ is given by,

$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

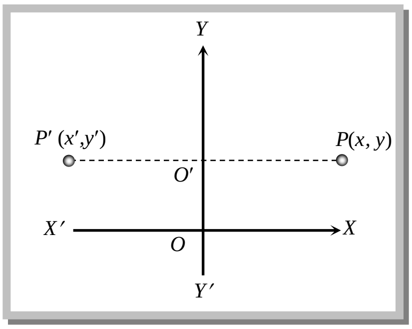
(2) **The image of a point with respect to x-axis :** Let $P(x, y)$ be any point and $P'(x', y')$ its image after reflection in the x-axis, then



$x' = x$ $\quad$ ($\because$ O' is the mid point of P and P')

$y' = -y$

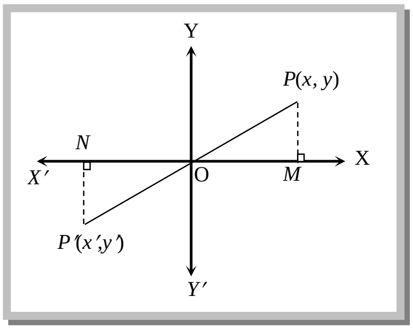
(3) **The image of a point with respect to y-axis :** Let $P(x, y)$ be any point and $P'(x', y')$ its image after reflection in the y-axis



then $\quad x' = -x$ $\quad$ ($\because$ O' is the mid point of P and P')

$y' = y$

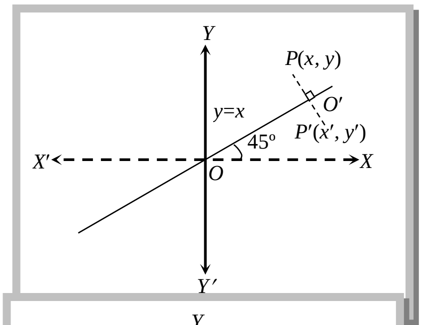
(4) **The image of a point with respect to the origin :** Let $P(x, y)$ be any point and $P'(x', y')$ be its image after reflection through the origin, then



$x' = -x$ $\quad$ ($\because$ O' is the mid point of P and P')

$y' = -y$

(5) **The image of a point with respect to the line $y = x$:** Let $P(x, y)$ be any point and $P'(x', y')$ be its image after reflection in the line $y = x$, then

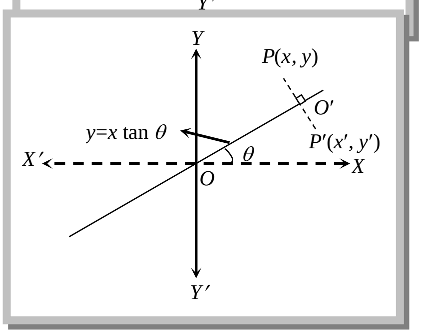


$x' = y$ $\quad$ ($\because$ O' is the mid point of P and P')

$y' = x$

(6) **The image of a point with respect to the line $y = x \tan\theta$:**

Let $P(x, y)$ be any point and $P'(x', y')$ be its image after reflection in the line $y = x\tan\theta$ then



$x' = x\cos 2\theta + y\sin 2\theta$ $\quad$ ($\because$ O' is the mid point of P and P')

$y' = x\sin 2\theta - y\cos 2\theta$

Example: 29 The reflection of the point (4, –13) in the line $5x + y + 6 = 0$ is **[EAMCET 1994]**

(a) (–1, –14) (b) (3, 4) (c) (1, 2) (d) (–4, 13)

Solution: (a) Let $Q(a,b)$ be the reflection of $P(4,-13)$ in the line $5x + y + 6 = 0$. Then the point $R\left(\frac{a+4}{2}, \frac{b-13}{2}\right)$ lies on $5x + y + 6 = 0$

. $\therefore$ $5\left(\frac{a+4}{2}\right) + \left(\frac{b-13}{2}\right) + 6 = 0 \Rightarrow 5a + b + 19 = 0$ .....(i)

Also PQ is perpendicular to $5x + y + 6 = 0$. Therefore $\left(\frac{b+13}{a-4}\right) \times \left(\frac{-5}{1}\right) \Rightarrow a - 5b - 69 = 0$ .....(ii)

Solving (i) and (ii), we get $a = -1, b = -14$.

Example: 30 The image of a point $A(3,8)$ in the line $x + 3y - 7 = 0$, is **[Rajasthan PET 1991]**

(a) (–1, –4) (b) (–3, –8) (c) (1, –4) (d) (3, 8)

Solution: (a) Equation of the line passing through (3, 8) and perpendicular to $x + 3y - 7 = 0$ is $3x - y - 1 = 0$. The intersection point of both the lines is (1, 2). Now let the image of $A(3,8)$ be $A'(x_1, y_1)$.

The point (1, 2) will be the midpoint of AA'. $\frac{x_1 + 3}{2} = 1 \Rightarrow x_1 = -1$ and $\frac{y_1 + 8}{2} = 2 \Rightarrow y_1 = 4$. Hence the image is (–1, –4).

2.19 Some Important Results

(1) Area of the triangle formed by the lines $y = m_1x + c_1$, $y = m_2x + c_2, y = m_3x + c_3$ is $\frac{1}{2}\left|\sum \frac{(c_1 - c_2)^2}{m_1 - m_2}\right|$.

(2) Area of the triangle made by the line $ax + by + c = 0$ with the co-ordinate axes is $\frac{c^2}{2|ab|}$.

