DAY TWO

Kinematics

Learning & Revision for the Day

- Frame of Reference
- Motion in a Straight Line
- Uniform and Non-uniform Motion
- Uniformly Accelerated Motion
- Graphs
- Elementary Concept of Differentiation and Integration for Describing Motion

Frame of Reference

The frame of reference is a suitable coordinate system involving space and time used as a reference to study the motion of different bodies. The most common reference frame is the cartesian frame of reference involving (x, y, z and t).

(i) **Inertial Frame of Reference** A frame of reference which is either at rest or moving with constant velocity is known as inertial frame of reference. Inertial frame of reference is one in which Newton's first law of motion holds good.

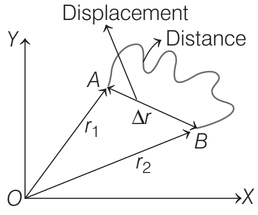
(ii) **Non-Inertial Frame of Reference** A frame of reference moving with some acceleration is known as non-inertial frame of reference. Non-inertial frame of reference in one which Newton's law of motion does not hold good.

Motion in a Straight Line

The motion of a point object in a straight line is one dimensional motion. During such a motion the point object occupies definite position on the path at each instant of time. Different terms used to described motion are defined below:

Distance and Displacement

- **Distance** is the total length of the path travelled by a particle in a given interval of time. It is a scalar quantity and its SI unit is metre (m).
- **Displacement** is shortest distance between initial and final positions of a moving object. It is a vector quantity and its SI unit is metre.



From the given figure, mathematically it is expressed as,

$$\Delta r = r_2 - r_1$$

- Displacement of motion may be zero or negative but path length or distance can never be negative.
- For motion between two points displacement is single valued while distance depends on actual path and so can have many values.
- Magnitude of displacement can never be greater than distance. However, it can be equal, if the motion is along a straight line without any change in direction.

Speed and Velocity

- **Speed** is defined as the total path length (or actual distance covered) by time taken by object.

  $$\text{Speed} = \frac{\text{Distance}}{\text{Time taken}}$$

  It is scalar quantity. Its SI unit is m/s.
- Average Speed, $v_{av} = \dfrac{\text{Total distance travelled}}{\Delta t}$
- When a body travels equal distance with speeds v_1 and v_2, the average speed (v) is the harmonic mean of the two speeds.

  $$\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$$
- When a body travels for equal time with speeds v_1 and v_2, the average speed v is the arithmetic mean of the two speeds.

  $$v_{av} = \frac{v_1 + v_2}{2}$$
- **Velocity** is defined as ratio of displacement and corresponding time interval taken by an object.

  i.e. velocity $= \dfrac{\text{Displacement}}{\text{time interval}}$
- Average velocity $= \dfrac{\text{Total displacement}}{\text{Total time taken}} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{\Delta x}{\Delta t}$

  Here, x_2 and x_1 are the positions of a particle at the time t_2 and t_1 respectively, with respect to a given frame of reference.
- For a moving body speed can never be negative or zero while velocity can be negative and zero.
- The **instantaneous speed** is average speed for infinitesimal small time interval (i.e. $\Delta t \to 0$)

  i.e. Instantaneous speed $v = \lim_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$
- The instantaneous velocity (or simply velocity) v of a moving particle is $v = \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$

  It (at a particular time) can be calculated as the slope (at that particular time) of the graph of x *versus* t.

Uniform and Non-uniform Motion

- An object is said to be in uniform motion if its velocity is uniform i.e. it undergoes equal displacement in equal may be intervals of time, however small these interval.
- An object is said to be in non-uniform motion if its undergoes equal displacement in unequal intervals of time., however small these intervals may be.

Acceleration

Acceleration of an object is defined as rate of change of velocity. It is a vector quantity having unit m/s^2 or ms^{-2}. It can be positive, zero or negative.

Average and Instantaneous Acceleration If velocity of a particle at instant t is v_1 and at instant t_2 is v_2, then

- Average acceleration, $a_{av} = \dfrac{v_2 - v_1}{t_2 - t_1} = \dfrac{\Delta v}{\Delta t}$
- Instantaneous acceleration, $a = \lim_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

Uniformly Accelerated Motion

- A motion, in which change in velocity in each unit of time is constant, is called an uniformly accelerated motion. So, for an uniformly accelerated motion, acceleration is constant.
- For uniformly accelerated motion are given below

  Equations of motion, $v = u + at$ ...(i)

  $$s = ut + \frac{1}{2}at^2 \quad \text{...(ii)}$$

  and $v^2 = u^2 + 2as$ ...(iii)

  where, u = initial velocity, v = velocity at time t and s = displacement of particle at time t.
- Equation of uniformaly accelerated motion under gravity are

  (i) $v = u - gt$ (ii) $h = ut - \dfrac{1}{2}gt^2$ (iii) $v^2 = u^2 - 2gh$

Elementary Concept of Differentiation and Integration for Describing Motion

- At an instant t, the body is at point $P(x, y, z)$.

  Thus, velocity along X-axis, $v_x = \dfrac{dx}{dt}$

  Acceleration along X-axis is $a_x = \dfrac{dv_x}{dt}$

  Velocity along Y-axis is $v_y = \dfrac{dy}{dt}$

  Acceleration along Y-axis is $a_y = \dfrac{dv_y}{dt}$

  Similarly, $v_z = \dfrac{dz}{dt}$ and $a_z = \dfrac{dv_z}{dt}$
- **For a accelerating body**

  (i) If a_x variable, $x = \int v_x\, dt$, $\int dv_x = \int a_x\, dt$

  (ii) If a_y is variable, $y = \int v_y\, dt$, $\int dv_y = \int a_y\, dt$

  (iii) If a_z is variable, $z = \int v_z\, dt$, $\int dv_z = \int a_z dt$

Also, distance travelled by a particle is $s = \int |v|\, dt$

(i) x-component of displacement is $\Delta x = \int v_x\, dt$

(ii) y-component of displacement is $\Delta y = \int v_y\, dt$

(iii) z-component of displacement is $\Delta z = \int v_z\, dt$

Graphs

During motion of the particle, its parameters of kinematical analysis changes with time. These can be represented on the graph, which are given as follows:

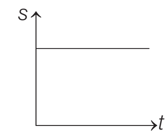
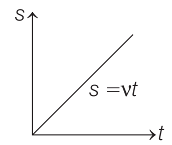
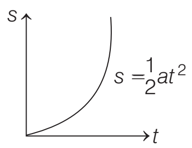
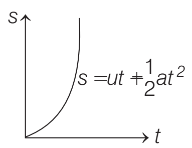
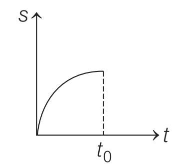
Position-Time Graph

(i) Position-time graph gives instantaneous value of displacement at any instant.

(ii) The slope of tangent drawn to the graph at any instant of time gives the instantaneous velocity at that instant.

(iii) The s-t graph cannot make sharp turns.

Different Cases of Position-Time Graph

Different Cases	s- t Graph	The main Features of Graph
At rest		Slope $= v = 0$
Uniform motion		Slope = constant, v = constant $a = 0$
Uniformly accelerated motion with $u = 0, s = 0$ at $t = 0$		$u = 0$, i.e. Slope of s-t graph at $t = 0$, should be zero.
Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$		Slope of s-t graph gradually goes on increasing
Uniformly retarded motion		θ is decreasing so, v is decreasing, a is negative

Velocity-Time Graph

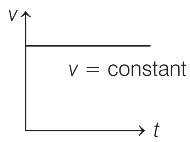
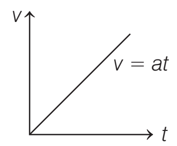
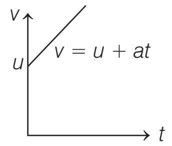
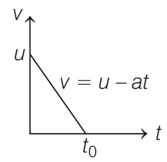
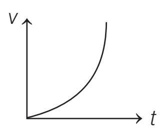
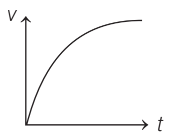
(i) Velocity-time graph gives the instantaneous value of velocity at any instant.

