Chapter 5

Quadratic Equations in one variable

Exercise 5.1

1. In each of the following, determine whether the given numbers are roots of the given equations or not;

(i) $x^2 – 5x + 6 = 0; 2 – 3$

(ii) $3x^2 – 13x – 10 = 0; 5, -\frac{2}{3}$

Solution

(i) $x^2 – 5x + 6 = 0; 2 – 3$

Let's us substitute the given values in the expression and check,

When x = 2

$X^2 – 5x + 6 = 0$

$(2)^2 – 5(2) + 6 = 0$

$4 – 10 + 6 = 0$

$0 = 0$

$\therefore X = 0$

When , x = -3

$X^2 – 5x + 6 = 0$

$(-3)^2 -5(-3) + 6 = 0$

$30 = 0$

$X \neq 0$

Hence the value x = 2 is the root of the equation

And value x = - 3 is not a root of the equation

(ii) $3x^2 - 13x - 10 = 0; 5, -\frac{2}{3}$

Let us substitute the given values in the expression and check,

when x = 5

$3x^2 - 13x - 10 = 0$

$3(5)^2 - 13(5) - 10 = 0$

$3(25) - 65 - 10 = 0$

$75 - 75 = 0$

$0 = 0$

$\therefore x = 0$

$when\ x = -\frac{2}{3}$

$3x^2 - 13x - 10 = 0$

$3\left(-\frac{2}{3}\right)^2 - 13\left(-\frac{2}{3}\right) - 10 = 0$

$\frac{4}{9} + \frac{26}{3} - 10 = 0$

$\frac{4}{3} + \frac{26}{3} - 10 = 0$

$\frac{30}{3} - 10 = 0$

$10 - 10 = 0$

$\therefore x = 0$

$hence\ , the\ value\ x = 5\ , -\frac{2}{3}$ are the roots of the equation

2. In each of the following, determine whether the given numbers are solutions of the given equation or not:

(i) $x^2 - 3\sqrt{3}x + 6 = 0$; $x = \sqrt{3}, -2\sqrt{3}$

(ii) $x^2 - \sqrt{2}x - 4 = 0$; $x = -\sqrt{2}, 2\sqrt{2}$

Solution

(i) $x^2 - 3\sqrt{3}x + 6 = 0$; $x = \sqrt{3}, -2\sqrt{3}$

Let us substitute the given values in the expression and check,

When $x = \sqrt{3}$

$X^2 - 3\sqrt{3}x + 6 = 0$

$(\sqrt{3})^2 - 3\sqrt{3}(\sqrt{3}) + 6 = 0$

$3 - 9 + 6 = 0$

$-9 + 9 = 0$

$0 = 0$

$\therefore \sqrt{3}$ is the solution of the equation.

When $x = -2\sqrt{3}$

$X^2 - 3\sqrt{3}x + 6 = 0$

$(-2\sqrt{3})^2 - 3\sqrt{3}\,(-2\sqrt{3}) + 6 = 0$

$4(3) + 18 + 6 = 0$

$12 + 18 + 6 = 0$

$36 = 0$

∴ $-2\sqrt{3}$ is not the solution of the equation

(ii) $x^2 - \sqrt{2}x - 4 = 0$; $x = -\sqrt{2}, 2\sqrt{2}$

Let us substitute the given values in the expression and check,

When $x = -\sqrt{2}$

$X^2 - \sqrt{2}x - 4 = 0$

$(-\sqrt{2})^2 - \sqrt{2}(-\sqrt{2}) - 4 = 0$

$4 - 4 = 0$

$0 = 0$

∴ $-\sqrt{2}$ is the solution of the equation

When $x = 2\sqrt{2}$

$X^2 - \sqrt{2}x - 4 = 0$

$(2\sqrt{2})^2 - \sqrt{2}(2\sqrt{2}) - 4 = 0$

$4(2) - 4 - 4 = 0$

$4 - 4 = 0$

$0 = 0$

$\therefore 2\sqrt{2}$ is the solution of the equation

3. (i) If $-\frac{1}{2}$ is a solution of the equation $3x^2 + 2kx - 3 = 0$, find the value of k.

(ii) If $\frac{2}{3}$ is a solution of the equation $7x^2 + kx - 3 = 0$, find the value of k

Solution

(i) If $-\frac{1}{2}$ is a solution of the equation $3x^2 + 2kx - 3 = 0$, find the value of k

Let us substitute the given value $x = -\frac{1}{2}$ in the expression we get

$3x^2 + 2kx - 3 = 0$

$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$

$\frac{3}{4} - k - 3 = 0$

$\frac{3}{4} - 3 = k$

By taking LCM

$K = \frac{3-12}{4}$

$= -\frac{9}{4}$

∴ value of k = $-\frac{9}{4}$

(ii) If $\frac{2}{3}$ is a solution of the equation $7x^2 + kx - 3 = 0$, find the value of k

Let us substitute the given value x = $\frac{2}{3}$ in the expression we get

$7x^2 + kx \ -3 = 0$

$7\left(\frac{2}{3}\right)^2 + \ k\left(\frac{2}{3}\right) - \ 3 = 0$

$7\left(\frac{4}{9}\right) + \frac{2k}{3} - \ 3 = 0$

$\frac{28}{9} - \ 3 + \frac{2k}{3} = 0$

$\frac{2k}{3} = 3 - \frac{28}{9}$

By taking LCM on the RHS

$\frac{2k}{3} = \frac{27-28}{9}$

$= -\frac{1}{9}$

K = $-\frac{1}{9} \times \left(\frac{3}{2}\right)$

$= -\frac{1}{6}$

∴ value of k = $-\frac{1}{6}$

4.(i) if $\sqrt{2}$ is a root of the equation $kx^2 + \sqrt{2}x - 4 = 0$, find the value of k.

(ii) If a is a root of the equation $x^2 - (a + b)x + k = 0$, find the value of k.

Solution

(i) if $\sqrt{2}$ is a root of the equation $kx^2 + \sqrt{2}x - 4 = 0$, find the value of k.

Let us substitute the given value $x = \sqrt{2}$ in the expression, we get

$Kx^2 + \sqrt{2}x - 4 = 0$

$K(\sqrt{2})^2 + \sqrt{2}(\sqrt{2}) - 4 = 0$

$2k + 2 - 4 = 0$

$2k - 2 = 0$

$K = \frac{2}{2}$

$= 1$

∴ value of $k = 1$

(ii) If a is a root of the equation $x^2 - (a + b)x + k = 0$, find the value of k.

Let us substitute the given value $x = a$ in the expression, we get

$X^2 - (a + b)x + k = 0$

$a^2 – (a + b)a + k = 0$

$a^2 – a^2 – ab + k = 0$

$-ab + k = 0$

$K = ab$

∴ value of $k = ab$

5. If $\frac{2}{3}$ and -3 are the roots of the equation $px^2 + 7x + q = 0$, find the values of p and q.

Solution

Let us substitute the given value $x = \frac{2}{3}$ in the expression we get

$Px^2 + 7x + q = 0$

$p\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + q = 0$

$\frac{4p}{9} + \frac{14}{3} + q = 0$

by taking LCM

$4p + 42 + 9q = 0$

$4p + 9q = - 42$...(i)

Now substitute the values $x = - 3$ in the expression, we get

$Px^2 + 7x + q = 0$

$P(-3)^2 + 7(-3) + q = 0$

$9p + q - 21 = 0$

$9p + q = 21$

$q = 21 - 9p$...(2)

by substituting the value of q in equation (1) we get

$4p + 9q = - 42$

$4p + 9(21 - 9p) = -42$

$4p + 189 - 81p = -42$

$189 - 77p = - 42$

$189 + 42 = 77p$

$231 = 77p$

$P = \frac{231}{77}$

$P = 3$

Now, substitute the value of p in equation (2) we get

$Q = 21 - 9p$

$= 21 - 9(3)$

$= 21 - 27$

$= -6$

∴ value of p is 3 and q is -6.

Exercise 5.2

Solve the following equation (1 to 24) by factorization:

1.(i) $x^2 - 3x - 10 = 0$

(ii) $x(2x + 5) = 3$

Solution

(i) $x^2 - 3x - 10 = 0$

Let us simplify the given expression

$X^2 - 5x + 2x - 10 = 0$

$X(x - 5) + 2(x - 5) = 0$

$(x + 2)(x - 5) = 0$

So now,

$(x + 2) = 0$ or $(x - 5) = 0$

$X = -2$ or $x = 5$

∴ value of $x = -2, 5$

(ii) $x(2x + 5) = 3$

Let us simplify the given expression

$2x^2 + 5x - 3 = 0$

Now, let us factorize

$2x^2 + 6x – x – 3 = 0$

$2x(x+ 3) – 1(x + 3) = 0$

$(2x -1) (x + 3) = 0$

So now,

$(2x – 1) = 0$ or $(x + 3) = 0$

$2x = 1$ or $x = -3$

$X = \frac{1}{2}$ or $x = -3$

∴ value of $x = \frac{1}{2} , -3$

2. (i) $3x^2 – 5x – 12 =0$

(ii) $21x^2 – 8x – 4 = 0$

Solution

(i) $3x^2 – 5x – 12 =0$

Let us simplify the given expression,

$3x^2 – 9x + 4x – 12 = 0$

$3x(x -3) + 4(x – 3) = 0$

$(3x + 4)(x -3) = 0$

So now,

(3x + 4) = 0 or (x – 3) = 0

3x = -4 or x = 3

X = $-\frac{4}{3}$ $or\ x = 3$

∴ value of x = $-\frac{4}{3}, 3$

(ii) $21x^2$ – 8x – 4 = 0

Let us simplify the given expression

$21x^2$ – 14x + 6x – 4 = 0

7x(3x -2) + 2(3x -2) = 0

(7x + 2) (3x -2) = 0

So now,

(7x + 2) = 0 or (3x – 2) = 0

7x = -2 or 3x = 2

X = $-\frac{2}{7}$ or x = $\frac{2}{3}$

∴ value of x = $-\frac{2}{7}$, $\frac{2}{3}$

3. (i) $3x^2 = x + 4$

(ii) $x(6x - 1) = 35$

Solution

(i) $3x^2 = x + 4$

Let us simplify the given expression

$3x^2 – x – 4 = 0$

Now let us factorize

$3x^2 – 4x + 3x – 4 = 0$

$X(3x – 4) + 1(3x – 4) = 0$

$(x + 1) (3x – 4) = 0$

So now,

$(x +1) = 0$ or $(3x – 4) = 0$

$X = -1$ or $3x = 4$

$X = -1$ or $x = \frac{4}{3}$

$\therefore$ value of $x = -1, \frac{4}{3}$

(ii) $x(6x - 1) = 35$

Let us simplify the given expression

$6x^2 – x – 35 = 0$

Now, let us factorize

$6x^2 - x - 35 = 0$

Now, let us factorize

$6x^2 - 15x + 14x - 35 = 0$

$3x(2x - 5) + 7(2x - 5) = 0$

$(3x + 7)(2x - 5) = 0$

So now,

$(3x + 7) = 0$ or $(2x - 5) = 0$

$3x = -7$ or $2x = 5$

$X = -\frac{7}{3}$ or $x = \frac{5}{2}$

$\therefore$ value of $x = -\frac{7}{3}, \frac{5}{2}$

4. (i) $6p^2 + 11p - 10 = 0$

(ii) $\frac{2}{3}x^2 - \frac{1}{3}x = 1$

Solution

(i) $6p^2 + 11p - 10 = 0$

Let us factorize the given expression,

$6p^2 + 15p - 4p - 10 = 0$

$3p(2p + 5) - 2(2p + 5) = 0$

$(3p - 2)(2p + 5) = 0$

So now,

$(3p - 2) = 0$ or $(2p + 5) = 0$

$3p = 2$ or $2p = -5$

$P = \frac{2}{3}$ or $p - \frac{5}{2}$

$\therefore$ value of $p = \frac{2}{3}, -\frac{5}{2}$

(ii) $\frac{2}{3}x^2 - \frac{1}{3}x = 1$

Let us simplify the given expression,

$2x^2 - x = 3$

$2x^2 - x - 3 = 0$

Let us factorize the given expression,

$2x^2 - 3x + 2x - 3 = 0$

$X(2x - 3) + 1(2x - 3) = 0$

$(x + 1)(2x - 3) = 0$

So now,

$(x + 1) = 0$ or $(2x - 3) = 0$

$X = -1$ or $2x = 3$

$X = -1$ or $x = \frac{3}{2}$

$\therefore$ value of $x = -1, \frac{3}{2}$

5. (i) $3(x-2)^2 = 147$

(ii) $\frac{1}{7}(3x-5)^2 = 28$

Solution

(i) $3(x-2)^2 = 147$

Firstly let us expand the given expression,

$3(x^2 - 4x + 4) = 147$

$3x^2 - 12x + 12 = 147$

$3x^2 - 12x + 12 - 147 = 0$

$3x^2 - 12x - 135 = 0$

Divide by 3, we get

$X^2 - 4x - 45 = 0$

Let us factorize the expression

$X^2 - 9x + 5x - 45 = 0$

$X(x - 9) + 5(x - 9) = 0$

$(x + 5)(x - 9) = 0$

So now,

$(x + 5) = 0$ or $(x - 9) = 0$

$X = -5$ or $x = 9$

$\therefore$ value of $x = -5, 9$

(ii) $\frac{1}{7}(3x-5)^2 = 28$

Let us simplify the expression,

$(3x - 5)^2 = 28 \times 7$

$(3x - 5)^2 = 196$

Now let us expand

$9x^2 - 30x + 25 = 196$

$9x^2 - 30x + 25 - 196 = 0$

$9x^2 - 30x - 171 = 0$

Dividing by 3, we get

$3x^2 - 10x - 57 = 0$

Let us factorize the expression,

$3x^2 - 19x + 9x - 57 = 0$

$X(3x - 19) + 3(3x - 19) = 0$

$(x + 3)(3x - 19) = 0$

So now,

$(x + 3) = 0$ or $(3x - 19) = 0$

$X = -3$ or $3x = 19$

$X = -3$ or $x = \frac{19}{3}$

$\therefore$ value of $x = -3, \frac{19}{3}$

6. $x^2 - 4x - 12 = 0$ where $x \in N$

Solution

Let us factorize the expression,

$X^2 - 4x - 12 = 0$

$X^2 - 6x + 2x - 12 = 0$

$X(x - 6) + 2(x - 6) = 0$

$(x + 2)(x - 6) = 0$

So now,

$(x + 2) = 0$ or $(x - 6) = 0$

$X = -2$ or $x = 6$

∴ value of $x = 6$ (since , -2 is not a natural number)

7. $2x^2 - 9x + 10 = 0$, when

(i) $x \in N$

(ii) $x \in Q$

Solution

Let us factorize the expression,

$2x^2 - 9x + 10 = 0$

$2x^2 - 4x - 5x + 10 = 0$

$2x(x – 2) – 5(x - 2) = 0$

$(2x – 5)(x - 2) = 0$

So now,

$(2x – 5) = 0$ or $(x – 2) = 0$

$2x = 5$ or $x = 2$

$X = \frac{5}{2}$ or $x = 2$

(i) when $x \in N$ then $x = 2$

(ii) when $x \in Q$ then, $x = 2, \frac{5}{2}$

8. (i) $a^2x^2 + 2ax + 1 = 0, a \neq 0$

(ii) $x^2 – (p + q)x + pq = 0$

Solution

(i) $a^2x^2 + 2ax + 1 = 0, a \neq 0$

Let us factorize the expression,

$a^2x^2 + 2ax + 1 = 0$

$a^2x^2 + ax + ax + 1 = 0$

$ax(ax + 1) + 1(ax + 1) = 0$

$(ax + 1)(ax + 1) = 0$

So now,

(ax +1) = 0 or (ax + 1) = 0

ax = -1 or ax = -1

x = $-\frac{1}{a}$ or x = $-\frac{1}{a}$

∴ value of x = $-\frac{1}{a}$, $-\frac{1}{a}$

(ii) x^2 – (p + q)x + pq = 0

Let us simplify the expression,

X^2 – (p + q)x + pq = 0

X^2 – px – qx + px = 0

X(x – p) – q(x – p) = 0

(x – q) (x – p) = 0

So now,

(x – q) = 0 or (x – p) = 0

X = q or x = p

∴ value of x = q,p

9. a^2x^2 +(a^2 + b^2)x +b^2 = 0, a ≠ 0

Solution

Let us simplify the expression

$a^2x^2 + (a^2 + b^2)x + b^2 = 0$

$a^2x^2 + a^2x + b^2x + b^2 = 0$

$a^2x(x + 1) + b^2(x + 1) = 0$

$(a^2x + b^2)(x + 1) = 0$

So now,

$(a^2x + b^2) = 0$ or $(x + 1) = 0$

$a^2x = -b^2$ or $x = -1$

$x = -\frac{b^2}{a^2}$ or $x = -1$

∴ value of $x = -\frac{b^2}{a^2}$, -1

10. (i) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$ (ii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Solution

