CBSE Board
Class XII Mathematics
Board Paper 2010
Delhi Set – 2

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three Section A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.
3. All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Evaluate: $\int \frac{\log x}{x} dx$

2. If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α is A an identity matrix?

3. What is the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

4. What is the cosine of the angle which the vector $\sqrt{2}\hat{i}+\hat{j}+k$ makes with y-axis?

5. Write a vector of magnitude 15 units in the direction of vector $\hat{i}-2\hat{j}+2k$

6. What is the range of the function f(x) = $\frac{|x-1|}{(x-1)}$?

7. Find the minor of the element of second row and third column (a_{23}) in the following determinant:

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

8. Write the vector equation of the following line:

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$$

9. What is the degree of the following differential equation?

$$5x\left(\frac{dy}{dx}\right)^2-\frac{d^2y}{dx^2}-6y=\log x$$

10. If $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & 5\end{bmatrix}=\begin{bmatrix}7 & 11\\ k & 23\end{bmatrix}$, then write the value of k.

SECTION – B

11. Find all points of discontinuity of f, where f is defined as following:

$$f(x) = \begin{cases}|x|+3 & , x\leq -3\\ -2x & , -3<x<3\\ 6x+2 & , x\geq 3\end{cases}$$

OR

Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$

12. Prove the following:

$$\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in(0,1)$$

OR

Prove the following:

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$$

13. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

14. Let $*$ be a binary operation on Q defined by $a * b = \frac{3ab}{5}$

Show that $*$ is commutative as well as associative. Also find its identity element, if it exists.

15. Using elementary row operations, find the inverse of the following matrix:

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

16. Find the Cartesian equation of the plane passing through the points A(0, 0, 0) and B(3, -1, 2) and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$

17. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-3\vec{b}\right)$ respectively, externally in the ratio 1:2. Also, show that P is the midpoint of the line segment R.

18. Evaluate: $\int_0^{\pi} \frac{x}{1+\sin x}dx$

19. Evaluate: $\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$

OR

Evaluate: $\int \frac{1-x^2}{x(1-2x)}dx$

20. Find the equations of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

21. Find the particular solution of the differential equation satisfying the given conditions:

$x^2dy + (xy + y^2)dx = 0$; $y = 1$ when $x = 1$.

22. Find the general solution of the differential equation,

$$x\log x \frac{dy}{dx} + y = \frac{2}{x}\log x$$

OR

Find the particular solution of the differential equation satisfying the given conditions:

$\frac{dy}{dx} = y\tan x$, given that $y = 1$ when $x = 0$.

SECTION – C

23. Evaluate $\int_{1}^{3}(3x^2+2x)dx$ as limit of sums.

OR

Using integration, find the area of the following region:

$$\left\{(x,y):\frac{x^2}{9}+\frac{y^2}{4}\le 1\le\frac{x}{3}+\frac{y}{2}\right\}$$

24. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs. 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

25. A card form a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both clubs. Find the probability of the lost card being of clubs.

OR

From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

26. Using properties of determinants show the following:

$$\begin{vmatrix}(b+c)^2 & ab & ca\\ ab & (a+c)^2 & bc\\ ac & bc & (a+b)^2\end{vmatrix} = 2abc\,(a+b+c)^3$$

27. Find the values of x for which $f(x) = [x(x - 2)]^2$ is an increasing function. Also, find the points on the curve where the tangent is parallel to x-axis.

28. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

29. Write the vector equations of the following lines and hence determine the distance between them:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6};\frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$$

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SECTION – A

1. $I=\int\frac{\log x}{x}dx$

Substitute $\log x = t$

Differentiating w.r.t. x, we get

$\frac{1}{x}dx=dt$

On substitution, we get

$I=\int \frac{\log x}{x}dx=\int t.dt=\frac{t^2}{2}+C=\frac{(\log x)^2}{2}+C$

2. Matrix A is a matrix of order 2.

Identity matrix of second order is $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

For A to be an identity matrix,

$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$\Rightarrow \cos\alpha=1$ and $\sin\alpha=0$

$\Rightarrow \cos\alpha=\cos 0^\circ$ and $\sin\alpha=\sin 0^o$

$\Rightarrow \alpha=0^\circ$

Thus, for $\alpha=0^\circ$, A is an identity matrix

3. Let $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=x$

$\Rightarrow \cos x=-\frac{\sqrt{3}}{2}$

$\Rightarrow \cos x=-\cos\left(\frac{\pi}{6}\right)$

$\Rightarrow \cos x=\cos\left(\pi-\frac{\pi}{6}\right)$

$\Rightarrow \cos x=\cos\left(\frac{5\pi}{6}\right)$

$\Rightarrow x=\frac{5\pi}{6}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is $\frac{5\pi}{6}$

4. The y-axis can be represented in vector form by $\hat{j}$ and $-\hat{j}$.

Let $\vec{a}=\sqrt{2}\hat{i}+\hat{j}+k$ and $\vec{b}=\hat{j}$ or $-\hat{j}$

$$\cos\theta=\frac{\hat{a}\cdot b}{|\hat{a}||b|}$$

$$\therefore \cos\theta=\frac{(\sqrt{2}\hat{i}+\hat{j}+k)\cdot(\pm\hat{j})}{|\sqrt{2}\hat{i}+\hat{j}+k||\hat{j}|}=\pm\frac{1\times1}{\sqrt{(\sqrt{2})^2+(1)^2+(1)^2}\times\sqrt{1^2}}=\pm\frac{1}{\sqrt{4}}=\pm\frac{1}{2}$$

So, the cosine of the angle which the vector $\sqrt{2}\hat{i}+\hat{j}+k$ makes with y axis is $\pm\frac{1}{2}$

5. Unit vector along the direction of vector $\vec{a}$, $\hat{a}=\frac{\vec{a}}{|\vec{a}|}$

Let $\vec{a}=\hat{i}-2\hat{j}+2k$

$$|\vec{a}|=\sqrt{(1)^2+(-2)^2+(2)^2}=\pm3$$

i.e. $\hat{a}=\frac{1}{3}(\hat{i}-2\hat{j}+2k)$

So, the vector whose magnitude is 15 and has direction along the vector $\hat{i}-2\hat{j}+2k$ is given by,

$$15\times\left(\frac{1}{3}\right)(i-2\hat{j}+2k)$$

$$=5(\hat{i}-2\hat{j}+2k)$$

$$=(5\hat{i}-10\hat{j}+10k)$$

So the required vector is $5\hat{i}-10\hat{j}+10k$

6.

f(x) is not defined at $x=1$.