(3) Area of the rhombus formed by the lines $ax \pm by \pm c = 0$ is $\left|\frac{2c^2}{ab}\right|$

(4) Area of the parallelogram formed by the lines $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$, $a_1x + b_1y + d_1$ and $a_2x + b_2y + d_2 = 0$ is $\left|\frac{(d_1 - c_1)(d_2 - c_2)}{a_1b_2 - a_2b_1}\right|$.

(5) The foot of the perpendicular (h,k) from (x_1, y_1) to the line $ax + by + c = 0$ is given by $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$. Hence, the coordinates of the foot of perpendicular is $\left(\frac{b^2x_1 - aby_1 - ac}{a^2 + b^2}, \frac{a^2y_1 - abx_1 - bc}{a^2 + b^2}\right)$

(6) Area of parallelogram $A = \frac{p_1p_2}{\sin\theta}$, where p_1 and p_2 are the distances between parallel sides and θ is the angle between two adjacent sides.

(7) The equation of a line whose mid-point is (x_1, y_1) in between the axes is $\frac{x}{x_1} + \frac{y}{y_1} = 2$

(8) The equation of a straight line which makes a triangle with the axes of centroid (x_1, y_1) is $\frac{x}{3x_1} + \frac{y}{3y_1} = 1$.

Example: 31 The coordinates of the foot of perpendicular drawn from (2, 4) to the line $x + y = 1$ is **[Roorkee 1995]**

(a) $\left(\frac{1}{3}, \frac{3}{2}\right)$ (b) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (c) $\left(\frac{4}{3}, \frac{1}{2}\right)$ (d) $\left(\frac{3}{4}, \frac{-1}{2}\right)$

Solution: (b) Applying the formula, the required co-ordinates is $\left(\frac{1^2 \times 2 - 1 \times 1 \times 4 + 1}{1^2 + 1^2}, \frac{1^2 \times 4 - 1 \times 1 \times 2 + 1}{1^2 + 1^2}\right) = \left(\frac{-1}{2}, \frac{3}{2}\right)$

Example: 32 The area enclosed within the curve $|x| + |y| = 1$ is **[Rajasthan PET 1990, 97; IIT 1981; UPSEAT 2003]**

(a) $\sqrt{2}$ (b) 1 (c) $\sqrt{3}$ (d) 2

Solution: (d) The given lines are $\pm x \pm y = 1$ *i.e.*, $x + y = 1$, $x - y = 1$, $x + y = -1$ and $x - y = -1$. These lines form a quadrilateral whose vertices are $A(-1, 0)$, $B(0, -1)$, $C(1, 0)$ and $D(0, 1)$. Obviously $ABCD$ is a square. Length of each side of this square is $\sqrt{1^2 + 1^2} = \sqrt{2}$. Hence, area of square is $\sqrt{2} \times \sqrt{2} = 2$ sq. units.

Example: 33 If x_1, x_2, x_3 and y_1, y_2, y_3 are both in G.P. with the same common ratio, then the point (x_1, y_1), (x_2, y_2) and (x_3, y_3) **[AIEEE 2003]**

(a) Lie on a straight line (b) Lie on an ellipse (c) Lie on a circle (d) Are vertices of a triangle

Solution: (a) Taking co-ordinates as $\left(\frac{x}{r}, \frac{y}{r}\right)$, (x, y) and (x_r, y_r). Above co-ordinates satisfy the relation $y = mx$, $\therefore$ the three points lie on a straight line.

Example: 34 A square of side a lies above the *x*-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of *x*-axis. The equation of its diagonal not passing through the origin is **[AIEEE 2003]**

(a) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$ (b) $y(\cos\alpha + \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

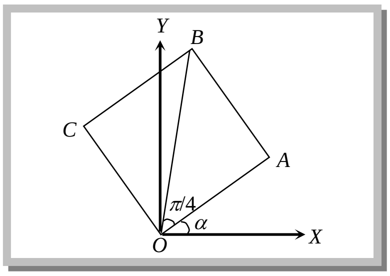
(c) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$ (d) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

Solution: (b) Co-ordinates of $A = (a\cos\alpha, a\sin\alpha)$; Equation of OB $y = \tan\left(\frac{\pi}{4} + \alpha\right)x$

$\because$ $CA \perp$ to OB; $\therefore$ slope of $CA = -\cot\left(\frac{\pi}{4} + \alpha\right)$

Equation of *CA*, $y - a\sin\alpha = -\cot\left(\frac{\pi}{4} + \alpha\right)(x - a\cos\alpha)$

$\Rightarrow$ $y(\sin\alpha + \cos\alpha) + x(\cos a - \sin\alpha) = a$.



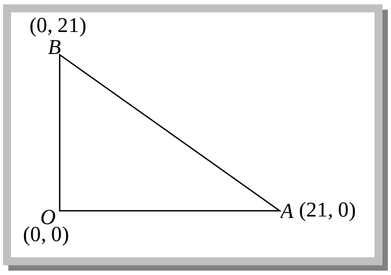
Example: 35 The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0) is **[IIT Screening 2003]**

(a) 133 (b) 190 (c) 233 (d) 105

Solution: (b) $x + y = 21$

The number of integral solution to the equation $x + y < 21$ *i.e.*, $x < 21 - y$

Number of integral co-ordinates $= 19 + 18 + ... + 1 = \frac{19 \times 20}{2} = 190$.



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