(i) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

Let u factorize the given expression,

$\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$ [as $\sqrt{3} \times 7\sqrt{3} = 3 \times 7 = 21$ and 3 + 7 = 10]

$\sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$

So now,

$(\sqrt{3}x + 7) = 0$ or $(x + \sqrt{3}) = 0$

$\sqrt{3}$ x = - 7 or x = -$\sqrt{3}$

X = $-\frac{7}{\sqrt{3}}$ or x = -$\sqrt{3}$

∴ value of x = $-\frac{7}{\sqrt{3}}, -\sqrt{3}$

(ii) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Let us factorize the given expression

$4\sqrt{3}x^2 + 8x – 3x - 2\sqrt{3} = 0$ [As, $4\sqrt{3} \times (-2\sqrt{3}) = -8 \times 3 = -24$ *and* $8 \times (-3) = -24$]

$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$(4x - \sqrt{3})(\sqrt{3}x + 2) = 0$

So now,

$(4x - \sqrt{3}) = 0$ or $(\sqrt{3}x + 2) = 0$

$4x = \sqrt{3}$ or $\sqrt{3}x = -2$

X = $\frac{\sqrt{3}}{4}$ or $x = -\frac{2}{\sqrt{3}}$

∴ value of x = $\frac{\sqrt{3}}{4}, -\frac{2}{\sqrt{3}}$

11. (i) $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$ (ii) $x + \frac{1}{x} = 2\left(\frac{1}{20}\right)$

Solution

(i) $x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$

Let us expand the given expression,

$X^2 - x - \sqrt{2}x + \sqrt{2} = 0$

Taking common, we have

$X(x - 1) - \sqrt{2}(x - 1) = 0$

$(x - 1)(x - \sqrt{2}) = 0$

So now,

$(x - 1) = 0$ or $(x + \sqrt{2}) = 0$

$X = 1$ or $x = -\sqrt{2}$

∴ Value of $x = 1, -\sqrt{2}$

(ii) $x + \frac{1}{x} = 2\left(\frac{1}{20}\right)$

Rewriting the given expression, we have

$\frac{x^2 + 1}{x} = \frac{41}{20}$

On cross multiplication we get

$20(x^2 + 1) = 41x$

$20x^2 + 20 = 41x$

$20x^2 - 41x + 20 = 0$

Let us factorize the expression now,

$20x^2 - 25x - 16x + 20 = 0$

$5x(4x - 5) - 4(4x - 5) = 0$

$(5x - 4)\ (4 - 5) = 0$

So,

$(5x - 4) = 0$ or $(4x - 5) = 0$

$5x = 4$ or $4x = 5$

$X = \frac{4}{5}$ or $x = \frac{5}{4}$

$\therefore$ Value of $x = \frac{4}{5}, \frac{5}{4}$

12. (i) $\boldsymbol{\frac{2}{x^2} - \frac{5}{x} + 2 = 0,\ x \neq 0}$ **(ii)** $\boldsymbol{\frac{x^2}{15} - \frac{x}{3} - 10 = 0}$

Solution

(i) $\boldsymbol{\frac{2}{x^2} - \frac{5}{x} + 2 = 0,\ x \neq 0}$

Taking L.C.M for the given expression,

$\frac{2-5x+2x^2}{x^2} = 0$

$2x^2 - 5x + 2 = 0$

Now, on factorizing the above expression we get

$2x^2 - 4x - x + 2 = 0$

$2x(x - 2) - 1(x - 2) = 0$

$(2x - 1)(x - 2) = 0$

So,

$(2x - 1) = 0$ or $(x - 2) = 0$

$2x = 1$ or $x = 2$

$X = \frac{1}{2}$ or $x = 2$

$\therefore$ value of $x = \frac{1}{2}, 2$

(ii) $\boldsymbol{\frac{x^2}{15} - \frac{x}{3} - 10 = 0}$

Taking L.C.M for the given expression,

$\frac{x^2 - 5x - 150}{15} = 0$

$X^2 - 5x - 150 = 0$

Now on factorizing the above expression we get

$X^2 - 15x + 10x - 150 = 0$

$X(x - 15) + 10(x - 15) = 0$

$(x - 15)(x + 10) = 0$

So,

$(x - 15) = 0$ or $(x + 10) = 0$

$X = 15$ or $x = -10$

$\therefore$ value of $x = 15, -10$

13. (i) $3x - \frac{8}{x} = 2$ (ii) $\frac{x+2}{x+3} = \frac{2x-3}{3x-7}$

Solution

(i) $3x - \frac{8}{x} = 2$

Taking L.C.M we have

$\frac{3x^2 - 8}{x} = 2$

$3x^2 - 8 = 2x$

$3x^2 - 2x - 8 = 0$

On factorizing the above expression we get

$3x^2 - 6x + 4x - 8 = 0$

$3x(x - 2) + 4(x - 2) = 0$

$(3x + 4)(x - 2) = 0$

So,

$(3x - 4) = 0$ or $(x - 2) = 0$

$3x = 4$ or $x = 2$

$X = \frac{4}{3}$ *or* $x = 2$

$\therefore$ value of $x = \frac{4}{3}, 2$

(ii) $\frac{x+2}{x+3} = \frac{2x-3}{3x-7}$

Upon cross multiplication we get

$(x + 2) (3x – 7) = (2x – 3) (x + 3)$

$3x^2 – 7x + 6x – 14 = 2x^2 + 6x – 3x – 9$

$3x^2 – x - 14 = 2x^2 + 3x – 9$

$3x^2 – 2x^2 – x – 3x – 14 + 9 = 0$

$X^2 – 4x – 5 = 0$

Factorizing the above expression , we get

$X^2 – 5x + x – 5 = 0$

$X(x – 5) + 1(x – 5) = 0$

$(x + 1) (x – 5) = 0$

So,

$X + 1 = 0$ or $x – 5 = 0$

$X = -1$ or $x = 5$

∴ Value of $x = -1,5$

14. (i) $\frac{8}{x+3} - \frac{3}{2-x} = 2$ **(ii)** $\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$

Solution

(i) $\frac{8}{x+3} - \frac{3}{2-x} = 2$

Taking L.C.M we have

$\frac{[8(2-x)-3(x+3)]}{[(x+3)(2-x)]} = 2$

Upon cross – multiplication

16 – 8x – 3x – 9 = 2(x + 3) (2 –x)

7 – 11x = 2(2x + 6 – x^2 – 3x)

7 – 11x = 2(6 – x^2 – x)

7 – 11x = 12 – $2x^2$ – 2x

$2x^2$ – 11x + 2x + 7 – 12 = 0

$2x^2$ – 9x – 5 = 0

Now, let's factorize the above equation to find x

$2x^2$ – 10x + x – 5 = 0

2x(x – 5) + 1(x – 5) = 0

(2x + 1) (x -5) = 0

So,

2x + 1 = 0 or x – 5 = 0

X = $-\frac{1}{2}$ or x = 5

$\therefore$ value of x = $-\frac{1}{2}$, 5

(ii) $\boldsymbol{\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}}$

Taking L.C.M we have

$\frac{[x^2+(x-1)^2]}{x(x-1)} = \frac{5}{2}$

$\frac{x^2+ x^2- 2x+1}{x^2-x} = \frac{5}{2}$

$\frac{2x^2- 2x+1}{x^2- x} = \frac{5}{2}$

Upon cross- multiplication we get

$2(2x^2 – 2x + 1) = 5(x^2 – x)$

$4x^2 – 4x + 2 = 5x^2 – 5x$

$5x^2- 4x^2 – 5x + 4x – 2 = 0$

$X^2 – x – 2 = 0$

Now, let's factorize the above equation to find x

$X^2 – 2x + x – 2 = 0$

X(x -2) + 1(x -2) = 0

(x + 1) (x -2) = 0

So,

X + 1 = 0 or x – 2 = 0

X = -1 or x = 2

∴ value of x = -1, 2

15. (i) $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 3$

(ii) $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$

Solution

(i) $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 3$

$\frac{[(x+1)(x+2)+(x-2)(x-1)]}{[(x-1)(x+2)]} = 3$ [taking L.C.M]

On expanding we get

$X^2 + 3x + 2 + x^2 – 3x + 2 = 3(x -1)(x + 2)$

$2x^2 + 4 = 3(x^2 +x -2)$

$2x^2 + 4 = 3x^2 + 3x – 6$

$3x^2 – 2x^2 + 3x – 6 – 4 = 0$

$X^2 + 3x – 10 = 0$

Now, let’s factorize the above equation to find x

$X^2 + 5x – 2x – 10 = 0$

X(x + 5) – 2(x – 5) = 0

(x + 5) (x -5) = 0

So,

X + 5 = 0 or x – 5 = 0

X = -5 or x = 5

∴ value of x = -5, 5

(ii) $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$

Taking L.C.M we have

$\frac{[x+5-(x-3)]}{[(x-3)(x+5)]} = \frac{1}{6}$

$\frac{x+5-x+3}{[(x-3)(x+5)]} = \frac{1}{6}$

$\frac{8}{[(x-3)(x+5)]} = \frac{1}{6}$

Upon cross multiplying we have

$8 \times 6 = (x-3)(x+5)$

$48 = x^2 + 5x – 3x – 15$

$X^2 + 2x – 15 – 48 = 0$

$X^2 + 2x – 63 = 0$

Now, let's factorize the above equation to find x

$X^2 + 9x – 7x – 63 = 0$

X(x + 9) – 7(x+9) = 0

(x – 7) (x+ 9) = 0

So,

x-7 = 0 or x + 9 = 0

x = 7 or x = -9

∴ value of x = 7, -9

16. (i) $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b, a + b \neq 0, ab \neq 0$

(ii) $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

Solution

(i) $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b, a + b \neq 0, ab \neq 0$

Let's rearrange the equation for simple solving

$$\left[\frac{a}{ax-1} - b\right] + \left[\frac{b}{bx-1} - a\right] = 0$$

$$\frac{[a-b(ax-1)]}{ax-1} + \frac{[b-a(bx-1)]}{bx-1} = 0$$

$$\frac{a-abx+b}{ax-1} + \frac{b-abx+a}{bx-1} = 0$$

$$\frac{a-abx+b}{ax-1} + \frac{b-abx+a}{bx-1} = 0$$

$(a - abx + b)\left[\frac{1}{ax-1} + \frac{1}{bx-1}\right] = 0$ {taking common terms out}

$$(a - abx + b)\left[\frac{bx-1+ax-1}{(ax-1)(bx-1)}\right] = 0$$

$$(a - abx + b)\left[\frac{ax+bx-2}{(ax-1)(bx-1)}\right] = 0$$

So,

$(a - abx + b) = 0 \ or \ \frac{ax+bx-2}{[(ax-1)(bx-1)]} = 0$

If (a –abx + b) = 0

a + b = abx

x = $\frac{a+b}{ab}$

and

if $\frac{ax+bx-2}{[(ax-1)(bx-1)]} = 0$

ax + bx – 2 = 0

(a + b)x = 2

X = $\frac{2}{a+b}$

∴ Value of x = $\frac{a+b}{ab}$, $\frac{2}{a+b}$

(ii) $\frac{\mathbf{1}}{\mathbf{2a+b+2x}} = \frac{\mathbf{1}}{\mathbf{2a}} + \frac{\mathbf{1}}{\mathbf{b}} + \frac{\mathbf{1}}{\mathbf{2x}}$

$\frac{1}{2a+b+2x} = \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$

Taking L.C.M on both sides, we have

$\frac{2x-2a-b-2x}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$

$\frac{-2a-b}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$

$\frac{-(2a+b)}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$

$\frac{-1}{x(2a+b+2x)} = \frac{1}{ab}$

$-ab = x(2a + b) + 2x^2$ [after cross multiplication]

$0 = 2x^2 + 2ax + bx + ab$

$2x(x + a) + b(x + a) = 0$

$(x + a)(2x + b) = 0$

$(x + a) = 0$ or $2x + b = 0$

X = -a or 2x = -b

$= x = -a$ or $x = -\frac{b}{2}$

$\therefore$ value of x = -a, $-\frac{b}{2}$

17. $\frac{1}{x+6} + \frac{1}{x-10} = \frac{3}{x-4}$

Solution

Given equation

$\frac{1}{x+6} + \frac{1}{x-10} = \frac{3}{x-4}$

Taking L.C.M for the R.H.S of the equation

$$\frac{[(x-10)+(x+6)]}{[(x+6)(x-10)]} = \frac{3}{x-4}$$

$$\frac{2x-4}{x^2-4x-60} = \frac{3}{x-4}$$

On cross multiplying we get

$(2x - 4)(x-4) = 3(x^2 - 4x - 60)$

$2x^2 - 8x - 4x + 16 = 3x^2 - 12x - 180$

$2x^2 - 12x + 16 = 3x^2 - 12x - 180$

$3x^2 - 2x^2 - 12x + 12x - 180 - 16 = 0$

$X^2 - 196 = 0$

$X^2 = 196$

$X = \sqrt{196}$

$\therefore x = \pm 14$

18. (i) $\sqrt{(3x+4)} = x$ **(ii)** $\sqrt{[x(x-7)]} = 3\sqrt{2}$

Solution

(i) $\sqrt{(3x+4)} = x$

On squaring on both sides, we get

$3x + 4 = x^2$

$X^2 - 3x - 4 = 0$

Let us factorize the above expression

$X^2 - 4x + x - 4 = 0$

$X(x-4) + 1(x-4) = 0$

$(x-4)(x+1) = 0$

So,

$X - 4 = 0$ or $x + 1 = 0$

$X = 4$ or $x = -1$

∴ value of x = 4, -1

(ii) $\sqrt{[x(x-7)]} = 3\sqrt{2}$

On squaring on both sides, we get

$X(x - 7) = (3\sqrt{2})^2$

$X^2 - 7x = 9 \times 2$

$X^2 - 7x - 18 = 0$

Let us factorize the above expression

$X^2 - 9x + 2x - 18 = 0$

$X(x - 9) + 2(x - 9) = 0$

$(x - 9)(x + 2) = 0$

So,

$X - 9 = 0$ or $x + 2 = 0$

X = 9 or x = -2

∴value of x = 9, -2

19. Use the substitution y = 3x + 1 to solve for x:

$5(3x +1)^2 + 6(3x + 1) – 8 = 0$

Solution

Given equation

$5(3x + 1)^2 + 6(3x +1) – 8 = 0$

Upon substituting y = 3x + 1

$5y^2 + 6y – 8 = 0$

We get a quadratic equation in y

Now, solving for y by factorization, we get

$5y^2 + 10y – 4y – 8 = 0$

5y(y + 2) – 4(y + 2) = 0

(5y – 4) (y + 2) = 0

So,

5y – 4 = 0 or y + 2 = 0

5y = 4 or y = -2

Y = $\frac{4}{5}$ or y = -2

Now, to find the value of x let's back substitute y

$3x + 1 = \frac{4}{5}$ or $3x + 1 = -2$

$3x = \frac{4-5}{5}$ or $3x = -3$

$3x = -\frac{1}{5}$ or $x = -\frac{3}{3}$

$X = -\frac{1}{15}$ or $x = -1$

$\therefore$ value of $x = -1, -\frac{1}{15}$

20. Find the values of x if $p + 1 = 0$ and $x^2 + px - 6 = 0$

Solution

Given quadratic equation : $x^2 + px - 6 = 0$

And $p + 1 = 0$

So,

$P = -1$

Substituting the value of p in the given quadratic equation we get

$X^2 + (-1)x - 6 = 0$

$X^2 - x - 6 = 0$

Solving for x by factorization we have

$X^2 - 3x + 2x - 6 = 0$

$X(x – 3) + 2(x -3) = 0$

$(x + 2) (x – 3) = 0$

So,

$X + 2 = 0$ or $x – 3 = 0$

$X = -2$ or $x = 3$

∴ value of $x = -2, 3$

21. Find the values of x if $p + 7 = 0$, $q – 12 = 0$ and $x^2 + px + q = 0$

Solution

Given quadratic equation : $x^2 + px + q = 0$

And $p + 7 = 0$ and $q – 12 = 0$

So,

$P = -7$ and $q = 12$

Substituting the value of p and q in the given quadratic equation we get

$X^2 + (-7)x + 12 = 0$

$X^2 – 7x + 12 = 0$

Solving for x by factorization we have

X^2 - 4x – 3x + 12 = 0

X(x – 4) – 3(x – 4) = 0

(x – 3) (x -4) = 0

So,

X – 3 = 0 or x – 4 = 0

X = 3 or x = 4

∴ value of x = 3,4

22. if x = p is a solution of the equation x(2x + 5) = 3, then find the value of p

