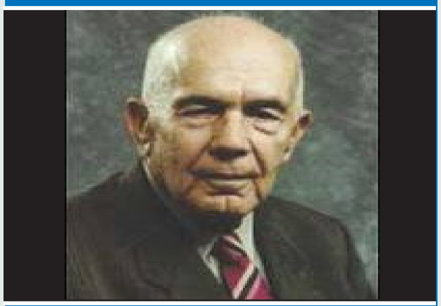
10 Operations Research



Frank Lauren Hitchcock
(1875-1957)

Introduction

Operations reserach (O.R.) is an analytical method of problem solving and decision-making, that is useful in management organisations. The transportation problem involves certain origins (sources) which may represent factories where we produce homogeneous items and a number of destinations where we supply a required quantity of the products. Each factory has a certain capacity constraint and each destination (dealer or customer) has a certain requirement. The unit cost of transportation of the items from the factory to the dealer/ customer is known. American mathematician and physicist Frank Lauren Hitchcock (1875-1957) known for his formulation of transportation problem in 1941.

Learning Objectives

After studying this chapter students are able to understand

- formulate the transportation and assignment problems
- distinguish between transportation and assignment problems
- find an initial basic feasible solution of a transportation problem
- identify the degeneracy and non-degeneracy in a transportation problem
- find the solution of an assignment problem by Hungarian method.
- distinguish between tactic and strategic decisions
- find the best alternatives using maximin and minimax criteria

10.1 Transportation Problem

The objective of transportation problem is to determine the amount to be transported from each origin to each destinations such that the total transportation cost is minimized.

GYPQK

10.1.1 Definition and formulation

The Structure of the Problem

Let there be m origins and n destinations. Let the amount of supply at the i^{th} origin is a_i. Let the demand at j^{th} destination is b_j.

The cost of transporting one unit of an item from origin i to destination j is c_{ij} and is known for all combinations (i,j). Quantity transported from origin i to destination j be x_{ij}

The objective is to determine the quantity x_{ij} to be transported over all routes (i,j) so as to

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minimize the total transportation cost. The supply limits at the origins and the demand requirements at the destinations must be satisfied.

The above transportation problem can be written in the following tabular form:

		Destinations					
		1	2	3	…	n	supply
Origins	1	(x_{11}) C_{11}	(x_{12}) C_{12}	(x_{13}) C_{13}	…	(x_{1n}) C_{1n}	a_1
	2	(x_{21}) C_{21}	(x_{22}) C_{22}	(x_{23}) C_{23}	…	(x_{2n}) C_{2n}	a_2
	⋮		⋮	⋮	⋮	⋮	⋮
	m	(x_{m1}) C_{m1}	(x_{m2}) C_{m2}	(x_{m3}) C_{m3}	…	(x_{mn}) C_{mn}	a_m
	demand	b_1	b_2	b_3	…	b_n	

Table-10.1

Now the linear programming model representing the transportation problem is given by

The objective function is Minimize Z = $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ Subject to the constraints

$\sum_{j=1}^{n} x_{ij}=a_i$, i=1,2,…….m (supply constraints)

$\sum_{i=1}^{m} x_{ij}=b_j$, j=1,2,…….$n$ (demand constraints)

$x_{ij} \geq 0$ for all i,j. (non-negative restrictions)

Some Definitions

Feasible Solution: A feasible solution to a transportation problem is a set of non-negative values x_{ij}(i=1,2,..,m, j=1,2,…n) that satisfies the constraints.

Basic Feasible Solution: A feasible solution is called a basic feasible solution if it contains not more than $m+n-1$ allocations, where m is the number of rows and n is the number of columns in a transportation problem.

Optimal Solution: Optimal Solution is a feasible solution (not necessarily basic) which optimizes(minimize) the total transportation cost.

Non degenerate basic feasible Solution: If a basic feasible solution to a transportation problem contains exactly $m+n-1$ allocations in independent positions, it is called a Non degenerate basic feasible solution. Here m is the number of rows and n is the number of columns in a transportation problem.

Degeneracy : If a basic feasible solution to a transportation problem contains less than $m+n-1$ allocations, it is called a degenerate

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basic feasible solution. Here m is the number of rows and n is the number of columns in a transportation problem.

10.1.2 Methods of finding initial Basic Feasible Solutions

There are several methods available to obtain an initial basic feasible solution of a transportation problem. We discuss here only the following three. For finding the initial basic feasible solution total supply must be equal to total demand.

$$\text{(i.e)} \quad \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Method:1North-West Corner Rule (NWC)

It is a simple method to obtain an initial basic feasible solution. Various steps involved in this method are summarized below.

Step 1: Choose the cell in the north-west corner of the transportation Table10.1 and allocate as much as possible in this cell so that either the capacity of first row (supply)is exhausted or the destination requirement of the first column(demand) is exhausted.(i.e) $x_{11} = \min(a_1,b_1)$

Step 2: If the demand is exhausted ($b_1 < a_1$), move one cell right horizontally tothe second column and allocate as much as possible.(i.e) $x_{12}= \min (a_1 - x_{11}, b_2)$

If the supply is exhausted ($b_1 > a_1$), move one cell down vertically to the second row and allocateas much as possible.(i.e)$x_{21} = \min (a_2,b_1 - x_{11})$

If both supply and demand are exhausted move one cell diagonally and allocate as much as possible.

Step 3: Continue the above procedure until all the allocations are made

Example 10.1

Obtain the initial solution for the following problem.

		Destination A	B	C	Supply
Sources	1	2	7	4	5
	2	3	3	1	8
	3	5	4	7	7
	4	1	6	2	14
Demand		7	9	18	

Solution:

Here total supply = 5+8+7+14=34,
Total demand = 7+9+18=34

(i.e) Total supply =Total demand. Therefore the given problem is balanced transportation problem.

∴ we can find an initial basic feasible solution to the given problem.

From the above table we can choose the cell in the North West Corner. Here the cell is (1,A)

Allocate as much as possible in this cell so that either the capacity of first row is exhausted or the destination requirement of the first column is exhausted.

i.e. $x_{11} = \min (5,7) = 5$

	A	B	C	Supply (a_i)
1	(5) 2	7	4	5/0
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand (b_j)	7/2	9	18	

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Reduced transportation table is

	A	B	C	a_i
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
b_j	2	9	18	

Now the cell in the North west corner is (2, A).

Allocate as much as possible in the first cell so that either the capacity of second row is exhausted or the destination requirement of the first column is exhausted.

i.e. x_{12} = min (2,8) = 2

	A	B	C	a_i
2	(2) 3	3	1	8/6
3	5	4	7	7
4	1	6	2	14
b_j	2/0	9	18	

Reduced transportation table is

	B	C	a_i
2	3	1	6
3	4	7	7
4	6	2	14
b_j	9	18	

Here north west corner cell is (2,B) Allocate as much as possible in the first cell so that either the capacity of second row is exhausted or the destination requirement of the second column is exhausted.

i.e. x_{22} = min (6,9) = 6

	B	C	a_i
2	(6) 3	1	6/0
3	4	7	7
4	6	2	14
b_j	9/3	18	

Reduced transportation table is

	B	C	a_i
3	4	7	7
4	6	2	14
b_j	3	18	

Here north west corner cell is (3,B).