For $x\geq1$, $f(x)=\frac{|x-1|}{(x-1)}=\frac{(x-1)}{(x-1)}=1$

For $x<1$, $f(x)=\frac{|x-1|}{(x-1)}=\frac{-(x-1)}{(x-1)}=\frac{(1-x)}{(x-1)}=-1$

Thus, range of the function is either -1 or 1 at all the points and is undefined at x = 1

7.

$$\text{Given determinant} = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

Minor of the element a_{23} is M_{23}

Obtained by deleting III column and II row

$$M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix}$$

$$= 10 - (-3)$$

$$= 13$$

8. The given equation of line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$

The given equation of line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$

i.e in standard form $\frac{x-5}{3} = \frac{y-(-4)}{7} = \frac{z-6}{-2}$

Comparing this equation with standard form $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

We get, $x_1 = 5$, $y_1 = -4$, $z_1 = 6$, $a = 3$, $b = 7$, $c = -2$

Thus, the required line is parallel to the vector $3\hat{i} + 7\hat{j} - 2k$ and passes through the point (5, -4, 6).

The vector form of the line can be written as $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a constant.

Thus, the required equation is $\vec{r} = (5\hat{i} - 4\hat{j} + 6k) + \lambda(3\hat{i} + 7\hat{j} - 2k)$

9. Given differential equation is $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$

In the above equation the highest order derivate is $\frac{d^2y}{dx^2}$ and its power is 1.

Thus, the degree of differential equation $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$ is 1.

10. Given $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & 5\end{bmatrix}=\begin{bmatrix}7 & 11\\ k & 23\end{bmatrix}$

Now using matrix multiplication in LHS, we get

$$\begin{bmatrix}1\times3+2\times2 & 1\times1+2\times5\\ 3\times3+4\times2 & 3\times1+4\times5\end{bmatrix}=\begin{bmatrix}7 & 11\\ k & 23\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}3+4 & 1+10\\ 9+8 & 3+20\end{bmatrix}=\begin{bmatrix}7 & 11\\ k & 23\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}7 & 11\\ 17 & 23\end{bmatrix}=\begin{bmatrix}7 & 11\\ k & 23\end{bmatrix}$$

Now on equating the corresponding elements we get the value of k = 17

SECTION – B

11. Here, f(x) = $\begin{cases}|x|+3, & x\le 3\\ -2x\ , & -3<x<3\\ 6x+2, & x\ge 3\end{cases}$

The function is defined on all the points and hence continuous possible points of discontinuity are 3 and -3 . We need to check the continuity of the function at two points x = 3 and x = -3.

Case 1: For x = -3, f(-3) = -(-3) + 3 = 6

$$\text{LHL}=\lim_{x\to-3^-}f(x)=\lim_{h\to0}(-(-3-h)+3)=6$$

$$\text{RHL}=\lim_{x\to-3^+}f(x)=\lim_{h\to0}\left(-2(-3+h)\right)=(-2)\times(-3)=6$$

$$\text{Since, }\lim_{x\to-3^-}f(x)=\lim_{x\to-3^+}f(x)=f(-3)$$

$$\text{So, f is continuous at } x=-3$$

Case2: For x = 3, f(c) = 6(3) + 2 = 20

$$\text{LHL}=\lim_{x\to3^-}f(x)=\lim_{h\to0}(-2(3-h)=-2\times3=-6$$

$$\text{RHL}=\lim_{x\to3^+}f(x)=\lim_{h\to0}(6(3+h)+2)=6\times3+2=20$$

$$\text{Since, }\lim_{x\to3^-}f(x)\ne\lim_{x\to3^+}f(x)$$

$$\text{Therefore, function f is not continuous at point } x=3$$

Hence x = 3 is the only point of discontinuity.

OR

$y = (\cos x)^{x} + (\sin x)^{\frac{1}{x}}$

For simplification, Let us consider y = A + B such that $A = (\cos x)^{x}$ and $B = (\sin x)^{\frac{1}{x}}$

Then, $\frac{dy}{dx} = \frac{dA}{dx} + \frac{dB}{dx}$ ... (1)

$A = (\cos x)^{x}$

Taking logarithms on both the sides, we have,

logA = x log(cosx)

$$\frac{1}{A}\frac{dA}{dx} = \frac{d}{dx}[x\log(\cos x)]$$

$$\Rightarrow \frac{dA}{dx} = A\frac{d}{dx}[x\log(\cos x)]$$

$$= (\cos x)^{x}\left[x\frac{d}{dx}(\log(\cos x)) + \log(\cos x)\frac{d}{dx}(x)\right]$$

$$= (\cos x)^{x}\left[x\frac{1}{\cos x}(-\sin x) + \log(\cos x)(1)\right]$$

$$= (\cos x)^{x}[-x\tan x + \log(\cos x)] \quad ...(2)$$

Now, B= $(\sin x)^{\frac{1}{x}}$

$B = (\sin x)^{\frac{1}{x}}$

Taking logarithms on both the sides, we have,

logB = $\frac{1}{x}$ log(sinx)

$$\frac{1}{B}\frac{dB}{dx} = \frac{d}{dx}\left[\frac{1}{x}\log(\sin x)\right]$$

$$\Rightarrow \frac{dB}{dx} = B\frac{d}{dx}\left[\frac{1}{x}\log(\sin x)\right]$$

$$= (\sin x)^{\frac{1}{x}}\left[\frac{1}{x}\frac{d}{dx}(\log(\sin x)) + \log(\sin x)\frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= (\sin x)^{\frac{1}{x}}\left[\frac{1}{x}\frac{1}{\sin x}(\cos x) + \log(\sin x)\left(-\frac{1}{x^2}\right)\right]$$

$$= (\sin x)^{\frac{1}{x}}\left[\frac{1}{x}\cot x - \left(\frac{1}{x^2}\right)\log(\sin x)\right]$$

$$= (\sin x)^{\frac{1}{x}}\left[\frac{x\cot x - \log(\sin x)}{x^2}\right] \quad ....(3)$$

