ICSE 2024 EXAMINATION

MATHEMATICS

SAMPLE PAPER - 7

Time Allowed : 2½ hours ***Max. Marks : 80***

General Instructions :

Attempt all questions from Section A and any four questions from Section B.
All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ]
Mathematical tables are provided.

SECTION - A (40 Marks)

(Attempt *all* questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: [15]

(i) ₹100 shares of a company are selling at ₹80. If the company is paying a dividend of 12%, then the rate of return is:

(a) 12% (b) 15% (c) 16% (d) 18%

(ii) The roots of $3x^2 - 5x + 1 = 0$ are :

(a) irrational (b) equal (c) imaginary (d) none of these

(iii) On dividing $x^3 - x^2 + x - 1$ by $(x - 1)$, the remainder is :

(a) 1 (b) –1 (c) 0 (d) 4

(iv) A is a matrix of order 2×3. The order of A^t is :

(a) 2×3 (b) 2×2 (c) 3×2 (d) 3×3

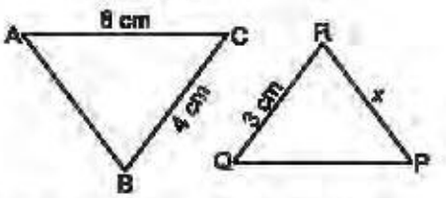
(v) The numbers x, y, z are in GP, then:

(a) $y = xz$ (b) $y^2 = x + z$ (c) $y = \frac{x+z}{z}$ (d) $y^2 = xz$

(vi) If the image of the point P under reflection in the x-axis is (–3, 4), then the coordinates of the point P are:

(a) (3, 4) (b) (–3, –4) (c) (3, –4) (d) (–4, 3)

(vii) In the figure, if $\Delta ABC \sim \Delta QPR$, then the value of x is :



(a) 2 cm

(b) 4 cm

(c) 3.5 cm

(d) 2.5 cm

(viii) A rectangular paper is folded into a cylinder. The length and breadth of the paper are L and B respectively. Which of the following represents its curved surface area?

(a) $2\pi LB$ (b) LB (c) $\frac{L}{B}$ (d) πLB

(ix) If $2(x - 1) \leq 9$, $x \in W$, then the greatest value of x is :

(a) 3 (b) 2 (c) 4 (d) 5

(x) Which of the following cannot be the probability of an event?

(a) $\frac{3}{4}$ (b) $\frac{3}{5}$ (c) 1.001 (d) 0.6

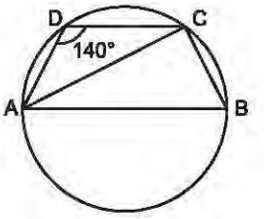
(xi) For a size transformation, if the scale factor is k and $k < 1$, it means:

(a) an enlargement (b) a reduction (c) an identity transformation (d) none of these

(xii) If P(–1, 1) is the mid-point of the line segment joining A(–3, b) and B(1, $b + 4$), then the value of b is:

(a) 1 (b) –1 (c) 2 (d) 0

(xiii) ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribling it and $\angle ADC = 140°$, then $\angle BAC$ is equal to:



(a) 80°

(b) 50°

(c) 40°

(d) 30°

(xiv) The nth term of an AP is $T_n = 4n - 1$. The common difference of the AP is :

(a) 2 (b) 3 (c) –3 (d) 4

(xv) The relation between mean, median, and mode is:

(a) Mean – Mode = 3(Mean – Median) (b) 3 (Mean – Mode) = (Mean – Median)

(c) Mean + Mode = 3(Mean – Median) (d) Mean – Mode = 3 (Mean + Median)

Question 2 :

(i) Mr. Richard has a recurring deposit account in a bank for 3 years at 7.5% p.a. simple interest. If he gets ₹ 8325 as interest at the time of maturity, find : [4]

(a) The monthly deposit

(b) The maturity value.

(ii) Using ruler and compass only, construct a ΔABC such that BC = 5 cm and AB = 6.5 cm and $\angle ABC = 120°$. Construct a circum-circle of ΔABC. [4]

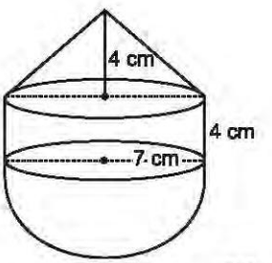
(iii) Sameer invests ₹4500 in 8%, ₹10 shares at ₹15. He sells the shares when the price rises to ₹30 and invests the proceeds in 12%, ₹100 shares at ₹125. Calculate: [4]

(a) the sale proceeds (b) the number of ₹125 shares he buys

(c) the change in his annual income from dividend.

Question 3 :

(i) The adjoining figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid. [4]



(ii) A(2, 5), B(–1, 2) and C(5, 8) are the vertices of a triangle ABC, 'M' is a point on AB such that AM : MB = 1 : 2. Find the co-ordinates of 'M'. Hence find the equation of the line passing through the points C and M. [4]

(iii) Use a graph paper for this question (Take 2 cm = 1 unit on both x and y axis) [5]

(a) Plot the following points : A(0, 4), B(2, 3), C(1, 1) and D(2, 0)

(b) Reflect points B, C, D on the y-axis and write down their coordinates. Name the images as B', C', D' respectively.

(c) Join the points A, B, C, D, D', C', B' and A in order, so as to form a closed figure. Write down the equation of the line of symmetry of the figure formed.

