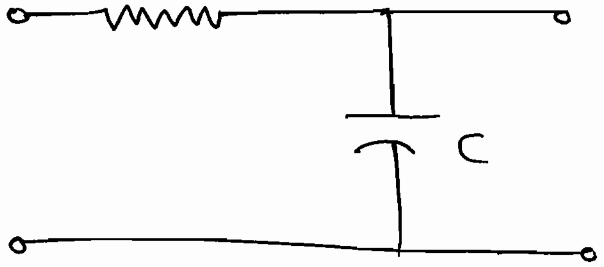
Ch- - Z- Transform

⇒ Generation (or) Generalization of DTFT is Z- transform.

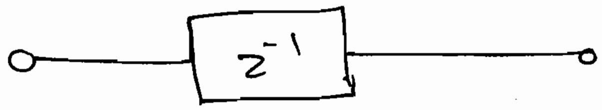
⇒ Discrete version of L.T.

⇒ $R = 1K\Omega \pm 2\%$ tolerance



$$H(s) = \frac{1}{1+SCR}.$$

⇓



⇒ $S = \sigma + j\omega$

↓

Complex variable

↓

$$\boxed{z = r e^{j\omega}}$$

⇒ In L.T. if i/p $x(t) = e^{st}$ ⇒ $\boxed{y(t) = e^{st}.H(s)}$

⇒ In Z.T. $x(n) = z^n \rightarrow \boxed{h(n) \downarrow H(z)} \rightarrow$ o/p $\boxed{y(n) = z^n . H(z)}$

⇒ Z.T. of general D.T. signal $x(n)$ is

$$\boxed{X(z) = \sum_{n=-\infty}^{+\infty} x(n). z^{-n}}.$$

Now $z = r \cdot e^{j\omega}$.

$$\therefore X(r \cdot e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left[x[n] \cdot r^{-n}\right] e^{-j\omega n}$$

$$\Rightarrow \boxed{X(z) = F.T.\{x(n) \cdot r^{-n}\}}$$

if $r = 1$

$$\Rightarrow X(z) = F.T.\{x(n)\}$$

$$\Rightarrow \boxed{Z.T = D.T.F.T.}$$

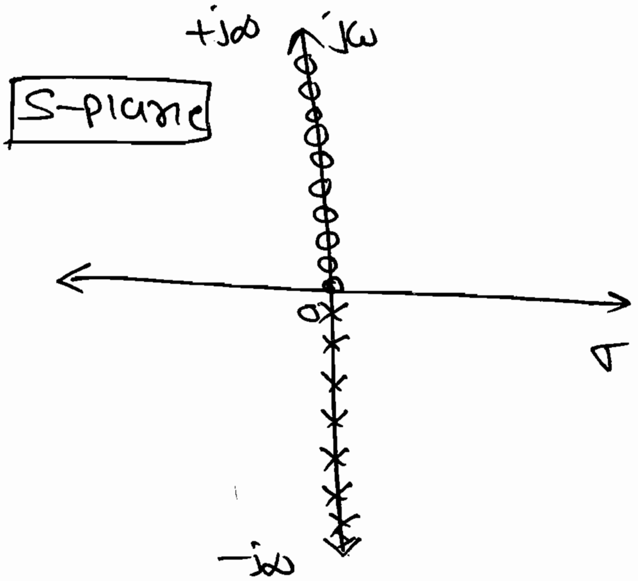
L.T.

$$X(s) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-st} dt$$

$$X(s) = F.T\{x(t) \cdot e^{-\sigma t}\}$$

if $\sigma = 0 \Rightarrow s = j\omega$

L.T = C.T.F.T



$\Rightarrow$ L.T. calculated on $j\omega$ axis is C.T.F.T.

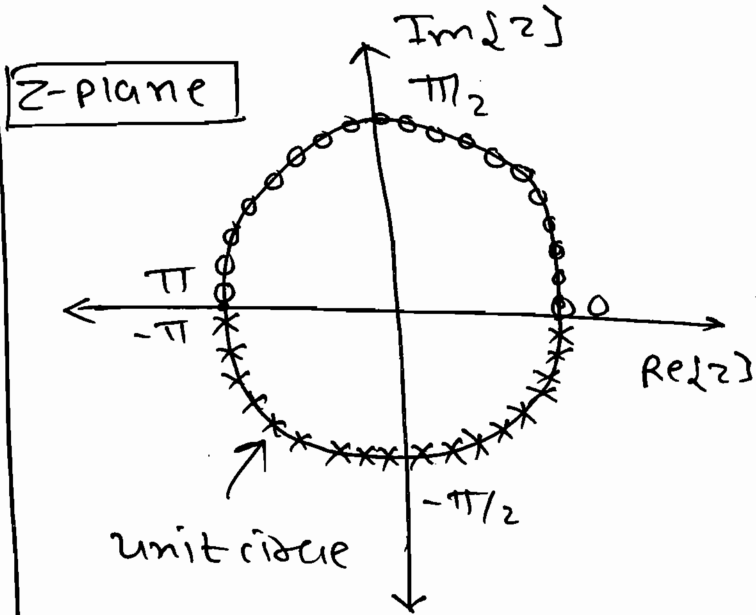
Z.T

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}.$$

$$X(z) = \sum F.T.\{x(n) \cdot r^{-n}\}$$

if $r = 1$

Z.T = D.T.F.T.



$\Rightarrow$ Z.T. calculated on unit circle is D.T.F.T.

Note:

⇒ +ve part of the 'jω' axis is corresponds to upper half of the unit circle. (ω varies from 0 to π).

⇒ −ve part of the 'jω' axis is corresponds to lower half of the unit circle (ω varies from 0 to −π).

⇒

$$x(t) = e^{st} \longrightarrow \text{Cont}^n.$$

$$\downarrow t = nT_s$$

$$x(nT_s) = e^{snT_s} \longrightarrow \text{Discrete}$$

$$\Rightarrow \quad x[n] = (e^{sT_s})^n \Leftrightarrow x[n] = z^n$$

$$\Downarrow$$

$$\boxed{z = e^{sT_s}}$$

⇒ The range of values of 'z' for which eq (1) is defined

$$\left[\sum_{n=-\infty}^{\infty} |x(n) z^{-n}| < \infty \right]$$ is R.O.C. of Z.T.