Now, on substituting (2) and (3) in (1) we get

$$\frac{dy}{dx}=(\cos x)^{x}\left[-x\tan x+\log(\cos x)\right]+(\sin x)^{\frac{1}{x}}\left[\frac{x\cot x-\log(\sin x)}{x^{2}}\right]$$

12. Let t = $\tan^{-1}\sqrt{x}$

So $\sqrt{x}$ = tan t

i.e. $\tan^2 t = x$

On substituting x in the R.H.S. of equation $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$,

we get $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\cos^{-1}\left(\frac{1-\tan^{2}t}{1+\tan^{2}t}\right)$

Now, using the formula $\cos 2\theta=\left(\frac{1-\tan^{2}\theta}{1+\tan^{2}\theta}\right)$ we have

$$\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\cos^{-1}\left(\cos(2t)\right)$$

$$=t=\tan^{-1}\sqrt{x}=\text{LHS}$$

Hence Proved.

OR

Let a be in I quadrant such that

$\cos^{-1}\left(\frac{12}{13}\right)$ = a

So cos a = $\frac{12}{13}$

$$\Rightarrow \sin a=\sqrt{1-\left(\frac{12}{13}\right)^{2}}$$

$$=\sqrt{1-\frac{144}{169}}$$

$$=\sqrt{\frac{169-144}{169}}$$

$$=\sqrt{\frac{25}{169}}=\frac{5}{13}$$

And tan a = $\frac{5}{12}$

So, a = $\tan^{-1}\left(\frac{5}{12}\right)$ ...(1)

Again $b\in$ I quadrant such that $\sin^{-1}\left(\frac{3}{5}\right)=b$

So, sin b = $\frac{3}{5}$

$$\Rightarrow \cos b = \sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$= \sqrt{1-\frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}} = \frac{4}{5}$$

And tan b = $\frac{3}{4}$

So, b = $\tan^{-1}\left(\frac{3}{4}\right)$ ...(2)

Now, let $\sin^{-1}\left(\frac{56}{65}\right) = c$ where c is in I quadrant

So, sin c = $\frac{56}{65}$

$$\Rightarrow \cos c = \sqrt{1-\left(\frac{56}{65}\right)^2}$$

$$= \sqrt{1-\frac{3136}{4225}}$$

$$= \sqrt{\frac{4225-3136}{4225}}$$

$$= \sqrt{\frac{1089}{4225}} = \frac{33}{65}$$

And, tan c = $\frac{56}{33}$

So $c = \tan^{-1}\left(\frac{56}{33}\right)$

$$\Rightarrow \sin^{-1}\left(\frac{56}{65}\right) = \tan^{-1}\left(\frac{56}{33}\right) \text{....(3)}$$

Now, we need to prove $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

Consider a + b

$$= \cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right)$$

$$= \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{3}{4}\right) \quad \begin{bmatrix} \cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right) \text{ and} \\ \sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right) \end{bmatrix}$$

$$= \tan^{-1}\left(\frac{\frac{5}{12} + \frac{3}{4}}{1 - \left(\frac{5}{12} \times \frac{3}{4}\right)}\right) \quad \left[\text{Using, } \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \tan^{-1}\left(\frac{20+36}{48-15}\right)$$

$$= \tan^{-1}\left(\frac{56}{33}\right)$$

$$= c = \sin^{-1}\left(\frac{56}{65}\right) \qquad [\text{Using,eq(3)}]$$

Hence Proved.

13. Let X denote the number of questions answered correctly by guessing in multiple choice examinations.

Probability of getting a correct answer by guessing, p = $\frac{1}{3}$

Therefore, q, the probability of an incorrect answer by guessing is = 1 - $\frac{1}{3}$ = $\frac{2}{3}$

There are in 5 questions in all.

So X follows binomial distribution with n = 5, p = $\frac{1}{3}$ and q = $\frac{2}{3}$

$$P(X = x) = {}^{n}C_{x}.q^{n-x}.p^{x} = {}^{5}C_{x}.\left(\frac{2}{3}\right)^{5-x}.\left(\frac{1}{3}\right)^{x}$$

P (guessing more than 4 correct answers) = $P(X \geq 4)$

$$= P(X = 4) + P(X = 5)$$

$$= {}^{5}C_{4}.\left(\frac{2}{3}\right)^{5-4}.\left(\frac{1}{3}\right)^{4} + {}^{5}C_{5}.\left(\frac{2}{3}\right)^{5-5}.\left(\frac{1}{3}\right)^{5}$$

$$= 5\times\left(\frac{2}{3}\right).\left(\frac{1}{81}\right) + 1\times1\times\left(\frac{1}{243}\right) \left[\text{Using } {}^{n}C_{r} = \frac{n!}{(n-r)!r!}\right]$$

$$= \frac{11}{243}$$

14. For a, b ∈ Q, * is a binary operation on Q defined as: $a*b=\frac{3ab}{5}$

$\text{Now, } b*a=\frac{3ba}{5}$

$\text{As, } ab=ba$

$\Rightarrow \frac{3ab}{5}=\frac{3ba}{5}$

$\therefore a*b=b*a$

So, the binary operation * is commutative.

Let a, b, c ∈ Q

$a*(b*c)=a*\frac{3bc}{5}$

$$\Rightarrow a*(b*c)=\frac{3a\frac{3bc}{5}}{5} \quad \text{... (1)}$$

$\Rightarrow a*(b*c)=\frac{9abc}{25}$

$\text{Now, } (a*b)*c=\frac{3ab}{5}*c$

$$\Rightarrow (a*b)*c=\frac{3\frac{3ab}{5}c}{5} \quad \text{... (2)}$$

$\Rightarrow (a*b)*c=\frac{9abc}{25}$

From equations (1) and (2):

a * (b * c) = (a * b) * c

So, the binary operation * is associative.