SECTION - B (40 Marks)

(Attempt *any four* questions from this Section)

Question 4 :

(i) Ms. Roy went to a departmental store and bought the following items. The GST rates and the quantity of each item and market price of each are given below :

S.No.	Items	Price per item in ₹	Quantity	GST rate	Amount
1.	Walnut	650	1	5%	
2.	Potato Chips	50	2	0%	
3.	Coffee	80	2	18%	

Find the :

(a) The total amount of SGST paid. (b) The total amount of the bill. [3]

(ii) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. [3]

(iii) Draw a histogram for the given data, using a graph paper : [4]

Class-interval	Frequency
0 – 10	14
10 – 20	22
20 – 30	27
30 – 40	22
40 – 50	23
50 – 60	20
60 – 70	15

Estimate the mode from the graph.

Question 5 :

(i) Find the values of x and y if $\begin{bmatrix}1 & 2\\3 & 3\end{bmatrix}\begin{bmatrix}x & 0\\0 & y\end{bmatrix}=\begin{bmatrix}x & 0\\9 & 0\end{bmatrix}$ [3]

(ii) The 4th term of a GP is 16 and the 7th term is 128. Find the first term and the common ratio of the GP. [3]

(iii) Using the Remainder Theorem, find the remainders obtained when $x^3 + (kx + 8)\,x + k$ is divided by $x+1$ and $x-2$ [4]
Hence find k if the sum of the two remainders is 1.

Question 6 :

(i) A(–1, 3), B(4, 2) and C(3, – 2) are the vertices of a triangle.

(a) Find the coordinates of the centroid of the triangle.

(b) Find the equation of the line through G and parallel to AC. [3]

(ii) Prove that : $\dfrac{\cot A - 1}{2 - \sec^2 A} = \dfrac{\cot A}{1 + \tan A}$. [3]

(iii) The 4th term of an A.P. is 22 and 15th term is 66. Find the first term and the common difference. Hence find the sum of the series to 8 terms. [4]

Question 7 :

(i) If one card is drawn at random from a pack of 52 playing cards, what is the probability that it is:

(a) a king? (b) the ace of clubs? (c) a heart? [3]

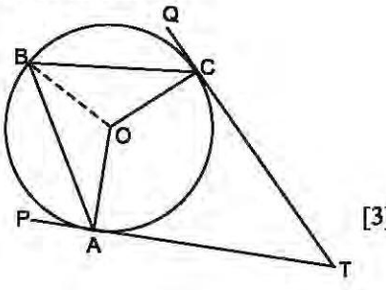
(ii) The scale of a map is 5 m to 1 cm. [3]

(a) what area in m^2 is represented by a square of side 3 cm on the map?

(b) What is the length of the side of the square on the map which represents an area of 50,625 m^2?

(iii) In the given figure, TP and TQ are two tangents to the circle with centre O, touching at A and C respectively. If $\angle BCQ = 55°$ and $\angle BAP = 60°$, find : [4]

(a) $\angle OBA$ and $\angle OBC$ (b) $\angle AOC$ (c) $\angle ATC$



Question 8 :

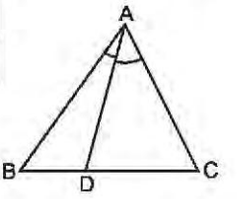
(i) Solve the following inequation and represent the solution set on a number line. [3]

$-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2},\ x \in I$

(ii) Find the mean of the following data: [3]

Class-interval	0–10	10–20	20–30	30–40	40–50
Frequency	12	16	6	7	9

(iii) In ΔABC, AD is the bisector of ∠A. If BC = 10 cm, BD = 6 cm and AC = 6 cm, find AB. [4]



Question 9 :

(i) The duration of telephone calls in a week at a school were recorded as below :

Time in seconds	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of calls	5	14	19	27	43	29	16	12	5

Draw an ogive and locate the median and quartiles. [6]

(ii) ₹ 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹ 100 more. Find the original number of children. [4]

Question 10 :

(i) Construct a right angled triangle PQR such that PQ = 6 cm, QR = 3.5 cm and ∠PQR = 90°. Bisect ∠P internally and mark a point Z on this bisector such that PZ = 4 cm. Find the points which are 4 cm from Z and also 4 cm from the line QR. [3]

(ii) If b is the mean proportion between a and c, show that : $\frac{a^4+a^2b^2+b^4}{b^4+b^2c^2+c^4} = \frac{a^2}{c^2}$ [3]

(iii) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from the point Q is 30°. Find the height of the tower PT, correct to the nearest metre. [4]

ANSWERS

1. (i) (b) (ii) (a) (iii) (c) (iv) (c) (v) (d) (vi) (b) (vii) (a) (viii) (b) (ix) (d)
(x) (c) (xi) (b) (xii) (b) (xiii) (b) (xiv) (d) (xv) (a)

2. (i) (a) ₹2000 (b) ₹80, 325 (iii) (a) ₹9000 (b) 72 (c) ₹624

3. (i) 1540 cm^3 (ii) (1, 4), $x - y + 3 = 0$ (iii) (b) (–2, 3), (–1, 1), (–2, 0) (c) $x = 0$

4. (i) (a) ₹30.65 (b) ₹921.20 (ii) 0.89 or –7.89 (iii) 25

5. (i) 3, 0 (ii) 2, 2 (iii) –2

6. (i) (a) (2, 1) (b) $5x + 4y = 14$ (iii) 192

7. (i) (a) $\frac{1}{13}$ (b) $\frac{1}{52}$ (c) $\frac{1}{4}$ (ii) (a) 2.25 m^2 (b) 45 cm (iii) (a) 30° (b) 130° (c) 50°

8. (i) {–2, –1, 0, 1} (ii) 22 (iii) 9 cm

9. (i) 55, 42, 66 (ii) 50 **10.** (iii) 17 m.

SOLUTION

Time Allowed : 2½ hours *Max. Marks* : **80**

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Mathematical tables are provided.