Allocate as much as possible in the first cell so that either the capacity of third row is exhausted or the destination requirement of the second column is exhausted.

i.e. x_{32} = min (7,3) = 3

	B	C	a_i
3	(3) 4	7	7/4
4	6	2	14
b_j	3/0	18	

Reduced transportation table is

	C	a_i
3	7	4
4	2	14
b_j	18	

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Here north west corner cell is (3,C) Allocate as much as possible in the first cell so that either the capacity of third row is exhausted or the destination requirement of the third column is exhausted.

i.e. x_{33} = min (4,18) = 4

	C	a_i
3	(4) 7	4/0
4	2	14
b_j	18/14	

Reduced transportation table and final allocation is x_{44} = 14

	C	a_i
4	(14) 2	14/0
b_j	14/0	

Thus we have the following allocations

	A	B	C	a_i
1	(5) 2	7	4	5
2	(2) 3	(6) 3	1	8
3	5	(3) 4	(4) 7	7
4	1	6	(14) 2	14
b_j	7	9	18	

Transportation schedule : 1→ A, 2→A, 2→B, 3→B, 3→C, 4→C

The total transportation cost.

$= (5\times2) + (2\times3) + (6\times3) + (3\times4) + (4\times7) + (14\times2)$

= ₹ 102

Example 10.2

Determine an initial basic feasible solution to the following transportation problem using North West corner rule.

	D_1	D_2	D_3	D_4	Availability
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Requirement	6	10	15	4	35

Here O_i and D_j represent i^{th} origin and j^{th} destination.

Solution :

Given transportation table is

	D_1	D_2	D_3	D_4	Availability (a_i)
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Requirement (b_j)	6	10	15	4	35

Total Availability = Total Requirement. Therefore the given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

First allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	4	1	5	14/8
O_2	8	9	2	7	16
O_3	4	3	6	2	5
b_j	6/0	10	15	4	35

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Second allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	9	2	7	16
O_3	4	3	6	2	5
b_j	6/0	10/2	15	4	35

Third Allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	(2) 9	2	7	16/14
O_3	4	3	6	2	5
b_j	6/0	10/2/0	15	4	35

Fourth Allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	(2) 9	(14) 2	7	16/14/0
O_3	4	3	6	2	5
b_j	6/0	10/2/0	15/1	4	35

Fifth allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	(2) 9	(14) 2	7	16/14/0
O_3	4	3	(1) 6	2	5/4
b_j	6/0	10/2/0	15/1/0	4	35

Final allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	(2) 9	(14) 2	7	16/14/0
O_3	4	3	(1) 6	(4) 2	5/4/0
b_j	6/0	10/2/0	15/1/0	4/0	35

Transportation schedule : $O_1 \rightarrow D_1$, $O_1 \rightarrow D_2$, $O_2 \rightarrow D_2$, $O_2 \rightarrow D_3$, $O_3 \rightarrow D_3$, $O_3 \rightarrow D_3$.

$$= (6\times6)+(8\times4)+(2\times9)+(14\times2)$$
$$+(1\times6)+(4\times2) = ₹128$$

Method:2 Least Cost Method (LCM)

The least cost method is more economical than north-west corner rule,since it starts with a lower beginning cost. Various steps involved in this method are summarized as under.

Step 1: Find the cell with the least(minimum) cost in the transportation table.

Step 2: Allocate the maximum feasible quantity to this cell.

Step:3 Eliminate the row or column where an allocation is made.

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Step:4 Repeat the above steps for the reduced transportation table until all the allocations are made.

Note

If the minimum cost is not unique then the choice can be made arbitrarily.

Example 10.3

Obtain an initial basic feasible solution to the following transportation problem using least cost method.

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	3	4	6
O_2	4	3	2	5	8
O_3	5	2	2	1	10
Demand	4	6	8	6	

Here O_i and D_j denote i^{th} origin and j^{th} destination respectively.

Solution:

Total Supply = Total Demand = 24

∴ The given problem is a balanced transportation problem.

Hence there exists a feasible solution to the given problem.

Given Transportation Problem is:

	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	1	2	3	4	6
O_2	4	3	2	5	8
O_3	5	2	2	1	10
Demand (b_j)	4	6	8	6	

The least cost is 1 corresponds to the cells (O_1, D_1) and (O_3, D_4)

Take the Cell (O_1, D_1) arbitrarily.

Allocate min (6,4) = 4 units to this cell.

	D_1	D_2	D_3	D_4	a_i
O_1	(4) 1	2	3	4	6/2
O_2	4	3	2	5	8
O_3	5	2	2	1	10
b_j	4/0	6	8	6	

The reduced table is

	D_2	D_3	D_4	a_i
O_1	2	3	4	2
O_2	3	2	5	8
O_3	2	2	1	10
b_j	6	8	6	

The least cost corresponds to the cell (O_3, D_4). Allocate min (10,6) = 6 units to this cell.

	D_2	D_3	D_4	a_i
O_1	2	3	4	2
O_2	3	2	5	8
O_3	2	2	(6) 1	10/4
b_j	6	8	6/0	

The reduced table is

	D_2	D_3	a_i
O_1	2	3	2
O_2	3	2	8
O_3	2	2	4
b_j	6	8	

The least costis 2 corresponds to the cells (O_1, D_2), (O_2, D_3), (O_3, D_2), (O_3, D_3)

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Allocate min (2,6) = 2 units to this cell.

	D_2	D_3	a_i
O_1	(2) 2	3	2/0
O_2	3	2	8
O_3	2	2	4
b_j	6/4	8	

The reduced table is

	D_2	D_3	a_i
O_2	3	2	8
O_3	2	2	4
bj	4	8	

The least cost is 2 corresponds to the cells (O_2, D_3), (O_3, D_2), (O_3, D_3)

Allocate min (8,8) = 8 units to this cell.

	D_2	D_3	a_i
O_2	3	(8) 2	8/0
O_3	2	2	4
b_j	4	8/0	

The reduced table is

	D_2	a_i
O_3	2	4
b_j	4	

Here allocate 4 units in the cell (O_3, D_2)

	D_2	a_i
O_3	(4) 2	4/0
b_j	4/0	

Thus we have the following allocations:

	D_1	D_2	D_3	D_4	a_i
O_1	(4) 1	(2) 2	3	4	6/2/0
O_2	4	3	(8) 2	5	8/0
O_3	5	(4) 2	2	(6) 1	10/4/0
b_j	4/0	6/4/0	8/0	6/0	

Transportation schedule :

$O_1 \rightarrow D_1$, $O_1 \rightarrow D_2$, $O_2 \rightarrow D_3$, $O_3 \rightarrow D_2$, $O_3 \rightarrow D_4$

Total transportation cost

$= (4\times1)+(2\times2)+(8\times2)+(4\times2)+(6\times1)$

$= 4+4+16+8+6$

= ₹ 38.