Element e is the identity element on set A for the binary operation * if

a * e = e * a = a   ∀ a∈A

Consider $\frac{5}{3}\in Q$

$a*\frac{5}{3}=\frac{3a\frac{5}{3}}{5}=a$

And $\frac{5}{3}*a=\frac{3\frac{5}{3}a}{5}=a$

$\text{Now,} a*\frac{5}{3}=\frac{5}{3}*a=a$

Therefore, $\frac{5}{3}$ is the identity element of the binary operation * on Q.

15. Let $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$

We can write A = IA

i.e $\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$

Applying $R_1\to\frac{1}{2}R_1$, we get

$\begin{bmatrix}1 & \frac{5}{2}\\ 1 & 3\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0\\ 0 & 1\end{bmatrix}A$

Applying $R_2\to R_2-R_1$, gives

$\begin{bmatrix}1 & \frac{5}{2}\\ 0 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & 0\\ \frac{-1}{2} & 1\end{bmatrix}A$

Applying, $R_1\to R_1-5R_2$ we obtain

$\begin{bmatrix}1 & 0\\ 0 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}3 & -5\\ \frac{-1}{2} & 1\end{bmatrix}A$

Applying $R_2\to 2R_2$, gives

$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}3 & -5\\ -1 & 2\end{bmatrix}A$

Therefore, $A^{-1}=\begin{bmatrix}3 & -5\\ -1 & 2\end{bmatrix}$

16. Let the equation of plane be ax + by + cz + d = 0 .... (1)

Since the plane passes through the point A (0, 0, 0) and B(3, -1, 2), we have

a × 0 + b × 0 + c × 0 + d = 0

$\Rightarrow$ d = 0 .... (2)

Similarly for point B (3, -1 , 2), a x 3 + b x (-1) + c x 2 + d = 0

3a – b + 2c = 0 (Using, d = 0) ...(3)

Given equation of the line is $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$

We can also write the above equation as $\frac{x-4}{1}=\frac{y-(-3)}{-4}=\frac{x-(-1)}{7}$

The required plane is parallel to the above line.

Therefore, a × 1 + b × (-4) + c × 7= 0

$\Rightarrow$ a – 4b + 7c = 0 ... (4)

Cross multiplying equations (3) and (4), we obtain:

$$\frac{a}{(-1)\times 7-(-4)\times 2}=\frac{b}{2\times 1-3\times 7}=\frac{c}{3\times(-4)-1\times(-1)}$$

$$\Rightarrow \frac{a}{-7+8}=\frac{b}{2-21}=\frac{c}{-12+1}$$

$$\Rightarrow \frac{a}{1}=\frac{b}{-19}=\frac{c}{-11}=k$$

$$\Rightarrow a=k, b=-19k, c=-11k$$

Substituting the values of a, b and c in equation (1), we obtain the equation of plane as:

$$kx-19ky-11kz+d=0$$

$$\Rightarrow k(x-19y-11z)=0 \qquad \text{(From equation(2))}$$

$$\Rightarrow x-19y-11z=0$$

So, the equation of the required plane is x – 19y - 11z = 0

17. Position vector of P is $\left(2\vec{a}+\vec{b}\right)$

Position vector of point Q is $\left(\vec{a}-3\vec{b}\right)$

Point R divides the line segment PQ externally in a ratio of 1 : 2.

Position vector of R = $\dfrac{1\left(\vec{a}-3\vec{b}\right)-2\left(2\vec{a}+\vec{b}\right)}{1-2}$

$$= \frac{\vec{a}-3\vec{b}-4\vec{a}-2\vec{b}}{1-2}$$

$$= 3\vec{a}+5\vec{b}$$

Now, we need to show that P is the mid-point of RQ.

So, Position vector of P = $\dfrac{\text{Position vector of R+Position vector of Q}}{2}$

$$= \frac{\left(3\vec{a}+5\vec{b}\right)+\left(\vec{a}-3\vec{b}\right)}{2} = \left(2\vec{a}+\vec{b}\right) = \text{Position vector of P (given)}$$

Hence proved.

18. Let $I=\int_0^{\pi}\frac{x}{1+\sin x}dx$ ...(1)

Using the property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$$\Rightarrow I=\int_0^{\pi}\frac{\pi-x}{1+\sin(\pi-x)}dx$$

$$=\int_0^{\pi}\frac{\pi-x}{1+\sin x}dx \quad ....(2)$$

Now adding (1) and (2), we get

$$2I=\int_0^{\pi}\frac{x}{1+\sin x}dx+\int_0^{\pi}\frac{\pi-x}{1+\sin x}dx$$

$$=\int_0^{\pi}\frac{\pi}{1+\sin x}dx$$

$$=\pi\int_0^{\pi}\frac{1}{1+\sin x}dx$$

$$=\pi\int_0^{\pi}\frac{(1-\sin x)}{(1-\sin^2 x)}dx$$

$$=\pi\int_0^{\pi}\frac{(1-\sin x)}{(\cos^2 x)}dx$$

$$=\pi\left[\int_0^{\pi}\left(\frac{1}{\cos^2 x}-\frac{\sin x}{\cos^2 x}\right)dx\right]$$

$$=\pi\left[\int_0^{\pi}\sec^2 x-\sec x\tan x dx\right]$$

$$=\pi\left[\int_0^{\pi}\sec^2 x dx-\int_0^{\pi}\sec x\tan x dx\right]$$

$$=\pi\left([\tan x]_0^{\pi}-[\sec x]_0^{\pi}\right)$$

$$\Rightarrow 2I=\pi(2)$$

$$\Rightarrow I=\pi$$

So, $\int_0^{\pi}\frac{x}{1+\sin x}dx=\pi$

19. Let $I=\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$

$$=\int e^x\left(\frac{\sin 2(2x)-4}{1-\cos 2(2x)}\right)dx$$

$$=\int e^x\left(\frac{2\sin 2x\cos 2x-4}{2\sin^2(2x)}\right)dx \qquad [\text{Using}, \sin 2x = 2\sin x.\cos x \text{ and } 2\sin^2 x = 1-\cos(2x)$$

$$=\int e^x\left(\frac{2(\sin(2x)\cos(2x)-4}{2\sin^2 2x}\right)dx = \int e^x\left(\frac{\sin(2x)\cos(2x)}{\sin^2 2x}-\frac{2}{\sin^2 2x}\right)dx$$

$$=\int e^x\left(\cot(2x)-2\operatorname{cosec}^2 2x\right)dx$$

Now, let f(x) = cot(2x) then f'(x) = $-2\operatorname{cosec}^2 2x$

$I = \int e^x\left(f(x)+f'(x)\right)dx$

So, I = $e^x f(x) + C = e^x \cot 2x + C$, where C is a constant

Therefore, $\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx = e^x\cot(2x) + C$