Solution

Given that x = p is a solution of the equation x(2x + 5) = 3

Then upon substituting x = p in must satisfy the equation

P(2p + 5) = 3

$2p^2 + 5p = 3$

$2p^2 + 5p – 3 = 0$

Factorizing the above expression, we get

$2p^2 + 6p – p – 3 = 0$

2p(p + 3) – 1(p + 3) = 0

(2p – 1) (p + 3) = 0

So,

$2p - 1 = 0$ or $p + 3 = 0$

$2p = 1$ or $p = -3$

$P = \frac{1}{2}$ or $p = -3$

$\therefore$ value of $P = \frac{1}{2}, -3$

23. If $x = 3$ is a solution of the equation $(k + 2)x^2 - kx + 6 = 0$, find the value of k. Hence, find the other root of the equation

Solution

Given equation : $(k + 2)x^2 - kx + 6 = 0$

And $x = 3$ is a solution of the equation

So, upon substituting $x = 3$ it must satisfy the equation

$(k + 2)(3)^2 - k(3) + 6 = 0$

$(k + 2)(9) - 3k + 6 = 0$

$9k + 18 - 3k + 6 = 0$

$6k + 24 = 0$

$6(k + 4) = 0$

So,

$K + 4 = 0$

$K = -4$

Now putting $k = -4$ in the given equation we have

$(-4 + 2)x^2 \; -(-4)x + 6 = 0$

$-2x^2 + 4x + 6 = 0$

$X^2 – 2x – 3 = 0$ [dividing by – 2 on both sides]

Factorizing the above expression we get

$X^2 – 3x + x – 3 = 0$

$X(x – 3) + 1(x – 3) = 0$

$(x + 1)(x – 3) = 0$

So, $x + 1 = 0 \;\; x – 3 = 0$

$X = -1$ or $x = 3$

Hence the other root of the given equation is -1.

Exercise 5.3

Solve the following (1 to 8) equation by using the formula:

1. (i) $2x^2 – 7x + 6 = 0$

(ii) $2x^2 – 6x + 3 = 0$

Solution

(i) $2x^2 – 7x + 6 = 0$

Let us consider

A = 2, b = - 7, c = 6

So, by using the formula

$X = -b \pm \frac{\sqrt{b^2-4ac}}{2a}$

So let, $b^2 – 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 – 4ac$

$= (-7)^2 – 4(2)(6)$

$= 49 – 48$

$= 1$

So,

X = $\frac{[-(7) \pm \sqrt{1}]}{2(2)}$

= $\frac{[7+1]}{4}$ or $\frac{[7-1]}{4}$

= $\frac{8}{4}$ or $\frac{6}{4}$

= 2 or $\frac{3}{2}$

∴ value of x = 2, $\frac{3}{2}$

(ii) $2x^2 - 6x + 3 = 0$

Let us consider

A = 2, b = -6, c = 3

So, by using the formula

X = -b $\pm \frac{\sqrt{b^2-4ac}}{2a}$

So let, $b^2 - 4ac = D$

X = $\frac{-b \pm \sqrt{D}}{2a}$

D = $b^2 - 4ac$

= $(-6)^2 - 4(2)(3)$

= 36 – 24

= 12

So

$X = \frac{[-(-6) \pm \sqrt{12}]}{2(2)}$

$= \frac{[6 \pm 2\sqrt{3}]}{4}$

$= \frac{[6+2\sqrt{3}]}{4}$ or $\frac{2(3-\sqrt{3})}{4}$

$= \frac{2(3+\sqrt{3})}{4}$ or $\frac{2(3-\sqrt{3})}{4}$

$= \frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$

$\therefore$ value of x = $\frac{3+\sqrt{3}}{2}$, $\frac{3-\sqrt{3}}{2}$

2. (i) $256x^2 - 32x + 1 = 0$

(ii) $25x^2 + 30x + 7 = 0$

Solution

(i) $256x^2 - 32x + 1 = 0$

Let us consider

a = 256, b = -32 , c = 1

so, by using the formula

$X = -b \pm \frac{\sqrt{b^2-4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (-32)^2 - 4(256)(1)$

= 1024 – 1024

= 0

So,

$X = \frac{[-(-32) \pm \sqrt{0}]}{2(256)}$

$= \frac{[32]}{512}$

$= \frac{1}{16}$

$\therefore$value of $x = \frac{1}{16}$

(ii) $25x^2 + 30x + 7 = 0$

Let us consider

a = 25, b = 30 c = 7

so, by using the formula

$X = -b \pm \frac{\sqrt{b^2-4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (30)^2 - 4(25)(7)$

$= 900 - 700$

$=200$

So,

$X = \frac{[-(30) \pm \sqrt{200}]}{2(25)}$

$= \frac{\left[-30 \pm \sqrt{(100\times2)}\right]}{50}$

$= \frac{[-30 \pm 10\sqrt{2}]}{50}$

$= \frac{[-3 \pm \sqrt{2}]}{5}$

$= \frac{[-3+\sqrt{2}]}{5} \quad or \ \frac{[-3-\sqrt{2}]}{5}$

$\therefore$ value of x $= \frac{[-3+\sqrt{2}]}{5}, \frac{[-3-\sqrt{2}]}{5}$

3. (i) $2x^2 + \sqrt{5}x - 5 = 0$

(ii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Solution

(i) $2x^2 + \sqrt{5}x - 5 = 0$

Let us consider

a = 2, b = $\sqrt{5}$, c = - 5

so, by using the formula

$X = -b \pm \frac{\sqrt{b^2-4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (\sqrt{5})^2 - 4(2)(-5)$

= 5 + 40

= 45

So,

$X = \frac{[-(\sqrt{5}) \pm \sqrt{45}]}{2(2)}$

$= \frac{[-\sqrt{5} \pm 3\sqrt{5}]}{4}$

$= \frac{[-\sqrt{5}+ 3\sqrt{5}]}{4} \quad or \quad \frac{[-\sqrt{5}-3\sqrt{5}]}{4}$

$= \frac{2\sqrt{5}}{4} \quad or - \frac{4\sqrt{5}}{4}$

$= \frac{\sqrt{5}}{2}\ or - \sqrt{5}$

∴ value of x = $\frac{\sqrt{5}}{2}$, $-\sqrt{5}$

(ii) $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

Let us consider,

a = $\sqrt{3}$ b = 10 c = -8$\sqrt{3}$

so, by using the formula

x = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

So let, $b^2 - 4ac = D$

X = -b $\pm \frac{\sqrt{D}}{2a}$

D = $b^2 - 4ac$

= $(10)^2 - 4(\sqrt{3})(-8\sqrt{3})$

= 100 + 96

= 196

So,

$$X = \frac{[-(10) \pm \sqrt{196}]}{2(\sqrt{3})}$$

$$= \frac{[-10 \pm 14]}{2(\sqrt{3})}$$

$$= \frac{[-10+14]}{2\sqrt{3}} \quad or \frac{[-10-14]}{2\sqrt{3}}$$

$$= \frac{4}{2\sqrt{3}} \quad or - \frac{24}{2\sqrt{3}}$$

∴ value of x = $\frac{4}{2}\ \sqrt{3}\ , -\frac{24}{2\sqrt{3}}$

4. (i) $\frac{x-2}{x+2} + \frac{x+2}{x-2} = 4$

(ii) $\frac{x+1}{x+3} = \frac{3x+2}{2x+3}$

Solution

(i) $\frac{x-2}{x+2} + \frac{x+2}{x-2} = 4$

By taking LCM

$$\frac{\left((x-2)^2+x+2\right)^2}{(x+2)(x-2)} = 4$$

$$\frac{x^2-4x+4+x^2+4x+4}{x^2-4} = 4$$

By simplifying the equation we get

$2x^2 + 8 = 4x^2 – 16$

$2x^2 + 8 – 4x^2 + 16 = 0$

$-2x^2 + 24 = 0$

$X^2 – 12 = 0$

Let us consider

a =1, b = 0, c = -12

so, by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 – 4ac = D$

$$X = -b \pm \frac{\sqrt{D}}{2a}$$

$D = b^2 – 4ac$

$= (0)^2 -4(1) (-12)$

$= 0 + 48$

$= 48$

So,

$$X = \frac{[-(0) \pm \sqrt{48}]}{2(1)}$$

$$= \frac{[\pm\sqrt{48}]}{2}$$

$$= \frac{[\pm\sqrt{(16\times3)}]}{2}$$

$= \pm\frac{4\sqrt{3}}{2}$

$= \pm 2\sqrt{3}$

$= 2\sqrt{3}\ \ or - 2\sqrt{3}$

∴ value of x = $2\sqrt{3}\ \ or - 2\sqrt{3}$

(ii) $\frac{x+1}{x+3} = \frac{3x+2}{2x+3}$

Let us cross multiply, we get

(x + 1) (2x +3) = (x + 3) (3x + 2)

Now by simplifying we get

$2x^2 + 3x + 2x + 3 = 3x^2 + 9x + 2x + 6$

$2x^2 + 5x + 3 – 3x^2 – 11x – 6 = 0$

$-x^2 – 6x – 3 = 0$

$X^2 + 6x + 3 = 0$

Let us consider

a = 1, b = 6 c = 3

so, by using the formula

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 – 4ac = D$

X = -b $\pm \frac{\sqrt{D}}{2a}$

$D = b^2 – 4ac$

$= (6)^2 \; -4(1)(3)$

$= 36 – 12$

$= 24$

So,

$X = \frac{[-(6) \pm \sqrt{24}]}{2(1)}$

$= \frac{[-6 \pm \sqrt{4 \times 6}]}{2}$

$= \frac{[-6 \pm 2\sqrt{6}]}{2}$

$= -3 \pm \sqrt{6}$

$= -3 + \sqrt{6}$ or $-3 - \sqrt{6}$

∴ value of $x = -3 + \sqrt{6}$, $-3 - \sqrt{6}$

5. (i) $a(x^2 + 1) = (a^2 + 1)x$, $a \neq 0$

(ii) $4x^2 – 4ax + (a^2 – b^2) = 0$

Solution

(i) $a(x^2 + 1) = (a^2 + 1)x$, $a \neq 0$

Let us simplify the expression

$ax^2 + a – a^2x + x = 0$

$ax^2 – (a^2 + 1)x + a = 0$

let us consider

a = a, b = -(a^2 + 1), c = a

so, by using the formula

x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

So let, b^2 – 4ac = D

X = -b $\pm \frac{\sqrt{D}}{2a}$

D = b^2 – 4ac

= $(-(a^2 + 1))^2$ – 4(a)(a)

= a^4 – $2a^2$ + 1

= $(a^2 – 1)^2$

So,

X = $\frac{[-(-(a2+1)) \pm \sqrt{(a^2-1)}^2]}{2(a)}$

= $\frac{[(a^2+1) \pm (a^2-1)]}{2a}$

= $\frac{[(a^2+1)+(a^2-1)]}{2a}$ or $\frac{[(a^2+1)-(a^2-1)]}{2a}$

= $\frac{[a^2+1+a^2-1]}{2a}$ or $\frac{[a^2+1-a^2+1]}{2a}$

= $\frac{2a^2}{2a}$ or $\frac{2}{2a}$

= a or $\frac{1}{a}$

∴value of x = a, $\frac{1}{a}$

(ii) $4x^2 - 4ax + (a^2 - b^2) = 0$

Let us consider,

$a = 4$, $b = -4a$, $c = (a^2 - b^2)$

so, by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$X = -b \pm \frac{\sqrt{D}}{2a}$$

$D = b^2 - 4ac$

$= (-4a)^2 - 4(4)(a^2 - b^2)$

$= 16a^2 - 16(a^2 - b^2)$

$= 16a^2 - 16a^2 + 16b^2$

$= 16b^2$

So,

$$X = \frac{[-(-4a) \pm \sqrt{16b^2}]}{2(4)}$$

$$= \frac{[4a \pm 4b]}{8}$$

$$= \frac{4[a \pm b]}{8}$$

$$= \frac{[a \pm b]}{2}$$

$= \frac{[a+b]}{2}\ or \frac{[a-b]}{2}$

∴ value of x = $\frac{[a+b]}{2}$, $\frac{[a-b]}{2}$

6. (i) $x - \frac{1}{x} = 3,\ x \neq 0$

(ii) $\frac{1}{x} + \frac{1}{x-2} = 3,\ x \neq 0, 2$

Solution

(i) $x - \frac{1}{x} = 3,\ x \neq 0$

Let us simplify the given expression

By taking LCM

$X^2 – 1 = 3x$

$X^2 – 3x – 1 = 0$

Let us consider,

a = 1, b = -3, c = -1

so, by using the formula

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

So let, $b^2 – 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 – 4ac$

$= (-3)^2 – 4(1) (-1)$

$= 9 + 4$

$= 13$

So,

$X = \frac{[-(-3)\pm\sqrt{13}]}{2(1)}$

$= \frac{[3\pm\sqrt{13}]}{2}$

$= \frac{[3+\sqrt{13}]}{2} \quad or \frac{[3-\sqrt{13}]}{2}$

$\therefore$value of x $= \frac{[3 +\sqrt{13}]}{2} \quad or \frac{[3-\sqrt{13}]}{2}$

(ii) $\frac{1}{x} + \frac{1}{x-2} = 3, \; x \neq 0, 2$

Let us simplify the given expression

By taking LCM

$\frac{[(x-2)+x]}{[x(x-2)]} = 3$

$\frac{[x-2+x]}{[x^2-2x]} = 3$

$2x – 2 = 3(x^2 – 2x)$

$2x – 2 = 3x^2 – 6x$

$3x^2 – 6x – 2x + 2 = 0$

$3x^2 – 8x + 2$

Let us consider

a = 3, b = - 8 , c = 2

so, by using the formula

x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

So let, b^2 – 4ac = D

X = -b $\pm \frac{\sqrt{D}}{2a}$

D = b^2 – 4ac

= $(-8)^2$ – 4(3) (2)

= 64 – 24

= 40

So,

X = $\frac{[-(-8) \pm \sqrt{40}]}{2(3)}$

= $\frac{[8 \pm 2\sqrt{10}]}{6}$

= $\frac{2[4 \pm \sqrt{10}]}{6}$

= $\frac{[4 \pm \sqrt{10}]}{3}$

= $\frac{[4+\sqrt{10}]}{3}$ or $\frac{[4-\sqrt{10}]}{3}$

∴value of x = $\frac{[4+\sqrt{10}]}{3}$ or $\frac{[4-\sqrt{10}]}{3}$

7. solve for x:

$$\mathbf{2\left(\frac{2x-1}{x+3}\right)-\ 3\left(\frac{x+3}{2x-1}\right)=5\ ,x\neq\ -3,\frac{1}{2}}$$

Solution

Let us consider, $\left(\frac{2x-1}{x+3}\right)=x$ then, $\left(\frac{x+3}{2x-1}\right)=\frac{1}{x}$

So the equation becomes,

$\frac{2x-3}{x}=5$

By taking LCM

$X^2 - 3 = 5x$

$2x^2 - 5x - 3 = 0$

Let us consider

a = 2, b = -5 , c = -3

so, by using the formula

x = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

So let, $b^2 - 4ac = D$

X = -b $\pm\frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (-5)^2 - 4(2)(-3)$

