BCECE

Engineering Entrance Exam

Solved Paper 2013

Physics

1. The dimensions of universal gravitational constant are

(a) $[M^{-2} L^{-3} T^{-2}]$ (b) $[M^{-2} L^{2} T^{-1}]$

(c) $[M^{-1} L^{3} T^{-2}]$ (d) $[ML^{2} T^{-1}]$

2. If $|\mathbf{A} \times \mathbf{B}| = \sqrt{3}\ A \cdot B$ then the value of $|\mathbf{A} + \mathbf{B}|$ is

(a) $A+B$

(b) $(A^2 + B^2 + \sqrt{3}AB)^{1/2}$

(c) $\left(A^2 + B^2 + \frac{AB}{\sqrt{3}}\right)^{1/2}$

(d) $(A^2 + B^2 + AB)^{1/2}$

3. Two spheres of masses m and M are situated in air and the gravitional force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The new gravitational force will be

(a) F (b) $\frac{F}{3}$

(c) $3F$ (d) $\frac{F}{9}$

4. A body floats in water with one fourth of its volume above the surface of water. If placed in oil it floats with one third of its volume above the surface of oil. The density of oil is

(a) $\frac{3}{4}$ (b) $\frac{4}{9}$

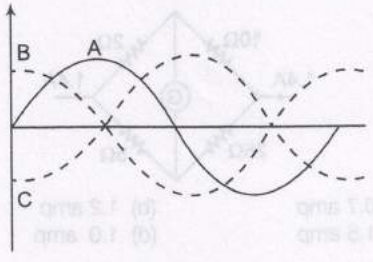
(c) $\frac{2}{3}$ (d) $\frac{9}{8}$

5. When capillary tubes of different radii 'r' dipped in water, water rises to different heights 'h' in them, then

(a) $hr^2 = \text{constant}$ (b) $hr = \text{constant}$

(c) $\frac{h}{r} = \text{constant}$ (d) $\frac{h}{r^2} = \text{constant}$

6. In the adjoining figure A, B, and C represents three progressive waves. Which of the following statement about the waves is correct?



(a) Wave C lags behind in phase by $\frac{\pi}{2}$ from A and B leads by $\frac{\pi}{2}$

(b) Wave C leads in phase by π from A and B lags behind by π

(c) Wave C leads in phase by $\frac{\pi}{2}$ from A and lags behind by $\frac{\pi}{2}$

(d) Wave C lags behind in phase by π from A and B leads by π

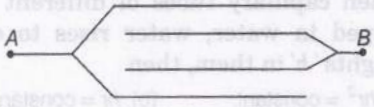
7. If 110 J of heat are added to a gaseous system, whose internal energy is 40J, then the amount of external work done is

(a) 40 J (b) 70 J (c) 110 J (d) 150 J

8. A ray is incident at an angle of incidence i on one surface of a prism of small angle A and emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, the angle of incidence i is nearly equals to

(a) $\mu A/2$ (b) $A/2\mu$ (c) μA (d) A/μ

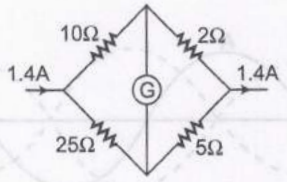
9. Figure below shows four plates each of area S separated from one another by a distance d. What is the capacitance between A and B?



(a) $\frac{4\varepsilon_0 3}{d}$ (b) $\frac{3\varepsilon_0 5}{d}$

(c) $\frac{2\varepsilon_0 5}{d}$ (d) $\frac{\varepsilon_0 5}{d}$

10. What is the value of current in the arm containing 2π resistor in the adjoining circuit?



(a) 0.7 amp (b) 1.2 amp

(c) 1.5 amp (d) 1.0 amp

11. For ohmic conductor the drift velocity v_d and the electric field applied across it are related as

(a) $v_d \propto \sqrt{E}$ (b) $v_d \propto E^2$

(c) $v_d \propto E$ (d) $v_d \propto \frac{1}{E}$

12. If **E** and **B** be the electric and magnetic field vectors of electromagnetic waves, the direction of propagation of electromagnetic waves is that of

(a) **E** (b) **B** (c) **E**×**B** (d) **B**×**E**

13. The current in RCL circuit is maximum where

(a) $X_L = 0$ (b) $X_L = X_C$

(c) $X_C = 0$ (d) $X_L^2 + X_C^2 = 1$

14. What is the value of $\overline{A} + A$ in the boolean algebra?

(a) 0 (b) 1 (c) A (d) $\overline{A}$

15. The truth table given below is for which gate?

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(a) XOR (b) OR

(c) AND (d) NAND

16. The magnetic flux through a circuit of resistance R changes by an amount $\Delta\phi$ in a time Δt. Then the total quantity of electric charge Q that passes any point in the circuit during the time Δt is represented by

(a) $Q = \frac{\Delta\phi}{\Delta t}$

(b) $Q = \frac{\Delta\phi}{R}$

(c) $Q = R.\frac{\Delta\phi}{\Delta t}$

(d) $Q = \frac{1}{R}\frac{\Delta\phi}{\Delta t}$

17. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun the tangential acceleration will be

(a) 40π m/s^2 (b) 40 m/s^2

(c) 160π m/s^2 (d) 640π m/s^2

18. The output of a OR gate is 1

(a) if either input is zero

(b) if both inputs are zero

(c) only if both inputs are 1

(d) if either or both inputs are 1

19. Two 220 V, 100 W bulbs are connected first in series and then in parallel. Each time the combination is connected to a 220 V AC supply line, the power drawn by the combination in each case respectively will be

(a) 50 W, 200 W (b) 50 W, 20 W
(c) 100 W, 50 W (d) 200 W, 150 W

20. Barrier potential of a *p-n* junction diode does not depend on

(a) diode design (b) doping density
(c) temperature (d) farward bias

21. A projectile is projected with a linear momentum **p** making an angle θ with the horizontal. The change in momentum of the projectile on return to the ground will be

(a) $2p\tan\theta$ (b) $2p\sin\theta$
(c) $2p\cos\theta$ (d) $2p$

22. A weigthtless thread can bear tension up to 3.7 kg wt. A stone of mass 500 g is tied to it and revolved in a circular path of radius 4m in a vertical plane. If $g = 10\,m/s^2$, then the maximum angular velocity of the stone will be

(a) 2 rad/s (b) 4 rad/s
(c) 16rad/s (d) $\sqrt{21}$ rad/s

23. Under a constant torque, the angular momentum of a body changes from A to 4 A in 4 s. The torque is

(a) 3 A (b) $\frac{1}{4}A$ (c) $\frac{3}{4}A$ (d) $\frac{4}{3}A$

24. The mass of a planet is double and its radius is half compared to that of earth. Assuming $g = 10\,m/s^2$ on earth, the acceleration due to gravity at the planet will be

