Chapter 4

Theories of Failure, Static and Dynamic Loading

CHAPTER HIGHLIGHTS

- *Static and Dynamic Loading*
- *Young's Modulus or Modulus of Elasticity*
- *Shear Stress Due to Transverse Loading in a Beam*
- *Stress Due to Torsional Moment*
- *Combined Bending Moment and Axial Load*
- *Combined Normal and Shear Stresses*
- *Theories of Elastic Failure*
- *Maximum Principal Stress Theory (Rankine's Theory)*
- *Maximum Shear Stress Theory (Guest and Tresca's Theory)*
- *Maximum Principal Strain Theory*
- *Maximum Total Strain Energy Theory*
- *Aliter*
- *Design for Variable or Fluctuating Loads*
- *Fatigue and Endurance Limit*

Static and Dynamic Loading

Machine design involves the proper sizing of a machine component to withstand safely various static and dynamic stresses induced in the member. These stresses induced may be due to various loadings, such as axial, transverse, torsional or bending, or may be due to fluctuating loads. If the induced stresses exceed certain limits, failure of the component occurs.

Induced Stress

The resistance offered by a component per unit area is termed as induced stress.

Strength

A material can resist the induced stress only upto certain limited values. This limiting value is termed as strength. Thus, strength is the point of induced stress at which failure of the material occurs. Strength is represented by the letter S – Thus,

S_y – Yield point strength
S_{ut} – Ultimate tensile strength
S_{uc} – Ultimate compressive strength
S_{us} – Ultimate shear strength
S_{sy} – Yield point shear strength, etc.

Factor of Safety

For the safe working of a component, the capacity of the member to resist failure should be greater than the effect of load or strength per unit area of cross section should be greater than the induced stress.

Therefore, a limit is set for the induced stress for the safe working or design purposes known as permissible or allowable stress.

The ratio of strength to permissible stress is called **factor of safety**.

$$\therefore \text{Factor of safety (FOS)} = \frac{\text{Failure stress or strength}}{\text{Allowable stress}}$$

Factor of safety is arbitrarily selected as 3 to 5 based on yield point or 5 to 7 based on fracture failure.

Strain

The ratio of change in dimension due to a loading to the original dimension is called strain.

For axial loading of a component,

$$\text{Strain } \varepsilon = \frac{\delta\ell}{\ell}$$

Young's Modulus or Modulus of Elasticity (E)

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{p\ell}{\delta\ell} = \frac{P\ell}{A\,\delta\ell}$$

where p = normal stress
P = Force or load.

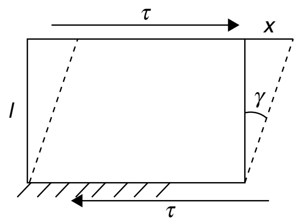
From the above,

Elongation $\delta\ell = \frac{P\ell}{AE}$

Shear Stress and Shear Strain

A section of a component is subjected to a shear stress when a force is acting parallel to the section.

$$\text{Shear stress } \tau = \frac{\text{shear force}}{\text{sectional area}}$$



Shear stress causes an angular deformation. The change in the right angle of the element is known as shear strain. It is denoted by the angle γ. From the figure,

$$\tan\gamma = \frac{x}{\ell}$$

For very small angles $\tan\gamma \simeq \gamma$

$$\therefore \gamma = \frac{x}{\ell}$$

Modulus of Rigidity (*G*)

Ratio of shear stress to shear strain is known as modulus of rigidity.

$$\therefore G = \frac{\tau}{\gamma}$$

Poisson's Ratio (μ)

Ratio of lateral strain to linear strain is known as Poisson's ratio. Thus, $\mu = \frac{\delta b/b}{\delta\ell/\ell}$.

Volumetric Strain

When a member is subjected to stresses it undergoes deformation in all directions. Owing to this there is a change in volume. The ratio of change in volume to the original volume is known as volumetric strain (e_v).

$$\therefore e_v = \frac{\delta V}{V}$$

It can be shown that volumetric strain is the sum of linear strains in three mutually perpendicular directions

$$\text{i.e. } e_v = e_x + e_y + e_z$$

Bulk Modulus

When a body is subjected to identical stresses in all three directions, it undergoes uniform changes in all the three directions without any distortion. The ratio of change in volume to original volume is the volumetric strain. Bulk modulus is given as the ratio of the applied normal stress to the volumetric strain.

$$\text{Bulk modulus } (K) = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{p}{e_v}$$

Relation between *E*, *G* and *K*

The following relationships between the elastic constants E, G and K can be proved.

1. $E = 2G(1+\mu)$
2. $E = 3K(1-2\mu)$
3. $\frac{9}{E} = \frac{3}{G} + \frac{1}{K}$ or $E = \frac{9GK}{3K+G}$

Stress Due to Bending Moment

When a body is subjected to a bending moment, the following equation can be used.

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

where

M = Bending moment

I = Moment of inertia of the section

σ_b = Bending stress

y = Distance from neutral axis of the section

E = Young's modulus

R = Radius of curvature due to bending

From the above,

Bending stress $\sigma_b = \frac{My}{I} = \frac{M}{Z}$ where

Z = section modulus $= \frac{I}{y}$

Shear Stress Due to Transverse Loading in a Beam

At a cross section of the beam shear stress is given by

$$q = \frac{F}{bI} a\bar{y} \quad \text{where } F = \text{Shear force}$$

q is maximum at neutral axis and zero at extreme fibres.

For rectangular sections,

$$q_{\max} = 1.5\, q_{av} = 1.5\, \frac{F}{bd}$$

For circular sections,

$$q_{\max} = \frac{4}{3}\, q_{av} = \frac{4}{3}\, \frac{F}{\left(\frac{\pi d^2}{4}\right)}$$

Stress Due to Torsional Moment

When a body is subjected to torsional moment or twisting moment or torque, the following equations are used:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{\ell}$$

Where T = torsional moment

J = Polar moment of inertia

τ = shear stress

r = radial distance of the element from the centre of the body

G = Modulus of rigidity

θ = angle of twist

ℓ = length of the body

Design of Machine Elements Under Static Loading

In the design of simple machine parts, dimensions are determined on the basis of pure tensile stress, pure compressive stress, direct shear stress, torsional shear stress, etc. Factors such as stress concentration, stress reversal, principal planes, etc are not considered. To account for these, a high factor of safety is used.

Shear stress at yield point (S_{sy}) is taken as half of the tensile stress at yield point.

i.e. $S_{sy} = \frac{1}{2} S_{yt}$

$\left[\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}\right.$ where σ_1 and σ_2 are the principal stresses. When $\sigma_2 = 0$, $\left.\tau_{max} = \frac{\sigma_1}{2}\right]$

Permissible shear stress $\tau = \frac{S_{sy}}{\text{Factor of safety}}$.

Solved Examples

Example 1: A shaft of 12 mm diameter and 1 m long is subjected to an axial load of 60 kN in tension. Find the elongation of the shaft. (Modulus of elasticity $E = 2 \times 10^5$ MPa)

Solution:

$d = 12$ mm; $L = 1$ m $= 1000$ mm

$P = 60$ kN; $E = 2 \times 10^5$ N/mm²

$$\delta L = \frac{PL}{AE}$$

$$= \frac{60 \times 10^3 \times 1000}{\frac{\pi}{4}(12)^2 \times 2 \times 10^5} = 2.65 \text{ mm.}$$

Example 2: An M.S bar of 16 mm diameter and 1 m length is subjected to an axial pull of 60 kN. If Young's modulus is 200 GPa and Poisson's ratio for the material is 0.3, find the change in diameter of the bar.

Solution:

$d = 16$ mm; $\ell = 1\text{m} = 1000$ mm

$P = 60$ kN; $E = 200$ GPa

$= 200 \times 10^9$ N/m²

$= 2 \times 10^5$ N/mm²

$\mu = 0.3$

$$\text{Longitudinal strain} = \frac{\delta L}{L} = \frac{P}{AE}$$

$$= \frac{60 \times 10^3}{\frac{\pi}{4}(16)^2 \times 2 \times 10^5} = 1.492 \times 10^{-3}$$

$$\text{Lateral strain } \frac{\delta d}{d} = \mu \frac{\delta L}{L}$$

$$= 0.3 \times 1.492 \times 10^{-3}$$

Decrease in diameter δd

$$= 16 \times 0.3 \times 1.492 \times 10^{-3}$$

$$= 7.16 \times 10^{-3} \text{ mm.}$$

Example 3: A steel flat of 10 mm width and 12 mm thick is bent into an arc of 10 m radius. If the modulus of elasticity of steel is 2×10^5 MPa, find the maximum intensity of stress induced in the cross section

Solution:

$$\frac{\sigma_t}{y} = \frac{E}{R}; y = \frac{t}{2} = \frac{12}{2} = 6 \text{ mm}$$

$$R = 10 \text{ m} = 10^4 \text{ mm}$$

$$\text{or } \sigma_t = \frac{Ey}{R} = \frac{2 \times 10^5 \times 6}{10 \times 10^3} = 120 \text{ MPa}$$

Example 4: A pillar of hollow circular cross section has to support a load of 800 kN. The ultimate strength in compression for the material is 630 MPa. Find the outer diameter of the pillar using a factor of safety 6, if the diameter ratio is 0.5.

Solution:

$$\text{Area of cross section } A = \frac{\pi}{4}\left(D^2 - d^2\right)$$

$$= \frac{\pi}{4} D^2 \left(1 - k^2\right)$$

$$\text{where } k = \frac{d}{D} = 0.5$$

$$= \frac{\pi}{4} D^2 \left(1 - 0.5^2\right) = 0.1875\, \pi D^2$$

$$\sigma_c = \frac{S_{uc}}{\text{FOS}} = \frac{P}{A}$$

$$\therefore \frac{630}{6} = \frac{800 \times 10^3}{0.1875 \pi D^2}$$

$$\Rightarrow D = 113.73 \text{ mm.}$$

Example 5: A shaft of 100 mm diameter, subjected to a radial load of 80 kN is supported on two bearings. If the permissible bearing pressure is 1.6 N/mm², find the length of the bearings.

Solution:

$$\text{Load per bearing } W = \frac{80}{2} = 40 \text{ kN}$$

Permissible bearing pressure

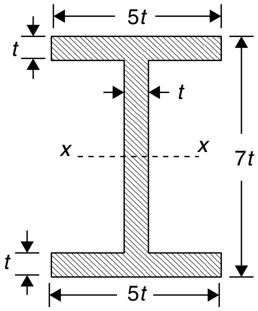
$$p = \frac{W}{\text{Projected area of bearing}}$$

$$= \frac{W}{d \times \ell} \text{ where } \ell = \text{length of bearing}$$

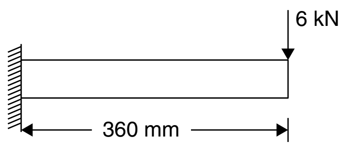
$$\therefore 1.6 = \frac{40 \times 10^3}{100 \times \ell}$$

$$\Rightarrow \ell = 250 \text{ mm.}$$

Example 6: A machine element of length 360 mm and cross section as shown in figure is fixed at one end and the other end has a load of 6 kN. If the yield strength in tension for the material is 360 MPa, find the dimension t for a factor of safety of 3.