SECTION - A (40 Marks)

(Attempt *all* questions from this Section)

Question 1 : Choose the correct answers to the questions from the given options: **[15]**

(i) ₹100 shares of a company are selling at ₹80. If the company is paying a dividend of 12%, then the rate of return is:

(a) 12% **(b) 15%** **(c) 16%** **(d) 18%**

(ii) The roots of $3x^2 - 5x + 1 = 0$ are :

(a) irrational **(b) equal** **(c) imaginary** **(d) none of these**

(iii) On dividing $x^3 - x^2 + x - 1$ by $x - 1$, the remainder is :

(a) 1 **(b) –1** **(c) 0** **(d) 4**

(iv) A is a matrix of order 2 × 3. The order of A^t is :

(a) 2 × 3 **(b) 2 × 2** **(c) 3 × 2** **(d) 3 × 3**

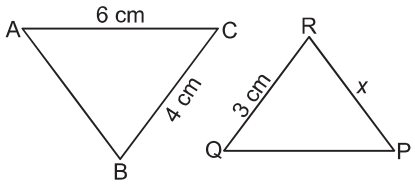
(v) The numbers *x, y, z* are in GP, then:

(a) $y = xz$ **(b) $y^2 = x + z$** **(c) $y = \frac{x+z}{z}$** **(d) $y^2 = xz$**

(vi) If the image of the point P under reflection in the *x*-axis is (–3, 4), then the coordinates of the point P are:

(a) (3, 4) **(b) (–3, –4)** **(c) (3, –4)** **(d) (–4, 3)**

(vii) In the figure, if $\Delta ABC \sim \Delta QPR$, then the value of *x* is :



(a) 2 cm

(b) 4 cm

(c) 3.5 cm

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(viii) A rectangular paper is folded into a cylinder. The length and breadth of the paper are L and B respectively. Which of the following represents its curved surface area?

(a) $2\pi LB$ **(b) LB** **(c) $\frac{L}{B}$** **(d) πLB**

(ix) If $2(x - 1) \leq 9$, $x \in W$, then the greatest value of *x* is :

(a) 3 **(b) 2** **(c) 4** **(d) 5**

(x) Which of the following cannot be the probability of an event?

(a) $\frac{3}{4}$ **(b) $\frac{3}{5}$** **(c) 1.001** **(d) 0.6**

(xi) For a size transformation, if the scale factor is k and $k < 1$, it means:

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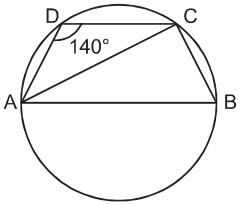
(xii) If P(–1, 1) is the mid-point of the line segment joining A(–3, b) and B(1, $b + 4$), then the value of b is:

(a) 1 (b) –1 (c) 2 (d) 0

(xiii) ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribling it and $\angle ADC = 140°$, then $\angle BAC$ is equal to:



(a) 80°

(b) 50°

(c) 40°

(d) 30°

(xiv) The nth term of an AP is $T_n = 4n - 1$. The common difference of the AP is :

(a) 2 (b) 3 (c) –3 (d) 4

(xv) Assertion (A) : Median of first eleven prime numbers is 13.

Reason (R) : For odd number of observations, n, median = value of $\left(\frac{n+1}{2}\right)$th observation.

(a) A is true, R is false (b) A is false, R is true (c) both A and R are true (d) both A and R are false

Solution :

(i) (b) Rate of return $= \frac{12 \times 100}{80} = 15\%$

(ii) (a) $D = (-5)^2 - 4 \times 3 \times 1 = 25 - 12 = 13 > 0$; So, roots are real, distinct and irrational.

(iii) (c) Required remainder $= 1^3 - 1 + 1 - 1 = 0$

(iv) (c)

(v) (d) Here, $\frac{y}{x} = \frac{z}{y} \Rightarrow y^2 = xz$

(vi) (b) R_x (–3, 4) gives (–3 – 4)

(vii) (a) $\frac{AC}{QR} = \frac{BC}{PR} = \frac{6}{3} = \frac{4}{x} \Rightarrow x = 2$

(viii) (b) CSA = area of the rectangular sheet = LB

(ix) (d) $2x - 2 \le 9$, $x \in W$

$\Rightarrow 2x \le 11$, $x \in W \Rightarrow x \le 5.5$, $x \in W$

So, solution set is {0, 1, 2, 3, 4, 5}

(x) (c) $0 \le P(x) \le 1$

(xi) (b)

(xii) (b) $1 = \frac{b + b + 4}{2} \Rightarrow 2b = 2 - 4 = -2 \Rightarrow b = -1$

(xiii) (b) $\angle ADC + \angle ABC = 180°$

$\Rightarrow 140° + \angle ABC = 180° \Rightarrow \angle ABC = 180° - 140° = 40°$

$\Rightarrow LCB = 90° \Rightarrow \angle BAC = 180° - (90° + 40°) = 50°$

(xiv) (d) $d = T_2 - T_1 = (8 - 1) - (4 - 1) = 4$

(xv) (c) First 11 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31

$n = 11$ (odd)

So, median $= \left(\frac{11+1}{2}\right)$th observation = 6th observation = 13.

Question 2 :

(i) Mr. Richard has a recurring deposit account in a bank for 3 years at 7.5% p.a. simple interest. If he gets ₹8325 as interest at the time of maturity, find : [4]

(a) The monthly deposit

(b) The maturity value.

(ii) Using ruler and compass only, construct a ΔABC such that BC = 5 cm and AB = 6.5 cm and $\angle ABC = 120°$. Construct a circum-circle of ΔABC. [4]

(iii) Sameer invests ₹4500 in 8%, ₹10 shares at ₹15. He sells the shares when the price rises to ₹30 and invests the proceeds in 12%, ₹100 shares at ₹125. Calculate: [4]

(a) the sale proceeds

(b) the number of ₹125 shares he buys

(c) the change in his annual income from dividend.