(a) 10 m/s^2 (b) 20 m/s^2
(c) 40 m/s^2 (d) None of these

25. A soap bubble in vacuum, has a radius of 3 cm and another soap bubble in vacuum has a radius of 4 cm. If the two bubbles coalesce under isothermal condition, then the radius of new bubble is

(a) 4.3 cm (b) 4.5 cm
(c) 5 cm (d) 7 cm

26. Deuteron and α-particle are put 1Å apart in air. Magnitude of intensity of electric field due to deuteron of α-particle is

(a) zero
(b) 2.88×10^{11} N/C
(c) 1.44×10^{11} N/C
(d) 5.76×10^{11} N/C

20. The air column in pipe, which is closed at one end will be in resonance with a vibrating turning fork at a frequency of 260 Hz, if the length of the air column is

(a) 31.73 cm (b) 62.5 cm
(c) 35.75 cm (d) 12.5 cm

28. A ball strikes against the floor and returns with double the velocity. In which type of collision is it possible?

(a) Inelastic
(b) Perfectly inclastic
(c) Perfectly elastic
(d) Not possible

29. A body is under the action of three force $\mathbf{F_1}, \mathbf{F_2}$, and $\mathbf{F_3}$. In which case the body cannot under go angular acceleration?

(a) $F_1 + F_2 + F_3 = 0$
(b) F_1, F_2 and F_3 are concurrent
(c) F_1, and F_2 act at the same point but, F_3 acts at different point
(d) $F_1 + F_2$ is parallel to F_3, but the three forces are not concurrent

30. Two beams of protons moving parallel in same direction will

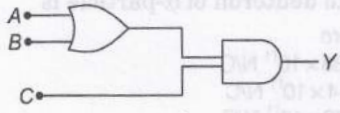
(a) repel each other
(b) exert no force
(c) attract each other
(d) deflect perpendicular to the plane of the beams

31. A photon and an electron possess same de-Broglie wavelength given that C = speed of light and v = space of electron, which of the following relation is correct?

(here, E_e = K.E of electron, E_{Ph} = K.E of photon, P_e = momentum of electron, P_{ph} = momentum of photon)

(a) $\frac{P_e}{P_{Pn}} = \frac{C}{2v}$ (b) $\frac{E_e}{E_{Ph}} = \frac{C}{2v}$
(c) $\frac{E_{ph}}{E_e} = \frac{2c}{v}$ (d) $\frac{P_e}{P_{Ph}} = \frac{2c}{v}$

32. To get an output $Y = 1$ from circuit of adjoining figure, the input must be



	A	B	C
(a)	0	1	0
(b)	1	0	0
(c)	1	0	1
(d)	1	1	0

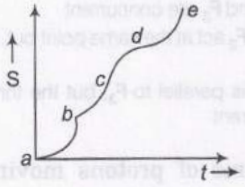
33. Hubble's law is expressed as

(here, v = speed of recession, r = distance of galaxy, H = Hubble constant)

(a) $v = Hr$ (b) $v = H^2r$

(c) $v = \frac{H}{r^2}$ (d) $v = Hr^2$

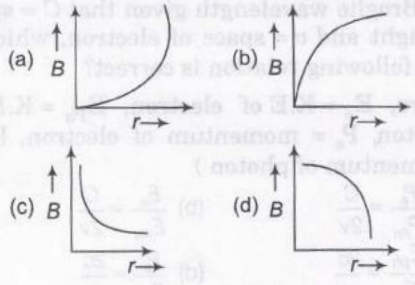
34. The displacement *versus* time graph for a body moving in a straight line is shown in figure. Which of the following regions represents motor when no force is acting on the body?



(a) *ab* (b) *bc*

(c) *cd* (d) *be*

35. Which of the following graphs shows the variation of magnetic field B, with distance from a long current carrying conductor?



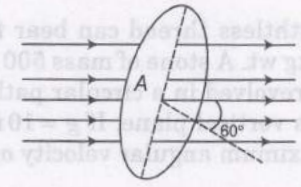
36. The time period of a freely suspended magnet does not depend upon

(a) length of the magnet
(b) the pole strength of the magnet
(c) the horizontal component of magnetic field of earth
(d) the length of the suspension

37. The magnetic permeability is defined as the ratio of

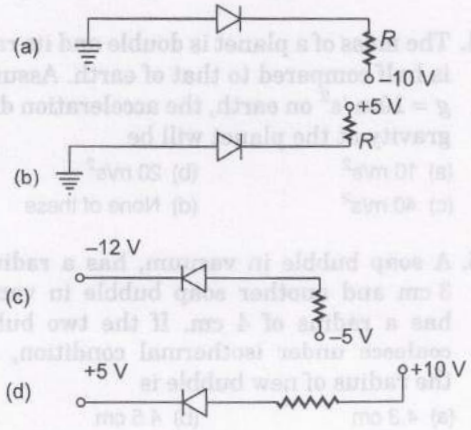
(a) magnetic induction and magnetising field
(b) intensity of magnetisation and magnetising field
(c) intensity of magnetisation and magnetic field
(d) None of the above

38. Figure represents an area $A = 0.5\,m^2$ situated in a uniform magnetic field $B = 2.0\,Wb/m^2$ and making an angle of 60° with respect to magnetic field. The value of magnetic flux through the area will be

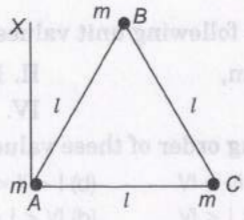


(a) 0.5 Wb
(b) $\sqrt{3}$ Wb
(c) $\frac{3}{2}$ Wb
(d) 2.0 Wb

39. Of the given diodes, shown in the adjoining diagrams, which one is reverse biased?



40. Three particles each of mass m gram, are situated at the vertices of an equilateral triangle ABC of the side 1 cm. The moment of inertia of the system (as shown in figure) about a line AX perpendicular to AB and in the plane of ABC in gram-cm^2 units will be



(a) $\frac{3}{2}ml^2$ (b) $\frac{3}{4}ml^2$ (c) $2\,ml^2$ (d) $\frac{5}{4}ml^2$

41. A galvanometer of 50π resistance has 25 divisions. A current of 4×10^{-4}A gives a deflection of one division. To convert this galvanometer into a voltmeter having a range of 25 volt, it should be connected with a resistance of