Solution:



Allowable stress $\sigma_t = \frac{360}{3}$

$= 120 \text{ N/mm}^2$

Maximum bending moment M

$= 6000 \times 360$

$= 216 \times 10^4 \text{ N mm}$

Moment of inertia about neutral axis

$$I_{XX} = \frac{5t \times (7t)^3}{12} - \frac{4t \times 5t^3}{12} = 101.25t^4$$

$$Z_{XX} = \frac{I}{y} = \frac{101.25t^4}{3.5t} = 28.93t^3$$

$$\sigma_t = \frac{M}{Z}$$

$$\Rightarrow 120 = \frac{216 \times 10^4}{28.93t^3}$$

$\Rightarrow t = 8.54$ mm

Example 7: The outside diameter of a hollow shaft is twice its inside diameter. Find the ratio of its power transmission capacity to that of a solid shaft of the same outside diameter, the same material and running at the same rpm.

Solution:

Power $P = \frac{2\pi NT}{60}$

$P \propto T$ where T is the torque transmitted

For shafts under torsion,

$$\frac{T}{J} = \frac{\tau}{r}$$

For hollow shaft, $\frac{32T}{\pi\left(D^4 - d^4\right)} = \frac{2\tau}{D}$

$$\Rightarrow T_h = \frac{\tau\pi(D^4 - d^4)}{16D}$$

$$= \tau\frac{\pi D^3}{16}\left(1 - k^4\right) \text{ where } k = \frac{d}{D}$$

For solid shaft, $T_s = \frac{\tau\pi D^3}{16}$

$$\therefore \frac{P_h}{P_s} = \frac{T_h}{T_s} = 1 - k^4$$

$$= 1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}.$$

Example 8: A solid circular shaft is to be designed to transmit a torque of 50 Nm. If the yield shear stress of the material is 140 MPa, assuming a factor of safety of 2, the minimum allowable design diameter in mm is

(A) 8 (B) 16 (C) 24 (D) 32

Solution:

$$\tau = \frac{16T}{\pi d^3}; T = 50 \text{ Nm}$$

$$= 50 \times 10^3 \text{ N mm}$$

But $\tau = \frac{S_{sy}}{\text{FOS}} = \frac{140}{2} = 70$ MPa

$$\therefore 70 = \frac{16 \times 50 \times 10^3}{\pi d^3}$$

$\Rightarrow d = 15.38$ mm $= 16$ mm.

Example 9: A hollow shaft with external diameter twice the internal diameter is required to transmit 250 kW power at 240 rpm. The maximum torque may be 1.5 times the mean torque. The allowable shear stress of the material is 40 N/mm^2. The twist per metre length is 1°. Determine the diameter of the shaft. (Take modulus of rigidity = 80 kN/mm^2)

Solution:

$P = 250$ kW; $N = 240$ rpm; $T_{max} = 1.5 \times T$; $\tau = 40$ N/mm^2;

$G = 80 \times 10^3$ N/mm^2; $\theta = 1° = \frac{\pi}{180}$ radian

$\lambda = 1$ m $= 1000$ mm; $P = \frac{2\pi NT}{60}$

$$250 \times 10^3 = \frac{2\pi \times 240 \times T}{60}$$

$\Rightarrow T = 9947$ Nm

$T_{max} = T \times 1.5 = 14921$ Nm

From the consideration of stress,

$$\frac{T_{max}}{J} = \frac{\tau}{D/2}$$

$$\tau = \frac{16T}{\pi D^3(1 - k^4)}$$

$$40 = \frac{16 \times 14921 \times 10^3}{\pi D^3 \left[1 - \left(\frac{1}{2}\right)^4\right]}$$

$\Rightarrow D = 126.55$ mm

From the consideration of the angle of twist,

$$\frac{T_{max}}{J} = \frac{G\theta}{\ell}$$

$$\frac{32 \times 14921 \times 10^3}{\pi D^4 \left[1 - 0.5^4\right]} = \frac{80 \times 10^3}{1000} \times \frac{\pi}{180}$$

$\Rightarrow D = 103.8$ mm

Choosing the larger value,

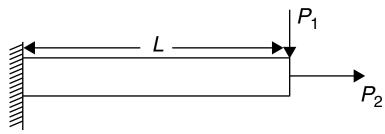
$D = 126.55$ mm

Combined Stresses

A component may be subjected to two types of loading simultaneously. The design is based on the combination of stresses.

Combined Bending Moment and Axial Load

The stresses due to each loading is found separately and then superimposed.



Consider a beam subjected to loads as shown in the figure.

Bending moment $M = P_1 L$

Bending stress $\sigma_{t_1} = \frac{M}{Z} = \frac{P_1 L}{Z}$

σ_{t_1} is tensile at the top fibre and compressive (or negative) at the bottom fibre.

Stress due to P_2

$$\sigma_{t_2} = \frac{P_2}{A}$$

Total stress at top fibre $= \sigma_{t_1} + \sigma_{t_2}$

$$= \frac{P_2}{A} + \frac{P_1 L}{Z}$$

Total stress at bottom fibre $= -\sigma_{t_1} + \sigma_{t_2}$

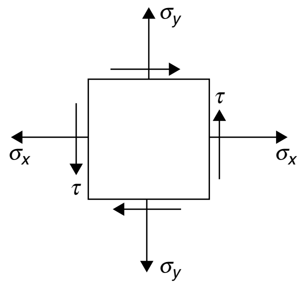
$$= \frac{P_2}{A} - \frac{P_1 L}{Z}$$

Design equation is

$$\frac{P_1 L}{Z} + \frac{P_2}{A} = \frac{S_{yt}}{\text{FOS}}$$

Combined Normal and Shear Stresses

Components subjected to bending moment and torque or direct tension and torque have both normal and shear stresses acting on mutually perpendicular planes of a small element. In such loads, principal stresses and maximum shear stresses result on some other planes. Design is based on these stresses.



For the stresses on a small element as shown above, the principal stresses are,

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$ and maximum shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

The planes on which principal stresses act are called principal planes. On these planes there are no shear stresses. Plane of maximum shear stress is at 45° to the principal planes.

From the above, it can be seen that

Maximum shear stress $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$.

Theories of Elastic Failure

The principal theories of elastic failure are

1. Maximum principal stress theory.
2. Maximum shear stress theory.
3. Maximum principal strain theory.
4. Maximum distortion energy theory.
5. Maximum total strain energy theory.

Maximum Principal Stress Theory (Rankine's Theory)

According to this theory, failure of a component takes place if maximum principal stress at any point exceeds the value of stress at elastic limit in simple tension. The design equation is

$$\sigma_1 = \frac{S_{yt}}{\text{FOS}}$$

Where FOS = factor of safety

It is used for brittle materials, which do not fail by yielding but fail by brittle fracture.

Maximum Shear Stress Theory (Guest and Tresca's Theory)

This theory is also known as Coulomb's theory, in the name of the original proposer. According to this theory, failure of a component takes place when maximum shearing stress in it reaches the value of shearing stress at elastic limit in uniaxial tension test. In two-dimensional stress system, the maximum shear stress is given by

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{\sigma_1 - \sigma_2}{2}$$

The design equation is

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{S_{sy}}{\text{FOS}} = \frac{1}{2}\left(\frac{S_{yt}}{\text{FOS}}\right)$$

$$\therefore \; \sigma_1 - \sigma_2 = \frac{S_{yt}}{\text{FOS}}$$

This theory **gives better results for ductile materials.**

Maximum Principal Strain Theory (or St. Venant's Theory)

According to this theory, failure of a component occurs when maximum strain in it reaches the value of strain in uniaxial stress at elastic limit.

Maximum Strain

$$e_1 = \frac{\sigma_1 - \mu(\sigma_2 + \sigma_3)}{E}$$

$e_1 = e_{max}$, i.e. $e_1 \geq e_2$ or e_3

The design equation is

$$\frac{\sigma_1 - \mu(\sigma_2 + \sigma_3)}{E} = \frac{S_{yt}}{E \times \text{FOS}}$$

or

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) = \frac{S_{yt}}{\text{FOS}}.$$

This theory is suitable for ductile materials.

Maximum Distortion Energy Theory/ Maximum Shear Strain Energy Theory (Von Mises-Henky Theory)

According to this theory, only a part of strain energy causes changes in the volume of the material, and the rest of it causes distortion or shearing action. Distortion is the cause of failure. Thus, the failure of a component occurs when maximum energy of distortion per unit volume under actual loading exceeds the value of maximum energy of distortion per unit volume in uni axial state of stress at elastic limit. The design equation is

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\left(\frac{S_{yt}}{\text{FOS}}\right)^2$$

For two-dimensional stress system, the equation reduces to

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \cdot \sigma_2 = \left(\frac{S_{yt}}{\text{FOS}}\right)^2$$

This theory is suitable for ductile materials, but it cannot be applied to materials under hydrostatic pressure.

Maximum Total Strain Energy Theory (Beltrami and Haigh's Theory)

According to this theory, failure of a component occurs when maximum strain energy per unit volume at a point reaches the value of strain energy per unit volume at elastic limit in simple tension test

For an element subjected to principal stresses σ_1, σ_2 and σ_3, strain energy per unit volume is $\frac{\sigma_1 e_1}{2} + \frac{\sigma_2 e_2}{2} + \frac{\sigma_3 e_3}{2}$

where $e_1 = [\sigma_1 - \mu(\sigma_2 + \sigma_3)]\frac{1}{E}$

$$e_2 = [\sigma_2 - \mu(\sigma_3 + \sigma_1)]\frac{1}{E}$$

$$e_3 = [\sigma_3 - \mu(\sigma_1 + \sigma_2)]\frac{1}{E}$$

From the above, the design equation is

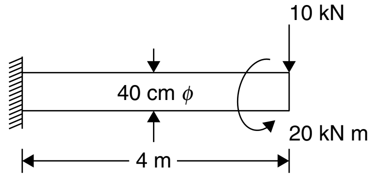
$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] = \left(\frac{S_{yt}}{\text{FOS}}\right)^2 \frac{1}{2E}$$

or

$$\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] = \left(\frac{S_{yt}}{\text{FOS}}\right)^2$$

This theory does not apply to brittle materials for which elastic limit stress in tension and in compression are quite different. However, this theory suits ductile materials.

Example 10: A cantilever beam of circular cross section as shown in figure is subjected to a vertical load of 10 kN and a twisting moment of 20 kN m at the free end. Find the maximum shear stress and the maximum principal stress.



Solution:

Maximum bending moment $M = 10 \times 4 = 40$ kN m

$$\frac{\sigma_b}{(d/2)} = \frac{M}{I} = \frac{64M}{\pi d^4}$$

$$\Rightarrow \sigma_b = \frac{32M}{\pi d^3} = \frac{32 \times 40}{\pi (0.4)^3} = 6366 \text{ kN/m}^2$$

Twisting moment $T = 20$ kN m

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\Rightarrow \frac{32T}{\pi d^4} = \frac{2\tau}{d}$$

$$\Rightarrow \tau = \frac{16T}{\pi d^3} = \frac{16\times 20}{\pi \times 0.4^3} = 1592 \text{ kN/m}^2$$

Maximum shear stress

$$= \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$$

$$= \sqrt{\left(\frac{6366}{2}\right)^2 + (1592)^2} = 3559 \text{ kN/m}^2$$

Maximum principal stress

$$\sigma_1 = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2}$$

$$= \frac{6366}{2} + 3559 = 6742 \text{ kN/m}^2$$

Aliter

$$\text{Maximum shear stress} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$= \frac{16}{\pi \times (0.4)^3}\sqrt{40^2 + 20^2}$$

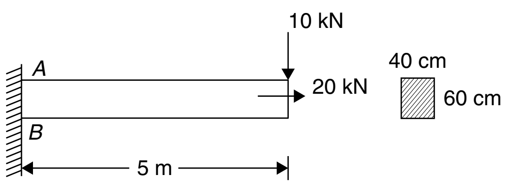
$$= 3559 \text{ kN/m}^2$$

$$\text{Maximum principal stress} = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right]$$

$$= \frac{16}{\pi \times (0.4)^3}\left[40 + \sqrt{40^2 + 20^2}\right]$$

$$= 6742 \text{ kN/m}^2$$

Example 11: A cantilever beam of rectangular cross section is loaded as shown in the figure. Find stresses at A and B.