(ii) The slope of tangent drawn on graph gives instantaneous acceleration.

(iii) Area under v-t graph with time axis gives the value of displacement covered in given time.

(iv) The v-t curve cannot take sharp turns.

Different Cases in Velocity-Time Graph

Different Cases	v- t Graph	The main Features of Graph
Uniform motion		(i) $\theta = 0^{\circ}$ (ii) v = constant (iii) Slope of v-t graph $= a = 0$
Uniformly accelerated motion with $u = 0$ and $s = 0$ at $t = 0$		So slope of v-t graph is constant $u = 0$ i.e. so, a = constant $u = 0$ i.e. $v = 0$ at $t = 0$
Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$		Positive constant acceleration because θ is constant and $<90^{\circ}$ but the initial velocity of the particle is positive
Uniformly decelerated motion		Slope of v-t graphs $= -a$ (retardation)
Non-uniformly accelerated motion		Slope of v-t graph increases with time. θ is increasing, so, acceleration is increasing
Non-uniformly decelerating motion		θ is decreasing, so acceleration decreasing

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 A wheel of radius 1m rolls forward half a revolution on a horizontal ground. The magnitude of the displacement of the point of the wheel initially in contact with the ground is

(a) 2π (b) $\sqrt{2}\,x$ (c) $\sqrt{\pi^2+4}$ (d) π

2 In one dimensional motion, instantaneous speed v statisfies $0 \le v \le v_0$.

(a) The displacement x in time T must always take non-negative values
(b) The displacement x in time T satisfies $-v_0T < x < v_0T$
(c) The acceleration is always a non-negative number
(d) The motion has no turning point

3 A particle moving in a straight line covers half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time intervals and with speeds of 4.5 m/s and 7.5 m/s, respectively. The average speed of the particle during this motion is

(a) 4 m/s (b) 5 m/s (c) 5.5 m/s (d) 4.8 m/s

4 Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be ➜ NEET 2017

(a) $\frac{t_1+t_2}{2}$ (b) $\frac{t_1t_2}{t_2-t_1}$ (c) $\frac{t_1t_2}{t_2+t_1}$ (d) t_1-t_2

5 A body travelling along a straight line traversed one-third of the total distance with a velocity $4\,\text{ms}^{-1}$. The remaining part of the distance was covered with a velocity $2\,\text{ms}^{-1}$ for half the time and with velocity 6ms^{-1} for the other half of time. The average velocity over the whole time of motion is

(a) $5\,\text{ms}^{-1}$ (b) $4\,\text{ms}^{-1}$ (c) $4.5\,\text{ms}^{-1}$ (d) $3.5\,\text{ms}^{-1}$

6 A body is moving with velocity 30 m/s towards East. After 10 s its velocity becomes 40 m/s towards North. The average acceleration of body is

(a) 7 m/s^2 (b) $\sqrt{7}$ m/s^2 (c) 5 m/s^2 (d) 1 m/s^2

7 A body initially at rest is moving with uniform acceleration a m/s^2. Its velocity after n second is v. The displacement of the body in last 2 s is

(a) $\frac{2v(n-1)}{n}$ (b) $\frac{v(n-1)}{n}$
(c) $\frac{v(n+1)}{n}$ (d) $\frac{2v(2n+1)}{n}$

8 The velocity of a particle at an instant is $10\,\text{ms}^{-1}$. After 3 s its velocity will become $16\,\text{ms}^{-1}$. The velocity at 2 s before the given instant, will be

(a) $6\,\text{ms}^{-1}$ (b) $4\,\text{ms}^{-1}$ (c) $2\,\text{ms}^{-1}$ (d) $1\,\text{ms}^{-1}$

9 An object moves, starting from rest through a resistive medium such that its acceleration is related to velocity as, $a = 3 - 2v$. Then,

(a) the terminal velocity is 1.5 unit
(b) the terminal velocity is 3 unit
(c) the slope of a-v graph is not constant
(d) initial acceleration is 2 unit

10 A particle moves in a straight line with a constant acceleration. It changes its velocity from 10 ms^{-1} to 20 ms^{-1}, while passing through a distance 135 m in t second. The value of t (in second) is

(a) 12 (b) 9
(c) 10 (d) 1.8

11 A particle moving in a straight line with uniform acceleration is observed to be a distance a from a fixed point initially. It is at distance b, c, d from the same point after n, $2n$, $3n$ seconds. The acceleration of the particle is

(a) $\frac{c-2b+a}{n^2}$ (b) $\frac{c+b+a}{9n^2}$
(c) $\frac{c+2b+a}{4n^2}$ (d) $\frac{c-b+a}{n^2}$

12 A car accelerates from rest at constant rate for first 10 s and covers a distance x. It covers a distance y in next 10 s at the same acceleration. Which of the following is true?

(a) $x = 3y$ (b) $y = 3x$ (c) $x = y$ (d) $y = 2x$

13 A car starts from rest, moves with an acceleration a and then decelerates at a constant rate b for sometime to come to rest. If the total time taken is t. The maximum velocity of car is given by

(a) $\frac{abt}{(a+b)}$ (b) $\frac{a^2t}{(a+b)}$
(c) $\frac{at}{(a+b)}$ (d) $\frac{b^2t}{(a+b)}$

14 A particle starts its motion from rest such that its velocity remains constant. If the distance covered in first 10 s is s_1 and that covered in the first 20 s is s_2, then

(a) $s_2 = 2s_1$ (b) $s_2 = 3s_1$ (c) $s_2 = 4s_1$ (d) $s_2 = s_1$

15 A bullet loses 1/20 of its velocity after penetrating a plank. How many planks are required to stop the bullet?

(a) 6 (b) 9 (c) 11 (d) 13

16 A man throws balls with the same speed vertically upwards one after the other at an interval of 2 s. What should be the speed of the throw, so that more than two balls are in the sky at any time? (Given, $g = 9.8\,ms^{-2}$)

(a) Any speed less than $19.6\,ms^{-1}$
(b) Only with speed $19.6\,ms^{-1}$
(c) Greater than $19.6\,ms^{-1}$
(d) At least $9.8\,ms^{-1}$

17 A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 s, the next 5 s and the next 5 s, respectively. The relation between h_1, h_2 and h_3 is ➜ NEET 2013

(a) $h_1 = 2h_2 = 3h_3$
(b) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$
(c) $h_2 = 3h_1$ and $h_3 = 3h_2$
(d) $h_1 = h_2 = h_3$

18 A boy standing at top of a tower of 20 m height drops a stone. Assuming $g = 10\,ms^{-2}$, the velocity with which it hits the ground is ➜ CBSE AIPMT 2011

(a) 20 m/s (b) 40 m/s (c) 5 m/s (d) 10 m/s

19 A ball is dropped from a high rise platform at $t = 0$ starting from rest. After 6 s another ball is thrown downwards from the same platform with speed v. The two balls meet at $t = 18$ s. What is the value of v? ➜ CBSE AIPMT 2010

(a) $74\,ms^{-1}$
(b) $55\,ms^{-1}$
(c) $40\,ms^{-1}$
(d) $60\,ms^{-1}$

20 A body is thrown vertically up with a velocity u. It passes three points A, B and C in its upward journey with velocity $\frac{u}{2}, \frac{u}{3}$ and $\frac{u}{4}$, respectively. The ratio of separations between points AB and between BC, i.e. $\frac{AB}{BC}$ is