$= 25 + 24$

$= 49$

So,

$X = \frac{[-(-5) \pm \sqrt{49}]}{2(2)}$

$= \frac{[5 \pm 7]}{4}$

$= \frac{[5+7]}{4}$ or $\frac{[5-7]}{4}$

$= \frac{[12]}{4}$ or $\frac{[-2]}{4}$

$= 3$ or $-\frac{1}{2}$

So, $x = 3$ or $-\frac{1}{2}$

Now

Let us substitute in the equations

When $x = 3$ then

$\left(\frac{2x-1}{x+3}\right) = 3$

By cross multiplying,

$2x - 1 = 3x + 9$

$3x + 9 - 2x + 1 = 0$

$X + 10 = 0$

$X = -10$

When x = $-\frac{1}{2}$ then

$\left(\frac{2x-1}{x+3}\right) = -\frac{1}{2}$

By cross multiplying

2(2x -1) = -(x + 3)

4x – 2 = -x – 3

4x – 2 = - x – 3

4x – 2 + x + 3 = 0

5x + 1 = 0

5x = -1

X = $-\frac{1}{5}$

∴value of x = -10 , $-\frac{1}{5}$

8. Solve the following quadratic equation for x and give your answer correct to 2 decimal places:

(i)x^2 – 5x – 10 = 0

(ii) x^2 + 7x = 7

Solution

(i)x^2 – 5x – 10 = 0

Let us consider

a = 1, b = -5 , c = -10

so, by using the formula

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

X = -b $\pm \frac{\sqrt{D}}{2a}$

D = $b^2 - 4ac$

= $(-5)^2$ -4(1) (-10)

= 25 + 40

= 65

So,

X = $\frac{[-(-5) \pm \sqrt{65}]}{2(1)}$

= $\frac{[5 \pm \sqrt{65}]}{2}$

= $\frac{[5 \pm 8.06]}{2}$

= $\frac{[5+8.06]}{2}$ or $\frac{[5-8.06]}{2}$

= $\frac{[13.06]}{2}$ or $\frac{[-3.06]}{2}$

= 6.53 or -1.53

∴value of x = 6.53 or -1.53

(ii) $x^2 + 7x = 7$

On rearranging the expression, we get

$X^2 + 7x - 7 = 0$

Let us consider

a = 1, b = 7, c = -7

so, by using the formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (7)^2 \ -4(1)(-7)$

$= 49 + 28$

$= 77$

So,

$X = \frac{[-7 \pm \sqrt{77}]}{2(1)}$

$= \frac{[-7 \pm 8.77]}{2}$

$= \frac{[-7+8.77]}{2} \ or \ \frac{[-7-8.77]}{2}$

$= \frac{1.77}{2} \ \ or - \frac{15.77}{2}$

= 0.885 or -7.885

∴vlue of x = 0.89 or – 7.89

9. solve the following equation by using quadratic formula and give your answer correct to 2 decimal places:

(i) $4x^2$ -5x – 3 = 0

(ii) $2x - \frac{1}{x} = 7$

Solution

(i) $4x^2$ -5x – 3 = 0

Let us consider

a = 4, b = -5 , c = -3

so, by using the formula

$D = b^2 - 4ac$

$= (-5)^2 - 4(4)(-3)$

= 25 + 48

= 73

So,

$X = \frac{[-(-5) \pm \sqrt{73}]}{2(4)}$

$= \frac{[5 \pm 8.54]}{8}$

$= \frac{[6+8.54]}{8} \ or \ \frac{[5-8.54]}{8}$

$= \frac{13.54}{8} \ or - \frac{3.54}{8}$

= 1.6925 or -0.4425

∴value of x = 1.69 or – 0.44

(ii) $\mathbf{2x - \frac{1}{x} = 7}$

By taking LCM

$2x^2 - 1 = 7x$

$2x^2 - 7x - 1 = 0$

Let us consider

a = 2, b = -7 , c = -1

so, by using the formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (-7)^2 - 4(2)(-1)$

= 49 + 8

= 57

So,

$X = \frac{[-(-7) \pm \sqrt{57}]}{2(2)}$

$= \frac{[7 \pm 7.549]}{4}$

$= \frac{[7+7.549]}{4}$ or $\frac{[7-7.549]}{4}$

$= \frac{14.549}{4}$ $or - \frac{0.549}{4}$

= 3.637 or – 0.137

= 3.64 or -0.14

∴value of x = 3.64 or – 0.14

12. Solve the following equations and give your answer correct to two significant figures.

(i) $x^2 - 4x - 8 = 0$ (ii) $x - \frac{18}{x} = 6$

Solution

(i) given equation

$X^2 - 4x - 8 = 0$

Let us consider

a = 1, b = -4 , c = -8

so, by using the formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

D = b^2 – 4ac

= $(-4)^2$ – 4(1) (-8)

= 16 + 32

= 48

So,

$=\frac{[-(-4)\pm\sqrt{48}]}{2(1)}$

$=\frac{[4\ \pm 6.93]}{2}$

$=\frac{[4+6.93]}{2}\ \ or\ \frac{[4-6.93]}{2}$

$=\frac{[10.93]}{2}\ \ or - \frac{2.93}{2}$

= 5.465 or -1.465

∴value of x = 5.47 or -1.47

(ii) given equation

$X - \frac{18}{x} = 6x$

$X^2 - 6x - 18 = 0$

Let us consider

a = 1, b = -6 , c = -18

so, by using the formula

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (-6)^2 - 4(1)\,(-18)$

$= 36 + 72$

$= 108$

So,

$X = \frac{[-(-6)\pm\sqrt{108}]}{2(1)}$

$= \frac{[6\pm10.39]}{2}$

$= \frac{[6+10.39]}{2}\ or\ \frac{[6-10.39]}{2}$

$= \frac{[16.39]}{2}\ \ or - \frac{4.39}{2}$

= 8.19 or -2.19

∴value of x = 8.19 or -2.19

13. solve the equation $5x^2 - 3x - 4 = 0$ and given your answer correct to 3 significant figures:

Solution

Given equation

$5x^2 - 3x - 4 = 0$

Let us consider

a = 5, b = - 3 , c = -4

so, by using the formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (-3)^2 - 4(5)(-4)$

= 9 + 80

= 89

So,

$X = \frac{[-(-3) \pm \sqrt{89}]}{2(5)}$

$= \frac{[3 \pm 9.43]}{10}$

$= \frac{[3+9.43]}{10}\ or \frac{[3-9.43]}{10}$

$= \frac{12.433}{10}\ \ or - \frac{6.43}{10}$

= 1.24 or – 0.643

∴value of x = 1.24 or -0.643

Exercise 5.4

1. find the discriminate of the following equations and hence find the nature of roots:

(i) $3x^2 – 5x – 2 = 0$

(ii) $2x^2 – 3x + 5 = 0$

(iii) $16x^2 – 40x + 25 = 0$

(iv) $2x^2 + 15x + 30 = 0$

Solution

(i) $3x^2 – 5x – 2 = 0$

let us consider

a = 3, b = -5 , c = -2

by using the formula

$D = b^2 – 4ac$

$= (-5)^2 – 4(3) (-2)$

= 25 + 24

= 49

So,

Discriminate , D = 49

D > 0

∴Roots are real and distinct

(ii) $2x^2 – 3x + 5 = 0$

Let us consider,

$a = 2, b = -3 , c = 5$

by using the formula

$D = b^2 – 4ac$

$= (-3)^2 – 4(2) (5)$

$= 9 – 40$

$= -31$

So,

Discriminate, $D = -31$

$D < 0$

∴Roots are not real

(iii) $16x^2 – 40x + 25 = 0$

Let us consider

$a = 16 , b = -40 , c = 25$

by using the formula,

$D = b^2 – 4ac$

$= (-40)^2 – 4(16) (25)$

$= 1600 – 1600$

$= 0$

So,

Discriminate D = 0

D = 0

∴Roots are real and equal

(iv) $2x^2 + 15x + 30 = 0$

Let us consider

a = 2, b = 15 , c = 30

by using the formula

$D = b^2 - 4ac$

$= (15)^2 - 4(2)(30)$

= 225 – 240

= -15

So,

Discriminate D = -15

$D < 0$

∴Roots are not real.

2. Discuss the nature of the roots of the following quadratic equations:

(i) $3x^2 - 4\sqrt{3}x + 4 = 0$

(ii) $x^2 - \frac{1}{2}x + 4 = 0$

(iii) $-2x^2 + x + 1 = 0$

(iv) $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Solution

(i) $3x^2 - 4\sqrt{3}x + 4 = 0$

Let us consider

$a = 3, b = - 4\sqrt{3}, c = 4$

by using the formula

$D = b^2 - 4ac$

$= (-4\sqrt{3})^2 \ -4(3)(4)$

$= 16(3) - 48$

$= 48 - 48$

$= 0$

So,

Discriminate D = 0

D = 0

∴Roots are real and equal

(ii) $x^2 - \frac{1}{2}x + 4 = 0$

Let us consider

a = 1, b = $-\frac{1}{2}$, $c = 4$

by using the formula,

D = $b^2 - 4ac$

= $\left(-\frac{1}{2}\right)^2 - 4(1)(4)$

= $\frac{1}{4} - 16$

= $-\frac{63}{4}$

So,

Discriminate D = $-\frac{63}{4}$

$D < 0$

∴Roots are not real

(iii) $-2x^2 + x + 1 = 0$

Let us consider

a = -2 , b = 1, c = 1

by using the formula

D = $b^2 - 4ac$

= $(1)^2 - 4(-2)(1)$

$= 1 + 8$

$= 9$

So,

Discriminate $D = 9$

$D > 0$

$\therefore$Roots are real and distinct

(iv) $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Let us consider

$a = 2\sqrt{3}$ $b = -5$ $c = \sqrt{3}$

by using he formula

$D = b^2 - 4ac$

$= (-5)^2 - 4(2\sqrt{3})(\sqrt{3})$

$= 25 - 24$

$= 1$

So,

Discriminate , $D = 1$

$D > 0$

$\therefore$Roots are real and distinct

3. Find the nature of the roots of the following quadratic equations:

(i) $\mathbf{x^2 - \frac{1}{2}x - \frac{1}{2} = 0}$

(ii) $\mathbf{x^2 - 2\sqrt{3}x - 1 = 0}$

If real roots exist, find them

Solution

(i) $\mathbf{x^2 - \frac{1}{2}x - \frac{1}{2} = 0}$

Let us consider

$a = 1,\ b = -\frac{1}{2},\ c = -\frac{1}{2}$

by using the formula

$D = b^2 - 4ac$

$= \left(-\frac{1}{2}\right)^2 - 4(1)\left(-\frac{1}{2}\right)$

$= \frac{1}{4} + 2$

$= \frac{1+8}{4}$

$= \frac{9}{4}$

So,

Discriminate , $D = \frac{9}{4}$

$D > 0$

∴Roots are real and unequal

(ii) $x^2 - 2\sqrt{3}x - 1 = 0$

Let us consider

$a = 1$, $b = 2\sqrt{3}$ $c = -1$

by using the formula

$D = b^2 – 4ac$

$= (2\sqrt{3})^2 – 4(1)(-1)$

$= 12 + 4$

$= 16$

So,

Discriminate $D = 16$

$D > 0$

∴Roots are real and unequal

4. without solving the following quadratic equation, find the value of ‘p’ for which the given equations have real and equal roots:

(i) $px^2 – 4x + 3 = 0$

(ii) $x^2 + (p - 3)x + p = 0$

Solution

(i) $px^2 - 4x + 3 = 0$

Let us consider,

a = p , b = -4 , c = 3

by using the formula

$D = b^2 - 4ac$

$= (-4)^2 - 4(p)(3)$

$= 16 - 12p$

Since, roots are real

$16 - 12p = 0$

$16 = 12p$

$P = \frac{16}{12}$

$= \frac{4}{3}$

$\therefore p = \frac{4}{3}$

(ii) $x^2 + (p - 3)x + p = 0$

Let us consider

a = 1, b = (p – 3), c = p

by using the formula

$D = b^2 - 4ac$

$= (p – 3)^2 – 4(1) (p)$

$= p^2 – 3^2 – 2(3) (p) – 4p$

$= p^2 + 9 - 6p – 4p$

$= p^2 – 10p + 9$

Since roots are real and have equal roots

$P^2 – 10p + 9 = 0$

Now let us factorize

$P^2 – 9p – p + 9 = 0$

$P(p -9) – 1(p – 9) = 0$

$(p – 9) (p – 1) = 0$

So,

$(p – 9) = 0$ or $(p – 1) = 0$

$P = 9$ or $p = 1$

$P = 1,9$

5. Find the value (s) of k for which each of the following quadratic equation has equal roots:

(i) $x^2 + 4kx + (k^2 – k + 2) = 0$

(ii) $(k – 4)x^2 + 2(k -4)x + 4 = 0$

Solution

(i) $x^2 + 4kx + (k^2 – k + 2) = 0$

Let us consider

$a = 1$, $b = 4k$ $c = k^2 – k + 2$

by using the formula

$D = b^2 – 4ac$

$= (4k)^2 – 4(1) (k^2 – k + 2)$

$= 16k^2 – 4k^2 + 4k – 8$

$= 12k^2 + 4k – 8$

As, roots are equal $D = 0$

$12k^2 + 4k – 8 = 0$

Dividing by 4 on both sides, we get

$3k^2 + k – 2 = 0$

$3k^2 + 3k – k – 2 = 0$

$3k (k + 1) – 1(k + 2) = 0$

$(3k – 1) (k + 2) = 0$

So,

$(3k – 1) = 0$ or $(k + 2) = 0$

$K = \frac{1}{3}$ $or\ k = -2$

$\therefore K = \frac{1}{3}, -2$

(ii) $(k - 4)x^2 + 2(k - 4)x + 4 = 0$

Let us consider

$a = (k - 4)$, $b = 2(k - 4)$, $c = 4$

by using the formula

$D = b^2 - 4ac$

$= (2(k - 4))^2 - 4(k - 4)(4)$

$= (4(k^2 + 16 - 8k)) - 16(k - 4)$

$= 4(k^2 - 8k + 16) - 16k + 64$

$= 4[k^2 - 8k + 16) - 16k + 64$

$= 4[k^2 - 8k + 16 - 4k + 16]$

$= 4[k^2 - 12k + 32]$

Since, roots are equal

$4[k^2 - 12k + 32] = 0$

$K^2 - 12k + 32 = 0$

Now let us factorize

$K^2 - 8k - 4k + 32 = 0$

$K(k - 8) - 4(k - 8) = 0$

$(k - 8)(k - 4) = 0$

So,

$(k - 8) = 0$ or $(k - 4) \neq 0$

$K = 8$ or $k \neq 4$

$\therefore k = 8$

6. find the values(s) of m for which each of the following quadratic equation has real and equal roots:

(i) $(3m + 1)x^2 + 2(m + 1)x + m = 0$

(ii) $x^2 + 2(m - 1)x + (m + 5) = 0$

Solution

(i) $(3m + 1)x^2 + 2(m + 1)x + m = 0$

Let us consider

$a = (3m +1)$ $b = 2(m +1)$ $c = m$

by using the formula

$D = b^2 - 4ac$

$= (2(m + 1))^2 - 4 (3m + 1)(m)$

$= 4(m^2 + 1 + 2m) - 4m(3m + 1)$

$= 4(m^2 + 2m + 1) - 12m^2 - 4m$

$= 4m^2 + 8m + 4 - 12m^2 - 4m$

$= -8m^2 + 4m + 4$

Since roots are equal

$D = 0$

$-8m^2 + 4m + 4 = 0$

Divide by 4 we get

$-2m^2 + m + 1 = 0$

$2m^2 - m - 1 = 0$

Now let us factorize

$2m^2 – 2m + m – 1 = 0$

$2m(m -1) + 1(m -1) = 0$

$(m -1) (2m + 1) = 0$

So,

$(m – 1) = 0$ or $(m + 1) = 0$

$m = 1$ or $m = -\frac{1}{2}$

$\therefore m = 1, -\frac{1}{2}$

(ii) $x^2 + 2(m – 1)x + (m + 5) = 0$

Let us consider

$a = 1, b = 2(m -1)\ c = (m + 5)$

by using the formula

$D = b^2 – 4ac$

$= (2(m -1))^2 – 4(1) (m + 5)$

$= [4(m^2 + 1- 2m)] – 4m – 20$

$= 4m^2 – 8m + 4\ \ - 4m – 20$

$= 4m^2 – 12m – 16$

Since roots are equal

$D = 0$

$4m^2 – 12m – 16 = 0$

Divide by 4, we get

$m^2 -3m – 4 = 0$

now let us factorize

$m^2 – 4m + m – 4 = 0$

$m(m – 4) + 1(m – 4) = 0$

$(m – 4) (m + 1) = 0$

So,

$(m – 4) = 0$ or $(m + 1) = 0$

$m = 4$ or $m = -1$

$\therefore m = 4, - 1$

7. Find the value of k for which each of the following quadratic equation has equal roots:

(i) $9x^2 + kx + 1 = 0$

(ii) $x^2 – 2kx + 7k -12 = 0$

Also find the roots for those values of k in each case.