Example 10.4

Determine how much quantity should be stepped from factory to various destinations for the following transportation problem using the least cost method.

		Destination				
		C	*H*	*K*	*P*	Capacity
Factory	*T*	6	8	8	5	30
	B	5	11	9	7	40
	M	8	9	7	13	50
	Demand	35	28	32	25	

Cost are expressed in terms of rupees per unit shipped.

Solution:

Total Capacity = Total Demand

∴ The given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

Given Transportation Problem is

		Destination				
		C	H	K	P	Capacity (a_i)
Factory	T	6	8	8	5	30
	B	5	11	9	7	40
	M	8	9	7	13	50
	Demand (b_j)	35	28	32	25	

First Allocation:

	C	H	K	P	a_i
T	6	8	8	(25) 5	30/5
B	5	11	9	7	40
M	8	9	7	13	50
b_j	35	28	32	25/0	

Second Allocation:

	C	H	K	P	a_i
T	6	8	8	(25) 5	30/5
B	(35) 5	11	9	7	40/5
M	8	9	7	13	50
b_j	35/0	28	32	25/0	

Third Allocation:

	C	H	K	P	a_i
T	6	8	8	(25) 5	30/5
B	(35) 5	11	9	7	40/5
M	8	9	(32) 7	13	50/18
b_j	35/0	28	32/0	25/0	

Fourth Allocation:

	C	H	K	P	a_i
T	6	(5) 8	8	(25) 5	30/5/0
B	(35) 5	11	9	7	40/5
M	8	9	(32) 7	13	50/18
b_j	35/0	28/23	32/0	25/0	

Fifth Allocation:

	C	H	K	P	a_i
T	6	(5) 8	8	(25) 5	30/5/0
B	(35) 5	11	9	7	40/5
M	8	(18) 9	(32) 7	13	50/18/0
b_j	35/0	28/23/5	32/0	25/0	

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Sixth Allocation:

	C	H	K	P	a_i
T	6	(5) 8	8	(25) 5	30/5/0
B	(35) 5	(5) 11	9	7	40/5/0
M	8	(18) 9	(32) 7	13	50/18/0
b_j	35/0	28/23/5/0	32/0	25/0	

Transportation schedule :

T→ *H*, *T*→*P*, *B*→*C*, *B*→*H*, M→*H*, M→*K*

The total Transportation cost = (5×8) + (25×5)+ (35×5) + (5×11)+ (18×9) + (32×7)

= 40+125+175+55+162+224

= ₹ 781

Method: 3 Vogel's Approximation Method (VAM)

Vogel's approximation method yields an initial basic feasible solution which is very close to the optimum solution. Various steps involved in this method are summarized as under

Step 1: Calculate the penalties for each row and each column. Here penalty means the difference between the least and the next higher cost in a row and in a column.

Step 2: Select the row or column with the largest penalty.

Step 3: In the selected row or column, allocate the maximum feasible quantity to the cell with the minimum cost.

Step 4: Eliminate the row or column where all the allocations are made.

Step 5: Write the reduced transportation table and repeat the steps 1 to 4.

Step 6: Repeat the procedure until all the allocations are made.

DO YOU KNOW?

Optimum solution for any transportation problem can be obtained by using MODI method.

Example 10.5

Find the initial basic feasible solution for the following transportation problem by VAM

		Distribution Centers				Availability
		D_1	D_2	D_3	D_4	
origin	S_1	11	13	17	14	250
	S_2	16	18	14	10	300
	S_3	21	24	13	10	400
	Requirement	200	225	275	250	

Solution:

Here $\sum a_i = \sum b_j = 950$

(i.e) Total Availability = Total Requirement

∴ The given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

First let us find the difference (penalty) between the first two smallest costs in each row and column and write them in brackets against the respective rows and columns.

	D_1	D_2	D_3	D_4	a_i	Penalty
S_1	(200) 11	13	17	14	250/50	2
S_2	16	18	14	10	300	4
S_3	21	24	13	10	400	3
b_j	200/0	225	275	250		
Penalty	(5)	5	1	4		

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Choose the largest difference. Here the difference is 5 which corresponds to column D_1 and D_2. Choose either D_1 or D_2 arbitrarily. Here we take the column D_1. In this column choose the least cost. Here the least cost corresponds to (S_1, D_1). Allocate min (250, 200) = 200units to this Cell.

The reduced transportation table is

	D_2	D_3	D_4	a_i	Penalty
S_1	(50) 13	17	14	50/0	1
S_2	18	14	10	300	4
S_3	24	13	10	400	3
b_j	225/175	275	250		
Penalty	(5)	1	4		

Choose the largest difference. Here the difference is 5 whichcorresponds to column D_2. In this column choose the least cost. Here the least cost corresponds to (S_1, D_2). Allocate $\min(50,175) = 50$ units to this Cell.

The reduced transportation table is

	D_2	D_3	D_4	a_i	Penalty
S_2	(175) 18	14	10	300/125	4
S_3	24	13	10	400	3
b_j	175/0	275	250		
Penalty	(6)	1	—		

Choose the largest difference. Here the difference is 6 which corresponds to column D_2. In this column choose the least cost. Here the least cost corresponds to (S_2, D_2). Allocate $\min(300,175) = 175$ units to this cell.

The reduced transportation table is

	D_3	D_4	a_i	Penalty
S_2	14	(125) 10	125/0	(4)
S_3	13	10	400	3
b_j	275	250/125		
Penalty	1	—		

Choose the largest difference. Here the difference is 4 corresponds to row S_2. In this row choose the least cost. Here the least cost corresponds to (S_2, D_4). Allocate $\min(125,250) = 125$ units to this Cell.

The reduced transportation table is

	D_3	D_4	a_i	Penalty
S_3	13	10	400	(3)
b_j	275	125		
Penalty	—	—		

The Allocation is

	D_3	D_4	a_i
S_3	(275) 13	(125) 10	400/275/0
b_j	275/0	125/0	

Thus we have the following allocations:

	D_1	D_2	D_3	D_4	a_i
S_1	(200) 11	(50) 13	17	14	250
S_2	16	(175) 18	14	(125) 10	300
S_3	21	24	(275) 13	(125) 10	400
b_j	200	225	275	250	

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Transportation schedule :

$S_1 \rightarrow D_1, S_1 \rightarrow D_2, S_2 \rightarrow D_2, S_2 \rightarrow D_4, S_3 \rightarrow D_3, S_3 \rightarrow D_4$

This initial transportation cost

$= (200 \times 11) + (50 \times 13) + (175 \times 18) +$

$(125 \times 10) + (275 \times 13) + (125 \times 10)$

= ₹ 12,075

Example 10.6

Obtain an initial basic feasible solution to the following transportation problem using Vogel's approximation method.