(a) 1 (b) 2 (c) $\frac{10}{7}$ (d) $\frac{20}{7}$

21 When a ball is thrown up vertically with velocity v_0, it reaches a maximum height of h. If one wishes to triple the maximum height, then the ball should be thrown with velocity

(a) $\sqrt{3}\,v_0$ (b) $3\,v_0$ (c) $9\,v_0$ (d) $3/2\,v_0$

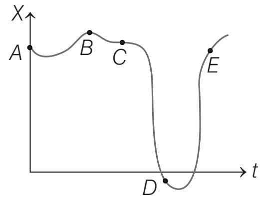
22 From the top of a tower two stones, whose masses are in the ratio 1 : 2 are thrown on straight up with an initial speed u and the second straight down with the same speed u. Then, neglecting air resistance,

(a) the heavier stone hits the ground with a higher speed
(b) the lighter stone hits the ground with a higher speed
(c) both the stones will have the same speed when they hit the ground
(d) the speed can not be determined with the given data

23 The velocity-time graph of particle comes out to be a non-linear curve. The motion is

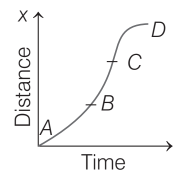
(a) uniform velocity motion
(b) uniformly accelerated motion
(c) non-uniform accelerated motion
(d) Nothing can be said about the motion

24 A graph of x *versus* t is shown in figure. Choose incorrect statements from below.



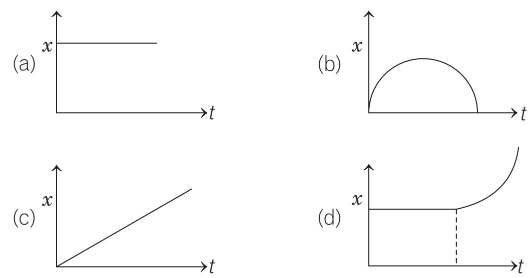
(a) The particle was released from rest at $t = 0$
(b) At B, the acceleration $a > 0$
(c) At C, the velocity and the acceleration
(d) The speed at D exceeds than at E

25 A particle shows distance-time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point

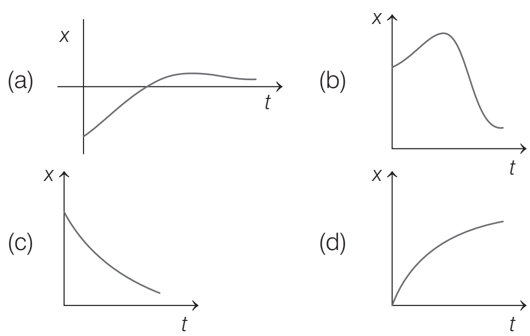


(a) B
(b) C
(c) D
(d) A

26 The position-time graph for a uniform motion is represented as



27 Among the four graphs, there is only one graph for which average velocity over the time interval $(0, T)$ can vanish for suitably chosen T. Which one is it?



28 A toy car with charge q moves on a frictionless horizontal plane surface under the influence of a uniform electric field E. Due to the force qE, its velocity increases from 0 to 6 m/s in are second duration. At that instant, the direction of the field is reversed. The car continues to move for two more second under the influence of this field. The average velocity and the average speed of the toy car between 0 to 3 seconds are respectively.

➜ NEET 2018

(a) 1 m/s, 3.5 m/s (b) 1 m/s, 3 m/s
(c) 2 m/s, 4 m/s (d) 1.5 m/s, 3 m/s

29 A particle moving along X-axis has acceleration f, at time t given by $f = f_0\left(1 - \frac{t}{T}\right)$, where f_0 and T are constants.

The particle at $t = 0$ has zero velocity. In the time interval between $t = 0$ and the instant when $f = 0$, the particle's velocity (v_x) is

(a) f_0T (b) $\frac{1}{2}f_0T^2$ (c) f_0T^2 (d) $\frac{1}{2}f_0T$

30 The position x of a particle with respect to time t along X-axis is given by $x = 9t^2 - t^3$, where x is in metre and t in second. What will be the position of this particle when it achieves maximum speed along the $+x$ direction?

(a) 32 m (b) 54 m (c) 81 m (d) 24 m

31 An object moving with a speed of 6.25 m/s, is deaccelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where v is the instantaneous speed. The time taken by the object, to come to rest would be

(a) 2 s (b) 4 s (c) 8 s (d) 1s

32 A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to

$$v(x) = \beta x^{-2n}$$

where, β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x, is given by

➜ CBSE AIPMT 2015

(a) $-2n\beta^2x^{-2n-1}$ (b) $-2n\beta^2x^{-4n-1}$
(c) $-2\beta^2x^{-2n+1}$ (d) $-2n\beta^2e^{-4n+1}$

33 A particle moves a distance x in time t according to equation $x = (t + 5)^{-1}$. The acceleration of particle is proportional to

➜ CBSE AIPMT 2010

(a) $(\text{velocity})^{3/2}$ (b) $(\text{distance})^2$
(c) $(\text{distance})^{-2}$ (d) $(\text{velocity})^{2/3}$

34 If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1s and 2s is

➜ NEET 2016

(a) $3A + 7B$ (b) $\frac{3}{2}A + \frac{7}{3}B$
(c) $\frac{A}{2} + \frac{B}{3}$ (d) $\frac{3}{2}A + 4B$

35 Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $X_P(t) = at + bt^2$ and $X_Q(t) = ft - t^2$. At what time do the cars have the same velocity?

➜ NEET 2016

(a) $\frac{a-f}{1+b}$ (b) $\frac{a+f}{2(b-1)}$
(c) $\frac{a+f}{2(1+b)}$ (d) $\frac{f-a}{2(1+b)}$

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

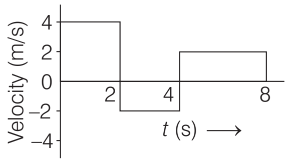
1 The motion of a particle along a straight line is described by equation, $x = 8 + 12t - t^3$, where x is in metre and t in sec. The retardation of the particle when its velocity becomes zero, is

(a) 24 ms^{-2} (b) zero (c) 6 ms^{-2} (d) 12 ms^{-2}

2 A body is thrown vertically upward in air when air resistance is taken into account, the time of ascent is t_1 and time of descent is t_2, then which of the following is true?

(a) $t_1 = t_2$ (b) $t_1 < t_2$ (c) $t_1 > t_2$ (d) $t_1 \geq t_2$

3 A body is moving in a straight line as shown in velocity-time graph. The displacement and distance travelled by body in 8 s are respectively



(a) 12 m, 20 m (b) 20 m, 12 m
(c) 12 m, 12 m (d) 20 m, 20 m

4 Two bodies A (of mass 1 kg) and B (of mass 3 kg) are dropped from heights of 16 m and 25 m, respectively. The ratio of the time taken by them to reach the ground is

(a) 5/4 (b) 12/5
(c) 5/12 (d) 4/5

5 A balloon rises from rest with a constant acceleration $\frac{9}{8}$.

A stone is released from it when it has risen to height h. The time taken by the stone to reach the ground is

(a) $4\sqrt{\frac{h}{g}}$ (b) $2\sqrt{\frac{h}{g}}$

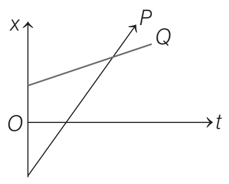
(c) $\sqrt{\frac{2h}{g}}$ (d) $\sqrt{\frac{g}{h}}$

6 The ratio of distance traversed in successive intervals of time when a body falls freely under gravity from certain height is