Solution:

Tensile stress due to axial force

$$\sigma_{t_A} = \sigma_{t_B} = \frac{20}{0.4\times 0.6} = 83.33 \text{ kN/m}^2$$

Bending moment at section AB

$M = 10 \times 5 = 50$ kNm

$$\sigma_b = \frac{My}{I} = \frac{50\times 0.6/2}{\left(\frac{0.4\times(0.6)^3}{12}\right)} = 2083.33 \text{ kN/m}^2$$

σ_b is tensile at A and compressive at B.

Total stress at point $A = \sigma_{t_A} + \sigma_b$

$= 83.33 + 2083.33$

$= 2166.66$ kN/m^2 (tensile)

Total stress at point $B = \sigma_{t_B} - \sigma_b$

$= 83.33 - 2083.33$

$= -2000$ kN/m^2

$= 2000$ kN/m^2 (compressive)

Example 12: A bolt is subjected to an axial force of 10 kN together with a transverse shear force of 6 kN. Determine the diameter of the bolt using

(i) Maximum principal stress theory.
(ii) Maximum shear stress theory.
(given $S_{yt} = 300$ N/mm^2 and FOS = 3)

Solution:

$$\text{Direct stress } \sigma_x = \frac{10\times 10^3}{\left(\frac{\pi d^2}{4}\right)}$$

$$= \frac{40\times 10^3}{\pi d^2}$$

Shear stress at the centre of the cross section of the bolt, $\tau = \frac{4}{3}\times q_{av}$

$$= \frac{4}{3}\times\frac{6\times 10^3}{\left(\frac{\pi d^2}{4}\right)} = \frac{32\times 10^3}{\pi d^2}$$

Maximum principal stress

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

$$= \frac{20\times 10^3}{\pi d^2} + \sqrt{\left(\frac{20\times 10^3}{\pi d^2}\right)^2 + \left(\frac{32\times 10^3}{\pi d^2}\right)^2}$$

$$= \frac{20\times 10^3}{\pi d^2} + \frac{10^3}{\pi d^2}\sqrt{20^2 + 32^2}$$

$$= \frac{20\times 10^3}{\pi d^2} + 37.74\times\frac{10^3}{\pi d^2} = 57.74\times\frac{10^3}{\pi d^2}$$

(i) $S_{yt} = 300$ N/mm^2

FOS = 3

Permissible stress in tension

$$\frac{S_{yt}}{\text{FOS}} = \frac{300}{3} = 100 \text{ N/mm}^2$$

According to maximum principal stress theory $\sigma_1 = \frac{S_{yt}}{\text{FOS}}$

$$\text{i.e. } \frac{57.74\times 10^3}{\pi d^2} = 100$$

$\Rightarrow d = 13.56$ mm.

(ii) Shear stress at elastic limit

$$S_{sy} = \frac{S_{yt}}{2} = \frac{300}{2}$$

Permissible shear stress $= \dfrac{S_{sy}}{\text{FOS}}$

$$= \frac{S_{yt}}{2\times 3} = \frac{300}{6}\ \text{N/mm}^2$$

$$= 50\ \text{N/mm}^2$$

$$\tau_{max} = 50\ \text{N/mm}^2$$

i.e. $\sqrt{\left(\dfrac{\sigma_x}{2}\right)^2 + \tau^2} = 50$

$$\Rightarrow \frac{37.74\times 10^3}{\pi d^2} = 50\ \text{N/mm}^2$$

$\Rightarrow d = 15.5$ mm.

Example 13: A solid circular shaft is subjected to a bending moment of 30 kN m and a torque of 10 kN m. Determine the diameter of the shaft according to maximum strain energy theory (Take Poisson's ratio = 0.25, yield strength of the shaft in tension = 200 N/mm^2 and Factor of safety = 2)

Solution:

Let d be the diameter of the shaft.

Principal stresses

$$\sigma_{1,2} = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{32M}{2\times \pi d^3} \pm \sqrt{\left(\frac{32M}{2\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16}{\pi d^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

$$\sigma_1 = \frac{16\times 10^6}{\pi d^3}\left[30 + \sqrt{30^2 + 10^2}\right]$$

$$= \frac{16\times 10^6}{\pi d^3}[30 + 31.623]$$

$$= \frac{985.97\times 10^6}{\pi d^3}\ \text{N/mm}^2$$

$$\sigma_2 = \frac{16\times 10^6}{\pi d^3}[30 - 31.623]$$

$$= -25.97 \times \frac{10^6}{\pi d^3}\ \text{N/mm}^2$$

$$\sigma_3 = 0$$

According to maximum strain energy theory,

$$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2) = \left(\frac{S_{yt}}{\text{FOS}}\right)^2$$

i.e. $\left(\dfrac{10^6}{\pi d^3}\right)^2 [(985.97)^2 + (-25.97)^2 + 2\times 0.25\times 985.97$

$$\times\ 25.97] = \left(\frac{200}{2}\right)^2$$

$$\Rightarrow \left(\frac{10^6}{\pi d^3}\right)^2 \times 985614 = 100^2$$

$$\Rightarrow \frac{10^6}{\pi d^3}\times 992.78 = 100$$

$\Rightarrow d = 146.75$ mm.

Example 14: At a point in a stressed body the principal stresses are 200 N/mm^2 tensile and 100 N/mm^2 compressive. Yield strength in tension for the material is 500 MPa. Determine the factor of safety based on maximum shear stress theory.

Solution:

$$\sigma_1 = 200\ \text{N/mm}^2$$

$$\sigma_2 = -100\ \text{N/mm}^2$$

Maximum shear stress $= \dfrac{\sigma_1 - \sigma_2}{2}$

$$= \frac{200 + 100}{2}$$

$$= 150\ \text{N/mm}^2$$

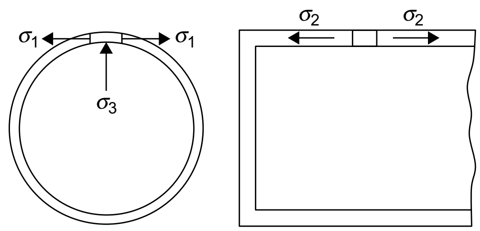
$$S_{sy} = \frac{S_{yt}}{2} = \frac{500}{2} = 250$$

Factor of safety $= \dfrac{250}{150} = 1.67$

Example 15: A cylindrical pressure vessel of 2 m diameter and 20 mm shell thickness is subjected to an internal pressure of 1.5 N/mm^2. The yield strength in tension for the material is 350 MPa and the Poisson's ratio is 0.25. Determine the factor of safety using

(i) Maximum strain theory.
(ii) Maximum distortion energy theory.

Solution:



The element on the shell is subjected to three principal stresses

$$\sigma_1 = \text{hoop stress} = \frac{pd}{2t}$$

$$= \frac{1.5\times 2000}{2\times 20}$$

$$= 75\ \text{N/mm}^2$$

$$\sigma_2 = \text{Longitudinal stress} = \frac{pd}{4t}$$

$$= \frac{\sigma_1}{2} = 37.5\ \text{N/mm}^2$$

$$\sigma_3 = \text{pressure} = -1.5\ \text{N/mm}^2$$

(i) According to maximum strain theory

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) = \frac{S_{yt}}{FOS}$$

$$\therefore\ 75 - 0.25(37.5 - 1.5) = \frac{350}{FOS}$$

$\Rightarrow$ FOS = 5.3.

(ii) According to maximum distortion energy theory,

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\left(\frac{S_{yt}}{FOS}\right)^2$$

i.e. $(75 - 37.5)^2 + (37.5 + 1.5)^2 + (-1.5 - 75)^2$

$$= 2 \times \left(\frac{350}{FOS}\right)^2$$

$\Rightarrow$ FOS = 5.28

Example 16: A bolt is subjected to a direct load of 25 kN and shear load of 15 kN. Yield strength of the material of the bolt is 200 N/mm^2 and factor of safety is 2. Considering Von mise's theory of failure, the minimum size of the bolt is

(A) 26.8 mm (B) 38.3 mm
(C) 31.8 mm (D) 22.3 mm

Solution:

Direct load $P_x = 25$ kN

Shear load $P_s = 15$ kN

Yield stress $S_{yt} = 200 = 0.2 \dfrac{\text{kN}}{\text{mm}^2}$

Factor of safety (FOS) = 2

Axial tensile stress $\sigma_x = \dfrac{P_x}{\left(\dfrac{\pi d^2}{4}\right)}$

$$= \frac{25}{0.7854 d^2} = \frac{31.83}{d^2} \text{kN/mm}^2$$

Shear stress $\tau = \dfrac{P_s}{A} = \dfrac{15}{0.7854 d^2}$

$$= \frac{19.098}{d^2} \text{ kN/mm}^2$$

Principal stresses σ_1, σ_2

$$= \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

$$= \frac{31.83}{2d^2} \pm \sqrt{\left(\frac{31.83}{2d^2}\right)^2 + \left(\frac{19.098}{d^2}\right)^2}$$

$$= \frac{15.92}{d^2} \pm \frac{24.86}{d^2}$$

$$\sigma_1 = \frac{40.78}{d^2}, \quad \sigma_2 = \frac{-8.94}{d^2}$$

According to Von mises theory, (Maximum distorsion energy theory)

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \times \sigma_2 = \left(\frac{S_{yt}}{FOS}\right)^2$$

$$\left(\frac{40.78}{d^2}\right)^2 + \left(\frac{-8.94}{d^2}\right)^2 - \frac{40.78}{d^2} \times \left(\frac{-8.94}{d^2}\right)$$

$$= \left(\frac{0.2}{2}\right)^2 = (0.1)^2$$

$$\Rightarrow \frac{2107.51}{d^4} = 0.1^2$$

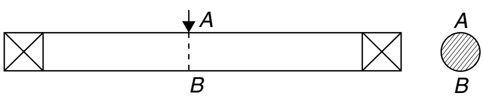
$$\Rightarrow d = 21.43 \text{ mm}$$

Nearest choice is $d = 22.3$ mm

Design for Variable or Fluctuating Loads

Static loading of machine parts happen only when the machine is idle. When the machine is running it is subjected to dynamic loads. A running machine develops variable or fluctuating stresses in its parts.

Consider a shaft supported at its ends by bearings. Due to its self weight or some external loading, let the shaft be sagging, however minute it is.



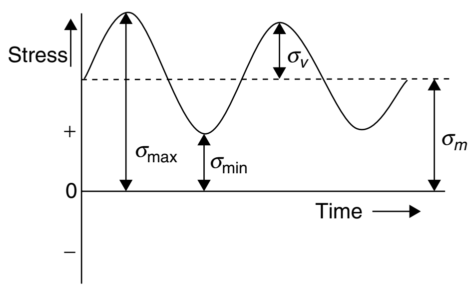
Owing to the loading, point A will be having compressive stresses and point B will be having tensile stresses. Now, when the shaft has rotated half a revolution point A comes to the position of B and stress there becomes tensile. So, the stresses are completely reversed, and also the stresses at a point fluctuates depending upon the revolution of the shaft.