Ware houses	Stores				
	I	II	III	IV	Availability (a_i)
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	1	4	5	19
Requirement (b_j)	21	25	17	17	

Solution:

Here $\sum a_i = \sum b_j = 80$

(i.e) Total Availability =Total Requirement

∴ The given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

First Allocation:

	I	II	III	IV	a_i	Penalty
A	5	1	3	3	34	2
B	3	3	5	4	15	1
C	6	4	4	3	12	1
D	4	(19) 1	4	5	19/0	(3)
b_j	21	25/6	17	17		
Penalty	1	2	1	1		

Second Allocation:

	I	II	III	IV	a_i	Penalty
A	5	(6) 1	3	3	34/28	(2)
B	3	3	5	4	15	1
C	6	4	4	3	12	1
b_j	21	6/0	17	17		
Penalty	2	2	1	1		

Third Allocation:

	I	III	IV	a_i	Penalty
A	5	(17) 3	3	28/11	(2)
B	3	5	4	15	1
C	6	4	3	12	1
b_j	21	17/0	17		
Penalty	2	1	1		

Fourth Allocation:

	I	IV	a_i	Penalty
A	5	3	11	2
B	3	4	15	1
C	6	(12) 3	12/0	(3)
b_j	21	17/5		
Penalty	2	1		

Fifth Allocation:

	I	IV	a_i	Penalty
A	5	(5) 3	11/6	(2)
B	3	4	15	1
b_j	21	5/0		
Penalty	2	1		

Sixth Allocation:

	I		Penalty
A	(6) 5	6/0	—
B	(15) 3	15/0	—
	21/0		
Penalty	(2)		

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Thus we have the following allocations:

	I	II	III	IV	a_i
A	(6) 5	(6) 1	(17) 3	(5) 3	34
B	(15) 3	3	5	4	15
C	6	4	4	(12) 3	12
D	4	(19) 1	4	5	19
b_j	21	25	17	17	

Transportation schedule :

A→ I, *A*→*II*, *A*→*III*, *A*→*IV*, *B*→*I*, *C*→*IV*, *D*→*II*

$= (6\times5)+(6+1)+(17\times3)+(5\times3)$

$(15\times3)+(12\times3)+(19\times1)$

$= 30+6+51+15+45+36+19$

$=$ ₹ 202

Exercise 10.1

1. What is transportation problem?
2. Write mathematical form of transportation problem.
3. what is feasible solution and non degenerate solution in transportation problem?
4. What do you mean by balanced transportation problem?
5. Find an initial basic feasible solution of the following problem using north west corner rule.

	D_1	D_2	D_3	D_4	Supply
O_1	5	3	6	2	19
O_2	4	7	9	1	37
O_3	3	4	7	5	34
Demand	16	18	31	25	

6. Determine an initial basic feasible solution of the following transportation problem by north west corner method.

	Bangalore	Nasik	Bhopal	Delhi	Capacity
Chennai	6	8	8	5	30
Madurai	5	11	9	7	40
Trichy	8	9	7	13	50
Demand (Units/day	35	28	32	25	

7. Obtain an initial basic feasible solution to the following transportation problem by using least- cost method.

	D_1	D_2	D_3	Supply
O_1	9	8	5	25
O_2	6	8	4	35
O_3	7	6	9	40
demand	30	25	45	

8. Explain Vogel's approximation method by obtaining initial feasible solution of the following transportation problem.

	D_1	D_2	D_3	D_4	Supply
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
Demand	7	5	3	2	

9. Consider the following transportation problem.

	D_1	D_2	D_3	D_4	Availability
O_1	5	8	3	6	30
O_2	4	5	7	4	50
O_3	6	2	4	6	20
Requirement	30	40	20	10	

Determine initial basic feasible solution by VAM.

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10. Determine basic feasible solution to the following transportation problem using North west Corner rule.

		Sinks					Supply
		A	*B*	*C*	*D*	*E*	
Origins	*P*	2	11	10	3	7	4
	Q	1	4	7	2	1	8
	R	3	9	4	8	12	9
Demand		3	3	4	5	6	

11. Find the initial basic feasible solution of the following transportation problem:

	I	II	III	Demand
A	1	2	6	7
B	0	4	2	12
C	3	1	5	11
Supply	10	10	10	

Using (i) North West Corner rule

(ii) Least Cost method

(iii) Vogel's approximation method

12. Obtain an initial basic feasible solution to the following transportation problem by north west corner method.

	D	*E*	*F*	*C*	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Required	200	225	275	250	

10.2 Assignment Problems

Introduction:

The assignment problem is a particular case of transportation problem for which more efficient (less-time consuming) solution method has been devised by KUHN (1956) and FLOOD (1956). The justification of the steps leading to the solution is based on theorems proved by Hungarian Mathematicians KONEIG (1950) and EGERVARY (1953), hence the method is named Hungarian Method.

Suppose that we have 'm' jobs to be performed on 'n' machines. The cost of assigning each job to each machine is C_{ij} $(i = 1,2,\ldots,n$ and $j = 1,2,\ldots,n)$. Our objective is to assign the different jobs to the different machines (one job per machine) to minimize the overall cost. This is known as **assignment problem.**

The assignment problem is a special case of transportation problem where the number of sources and destinations are equal. Supply at each source and demand at each destination must be one. It means that there is exactly one occupied cell in each row and each column of the transportation table. Jobs represent sources and machines represent destinations.

10.2.1 Definition and formulation

Consider the problem of assigning n jobs to n machines (one job to one machine). Let C_{ij} be the cost of assigning i^{th} job to the j^{th} machine and x_{ij} represents the assignment of i^{th} job to the j^{th} machine.

Then, $x_{ij} = \begin{cases} 1, \text{ if } i^{th} \text{ job is assigned to } j^{th} \text{ machine} \\ 0, \text{ if } i^{th} \text{ job is not assigned to } j^{th} \text{ machine.} \end{cases}$

		Machines				
		1	2	…	n	Supply
Jobs	1	(x_{11}) C_{11}	(x_{12}) C_{12}	…	(x_{1n}) C_{1n}	1
	2	(x_{21}) C_{21}	(x_{22}) C_{22}	…	(x_{2n}) C_{2n}	1
	⋮		⋮	⋮	⋮	⋮
	m	(x_{ij}) C_{n1}	(x_{ij}) C_{n2}	…	(x_{ij}) C_{nn}	1
	demand	1	1	…	1	

x_{ij} is missing in any cell means that no assignment is made between the pair of job and machine. $(i.e)$ $x_{ij} = 0$.

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x_{ij} presents in any cell means that an assignment is made their.In such cases $x_{ij} = 1$.

The assignment model can be written in LPP as follows:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}\, x_{ij}$$

Subject to the constrains

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1, 2, ...n$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1, 2, ...n \quad \text{and } x_{ij} = 0$$

(or) 1 for all i,j

Note

The optimum assignment schedule remains unaltered if we add or subtract a constant from all the elements of the row or column of the assignment cost matrix.

Note

If for an assignment problem all $C_{ij} > 0$ then an assignment schedule (x_{ij}) which satisfies $\sum C_{ij}\, x_{ij} = 0$ must be optimal.

10.2.2 Solution of assignment problems (Hungarian Method)

First check whether the number of rows is equal to the numbers of columns, if it is so, the assignment problem is said to be balanced.

Step :1 Choose the least element in each row and subtract it from all the elements of that row.

Step :2 Choose the least element in each column and subtract it from all the elements of that column. Step 2 has to be performed from the table obtained in step 1.