(a) 1 : 2 : 3 (b) 1 : 5 : 9

(c) 1 : 3 : 5 (d) $\sqrt{1} : \sqrt{2} : \sqrt{3}$

7 Figure shows the time-displacement curve of the particles *P* and *Q*. Which of the following statement is correct?



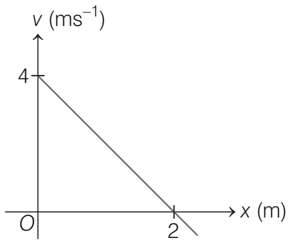
(a) Both *P* and *Q* move with uniform equal speed

(b) *P* is accelerated and *Q* moves with uniform speed, but the speed of *P* is more than the speed of *Q*

(c) Both *P* and *Q* moves with uniform speeds but the speed of *P* is more than the speed of *Q*

(d) Both *P* and *Q* moves with uniform speeds but the speed of *Q* is more than the speed of *P*

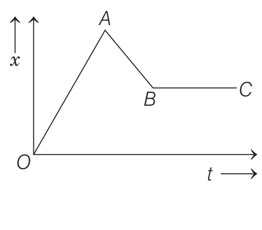
8 The velocity (v) of a particle moving along X-axis varies with its position x as shown in figure. The acceleration (a) of particle varies with position (x) as



(a) $a^2 = x + 3$ (b) $a = 2x^2 + 4$

(c) $2a = 3x + 5$ (d) $a = 4x - 8$

9 Given graph $(x\text{-}t)$ representing the motion of an object, match the terms of Column I with the items of Column II and choose the correct options from the codes given below



Then, match the following columns and choose the correct option from the codes given below.

	Column I		Column II
A.	Part *OA* of graph	1.	Positive velocity
B.	Part *AB* of graph	2.	Object at rest
C.	Part *BC* of graph	3.	Negative velocity
D.	Point '*A*' in the graph	4.	Change in direction of motion

	A	B	C	D
(a)	1	2	3	4
(b)	1	3	2	4
(c)	2	1	3	4
(d)	4	3	2	1

10 The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(a) decrease with time (b) be independent of α and β

(c) drop to zero when $\alpha = \beta$ (d) increase with time

11 A particle is released from rest from a tower of height $3h$. The ratio of times to fall equal height h, i.e. $t_1 : t_2 : t_3$ is

(a) $\sqrt{3} : \sqrt{2} : 1$ (b) 3 : 2 : 1

(c) 9 : 4 : 1 (d) $1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

12 A train accelerates from rest at a constant rate α for distance x_1 and time t_1, it retards to rest at constant rate β for distance x_2 and time t_2. Which of the relation is correct?

(a) $\frac{x_1}{x_2} = \frac{\alpha}{\beta} = \frac{t_1}{t_2}$ (b) $\frac{x_1}{x_2} = \frac{\beta}{\alpha} = \frac{t_1}{t_2}$

(c) $\frac{x_1}{x_2} = \frac{\alpha}{\beta} = \frac{t_2}{t_1}$ (d) $\frac{x_1}{x_2} = \frac{\beta}{\alpha} = \frac{t_2}{t_1}$

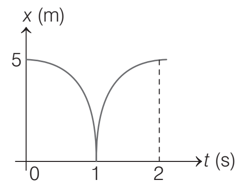
13 A dust packet is dropped from 5th storey of a multi-storeyed building. In the first second of its free fall another dust packet is dropped from 7th storey 15 m below the 9th storey. If both packets reach the ground at same time, then height of the building is

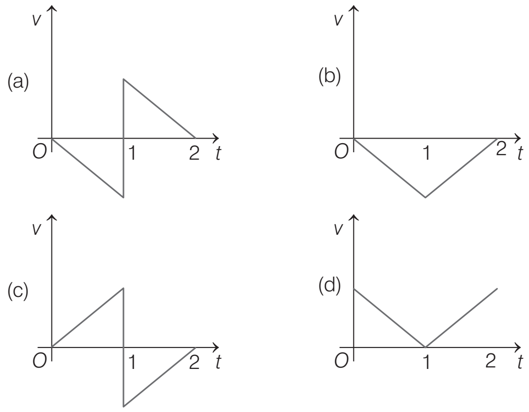
(a) 25 m (b) 15 m (c) 20 m (d) 16 m

14 A ball is dropped into a well in which the water level is at a depth h below the top. If the speed of sound is c, then the time after which the splash is heard will be given by

(a) $h\left[\sqrt{\frac{2}{gh}} + \frac{1}{c}\right]$ (b) $h\left[\sqrt{\frac{2}{gh}} - \frac{1}{c}\right]$ (c) $h\left[\frac{2}{g} + \frac{1}{c}\right]$ (d) $h\left[\frac{2}{g} - \frac{1}{c}\right]$

15 The displacement-time graph of a moving particle with constant acceleration is shown in figure. The velocity-time graph is given by





16 Water drops fall from a top on the floor 5 m below at regular intervals. The fifth drop is leaving the top at the instant, the first strikes the ground. The height at which the third drop will be from ground at that instant is (take, $g = 10\ \text{ms}^2$)

(a) 1.25 m (b) 2.15 m (c) 2.75 m (d) 3.75 m

17 From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground, is n times that taken by it to reach the highest point of its path. The relation among H, u and n is

(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2u^2$
(c) $2gH = nu^2(n-2)$ (d) $gH = (n-2)\,u^2$

18 A train is moving along a straight path with uniform acceleration. Its engine passes across a pole with a velocity of $60\ \text{kmh}^{-1}$ and the end (guard's van) passes across same pole with a velocity of $80\ \text{kmh}^{-1}$. The middle point of the train will pass across same pole with a velocity

(a) $70\ \text{kmh}^{-1}$ (b) $70.7\ \text{kmh}^{-1}$
(c) $65\ \text{kmh}^{-1}$ (d) $75\ \text{kmh}^{-1}$

19 A ball rolls off the top of stair way with a horizontal velocity of magnitude 1.8 m/s. The steps are 0.20 m high and 0.20 m wide. Which step will the ball hit first?

(a) First (b) Second
(c) Third (d) Fourth

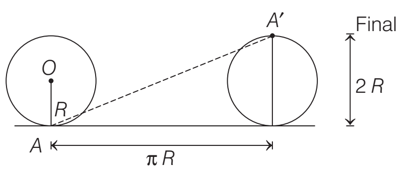
ANSWERS

SESSION 1	**1** (c)	**2** (b)	**3** (a)	**4** (c)	**5** (b)	**6** (c)	**7** (a)	**8** (a)	**9** (a)	**10** (b)
	11 (a)	**12** (b)	**13** (a)	**14** (c)	**15** (b)	**16** (c)	**17** (b)	**18** (a)	**19** (a)	**20** (d)
	21 (a)	**22** (c)	**23** (c)	**24** (b)	**25** (b)	**26** (a)	**27** (b)	**28** (b)	**29** (d)	**30** (b)
	31 (a)	**32** (b)	**33** (a)	**34** (b)	**35** (d)					
SESSION 2	**1** (d)	**2** (b)	**3** (a)	**4** (d)	**5** (b)	**6** (c)	**7** (c)	**8** (d)	**9** (b)	**10** (d)
	11 (d)	**12** (b)	**13** (c)	**14** (a)	**15** (d)	**16** (d)	**17** (c)	**18** (b)	**19** (d)	