Solution :

(i) (a) Let Mr Richard deposit ₹ P monthly.

n = 3 × 12 months = 36 months, R = 7.5 % p.a.

Amount deposited in 36 months = ₹ 36 P

$$\text{S. I.} = \text{P} \times \frac{n(n+1)}{2} \times \frac{1}{12} \times \frac{\text{R}}{100}$$

$$\Rightarrow 8325 = \text{P} \times \frac{36 \times 37}{2} \times \frac{1}{12} \times \frac{7.5}{100} \Rightarrow \text{P} = \frac{8325 \times 2 \times 12 \times 100}{36 \times 37 \times 7.5} = 2000$$

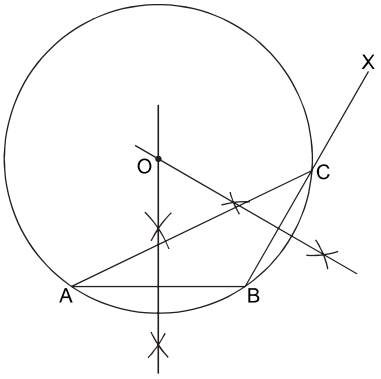
Hence, Mr Richard deposits ₹ 2000 per month. **Ans.**

(b) Amount deposited in 36 months = ₹ 36 P = ₹ 36 × 2000 = ₹ 72,000

∴ Maturity value = ₹ 72,000 + interest = ₹ (72,000 + 8,325) = ₹ 80,325 **Ans.**

(ii) **Steps of construction :**

1. Draw AB = 6.5 cm.
2. At B, draw ∠ABX = 120°.
3. From BX, cut off BC = 5 cm.
4. Join AC to get the required ΔABC.
5. Draw the perpendicular bisectors of AB and BC. These perpendicular bisectors meet at the point O.
6. With O as centre and radius equal to OA or OB or OC draw a circle, which passes through A, B and C. This circle is the required circumcircle of ΔABC.



(iii) Number of shares of first kind = $\frac{₹4500}{₹15}$ = 300.

Face value of 300 shares = ₹3000

Annual dividend = 8% of ₹3000 = ₹240.

(a) Sale proceeds = 300 × ₹30 = ₹9000 **Ans.**

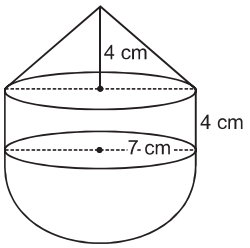
(b) Required number of shares = $\frac{₹9000}{₹125}$ = 72 **Ans.**

(c) New dividend = 12% of ₹100 × 72 = ₹864

Change in income = ₹(864 – 240) = ₹624.

Question 3 :

(i) The adjoining figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid. [4]



(ii) A(2, 5), B(–1, 2) and C(5, 8) are the vertices of a triangle ABC, 'M' is a point on AB such that AM : MB = 1 : 2. Find the co-ordinates of 'M'. Hence find the equation of the line passing through the points C and M. [4]

(iii) Use a graph paper for this question (Take 2 cm = 1 unit on both x and y axis) [5]

(a) Plot the following points : A(0, 4), B(2, 3), C(1, 1) and D(2, 0)

(b) Reflect points B, C, D on the y-axis and write down their coordinates. Name the images as B', C', D' respectively.

(c) Join the points A, B, C, D, D', C', B' and A in order, so as to form a closed figure. Write down the equation of the line of symmetry of the figure formed.

Solution :

(i) Common radius (r) = 7 cm

Height of the cone (h) = 4 cm

Height of the cylinder (H) = 4 cm

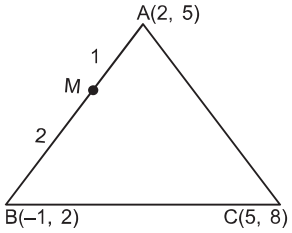
$\therefore$ Volume of the solid = volume of the hemisphere + volume of the cylinder + volume of the cone

$$= \frac{2}{3}\pi r^3 + \pi r^2 \mathrm{H} + \frac{1}{3}\pi r^2 h = \pi r^2\left(\frac{2}{3}r + \mathrm{H} + \frac{1}{3}h\right)$$

$$= \frac{22}{7} \times 7 \times 7\left(\frac{2}{3} \times 7 + 4 + \frac{4}{3}\right) \text{ cm}^3$$

$$= 154\left(\frac{14}{3} + 4 + \frac{4}{3}\right) \text{ cm}^3 = 154 \times \frac{30}{3} \text{ cm}^3 = 1540 \text{ cm}^3 \quad \textbf{Ans.}$$

(ii) Let the coordinates of M be (x, y).



Then, AM : MB = 1 : 2

$$\therefore \quad x = \frac{1 \times (-1) + 2 \times 2}{1 + 2} \text{ and } y = \frac{1 \times 2 + 2 \times 5}{1 + 2}$$

$$\Rightarrow \quad x = \frac{-1 + 4}{3} \text{ and } y = \frac{2 + 10}{3} \Rightarrow x = 1 \text{ and } y = 4$$

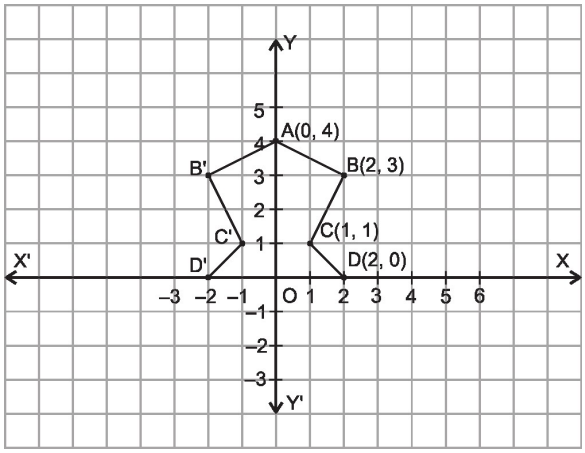
Hence, the coordinates of M are (1, 4) **Ans.**

Equation of the line passing through the points C(5, 8) and M(1, 4) is

$$y - 8 = \frac{4 - 8}{1 - 5}(x - 5)$$

$$\Rightarrow \quad y - 8 = \frac{-4}{-4}(x - 5) \Rightarrow y - 8 = x - 5 \Rightarrow x - y + 3 = 0 \quad \textbf{Ans.}$$

(iii) (a) The points A(0, 4), B(2, 3), C(1, 1) and D(2, 0) have been plotted as shown below.