Types of Fluctuating Stresses

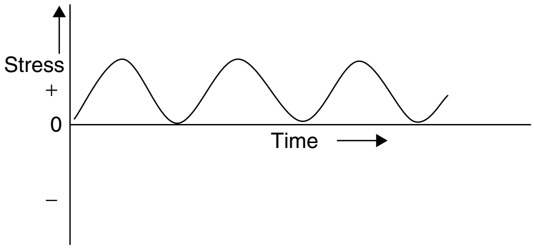
When the load or stress on a component varies in magnitude or direction or both, the loading is known as variable loading.

The variation types are as follows.

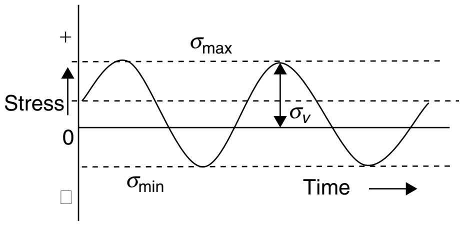
1. Stress variation only on positive side.



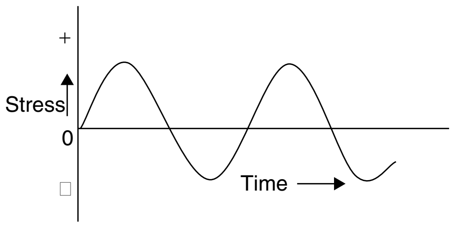
2. Stress variation on positive side, but with zero minimum stress.



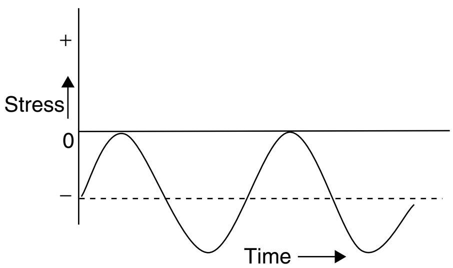
3. Stress variation in positive and negative directions.



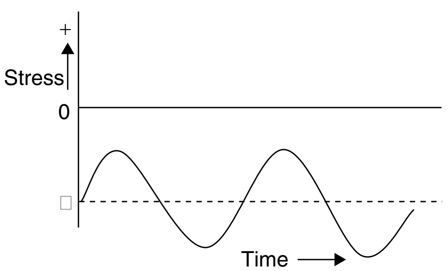
4. Stress variation equally in positive and negative directions.



5. Stress variation only on negative side but with zero maximum stress.



6. Stress variation only on negative side.



Type (3) is the general case of fluctuating stress. When the amplitude is equal in positive and negative sides as in the case (4), it is called a completely reversed stress.

When minimum or maximum stress is zero (2 or 5) it is a case of repeated stress.

Mean or Average Stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Variable Stress (or Stress Amplitude)

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

(σ_v is also denoted as σ_a, meaning stress amplitude)

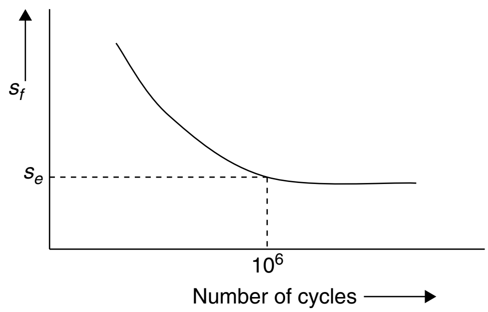
Stress Ratio

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

$$\text{Amplitude ratio} = \frac{\sigma_a}{\sigma_m}$$

Fatigue and Endurance Limit

When a material is subjected to fluctuating stresses the maximum value of fluctuating stress at which failure occurs is below the yield point stress at static loading. This fact is established experimentally. Such type of failure under fluctuating stresses is called **fatigue failure**. The failure takes place after some cycles of reversals. The curve relates to reversible stress and the number of cycles of reversals for failure is called the **endurance curve**. The value of stress at which the material takes 10^6 (one million) number of cycles before failure is called the **endurance limit (S_e)** of the material. The ordinate of the curve is termed as **fatigue strength S_f**.



Usually S_f is taken as 0.8 to $0.9S_u$ at 10^3 cycles and $S_f = S_e = 0.5S_u$ at 10^6 cycles for steel. S_e is further corrected using various derating factors.

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the design or working stress is found based on endurance limit.

$$\text{Design or working stress} = \frac{\text{Endurance limit stress}}{\text{Factor of safety (FOS)}}$$

$$\text{For normal loading, } \sigma = \frac{S_e}{\text{FOS}}$$

$$\text{For shear loading, } \tau = \frac{S_{es}}{\text{FOS}}$$

where σ and τ are maximum permissible induced stresses and $S_{es} = 0.6\ S_{us}$

Stress Concentration

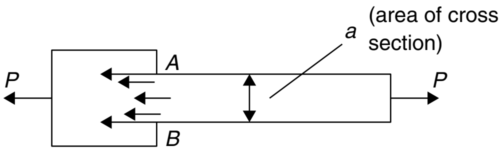
Stress concentration can be defined as the localisation of high stresses due to irregularities in the component and abrupt changes in cross section.

Stress Concentration Factor (k_t)

It is defined as the ratio of highest value of actual stress to the nominal stress for minimum cross section. It is denoted by k_t.

Thus, $$k_t = \frac{\sigma_{\max}}{\sigma_0} = \frac{\tau_{\max}}{\tau_0}$$

where σ_0, τ_0 = nominal tensile and shear stresses.

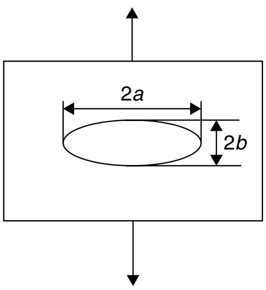


Thus, for the component shown above, maximum stresses are at A and B and

$$\sigma_{\max} = k_t \frac{P}{a}$$

k_t is known as theoretical or form stress concentration factor. k_t depends on the material and geometry of the component.

Stress concentration factor for an elliptical hole on a plate subjected to uniaxial tension is given as follows:



Stress concentration factor,

$$k_t = \left(1 + \frac{2a}{b}\right)$$

NOTE

For circular hole, $a = b \Rightarrow k_t = 1 + 2 = 3$

Fatigue Stress Concentration Factor k_f

Fatigue stress concentration factor is applied in design when the component is subjected to cyclic or fatigue loading. From the experimental tests it is defined as

$$k_f = \frac{\text{(endurance limit without stress concentration)}}{\text{(endurance limit with stress concentration)}}$$

Notch Sensitivity

Notch sensitivity is the susceptibility of a material to succumb to the damaging effects of stress raising notches in cyclic loading. The ratio of increase in fatigue stress over nominal stress to the increase in theoretical stress over nominal stress is called **notch sensitivity factor** (q).

$$\text{or } q = \frac{k_f\sigma_0 - \sigma_0}{k_t\sigma_0 - \sigma_0} = \frac{k_f - 1}{k_t - 1}$$

where σ_0 = nominal stress as obtained by nominal equation

From the above, $k_f = 1 + q(k_t - 1)$

The effect of stress concentration is more predominant in cyclic loading. In the case of static loading the increase in stress at the stress raiser causes local yielding of components, which results in the distribution of stresses. In cyclic loading, stress concentration results in the formation and propagation of cracks, which leads to final fracture.

Estimation of Endurance Limit

Endurance limit is found from the following equation

$$S_e = k_a\, k_b\, k_c\, k_d\, S_e'$$

Where S_e' = endurance limit of rotating beam specimen subjected to reversed bending stress

S_e = endurance limit stress of the particular component subjected to reversed bending stress

k_a = surface finish factor

k_b = size factor

k_c = reliability factor

k_d = modifying factor to account for stress concentration

$$k_d = \frac{1}{k_f} \text{ and } k_f = 1 + q(k_t - 1)$$

Combined Steady and Variable Stresses

The stress on a machine element subjected to a variable load other than completely reversible type can be split into two components — the mean stress and the variable stress. If the maximum and minimum values of stresses are $\sigma_{\max}$ and $\sigma_{\min}$ respectively,

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} \text{ and } \sigma_v = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

As already mentioned, σ_m = mean stress and σ_v = stress amplitude.

When $\sigma_v = 0$, the loading is purely static and failure occurs at S_y or S_u. When $\sigma_m = 0$, the load is completely reversible and failure occurs at S_e.

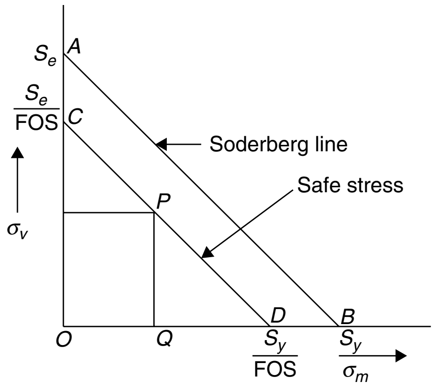
There are three important ways of solving the problems of combination stress which are as follows:

1. Soderberg method.
2. Goodman method.
3. Gerber method.

Soderberg Method

The failure points when $\sigma_m = 0$ and $\sigma_v = 0$ are marked by points A and B in the following diagram (σ_m Vs σ_v).

According to Soderberg any combination of σ_m and σ_v causing failure can be represented by a point on the line joining A and B.



A safe stress line *CD* can be drawn parallel to *AB* taking a suitable factor of safety.

Considering a design point *P* on the line *CD*, from the similar triangles,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD}$$

i.e. $\frac{\sigma_v}{S_e/\text{FOS}} = 1 - \frac{\sigma_m}{S_y/\text{FOS}}$

$$\Rightarrow \sigma_v = \frac{S_e}{\text{FOS}}\left[1 - \frac{\sigma_m}{(S_y/\text{FOS})}\right]$$

$$= S_e\left[\frac{1}{\text{FOS}} - \frac{\sigma_m}{S_y}\right]$$

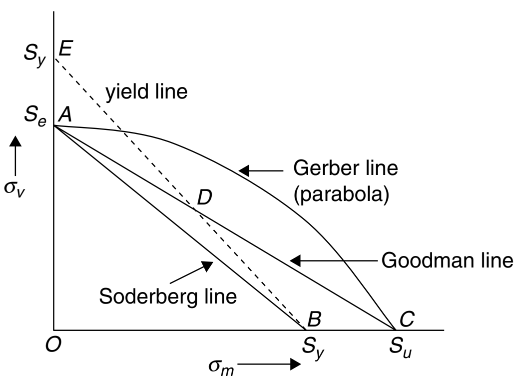
$$\Rightarrow \frac{\sigma_v}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{\text{FOS}}$$

The above equation is called the **Soderberg equation** and the line *AB* is called the **Soderberg line**.

Goodman Method

According to Goodman, the failure points lie on a straight line joining *A* (representing S_e) and point *C* (representing S_u) as shown in the figure. So **Goodman equation** is

$$\frac{\sigma_m}{S_u} + \frac{\sigma_v}{S_e} = \frac{1}{\text{FOS}}$$ and *AC* is called **Goodman line**.



Gerber Method

According to Gerber, the failure points lie not on a straight line, but on a parabola joining *A* and *C*. This parabola is known as the **Gerber line** and the **Gerber equation** is

$$\left(\frac{\sigma_m}{S_u}\right)^2 \times \text{FOS} + \frac{\sigma_v}{S_e} = \frac{1}{\text{FOS}}$$

Yield Line

Yield line is a straight line joining S_y on ordinate to S_y on abscissa.

The equation is $\sigma_m + \sigma_v = S_y$

From design considerations Goodman line is more safe as it is completely inside the Gerber line and inside the failure points. The Soderberg line is more conservative.