Hints and Explanations

SESSION 1

1 Horizontal distance covered by the wheel in half revolution πR.



So, the displacement of the point which was initially in contact with ground

$= AA'$

$= \sqrt{(\pi R^2) + (2R)^2}$

$= R\sqrt{\pi^2 + 4}$

$= \sqrt{\pi^2 + 4}$ $[\because R = 1\ \text{m}]$

2 The maximum distance covered in time T is $v_0 T$.

Therefore, for the object having one-dimensional motion, the displacement x in time T satisfies

$$-v_0 T < x < v_0 T.$$

3 If t_1 and $2t_2$ are the time taken by the particle to cover first and second half distance, respectively.

$$t_1 = \frac{x/2}{3} = \frac{x}{6}$$

Clearly $x_1 = 4.5t_2$ and $x_2 = 7.5t_2$

So, $x_1 + x_2 = \frac{x}{2} = 4.5t_2 + 7.5t_2$

or $t_2 = \frac{x}{24}$

Total time, $t = t_1 + 2t_2 = \frac{x}{6} + \frac{x}{12} = \frac{x}{4}$

So, average speed = 4 m/s

4 Speed of walking $= \dfrac{h}{t_1} = v_1$

Speed of escalator $= \dfrac{h}{t_2} = v_2$

Time taken when she walks over running escalator

$\Rightarrow t = \dfrac{h}{v_1 + v_2}$

$\Rightarrow \dfrac{1}{t} = \dfrac{v_1}{h} + \dfrac{v_2}{h} = \dfrac{1}{t_1} + \dfrac{1}{t_2} \Rightarrow t = \dfrac{t_1 t_2}{t_1 + t_2}$

5 Let total distance be s and the time taken to cover first one-third distance as t_1, then $t_1 = \dfrac{s/3}{4} = \dfrac{s}{12}$

Now, let t_2 be the time for the rest two journeys. Then,

$$\frac{2s}{3} = 2t_2 + 6t_2 = 8t_2$$

$\therefore \quad t_2 = \dfrac{2s}{24} = \dfrac{s}{12}$

$\therefore$ Average velocity

$$= \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{s}{t_1 + 2t_2} = \frac{s}{\frac{s}{12} + \frac{s}{6}}$$

$$= \frac{12 \times 6}{12 + 6} = 4 \text{ ms}^{-1}$$

6 Average acceleration

$$= \frac{\text{Change in velocity}}{\text{Total time}}$$

$a = \dfrac{|v_f - v_i|}{\Delta t}$,

$$a = \frac{\sqrt{(30)^2 + (40)^2}}{10}$$

$$= \frac{\sqrt{900 + 1600}}{10}$$

$$= \frac{\sqrt{2500}}{10} = 5 \text{ ms}^{-2}$$

7 Displacement in last 2 s

$$= \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2$$

$$= 2a(n-1)$$

Acceleration, $\quad a = \dfrac{v}{n} \quad [\because t = n \text{ second}]$

Displacement in last 2 s $= \dfrac{2v(n-1)}{n}$

8 Using equation of motion,

$v = u + at$, we get

$16 = 10 + 3a$

$\Rightarrow \quad a = 2 \text{ ms}^{-2}$

and $\quad 10 = u + 2 \times 2$

[u = required velocity]

$\Rightarrow \quad u = 6 \text{ ms}^{-1}$

9 The meaning of terminal velocity is constant velocity. For constant velocity, acceleration should be zero (i.e. $a = 0$).

$\because \quad a = 3 - 2v$

or $\quad 0 = 3 - 2v$

$\therefore$ Velocity, $\quad v = \dfrac{3}{2} = 1.5$ unit

10 From equation of motion,

$v^2 = u^2 + 2as$

$\Rightarrow \quad a = \dfrac{v^2 - u^2}{2s}$

$$= \frac{(20)^2 - (10)^2}{2 \times 135} = \frac{300}{270} = \frac{10}{9} \text{ ms}^{-2}$$

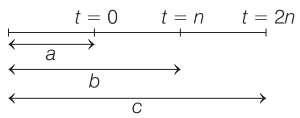
From first equation of motion,

$v = u + at$

$\Rightarrow \quad t = \dfrac{v - u}{a} = \dfrac{20 - 10}{10/9} = \dfrac{10}{10/9} = 9 \text{ s}$

11 Using equation of motion,

$s = ut + \dfrac{1}{2}at^2$, we get



$$b - a = un + \frac{1}{2}An^2$$

$\Rightarrow \quad 2b - 2a = 2un + An^2 \quad \ldots\text{(i)}$

Again, $c - a = u(2n) + \dfrac{1}{2}A(2n)^2 \quad \ldots\text{(ii)}$

On substracting, Eq. (i) from Eq. (ii), we get

$c - a - 2b + 2a = An^2$

$$A = \frac{c - 2b + a}{n^2}$$

12 From equation of motion, we have

$s = ut + \dfrac{1}{2}at^2$ where, u is initial velocity, t is time and a is acceleration.

Since, car accelerates from rest

$u = 0, t = 10$ s

$\therefore \quad s = 0 + \dfrac{1}{2} \times a \times (10)^2 = 50a \quad \ldots\text{(i)}$

Also, $\quad v = u + at$

where, v is final velocity.

$\therefore$ Velocity after 10 s is

$v = 0 + a \times 10$

$v = 10a = 10 \times \dfrac{s}{50} \quad \ldots\text{(ii)}$

In the next 10 s car moves with constant acceleration and with initial velocity v.

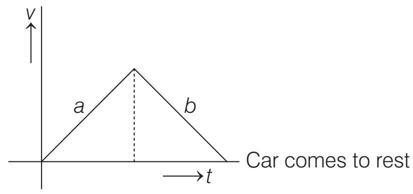
$\therefore s' = vt + \dfrac{1}{2}at^2$

$$= \frac{s}{50} \times 10 \times 10 + \frac{1}{2} \times \frac{s}{50} \times 100 = 3 \text{ s}$$

Given, $s = x$ and $s' = y$

$\therefore \quad y = 3x$

13 By equation of motion, we have



Since, body starts from rest $u = 0$.

Let t_1 be time when body accelerates and t_2 when it decelerates.

$\therefore \quad t = t_1 + t_2 \Rightarrow t_2 = t - t_1$

$\therefore \quad v = 0 + at_1 = at_1 \quad \ldots\text{(i)}$

When car finally comes to rest, $v = 0$

$\therefore \quad 0 = v - b(t - t_1) \quad \ldots\text{(ii)}$

From Eqs. (i) and (ii), we get

$t_1 = \dfrac{b}{(a+b)}t$ and $v = \dfrac{ab}{(a+b)}t$

14 We know that from second equation of motion

$$s = ut + \frac{1}{2}at^2 \quad \ldots\text{(i)}$$

Given distance, $s = s_1$ in first 10 sec and distance, $s = s_2$ in first 20 sec. and $u = 0$

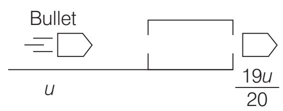
So, $\quad s_1 = \dfrac{1}{2}a(t_1)^2 \quad \ldots\text{(ii)}$

$s_2 = \dfrac{1}{2}a(t_2)^2 \quad \ldots\text{(iii)}$

From Eqn. (ii) and (iii), we get

$$\frac{s_1}{s_2} = \left(\frac{t_1}{t_2}\right)^2 = \left(\frac{10}{20}\right)^2 = \frac{1}{4} \Rightarrow s_2 = 4s_1.$$

15 The final velocity after it passes the plank is $\dfrac{19u}{20}$



Let x be the thickness of the plank, the deacceleration due to resistance of plank is given by $v^2 = u^2 + 2as$

where, v is final velocity, u is initial velocity, a is acceleration and s is displacement.

Here, $\quad v = \dfrac{19}{20}u$

$\therefore \left(\dfrac{19}{20}u\right)^2 = u^2 + 2ax \Rightarrow 2ax = \dfrac{-39}{400}u^2$

Suppose the bullet is stopped after passing through n such planks. Then, the distance covered by bullet is nx.