(a) 245π as shunt (b) 2450π as series
(c) 2500π as shunt (d) 2550π in series

42. A car is moving towards a high cliff. The car driver sounds a horn of frequency f. The reflected sound heard by the driver has the frequency $2f$. If v be the velocity of sound, then the velocity of the car, in the same velocity units will be

(a) $\frac{v}{\sqrt{2}}$ (b) $\frac{v}{2}$ (c) $\frac{v}{3}$ (d) $\frac{v}{4}$

43. A plane glass slab is kept over various coloured letters, the letter which appears least raised is

(a) violet (b) blue
(c) green (d) red

44. If the angle of incidence is i and that of refraction is r. Then the speed of light in the medium to which the light is refracted from air is

(a) $v = C\frac{\sin i}{\cos r}$ (b) $v = C\frac{\cos r}{\cos i}$
(c) $v = C\frac{\sin r}{\sin i}$ (d) $v = C\frac{\sin i}{\sin r}$

45. An electron moves with a velocity **v** in an electric field **E**. If the angle between **v** and **E** is neither 0 nor π, the path followed by the electron is

(a) parabola
(b) circle
(c) straight line
(d) ellipse

46. The internal resistance of a cell of emf 2 V is 0.1π. If it is connected to a resistance of $3.9\ \pi$, then the voltage across the cell will be (in volts)

(a) 2 V
(b) 0.5 V
(c) 1.95 V
(d) 2.5 V

47. An electric kettle takes 4 A current a t 220 V. How much time will it take to boil 1 kg of water from room temperature 20°C? (the temperature of boiling water is 100°C)

(a) 12.6 min
(b) 12.8 min
(c) 6.3 min
(d) 6.4 min

48. The phenomenon of pair production is

(a) ejection of an electron from a nucleus
(b) ejection of an electron from a metal surface
(c) ionisation of a neutral atom
(d) the production of an electron and a positron from γ-rays.

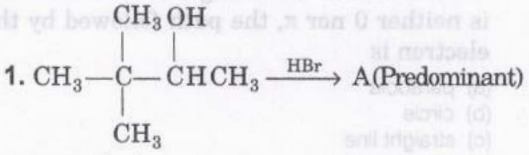
49. In an electron microscope if the potential is increased from 20kV to 80 kV, the resolving power R of the microscope will become

(a) $\frac{R}{2}$ (b) $2R$
(c) $4R$ (d) $5R$

50. Force acting upon a charged particle kept between the plates of changed capacitor is F. If one of the plates of the capacitor is removed force acting on the same particle will become

(a) 0
(b) F
(c) $\frac{F}{2}$
(d) $2F$

Chemistry

1. 

Identify *A*.

(a) $(CH_3)_2C$ (Br) CH $(CH_3)_2$
(b) $(CH_3)_3CCH$ (Br) CH_3
(c) $(CH_3)_3COHBr$ CH_3
(d) None of the above

2. A β-hydroxy carbonyl compound is obtained by the action of NaOH on

(a) C_6H_5CHO (b) HCHO
(c) CH_3CHO (d) $(CH_3)_3C \cdot CHO$

3. Which of the following statements is correct?

(a) + *I* effect stabilises a carbanion
(b) + *I* effect stabilises a carbocation
(c) – *I* effect stabilises a carbanion
(d) – *I* effect stabilises a carbocation

4. $C_4H_6O_4\ A \xrightarrow{\Delta} C_3H_6O_2\ B \xrightarrow[\Delta]{\text{sodalime}} C$

Compound '*C*' is a hydrocarbon, occupying approx 0.75 L volume per gram. Identify '*A*' and '*B*'.

(a) Tartaric acid, propanoic acid
(b) Succinic acid, succinic anhydride
(c) Maleic anhydride, maleic acid
(d) Methyl malonic acid, propanoic acid

5. What is effective nuclear charge and the periphery of nitrogen atom when an extra electron is added in the formation of an anion?

(a) 1.20 (b) 2.45
(c) 3.55 (d) 5.95

6. A certain metal sulphide, MS_2, is used extensively as a high temperature lubricant. If MS_2 has 40.06 % sulphur by weight, atomic mass of *M* will be

(a) 100 amu (b) 96 amu
(c) 60 amu (d) 30 amu

7. In HNC, which element has least value of formal charge?

(a) H (b) N
(c) C (d) All have same value

8. Consider following unit values of energy

I. 1 L atm, II. 1 erg,
III. 1 J IV. kcal,

Increasing order of these values is

(a) I = II = III = IV (b) I < II < III < IV
(c) II < III < I < IV (d) IV < I < III < II

9. The number of elements in the transition metal series Sc through Zn that have four unpaired electrons in their + 2 state are

(a) 2 (b) 4
(c) 5 (d) 6

10. In an atmosphere with industrial smog, copper corrodes to

I. $Cu_2(OH)_2SO_4$ II. $Cu_2(OH)_2CO_3$
III. $CuSO_4$ IV. $CuCO_3$

(a) I and III
(b) II and IV
(c) I and IV
(d) I and II

11. Mischmetal is

(a) an alloy of lanthanide and nickel
(b) an alloy of lanthanide and copper
(c) an alloy of lanthanide, iron and carbon
(d) an alloy of lanthanide, magnesium and nickel

12. What is the oxidation state of iron in $[Fe(H_2O)_5\ NO]^{2+}$?

(a) 0 (b) + 1
(c) + 2 (d) + 3

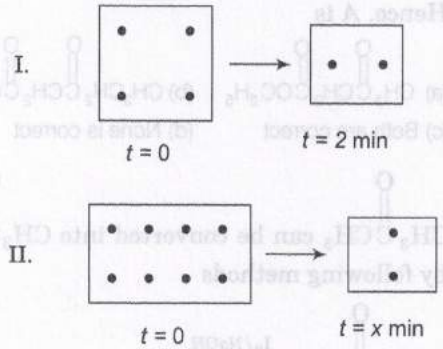
13. Identify the final product of the reaction

$$BCl_3 + NH_4Cl \xrightarrow[C_6H_5Cl]{140^\circ C} \xrightarrow{NaBH_4} ?$$

(a) $B_3N_3H_6$ (b) B_2H_6
(c) $NaBCl_4$ (d) BN

14. 2.56 g of sulphur in 100 g of CS_2 has depression in freezing point of 0.010°C. Atomicity of sulphur in CS_2 is
(Given, $K_f = 0.1°$ molal^{-1})
(a) 2 (b) 4
(c) 6 (d) 8