(b) Reflecting the points B(2, 3), C(1, 1) and D(2, 0) on the y-axis, we get the images as B'(–2, 3), C'(–1, 1) and D'(–2, 0) respectively. **Ans.**

(c) On joining the points A, B, C, D, D', C', B' and A in order, a closed figure is obtained, which is symmetrical about the y-axis.

So, equation of the line of symmetry of the figure is $x = 0$. **Ans.**

SECTION - B (40 Marks)

(Attempt *any four* questions from this Section)

Question 4 :

(i) Ms. Roy went to a departmental store and bought the following items. The GST rates and the quantity of each item and market price of each are given below :

S.No.	Items	Price per item in ₹	Quantity	GST rate	Amount
1.	Walnut	650	1	5%	
2.	Potato Chips	50	2	0%	
3.	Coffee	80	2	18%	

Find the :

(a) The total amount of SGST paid. **(b) The total amount of the bill.** **[3]**

(ii) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. **[3]**

(iii) Draw a histogram for the given data, using a graph paper : **[4]**

Class Interval	Frequency
0 – 10	14
10 – 20	22
20 – 30	27
30 – 40	22
40 – 50	23
50 – 60	20
60 – 70	15

Estimate the mode from the graph.

Solution :

(i) (a) GST paid on walnut = 5% of ₹ 650 = ₹32.50

GST paid on potato chips = ₹ 0

GST paid on coffee = 18% of (2 × ₹ 80) = 18% of ₹ 160 = ₹ 28.80

$\therefore$ Total amount of GST paid = ₹(32.50 + ₹28.80) = ₹ 61.30

Total amount of SGST paid = 50% of ₹61.30 = ₹ 30.65 **Ans.**

(b) Total amount of the bill = ₹(650 + 50 + 160) + GST = ₹860 + ₹61.30 = ₹921.30 **Ans.**

(ii) We have, $x^2 + 7x = 7$

$\Rightarrow x^2 + 7x - 7 = 0$ ...(1)

Comparing (1) with $ax^2 + bx + c = 0$, we get

$a = 1, b = 7, c = -7$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

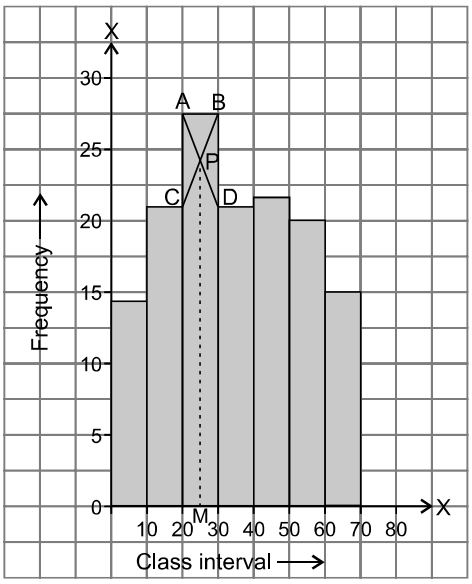
$$= \frac{-7 \pm \sqrt{7^2 - 4 \times 1 \times (-7)}}{2 \times 1} = \frac{-7 \pm \sqrt{77}}{2} = \frac{-7 \pm 8.77}{2}$$

$$\Rightarrow x = \frac{-7 + 8.77}{2} \text{ or } x = \frac{-7 - 8.77}{2} \Rightarrow x = \frac{1.77}{2} \text{ or } x = \frac{-15.77}{2}$$

$\Rightarrow x = 0.885$ or $x = -7.885 \Rightarrow x = 0.89$ or $x = -7.89$ **Ans.**

(iii) The required histogram is shown:

Mark the upper corners of the highest rectangle and then corners of the adjacent rectangles as A, B, C and D, as shown. Join AD and BC to intersect at P. From P, draw PM $\perp$ x-axis, meeting the x-axis at M. Abscissa of M is 25. Hence, mode of the data is ₹25.



Question 5 :

(i) Find the values of x and y if $\begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$ **[3]**

(ii) The 4th term of a GP is 16 and the 7th term is 128. Find the first term and the common ratio of the GP. [3]

(iii) Using the Remainder Theorem, find the remainders obtained when
$x^3 + (kx + 8)\,x + k$ is divided by $x + 1$ and $x - 2$. [4]
Hence find k if the sum of the two remainders is 1.

Solution :

(i) We have, $\begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x+0 & 0+2y \\ 3x+0 & 0+3y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x & 2y \\ 3x & 3y \end{bmatrix} = \begin{bmatrix} x & 0 \\ 9 & 0 \end{bmatrix}$

$\Rightarrow \quad 2y = 0$ and $3x = 9 \Rightarrow y = 0$ and $x = 3$

Hence, $x = 3$ and $y = 0$ **Ans.**

(ii) $ar^3 = 16$ and $ar^6 = 128$

$\Rightarrow r^3 = \dfrac{128}{16} = 8 \Rightarrow r = 2$

So, $a \times 2^3 = 16 \Rightarrow a = 2$

So, first term and common ratio of the GP are 2 and 2 respectively. **Ans.**

(iii) Let $p(x) = x^3 + kx^2 + 8x + k$,

When $p(x)$ is divided by $x + 1$, the remainder = $p(-1)$.