Solution

(i) $9x^2 + kx + 1 = 0$

Let us consider

$a = 9, b = k, c = 1$

by using the formula

$D = b^2 – 4ac$

$= (k)^2 – 4(9)(1)$

$= k^2 – 36$

Since roots are equal

$D =0$

$K^2 – 36 = 0$

$(k + 6) (k – 6) = 0$

So,

$(k + 6) = 0$ or $(k – 6) = 0$

$K = -6$ or $k = 6$

$\therefore k = 6, -6$

Now let us substitute in the equation

When $k = 6$ then

$9x^2 + kx + 1 = 0$

$9x^2 + 6x + 1 = 0$

$(3x)^2 + 2(3x)(1) + 1^2 = 0$

$(3x + 1)^2 = 0$

$3x + 1 = 0$

X = - 1

X = $-\frac{1}{3}, -\frac{1}{3}$

When k = -6 then

$9x^2 + kx + 1 = 0$

$9x^2 – 6x + 1 = 0$

$(3x)^2 – 2(3x)(1) + 1^2 = 0$

$(3x – 1)^2 = 0$

$3x – 1 = 0$

$3x = 1$

X = $\frac{1}{3}, \frac{1}{3}$

(ii) x^2 – 2kx + 7k -12 = 0

Let us consider

a = 1, b = -2k , c = (7k – 12)

by using the formula

$D = b^2 – 4ac$

$= (-2k)^2 – 4(1)(7k - 12)$

$= 4k^2 – 28k + 48$

Since roots are equal

D = 0

$4k^2 - 28k + 48 = 0$

Divide by 4 we get

$K^2 - 7k + 12 = 0$

Now let us factorize

$K^2 - 3k - 4k + 12 = 0$

$K(k - 3) - 4(k - 3) = 0$

$(k - 3)(k - 4) = 0$

So,

$(k - 3) = 0$ or $(k - 4) = 0$

K = 3 or k = 4

$\therefore K = 3,4$

Now, let us substitute in the equation

When k = 3 then

by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$X = -b \pm \frac{\sqrt{D}}{2a}$$

$$= \frac{[-(-2k) \pm \sqrt{0}]}{2(1)}$$

$= \frac{[2(3)]}{2}$

= 3

X = 3,3

by using the formula

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

X = -b $\pm \frac{\sqrt{D}}{2a}$

$= \frac{[-(-2k) \pm \sqrt{0}]}{2(1)}$

$= \frac{[2(4)]}{2}$

$= \frac{8}{2}$

= 4

X = 4, 4

8. find the value(s) of p for which the quadratic equation (2p +1)x^2 – (7p + 2)x + (7p – 3) = 0 has equal roots. Also find these roots.

Solution

Given:

(2p +1)x^2 – (7p + 2)x + (7p – 3) = 0

Let us compare with $ax^2 + bx + c = 0$

So we get

$a = (2p + 1)$, $b = -(7p + 2)$, $c = (7p - 3)$

by using the formula

$D = b^2 – 4ac$

$0 = (-(7p + 2))^2 – 4(2p + 1)(7p - 3)$

$= 49p^2 + 4 + 28p – 4 [14p^2 – 6p + 7p – 3]$

$= 49p^2 + 4 + 28p – 56p^2 – 4p + 12$

$= -7p^2 + 24p + 16$

Let us factorize

$-7p^2 + 28p – 4p + 16 = 0$

$-p(p - 4) – 4(p - 4) = 0$

$(p – 4) (-7p – 4) = 0$

So,

$(p – 4) = 0$ or $(-7p – 4) = 0$

$P = 4$ or $-7p = 4$

$P = 4$ or $p = -\frac{4}{7}$

$\therefore$value of $p = 4 , -\frac{4}{7}$

9. find the value(s) of p for which the equation $2x^2 + 3x + p =$ 0 has real roots.

Solution

Given

$2x^2 + 3x + p = 0$

Let us consider

a = 2, b = 3, c = p

by using the formula

$D = b^2 – 4ac$

$= (3)^2 – 4(2)(p)$

$= 9 - 8p$

Since roots are real

$9 – 8p \geq 0$

$9 \geq 8p$

$8p \leq 9$

$P \leq \frac{9}{8}$

10. find the least positive value of k for which the equation $x^2 + kx + 4 = 0$ has real roots

solution

given

$x^2 + kx + 4 = 0$

let us consider,

$a = 1, b = k , c = 4$

by using the formula

$D = b^2 – 4ac$

$= (k)^2 – 4(1) (4)$

$= k^2 – 16$

Since roots are real and positive

$K^2 – 16 \geq 0$

$K^2 \geq 16$

$K \geq 4$

$K = 4$

$\therefore$value of $k = 4$

11. find the value of p for which the equation $3x^2 - px + 5 = 0$ has real roots

Solution

Given

$3x^2 - px + 5 = 0$

Let us consider

a = 3, b = -p , c = 5

by using the formula

$D = b^2 - 4ac$

$= (-p)^2 - 4(3)(5)$

$= p^2 - 60$

Since roots are real

$P^2 - 60 \geq 0$

$P^2 \geq 60$

$P \geq \pm\sqrt{60}$

$p \geq \pm 2\sqrt{15}$

$p \geq 2\sqrt{15}$ or $p \leq -2\sqrt{15}$

$\therefore$ value of $p = 2\sqrt{15}$, $-2\sqrt{15}$

Exercise 5.5

1. (i) find two consecutive natural numbers such that the sum of their squares is 61.

(ii) Find two consecutive integers such that the sum of their square is 61.

Solution

(i) find two consecutive natural numbers such that the sum of their squares is 61.

Let us consider first natural number be 'x'

Second natural number be 'x + 1'

So according to the question

$X^2 + (x + 1)^2 = 61$

Let us simplify the expression,

$X^2 + x^2 + 1^2 + 2x – 61 = 0$

$2x^2 + 2x – 60 = 0$

Divide by 2, we get

$X^2 + x – 30 = 0$

Let us factorize

$X^2 + 6x – 5x – 30 = 0$

$X(x + 6) – 5(x + 6) = 0$

$(x + 6) (x – 5) = 0$

So,

$(x + 6) = 0$ or $(x – 5) = 0$

$X = -6$ or $x = 5$

$\therefore x = 5$ [since -6 is not a positive number]

Hence the first natural number = 5

Second natural number = 5 + 1 = 6

(ii) Find two consecutive integers such that the sum of their squares is 61.

Let us consider first integer number be ‘x’

Second integer number be ‘x + 1’

So according to the question

$X^2 + (x + 1)^2 = 61$

Let us simplify the expression,

$X^2 + x^2 + 1^2 + 2x – 61 = 0$

$2x^2 + 2x – 60 = 0$

Divide by 2, we get

$X^2 + x – 30 = 0$

Let us factorize

$X^2 + 6x - 5x - 30 = 0$

$X(x + 6) - 5(x + 6) = 0$

$(x + 6)(x - 5) = 0$

So,

$(x + 6) = 0$ or $(x - 5) = 0$

$X = -6$ or $x = 5$

$\therefore x = 5$ [since -6 is not a positive number]

Hence the first natural number = 5

Second natural number = 5 + 1 =6

(ii) Find two consecutive integers such that the sum of their squares is 61.

Let us consider first integer number be 'x'

Second integer number be 'x + 1'

So according to the question

$X^2 + (x + 1)^2 = 61$

Let us simplify the expression

$X^2 + x^2 + 1^2 + 2x - 61 = 0$

$2x^2 + 2x - 60 = 0$

Divide by 2, we get

$X^2 + x - 30 = 0$

Let us factorize

$X^2 + 6x - 5x - 30 = 0$

$X(x + 6) - 5(x + 6) = 0$

$(x + 6)(x - 5) = 0$

So,

$(x + 6) = 0$ or $(x - 5) = 0$

$X = -6$ or $x = 5$

Now,

If $x = -6$ then

First integer number = -6

Second integer number = - 6 + 1 = -5

If x = 5, then

First integer number = 5

Second integer number = 5 + 1 = 6

2. (i) If the product of two positive consecutive even integers is 288, find the integers.

(ii) if the product of two consecutive even integers is 224, find the integers.

(iii) Find two consecutive even natural numbers such that the sum of their squares is 340.

(iv) Find two consecutive odd integers such that the sum of their squares is 394.

Solution

(i) If the product of two positive consecutive even integers is 288, find the integers.

Let us consider first positive even integer number be ‘2x’

Second even integer number be ‘2x + 2’

So according to the question

$2x \times (2x + 2) = 288$

$4x^2 + 4x - 288 = 0$

Divided by 4, we get

$X^2 + x - 72 = 0$

Let us factorize

$X^2 + 9x - 8x - 72 = 0$

$X(x + 9) - 8(x + 9) = 0$

$(x + 9)(x - 8) = 0$

So,

$(x + 9) = 0$ or $(x - 8) = 0$

$X = -9$ or $x = 9$

∴value of x = 8 [since -9 is not positive]

First even integer = 2x = 2(8) = 16

Second even integer = 2x+ 2 = 2(8) + 2 = 18

(ii) if the product of two consecutive even integers is 244, find the integers.

Let us consider first positive even integer number be '2x'

Second even integer number be '2x + 2'

So according to the question

$2x \times (2x + 2) = 224$

$4x^2 + 4x – 224 = 0$

Divide by 4, we get

$X^2 + x – 56 = 0$

Let us factorize

$X^2 + 8x – 7x – 56 = 0$

$X(x + 8) – 7(x + 8) = 0$

$(x + 8) (x -7) = 0$

So,

$(x + 8) = 0$ or $(x – 7) = 0$

X = - 8 or x = 7

∴value of x = 7 [since -8 is not positive]

First even integer = $2x = 2(7) = 14$

Second even integer = $2x + 2 = 2(7) + 2 = 16$

(ii) find two consecutive even natural numbers such that the sum of their squares is 340

Let us consider first positive even natural number be ‘2x’

Second even number be ‘2x + 2’

So according to the question

$(2x)^2 + (2x + 2)^2 = 340$

$4x^2 + 4x^2 + 8x + 4 – 340 = 0$

$8x^2 + 8x – 336 = 0$

Divide by 8, we get

$X^2 + 7x – 6x – 56 = 0$

$X(x + 7) – 6(x + 7) = 0$

$(x + 7) (x – 6) = 0$

So,

$(x + 7) = 0$ or $(x – 6) = 0$

$X = -7$ or $x = 6$

∴value of $x = 6$ [since -7 is not positive]

First even natural number = $2x = 2(6) = 12$

Second even natural number = $2x + 2 = 2(6) + 2 = 14$

(iv) find two consecutive odd integers such that the sum of their squares is 394

Let us consider first odd integer number be ‘2x + 1’

Second odd integer number be ‘2x + 3’

So according to the question,

$(2x + 1)^2 + (2x + 3)^2 = 394$

$4x^2 + 4x + 1 + 4x^2 + 12x + 9 – 394 = 0$

$8x^2 + 16x – 384 = 0$

Divide by 8 we get

$X^2 + 2x – 48 = 0$

Let us factorize

$X^2 + 8x – 6x – 48 = 0$

$X(x + 8) – 6(x + 8) = 0$

$(x + 8) (x – 6) = 0$

So,

$(x + 8) = 0$ or $(x – 6) = 0$

$X = - 8$ or $x = 6$

When $x = -8$, then

First odd integer $= 2x + 1 = 2(-8) + 1 = -16 + 1 = -15$

Second odd integer $= 2x + 3 = 2(-8) + 3 = -16 + 3 = -13$

When x = 6 then

First odd integer = 2x + 1 = 2(6) + 1 = 12 + 1 = 13

Second odd integer = 2x + 3 = 2(6) + 3 = 12 + 3 = 15

∴the required odd integers are -15 , -13, 13, 15

3. the sum of two numbers is 9 and the sum of their square is 41. Taking one number as x, from ail equation in x and solve it to find the numbers.

Solution

Given:

Sum of two number = 9

Let us consider first number be ‘x’

Second number be ‘9 –x’

So according to the question

$(x)^2 + (9 - x)^2 = 41$

$X^2 + 81 - 18x + x^2 - 41 = 0$

$2x^2 - 18x + 40 = 0$

Divide by 2, we get

$X^2 - 9x + 20 = 0$

Let us factorize

$X^2 - 9x + 20 = 0$

$X(x - 4) - 5(x - 4) = 0$

$(x - 4)(x - 5) = 0$

So,

$(x - 4) = 0$ or $(x - 5) = 0$

$X = 4$ or $x = 5$

When $x = 4$, then

First number $= x = 4$

Second number $= 9 - x = 9 - 4 = 5$

When $x = 5$ then

First number $= x = 5$

Second number $= 9 - x = 9 - 5 = 4$

∴the required number are 4 and 5

4. Five times a certain whole number is equal to three less than twice the square of the number. Find the number

Solution

Let us consider the number be 'x'

So according to the question

$5x = 2x^2 - 3$

$2x^2 - 3 - 5x = 0$

$2x^2 - 5x - 3 = 0$

Let us factorize

$2x^2 - 6x + x - 3 = 0$

$2x(x - 3) + 1(x - 3) = 0$

$(x - 3)(2x + 1) = 0$

So,

$(x - 3) = 0$ or $(2x + 1) = 0$

X = 3 or 2x = -1

X = 3 or $x = -\frac{1}{2}$

∴ the required number is 3 [since , $-\frac{1}{2}$ cannot be a whole number]

5. Sum of two natural numbers is 8 and the difference of their reciprocals is $\frac{2}{15}$. Find the numbers

Solution

Let us consider two numbers as ‘x’ and ‘y’

So according to the question

$\frac{1}{x} - \frac{1}{y} = \frac{2}{15}$...(i)

It is given that x + y = 8

So, y = 8 – x ....(ii)

Now, substitute the value of y in equation (i) we get

$$\frac{1}{x} - \frac{1}{8-x} = \frac{2}{15}$$

By taking LCM

$$\frac{[8-x-x]}{x(8-x)} = \frac{2}{15}$$

$$\frac{8-2x}{x(8-x)} = \frac{2}{15}$$

By cross multiplying

15 (8 -2x) = 2x(8 – x)

$120 - 30x = 16x - 2x^2$

$120 - 30x - 16x + 2x^2 = 0$

$2x^2 - 46x + 120 = 0$

Divide by 2, we get

$X^2 - 23x + 60 = 0$

Let us factorize

$X^2 - 20x - 3x + 60 = 0$

X(x – 20) – 3(x -20) = 0

$(x - 20)(x - 3) = 0$

So,

$(x - 20) = 0$ or $(x - 3) = 0$

X = 20 or x = 3

Now,

Sum of two natural numbers $y = 8 - x = 8 - 20 = -12$ which is a negative value

So value of $x = 3$, $y = 8 - x = 8 - 3 = 5$

∴the value of x and y are 3 and 5

6. The difference between the squares of two numbers is 45. The square of the smaller number is 4 times the larger number. Determine the numbers.

Solution

let us consider the larger number be 'x'

smaller number be 'y'

so according to the question

$x^2 - y^2 = 45$....(i)

$y^2 = 4x$ ....(ii)

now substitute the value of y in equation (i) we get

$x^2 - 4x = 45$

$x^2 - 4x - 45 = 0$

let us factorize

$x^2 - 9x + 5x - 45 = 0$

$x(x - 9) + 5(x - 9) = 0$

$(x - 9)(x + 5) = 0$

So,

$(x - 9) = 0$ or $(x + 5) = 0$

X = 9 or x = -5

When x = 9 then

The larger number = x = 9

Smaller number = y = $y^2 = 4x$

$Y = \sqrt{4}x = \sqrt{4}(9) = \sqrt{36} = 6$

When x = -5 then

The larger number = x = - 5

Smaller number = y = $y^2 = 4x$

$Y = \sqrt{4}x = \sqrt{4}(-5) = \sqrt{-20}$ (*which is not possible*)

∴the value of x and y are 9, 6

7. there are three consecutive positive integers such that the sum of the square of the first and the product of the other two is 154. What are the integers?