15. For the zeroth order reaction, sets I and II are given, hence x is



(a) 2 min (b) 4 min
(c) 6 min (d) 8 min

16. Equilibrium constant for the reaction,
$NH_4OH + H^+ \rightleftharpoons NH_4^+ + H_2O$ is 1.8×10^9. Hence, equilibrium constant for
$NH_3(aq) + H_2O \rightleftharpoons NH_4^+ + OH^-$ is
(a) 1.8×10^{-5} (b) 1.8×10^5
(c) 1.8×10^{-9} (d) 5.59×10^{-10}

17. Calculate pH change when 0.01 mol CH_3KCOO Na solution is added to 1L of 0.01 M CH_3COOH solution.
K_a (CH_3COOH) = 1.8×10^{-5}, pK_a = 4.74
(a) 3.37 (b) 1.37
(c) 4.74 (d) 8.01

18. pK_b of NH_3 is 4.74 and pK_b of A^-, B^- and C^- are 4, 5 and 6 respectively. Aqueous solution of 0.01 M has pH in the increasing order
(a) $NH_4A < NH_4B < NH_4C$
(b) $NH_4C < NH_4B < NH_4A$
(c) $NH_4C < NH_4A < NH_4B$
(d) All have equal pH

19. Osmotic pressure of insulin solution at 298 K is found to be 0.0072 atm. Hence, height of water column due to this pressure is [given d (Hg) = 13.6 g / mL]
(a) 0.76 cm (b) 0.70 cm
(c) 7.4 cm (d) 76 cm

20. $\Delta G°$ and $\Delta H°$ for a reaction at 300 K is -66.9 kJ mol^{-1} and -41.8 kJ mol^{-1} respectively. $\Delta G°$ for the same reaction at 330 K is
(a) -25.1 kJ mol^{-1}
(b) $+25.1$ kJ mol^{-1}
(c) 18.7 kJ mol^{-1}
(d) -69.4 kJ mol^{-1}

21. At 1000 K, from the data
$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$;
$\Delta H = -123.77$ kJ mol^{-1}

Substance	N_2	H_2	NH_3
P/R	3.5	3.5	4

Calculate the heat of formation of NH_3 at 300 K.
(a) -44.42 kJ mol^{-1}
(b) -88.85 kJ mol^{-1}
(c) $+44.42$ kJ mol^{-1}
(d) $+88.85$ kJ mol^{-1}

22. Match the electrode (in column I) with its general name (in Column II) and choose the correct option given below.

	Column I		Column II
A.	Calomel	I.	Reference
B.	Glass	II.	Redox
C.	Hydrogen	III.	Membrane
D.	Quinhydrone	IV.	Gas

	A	B	C	D
(a)	III	III	IV	IV
(b)	IIII	IV	II	II
(c)	III	II	IV	I
(d)	II	IV	III	I

23. In a Daniell cell constructed in the laboratory, the voltage observed was 0.9 V instead of 1.1 V of the standard cell. A possible explanation is

(a) $[Zn^{2+}] > [Cu^{2+}]$

(b) $[Zn^{2+}] < [Cu^{2+}]$

(c) Zn electrode has twice the surface of Cu electrode

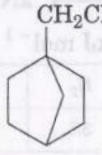
(d) mol ratio of Zn^{2+} : Cu^{2+} is 2 : 1

24. A quantity of electrical charge that brings about the deposition of 4.5 g Al from Al^{3+} at the cathode will also produce the following volume at (STP) of $H_2(g)$ from H^+ at the cathode

(a) 44.8 L (b) 22.4 L

(c) 11.2 L (d) 5.6 L

25. IUPAC name of the following compound is



(a) 3-propyl cyclo [3, 6] octane

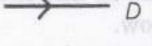
(b) 1-ethyl tricyclo [2, 3, 0] heptane

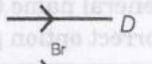
(c) 1-ethyl bicyclo [2, 2, 1] heptane

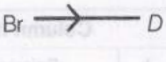
(d) 4-ethyl bicyclo [2, 2, 2] heptane

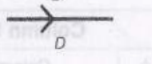
26. Identify *Y* in the following series of equation

$$\rightarrow\!\!-H + Br_2 \xrightarrow{h\nu} \text{'}X\text{'} \xrightarrow[D_2O]{Mg/\text{ether}} \text{'}Y\text{'}$$

(a) 

(b) 

(c) 

(d) 

27. Arrange the following in the order of rate of oxidation with periodic acid

I. $HOCH_2CH_2OH$,

II. $CH_3CHOHCHOHCH_3$

III. $(CH_3)_2COHCOH(CH_3)_2$

(a) I > II > III (b) II > I > III

(c) III > II > I (d) I > III = II

28. Phenol and NH_3 reacts in presence of $ZnCl_2$ at 300°C to produce

(a) tertiary amine (b) secondary amine

(c) primary amine (d) All of these

29. $A\ [C_6H_{10}O_3$ (keto ester)$] \xrightarrow[\Delta]{NaOH + I_2}$ yellow ppt $+ B \xrightarrow{H^+} C \xrightarrow[-CO_2]{\Delta} CH_3COOH.$

Hence, *A* is

(a) $CH_3\overset{O}{\overset{\|}{C}}CH_2\overset{O}{\overset{\|}{C}}OC_2H_5$ (b) $CH_3CH_2\overset{O}{\overset{\|}{C}}CH_2\overset{O}{\overset{\|}{C}}OCH_3$

(c) Both are correct (d) None is correct

30. $CH_3\overset{O}{\overset{\|}{C}}CH_3$ can be converted into $CH_3\overset{O}{\overset{\|}{C}}OH$ by following methods

I. $CH_3\overset{O}{\overset{\|}{C}}CH_3 \xrightarrow[\Delta, H^+]{I_2/NaOH}$

II. $CH_3\overset{O}{\overset{\|}{C}}CH_3 \xrightarrow{Cr_2O_7^{2-}/H^+}$

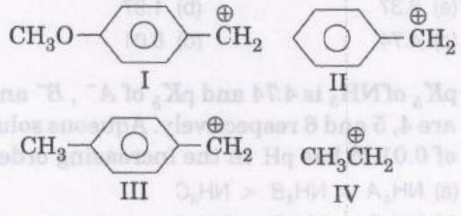
III. $CH_3\overset{O}{\overset{\|}{C}}CH_3 \xrightarrow{Ag(NH_3)_2^+}$

Which method are most effective?