OR

$$\int \frac{1-x^2}{x(1-2x)}dx$$

Here $\frac{1-x^2}{x(1-2x)}$ is an improper rational fraction.

Reducing it to proper rational fraction gives

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2}+\frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right) \text{............(1)}$$

Now, let $\frac{2-x}{x(1-2x)} = \frac{A}{x}+\frac{B}{(1-2x)}$

$$\Rightarrow \frac{2-x}{x(1-2x)} = \frac{A(1-2x)+Bx}{x(1-2x)} \Rightarrow 2-x = A-x(2A-B)$$

Equating the coefficients we get, $A = 2$ and $B = 3$

So, $\frac{2-x}{x(1-2x)} = \frac{2}{x}+\frac{3}{(1-2x)}$

Substituting in equation (1), we get

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{(1-2x)}\right)$$

i.e $\int \frac{1-x^2}{x(1-2x)}dx = \int\left[\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{(1-2x)}\right)\right]dx$

$$=\int\frac{dx}{2}+\int\frac{dx}{x}+\frac{3}{2}\int\frac{dx}{(1-2x)} = \frac{x}{2}+\log|x|+\frac{3}{2}\times\frac{1}{(-2)}\log|1-2x|+C$$

$$=\frac{x}{2}+\log|x|-\frac{3}{4}\log|1-2x|+C$$

20. Equation of the curve is $y = x^3 + 2x + 6$

Slope of the normal at point (x, y) = $\frac{-1}{\left(\frac{dy}{dx}\right)}$

$$\frac{dy}{dx} = 3x^2 + 2$$

On substitution, we get

Slope of the normal = $\frac{-1}{3x^2+2}$ ...(1)

Normal to the curve is parallel to the line x + 14y + 4 = 0,

i.e. $y = -\frac{1}{14}x - \frac{4}{14}$

So the slope of the line is the slope of the normal.

Slope of the line is $-\frac{1}{14} = \frac{-1}{3x^2+2}$

$$\Rightarrow 3x^2 + 2 = 14$$

$$\Rightarrow 3x^2 = 12$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

When x = 2, y = 18 and when x = -2, y =-6

Therefore, there are two normals to the curve $y = x^3 + 2x + 6$.

Equation of normal through point (2, 18) is given by:

$y - 18 = -\frac{1}{14}$ (x-2)

$$\Rightarrow 14y - 252 = -x + 2$$

$$\Rightarrow x + 14y - 254 = 0$$

Equation of normal through point (-2,-6) is given by:

$y - (-6) = -\frac{1}{14}$ (x- (-2))

$$\Rightarrow 14y + 84 = -x - 2$$

$$\Rightarrow x + 14y + 86 = 0$$

Therefore, the equation of normals to the curve are x + 14y – 254 = 0 and x + 14y + 86 = 0

21. $x^2dy + (xy + y^2)\,dx = 0$

$x^2dy = -(xy + y^2)dx$

$$\Rightarrow \frac{dy}{dx} = -\frac{-(xy+y^2)}{x^2} \quad .....(1)$$

This is a homogeneous differential equation.
Such type of equations can be reduced to variable separable form by the substitution y = vx.
Differentiating w.r.t. x we get,

$$\frac{d}{dx}(y)=\frac{d}{dx}(vx)\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting the value of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v+x\frac{dv}{dx}=\frac{-[x\times vx+(vx)^2]}{x^2}$$

$$\Rightarrow x\frac{dv}{dx}=-v^2-2v=-v(v+2)$$

$$\Rightarrow\frac{dv}{v(v+2)}=-\frac{dx}{x}$$

$$\Rightarrow\frac{1}{2}\left[\frac{1}{v}-\frac{1}{v+2}\right]dv=-\frac{dx}{x}$$

Integrating both sides, we get:

$$\frac{1}{2}[\log v-\log(v+2)]=-\log x+\log C$$

$$\Rightarrow\frac{1}{2}\log\left(\frac{v}{v+2}\right)=\log\frac{C}{x}$$

$$\Rightarrow\frac{v}{v+2}=\left(\frac{C}{x}\right)^2$$

Substituting $v=\frac{y}{x}$

$$\Rightarrow\frac{\frac{y}{x}}{\frac{y}{x}+2}=\left(\frac{C}{x}\right)^2$$

$$\Rightarrow\frac{y}{y+2x}=\frac{C^2}{x^2}$$

$$\Rightarrow\frac{x^2y}{y+2x}=D \qquad \text{......(2)}$$

Now, it is given that y = 1 at x = 1.

$$\Rightarrow\frac{1}{1+2}=D\Rightarrow D=\frac{1}{3}$$

Substituting $D=\frac{1}{3}$ in equation (2), we get

$$\frac{x^2y}{y+2x}=\frac{1}{3}\Rightarrow y+2x=3x^2y$$

So, the required solution is $y+2x=3x^2y$

22. $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$

Dividing all the terms of the equation by xlogx, we get

$$\Rightarrow \frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x^2}$$

This equation is in the form of a linear differential equation

$$\frac{dy}{dx}+Py=Q, \text{where } P=\frac{1}{x\log x} \text{and} Q=\frac{2}{x^2}$$

$$\text{Now, I.F}=e^{\int Pdx}=e^{\int\frac{1}{x\log x}dx}=e^{\log(\log x)}=\log x$$