$p(-1) = (-1)^3 + k(-1)^2 + 8 \times (-1) + k$

$= -1 + k - 8 + k = 2k - 9$ ...(i) **Ans.**

When $p(x)$ is divided by $x - 2$, the remainder = $p(2)$

$p(2) = (2)^3 + k(2)^2 + 8 \times (2) + k = 8 + 4k + 16 + k$

$= 5k + 24$ ...(ii) **Ans.**

Now, $2k - 9 + 5k + 24 = 1 \Rightarrow 7k = -14 \Rightarrow k = \dfrac{-14}{7} = -2$ **Ans.**

Question 6 :

(i) A(–1, 3), B(4, 2) and C(3, – 2) are the vertices of a triangle.

(a) Find the coordinates of the centroid of the triangle.

(b) Find the equation of the line through G and parallel to AC. **[3]**

(ii) Prove that : $\dfrac{\cot A - 1}{2 - \sec^2 A} = \dfrac{\cot A}{1 + \tan A}$. **[3]**

(iii) The 4th term of an A.P. is 22 and 15th term is 66. Find the first term and the common difference. Hence find the sum of the series to 8 terms. **[4]**

Solution :

(i) (a) Coordinates of the centroid (G) of the triangle are

$\left(\dfrac{-1+4+3}{3}, \dfrac{3+2-2}{3}\right)$ or (2, 1) **Ans.**

(b) Gradient of the line AC = $\dfrac{-2-3}{3+1} = \dfrac{-5}{4}$

$\therefore$ Equation of the line passing through G (2, 1) and parallel to AC is

$$y - 1 = \frac{-5}{4}(x - 2)$$

or $4y - 4 = -5x + 10$ or $5x + 4y = 14$ **Ans.**

(ii) L.H.S. = $\dfrac{\cot A - 1}{2 - \sec^2 A} = \dfrac{\cot A - 1}{2 - (1 + \tan^2 A)} = \dfrac{\cot A - 1}{1 - \tan^2 A} = \dfrac{\cot A - 1}{(1 - \tan A)(1 + \tan A)}$

$= \dfrac{\cot A - 1}{\left(1 - \dfrac{1}{\cot A}\right)(1 + \tan A)} = \dfrac{\cot A (\cot A - 1)}{(\cot A - 1)(1 + \tan A)} = \dfrac{\cot A}{1 + \tan A}$ = R.H.S. **Proved.**

(iii) We have $T_4 = a + 3d = 22$ ...(1)

$T_{15} = a + 14d = 66$ ...(2)

Subtracting (1) from (2), we get; $11d = 44 \Rightarrow d = 4$

From (1), $a + 3 \times 4 = 22 \Rightarrow a = 22 - 12 = 10$

Now, $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow S_8 = \dfrac{8}{2}\left[2 \times 10 + (8-1) \times 4\right] = 4[20 + 28] = 4 \times 48 = 192$

Hence, $a = 10$, $d = 4$, $S_8 = 192$ **Ans.**

Question 7 :

(i) The duration of telephone calls in a week at a school were recorded as below :

Time in seconds	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of calls	5	14	19	27	43	29	16	12	5

Draw an ogive and locate the median and quartiles. **[5]**

(ii) ₹ 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹ 100 more. Find the original number of children. **[5]**

Solution :

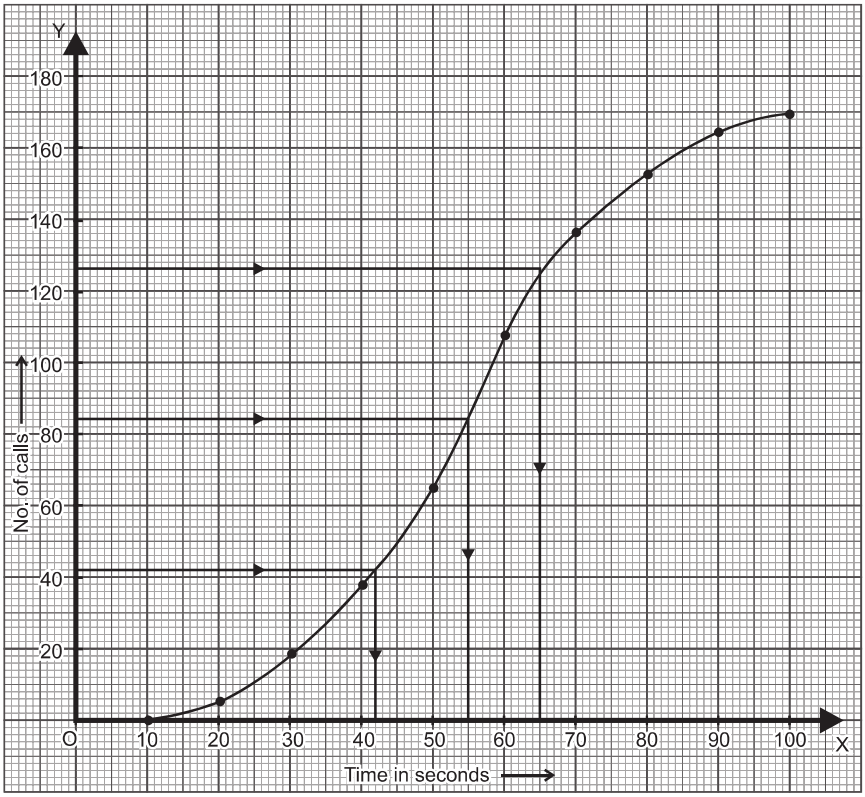
(ii) First we prepare a cumulative frequency table.