(a) I, III (b) II, III

(c) I, II (d) I, II, III

31. Relative stabilities of the following carbocation will be in order



(a) IV < III < II < I (b) IV < II < III < I

(c) II < IV < III < I (d) I < II < III < IV

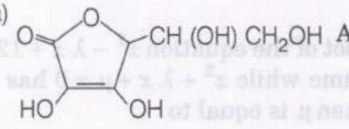
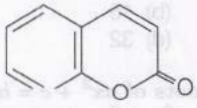
32. The correct statement in respect of protein haemoglobin is that it
(a) functions as a catalyst for biological reactions
(b) maintains blood sugar level
(c) acts as an oxygen carrier in the blood
(d) forms antibodies and offers resistance to diseases

33. $CH_3COOH + HCOOH \xrightarrow[570\ K]{MnO}$
Main product of above reaction is
(a) CH_3CHO (b) CH_3COCH_3
(c) HCHO (d) $(CH_3CO)_2O$

34. Aniline is reacted with bromine water and the resulting product is treated with an aqueous solution of sodium nitrite in presence of dilute hydrochloric acid. The compound so formed is converted to a tetrafluoroborate which is subsequently heated. The final product is
(a) *p*-bromoaniline
(b) *p*-bromofluorobenzene
(c) 1, 3, 5-tri bromobenzene
(d) 2, 4, 6-tribromofluorobenzene

35. With hard water, ordinary soap forms curdy precipitate of
(a) $(RCOO)_2Ca$ (b) $(RCOO)_2Mg$
(c) Both (a) and (b) (d) None of these

36. Which is matched incorrectly?

	Structure	Compound
(a)		Ascorbic acid
(b)		$(RCOO)_2Mg$ Coumarin

(c) Both (a) and (b)
(d) None of the above

37. Which base is normally found in RNA but not in DNA?
(a) Uracil
(b) Thymine
(c) Guanine
(d) Adenine

38. The dipole moment of HBr is 2.60×10^{-30} cm and the interatomic spacing is 1.41Å. What is the per cent ionic character of HBr?
(a) 50% (b) 11.5% (c) 4.01% (d) 1.19%

39. An element is oxidised by fluorine and not by chlorine. Identify the element.
(a) Sodium (b) Aluminium
(c) Oxygen (d) Sulphur

40. The reduction of an oxide by aluminium is called
(a) Kroll's process
(b) van Arkel process
(c) Ellingham process
(d) Goldschmidt's aluminothermite process

41. H_2 gas is liberated at cathode and anode both by electrolysis of the following solution except in
(a) NaCl (b) NaH
(c) LiH (d) HCOONa

42. Identify (*A*) and (*B*) in the following sequence of reactions.
$SnCl_2 + 2NaOH \longrightarrow (A)$ (white ppt)
$\xrightarrow[(excess)]{NaOH} (B)$
(a) $Sn(OH)_2$, Na_2SnO_3
(b) $Sn(OH)_2$, Na_2SnO_2
(c) $Sn(OH)_2$, $Na_2[Sn(OH)_6]$
(d) $Sn(OH)_2$, no effect

43. Nature of nitride ion is
(a) acidic (b) basic
(c) amphiprotic (d) cannot predict

44. Wave number of spectral line for a given transition is $x\ cm^{-1}$ for He^+, then its value for Be^{3+} (isoelectronic of He^+) for same transition is
(a) $\frac{x}{4}\ cm^{-1}$ (b) $x\ cm^{-1}$
(c) $4x\ cm^{-1}$ (d) $16x\ cm^{-1}$

45. H_2O_2 oxidises MnO_2 is MnO_4^- in basic medium, H_2O_2 and MnO_2 react in the molar ratio of
(a) 1 : 1 (b) 2 : 1 (c) 2 : 3 (d) 3 : 2

46. 25 mL of an aqueous solution of KCl was found to require 20 mL of 1 M $AgNO_3$ solution when titrated using a K_2CrO_4 as indicator. Depression in freezing point of KCl solution with 100% ionisation will be [$K_F = 2.0°\ mol^{-1} kg$, molarity = molality]

(a) 3.2° (b) 1.6°
(c) 0.8° (d) 5.0°

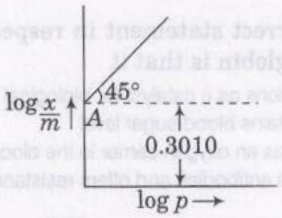
47. Out of the following, select Lux-Flood Acid

(a) CO_2 (b) BF_3
(c) H^+ (d) $Al(CH_3)_3$

48. On adding $AgNO_3$ solution into KI solution, a negatively charged colloidal sol. is obtained when they are in

(a) 100 mL of 0.1 M $AgNO_3$ + 100 mL of 0.1 M KI
(b) 100 mL of 0.1 M $AgNO_3$ + 100 mL of 0.2 mKI
(c) 100 mL of 0.2 M $AgNO_3$ + 100 mL of 0.1 M KI
(d) 100 mL of 0.15 M $AgNO_3$ + 100 mL of 0.15 M KI

49. Graph between $\log\left(\frac{x}{m}\right)$ and $\log p$ is a straight line at angle 45° with intercept OA as shown. Hence, $\left(\frac{x}{m}\right)$ at a pressure of 0.2 atm is



(a) 0.8 (b) 0.6
(c) 0.4 (d) 0.2

50. From the following reactions at 298 K,

(A) $CaC_2(s) + 2H_2O(l) \longrightarrow Ca(OH)_2(s) + C_2H_2(g)\ DH°(kJ\ mol^{-1}) - 127.9$

(B) $Ca(s) + \frac{1}{2}O_2(g) \longrightarrow CaO(s) - 635.1$

(C) $CaO(s) + H_2O(l) \longrightarrow Ca(OH)_2(s) - 65.2$

(D) $C(s) + O_2(s) \longrightarrow CO_2(s) - 393.5$

(E) $C_2H_2(g) + \frac{5}{2}O_2(g) \longrightarrow 2CO_2(g) + H_2O(e) - 1299.58$

Calculate the heat of formation of $CaC_2(s)$ at 298 K.