Both Goodman and Soderberg lines are linear equations in the form

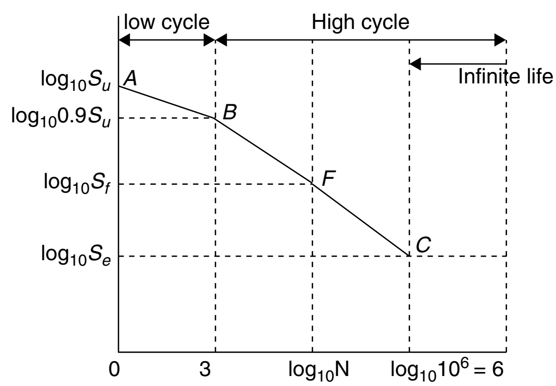
$$\frac{x}{a} + \frac{y}{b} = 1$$

Modified Goodman Line

Goodman line is modified by combining fatigue failure with failure by yielding. Refering to the figure, after the point of intersection *D* with the yield line, the modified Goodman line is *DB*, joining the yield point. Thus, the region *OADB* is called the **modified Goodman diagram**.

S-N Diagrams

S-N diagram is a graph plotted on a semi log paper or log log paper, life in number of cycles as abscissa and fatigue strength S_f as ordinate.



S-N Curve

For design for infinite life (10^6 cycle or more) endurance limit becomes the criterion for failure

Permissible variable stress (or stress amplitude) = $\frac{S_e}{\text{FOS}}$.

When the component is designed for finite life, SN curve can be used.

Example 17: A machine element is subjected to variable stress between 250 N/mm² (maximum) to 50 N/mm² (minimum), both tensile. The yield point stress of the material is 430 N/mm² and the endurance limit is 240 N/mm². According to Soderberg criterion, factor of safety is

(A) 2.5 (B) 2.3 (C) 1.8 (D) 1.3

Solution:

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_v}{S_e} = \frac{1}{\text{FOS}}$$

$$\sigma_m = \frac{250+50}{2} = 150 \text{ N/mm}^2$$

$$\sigma_v = \frac{250-50}{2} = 100 \text{ N/mm}^2$$

$$\therefore \frac{150}{430} + \frac{100}{240} = \frac{1}{\text{FOS}}$$

$$\Rightarrow \text{FOS} = 1.3.$$

Example 18: If the theoretical stress concentration factor and notch sensitivity of an element are 1.15 and 0.9, respectively, the fatigue stress concentration factor is

(A) 0.1015 (B) 1.118 (C) 1.200 (D) 1.135

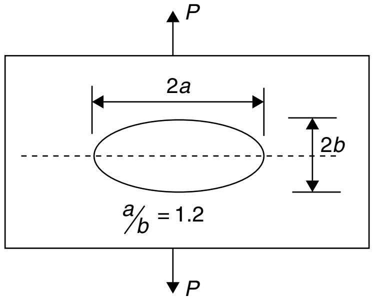
Solution:

$$q = \frac{k_f - 1}{k_t - 1}$$

$$\therefore 0.9 = \frac{k_f - 1}{1.15 - 1}$$

$$\Rightarrow k_f = 1.135$$

Example 19: An elliptical hole is provided on a rectangular thin plate which is loaded as shown in the figure.



The theoretical stress concentration factor k_t is

(A) 2.4 (B) 3.4 (C) 4.4 (D) 1.8

Solution:

$$\sigma_{max} = \sigma\left(1+\frac{2a}{b}\right)$$

$$= \sigma(1+2\times1.2)$$

$$= \sigma \times 3.4$$

$$k_t = \frac{\sigma_{max}}{\sigma} = 3.4.$$

Direction for questions (Examples 20 to 22): Bending stress in a machine part fluctuates between a tensile stress of 280 N/mm² and compressive stress of 140 N/mm². The factor of safety is 1.75 and the yield point is never likely to be less than 55% of ultimate tensile strength and not greater than 93% of it. The endurance strength is 50% of the ultimate strength.

Example 20: According to Gerber's formula, the minimum tensile strength of the part to carry this fluctuation indefinitely is

(A) 754.88 N/mm² (B) 948.28 N/mm²
(C) 678.6 N/mm² (D) 948.28 N/mm²

Solution:

$$\sigma_1 = 280 \text{ N/mm}^2$$

$$\sigma_2 = -140 \text{ N/mm}^2$$

$$\text{FOS} = 1.75$$

$$0.55\, S_u < S_y < 0.93\, S_u$$

$$S_e = 0.5\, S_u$$

$$\sigma_m = \frac{\sigma_1+\sigma_2}{2} = \frac{280-140}{2} = 70 \text{ N/mm}^2$$

Variable stress $\sigma_v = \dfrac{\sigma_1 - \sigma_2}{2}$

$$= \frac{280-(-140)}{2} = 210 \text{ N/mm}^2$$

According to Gerber equation,

$$\frac{1}{\text{FOS}} = \left(\frac{\sigma_m}{S_u}\right)^2 \text{FOS} + \frac{\sigma_v}{S_e}$$

$$\therefore \frac{1}{1.75} = \left(\frac{70}{S_u}\right)^2 \times 1.75 + \frac{210}{0.5\sigma_u}$$

$$\Rightarrow 1 = \frac{15006.25}{S_u^2} + \frac{735}{S_u}$$

$$\Rightarrow S_u^2 - 735\, S_u - 15006.25 = 0$$

$$S_u = \frac{735 \pm \sqrt{735^2 + 4\times15006.25}}{2}$$

$$= 367.5 \pm 387.4$$

$$= 754.88 \text{ N/mm}^2 \text{ (Taking positive value)}$$

Example 21: By Goodman's formula, the minimum ultimate tensile strength for indefinite fluctuation is

(A) 632 N/mm² (B) 983.5 N/mm²
(C) 857.5 N/mm² (D) 783.5 N/mm²

Solution:

By Goodman's formula,

$$\frac{1}{\text{FOS}} = \frac{\sigma_m}{S_u} + \frac{\sigma_v}{S_e}$$

$$\therefore \frac{1}{1.75} = \frac{70}{S_u} + \frac{210}{0.5 S_u} = \frac{490}{S_u}$$

$$\Rightarrow S_u = 857.5 \text{ N/mm}^2.$$

Example 22: By Soderberg formula, the minimum ultimate tensile strength for indefinite fluctuation is

(A) 1310.1 N/mm² (B) 1020.3 N/mm²
(C) 888.32 N/mm² (D) 957.7 N/mm²

Solution:
By Soderberg formula,

$$\frac{1}{\text{FOS}} = \frac{\sigma_m}{S_y} + \frac{\sigma_v}{S_e}$$

$$\therefore \frac{1}{1.75} = \frac{70}{0.55\sigma_u} + \frac{210}{0.5\sigma_u}$$

$$= \frac{547.27}{\sigma_u}$$

$$\Rightarrow \sigma_u = 957.73 \text{ N/mm}^2$$

Direction for questions 23 and 24: A **spherical pressure vessel** with 500 mm inner diameter is welded from steel plates of ultimate stress 440 N/mm². It is subjected to an internal pressure varying from 2 N/mm² to 6 N/mm². Factor of safety is 3. The uncorrected endurance stress is 50% of the ultimate stress.

Example 23: If surface finish factor = 0.85, size factor = 0.8, reliability factor = 0.87, stress concentration = 0.8 and temperature factor = 1.0, find the corrected endurance strength.

Solution:

$$S_e' = 0.5\, S_{ut}$$
$$= 0.5 \times 440$$
$$= 220 \text{ N/mm}^2$$

Endurance limit

$$S_e = 0.85 \times 0.8 \times 0.87 \times 1 \times 0.8 \times S_e'$$
$$= 0.47328 \times 220$$
$$= 104.12 \text{ N/mm}^2$$

Example 24: Using Soderberg equation, determine the thickness of the plate (Take yield stress = 0.55 ultimate stress)

Solution:
Stress developed $\sigma_t = \frac{P_i D_i}{4 \times t}$ (hoop stress for spherical vessel)

$$\therefore (\sigma_t)_{\max} = \frac{6 \times 500}{4 \times t} = \frac{750}{t} \text{ N/mm}^2$$

$$(\sigma_t)_{\min} = \frac{2 \times 500}{4t} = \frac{250}{t} \text{ N/mm}^2$$

$$\text{Mean stress } \sigma_m = \frac{(\sigma_t)_{\max} + (\sigma_t)_{\min}}{2}$$

$$= \frac{750 + 250}{2t}$$

$$= \frac{500}{t} \text{ N/mm}^2$$

$$\text{Variable stress } \sigma_v = \frac{(\sigma_t)_{\max} - (\sigma_t)_{\min}}{2}$$

$$= \frac{750 - 250}{2t} = \frac{250}{t} \text{ N/mm}^2$$

According to Soderberg equation,

$$\frac{\sigma_m}{S_y} + \frac{\sigma_v}{S_e} = \frac{1}{\text{FOS}}$$

$$S_y = 0.55\, S_u$$
$$= 0.55 \times 440$$
$$= 242 \text{ N/mm}^2$$

$$\therefore \frac{500}{t \times 242} + \frac{250}{t \times 104.12} = \frac{1}{3}$$

$$\Rightarrow t = 13.4 \text{ mm.}$$

Example 25: A machine element is subjected to a variable stress which varies from 50 to 100 MPa. Corrected endurance limit for the element is 270 MPa. Ultimate tensile strength and yield strength of the material are 600 MPa and 460 MPa, respectively. Find the value of factor of safety by Gerber method.

Solution:
$\sigma_{\max} = 100$ N/mm²; $\sigma_{\min} = 50$ N/mm²
$S_e = 270$ N/mm²; $S_{ut} = 600$ N/mm²; $S_{yt} = 460$ N/mm²

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$= \frac{100 + 50}{2} = 75 \text{ N/mm}^2$$

$$\sigma_v = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

$$= \frac{100 - 50}{2} = 25 \text{ N/mm}^2$$

According to Gerber equation

$$\frac{\sigma_v}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 \times \text{FOS} = \frac{1}{\text{FOS}}$$

$$\frac{25}{270} + \left(\frac{75}{270}\right)^2 \times N = \frac{1}{N}$$

where N = FOS

$$\Rightarrow \frac{25}{270} N + \left(\frac{75}{270}\right)^2 N^2 = 1$$

$$\Rightarrow N^2 + 1.2\, N - 12.96 = 0$$

$$\Rightarrow N = \frac{-1.2 \pm \sqrt{1.2^2 + 4 \times 12.96}}{2}$$

$$= \frac{-1.2 \pm 7.3}{2}$$

$$= 3.05. \quad \text{(Taking positive value)}$$

Example 26: A varying torque of 30 kN m to 80 kN m is applied at the end of a shaft. The shaft material has an yield strength of 350 MPa and the endurance limit (S_e') of 250 MPa with derating factors, $k_a = 0.85$, $k_b = 0.82$, $k_c = 0.6$. Notch sensitivity $q = 0.9$ and stress concentration factor $k_t = 1.39$. Find the shaft diameter (Take FOS = 1.6).

Solution:

$k_t = 1.39,\ q = 0.9$

$k_f = 1 + (k_t - 1) \times q$

$= 1 + (1.39 - 1) \times 0.9$

$= 1.351$

$$k_d = \frac{1}{k_f} = \frac{1}{1.351} = 0.74$$

Corrected endurance limit

$S_e = S_e{}' \times k_a \cdot k_b \cdot k_c \cdot k_d$

$= 250 \times 0.85 \times 0.82 \times 0.6 \times 0.74$

$= 77.367$ MPa

Using Soderberg equation,

$$\frac{1}{\text{FOS}} = \frac{\tau_m}{S_{ys}} + \frac{\tau_v}{S_e}$$

$$T_m = \frac{80+30}{2} = 55 \text{ kN m}$$

$$T_v = \frac{80-30}{2} = 25 \text{ kN m}$$

$$\tau = \frac{16T}{\pi D^3}$$

$$\therefore \frac{1}{1.6} = \frac{16\times10^6}{\pi D^3}\left[\frac{55}{350} + \frac{25}{77.367}\right]$$

$\Rightarrow D = 157.6$ mm

Example 27: A rotating element of a machine is subjected to a completely reversed bending stress. If the ultimate tensile strength of the material is 620 MPa and corrected endurance limit is 310 MPa, determine the fatigue strength for a life of 80,000 cycles.