Time in seconds	No. of calls	Cumulative frequency
10–20	5	5
20–30	14	19
30–40	19	38
40–50	27	65

50–60	43	108
60–70	29	137
70–80	16	153
80–90	12	165
90–100	5	170

On a graph paper, mark time along x-axis and number of calls along y-axis. Then we plot the points (10, 0), (20, 5), (30, 19), (40, 38), (50, 65), (60, 108), (70, 137), (80, 153), (90, 165) and (100, 70).

Join these points by a free hand curve to get the ogive.



Here, N = 170 $\Rightarrow \frac{N}{2} = 85$

From the graph, median = 55. **Ans.**

$\frac{N}{4} = 42.5$

From the graph lower quartile, $Q_1 = 42$ **Ans.**

$\frac{3N}{4} = 127.5$

From the graph, upper quartile, $Q_3 = 66$ **Ans.**

(ii) Let the original number of children be x.

Then, share of each child = $\frac{₹\,7500}{x}$

New number of children = $x - 20$

New share of each child = $\frac{₹\,7500}{x-20}$

But, $\frac{7500}{x-20} - \frac{7500}{x} = 100$ [Given]

$\Rightarrow 7500\frac{(x - x + 20)}{x(x-20)} = 100$

$\Rightarrow x^2 - 20x = 75 \times 20$

$\Rightarrow x^2 - 20x - 1500 = 0$

$\Rightarrow x^2 - 50x + 30x - 1500 = 0$

$\Rightarrow x(x - 50) + 30(x - 50) = 0$

$\Rightarrow (x - 50)(x + 30) = 0$

$\Rightarrow x = 50$ or $x = -30$

$\Rightarrow x = 50$ [Rejecting $x = -30$, as number of children cannot be negative]

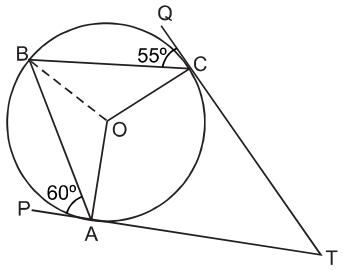
Hence, original number of children is 50. **Ans.**

Question 8 :

(i) If one card is drawn at random from a pack of 52 playing cards, what is the probability that it is:
(a) a king? (b) the ace of clubs? (c) a heart? [3]

(ii) The scale of a map is 5 m to 1 cm. [3]
(a) what area in m^2 is represented by a square of side 3 cm on the map?
(b) What is the length of the side of the square on the map which represents an area of 50,625 m^2?

(iii) In the given figure, TP and TQ are two tangents to the circle with centre O, touching at A and C respectively. If $\angle BCQ = 55°$ and $\angle BAP = 60°$, find:



(a) $\angle OBA$ and $\angle OBC$ (b) $\angle AOC$ (c) $\angle ATC$ [4]

Solution :

(i) Total number of cards = 52

(a) Total number of kings = 4

$\therefore$ P (a king) = $\frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}} = \frac{4}{52} = \frac{1}{13}$ **Ans.**

(b) There is only one ace of clubs in a pack of 52 cards

$\therefore$ P(an ace of clubs) = $\frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}} = \frac{1}{52}$ **Ans.**

(c) There are 13 cards of hearts in a pack of playing cards.

$\therefore$ P(a heart) = $\frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}} = \frac{13}{52} = \frac{1}{4}$ **Ans.**

(ii) Scale = 1 : 500

So, scale factor, $k = \frac{1}{500}$

Actual area $= \frac{1}{k^2} \times$ area on the map

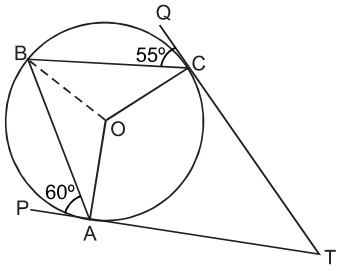
$= 2500 \times 9 \text{ cm}^2 = \frac{2500 \times 9}{100 \times 100} \text{ m}^2 = 2.25 \text{ m}^2$ **Ans.**

(b) Actual length of the side of the square = $\sqrt{50625}$ m = 225 m

So, side of on the map = k × Actual length

$= \frac{1}{500} \times 225$ m = 45 cm **Ans.**

(iii) (a) We know that the radius through the point of contact is perpendicular to the tangent.



$\therefore \angle OCQ = 90°$ and $\angle OAP = 90°$

Now, $\angle OCB = \angle OCQ - \angle BCQ = 90° - 55° = 35°$

$\angle OAB = \angle OAP - \angle BAP = 90° - 60° = 30°$

Now, OC = OB and OA = OB

$\Rightarrow \angle OBC = \angle OCB$ and $\angle OBA = \angle OAB$

[Angles opposite to equal sides are equal]

$\Rightarrow \angle OBC = 35°$ and $\angle OBA = 30°$ **Ans.**

(b) $\angle ABC = \angle OBA + \angle OBC = 35° + 30° = 65°$

$\angle AOC = 2\angle ABC$ [Angles at the centre is twice the angle at remaining part of the circle]

$\Rightarrow \angle AOC = 2 \times 65° = 130°$ **Ans.**

(c) In quadrilateral AOCT,

$\angle AOC + \angle OAT + \angle OCT + \angle ATC = 360°$ [Angle sum property of a quadrilateral]

$\Rightarrow 130° + 90° + 90° + \angle ATC = 360° \Rightarrow \angle ATC = 360° - 310° = 50°$ **Ans.**

Question 9 :

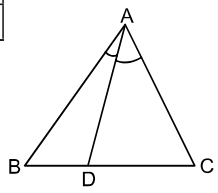
(i) Solve the following inequation and represent the solution set on a number line. **[3]**

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \leq 7\frac{1}{2}, x \in I$$