Step:3 Check whether there is atleast one zero in each row and each column and make an assignment as follows.

(i) Examine the rows successively until a row with exactly one zero is found. Mark that zero by □, that means an assignment is made there . Cross $(\times)$ all other zeros in its column. Continue this until all the rows have been examined.

(ii) Examine the columns successively until a columns with exactly one zero is found. Mark that zero by □, that means an assignment is made there . Cross $(\times)$ all other zeros in its row. Continue this until all the columns have been examined.

Step :4 If each row and each column contains exactly one assignment, then the solution is optimal.

DO YOU KNOW?

Hungarian method provides optimum assignment schedule in an assignment problem.

Example 10.7

Solve the following assignment problem. Cell values represent cost of assigning job A, B, C and D to the machines I, II, III and IV.

machines

jobs		I	II	III	IV
	A	10	12	19	11
	B	5	10	7	8
	C	12	14	13	11
	D	8	15	11	9

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Solution:

Here the number of rows and columns are equal.

∴ The given assignment problem is balanced.

Now let us find the solution.

Step 1: Select a smallest element in each row and subtract this from all the elements in its row.

	I	II	III	IV
A	0	2	9	1
B	0	5	2	3
C	1	3	2	0
D	0	7	3	1

Look for atleast one zero in each row and each column.Otherwise go to step 2.

Step 2: Select the smallest element in each column and subtract this from all the elements in its column.

	I	II	III	IV
A	0	0	7	1
B	0	3	0	3
C	1	1	0	0
D	0	5	1	1

Since each row and column contains atleast one zero, assignments can be made.

Step 3 (Assignment):

Examine the rows with exactly one zero. First three rows contain more than one zero. Go to row *D*. There is exactly one zero. Mark that zero by □ (i.e) job *D* is assigned to machine I. Mark other zeros in its column by ×.

	I	II	III	IV
A	~~0~~	0	7	1
B	~~0~~	3	0	3
C	1	1	0	0
D	[0]	5	1	2

Step 4: Now examine the columns with exactly one zero. Already there is an assignment in column I. Go to the column II. There is exactly one zero. Mark that zero by □. Mark other zeros in its row by ×.

	I	II	III	IV
A	~~0~~	[0]	7	1
B	~~0~~	3	0	3
C	1	1	0	0
D	[0]	5	1	2

Column III contains more than one zero. Therefore proceed to Column IV, there is exactly one zero. Mark that zero by □. Mark other zeros in its row by ×.

	I	II	III	IV
A	~~0~~	[0]	7	1
B	~~0~~	3	0	3
C	1	1	~~0~~	[0]
D	[0]	5	1	2

Step 5: Again examine the rows. Row B contains exactly one zero. Mark that zero by □.

	I	II	III	IV
A	~~0~~	[0]	7	1
B	~~0~~	3	[0]	3
C	1	1	~~0~~	[0]
D	[0]	5	1	2

Thus all the four assignments have been made. The optimal assignment schedule and total cost is

Job	Machine	cost
A	II	12
B	III	7
C	IV	11
D	I	8
Total cost		38

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The optimal assignment (minimum) cost

= ₹ 38

Example 10.8

Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows. Determine the optimum assignment schedule.

Job

Person		1	2	3	4	5
	A	8	4	2	6	1
	B	0	9	5	5	4
	C	3	8	9	2	6
	D	4	3	1	0	3
	E	9	5	8	9	5

Solution:

Here the number of rows and columns are equal.

∴ The given assignment problem is balanced.

Now let us find the solution.

Step 1: Select a smallest element in each row and subtract this from all the elements in its row.

The cost matrix of the given assignment problem is

Job

Person		1	2	3	4	5
	A	7	3	1	5	0
	B	0	9	5	5	4
	C	1	6	7	0	4
	D	4	3	1	0	3
	E	4	0	3	4	0

Column 3 contains no zero. Go to Step 2.

Step 2: Select the smallest element in each column and subtract this from all the elements in its column.

Job

Person		1	2	3	4	5
	A	7	3	0	5	0
	B	0	9	4	5	4
	C	1	6	6	0	4
	D	4	3	0	0	3
	E	4	0	2	4	0

Since each row and column contains atleast one zero, assignments can be made.

Step 3 (Assignment):

Examine the rows with exactly one zero. Row B contains exactly one zero. Mark that zero by □ (i.e) PersonB is assigned to Job 1. Mark other zeros in its column by × .

Job

Person		1	2	3	4	5
	A	7	3	0	5	0
	B	[0]	9	4	5	4
	C	1	6	6	0	4
	D	4	3	0	0	3
	E	4	0	2	4	0

Now, Row C contains exactly one zero. Mark that zero by □. Mark other zeros in its column by ×.

Job

Person		1	2	3	4	5
	A	7	3	0	5	0
	B	[0]	9	4	5	4
	C	1	6	6	[0]	4
	D	4	3	0	~~0~~	3
	E	4	0	2	4	0

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Now, Row D contains exactly one zero. Mark that zero by □. Mark other zeros in its column by ×.

Job

Person		1	2	3	4	5
	A	7	3	~~0~~	5	0
	B	[0]	9	4	5	4
	C	1	6	6	[0]	4
	D	4	3	[0]	~~0~~	3
	E	4	0	2	4	0

Row E contains more than one zero, now proceed column wise. In column 1, there is an assignment. Go to column 2. There is exactly one zero. Mark that zero by □. Mark other zeros in its row by ×.

Job

Person		1	2	3	4	5
	A	7	3	~~0~~	5	0
	B	[0]	9	4	5	4
	C	1	6	6	[0]	4
	D	4	3	[0]	~~0~~	3
	E	4	[0]	2	4	~~0~~

There is an assignment in Column 3 and column 4. Go to Column 5. There is exactly one zero. Mark that zero by □. Mark other zeros in its row by ×.

Job

Person		1	2	3	4	5
	A	7	3	~~0~~	5	[0]
	B	[0]	9	4	5	4
	C	1	6	6	[0]	4
	D	4	3	[0]	~~0~~	3
	E	4	[0]	2	4	~~0~~

Thus all the five assignments have been made. The Optimal assignment schedule and total cost is

Person	**Job**	**cost**
A	5	1
B	1	0
C	4	2
D	3	1
E	2	5
Total cost		9

The optimal assignment (minimum) cost = ₹ 9.

Example 10.9

Solve the following assignment problem.

Men

Task		1	2	3
	P	9	26	15
	Q	13	27	6
	R	35	20	15
	S	18	30	20

Solution:

Since the number of columns is less than the number of rows, given assignment problem is unbalanced one. To balance it, introduce a dummy column with all the entries zero. The revised assignment problem is

Men

Task		1	2	3	d
	P	9	26	15	0
	Q	13	27	6	0
	R	35	20	15	0
	S	18	30	20	0

Here only 3 tasks can be assigned to 3 men.

Step 1: is not necessary, since each row contains zero entry. Go to Step 2.