(a) – 59.82 KJ mol^{-1} (b) + 59.82 KJ mol^{-1}
(c) – 190.22 KJ mol^{-1} (d) +190.22 KJ mol^{-1}

Mathematics

1. If $x_r = \cos\left(\frac{\pi}{3^r}\right) - i\sin\left(\frac{\pi}{3^r}\right)$, (where $i = \sqrt{-1}$), then the value of $x_1 \cdot x_2 \ldots \infty$, is

(a) 1 (b) – 1
(c) $-i$ (d) i

2. Number of identical terms in the sequence 2, 5, 8, 11, ... upto 100 terms and 3, 5, 7, 9, 11, ... upto 100 terms are

(a) 17 (b) 33 (c) 50 (d) 147

3. If $1 + \lambda + \lambda^2 + \ldots + \lambda^n = (1+\lambda)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})$, then the value of n is (where, $n \in N$)

(a) 32 (b) 16
(c) 31 (d) 15

4. If one root of the equation $x^2 - \lambda x + 12 = 0$ is even prime while $x^2 + \lambda x + \mu = 0$ has equal roots, then μ is equal to

(a) 8 (b) 16
(c) 24 (d) 32

5. If α, and β are the roots of $ax^2 + c = bx$, then the equation $(a + cy)^2 = b^2 y$ in y has the roots

(a) α^{-1}, β^{-1} (b) α^2, β^2
(c) $\alpha\beta^{-1}, \alpha^{-1}\beta$ (d) α^{-2}, β^{-2}

6. If ${}^nC_{r-1} = 10$, ${}^nC_r = 45$ and ${}^nC_{r+1} = 120$, then r equals to

(a) 1 (b) 2
(c) 3 (d) 4

7. The number of six-digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 and 7, so that digits do not repeat and the terminal digits are even is

(a) 144 (b) 72
(c) 288 (d) 720

8. The coefficient of x^{20} in the expansion of $(1+x^2)^{40}\cdot\left(x^2+2+\frac{1}{x^2}\right)^{-5}$ is

(a) ${}^{20}C_{10}$ (b) ${}^{30}C_{25}$
(c) 1 (d) 0

9. If $\Delta=\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}$; $0\le\theta<2\pi$, then

(a) $\Delta=0$ (b) $\Delta\in(0,\infty)$
(c) $\Delta\in[-1,2]$ (d) $\Delta\in[2,4]$

10. If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$ and $A^2=8A+KI_2$, then K is equal to

(a) -1 (b) 1
(c) -7 (d) 7

11. If $x\in\left(\frac{3\pi}{2},2\pi\right)$, then the value of the expression

$\sin^{-1}[\cos\{\cos^{-1}(\cos x)\}+\sin^{-1}(\sin x)]$, is

(a) $-\frac{\pi}{2}$ (b) $\frac{\pi}{2}$
(c) 0 (d) π

12. The value of 'a' for which $ax^2+\sin^{-1}(x^2-2x+2)+\cos^{-1}(x^2-2x+2)=0$ has a real solution, is

(a) $-\frac{2}{\pi}$ (b) $\frac{2}{\pi}$
(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

13. If $\sin x+\cos x=\sqrt{\left(y+\frac{1}{y}\right)}$, $x\in[0,\pi]$, then

(a) $x=\frac{\pi}{4}, y=1$ (b) $y=0$
(c) $y=2$ (d) $x=\frac{3\pi}{4}$

14. If n be a positive integer such that $\sin\left(\frac{\pi}{2n}\right)+\cos\left(\frac{\pi}{2n}\right)=\frac{\sqrt{n}}{2}$, then

(a) $n=6$ (b) $n=2$
(c) $n=1$ (d) $n=3,4,5$

15. If $\tan x\cdot\tan y=a$ and $x+y=\frac{\pi}{6}$, then $\tan x$ and $\tan y$ satisfy the equation

(a) $x^2-\sqrt{3}(1-a)x+a=0$
(b) $\sqrt{3}x^2-(1-a)x+a\sqrt{3}=0$
(c) $x^2+\sqrt{3}(1+a)x-a=0$
(d) $\sqrt{3}x^2+(1+a)x-a\sqrt{3}=0$

16. The angle of elevation of the top of a tower at a point on the ground is 30°. If on walking 20 m toward the tower, the angle of elevation becomes 60°, then the height of the tower is

(a) 10 m (b) $\frac{10}{\sqrt{3}}$ m
(c) $10\sqrt{3}$ m (d) None of these

17. The centroid of the triangle is (3, 3) and the orthocentre is (–3, 5), then its circumcentre is

(a) (0, 4) (b) (0, 8)
(c) (6, 2) (d) (6, – 2)

18. Point $R(h,k)$ divides a line segment between the axes in the ratio 1 : 2. Find equation of the line.

(a) $2kx+hy=3hk$
(b) $2kx+hy=2hk$
(c) $2kx-hy=3hk$
(d) None of the above

19. If the slope of one of the lines represented by $ax^2+2hxy+by^2=0$ be the square of the other, then

(a) $a^2b+ab^2-6abh+8h^3=0$
(b) $a^2b+ab^2+6abh+8h^3=0$
(c) $a^2b+ab^2-3abh+8h^3=0$
(d) $a^2b+ab^2-6abh-8h^3=0$

20. The line $(x-2)\cos\beta+(y-2)\sin\theta=1$ touches a circle for all value of θ, then the equation of circle is

(a) $x^2+y^2-4x-4y+7=0$
(b) $x^2+y^2+4x+4y+7=0$
(c) $x^2+y^2-4x-4y-7=0$
(d) None of the above

21. If $(-3, 2)$ lies on the circle $x^2+y^2+2gx+2fy+c=0$ which is concentric with the circle $x^2+y^2+6x+8y-5=0$, then C is equal to

(a) 11 (b) -11
(c) 24 (d) 100

22. The angle between the tangents drawn from the origin to the parabola $y^2=4a(x-a)$ is

(a) 0 (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

23. If focii of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ coincide with the focii of $\frac{x^2}{25}+\frac{y^2}{9}=1$ and eccentricity of the hyperbola is 2, then

(a) $a^2+b^2=14$
(b) there is a director circle of the hyperbola
(c) centre of the director circle is (0, 0)
(d) length of latusrectum of the hyperbola is 12

24. $\dfrac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots\infty}{1+\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+\ldots\infty}$ is equal to

(a) $\frac{e+1}{e-1}$ (b) $\frac{e-1}{e+1}$
(c) $\frac{e^2+1}{e^2-1}$ (d) $\frac{e^2-1}{e^2+1}$

25. If n is even, then in the expansion of $\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\ldots\right)^2$, then the coefficient of x^n is

(a) $\frac{2^n}{n!}$ (b) $\frac{2^n-2}{n!}$
(c) $\frac{2^{n-1}-1}{n!}$ (d) $\frac{2^{n-1}}{n!}$

26. The angle between the line $\frac{x+1}{2!}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x+2y-11z=3$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\sin^{-1}\left(\frac{8}{21}\right)$

27. The area of the triangle whose vertices are $A(1, 2, 3)$, $B(2, -1, 1)$ and $C(1, 2, -4)$ is

(a) $7\sqrt{10}$ sq units (b) $\frac{1}{2}\sqrt{10}$ sq units
(c) $\frac{7}{2}\sqrt{10}$ sq units (d) None of these

28. If $\mathbf{a}$ and $\mathbf{b}$ are the vectors determined by two adjacent sides of regular hexagon, then vector $\mathbf{EO}$ is