The general solution of the given differential equation is given by

$$y\times \text{I.F.}=\int(Q\times \text{I.F.})dx+C$$

$$\Rightarrow y\log x=\int\left(\frac{2}{x^2}\log x\right)dx$$

$$\Rightarrow y\log x=2\int\left(\log x\times\frac{1}{x^2}\right)dx.$$

$$=2\left[\log x\times\int\frac{1}{x^2}dx-\int\left\{\frac{d}{dx}(\log x)\times\int\frac{1}{x^2}dx\right\}dx\right]$$

$$=2\left[\log x\left(-\frac{1}{x}\right)-\int\left(\frac{1}{x}\times\left(-\frac{1}{x}\right)\right)dx\right]$$

$$=2\left[-\frac{\log x}{x}+\int\frac{1}{x^2}dx\right]$$

$$=2\left[-\frac{\log x}{x}-\frac{1}{x}\right]+C$$

So the required general solution is $y\log x=-\frac{2}{x}(1+\log x)+C$

OR

$$\frac{dy}{dx}=y\tan x$$

$$\Rightarrow \frac{dy}{y}=\tan x dx$$

On integration, we get

$$\int \frac{dy}{y} = \int \tan x\, dx$$

$\Rightarrow \log y = \log(\sec x) + \log C$ ...(1)

$\Rightarrow \log y = \log(C\sec x)$

$\Rightarrow y = C\sec x$

Now, it is given that $y = 1$ when $x = 0$

$\Rightarrow 1 = C \times \sec 0$

$\Rightarrow 1 = C \times 1$

$\therefore C = 1$

Substituting C = 1 in equation (1), we get

y = sec x as the required particular solution.

SECTION – C

23. $I = \int_1^3 (3x^2 + 2x)dx$

Here a = 1, b = 3

$f(x) = 3x^2 + 2x$ $h = \frac{b-a}{n} = \frac{2}{n}$

Since, $\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + \ldots + f(a+(n-1)h)\right]$

So, $\int_1^3 (3x^2 + 2x) = \lim_{h\to 0} h[(3(1)^2 + 2(1)) + (3(1+h)^2 + 2(1+h)) +$

$3(1+2h)^2 + 2(1+2h))\ldots + 3(1+(n-1)h)^2 + 2(1+(n-1)h)]$

$= \lim_{h\to 0} h[3(n) + 3(h^2 + 4h^2 + \ldots(n-1)^2 h^2) + 3(2h + 4h + \ldots + 2(n-1)h) + 2n + 2(h + 2h$

$+ \ldots + (n-1)h)]$

$= \lim_{h\to 0} [5nh + 3h^3(1^2 + 2^2 + \ldots + (n-1)^2) + 6h^2(1 + 2 + \ldots + (n-1)) + 2h^2$

$(1 + 2 + (n-1))]$

$$= \lim_{h\to 0}\left[5nh + 3h^3 \times \frac{(n-1)n(2n-1)}{6} + \frac{8h^2(n)(n-1)}{2}\right]$$

$$= \lim_{h\to 0}\left[10 + \frac{(nh-h)nh(2nh-h)}{2} + 4(nh)(nh-h)\right]$$

$$= \left[10 + \frac{2\times \not{2} \times 4}{\not{2}} + 4\times 2\times 2\right]$$

$= 10 + 8 + 16 = 34$

OR

Given ellipse

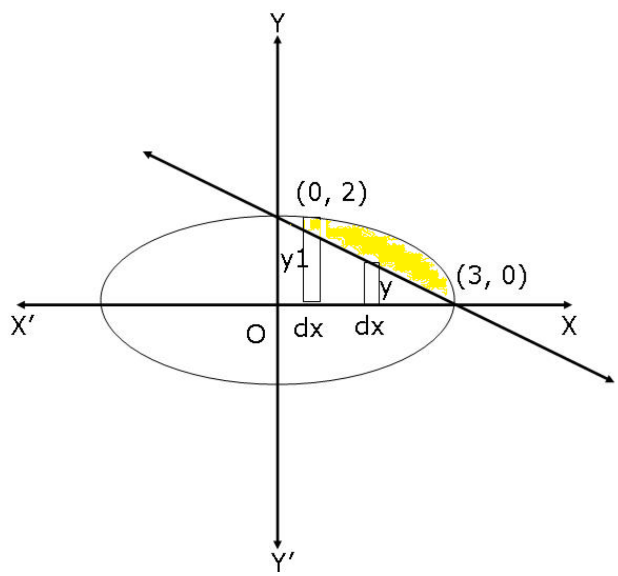
$$\Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$$

$$\Rightarrow y=\frac{2}{3}\sqrt{9-x^2}$$

Given line $\frac{x}{3}+\frac{y}{2}=1$

$$\Rightarrow y=\left(2-\frac{2x}{3}\right)$$

Required Area $\left\{(x,y):\frac{x^2}{9}+\frac{y^2}{4}\le 1\le \frac{x}{3}+\frac{y}{2}\right\}$ is given below



Required Area = $\int_0^3 (y_1-y_2)\,dx$

$$=\int_0^3\left[\frac{2}{3}\sqrt{9-x^2}-\left(2-\frac{2x}{3}\right)\right]dx$$

$$=\left[\frac{2}{3}\left(\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right)-2x+\frac{x^2}{3}\right]_0^3$$