Step 2 :

Men

Task		1	2	3	d
	P	0	6	9	0
	Q	4	7	0	0
	R	26	0	9	0
	S	9	10	14	0

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Step 3 (Assignment) :

Task \ Men	1	2	3	d
P	[0]	6	9	~~0~~
Q	4	7	[0]	~~0~~
R	26	[0]	9	~~0~~
S	9	10	14	[0]

Since each row and each columncontains exactly one assignment,all the three men have been assigned a task. But task *S* is not assigned to any Man. The optimal assignment schedule and total cost is

Task	Men	cost
P	1	9
Q	3	6
R	2	20
S	*d*	0
Total cost		35

The optimal assignment (minimum) cost = ₹ 35.

Exercise 10.2

1. What is the Assignment problem?
2. Give mathematical form of Assignment problem.
3. What is the difference between Assignment Problem and Transportation Problem?
4. Three jobs A, B and C one to be assigned to three machines U, V and W. The processing cost for each job machine combination is shown in the matrix given below. Determine the allocation that minimizes the overall processing cost.

Job \ Machine	*U*	*V*	*W*
A	17	25	31
B	10	25	16
C	12	14	11

(cost is in ₹ per unit)

5. A computer centre has got three expert programmers. The centre needs three application programmes to be developed. The head of the computer centre, after studying carefully the programmes to be developed, estimates the computer time in minitues required by the experts to the application programme as follows.

Programmers \ Programmes	*P*	*Q*	*R*
1	120	100	80
2	80	90	110
3	110	140	120

Assign the programmers to the programme in such a way that the total computer time is least.

6. A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimates of the time each man would take to perform each task is given below:

Sub-ordinates \ Tasks	1	2	3	4
P	8	26	17	11
Q	13	28	4	26
R	38	19	18	15
S	9	26	24	10

How should the tasks be allocated to subordinates so as to minimize the total man-hours?

7. Find the optimal solution for the assignment problem with the following cost matrix.

Salesman \ Area	1	2	3	4
P	11	17	8	16
Q	9	7	12	6
R	13	16	15	12
S	14	10	12	11

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8. Assign four trucks 1, 2, 3 and 4 to vacant spaces A, B, C, D, E and F so that distance travelled is minimized. The matrix below shows the distance.

	1	2	3	4
A	4	7	3	7
B	8	2	5	5
C	4	9	6	9
D	7	5	4	8
E	6	3	5	4
F	6	8	7	3

10. 3 Decision Theory

Introduction:

Decision theory is primarily concerned with helping people and organizations in making decisions. It provides a meaningful conceptual frame work for important decision making. The decision making refers to the selection of an act from amongst various alternatives, the one which is judged to be the best under given circumstances.

The management has to consider phases like planning, organization, direction, command and control. While performing so many activities, the management has to face many situations from which the best choice is to be taken. This choice making is technically termed as "decision making" or decision taking. A decision is simply a selection from two or more courses of action. Decision making may be defined as "a process of best selection from a set of alternative courses of action, that course of action which is supposed to meet objectives upto satisfaction of the decision maker."

The knowledge of statistical techniques helps to select the best action. The statistical decision theory refers to an optimal choice under condition of uncertainty. In this case probability theory has a vital role, as such, this probability theory will be used more frequently in the decision making theory under uncertainty and risk.

The statistical decision theory tries to reveal the logical structure of the problem into alternative action, states of nature, possible outcomes and likely pay-offs from each such outcome. Let us explain the concepts associated with the decision theory approach to problem solving.

10.3.1 Meaning

The decision maker: The decision maker refers to individual or a group of individual responsible for making the choice of an appropriate course of action amongst the available courses of action.

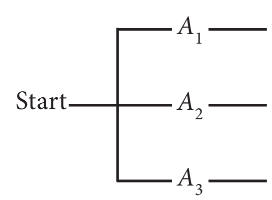
Acts (or courses of action): Decision making problems deals with the selection of a single act from a set of alternative acts. If two or more alternative courses of action occur in a problem, then decision making is necessary to select only one course of action. Let the acts or action be a_1, a_2, a_3,… then the totality of all these actions is known as action space denoted by A. For three actions a_1, a_2 a_3; A = action space = (a_1, a_2, a_3) or A = (A_1, A_2, A_3). Acts may be also represented in the following matrix form.

Acts
A_1
A_2
…
A_n

(or)

Acts	A_1	A_2	…	A_n

In a tree diagram the acts or action are shown as



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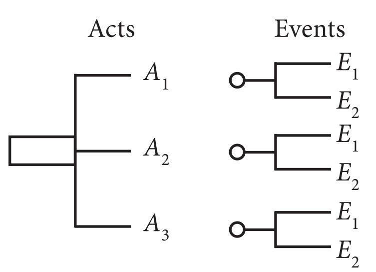
Events (or States of nature): The events identify the occurrences, which are outside of the decision maker' s control and which determine the level of success for a given act. These events are often called ' States of nature' or outcomes. An example of an event or states of nature is the level of market demand for a particular item during a stipulated time period.

A set of states of nature may be represented in any one of the following ways:

$S = \{S_1, S_2, \ldots, S_n\}$ or $E = \{E_1, E_2, \ldots, E_n\}$ or $\Omega = \{\theta_1, \theta_2, \theta_3\}$

For example, if a washing powder is marketed, it may be highly liked by outcomes (outcome θ_1) or it may not appeal at all (outcome θ_2) or it may satisfy only a small fraction, say 25% (outcome θ_3)

$\therefore \Omega = \{\theta_1, \theta_2, \theta_3\}$



In a tree diagram the places are next to acts. We may also get another act on the happening of events as follows:

In a matrix form, they may be represented as either of the two ways.

States of nature → Acts ↓	S_1	S_2
A_1 A_2		

or

Acts → States of nature ↓	$A_1 A_2, \ldots A_n$
S_1 S_2	

Pay-off: The result of combinations of an act with each of the states of nature is the outcome and monetary gain or loss of each such outcome is the pay-off. This means that the expression pay-off should be in quantitative form.

Pay -off may be also in terms of cost saving or time saving. In general, if there are k alternatives and n states of nature, there will be $k \times n$ outcomes or pay-offs. These $k \times n$ payoffs can be very conveniently represented in the form of a $k \times n$ pay-off table.

States of nature	Decision alternative			
	A_1	A_2	....	A_k
E_1	a_{11}	a_{12}	....	a_{1k}
E_2	a_{21}	a_{22}	....	a_{2k}
.	.	.	....	.
.	. .	. .	....	. .
E_n	a_{n1}	a_{n2}	....	a_{nk}

Where a_{ij} = conditional outcome (pay-off) of the i^{th} event when j^{th} alternative is chosen. The above pay-off table is called pay-off matrix.

10.3.2 Situations- Certainty and uncertainty

Types of decision making: Decisions are made based upon the information data available about the occurrence of events as well as the decision situation. There are two types of decision making situations: certainty and uncertainty.

Decision making under certainty: In this case the decision maker has the complete knowledge of consequence of every decision choice with certainty. In this decision model, assumed certainty means that only one possible state of nature exists.