$\therefore \quad 0 = \left(\dfrac{19}{20}\right)^2 u^2 + 2\,anx$

$$\Rightarrow -\left(\frac{19}{20}\right)^2 u^2 = n \times \frac{-39}{400}u^2 \Rightarrow n = \frac{361}{39} \approx 9$$

16 Time taken by ball to reach maximum height $v = u - gt$

At maximum height, final speed is zero i.e. $v = 0$

So, $u = gt$

In 2s, $u = 2 \times 9.8 = 19.6 \text{ ms}^{-1}$

If man throws the ball with velocity of 19.6 ms^{-1}, then after 2 s it will reach the maximum height. When he throws second ball, first is at top. When he throws third ball, first will come to ground and second will at the top.

Therefore, only 2 balls are in air. If he wants to keep more than 2 balls in air, he should throw the ball with a speed greater than 19.6 ms^{-1}.

17 For free fall from a height, $u = 0$

Distance covered by stone in first 5s,

$$h_1 = 0 + \frac{1}{2}g(5)^2 = \frac{25}{2}g \quad \ldots\text{(i)}$$

Distance covered in first 10s,

$$s_2 = 0 + \frac{1}{2}g(10)^2 = \frac{100}{2}g$$

$\therefore$ Distance covered in second 5s

$$h_2 = s_2 - h_1 = \frac{100g}{2} - \frac{25g}{2} = \frac{75g}{2} \quad \ldots\text{(ii)}$$

Distance covered in first 15s,

$$s_3 = 0 + \frac{1}{2}g(15)^2 = \frac{225}{2}g$$

Distance covered in last 5s,

$$h_3 = s_3 - s_2 = \frac{225}{2}g - \frac{100}{2}g = \frac{125}{2}g \quad \ldots\text{(iii)}$$

From Eqs. (i), (ii) and (iii) we get

$$h_1 : h_2 : h_3 = \frac{25g}{2} : \frac{75g}{2} : \frac{125g}{2} = 1 : 3 : 5$$

$$\Rightarrow \quad h_1 = \frac{h_2}{3} = \frac{h_3}{5}$$

18 From the equation of motion

$v^2 = u^2 + 2gh$

Given, $u = 0$,

$\therefore \quad v = \sqrt{2gh}$

Given, $g = 10 \text{ ms}^{-2}$, $h = 20$ m

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ ms}^{-1}$$

19 For first ball, $u = 0$

$$s_1 = ut + \frac{1}{2}gt^2 = \frac{1}{2}gt_1^2 = \frac{1}{2}g(18)^2$$

For second ball, $s_2 = vt_2 + \frac{1}{2}gt^2$

$\Rightarrow \quad t_2 = 18 - 6 = 12s$

$\Rightarrow \quad s_2 = v \times 12 + \frac{1}{2}g\,(12)^2$

Here, $s_1 = s_2$

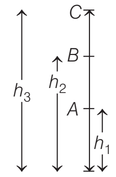
$$\frac{1}{2}g\,(18)^2 = 12v + \frac{1}{2}g\,(12)^2$$

$\Rightarrow \quad v = 73.5 \simeq 74 \text{ ms}^{-1}$

20 Here, for point $A = \frac{u^2}{4} - u^2 = -2gh_1$

for point $B = \frac{u^2}{9} - u^2 = -2gh_2$

and for point $C = \frac{u^2}{16} - u^2 = -2gh_3$



$$\therefore \quad AB = \frac{u^2}{2g}\left[\frac{8}{9} - \frac{3}{4}\right] = \frac{u^2}{2g} \cdot \frac{5}{36}$$

$$BC = \frac{u^2}{2g}\left[\frac{15}{16} - \frac{8}{9}\right] = \frac{u^2}{2g} \cdot \frac{7}{144}$$

$$\therefore \quad \frac{AB}{AC} = \frac{5}{36} \times \frac{144}{7} = \frac{20}{7}$$

21 We have, $v^2 = u^2 - 2gh$

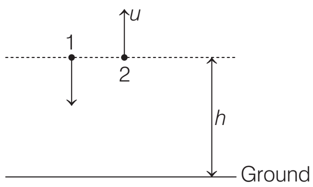
Here, $v = 0, u = v_0$

$\therefore \quad 0 = v_0^2 - 2gh \Rightarrow v_0 = \sqrt{2gh}$

When $h' = 3h$, then

$$v'_0 = \sqrt{2g \times 3h} = \sqrt{3}\sqrt{2gh} = \sqrt{3}\, v_0$$

22



Applying equation of motion for both particles 1 and 2

For particle 1,

$v^2 = u^2 + 2as \Rightarrow v_1^2 = u^2 - 2gh \quad \ldots\text{(i)}$

For particle 2, $v_2^2 = u^2 - 2gh \quad \ldots\text{(ii)}$

Hence, $v_1 = v_2$

Both the stones will have same speed when they hit the ground.

23 Velocity-time graph gives the instantaneous value of velocity at any instant. For non-uniformly accelerated motion, v-t graph is non-linear.

24 From graph, when $t = 0$, the particle is released from rest at A, hence, $v = 0$. At B, the graph is parallel to time axis, hence velocity is constant there. The acceleration a is zero.

At C, the graph changes slope, where velocity and acceleration vanish.

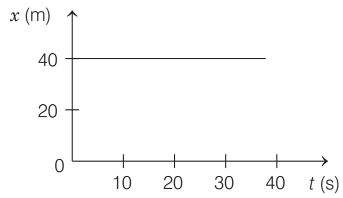
Average velocity for motion between A and D is negative, because the value of x is decreasing with time t. The slope of graph (which represents speed) is more at D than at E.

25 At instant instantaneous velocity of particle is given by

$$v = \frac{ds}{dt} = \tan\theta$$

From figure shows that slop of curve is maximum at point C.

26 For object at rest, the position-time graph is a straight line parallel to the time axis.



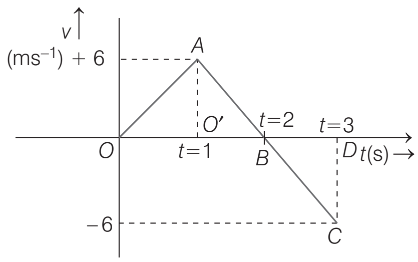
(a)

27 In graph (b), one value of displacement is obtained at two different time.

Thus, the average displacement of particle is zero. As average velocity $= \frac{\text{Displacement}}{\text{Time}}$

So, average velocity would be zero.

28 Given condition can be represented through graph also as shown below.



$\therefore$ Displacement in three seconds

= Area under the graph

= Area of $\Delta OAO'$ + Area of $\Delta AO'B$ − Area of ΔBCD

$$= \frac{1}{2} \times 1 \times 6 + \frac{1}{2} \times 1 \times 6 - \frac{1}{2} \times 6 \times 1$$

= 3 m

$\therefore$ Average velocity $= \frac{3}{3} = 1 \text{ ms}^{-1}$.