$$=\left[\frac{2}{3}\left(\frac{9}{2}\sin^{-1}1\right)-6+3\right]-0$$

$$=3\times\frac{\pi}{2}-3=\frac{3}{2}(\pi-2)\text{ sq units}$$

24. Let x be the number of gold rings and y be number of chains manufactured

L.P.P. is

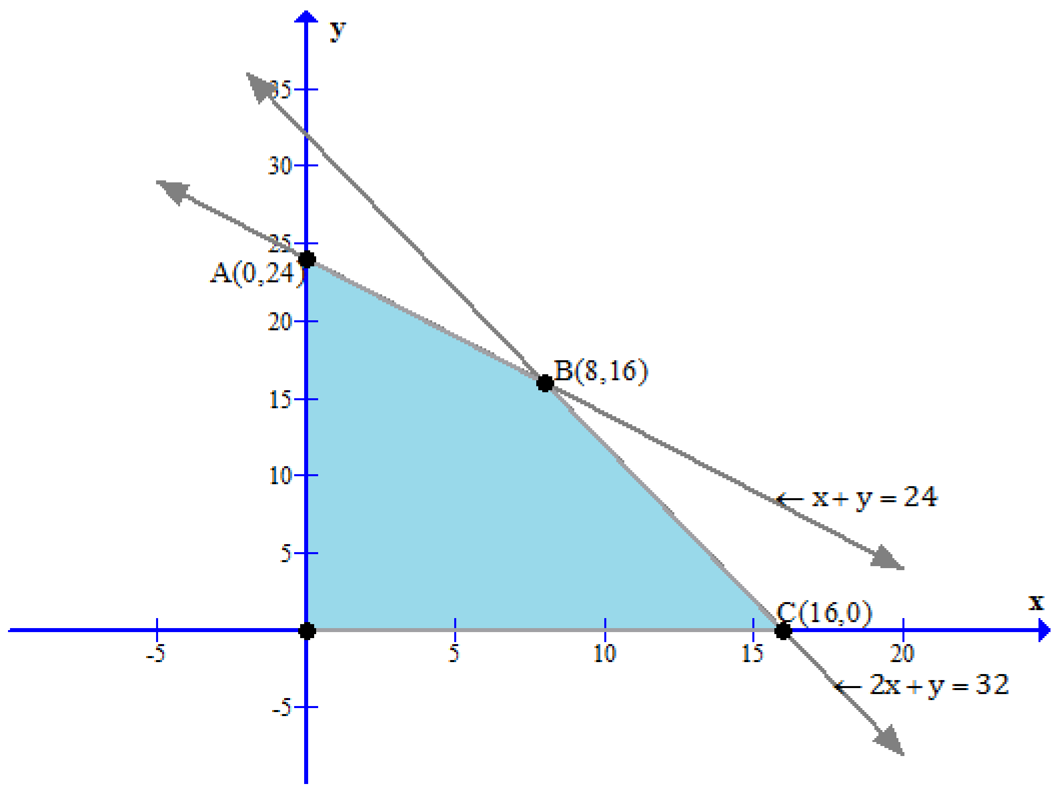
Max Z = 300x + 190y

Substitute in $x + y \le 24$

$x + \frac{y}{2} \le 16$ or $2x + y \le 32$

$x \ge 0, \quad y \ge 0$

Feasible region



Corner Points	Value of Z=300x+190y	
A(0, 24)	4560	
B(8, 16)	5440	Maximum
C(16, 0)	4800	
O(0, 0)	0	

Hence to make the maximum profit, 8 gold rings and 16 chains must be manufactured.

25. Let the events E_1, E_2, E_3,, E_4 and A be defined as follows:

E_1: Missing card is a diamond

E_2: Missing card is a spade

E_3: Missing card is a club

E_4: Missing card is a heart

A: Drawing two club cards

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$

$$P(A|E_1) = P(A|E_2) = P(A|E_4) = \frac{13}{51} \times \frac{12}{50}$$

$$P(A|E_3) = \frac{12}{51} \times \frac{11}{50}$$

$$P(E_3|A) = \sum_{i=1}^{4} \frac{P(E_3)P(A|E_3)}{P(E_i)P(A|E_i)}$$

$$= \frac{\frac{1}{4} \times \frac{12}{51} \times \frac{11}{50}}{\frac{1}{4}\left[\frac{13\times12+13\times12+13\times12+11\times12}{51\times50}\right]}$$

$$= \frac{12\times11}{3\times13\times12+12\times11} = \frac{11}{50}$$

OR

Total number of bulbs = 10

Number of defective bulbs = 3

Number of non-defective bulbs = 7

P(drawing a defective bulb), $p = \frac{3}{10}$

P(drawing a non-defective bulb), $q = \frac{7}{10}$

Two bulbs are drawn.

Let X denote the number of defective bulbs, then X can take values 0, 1, and 2.

P(X = 0) = P(drawing both non-defective bulbs) = $\left(\frac{7}{10}\right)^2$

P(X = 1) = P(drawing one defective and one non defective bulb)

= P (drawing a non-defective bulb and a defective bulb) + P (drawing a defective bulb and a non -efective bulb)

$$= \left(\frac{7}{10}\right)\left(\frac{3}{10}\right) + \left(\frac{3}{10}\right)\left(\frac{7}{10}\right) = \frac{21}{50}$$

P(X = 2) = P(drawing both defective bulbs)= $\left(\frac{3}{10}\right)^2$

Required probability distribution is

X	0	1	2
P(X)	$\frac{49}{100}$	$\frac{21}{50}$	$\frac{9}{100}$

26. Consider,

$$\Delta = \begin{vmatrix} (b+c)^2 & ab & ac \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix}$$

By performing $R_1 \to aR_1$, $R_2 \to bR_2$, $R_3 \to cR_3$ and dividing the determinant by abc, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(b+c)^2 & a^2b & a^2c \\ ab^2 & b(a+c)^2 & b^2c \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$$

Now, taking a, b, c common from C_1, C_2 and C_3

$$\Delta = \frac{abc}{abc}\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ b^2 - (c+a)^2 & (c+a)^2 - b^2 & b^2 \\ 0 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$\Delta = (a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & c+a-b & b^2 \\ 0 & c-a-b & (a+b)^2 \end{vmatrix}$$

Applying $R_3 \to R_3 - (R_1 + R_2)$

$$\Delta = (a+b+c)^2\begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & c+a-b & b^2 \\ 2a-2b & -2a & 2ab \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$

$$\Delta = (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

Applying $C_3 \to C_3 + bC_2$

$$\Delta = (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & bc+ab \\ -2b & -2a & 0 \end{vmatrix}$$