Decision making under uncertainty: Under conditions of uncertainty, only pay-offs are known and nothing is known about the lilkelihood of each state of nature.

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Such situations arise when a new product is introduced in the market or a new plant is set up. The number of different decision criteria available under the condition of uncertainty is given below.

10.3.3 Maximin and Minimax Strategy

Maximin criteria

This criterion is the decision to take the course of action which maximizes the minimum possible pay-off. Since this decision criterion locates the alternative strategy that has the least possible loss, it is also known as a pessimistic decision criterion. The working method is:

(i) Determine the lowest outcome for each alternative.

(ii) Choose the alternative associated with the maximum of these.

Minimax criteria

This criterion is the decision to take the course of action which minimizes the maximum possible pay-off. Since this decision criterion locates the alternative strategy that has the greatest possible gain. The working method is:

(iii) Determine the highest outcome for each alternative.

(iv) Choose the alternative associated with the minimum of these.

Example 10.10

Consider the following pay-off (profit) matrix Action States

Action	States			
	(S_1)	(S_2)	(S_3)	(S_4)
A_1	5	10	18	25
A_2	8	7	8	23
A_3	21	18	12	21
A_4	30	22	19	15

Determine best action using maximin principle.

Solution:

Action	States				Minimum
	(S_1)	(S_2)	(S_3)	(S_4)	
A_1	5	10	18	25	5
A_2	8	7	8	23	7
A_3	21	18	12	21	12
A_4	30	22	19	15	15

Max (5,7,12,15)=15 ∴ Action A4 is the best.

Example 10.11

A business man has three alternatives open to him each of which can be followed by any of the four possible events. The conditional pay offs for each action - event combination are given below:

Alternative	Pay–offs (Conditional events)			
	A	B	C	D
X	8	0	–10	6
Y	–4	12	18	–2
Z	14	6	0	8

Determine which alternative should the businessman choose, if he adopts the maximin principle.

Solution:

Alternative	Pay – offs (Conditional events)				Minimum pay off
	A	B	C	D	
X	8	0	–10	6	–10
Y	–4	12	18	–2	–4
Z	14	6	0	8	**0**

Max (–10,–4, 0) = 0. Since the maximum payoff is 0, the alternative Z is selected by the businessman.

Example 10.12

Consider the following pay-off matrix.

Alternative	Pay – offs (Conditional events)			
	A_1	A_2	A_3	A_4
E_1	7	12	20	27
E_2	10	9	10	25
E_3	23	20	14	23
E_4	32	24	21	17

Using minmax principle, determine the best alternative.

Solution:

Alternative	Pay-offs (Conditional events)				Maximum pay off
	A_1	A_2	A_3	A_4	
E_1	7	12	20	27	27
E_2	10	9	10	25	25
E_3	23	20	14	23	23
E_4	32	24	21	17	32

min(27, 25, 23, 32) = 23. Since the minimum cost is 23, the best alternative is E_3 according to minimax principle.

Exercise 10.3

1. Given the following pay-off matrix(in rupees) for three strategies and two states of nature.

Strategy	States-of-nature	
	E_1	E_2
S_1	40	60
S_2	10	–20
S_3	–40	150

Select a strategy using each of the following rule (i) Maximin (ii) Minimax.

2. A farmer wants to decide which of the three crops he should plant on his 100-acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high medium and low. His estimated profit for each is shown in the table.

Rainfall	Estimated Conditional Profit(Rs.)		
	Crop *A*	Crop *B*	Crop *C*
High	8000	3500	5000
Medium	4500	4500	5000
Low	2000	5000	4000

If the farmer wishes to plant only crop, decide which should be his best crop using (i) Maximin (ii)Minimax

3. The research department of Hindustan Ltd. has recommended to pay marketing department to launch a shampoo of three different types. The marketing types of shampoo to be launched under the following estimated pay-offs for various level of sales.

Types of shampoo	Estimated Sales (in Units)		
	15000	10000	5000
Egg shampoo	30	10	10
Clinic Shampoo	40	15	5
Deluxe Shampoo	55	20	3

What will be the marketing manager's decision if (i) Maximin and (ii) Minimax principle applied?

4. Following pay-off matrix, which is the optimal decision under each of the following rule (i) maxmin (ii) minimax

Act	States of nature			
	S_1	S_2	S_3	S_4
A_1	14	9	10	5
A_2	11	10	8	7
A_3	9	10	10	11
A_4	8	10	11	13

Exercise 10.4

Choose the correct Answer

1. The transportation problem is said to be unbalanced if __________.
   (a) Total supply ≠ Total demand
   (b) Total supply = Total demand
   (c) $m = n$
   (d) $m+n-1$

2. In a non – degenerate solution number of allocations is
   (a) Equal to $m+n-1$
   (b) Equal to $m+n+1$

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(c) Not equal to $m+n-1$

(d) Not equal to $m+n+1$

3. In a degenerate solution number of allocations is
   (a) equal to $m+n-1$
   (b) not equal to $m+n-1$
   (c) less than $m+n-1$
   (d) greather than $m+n-1$

4. The Penalty in VAM represents difference between the first ________
   (a) Two largest costs
   (b) Largest and Smallest costs
   (c) Smallest two costs
   (d) None of these

5. Number of basic allocation in any row or column in an assignment problem can be
   (a) Exactly one (b) at least one
   (c) at most one (d) none of these

6. North-West Corner refers to ________
   (a) top left corner
   (b) top right corner
   (c) bottom right corner
   (d) bottom left corner

7. Solution for transportation problem using __________method is nearer to an optimal solution.
   (a) NWCM (b) LCM
   (c) VAM (d) Row Minima

8. In an assignment problem the value of decision variable x_{ij} is _________.
   (a) 1 (b) 0
   (c) 1 or 0 (d) none of them

9. If number of sources is not equal to number of destinations, the assignment problem is called_______________
   (a) balanced (b) unsymmetric
   (c) symmetric (d) unbalanced

10. The purpose of a dummy row or column in an assignment problem is to
    (a) prevent a solution from becoming degenerate
    (b) balance between total activities and total resources
    (c) provide a means of representing a dummy problem
    (d) none of the above

11. The solution for an assignment problem is optimal if
    (a) each row and each column has no assignment
    (b) each row and each column has atleast one assignment
    (c) each row and each column has atmost one assignment
    (d) each row and each column has exactly one assignment

12. In an assignment problem involving four workers and three jobs, total number of assignments possible are
    (a) 4 (b) 3 (c) 7 (d) 12

13. Decision theory is concerned with
    (a) analysis of information that is available
    (b) decision making under certainty
    (c) selecting optimal decisions in sequential problem
    (d) All of the above

14. A type of decision –making environment is
    (a) certainty (b) uncertainty
    (c) risk (d) all of the above

Miscellaneous Problems

1. The following table summarizes the supply, demand and cost information for four factors S_1, S_2, S_3, S_4 shipping goods to three warehouses D_1, D_2, D_3

	D_1	D_2	D_3	Supply
S_1	2	7	14	5
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
Demand	7	9	18	

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Find an initial solution by using north west corner rule. What is the total cost for this solution?