Total distance travelled, $d = 9$ m

$\therefore$ Average speed $= \frac{9}{3} = 3 \text{ ms}^{-1}$

29 Acceleration,

$$f = \frac{dv}{dt} = f_0\left(1 - \frac{t}{T}\right)$$

or $$dv = f_0\left(1 - \frac{t}{T}\right)dt \quad \ldots\text{(i)}$$

On integrating Eq. (i) both sides, we get

$$\therefore \quad v = f_0 t - \frac{f_0}{T} \cdot \frac{t^2}{2} + C \quad \ldots\text{(ii)}$$

After applying boundary conditions $v = 0$ at $t = 0$, we get

$$C = 0$$

$$\Rightarrow \quad v = f_0 t - \frac{f_0}{T} \cdot \frac{t^2}{2} \quad \ldots\text{(iii)}$$

Given, $f = f_0 \left(1 - \frac{t}{T}\right)$

Substituting, $f = 0$, $t = T$ in Eq. (iii), then velocity,

$$v_x = f_0 T - \frac{f_0}{T} \cdot \frac{T^2}{2} = \frac{1}{2} f_0 T$$

30 Given, $x = 9t^2 - t^3$ ...(i)

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(9t^2 - t^3)$

$$= 18t - 3t^2 \quad \left[\because \frac{d}{dx} x^n = nx^{n-1}\right]$$

Also, Acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt}(18t - 3t^2) = 18 - 6t \quad \ldots\text{(ii)}$$

Now, speed of particle is maximum, when its acceleration is zero, i.e. $a = 0$

i.e. $18 - 6t = 0$ or $t = 3$ s

Putting in Eq. (i), we obtain position of particle at the time

$$x = 9(3)^2 - (3)^3 = 54\,\text{m}$$

31 Given, $\frac{dv}{dt} = -2.5\sqrt{v}$

$$\Rightarrow \quad \frac{dv}{\sqrt{v}} = -2.5\,dt$$

$$\Rightarrow \quad \int_{6.25}^{0} v^{-1/2} dv = -2.5 \int_0^t dt$$

$$\Rightarrow \quad -2.5\,[t]_0^t = [2v^{1/2}]_{6.25}^0$$

$$\Rightarrow \quad \text{Time } t = 2\text{ s}$$

32 $a = \frac{\partial v}{\partial t} = \frac{\partial v}{\partial x} \times \frac{\partial x}{\partial t} = \frac{\partial v}{\partial x} \times v$

We have $v = \beta x^{-2n}$

$$\frac{\partial v}{\partial x} = -2n\beta x^{-2n-1}$$

Acceleration,

$$a = (-2n\beta x^{-2n-1}) \times (\beta x^{-2n})$$

$$a = -2n\beta^2 x^{-4n-1}$$

33 Given, $x = (t + 5)^{-1}$

$$\therefore \quad v = \frac{dx}{dt} = \frac{-1}{(t+5)^2} \quad \ldots\text{(i)}$$

$$a = \frac{d^2x}{dt^2} = \frac{2}{(t+5)^3} \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get $a \propto (v)^{3/2}$

34 Velocity of the particle is given as $v = At + Bt^2$

where A and B are constants.

$$\Rightarrow \quad \frac{dx}{dt} = At + Bt^2 \quad \left[\because v = \frac{dx}{dt}\right]$$

$$\Rightarrow \quad dx = (At + Bt^2)\,dt$$

Integrating both sides, we get

$$\int_{x_1}^{x_2} dx = \int_1^2 (At + Bt^2)\,dt$$

$$\Rightarrow \Delta x = x_2 - x_1 = A\int_1^2 t\,dt + B\int_1^2 t^2 dt$$

$$= A\left[\frac{t^2}{2}\right]_1^2 + B\left[\frac{t^3}{3}\right]_1^2$$

$$= \frac{A}{2}(2^2 - 1^2) + \frac{B}{3}(2^3 - 1^3)$$

$\therefore$ Distance travelled between $1s$ and $2s$ is

$$\Delta x = \frac{A}{2} \times (3) + \frac{B}{3}(7)$$

$$= \frac{3}{2}A + \frac{7}{3}B$$

35 Velocity of each car is given by

$$v_P = \frac{dX_p(t)}{dt} = a + 2bt$$

and $v_Q = \frac{dX_Q(t)}{dt} = f - 2t$

It is given that $v_P = v_Q$

$$\Rightarrow \quad a + 2bt = f - 2t$$

$$\Rightarrow \quad t = \frac{f - a}{2(b+1)}$$

SESSION 2

1 Given, $x = 8 + 12t - t^3$

We know

$$v = \frac{dx}{dt} \text{ and acceleration } a = \frac{dv}{dt}$$

So, $v = 12 - 3t^2$ and $a = -6t$

At $t = 2$ s, $v = 0$ and $a = -6 \times 2$

$$a = -12\text{ m/s}^2$$

So, retardation of the particle

$$= 12\text{ m/s}^2$$

2 First of all note that, air resistance acts in direction opposite to the motion of body. So, when a body is thrown up, then both the deaccelerating forces, i.e. gravity and air resistance act in same direction. Thus, total deacceleration is $a_1 = g + a_0$, where a_0 is deacceleration due to air resistance which is assumed to be constant.

If u be the initial velocity and t_1 be the time of ascent, then

$$t_1 = \frac{u}{g + a_0} \text{ or } u = t_1(g + a_0)$$

and $h = \frac{u^2}{2(g + a_0)}$ ...(i)

Also, t_2 is time of descent, then

$$h = ut + \frac{1}{2}a_2 t_2^2$$

As, $a_2 = g - a_0$, during descent

$$\Rightarrow \frac{u^2}{2(g + a_0)} = 0 + \frac{1}{2} \times [g - a_0] \times t_2^2$$

[using Eq. (i)]

$$\Rightarrow t_2 = \frac{u}{\sqrt{(g + a_0)(g - a_0)}} = t_1\sqrt{\frac{g + a_0}{g - a_0}}$$

i.e. $\frac{t_2}{t_1} = \sqrt{\frac{g + a_0}{g - a_0}} > 1 \Rightarrow t_2 > t_1$

3 Displacement

$= (2 \times 4 - 2 \times 2 + 2 \times 4) = 12$ m

Distance $= 2 \times 4 + 2 \times 2 + 2 \times 4 = 20$ m

4 For a freely falling body,

$$h = \frac{1}{2}gt^2$$

$$\therefore \quad \frac{h_1}{h_2} = \left(\frac{t_1}{t_2}\right)^2$$

Given, $h_1 = 16$ m, $h_2 = 25$ m

$$\therefore \quad \frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

5 From equation, $v^2 = u^2 + 2as$

Given, $u = 0$, $a = \frac{9}{8}$

We have, $v = \sqrt{2\left(\frac{9}{8}\right)h}$

When the stone released from this balloon. It will go upward with velocity $v = \frac{\sqrt{gh}}{2}$

In this condition, time taken by stone to reach the ground

$$t = \frac{v}{g}\left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right]$$

$$= \frac{\sqrt{gh}/2}{g}\left[1 + \sqrt{1 + \frac{2gh}{gh/4}}\right]$$

$$= \frac{2\sqrt{gh}}{g} = 2\sqrt{\frac{h}{g}}$$

6 The distance, $x = ut + \frac{1}{2}at^2$

For free fall starting from rest,

$u = 0$, $a = g \Rightarrow x = \frac{1}{2}gt^2$

t	0	T	$2T$	$3T$
x	0	$\frac{g}{2}T^2$	$\frac{g}{2}(4T^2)$	$\frac{g}{2}(9T^2)$

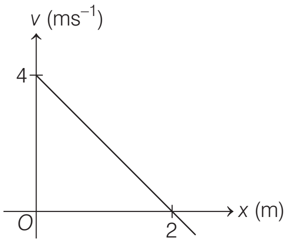
Δt	0 to T	T to $2T$	$2T$ to $3T$
Δx	$\frac{g}{2}T^2$	$\frac{g}{2}(4-1)T^2$	$\frac{g}{2}(9-4)T^2 = \frac{g}{2}5T^2$

$\therefore$ Required ratio,

$$\frac{g}{2}T^2 : \frac{g}{2}3T^2 : \frac{g}{2}5T^2 = 1 : 3 : 5$$

7 As x-t graph is a straight line in either case, velocity of both is uniform. As the slope of x-t graph for P is greater, therefore velocity of P is greater than that of Q.