(a) $(\mathbf{a}+\mathbf{b})$ (b) $(\mathbf{a}-\mathbf{b})$
(c) $2\mathbf{a}$ (d) $2\mathbf{b}$

29. If $\mathbf{a}=\mathbf{i}+\mathbf{j}+\mathbf{k}$, $\mathbf{b}=4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and $\mathbf{c}=\mathbf{i}+\alpha\mathbf{j}+\beta\mathbf{k}$ are linearly dependent vectors and $|\mathbf{c}|=\sqrt{3}$, then the values of α and β are respectively

(a) $\pm1, 1$ (b) $\pm2, 1$
(c) $0, \pm1$ (d) None of these

30. Let $\mathbf{u}=\mathbf{i}+\mathbf{j}$, $\mathbf{v}=\mathbf{i}-\mathbf{j}$ and $\mathbf{w}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$. If $\hat{\mathbf{n}}$ is a unit vector such that $\mathbf{u}\cdot\hat{\mathbf{n}}=0$ and $\mathbf{u}\cdot\hat{\mathbf{n}}=0$, then $|\mathbf{w}\cdot\hat{\mathbf{n}}|$ is equal to

(a) 3 (b) 0
(c) 1 (d) 2

31. If $g(x)=1+\sqrt{x}$ and $f\{g(x)\}=3+2\sqrt{x}+x$, then $f(x)$ is equal to

(a) $1+2x^2$ (b) $2+x^2$
(c) $1+x$ (d) $2+x$

32. Domain of $f(x)=y=\sqrt{\log_3\{\cos(\sin x)\}}$ is

(a) $\left\{\frac{n\pi}{2}: n\in I\right\}$
(b) $\{2n\pi: n\in I\}$
(c) $\{n\pi: n\in I\}$
(d) None of the above

33. $\lim_{x\to1}(\log_2 2x\log_x 5)$ is equal to

(a) $\log_2 5$ (b) $_e\log_2 5$
(c) e (d) 0

34. A function is defined as follows

$$f(x)=\begin{cases}1, & \text{when } -\infty<x<0\\ 1+\sin x, & \text{when } 0\le x<\dfrac{\pi}{2}\\ 2+\left(x-\dfrac{\pi}{2}\right)^2, & \text{when } \dfrac{\pi}{2}\le x<\infty\end{cases}$$

continuity of $f(x)$ is

(a) $f(x)$ is continuous at $x=\dfrac{\pi}{2}$

(b) $f(x)$ is continuous at $x=0$

(c) $f(x)$ is discontinuous at $x=0$

(d) $f(x)$ is continuous over the whole real number

35. If $f(x)=\sqrt{1-\sqrt{1-x^2}}$, then $f(x)$ is

(a) continuous on $[-1, 1]$

(b) differentiable on $(-1, 0)\cup(0, 1)$

(c) Both (a) and (b)

(d) None of the above

36. If $x=\sqrt{a^{\sin^{-1}t}}$, and $y=\sqrt{a^{\cos^{-1}t}}$, then the value of $\dfrac{dy}{dx}$ is

(a) $\dfrac{y}{x}$ (b) $\dfrac{x}{y}$ (c) $\dfrac{-y}{x}$ (d) $\dfrac{-x}{y}$

37. If f and g be differentiable function satisfying $g'(a)=2$, $g(a)=b$ and $fog=I$ (identity function), then $f'(b)$ is equal to

(a) $\dfrac{1}{2}$ (b) 2

(c) $\dfrac{2}{3}$ (d) None of these

38. If $x=at^2$ and $y=2at$, then $\dfrac{d^2y}{dx^2}$ at $t=2$, is

(a) $\dfrac{-1}{16a}$ (b) $\dfrac{1}{16a}$

(c) $\dfrac{1}{16}$ (d) $\dfrac{1}{a}$

39. The points at which the tangent to the curve $y=x^3-3x^2-9x+7$ is parallel to the x-axis are

(a) $(3, -20)$ and $(-1, 12)$ (b) $(3, 20)$ and $(1, 12)$

(c) $(1, -10)$ and $(2, 6)$ (d) None of these

40. $\int\dfrac{dx}{\sin^2 x\cdot\cos^2 x}$ is equal to

(a) $\tan x+\cot x+C$ (b) $\tan x-\cot x+C$

(c) $\tan x\cdot\cot x+C$ (d) $\tan x-\cot 2x+C$

41. $\int\dfrac{e^{2x}-1}{e^{2x}+1}dx$ is equal to

(a) $\dfrac{e^x+e^{-x}}{e^x-e^{-x}}+C$ (b) $\log\dfrac{e^x+e^{-x}}{e^x-e^{-x}}+C$

(c) $\log|e^x+e^{-x}|+C$ (d) $\log|e^x-e^{-x}|+C$

42. The value of $\int_0^1\tan^{-1}\left(\dfrac{2x-1}{1+x-x^2}\right)dx$ is

(a) 1 (b) 0

(c) -1 (d) $\dfrac{\pi}{4}$

43. If $\int_{\log 2}^{x}\dfrac{1}{\sqrt{e^x-1}}dx=\dfrac{\pi}{6}$, then the value of x is

(a) $\log 2$ (b) $\log 3$

(c) $\log 4$ (d) None of these

44. The area bounded by the curve $y=\sin x$ between $x=0$ and $x=2\pi$ is

(a) 1 sq unit (b) 2 sq units

(c) 4 sq units (d) 8 sq units

45. If the area above x-axis bounded by the curves $y=2^{Kx}$, $x=0$ and $x=2$ is $\dfrac{3}{\log 2}$, then the value of K is

(a) $\dfrac{1}{2}$ (b) 1

(c) -1 (d) 2

46. The solution of the differential equation $\sec^2 x\cdot\tan y\,dx+\sec^2 y\cdot\tan x\,dy=0$ is

(a) $\tan x\cdot\cot y=C$ (b) $\cot x\cdot\tan y=C$

(c) $\tan x\cdot\tan y=C$ (d) $\sin x\cdot\cos y=C$

47. Two cards are drawn at random from a pack of 52 cards. The probability of getting atleast a spade and an ace, is

(a) $\dfrac{1}{34}$ (b) $\dfrac{8}{221}$

(c) $\dfrac{1}{26}$ (d) $\dfrac{2}{51}$

48. The relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is

(a) reflexive (b) symmetric
(c) transitive (d) None of these

49. The proposition $\sim (p \Leftrightarrow q)$ is equivalent to

(a) $(p \vee \sim q) \wedge (q \wedge \sim p)$ (b) $(p \wedge \sim q) \vee (q \wedge \sim p)$
(c) $(p \wedge \sim q) \wedge (q \wedge \sim p)$ (d) None of the above