Solution

Let us consider the first integer be 'x'

Second integer be 'x + 1'

Third integer be 'x + 2'

So, according to the question,

$X^2 + (x + 1)(x + 2) = 154$

Let us simplify

$X^2 + x^2 + 3x + 2 - 154 = 0$

$2x^2 + 3x - 152 = 0$

Let us factorize,

$2x^2 + 19x - 16x - 152 = 0$

$X(2x + 19) - 8(2x + 19) = 0$

$(2x + 19)(x - 8) = 0$

So,

$(2x + 19) = 0$ $(x - 8) = 0$

$2x = -19$ or $x = 8$

$X = -\frac{19}{2}$ or $x = 8$

∴the value of x = 8 [since $-\frac{19}{2}$ is a negative value]

So,

First integer = x = 8

Second integer = x + 1 = 8 + 1 = 9

Third integer = x + 2 = 8 + 2 = 10

∴the number are 8, 9, 10.

8. (i) Find three successive even natural numbers, the sum of whose squares is 308.

(ii) find three consecutive odd integers, the sum of whose square is 83.

Solution

(i) Find three successive even natural numbers the sum of whose squares is 308

Let us consider first even natural number be '2x'

Second even number be '2x + 2'

Third even number be '2x + 4'

So according to the question

$(2x)^2 + (2x + 2)^2 + (2x + 4)^2 = 308$

$4x^2 + 4x^2 + 8x + 4 + 4x^2 + 16x + 16 – 308 = 0$

$12x^2 + 24x – 288 = 0$

Divide by 12 we get

$X^2 + 2x – 24 = 0$

Let us factorize

$X^2 + 6x – 4x – 24 = 0$

$X(x + 6) – 4(x + 6) = 0$

$(x + 6) (x – 4) = 0$

So,

$(x + 6) = 0$ or $(x – 4) = 0$

$X = -6$ or $x = 4$

∴value of $x = 4$ [since -6 is not positive]

First even natural number = $2x = 2(4) = 8$

Second even natural number = $2x + 2 = 2(4) + 2 = 10$

Third even natural number = $2x + 4 = 2(4) + 4 = 12$

∴the number are 8, 10, 12

(ii) Find three consecutive odd integers, the sum of whose square is 83.

Let the three numbers be 'x' , 'x + 2', 'x + 4'

So according to the question

$(x)^2 + (x +2)^2 + (x + 4)^2 = 83$

$X^2 + x^2 + 4x + 4 + x^2 + 8x + 16 – 83 = 0$

$3x^2 + 12x – 63 = 0$

Divide by 3, we get

$X^2 + 4x – 21 = 0$

Let us factorize

$X^2 + 7x – 3x – 21 = 0$

$X(x + 7) – 3(x + 7) = 0$

$(x + 7) (x – 3) = 0$

So

$(x + 7) = 0$ or $(x – 3) = 0$

$X = -7$ or $x = 3$

∴the numbers will be x , x + 2, x + 4 = -7, -7 + 2, -7 + 4 = -7, -5,-3

Or the numbers will be x, x + 2, x + 4 = 3, 3+2, 3+ 4 = 3, 5, 7

9. In a certain positive fraction, the denominator is greater than the numerator by 3. If 1 is subtracted from both the numerator and denominator the fraction is decreased by $\frac{1}{14}$. Find the fraction

Solution

Let the numerator be 'x'

Denominator be 'x +3'

So the fraction is $\frac{x}{x+3}$

According to the question

$$\frac{x-1}{x+3-1} = \frac{x}{x+3} - \frac{1}{14}$$

Firstly let us simplify RHS

$$\frac{x-1}{x+2} = \frac{14x-x-3}{14(x+3)}$$

$$\frac{x-1}{x+2} = \frac{13x-3}{14x+42}$$

By cross multiplying we get

(x -1) (14x + 42) = (x +2)(13x – 3)

$14x^2 + 42x – 14x – 42 = 13x^2 – 3x + 26x – 6$

$14x^2 + 42x – 14x – 42 – 13x^2 + 3x – 26x + 6 = 0$

$X^2 + 5x – 36 = 0$

Let us factorize

$X^2 + 9x – 4x – 36 = 0$

X(x + 9) – 4(x + 9) = 0

(x + 9) (x – 4) = 0

So,

(x + 9) = 0 or (x – 4) = 0

X = - 9 or x = 4

So the value of x = 4 [since – 9 is a negative number]

When substitute the value of x = 4 in the fraction $\frac{x}{x+3}$ we get

$\frac{4}{4+3} = \frac{4}{7}$

∴the required fraction is = $\frac{4}{7}$

10. the sum of the numerator and denominator of a certain positive fraction is 8. If 2 is added to both the numerator and denominator the fraction is increased by $\frac{4}{35}$. find the fraction

Solution

Let the denominator be 'x'

So the numerator will be '8-x'

The obtained fraction is $\frac{8-x}{x}$

So according to the question

$\frac{8-x+2}{x+2} = \frac{8-x}{x} + \frac{4}{35}$

$\frac{10-x}{x+2} = \frac{8-x}{x} + \frac{4}{35}$

$\frac{10-x}{x+2} - \frac{8-x}{x} = \frac{4}{35}$

By taking LCM

$$\frac{10x-x^2-8x+x^2-16+2x}{x(x+2)} = \frac{4}{35}$$

$$\frac{4x-16}{x^2+2x} = \frac{4}{35}$$

By cross multiplying

$35(4x - 16) = 4(x^2 + 2x)$

$140x – 560 = 4x^2 + 8x$

$4x^2 + 8x – 140x + 560 = 0$

$4x^2 – 132x + 560 = 0$

Divide by 4 we get

$X^2 – 33x + 140 = 0$

Let us factorize,

$X^2 – 28x – 5x + 140 = 0$

$X(x – 28) – 5(x – 28) = 0$

$(x – 28) (x – 5) = 0$

So,

$(x – 28) = 0$ or $(x – 5) = 0$

$X = 28$ or $x = 5$

So the value of $x = 5$ [since $x = 28$ is not possible as sum of numerator and denominator is 8]

When substitute the value of x = 5 in the fraction $\frac{8-x}{x}$ we get

$\frac{8-5}{5} = \frac{3}{5}$

∴the required fraction is = $\frac{3}{5}$

11. A two digit number contains the bigger at ten's place. The product of the digits is 27 and the difference between two digits is 6. Find the number

Solution

Let us consider unit's digit be 'x'

Ten's digit = x + 6

Number = x + 10(x + 6)

= x + 10x + 60

= 11x + 60

So according the question

X(x + 6) = 27

$X^2 + 6x – 27 = 0$

Let us factorize

$X^2 + 9x – 3x – 27 = 0$

X(x + 9) – 3(x + 9) = 0

(x + 9) (x – 3) = 0

So,

(x + 9) = 0 or (x – 3) = 0

X = -9 or x = 3

So, value of x = 3[since -9 is a negative number]

∴the number = 11x + 60

= 11(3) + 60

= 33 + 60

= 93

12. A two digit positive number is such that the product of its digits is 6. If 9 is added to the number, the digits interchange their places. Find the number (2014)

Solution

Let us consider 2 digit number be 'xy' = 10x + y

Reversed digits = yx = 10y + x

So according to the question

10x + y + 9 = 10y + x

It is given that

Xy = 6

Y = $\frac{6}{x}$

So, by substituting the value in above equation we get

$10x + \frac{6}{x} + 9 = 10\left(\frac{6}{x}\right) + x$

By taking LCM

$10x^2 + 6 + 9x = 60 + x^2$

$10x^2 + 6 + 9x - 60 - x^2 = 0$

$9x^2 + 9x - 54 = 0$

Divide by 9, we get

$X^2 + x - 6 = 0$

Let us factorize ,

$X^2 + 3x - 2x - 6 = 0$

$X(x + 3) - 2(x + 3) = 0$

$(x + 3)(x - 2) = 0$

So,

$(x + 3) = 0$ or $(x - 2) = 0$

$X = -3$ or $x = 2$

Value of x = 2 [since -3 is a negative value]

Now substitute the value of x in $y = \frac{6}{x}$ we get

$Y = \frac{6}{2} = 3$

∴ 2- digit number = $10x + y = 10(2) + 3 = 23$

13. A rectangle of area 105 cm^2 has its length equal to x cm. Write down its breath in terms of x. Given that the perimeter is 44cm, write down an equation in x and solve it to determine the dimension of the rectangle.

Solution

Given

Perimeter of rectangle = 44 cm

Length + breadth = $\frac{44}{2} = 22\ cm$

Let us consider length be ‘x’

Breadth be ’22-x’

So according to the question,

$X(22 - x) = 105$

$22x - x^2 - 105 = 0$

$X^2 - 22x + 105 = 0$

Let us factorize

$X^2 - 15x - 7x + 105 = 0$

$X(x - 15) - 7(x - 15) = 0$

$(x - 15)(x - 7) = 0$

So,

$(x - 15) = 0$ or $(x - 7) = 0$

$X = 15$ or $x = 7$

Since length > breath $x = 7$ is not admissible

$\therefore$lenght = 15cm

Breadth = $22 - x = 22 - 15 = 7$ cm

14. A rectangle garden 10m by 16m is to be surrounded by a concrete walk of uniform width. Given that the area of the walk is 120 square meters, assuming the width of the walk to be x, form an equation in x and solve it to find the value of x. (1992)

Solution

Given :

Length of garden = 16cm

Width = 10cm

Let the width of walk be 'x' meter

Outer length = $16 + 2x$

Outer width = $10 + 2x$

So according to the question

$(16 + 2x)(10 + 2x) - 16(10) = 120$

$160 + 32x + 20x + 4x^2 - 160 – 120 = 0$

$4x^2 + 52x – 120 = 0$

Divide by 4 we get

$X^2 + 13x – 30 = 0$

$X^2 + 15x – 2x – 30 = 0$

$X(x + 15) – 2(x + 15) = 0$

$(x + 15) (x -2) = 0$

So,

$(x + 15) = 0$ or $(x – 2) = 0$

$X = -15$ or $x = 2$

∴ value of x is 2 [since , -15 is a negative value]

15. The length of a rectangle exceeds its breadth by 5m. If the breadth was doubled and the length reduced by 9m, the area of the rectangle would have increased by 140m^2. Find its dimensions.

Solution

In first case:

Let us consider length of the rectangle be 'x' meter

Width = (x – 5) meter

Area = lb

= x(x – 5) sq.m

In second case

Length = (x – 9) meters

Width = 2(x – 5) meter

Area = (x – 9) 2(x – 5) = 2(x – 9) (x – 5) sq.m

So according to the question

$2(x - 9)(x - 5) = x(x - 5) + 140$

$2(x^2 - 14x + 45) = x^2 - 5x + 140$

$2x^2 - 28x + 90 - x^2 + 5x - 140 = 0$

$X^2 - 23x - 50 = 0$

Let us factorize

$X^2 - 25x + 2x - 50 = 0$

$X(x - 25) + 2(x - 25) = 0$

$(x - 25)(x + 2) = 0$

So,

$(x - 25) = 0$ or $(x + 2) = 0$

$X = 25$ or $x = -2$

∴length of the first rectangle = 25 meters [since -2 is a negative value]

Width = x – 5 = 25 – 5 = 20 meters

Area = lb

$= 25 \times 20 = 500m^2$

16. the perimeter of a rectangle plot is 180 m and its area is $1800m^2$. Take the length of the plot as x m. Use the perimeter 180 m to write the value of the breadth in terms of x. Use the values of length, breadth and the area to write an equation in x. Solve the equation to calculate the length and breadth of the plot. (1993)

Solution

Given:

The perimeter of a rectangular field = 180 m

And area = $1800m^2$

Let's assume the length of the rectangular field as 'x' m

We know that,

Perimeter of rectangular field = 2(length + breadth)

So, (length + breadth) = $\frac{perimeter}{2}$

X + breath = $\frac{180}{2}$

= breadth = 90 – x

Now, the area of the area of the rectangular field is given as

Length × breadth = 1800

$X \times (90 - x) = 1800$

$90x - x^2 = 1800$

$X^2 - 90x + 1800 = 0$

Upon factorization we have

$X^2 - 60x - 30x + 1800 = 0$

$X(x - 60) - 30(x - 60) = 0$

$(x - 30)(x - 60) = 0$

So,

$X - 30 = 0$ or $x - 60 = 0$

$X = 30$ or $x = 60$

As length is greater than its breadth,

Therefore for the rectangular field

Length = 60m and breath = (90 – 60) = 30m

17. The lengths of the parallel sides of a trapezium are (x + 9) cm and (2x – 3)cm and the distance between them is (x + 4) cm. If its area is 540 cm^2 find x.

Solution

We know that,

Area of a trapezium = $\frac{1}{2} \times (sum\ of\ parallel\ sides) \times (height)$

Given the length of parallel sides are (x + 9) and (2x – 3)

And height = (x + 4)

Now, according the conditions in the problem

$\frac{1}{2} \times (x + 9 + 2x - 3) \times (x + 4) = 540$

$(3x + 6)(x + 4) = 540 \times 2$

$3x^2 + 12x + 6x + 24 = 1080$

$3x^2 + 18x - 1056 = 0$

$3x^2 + 18x - 1056 = 0$

$X^2 + 6x - 352 = 0$ [dividing by 3]

By factorization method, we have

$X^2 + 22x - 16x - 352 = 0$

X(x + 22) – 16(x + 22) = 0

(x – 16) (x + 22) = 0

So,

X – 16 = 0 or x + 22 = 0

X = 16 or x = - 22

As measurements cannot be negative x = -22 is not possible

Therefore x = 16

18. If the perimeter of a rectangular plot is 68m and the length of its diagonal is 26m, find its area.

Solution

Given

Perimeter = 68m and diagonal = 26m

So, length + breadth = $\frac{perimeter}{2}$

= $\frac{68}{2}$

= 34m

Let's consider the length of the rectangular plot to be 'x' m

Then breadth = (34 – x) m

Now, the diagonal of the rectangular plot is given by

Length^2 + breadth^2 = diagonal^2 [by Pythagoras theorem]

$X^2 + (34 - x)^2 = 26^2$

$X^2 + 1156 + x^2 - 68x = 676$

$2x^2 - 68x + 1156 - 676 = 0$

$2x^2 - 68x + 480 = 0$

$X^2 - 34x + 240 = 0$ [dividing by 2]

By factorization method we have

$X^2 – 24x – 10x + 240 = 0$

$X(x – 24) – 10(x – 24) = 0$

$(x – 10) (x – 24) = 0$

So,

$X – 10 = 0$ or $x – 24 = 0$

$X = 10$ or $x = 24$

As length is greater than breadth,

Thus, length = 24m and breath = (34 – 24)m = 10m

And area of the rectangular plot = $24 \times 10 = 240m^2$

19. If the sum of two smaller sides of a right angled triangle is 17cm and the perimeter is 30cm, then find the area of the triangle

Solution

Given

The perimeter of the triangle = 30 cm

Let's assume the length of one of the two small sides as x cm

Then, the other side will be (17 – x) cm

Now, length of hypotenuse = perimeter – sum of other two sides

= (30 – 17)cm

= 13cm

According to the problem, by Pythagoras theorem we have

$X^2 + (17 – x)^2 = 13^2$

$X^2 + 289 + x^2 – 34x = 169$

$2x^2 – 34x + 289 – 169 = 0$

$2x^2 – 34x + 120 = 0$

$X^2 – 17x + 60 = 0$ [dividing by 2]

By factorization method we have

$X^2 – 12x – 5x + 60 = 0$

$X(x – 12) – 5(x – 12) = 0$

$(x – 5) (x – 12) = 0$

So,

$(x – 5) = 0$ or $(x – 12) = 0$

X = 5 or x = 12

When x = 5

First side = 5cm and second side = (17 – 5) = 12cm

And when x = 12

First side = 12 cm and second side = (17 – 12) = 5 cm

Thus,

Area of the triangle = $\frac{1}{2}$ (5×12)

= $\frac{60}{2}$

= $30cm^2$

20. the hypotenuse of grassy land in the shape of a right triangle is 1 meter more than twice the shortest side. If the third side is 7 metres more than the shortest side, find the sides of the grassy land.