8 Given velocity (v)-position (x) graph is shown in figure.



We know that, acceleration,

$$a = \frac{vdv}{dx} \quad \ldots \text{(i)}$$

From the graph, we have

$$\frac{x}{2} + \frac{v}{4} = 1 \Rightarrow 2x + v = 4 \quad \ldots \text{(ii)}$$

$$\Rightarrow \quad \frac{dv}{dx} = -2 \quad \ldots \text{(iii)}$$

From Eqs. (i), (ii) and (iii), we get

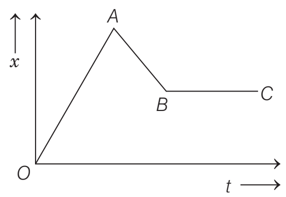
$$a = (4 - 2x)(-2) = -8 + 4x$$

$$\Rightarrow \quad a = 4x - 8$$

9 In $(x$ - $t)$ graph $OA \rightarrow$ Positive slope $\rightarrow$ Positive velocity

$AB \rightarrow$ Negative slope $\rightarrow$ Negative velocity

$BC \rightarrow$ Zero slope $\rightarrow$ Object at rest



At point 'A', there is change in sign of velocity, hence the direction of motion must have changed at 'A'.

10 Given, $x = ae^{-\alpha t} + be^{\beta t}$

Velocity, $v = \frac{dx}{dt}$

$$= -a\alpha e^{-\alpha t} + b\beta e^{\beta t} = A + B$$

where, $A = -a\alpha e^{-\alpha t}$, $B = b\beta e^{\beta t}$

The value of term $A = -a\alpha e^{-\alpha t}$ decreases and $B = b\beta e^{\beta t}$ increases with time. As a result, velocity goes on increasing with time.

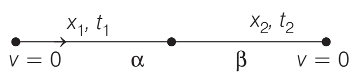
11 $h = \frac{1}{2} g\, t_1^2 \Rightarrow 2h = \frac{1}{2} g\, (t_1 + t_2)^2$

and $\quad 3\,h = \frac{1}{2} g\, (t_1 + t_2 + t_3)^2$

i.e. $t_1 : (t_1 + t_2) : (t_1 + t_2 + t_3) = 1 : \sqrt{2} : \sqrt{3}$

or $\quad t_1 : t_2 : t_3 = 1 : (\sqrt{2} - 1) : (\sqrt{3} - \sqrt{2})$

12 Consider the diagram



Velocity, $v_1 = 0 + \alpha\, t_1 = \alpha\, t_1$

Similarly, $v_2 = \beta\, t_2$

As, $\quad \alpha t_1 = \beta t_2 \Rightarrow \frac{\alpha}{\beta} = \frac{t_2}{t_1} \quad \ldots$(i)

$$x_1 = 0\,(t_1) + \frac{1}{2}\alpha\,(t_1)^2 \quad \ldots \text{(ii)}$$

$$x_2 = (\alpha t_1)\, t_2 + \frac{1}{2}(-\beta)\,(t_2)^2$$

$$x_2 = (\beta t_2)(t_2) - \frac{1}{2}\beta\,(t_2)^2$$

[using Eq. (i)]

$$\Rightarrow \quad x_2 = \frac{1}{2}\beta t_2^2 \quad \ldots \text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$\frac{x_1}{x_2} = \frac{\alpha t_1^2}{\beta t_2^2} = \left(\frac{\alpha t_1}{\beta t_2}\right)\left(\frac{\beta}{\alpha}\right) = \frac{t_1}{t_2} \quad \ldots \text{(iv)}$$

[using Eq. (i)]

From Eqs. (i) and (iv), we get

$$\frac{\alpha}{\beta} = \frac{t_2}{t_1} = \frac{x_2}{x_1}$$

$$\Rightarrow \quad \frac{\beta}{\alpha} = \frac{t_1}{t_2} = \frac{x_1}{x_2} \Rightarrow \frac{x_1}{x_2} = \frac{\beta}{\alpha} = \frac{t_1}{t_2}$$

13 From the relation,

$$h = \frac{1}{2} gt^2 \quad \ldots \text{(i)}$$

$$h - 15 = \frac{1}{2} g\,(t-1)^2 \quad \ldots \text{(ii)}$$

$$\frac{1}{2} gt^2 - 15 = \frac{1}{2} g\,(t-1)^2$$

[from Eq. (i)]

$$\frac{1}{2} g\,[t^2 - (t-1)^2] = 15$$

$$(t + t - 1)(t - t + 1) = \frac{15 \times 2}{g} = 3$$

$[\because g = 10 \text{ m/s}^2]$

$$2t - 1 = 3 \Rightarrow t = 2\text{s}$$

$$\therefore \quad h = \frac{1}{2} \times 10 \times 2 \times 2 = 20 \text{ m}$$

14 Time of fall $= \sqrt{\frac{2h}{g}}$

Time taken by the sound to come out $= \frac{h}{c}$

$$\text{Total time} = \sqrt{\frac{2h}{g}} + \frac{h}{c} = h\left[\sqrt{\frac{2}{gh}} + \frac{1}{c}\right]$$

15 From the given graph, for $0 < t < 1$s, slope of x-t graph is decreasing, this implies v (velocity) is increasing.

For $1 < t < 2$s, slope of x-t graph is increasing, this implies v is increasing.

Thus, the above conditions is only satisfied by the graph in option (d).

16 By the time fifth water drop starts falling, the first water drop reaches the ground. This means,

$$u = 0,\, h = \frac{1}{2} gt^2$$

$$\Rightarrow \quad 5 = \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow \quad t = 1 \text{ s}$$

Hence, the interval of falling of each water drop is $\frac{1}{4} = 0.25$ s

When the fifth drop starts its journey towards ground, the third drop would be in air for $0.25 + 0.25 = 0.5$ s

Height (distance) covered by third drop in air is

$$h_1 = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times (0.5)^2$$

$$= 5 \times 0.25 = 1.25$$

Therefore, the third water drop will be at a height $= 5 - 1.25 = 3.75$ m

17 At highest point of path, velocity $= 0$

$$\Rightarrow 0 = u - gt \Rightarrow t = \frac{u}{g}$$

Given, time for the particle to hit the ground $= nt$ $\quad \ldots$(i)

Now, $-H = u\,(nt) - \frac{1}{2} g\,(nt)^2$

$$\Rightarrow \quad -H = un\left(\frac{u}{g}\right) - \frac{1}{2} gn^2 \frac{u^2}{g^2}$$

$$\Rightarrow \quad -H = n\frac{u^2}{g} - \frac{n^2}{2}\frac{u^2}{g}$$

$$\Rightarrow \quad -2gH = 2nu^2 - n^2u^2$$

$$\Rightarrow \quad 2gH = n^2u^2 - 2nu^2$$

$$\Rightarrow \quad 2gH = nu^2\,(n-2)$$

18 From $v^2 - u^2 = 2as$, we get

$$\frac{(80)^2 - (60)^2}{2a} = s$$

$$\therefore \text{ Distance, } s = \frac{6400 - 3600}{2a} = \frac{1400}{a}$$

The middle point of the train is to cover a distance

$$\frac{s}{2} = \frac{700}{a}$$

Again, from $v^2 - u^2 = 2as$,

$$v^2 - (60)^2 = 2a \times \frac{700}{a} = 1400$$

$$v^2 = 1400 + 3600$$

$$\text{Velocity,} \quad v = \sqrt{5000} = 70.7 \text{ kmh}^{-1}$$

19 Let ball strike the nth step of stairs.

Vertical distance travelled

$$= ny = n \times 0.20 = \frac{1}{2} gt^2$$

Horizontal distance travelled

$$= nx = ut$$

$$\Rightarrow \quad t = \frac{nx}{u}$$

$$ny = \frac{1}{2} gt^2 = \frac{1}{2} g \cdot \left(\frac{nx}{u}\right)^2 = \frac{1}{2} g\,\frac{n^2x^2}{u^2}$$

$$\Rightarrow n = \frac{2u^2}{g}\,\frac{y}{x}$$

$$= \frac{2 \times (1.8)^2 \times 0.2}{9.8 \times (0.2)^2} \approx 4$$