(ii) Find the mean of the following data: **[3]**

Class-interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	12	16	6	7	9

(iii) In ΔABC, AD is the bisector of $\angle A$. If BC = 10 cm, BD = 6 cm and AC = 6 cm, find AB. **[4]**



Solution :

(i) We have, $-8\frac{1}{2} < -\frac{1}{2} - 4x \leq 7\frac{1}{2}, x \in I$

$\Rightarrow -\frac{17}{2} < -\frac{1}{2} - 4x$ and $-\frac{1}{2} - 4x \leq \frac{15}{2}, x \in I$

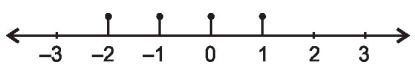
$\Rightarrow -\frac{17}{2} + \frac{1}{2} < -4x$ and $-4x \leq \frac{15}{2} + \frac{1}{2}, x \in I$

$\Rightarrow -8 < -4x$ and $-4x \leq 8, x \in I \Rightarrow -2 < -x$ and $-x \leq 2, x \in I$

$\Rightarrow x < 2$ and $x \geq -2, x \in I \Rightarrow -2 \leq x < 2, x \in I$

$\therefore$ Solution set = {–2, –1, 0, 1} **Ans.**

The solution set has been represented on the number line as below :



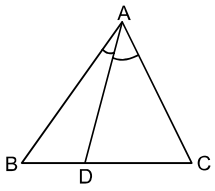
(ii) We may prepare the table given below:

Class-interval	Frequency (f)	Class-marks (x)	fx
0 – 10	12	5	60
10 – 20	16	15	240
20 – 30	6	25	150
30 – 40	7	35	245
40 – 50	9	45	405
	$\Sigma f = 50$		$\Sigma fx = 1100$

$$\therefore \text{ Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{1100}{50} = 22$$

Thus, the mean of the given data is 22.

(iii) We know that the internal bisector of an angle of a triangle divides the opposite sides in the ratio of the sides containing the angle.



$$\therefore \quad \frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{BD}{BC - BD} = \frac{AB}{AC} \Rightarrow \frac{6}{10-6} = \frac{AB}{6} \Rightarrow \frac{6}{4} = \frac{AB}{6} \Rightarrow AB = \frac{36}{4} = 9 \text{ cm} \quad \textbf{Ans.}$$

Question 10 :

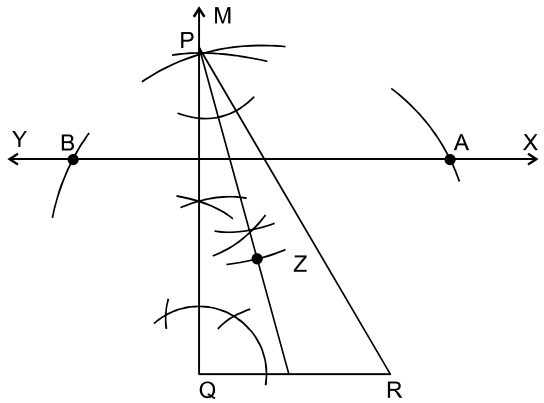
(i) Construct a right angled triangle PQR such that PQ = 6 cm, QR = 3.5 cm and ∠PQR = 90°. Bisect ∠P internally and mark a point Z on this bisector such that PZ = 4 cm. Find the points which are 4 cm from Z and also 4 cm from the line QR. [3]

(ii) If b is the mean proportion between a and c, show that : $\frac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4} = \frac{a^2}{c^2}$ [3]

(iii) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from the point Q is 30°. Find the height of the tower PT, correct to the nearest metre. [4]

Solution :

(i) A and B are two points which are 4 cm from Z and also 4 cm from QR.



(ii) If b is the mean proportion between a and c, then

$$\frac{a}{b} = \frac{b}{c} = k \text{ (say)}$$

$\Rightarrow a = bk, \quad b = ck$

$\Rightarrow a = ck^2, \quad b = ck$ ... (1)

Now, LHS $= \dfrac{a^4 + a^2b^2 + b^4}{b^4 + b^2c^2 + c^4}$

$= \dfrac{\left(ck^2\right)^4 + \left(ck^2\right)^2 \left(ck\right)^2 + \left(ck\right)^4}{\left(ck\right)^4 + \left(ck\right)^2 c^2 + c^4}$ [From (1)]

$= \dfrac{c^4k^8 + c^4k^6 + c^4k^4}{c^4k^4 + c^4k^2 + c^4} = \dfrac{k^4c^4\left(k^4 + k^2 + 1\right)}{c^4\left(k^4 + k^2 + 1\right)} = k^4$

RHS $= \dfrac{a^2}{c^2} = \dfrac{\left(ck^2\right)^2}{c^2} = k^4$

Hence, LHS = RHS **Proved**

(iii) Let the horizontal distance between the towers be x m.

Then, in ΔPQR, we have,

$\tan 60° = \dfrac{RQ}{PQ} \Rightarrow \sqrt{3} = \dfrac{50}{x}$

$\Rightarrow x = \dfrac{50}{\sqrt{3}}$ ...(1)

In ΔPTQ, we have,

$\tan 30° = \dfrac{PT}{x} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{PT}{x}$

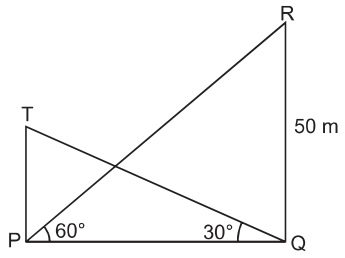
$\Rightarrow x = PT\sqrt{3}$ ...(2)

From (1) and (2), we get

$\dfrac{50}{\sqrt{3}} = PT\sqrt{3}$

$\Rightarrow PT = \dfrac{50}{3} = 16\dfrac{2}{3}$ m = 16.67 m

Hence, required height = 17 m **Ans.**



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