Solution:

$S_{ut} = 620$ N/mm^2

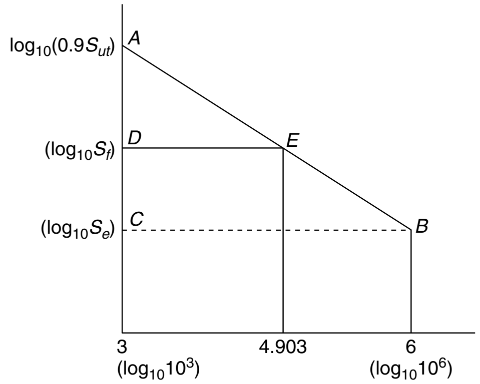
$S_e = 310$ N/mm^2

$N = 80{,}000$ cycles

$\log_{10}N = \log_{10}80{,}000$

$= \log_{10}8 + \log_{10}10^4$

$= 0.903 + 4 = 4.903$



$\log_{10}(0.9\ S_{ut}) = \log_{10}(0.9 \times 620) = 2.7466$

$\log_{10}S_e = \log_{10}310 = 2.4914$

From similar triangular ABC and AED

$$\frac{AC}{BC} = \frac{AD}{DE}$$

$$\frac{(2.7466 - 2.4914)}{6-3} = \frac{2.7466 - \log_{10} S_f}{4.903 - 3}$$

$$\frac{0.2552}{3} = \frac{2.7466 - \log_{10} S_f}{1.903}$$

$\Rightarrow S_f = 384.34$ N/mm^2.

Exercises

Practice Problems I

Direction for questions 1 to 5: A part is subjected to a fluctuating stress that varies from 45 to 95 N/mm^2. The corrected endurance limit stress for the machine part is 225 N/mm^2. The ultimate tensile strength and yield strength of the material are 600 MPa and 450 MPa, respectively.

1. The mean stress is
(A) 85 N/mm^2 (B) 70 N/mm^2
(C) 68 N/mm^2 (D) 91 N/mm^2

2. The stress amplitude is
(A) 25 N/mm^2 (B) 30 N/mm^2
(C) 45 N/mm^2 (D) 51 N/mm^2

3. FOS (factor of safety) according to Gerber line theory is
(A) 5.41 (B) 6.2 (C) 3.8 (D) 5.9

4. FOS as per Soderberg line theory is
(A) 5.85 (B) 4.25 (C) 3.75 (D) 7.1

5. FOS as per Goodman theory is
(A) 7.2 (B) 6.5 (C) 3.2 (D) 4.39

Direction for questions 6 to 8: A rod of a mechanism made of steel is subjected to a reversed axial load of 100 kN. Use factor of safety 2. For an infinite life condition, $S_{ut} = 550$ N/mm^2 K_a, K_b, K_c, are 0.78, 0.85 and 0.868, respectively. The endurance limit in reversed bending may be assumed to be one-half of ultimate tensile strength and correction factor for axial loading = 0.8

6. The endurance limit stress for the rod is
(A) 158.26 N/mm^2 (B) 169.3 N/mm^2
(C) 145.1 N/mm^2 (D) 136.3 N/mm^2

7. Permissible stress amplitude is
(A) 48.3 N/mm^2 (B) 63.5 N/mm^2
(C) 55 N/mm^2 (D) 49 N/mm^2

8. Diameter of rod is
(A) 49.3 mm (B) 41.1 mm
(C) 44.78 mm (D) 35.6 mm

9. A rectangular plate 40 mm width and 10 mm thickness is subjected to an axial load of 10 kN. The plate has a 10 mm diameter hole in it. If the stress concentration factor is 2.5, the maximum stress in the plate is

(A) 62.5 MPa (B) 52.5 MPa
(C) 40 MPa (D) 25 MPa

10. In cyclic loading, stress concentration is more serious in

(A) Brittle materials (B) Ductile materials
(C) Elastic materials (D) Brittle and ductile materials

Direction for questions 11 and 12: A 50 mm diameter shaft is made of carbon steel having an ultimate tensile strength of 630 MPa. It is subjected to a torque which fluctuates between 2000 Nm and −800 Nm.

Given $S_e = 0.25\ S_u$ and $S_y = 510$ N/mm^2

11. The variable shear stress is

(A) 43 MPa (B) 47 MPa
(C) 57 N/mm^2 (D) 66 N/mm^2

12. If fatigue stress concentration factor is 1.32, the factor of safety according to Soderberg criterion is

(A) 1.32 (B) 1.58 (C) 1.73 (D) 1.90

Direction for questions 13 to 15: A leaf spring in automobile is subjected to cyclic stresses. The average stress is 150 MPa. The variable stress is 50 MPa. The ultimate stress is 630 MPa and the yield point stress is 350 MPa. The endurance limit is 150 MPa.

13. Factor of safety according to Goodman's criterion is

(A) 2.5 (B) 1.75 (C) 1.25 (D) 1.12

14. Factor of safety according to Soderberg criterion is

(A) 1.3 (B) 1.8 (C) 2.5 (D) 2.75

15. If the surface finish factor is 0.89 and the size factor 0.85, then the factor of safety according to Soderberg criterion is

(A) 1.3 (B) 1.25 (C) 1.15 (D) 1.09

Direction for questions 16 to 19: A simply supported beam has a concentrated load (in *N*) at the centre which fluctuates from *P* to 4*P*. The span of the beam is 750 mm. The diameter of the beam is 50 mm. Given: $S_u = 750$ MPa, $S_y =$ 600 MPa, $S_e = 330$ MPa for reverse bending. Derating factors $k_a = 0.9$, $k_b = 0.85$. Factor of safety = 1.3

16. The mean bending stress (MPa) is nearly

(A) $\frac{P}{68}$ (B) $\frac{P}{52}$ (C) $\frac{P}{42}$ (D) $\frac{P}{26}$

17. The variable bending stress (MPa) is nearly

(A) $\frac{P}{43}$ (B) $\frac{P}{57}$ (C) $\frac{P}{63}$ (D) $\frac{P}{71}$

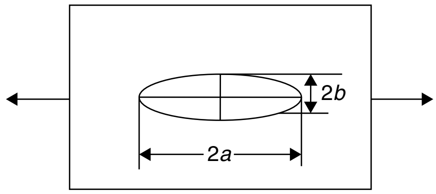
18. The value of *P* according to Goodman's line is (kN)

(A) 6.12 (B) 5.43
(C) 5.31 (D) 4.87

19. The value of *P* according to Soderberg criterion is (kN)

(A) 6.18 (B) 5.9 (C) 4.98 (D) 4.63

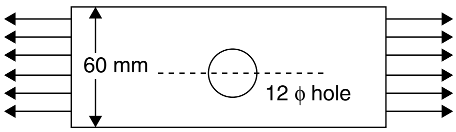
20. In the figure given below $\frac{b}{a} = \frac{1}{3}$. The theoretical stress concentration factor is



(A) 2 (B) $\frac{5}{3}$

(C) 2 (D) $\frac{15}{2}$

21.



A completely reversible load of 15 kN acts on a plate with a 12 mm diameter hole as shown in figure. The plate has ultimate strength S_u = 440 N/mm^2, notch sensitivity $q = 0.8$, surface finish factor = 0.67, size factor = 0.85, reliability factor for 90% reliability = 0.897. If $k_t = 2.35$ and factor of safety = 2, determine the required minimum plate thickness in millimeter. (Take $S_e' = 0.5\ S_u$).

22. A shaft of 20 mm diameter is subjected to an axial pull of 40 kN. The shaft material has a tensile yield strength of 310 N/mm^2. If the shaft is designed based on maximum shear stress theory and a factor of safety of 2, determine the maximum torque (in N m) that can be applied before yielding

23. An element in a strained body is subjected to a tensile stress of 60 MPa and shear stress of 40 MPa. If the yield stress in tension for the material is 320 MPa, the factor of safety applying maximum principal stress theory is

(A) 2.5 (B) 3 (C) 3.5 (D) 4

24. A thin spherical vessel of 200 mm diameter and 1 mm shell thickness is subjected to an internal pressure varying from 3 MPa to 7 MPa. If yield, ultimate and endurance strength of the material are 600, 800 and 400 N/mm^2, respectively, determine the factor of safety as per Goodman's relation.

25. A tension member of 50 mm diameter is to be replaced by a square bar of same material. Side of the square will be

(A) 44.31 mm (B) 42.42 mm
(C) 40.34 mm (D) 38.24 mm

26. The relationship between theoretical stress concentration factor k_t, fatigue stress factor k_f and notch sensitivity *q* is given by

(A) $q = \dfrac{k_t - 1}{k_f - 1}$ (B) $q = \dfrac{k_f - 1}{k_t - 1}$

(C) $q = \dfrac{k_f - 1}{k_t + 1}$ (D) $q = \dfrac{k_t + 1}{k_f + 1}$

27. Maximum shear stress developed in a shaft subjected to pure torsional moment is 120 MPa. The yield and ultimate strengths of the shaft material in tension are 300 MPa and 450 MPa, respectively. Using Von-mises theory of failure, the factor of safety is
(A) 1.44 (B) 1.86
(C) 2.42 (D) 2.92

28. In the case of brittle materials, the most appropriate theory of failure applied is
(A) maximum shear stress theory.
(B) maximum principal stress theory.
(C) maximum strain theory.
(D) maximum total strain energy theory.

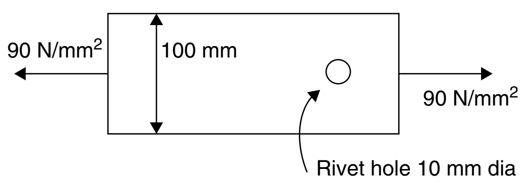
29. A compressed air cylinder has 700 N/cm^2 air pressure at the time of delivery. The diameter of the cylinder inside is 250 mm. The yield strength of the steel in tension is 23000 N/cm^2. For a factor of safety of 2.5, the minimum wall thickness of the cylinder should be nearly
(A) 6 mm (B) 8 mm
(C) 10 mm (D) 12 mm

30. A spherical metal pressure vessel is constructed by riveting. The diameter of the vessel is 1.2 m. The maximum internal pressure expected is 1.5 MPa. The permissible tensile stress of the vessel material is 62.5 N/mm^2. If the efficiency of the riveted joint is 75%, the required minimum thickness of the plate is nearly
(A) 6 mm (B) 8 mm
(C) 10 mm (D) 12 mm

Practice Problems 2

1. The ratio of torque transmitting capacity to weight of a circular shaft is directly proportional to
(A) diameter
(B) square of diameter
(C) cube of diameter
(D) square root of diameter

2. A uniform plate of 100 mm width containing a rivet hole of 10 mm dia is subjected to a uniaxial tensile stress of 90 N/mm^2 as shown in the figure. Maximum stress developed in the plate is



(A) 100 MPa (B) 270 MPa
(C) 300 MPa (D) 200 MPa

3. The principal stresses in an element of a strained body subjected to bi-axial stresses are 350 MPa and 140 MPa. Maximum working stress as per distortion energy theory is
(A) 210 MPa (B) 305 MPa
(C) 360 MPa (D) 390 MPa

Direction for questions 4 to 6: A solid circular shaft is subjected to a bending moment of 50 kN m and a torque of 10 kN m. Given that stress at elastic limit = 200 N/mm^2, Poisson's ratio = 0.25 and factor of safety = 2.