Solution

Let's consider the shortest side to be 'x' cm

Hypotenuse = 2x + 1

And third side = x + 7

Now, by Pythagoras theorem we have

$(2x + 1)^2 = x^2 + (x + 7)^2$

$4x^2 + 1 + 4x = x^2 + x^2 + 49 + 14x$

$4x^2 - 2x^2 + 4x – 14x + 1 – 49 = 0$

$2x^2 – 10x – 48 = 0$

$X^2 – 5x – 24 = 0$ [dividing by 2]

By factorization method, we have

$X^2 – 8x + 3x – 24 = 0$

$X(x – 8) + 3(x – 8) = 0$

$(x – 8)(x + 3) = 0$

So,

$x - 8 = 0$ or $x + 3 = 0$

$x = 8$ or $x = -3$

As measurement of side cannot be negative $x = 8$

Therefore

The shortest side = 8m

Third side = $x + 7 = 8 + 7 = 13$m

And hypotenuse = $2x + 1 = 8 \times 2 + 1 = 16 + 1 = 17$ m

Chapter test

Solve the following equation (1 to 4) by factorisation :

1.(i) $x^2 + 6x - 16 = 0$ (ii) $3x^2 + 11x + 10 = 0$

Solution

(i) $x^2 + 6x - 16 = 0$

Let us factorize the given expression

$X^2 + 8x - 2x - 16 = 0$ [as $8 \times (-2) = -16$ and $8 - 2 = 6$]

$X(x + 8) - 2(x + 8) = 0$

$(x - 2)(x + 8) = 0$

So now,

$(x - 2) = 0$ or $(x + 8) = 0$

$X = 2$ or $x = - 8$

$\therefore$value of $x = 2, -8$

(ii) $3x^2 + 11x + 10 = 0$

Let us factorize the given expression,

$3x^2 + 6x + 5x + 10 = 0$ [As $3 \times 10 = 30$ and $6 + 5 = 11$]

$3x(x + 2) + 5(x + 2) = 0$

$(3x + 5)(x + 2) = 0$

So now,

(3x + 5) = 0 or (x + 2) = 0

3x = -5 or x = -2

X = - $\frac{5}{3}$ or x = -2

∴value of x $= -\frac{5}{3}, -2$

2. (i) $2x^2 + ax - a^2 = 0$ (ii) $\sqrt{3x^2} + 10x + 7\sqrt{3} = 0$

Solution

(i) $2x^2 + ax - a^2 = 0$

Let us factorize the given expression

$2x^2 + 2ax - ax - a^2 = 0$ [As $2 \times (-a^2) = -2a^2$ and 2a – a = a]

2x(x + a) – a(x + a) = 0

(2x – a) (x + a) = 0

So now,

(2x – a) = 0 or (x + a) = 0

2x = a or x = -a

X = $\frac{a}{2}$ or x = -a

∴value of x = $\frac{a}{2}$, -a

(ii) $\sqrt{3x^2} + 10x + 7\sqrt{3} = 0$

Let us factorize the given expression,

$\sqrt{3x^2} + 3x + 7x + 7\sqrt{3} = 0$ [$as\ \sqrt{3} \times (7\sqrt{3}) = 7 \times (\sqrt{3})^2 =$ 21 and 7 + 3 = 10]

$\sqrt{3x}(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$

$(\sqrt{3}x + 7)(x + \sqrt{3}) = 0$

So now,

$(\sqrt{3x} + 7) = 0\ (x + \sqrt{3}) = 0$

$\sqrt{3}x = -7\ or\ x = -\sqrt{3}$

X = $-\frac{7}{\sqrt{3}}\ \ or\ x = -\sqrt{3}$

∴ value of x = $-\frac{7}{\sqrt{3}}, -\sqrt{3}$

3. (i) x(x + 1) + (x + 2) (x + 3) = 42 (ii) $\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x-2}$

Solution

(i) x(x + 1) + (x + 2) (x + 3) = 42

Let us simplify the given expression

$X^2 + x + x^2 + 2x + 3x + 6 = 42$

$2x^2 + 6x + 6 – 42 = 0$

$2x^2 + 6x – 36 = 0$

$X^2 + 3x – 18 = 0$ [dividing by 2]

Now, let us factorize

$X^2 + 6x – 3x – 18 = 0$ [As $6 \times(-3) = - 18$ and $6 – 3 = 3$]

$X(x + 6) – 3(x + 6) = 0$

$(x + 6) (x – 3) = 0$

So now,

$(x + 6) = 0$ or $(x – 3) = 0$

$X = -6$ or $x = 3$

$\therefore$value of $x = -6,3$

(ii) $\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x-2}$

Let us simplify the given expression

$\frac{[6(x-1)-2x]}{[x(x-1)]} = \frac{1}{x-2}$ [taking LCM]

$\frac{6x-6-2x}{x^2-x} = \frac{1}{x-2}$

$(4x -6)(x-2) = (x^2 –x)$

$4x^2 – x^2 – 14x + x + 12 = 0$

$3x^2 – 13x + 12 = 0$

Now let us factorize

$3x^2 - 9x - 4x + 12 = 0$

$3x(x - 3) - 4(x - 3) = 0$

$(3x - 4)(x - 3) = 0$

So now,

$(3x - 4) = 0$ or $(x - 3) = 0$

$X = 4$ or $x = 3$

$X = \frac{4}{3}$ or $x = 3$

$\therefore$value of $x = \frac{4}{3}, 3$

4. (i) $\sqrt{(x + 15)} = x + 3$ **(ii)** $\sqrt{(3x^2 - 2x - 1)} = 2x - 2$

Solution

(i) $\sqrt{(x + 15)} = x + 3$

Let us simplify the given expression,

$X + 15 = (x + 3)^2$ [squaring on both sides]

$X + 15 = x^2 + 9 + 6x$

$X^2 + 6x - x + 9 - 15 = 0$

$X^2 + 5x - 6 = 0$

Now let us factorize

$X^2 + 6x - x - 6 = 0$

$X(x + 6) - 1(x + 6) = 0$

$(x - 1)(x + 6) = 0$

So now,

$(x - 1) = 0$ or $(x + 6) = 0$

$X = 1$ or $x = -6$

∴value of $x = 1, -6$

Let's check

When x = 6 then

LHS = $\sqrt{(x + 15)}$

$= \sqrt{(-6 + 15)}$

$= \sqrt{9}$

$= 3$

RHS = $x + 3$

$= -6 + 3$

$= -3$

Thus LHS ≠ RHS

So $x = -6$ is not a root

And when $x = 1$ then

LHS = $\sqrt{(x + 15)}$

= $\sqrt{1 + 15}$

= $\sqrt{16}$

= 4

RHS = x + 3

= 1 + 3

= 4

Thus LHS = RHS

So x = 1 is a root of this equation

Therefore x = 1

(ii) $\boldsymbol{\sqrt{(3x^2 - 2x - 1)} = 2x - 2}$

Let us simplify the given expression,

$3x^2 – 2x – 1 = (2x – 2)^2$ [squaring on both sides]

$3x^2 – 2x – 1 = 4x^2 + 4 – 8x$

$4x^2 – 3x^2 – 8x + 2x + 4 + 1 = 0$

$X^2 - 6x + 5 = 0$

Now, let us factorize

$X^2 – 5x – x + 5 = 0$

X(x – 5) – 1(x – 5) = 0

(x – 1) (x – 5) = 0

So now

(x – 1) = 0 or (x – 5) = 0

X = 1 or x = 5

∴value of x = 1,5

Let's check:

When x = 5 then

LHS = $\sqrt{(3x^2 - 2x - 1)}$

= $\sqrt{(3(5)^2 - 2(5) - 1)}$

= $\sqrt{(3 \times 25 - 2 \times 5 - 1)}$

= $\sqrt{64} = 8$

RHS = 2x – 2

= 2(5) – 2

= 10 – 2 = 8

Thus LHS = RHS

So, x = 5 is a root

And when x = 1 then

LHS = $\sqrt{(3x^2 - 2x - 1)}$

= $\sqrt{3(1)^2 - 2(1) - 1}$

= $\sqrt{(3 \times 1 - 2 \times 1 - 1)}$

$= \sqrt{0} = 0$

RHS = 2x – 2

= 2(1) – 2

= 0

Thus LHS = RHS

So , x = 1 is a root

Therefore x = 1, 5

Solve the following equation (5 to 8) by using formula:

5 (i) $2x^2 - 3x - 1 = 0$ (ii) $x\left(3x + \frac{1}{2}\right) = 6$

Solution

(i) $2x^2 - 3x - 1 = 0$

Let us consider

A = 2, b = -3, c = -1

So by using the formula,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (-3)^2 – 4(2)(-1)$

$= 9 + 8$

$= 17$

So,

$X = \frac{[-(-3)\pm\sqrt{17}]}{2(2)}$

$= \frac{[3\pm\sqrt{17}]}{4}$

$= \frac{[3+\sqrt{17}]}{4} \ or \frac{[3-\sqrt{17}]}{4}$

∴value of x = $\frac{3+\sqrt{17}}{4}, \frac{3-\sqrt{17}}{4}$

(ii) x$\left(3x + \frac{1}{2}\right) = 6$

Let us simplify the given expression

$X^2 + \frac{x}{2} = 6$

$\frac{6x^2+x}{2} = 6$ [taking LCM]

$6x^2 + x = 12$

$6x^2 + x – 12 = 0$

Let us consider

A = 6, b = 1, c = -12

So by using the formula,

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (1)^2 - 4(6)\ (-12)$

$= 1 + 288$

$= 289$

So,

$X = \frac{[-(1) \pm \sqrt{289}]}{2(6)}$

$= \frac{[-1 \pm 17]}{12}$

$= \frac{[-1+17]}{12}\ \ or \frac{[-1-17]}{12}$

$= \frac{16}{12}\ \ or - \frac{18}{12}$

$= \frac{4}{3}\ \ or - \frac{3}{2}$

$\therefore$value of x = $\frac{4}{3}\ , -\frac{3}{2}$

6. (i) $\frac{2x+5}{3x+4} = \frac{x+1}{x+3}$

(ii) $\frac{2}{x+2} - \frac{1}{x+1} = \frac{4}{x+4} - \frac{3}{x+3}$

Solution

(i) $\frac{2x+5}{3x+4} = \frac{x+1}{x+3}$

Let's simply the given expression

(2x +5)(x +3) = (x +1)(3x +4)

$2x^2 + 6x + 5x + 15 = 3x^2 + 3x + 4x + 4$

$2x^2 + 11x + 15 = 3x^2 + 7x + 4$

$3x^2 – 2x^2 + 7x – 11x + 4 – 15 = 0$

$X^2 - 4x – 11 = 0$

Let us consider

a = 1, b = -4 , c = -11

so by using the formula

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

So let, $b^2 – 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 – 4ac$

$= (-4)^2 - 4(1) (-11)$

= 16 + 44

= 60

So,

$X = \frac{[-(-4) \pm \sqrt{60}]}{2(1)}$

$= \frac{[4 \pm 2\sqrt{15}]}{2}$

$= \frac{[4+2\sqrt{15}]}{2} \quad or \frac{[4-2\sqrt{15}]}{2}$

$= \frac{2(2+\sqrt{15})}{2} \quad or \frac{2(2-\sqrt{15})}{2}$

= (2 + $\sqrt{15}$) or (2 - $\sqrt{15}$)

∴Value of x = (2 + $\sqrt{15}$), (2 - $\sqrt{15}$)

(ii) $\frac{\mathbf{2}}{\boldsymbol{x}+\mathbf{2}} - \frac{\mathbf{1}}{\boldsymbol{x}+\mathbf{1}} = \frac{\mathbf{4}}{\boldsymbol{x}+\mathbf{4}} - \frac{\mathbf{3}}{\boldsymbol{x}+\mathbf{3}}$

$\frac{2x+2-x-2}{(x+2)(x+1)} = \frac{4x+12-3x-12}{(x+4)(x+3)}$

$\frac{x}{(x+2)(x+1)} = \frac{x}{(x+4)(x+3)}$

$\frac{1}{(x+2)(x+1)} = \frac{1}{(x+4)(x+3)}$ [dividing by x if x ≠ 0]

$\frac{1}{x^2+3x+2} = \frac{1}{x^2+7x+12}$

So we have

$X^2 + 7x + 12 – x^2 – 3x – 2 = 0$

4x + 10 = 0

2x + 5 = 0

X = $-\frac{5}{2}$

But if x = 0 then

$$\frac{0}{(x+2)(x+1)} = \frac{0}{(x+4)(x+3)}$$

Which is actually true

Therefore x = 0, $-\frac{5}{2}$

7. (i) $\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2}$, $x \neq \frac{4}{3}$

(ii) $\frac{4}{x} - 3 = \frac{5}{2x+3}$, $x \neq 0, -\frac{3}{2}$

Solution

(i) $\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2}$, $x \neq \frac{4}{3}$

Taking LCM we get

$$\frac{[(3x-4)^2 + 7^2]}{[7(3x-4)]} = \frac{5}{2}$$

$2[(3x - 4)^2 + 7^2] = 5 \times [7(3x - 4)]$

$2(9x^2 + 16 - 24x + 49) = 35(3x - 4)$

$2(9x^2 - 24x + 65) = 35(3x - 4)$

$18x^2 - 48x + 130 = 105x - 140$

$18x^2 - 153x + 270 = 0$

$2x^2 - 17x + 30 = 0$ [dividing by 9]

Let us consider

A = 2, b = -17 , c = 30

so by using the formula

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

X = -b $\pm \frac{\sqrt{D}}{2a}$

D = $b^2 - 4ac$

= $(-17)^2 - 4(2)(30)$

=289 – 240

= 49

So,

X = $\frac{[-(-17) \pm \sqrt{49}]}{2(2)}$

= $\frac{[17 \pm 7]}{4}$

= $\frac{[17+7]}{4}$ or $\frac{[17-7]}{4}$

= $\frac{24}{4}$ or $\frac{10}{4}$

= 6 or $\frac{5}{2}$

∴value of x = 6, $\frac{5}{2}$

(ii) $\frac{4}{x} - 3 = \frac{5}{2x+3}$, $x \neq 0, -\frac{3}{2}$

Let's simplify the given equation

$\frac{(4-3x)}{x} = \frac{5}{2x+3}$ [taking LCM]

(4 -3x)(2x + 3) = 5x

$8x + 12 – 6x^2 – 9x = 5x$

$6x^2 + 5x + x – 12 = 0$

$6x^2 + 6x – 12 = 0$

$X^2 + x – 2 = 0$ [dividing by 6]

Let us consider

a = 1, b = 1, c = -2

so by using the formula

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

So let, $b^2 – 4ac = D$

X = -b $\pm \frac{\sqrt{D}}{2a}$

$D = b^2 – 4ac$

$= (1)^2 – 4(1) (-2)$

= 1 + 8

= 9

So,

X = $\frac{[-(1) \pm \sqrt{9}]}{2(1)}$

= $\frac{[-1 \pm 3]}{2}$

= $\frac{[-1 + 3]}{2}$ or $\frac{[-1 - 3]}{2}$

= $\frac{2}{2}$ $or - \frac{4}{2}$

= 1 or -2

∴value of x = 1, - 2

8. (i) $x^2 + (4 - 3a)x - 12a = 0$

(ii) $10ax^2 - 6x + 15ax - 9 = 0, a \neq 0$

Solution

(i) $x^2 + (4 - 3a)x - 12a = 0$

Let us consider

a = 1, b = (4 -3a) , c = -12a

so by using the formula

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

X = -b $\pm \frac{\sqrt{D}}{2a}$

D = $b^2 - 4ac$

$= (4 - 3a)^2 - 4(1)(-12a)$

$= 16 + 9a^2 - 24a + 48a$

$= 16 + 9a^2 + 24a$

$= (4 + 3a)^2$

So,

$X = \frac{[-(4-3a) \pm \sqrt{(4+3a)^2}]}{2(1)}$

$= \frac{[-4+3a \pm (4+3a)]}{2}$

$= \frac{[-4+3a+(4+3a)]}{2} \quad or \quad \frac{[-4+3a-(4+3a)]}{2}$

$= \frac{6a}{2} \quad or - \frac{8}{2}$

$= 3a$ or -4

$\therefore$ value of $x = 3a, -4$

(ii) $10ax^2 - 6x + 15ax - 9 = 0, a \neq 0$

$10ax^2 - (6 - 15a)x - 9 = 0$

Let us consider

$a = 10, b = -(6 - 15a), c = -9$

so by using the formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So let, $b^2 - 4ac = D$