Applying $C_1 \to aC_1$ and $C_2 \to bC_2$

$$\Delta = \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac-a^2 & 0 & a^2 \\ 0 & bc+ab-b^2 & bc+ab \\ -2ab & -2ab & 0 \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$

$$\Delta = \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab+ac-a^2 & 0 & a^2 \\ -bc-ab+b^2 & bc+ab-b^2 & bc+ab \\ 0 & -2ab & 0 \end{vmatrix}$$

Expanding along R_3

$$= \frac{(a+b+c)^2}{ab}\left(2ab\left(ab^2c + a^2b^2 + abc^2 + a^2bc - a^2bc - a^3b + a^2bc + a^3b - a^2b^2\right)\right)$$

$$= 2(a+b+c)^2\left(ab^2c + abc^2 + a^2bc\right)$$

$$= 2(a+b+c)^3 abc = \text{R.H.S.}$$

27. $f(x) = (x(x-2))^2 = x^2(x^2 - 4x + 4) = x^4 - 4x^3 + 4x^2$

$f'(x) = 4x^3 - 12x^2 + 8x$

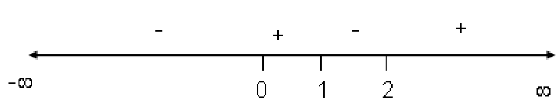
$f'(x) = 4x(x^2 - 3x + 2)$

$\quad = 4x(x-2)(x-1)$

$f'(x) = 0 \quad \Rightarrow x = 0$ or $1, 2$

So, the tangent to curve $f(x)$ is parallel to the x-axis if $x = 0, x = 1$ or $x = 2$.

Now points 0, 1 and 2 will divide the number line into 4 disjoint intervals $(-\infty, 0)$ $(0, 1)$ $(1, 2)$ $(2, \infty)$



Now in the interval (-∞, 0) and (1, 2) f'(x) < 0. So the function f(x) is strictly decreasing in these intervals.

f'(x) >0 in interval (0, 1) and (2, ∞)

So the function f(x) is strictly increasing in intervals (0, 1) and (2, ∞)

Tangent is parallel to X- axis if $\frac{dy}{dx}=0$

Which gives us x = 0, 1, 2

Hence, x = 0, y = 0

X = 1, y = 1

x = 2, y = 0

Required points are (0, 0), (1, 1), (2, 0).

28. Let r and h be the radius and height of the right circular cylinder with the open top.

So surface area of the cylinder S is given by,

$S = \pi r^2 + 2\pi rh$

$\Rightarrow h = \frac{S-\pi r^2}{2\pi r}$ ...(i)

Let V be the volume, so

$$V=\pi r^2 h=\pi r^2\frac{(S-\pi r^2)}{2\pi r}=\frac{r(S-\pi r^2)}{2}$$

$$\frac{dV}{dr}=\frac{S}{2}-\frac{3\pi r^2}{2} \quad \text{...(ii)}$$

for maxima or minima $\frac{dV}{dr}=0$

$$\Rightarrow S=3\pi r^2 \quad \text{or} \quad r=\sqrt{\frac{S}{3\pi}}$$

Using this (i)

$$h=\frac{2\pi r^2}{2\pi r}=r$$

$$\frac{d^2V}{dr^2}=-3\pi r$$

$$=-3\pi\sqrt{\frac{S}{3\pi}}<0$$

So, $r=\sqrt{\frac{S}{3\pi}}$ is a point of maxima

And in this case radius of base = height

29. Given equation of line is $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$

This can also be written in the standard form as $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-(-4)}{6}$

The vector form of the above equation is,

$\vec{r}=(\hat{i}+2\hat{j}-4k)+\lambda(2\hat{i}+3\hat{j}+6k)$

$\Rightarrow \vec{r}=\overrightarrow{a_1}+\lambda\vec{b}$ ...(1)

where, $\overrightarrow{a_1}=\hat{i}+2\hat{j}-4k$ and $\vec{b}=2\hat{i}+3\hat{j}+6k$

The second equation of line is $\frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$

The above equation can also be written as $\frac{x-3}{4}=\frac{y-3}{6}=\frac{z-(-5)}{12}$

The vector form of this equation is

$\vec{r}=(3\hat{i}+3\hat{j}-5k)+\mu(4\hat{i}+6\hat{j}+12k)$

$\Rightarrow \vec{r}=(3\hat{i}+3\hat{j}-5k)+2\mu(2\hat{i}+3\hat{j}+6k)$

$\Rightarrow \vec{r}=\overrightarrow{a_2}+2\mu\vec{b}$...(2)

where $\overrightarrow{a_2}=3\hat{i}+3\hat{j}-5k$ and $\vec{b}=2\hat{i}+3\hat{j}+6k$

Since $\vec{b}$ is same in equations (1) and (2), the two lines are parallel.
Distance d, between the two parallel lines is given by the formula,

$$d=\left|\frac{\vec{b}\times(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\vec{b}|}\right|$$

Here, $\vec{b}=(2\hat{i}+3\hat{j}+6k)$ $\overrightarrow{a_2}=(3\hat{i}+3\hat{j}-5k)$ and $\overrightarrow{a_1}=(\hat{i}+2\hat{j}-4k)$

On substitution, we get

$$d=\left|\frac{(2\hat{i}+3\hat{j}+6k)\times(3\hat{i}+3\hat{j}-5k-(\hat{i}+2\hat{j}-4k))}{\sqrt{4+9+36}}\right|$$

$$=\frac{1}{\sqrt{49}}\left|(2\hat{i}+3\hat{j}+6k)\times(2\hat{i}+\hat{j}-k)\right|$$

$$=\frac{1}{7}\begin{vmatrix}\hat{i} & \hat{j} & k\\ 2 & 3 & 6\\ 2 & 1 & -1\end{vmatrix}$$

$$=\frac{1}{7}\left|\hat{i}(-3-6)-\hat{j}(-2-12)+k(2-6)\right|$$

$$=\frac{1}{7}\left|-9\hat{i}+14\hat{j}-4k\right|$$

$$= \frac{1}{7}\left|\sqrt{81+196+16}\right|$$

$$= \frac{\sqrt{293}}{7}$$

Thus, the distance between the two given lines is $\frac{\sqrt{293}}{7}$