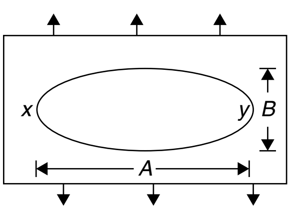
4. The diameter of the shaft according to maximum principal stress theory is (mm) nearly
(A) 142.8 (B) 156.3
(C) 172.6 (D) 189.5

5. The diameter of the shaft according to maximum shear stress theory is (mm) nearly
(A) 173 (B) 168.4 (C) 174.6 (D) 169.3

6. The diameter of the shaft according to maximum strain energy theory is (mm) nearly.
(A) 180.6 (B) 173 (C) 154 (D) 148.8

7. The theoretical stress concentration factor at a section of a loaded element is 1.63. The fatigue stress concentration factor is evaluated to be 1.42. Then, the notch sensitivity at the section is
(A) 0.21 (B) 1.05 (C) 0.67 (D) 0.52

8. A loaded semi-infinite flat plate is having an elliptical hole ($A/B = 2$) in the middle as shown in the figure.



The stress concentration factors at points either X or Y is
(A) 1 (B) 2 (C) 3 (D) 5

Direction for questions 9 to 11: A mild steel cylindrical shell of diameter 120 cm is subjected to an interal pressure 1.5 MPa. The material of the shell yields at 200 MPa and the factor of safety = 3.

9. The thickness of the plate according to the maximum principal stress theory is
(A) 13.5 mm (B) 15 mm
(C) 14.71 mm (D) 18.25 mm

10. The thickness according to maximum shear stress theory is
(A) 9.3 mm (B) 13.5 mm
(C) 8.5 mm (D) 6.75 mm

11. The thickness according maximum shear strain theory is
(A) 14.28 mm (B) 12.8 mm
(C) 11.7 mm (D) 8.2 mm

12. The equivalent Bending Moment for design of a circular shaft on the basis of principal stress, when it is subjected to a bending moment M and a Torque T simultaneously is
(A) $\sqrt{M^2+T^2}$ (B) $M+\sqrt{M^2+T^2}$
(C) $\frac{\left(M+\sqrt{M^2+T^2}\right)}{2}$ (D) $\sqrt{2+4T^2}$

13. The stress level below which the material has high probability of not failing under repeated reversal of stress is called
(A) Elastic limit (B) Proportional limit
(C) Tolerance limit (D) Endurance limit

14. The maximum shear stress developed in a thin cylindrical shell of radius r and thickens t, when subjected to internal pressure p will be
(A) pr/t (B) $pr/2t$ (C) $pr/3t$ (D) $pr/4t$

15. A thin cylinder of diameter D and thickness 't' is subjected to an internal pressure 'p'. The Youngs modulus of the material is E and Poission's ratio is $\frac{1}{m}$, then the diametral strain is
(A) $\frac{pD}{2tE}\left[1-\frac{2}{m}\right]$ (B) $\frac{pD}{4tE}\left[1-\frac{2}{m}\right]$
(C) $\frac{pD}{2tE}\left[1-\frac{1}{m}\right]$ (D) $\frac{pD}{4tE}\left[2-\frac{1}{m}\right]$

Direction for questions 16 to 21: A mild steel specimen tested in a laboratory gave the following data.

Diameter of the specimen = 25 mm, length = 300 mm, extension under a load of 15 kN = 0.045 mm.

Load at yield point = 127.5 kN, maximum load = 208.6 kN. length of specimen at failure = 375 mm, Neck diameter = 17.75 mm. Factor of safety: 2

16. The Young's modulus in kN/mm^2 is
(A) 203.7 (B) 301.2 (C) 518.8 (D) 415.5

17. The yield point stress is
(A) 0.29 N/mm^2 (B) 0.26 N/mm^2
(C) 0.3 N/mm^2 (D) 0.41 N/mm^2

18. The ultimate stress is
(A) 0.628 kN/mm^2 (B) 0.425 kN/mm^2
(C) 0.53 kN/mm^2 (D) 0.695 kN/mm^2

19. Percentage elongation at failure is
(A) 20% (B) 25%
(C) 30% (D) 45%

20. Percentage reduction in area at failure is
(A) 50.7 (B) 49.5
(C) 80.5 (D) 60.8

21. Safe stress in kN/mm^2 is
(A) 0.33 (B) 0.414
(C) 0.520 (D) 0.212

Direction for questions 22 to 26: Tensile stress acting on a member in X direction varies from 40 to 100 N/mm^2 and tensile stress in Y direction varies from 10 to 80 N/mm^2. The frequency of variation is equal. If the corrected edurance limit is 270 N/mm^2 and S_{ut} = 660 N/mm^2

22. The mean stress is
(A) 61.44 N/mm^2 (B) 85.31 /mm^2
(C) 72.35 N/mm^2 (D) 89.90 N/mm^2

23. Stress amplitude is
(A) 32.79 N/mm^2 (B) 41.11 N/mm^2
(C) 53.8 N/mm^2 (D) 61.3 N/mm^2

24. Stress ratio R_x in the x-direction is
(A) 2.5 (B) 0.4
(C) 0.3 (D) 0.7

25. Stress ratio R_y in the y-direction is
(A) 8 (B) 0.4
(C) 0.125 (D) 0.3

26. The amplitude ratio is
(A) 0.3125 (B) 0.4
(C) 0.625 (D) 0.534

27. For a shaft subjected to bending moment M and twisting moment T simultaneously, the equivalent torque is given by
(A) $\sqrt{M^2+T^2}$ (B) $M+\sqrt{M^2+T^2}$
(C) $M+\sqrt{M^2+\frac{T^2}{2}}$ (D) $T+\sqrt{M^2+T^2}$

28. A ductile material subjected to variable loading has maximum and minimum stresses 150 N/mm^2 and 50 N/mm^2, respectively. The endurance limit and yield point of the material are 200 N/mm^2 and 300 N/mm^2, respectively. If the fatigue stress concentration factor is 1.29, the available factor of safety for loading is
(A) 1.352 (B) 1.525
(C) 1.934 (D) 2.124

29. A shaft with a pulley at the centre is supported on two bearings 400 mm apart and rotates at 1400 rpm. The belt on the pulley has tight side and slack side tensions of 1000 N and 500 N, respectively. The ultimate strength and endurance limit of the shaft material is 600 N/mm^2 and 280 N/mm^2, respectively. Find the minimum diameter of the shaft (in mm) with bending consideration only. The stress concentration factor due to key way is 1.85 and a factor of safety 3 based on endurance limit can be assumed.

Direction for questions 30 to 31: A specimen of steel has a fatigue strength of 280 N/mm^2 and an ultimate strength of 600 N/mm^2. The fatigue strength for 10^3 cycles is 0.9 S_u.

30. The fatigue strength for a life of 200×10^3 cycles of stress reversals is nearly

(A) 326 MPa (B) 432 MPa
(C) 492 MPa (D) 512 MPa

31. If the fluctuating reversible stress is 420 MPa, the life of the specimen will be

(A) 21434 cycle (B) 29785 cycles
(C) 32434 cycles (D) 37940 cycles

32. A component made of carbon steel is designed on the strength basis by

(A) ultimate tensile strength
(B) yield strength
(C) modulus of elasticity
(D) modulus of rigidity

33. In the maximum principal stress theory, the shape of the region of safety on a coordinate system whose σ_1 and σ_2 are represented as co-ordinate axes, is

(A) Square (B) Hexagon
(C) Ellipse (D) Circle

34. An element in the critical section of a strained body is subjected to two principal stresses 400 MPa and 150 MPa. If the factor of safety is 3, the maximum permissible stress according to distortion energy theory is

(A) 103.33 MPa (B) 116.67 MPa
(C) 350 MPa (D) 390 MPa

35. A rod of 30 mm diameter and 800 mm length is fixed at one end and the other end is hinged. It is subjected to a compressive load of 40 kN axially. The young's modulus of the rod material is 200 GPa. The factor of safety of the loading is

(A) 4.85 (B) 6.13
(C) 6.48 (D) 6.92

36. A rotating steel shaft supported at both ends, is subjected to a concentrated load at the middle. The maximum bending stress developed is 120 MPa. The corrected endurance strength, yield strength and the ultimate strength of the shaft material are 200 MPa, 300 MPa and 500 MPa, respectively. The factor of safety according to Soderberg criterion is

(A) 1.45 (B) 1.67
(C) 1.86 (D) 1.92

37. The maximum shear stress developed in a shaft due to a purely torsional moment is 150 MPa. The yield and ultimate strengths in tension for the shaft material are 320 and 460 MPa, respectively. Determine the factor of safety as per maximum distortion energy theory.

38. A ductile material has an endurance limit of 200 N/mm^2, with yield strength and ultimate tensile strength of 300 and 330 N/mm^2, respectively. The material is subjected to variable stresses of 150 N/mm^2 maximum and 50 N/mm^2 minimum. If the fatigue stress concentration factor is 1.3, determine the factor of safety of the loading.

39. A shaft is subjected to a completely reversible load. The fatigue strength to sustain 1000 cycles is 500 MPa and the corrected endurance limit is 80 MPa. If the maximum stress is 100 MPa, determine the life of the shaft in number of cycles.

40. A uniform circular shaft is subjected to axial force varying from 15 kN to 90 kN. If the yield strength and endurance limit of the material are 240 MPa and 60 MPa, respectively, determine the minimum diameter of shaft (in mm) for a factor of safety of 2.5 (Apply Soderberg principle).

41. A 12 mm diameter bar is subjected to an axial load of 60 kN in tension. The modulus of elasticity of the material of the bar is 2×10^5 MPa. If Poisson's ratio for the material is 0.3, determine the change in diameter of the bar (in mm).

42. A steel wire of 6 mm diameter is used to lift a load of 2 kN. The length of the hanging wire is 100 m and its unit weight is 7.7 N/m^3. If Young's modulus of the wire material is 2×10^5 N/mm^2, determine the total elongation of the wire.

43. A steel flat 100 mm wide and 12 mm thick is bent into a circular arc of 10 m radius. Determine the value of the applied bending moment (Young's modulus = 2×10^5 MPa).

44. Principal stresses at a point in a strained body consists of a tensile stress $\sigma_1 = 200$ N/mm^2 a compressive stress $\sigma_2 =$ 100 N/mm^2 and $\sigma_3 = 0$. If yield strength of the material is 600 MPa, determine the factor of safety based on maximum shear stress theory.

45. A cylindrical shaft has an outside diameter double the inner diameter. It is subjected to a bending moment of 15000 Nm and a torque of 25000 Nm. If the yield strength in shear for the material is 350 MPa, determine the outside diameter of the shaft for a factor of safety of 2.