2. Consider the following transportation problem

Destination

	D_1	D_2	D_3	D_4	Availability
O_1	5	8	3	6	30
O_2	4	5	7	4	50
O_3	6	2	4	6	20
Requirement	30	40	20	10	

Determine an initial basic feasible solution using (a) Least cost method (b) Vogel's approximation method.

3. Determine an initial basic feasible solution to the following transportation problem by using (i)North West Corner rule (ii) least cost method.

Destination

		D_1	D_2	D_3	Supply
Source	S_1	9	8	5	25
	S_2	6	8	4	35
	S_3	7	6	9	40
	Requirement	30	25	45	

4. Explain Vogel's approximation method by obtaining initial basic feasible solution of the following transportation problem.

Destination

		D_1	D_2	D_3	D_4	Supply
Origin	O_1	2	3	11	7	6
	O_2	1	0	6	1	1
	O_3	5	8	15	9	10
	Demand	7	5	3	2	

5. A car hire company has one car at each of five depots a,b,c,d and e. A customer in each of the fine towers A,B,C,D and E requires a car. The distance (in miles) between the depots (origins) and the towers(destinations) where the customers are given in the following distance matrix.

	a	b	c	d	e
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How should the cars be assigned to the customers so as to minimize the distance travelled?

6. A natural truck-rental service has a surplus of one truck in each of the cities 1,2,3,4,5 and 6 and a deficit of one truck in each of the cities 7,8,9,10,11 and 12. The distance(in kilometers) between the cities with a surplus and the cities with a deficit are displayed below:

To

		7	8	9	10	11	12
From	1	31	62	29	42	15	41
	2	12	19	39	55	71	40
	3	17	29	50	41	22	22
	4	35	40	38	42	27	33
	5	19	30	29	16	20	33
	6	72	30	30	50	41	20

How should the truck be dispersed so as to minimize the total distance travelled?

7. A person wants to invest in one of three alternative investment plans: Stock, Bonds and Debentures. It is assumed that the person wishes to invest all of the funds in a plan. The pay-off matrix based on three potential economic conditions is given in the following table:

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Alternative	Economic conditions		
	High growth(Rs.)	Normal growth(Rs.)	Slow growth (Rs.)s
Stocks	10000	7000	3000
Bonds	8000	6000	1000
Debentures	6000	6000	6000

Determine the best investment plan using each of following criteria i) Maxmin ii) Minimax.

Summary

- In a transportation problem if the total supply equals the total demand, it is said to be balanced transportation problem. Otherwise it is said to be unbalanced transportation problem
- **Feasible Solution**: A feasible solution to a transportation problem is a set of non-negative values x_{ij}(i=1,2,..,m, j=1,2,…n) that satisfies the constraints.
- **Basic Feasible Solution**: A feasible solution is called a basic feasible solution if it contains not more than m+n-1 allocations, where m is the number of rows and n is the number of columns in a transportation table.
- **Optimal Solution**: Optimal Solution is a feasible solution (not necessarily basic) which optimizes(minimize) the total transportation cost.
- **Non degenerate basic feasible Solution**: If a basic feasible solution to a transportation problem contains exactly m+n-1 allocations in independent positions, it is called a Non degenerate basic feasible solution.
- **Degeneracy : If** : If a basic feasible solution to a transportation problem contains less than m+n-1 allocations , it is called a degenerate basic feasible solution.
- In an assignment problems number of rows and columns must be equal
- The optimum assignment schedule remains unaltered if we add or subtract a constant from all the elements of the row or column of the assignment cost matrix.
- If for an assignment problem all $C_{ij} > 0$ then an assignment schedule (x_{ij}) which satisfies $\sum C_{ij}\, x_{ij} = 0$ must be optimal.

GLOSSARY (கலைச்சொற்கள்)	
Approximation	தோராயமாக
Assignment problems	ஒதுக்கீடு கணக்குகள்
Decision theory	முடிவு கோட்பாடுகள்
Degenerate	சிதைந்த
Destination	சேருமிடம்
Feasible solution	ஏற்புடையத் தீர்வு

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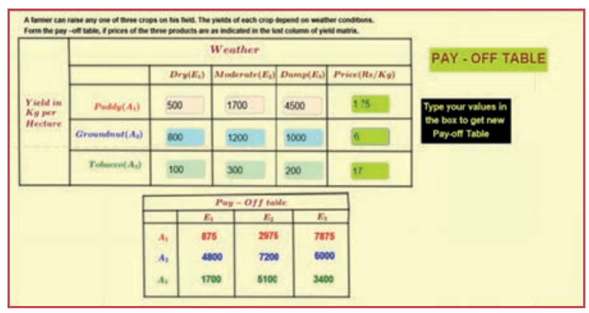
Initial basic feasible solution	ஆரம்ப அடிப்படை ஏற்புடையத் தீர்வு
Least cost method	மீச்சிறு செலவு முறை
Non-degenerate	சிதைவற்ற
North west- Conner method	வட மேற்கு மூலை முறை
Optimum solution	உகந்ததீர்வு
Pay off	இழப்பு ஈட்டியப்பு
Strategy	உத்தி
Transportation cost	போக்குவரத்து செலவு
Transportation problems	போக்குவரத்து கணக்குகள்

ICT Corner

Expected Result is shown in this picture

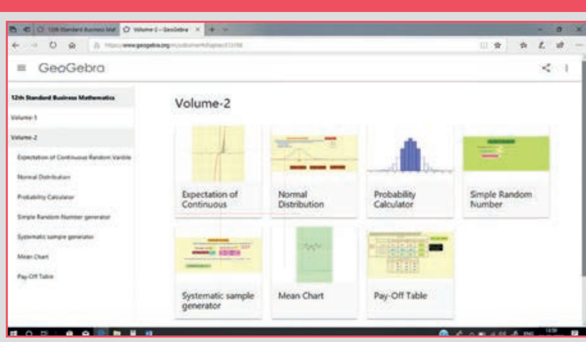


Step – 1 : Open the Browser, type the URL Link given below (or) Scan the QR Code. GeoGebra work Book named "12th Standard Business Mathematics and Statistics" will open. In the work book there are two Volumes. Select "Volume-2".

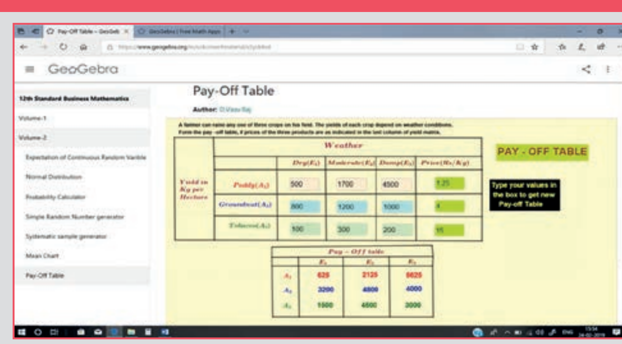
Step - 2 : Select the worksheet named "Pay-Off Table."

You can enter your own data in the respective boxes. Your Pay-Off table will be generated below. Calculate and check your answer.

Step 1



Step 2



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