$X = -b \pm \frac{\sqrt{D}}{2a}$

$D = b^2 - 4ac$

$= (6 - 15a)^2 - 4(10a)(-9)$

$= 36 + 225a^2 - 180a + 360a$

$= 36 + 225a^2 + 180a$

$= (6 + 15a)^2$

So,

$$X = \frac{\left[-(-(6 - 15a)) \pm \sqrt{(6+15a)^2}\right]}{2(10a)}$$

$$= \frac{[6 - 15a \pm (6 + 15a)]}{20a}$$

$$= \frac{[6 - 15a + (6 + 15a)]}{20a} \quad or \quad \frac{[6 - 15a - (6 + 15a)]}{20a}$$

$$= \frac{12}{20}a \text{ or } -\frac{30}{20}\text{a}$$

$$= \frac{3}{5}\text{a or } -\frac{3}{2}$$

$\therefore$value of x $= \frac{3}{5}$ a, $-\frac{3}{2}$

9. solve for x using the quadratic formula. Write your answer correct to two significant figures: $(x - 1)^2 - 3x + 4 = 0$

Solution

Given quadratic equation

$(x - 1)^2 - 3x + 4 = 0$

$X^2 - 2x - 3x + 1 + 4 = 0$

$X^2 - 5x + 5 = 0$

Let us consider

a = 1, b = -5 , c = 5

so by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So let, $b^2 - 4ac = D$

$$X = -b \pm \frac{\sqrt{D}}{2a}$$

$D = b^2 - 4ac$

$= (-5)^2 - 4(1)\ (5)$

$= 25 - 20$

$= 5$

So,

$$X = \frac{[-(-5) \pm \sqrt{5}]}{2(1)}$$

$$= \frac{[5 \pm \sqrt{5}]}{2}$$

$$= \frac{[5 + \sqrt{5}]}{2} \quad or \; \frac{[5 - \sqrt{5}]}{2}$$

$$= \frac{5 + 2.236}{2} \quad or \quad \frac{5 - 2.236}{2}$$

$= \frac{7.236}{2} \quad or \frac{2.764}{2}$

= 3.618 or 1.382

∴value of x = 3.618 , 1.382

10. Discuss the nature roots of the following equations:

(i) $3x^2 - 7x + 8 = 0$ (ii) $x^2 - \frac{1}{2}x - 4 = 0$

(iii) $5x^2 - 6\sqrt{5}x + 9 = 0$ (iv) $\sqrt{3}x^2 - 2x - \sqrt{3} = 0$

In case the real roots exist, then find them.

Solution

(i) $3x^2 - 7x + 8 = 0$

Let us consider

a = 3, b = -7 ,c = 8

by using the formula

$D = b^2 - 4ac$

$= (-7)^2 - 4(3)(8)$

= 49 – 96

= -47

So,

Discriminate D = -47

D $<$ 0

∴Roots are not real.

(ii) $x^2 - \frac{1}{2}x - 4 = 0$

Let us consider

a = 1, b = $-\frac{1}{2}$ c = -4

by using the formula

D = $b^2 - 4ac$

$= \left(-\frac{1}{2}\right)^2 - 4(1)(-4)$

$= \frac{1}{4} + 16$

$= \frac{65}{16}$

So,

Discriminate D = $\frac{65}{16}$

D $>$ 0

∴Roots are real and distinct

So,

$$X = \frac{\left[-\left(-\frac{1}{2}\right) \pm \sqrt{\left(\frac{65}{16}\right)}\right]}{2(1)}$$

$= \frac{\left[\frac{1}{2} \pm \frac{\sqrt{65}}{4}\right]}{2}$

$= \frac{\left[\frac{1}{2} + \frac{\sqrt{65}}{4}\right]}{2} \quad or \frac{\left[\frac{1}{2} - \frac{\sqrt{65}}{4}\right]}{2}$

$= \frac{\frac{2+\sqrt{65}}{4}}{2} \quad or \quad \frac{\frac{2-\sqrt{65}}{4}}{2}$

$= \frac{(2+\sqrt{65})}{8} \quad or \quad \frac{2-\sqrt{65}}{8}$

∴ value of x = $\frac{(2+\sqrt{65})}{8}, \frac{(2-\sqrt{65})}{8}$

(iii) $5x^2 - 6\sqrt{5}x + 9 = 0$

Let us consider

a = 5, b = $-6\sqrt{5}$, c = 9

by using the formula

$D = b^2 - 4ac$

$= (-6\sqrt{5})^2 - 4(5)(9)$

= 180 – 180

= 0

So,

Discriminate D = 0

D = 0

∴Roots are equal and real

So,

X = $\frac{[-(-6\sqrt{5})\pm\sqrt{0}]}{2(5)}$

= $\frac{6\sqrt{5}}{10}$

= $\frac{3\sqrt{5}}{5}$

∴value of x = $\frac{3\sqrt{5}}{5}$

(iv) $\sqrt{3}x^2 - 2x - \sqrt{3} = 0$

Let us consider

a = $\sqrt{3}$, b = -2 , c = - $\sqrt{3}$

by using the formula

D = $b^2 - 4ac$

= $(-2)^2 - 4(\sqrt{3})(-\sqrt{3})$

= 4 + 4(3)

= 4 + 12

= 16

So,

Discriminate D = 16

$D > 0$

∴Roots are real and distinct.

So,

$X = \frac{[-(-2)\pm\sqrt{16}]}{2(\sqrt{3})}$

$= \frac{[2 \pm 4]}{2\sqrt{3}}$

$= \frac{[2+4]}{2\sqrt{3}} \ \ or \ \frac{[2-4]}{2\sqrt{3}}$

$= \ \frac{6}{2\sqrt{3}} \ \ or \ -\frac{2}{2\sqrt{3}}$

$= \frac{3}{\sqrt{3}} \ \ \ or - \frac{1}{\sqrt{3}}$

$= \sqrt{3} \ \ or - \frac{1}{\sqrt{3}}$

$=$ ∴ value of $x = \sqrt{3}, -\frac{1}{\sqrt{3}}$

11. Find the values of k so that the quadratic equation $(4 - k)x^2 + 2(k + 2)x + (8k + 1) = 0$ has equal roots.

Solution

Given quadratic equation

$(4 - k)x^2 + 2(k + 2)x + (8k + 1) = 0$

Let us consider

a = (4 – k) , b = 2(k + 2) , c = (8k + 1)

by using the formula

$D = b^2 - 4ac$

$= [2(k+2)]^2 - 4(4-k)(8k+1)$

$= 4(k^2 + 4k + 4) - 4(32k - 8k^2 + 4 - k)$

$= 4k^2 + 16k + 16 - 128k + 32k^2 - 16 + 4k$

$= 36k^2 \ -108k$

= 36k(k -3)

So,

Discriminate D = 36k(k -3)

As the roots are equal

Hence D = 0

36k(k – 3) = 0

So,

36k = 0 or k – 3 = 0

K = 0 or k = 3

Therefore , the value of x = 0, 3

12. Find the values of m so that the quadratic equation $3x^2$ - $5x – 2m = 0$ has two distinct real roots.

Solution

Given quadratic equation

$3x^2 – 5x – 2m = 0$

Let us consider

a = 3, b = -5, c = -2m

by using the formula

$D = b^2 – 4ac$

$= (-5)^2 – 4(3)(-2m)$

= 25 + 24m

So,

Discriminate D = 25 + 24m

As the roots are real and distinct

Hence D > 0

25 + 24m > 0

24m > -25

So,

$m > -\frac{25}{24}$

therefore the value of m must be greater than $-\frac{25}{24}$

13. find the value(s) of k for which each of the following quadratic equation has equal roots:

(i) $3kx^2 = 4(kx - 1)$ (ii) $(k + 4)x^2 + (k + 1)x + 1 = 0$

Also find the roots for that values(s) of k in each case

Solution

For a quadratic equation to have equal roots, discriminate (D) = 0

(i) $3kx^2 = 4(kx - 1)$

Let's rearrange the given equation

$3kx^2 = 4kx - 4$

$3kx^2 - 4kx + 4 = 0$

Let us consider

$a = 3k$, $b = -4k$, $c = 4$

by using the formula

$D = b^2 - 4ac$

$= (-4k)^2 \ -4(3k)(4)$

$= 16k^2 - 48k$

Now,

$16k^2 - 48 = 0$

$16k(k - 3) = 0$

So,

$K = 0$ or $k - 3 = 0$

Thus k = 0 or 3

As D = 0 by the quadratic formula we have

X = $-\frac{b}{2a}$

= $\frac{4k}{2\times 3k}$

= $\frac{4k}{6k}$

= $\frac{2}{3}$

Hence the roots are $\frac{2}{3}, \frac{2}{3}$

(ii) (k + 4)x^2 + (k + 1)x +1 =0

Let us consider

a = k + 4, b = k + 1, c = 1

by using the formula

D = b^2 – 4ac

= $(k+1)^2 - 4(k+4)(1)$

= k^2 + 1 + 2k – 4k – 16

= k^2 – 2k – 15

Now,

K^2 – 2k – 15 = 0

K^2 – 5k + 3k – 15 = 0

K(k – 5) + 3(k -5) = 0

(k +3) (k -5) = 0

So,

K + 3 = 0 or k – 5 = 0

K = -3 or k = 5

Thus k = -3, 5

As D = 0 by the quadratic formula we have

X = $-\frac{b}{2a}$

= $-\frac{k+1}{[2\times(k+4)]}$

= $\frac{-k-1}{2k+8}$

Now when k = 5 we get

X = $\frac{-5-1}{2\times5+8}$

= $-\frac{6}{18}$

= $-\frac{1}{3}$

Hence the roots are $-\frac{1}{3}$, $-\frac{1}{3}$

And when k = -3 we get

X = $\frac{-(-3)-1}{(2\times(-3)+8)}$

= $\frac{3-1}{-6+8}$

= $\frac{2}{2}$ $= 1$

Hence the roots are 1,1

14. Find two natural number which differ by 3 and whose squares have the sum 117.

Solution

Let the first natural number be x

Then second the natural number will be x + 3

According to the condition given in the problem,

$X^2 + (x + 3)^2 = 117$

$X^2 + x^2 + 9 + 6x = 117$

$2x^2 + 9 + 6x = 117$

$2x^2 + 6x – 108 = 0$

$X^2 + 3x – 54 = 0$ [dividing by 2]

$X(x + 9) - 6(x + 9) = 0$

$(x – 6)(x + 9) = 0$

So,

$X – 6 = 0$ or $x + 9 = 0$

$X = 6$ or $x = -9$

As x should be a natural number

$X = 6$

Hence first number = 6 and second number = 6 + 3 = 9

15. Divide 16 into two parts such that the twice the square of the larger part exceeds the square of the smaller part by 164.

Solution

Let the larger part be considered as x

Then, the smaller part will be (16 –x)

According to the conditions given in the problem we have

$2x^2 - (16 - x)^2 = 164$

$2x^2 - (256 - 32x + x^2) = 164$

$2x^2 - 256 + 32x - x^2 - 164 = 0$

$X^2 + 32x - 420 = 0$

Now, by factorization method

$X^2 + 42x - 10x - 420 = 0$

$X(x + 42) - 10(x + 42) = 0$

$(x - 10)(x + 42) = 0$

So,

$X - 10 = 0$ or $x + 42 = 0$

$X = 10$ or $x = -42$ which is not possible as its negative

Thus $x = 10$

Therefore the larger part = 10 and the smaller part = 16 – 10 = 6

16. two natural number are in the ratio 3:4 . find the numbers if the difference between their squares is 175

Solution

Given ratio of two natural number is 3:4

Let the numbers be taken as 3x and 4x

Then according to the conditions in the problem we have

$(4x)^2 - (3x)^2 = 175$

$16x^2 - 9x^2 = 175$

$7x^2 = 175$

$X^2 = \frac{175}{7} = 25$

So,

$X = \sqrt{25} = \pm 5$

But the value of x cannot be -5 as its not a natural number

Thus, x = 5

Therefore

The natural numbers are 3(5) and 4(5) i.e. 15 and 20

17. two squares have sides x cm and (x + 4)cm . the sum of their area is 656 sq. cm . express this as an algebraic equation and solve it to find the sides of the squares.

Solution

We have

Side of first square = x cm

And the side of second square = (x + 4) cm

Now according to the given condition in the problem, we have

$X^2 + (x + 4)^2 = 656$

$X^2 + x^2 + 16 + 8x = 656$

$2x^2 + 8x + 16 – 656 = 0$

$2x^2 + 8x – 640 = 0$

$X^2 + 4x – 320 = 0$[dividing by 2]

By factorization method, we have

$X^2 + 20x – 16x – 320 = 0$

$X(x + 20) -16(x + 20) = 0$

$(x + 20) (x – 16) = 0$

So,

$X + 20 = 0$ or $x – 16 = 0$

$X = -20$ or $x = 16$

Since side of a square cannot be negative

Thus, x = 16

Therefore,

Side of first square = 16cm

And the side of the second square = (16 + 4) = 20 cm

18. The length of a rectangular garden is 12m more than its breadth. The numerical value of its area is equal to 4 times the numerical value of its perimeter. Find the dimensions of the garden

Solution

Let's assume the breadth of the rectangular garden as x m

Then length = (x + 12)m

So,

Area = $l \times b$ m^2

= $x \times (x + 12)m^2$

And perimeter = 2(l + b)

= 2 (x + x + 12)

= 2(2x + 12)m

Now according to the given condition in the problem, we have

$X(x + 12) = 4 \times 2(2x + 12)$

$X^2 + 12x = 16x + 96$

$X^2 – 4x – 96 = 0$

$X^2 – 12x + 8x – 96 = 0$

$X(x – 12) + 8(x – 12) = 0$

$(x + 8) (x – 12) = 0$

So,

$X + 18 = 0$ or $x – 12 = 0$

$X = -18$ or $x = 12$

But $x = -18$ is not possible as it negative

Thus $x = 12$

Therefore

Breadth = 12 m and length = 12 + 12 = 24m

19. AA farmer wishes to grow a $100m^2$ rectangular vegetable garden. Since he has with him only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of his house act as the fourth side fence. Find the dimensions of his garden

Solution

Given

Area of rectangular garden = $100cm^2$

Length of barbed wire = 30m

Let's assume the length of the side opposite to wall to be x

And the length of other side = $\frac{30-x}{2}$

So, the area = $\frac{30-x}{2} \times x$

$= \frac{30x-x^2}{2}$

$= \frac{30x-x^2}{2} = 100$

$30x - x^2 = 200$

$X^2 - 30x + 200 = 0$

By factorization method we have

$X^2 - 20x - 10x + 200 = 0$

$X(x - 20) (x - 10) =0$

So,

X – 20 or x – 10 = 0

X = 20 or x = 10

Hence

(i) If x = 20 then side opposite to the wall = 20 m

And other side will be = $\frac{30-20}{2} = \frac{10}{2} = 5m$

(ii) If x = 10 then side opposite to the wall = 10m

And other sides will be = $\frac{30-10}{2} = \frac{20}{2} = 10m$

Therefore

Sides of the rectangular can be 20m , 5m or 10m , 10m

20. the hypotenuse of a right angled triangle is 1 m less than twice the shortest side. If the third side is 1 m more than the shortest side, find the sides of the triangle.

Solution

Let's consider the length of the shortest side = x m

Length of hypotenuse = 2x – 1

And third side = x + 1

Now according to the given condition in the problem, we have

$X^2 + (x+1)^2 = (2x-1)^2$ [by Pythagoras theorem]

$X^2 + x^2 + 2x + 1 = 4x^2 + 1 - 4x$

$4x^2 - 2x^2 - 4x - 2x - 1 + 1 = 0$

$2x^2 - 6x = 0$

$X^2 - 3x = 0$ [dividing by 2]

$X(x-3) = 0$

So,

X = 0 or x – 3 = 0

X = 0 or x = 3

But x = 0 is not possible

Hence , x = 3

So,

Shortest side = 3m

Hypotenuse = 2 × 3 – 1 = 6 – 1 = 5m

And third side = x + 1 = 3 + 1 = 4m

Therefore the sides of the triangle are 3m, 5m, and 4m.