46. For an aluminium component under steady load, the recommended theory of failure is

(A) principal stress theory
(B) principal strain theory
(C) Maximum strain energy theory
(D) maximum shear stress theory

47. A machine component is subjected uni axial normal stress σ and shear stress τ. Design equation as per maximum distortion energy theory is

(A) $\sqrt{\sigma^2 + 3\tau^2} = \dfrac{S_y}{N}$ (B) $\sqrt{\sigma^2 + 4\tau^2} = \dfrac{S_y}{N}$

(C) $\dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \dfrac{S_y}{N}$ (D) $\dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2} = \dfrac{S_y}{2N}$

Previous Years' Questions

1. In terms of theoretical stress concentration, factor (K_t) and fatigue stress concentration factor (K_f) the notch sensitivity 'q' is expressed as **[2004]**

(A) $\frac{(K_f - 1)}{(K_t - 1)}$ (B) $\frac{(K_f - 1)}{(K_t + 1)}$

(C) $\frac{(K_f - 1)}{(K_t - 1)}$ (D) $\frac{(K_f + 1)}{(K_t - 1)}$

2. The S-N curve for steel becomes asymptotic nearly at **[2004]**

(A) 10^3 cycles (B) 10^4 cycles

(C) 10^6 cycles (D) 10^9 cycles

3. A cylindrical shaft is subjected to an alternating stress of 100 MPa. Fatigue strength to sustain 1000 cycle is 490 MPa. If the corrected endurance strength is 70 MPa, estimated shaft life will be **[2006]**

(A) 1071 cycles (B) 15000 cycles

(C) 281914 cycles (D) 928643 cycles

4. According to Von-Mises distortion energy theory, the distortion energy under three-dimensional stress state is represented by **[2006]**

(A) $\frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$

(B) $\frac{1-2\mu}{6E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$

(C) $\frac{1-\mu}{3E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$

(D) $\frac{1}{3E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \mu(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$

5. A thin spherical pressure vessel of 200 mm diameter and 1 mm thickness is subjected to an internal pressure varying from 4 to 8 MPa. Assume that the yield, ultimate, and endurance strength of material are 600, 800 and 400 MPa, respectively. The factor of safety as per Goodman's relation is **[2007]**

(A) 2.0 (B) 1.6 (C) 1.4 (D) 1.2

6. The transverse shear stress acting in a beam of rectangular cross section, subjected to a transverse shear load, is **[2008]**

(A) variable with maximum at the bottom of the beam

(B) variable with maximum at the top of the beam

(C) uniform

(D) variable with maximum on the neutral axis

7. An axial residual compressive stress due to manufacturing process is present on the outer surface of a rotating shaft subjected to bending. Under a given bending load, the fatigue life of the shaft in the presence of the residual compressive stress is **[2008]**

(A) decreased

(B) increased or decreased, depending on the external bending load

(C) neither decreased nor increased

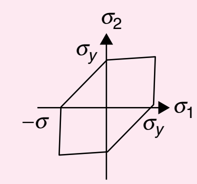
(D) increased

8. A forged steel link with uniform diameter of 30 mm at the centre is subjected to an axial force that varies from 40 kN in compression to 160 kN in tension. The tensile (S_u), yield (S_y) and corrected endurance (S_e) strengths of the steel material are 600 MPa, 420 MPa and 240 MPa, respectively. The factor of safety against fatigue endurance as per Soderberg's criterion is **[2009]**

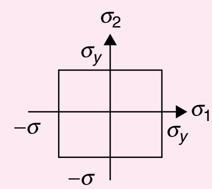
(A) 1.26 (B) 1.37 (C) 1.45 (D) 2.00

9. Match the following criteria of material failure, under biaxial stresses σ_1 and σ_2 and yield stress σ_y, with their corresponding graphic representation: **[2011]**

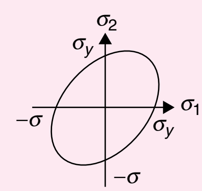
P. Maximum-normal-stress criterion



Q. Maximum-distortion-energy criterion



R. Maximum-shear-stress criterion



(A) P-M, Q-L, R-N (B) P-N, Q-M, R-L

(C) P-M, Q-N, R-L (D) P-N, Q-L, R-M

10. A bar is subjected to a fluctuating tensile load from 20 kN to 100 kN. The material has an yield strength of 240 MPa and the endurance limit in reversed bending is 160 MPa. According to the Soderberg principle, the area of cross section in mm^2 of the bar for a factor of safety of 2 is **[2013]**

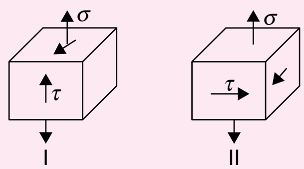
(A) 400 (B) 600 (C) 750 (D) 1000

11. In a simple concentric shaft-bearing arrangement, the lubricant flows in the 2 mm gap between the shaft and the bearing. The flow may be assumed to be a plane Couette flow with zero pressure gradient. The diameter of the shaft is 100 mm and its tangential speed is 10 m/s. The dynamic viscosity of the lubricant is 0.1 kg/m.s. The frictional resisting force (in newton) per 100 mm length of the bearing is ______ **[2014]**

12. A rotating steel shaft is supported at the ends. It is subjected to a point load at the centre. The maximum bending stress developed is 100 MPa. If the yield, ultimate and corrected endurance strength of the shaft material are 300 MPa, 500 MPa and 200 MPa, respectively, then the factor of safety for the shaft is ______. **[2014]**

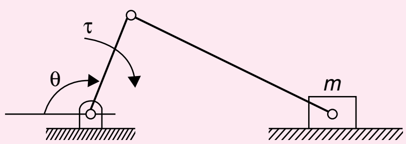
13. A shaft is subjected to pure torsional moment. The maximum shear stress developed in the shaft is 100 MPa. The yield and ultimate strengths of the shaft material in tension are 300 MPa and 450 MPa, respectively. The factor of safety using maximum distortion energy (von-Mises) theory is ______ **[2014]**

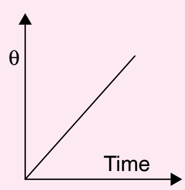
14. Consider the two states of stress as shown in configurations I and II in the figure below. From the standpoint of distortion energy (von-Mises) criterion, which one of the following statements is true? **[2014]**

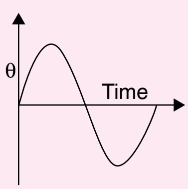


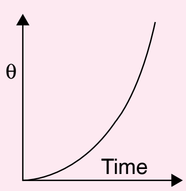
(A) I yields after II
(B) II yields after I
(C) Both yield simultaneously
(D) Nothing can be said about their relative yielding.

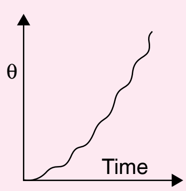
15. Consider a slider crank mechanism with nonzero masses and inertia. A constant torque τ is applied on the crank as shown in the figure. Which of the following plots best resembles variation of crank angle, θ versus time **[2015]**



(A)


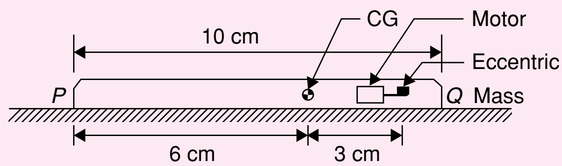
(B)


(C)


(D)


16. Which one of the following is the most conservative fatigue failure criterion? **[2015]**
(A) Soderberg (B) Modified Goodman
(C) ASME Elliptic (D) Gerber

17. A mobile phone has a small motor with an eccentric mass used for vibrator mode. The location of the eccentric mass on motor with respect to center of gravity (CG) of the mobile and the rest of the dimensions of the mobile phone are shown. The mobile is kept on a flat horizontal surface.

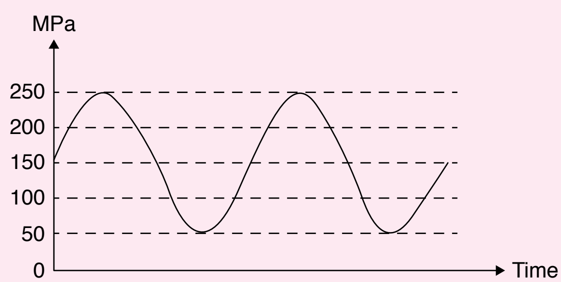


Given in addition that the eccentric mass = 2 grams, eccentricity = 2.19 mm, mass of mobile = 90 grams, $g = 9.81$ m/s^2. Uniform speed of the motor in RPM for which the mobile will get just lifted off the ground at the end Q is approximately **[2015]**
(A) 3000 (B) 3500
(C) 4000 (D) 4500

18. A machine element is subjected to the following bi-axial state of stress: σ_x = 80 MPa; σ_y = 20 MPa; τ_{xy} = 40 MPa. If the shear strength of the material is 100 MPa, the factor of safety as per Tresca's maximum shear stress theory is: **[2015]**
(A) 1.0 (B) 2.0
(C) 2.5 (D) 3.3

19. The uniaxial yield stress of a material is 300 MPa. According to Von Mises criterion, the shear yield stress (in MPa) of the material is ______. **[2015]**

20. For the given fluctuating fatigue load, the values of stress amplitude and stress ratio are respectively: **[2015]**



(A) 100 MPa and 5
(B) 250 MPa and 5
(C) 100 MPa and 0.20
(D) 250 MPa and 0.20

21. The principal stresses at a point inside a solid object are σ_1 = 100 MPa, σ_2 = 100 MPa and σ_3 = 0 MPa. The yield strength of the material is 200 MPa. The factor of safety calculated using Tresca (maximum shear stress) theory is n_T and the factor of safety calculated using von Mises (maximum distortional energy) theory is n_V. Which one of the following relations is TRUE? **[2016]**

(A) $n_T = (\sqrt{3}/2)\,n_V$ (B) $n_T = (\sqrt{3})\,n_V$

(C) $n_T = n_V$ (D) $n_V = (\sqrt{3})\,n_T$

22. In a structural member under fatigue loading, the minimum and maximum stresses developed at the critical point are 50 MPa and 150 MPa, respectively. The endurance, yield, and the ultimate strengths of the material are 200 MPa, 300 MPa and 400 MPa, respectively. The factor of safety using modified Goodman criterion is: **[2016]**

(A) $\frac{3}{2}$ (B) $\frac{8}{5}$

(C) $\frac{12}{7}$ (D) 2

Answer Keys

Exercises

Practice Problems 1

1. B	**2.** A	**3.** A	**4.** C	**5.** D	**6.** A	**7.** B	**8.** C	**9.** B	**10.** B
11. C	**12.** D	**13.** B	**14.** A	**15.** C	**16.** D	**17.** A	**18.** B	**19.** C	**20.** B
21. 12 mm		**22.** 69.4 Nm		**23.** D	**24.** 1.778	**25.** A	**26.** B	**27.** A	**28.** B
29. C	**30.** C								

Practice Problems 2

1. A	**2.** C	**3.** B	**4.** C	**5.** A	**6.** B	**7.** C	**8.** D	**9.** A	**10.** D
11. C	**12.** C	**13.** D	**14.** D	**15.** D	**16.** A	**17.** B	**18.** B	**19.** B	**20.** B
21. D	**22.** A	**23.** A	**24.** B	**25.** C	**26.** D	**27.** A	**28.** B	**29.** 32 mm	
30. A	**31.** B	**32.** A	**33.** A	**34.** B	**35.** B	**36.** B	**37.** 1.23	**38.** 1.519	
39. 430900		**40.** 38 mm		**41.** 9.55×10^{-3}		**42.** 37.3 mm		**43.** 288 Nm	
44. 2	**45.** 96.7 mm		**46.** D	**47.** A					

Previous Years' Questions

1. A	**2.** C	**3.** C	**4.** C	**5.** B	**6.** D	**7.** D	**8.** A	**9.** C	**10.** D
11. 15 to 16		**12.** 1.9 to 2.1		**13.** 1.7 to 1.8		**14.** C	**15.** D	**16.** A	**17.** B
18. B	**19.** 171 to 175		**20.** C	**21.** C	**22.** D				