50. When two equal forces are inclined at an angle 2α, their resultant is twice as great as when they act at an angle 2β, then

(a) $\cos\alpha = 2\cos\beta$
(b) $\cos\alpha = 2\sin\beta$
(c) $\cos\beta = 2\cos\alpha$
(d) $\sin\beta = 2\cos\alpha$

Answers

Physics

1.	(c)	2.	(d)	3.	(a)	4.	(d)	5.	(b)	6.	(a)	7.	(b)	8.	(c)	9.	(c)	10.	(d)
11.	(c)	12.	(c)	13.	(b)	14.	(b)	15.	(d)	16.	(b)	17.	(b)	18.	(d)	19.	(a)	20.	(a)
21.	(b)	22.	(b)	23.	(c)	24.	(d)	25.	(b)	26.	(c)	27.	(a)	28.	(d)	29.	(b)	30.	(c)
31.	(c)	32.	(c)	33.	(a)	34.	(b)	35.	(c)	36.	(d)	37.	(a)	38.	(a)	39.	(b)	40.	(d)
41.	(b)	42.	(c)	43.	(d)	44.	(c)	45.	(a)	46.	(c)	47.	(c)	48.	(d)	49.	(b)	50.	(c)

Chemistry

1.	(a)	2.	(c)	3.	(b)	4.	(d)	5.	(c)	6.	(b)	7.	(c)	8.	(c)	9.	(a)	10.	(d)
11.	(c)	12.	(b)	13.	(a)	14.	(d)	15.	(c)	16.	(a)	17.	(b)	18.	(b)	19.	(c)	20.	(d)
21.	(a)	22.	(b)	23.	(a)	24.	(d)	25.	(c)	26.	(b)	27.	(a)	28.	(c)	29.	(a)	30.	(c)
31.	(b)	32.	(c)	33.	(a)	34.	(d)	35.	(c)	36.	(d)	37.	(a)	38.	(b)	39.	(c)	40.	(d)
41.	(b)	42.	(b)	43.	(b)	44.	(c)	45.	(d)	46.	(a)	47.	(a)	48.	(b)	49.	(a)	50.	(a)

Mathematics

1.	(c)	2.	(b)	3.	(c)	4.	(b)	5.	(d)	6.	(b)	7.	(d)	8.	(b)	9.	(d)	10.	(c)
11.	(b)	12.	(c)	13.	(a)	14.	(a)	15.	(b)	16.	(c)	17.	(c)	18.	(a)	19.	(a)	20.	(a)
21.	(b)	22.	(b)	23.	(d)	24.	(b)	25.	(d)	26.	(d)	27.	(c)	28.	(b)	29.	(a)	30.	(a)
31.	(b)	32.	(c)	33.	(b)	34.	(d)	35.	(a)	36.	(c)	37.	(a)	38.	(a)	39.	(a)	40.	(b)
41.	(c)	42.	(b)	43.	(c)	44.	(c)	45.	(b)	46.	(c)	47.	(c)	48.	(d)	49.	(b)	50.	(a)

Hints & Solutions

Physics

1. $F = \frac{Gm_1m_2}{r^2}$

$\Rightarrow \quad G = \frac{Fr^2}{m_1m_2} = \frac{[MLT^{-2}][L^2]}{[M^2]}$

$= [M^{-1}\, L^3\, T^{-2}]$

2. Given, $|\mathbf{A} \times \mathbf{B}| = \sqrt{3}\, \mathbf{A} \cdot \mathbf{B}$

$\Rightarrow \quad AB \sin\theta = \sqrt{3}\, AB \cos\theta$

$\Rightarrow \quad \tan\theta = \sqrt{3} \text{ or } \theta = 60°$

Hence, $|\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{A^2 + B^2 + 2AB\cos 60°}$

$= \sqrt{A^2 + B^2 + 2AB\frac{1}{2}}$

$= (A^2 + B^2 + AB)^{1/2}$

3. The gravitational force will remain the same as it is independent of density of medium.

4. Let the volume of the body be V and density be d. Then for the case of water

$$V \times d \times g = \frac{3}{4} \times v \times d_{water} \times g$$

For oil $V \times d \times g = \frac{2}{3} \times V \times d_{oil} \times g$

$\Rightarrow \quad \frac{d_{oil}}{d_{water}} = \frac{9}{8}$

5. We have, $h = \frac{2\sigma\cos\theta}{r\rho g}$

$\Rightarrow \quad h.r = \text{constant}$

6. According to the given figure, A attains mean position $T/4$ time earlier than C. So, C lags behind A by $\frac{\pi}{2}$. Also B attains the mean position $\frac{T}{2}$ time earlier.

Hence, B leads A by $\frac{\pi}{2}$.

7. We have, $\Delta Q = \Delta U + \Delta W$

Here, $\Delta Q = 110$ J, $\Delta U = 40$ J

$\therefore \quad \Delta W = 70$ J

8. For a small angle prism, $\delta = (u - 1)A$ ...(i)

Since, the ray emerges normally, $\therefore e = 0$

By the relation,

$$A + \delta = i + e$$

We have, $i = A + \delta$

or $6 = (i - A)$ ...(ii)

From Eqs. (i) and (ii)

$(i - A) = (\mu - i)A$

$\Rightarrow \quad i = \mu A$

9. Two plates of B are joined together and therefore cannot form a capacitor. Thus, there are only two capacitors in parallel – one between the upper most plate and the next one and second between the lower most plate and the one above it.

Hence, $C = C_1 + C_2 = \frac{\varepsilon_0 5}{d} + \frac{\varepsilon_0 5}{d} = \frac{2\varepsilon_0 5}{d}$

10. As the given condition is of balanced wheat stone bridge.

$\therefore \quad I_1 \times 12 = (1.4 - I_1) \times 30$

$\Rightarrow \quad I_1 \times 42 = 1.4 \times 30$

$\Rightarrow \quad 1.4 \times \frac{30}{42} = 1\,\text{A}.$

11. We have $I = ne\ Av_d$

and $I = \frac{V}{R}$

$\therefore \quad \frac{V}{R} = ne\ A v_d$

or $V = ne\ A v_d \cdot R$

$= neA v_d \left(\rho \frac{l}{A}\right)$

$\therefore \quad \frac{V}{l} = E = ne\rho l v_d$

$\Rightarrow \quad E \propto v_d$

12. Poynting vector which is in the direction of travel of electromagnetic wave is along the vector $\mathbf{E} \times \mathbf{B}$.

13. The current an a LCR circuit is given as

$$I = \frac{E_0 \sin(\omega t \pm \phi)}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Current is maximum if,

$R^2 + (X_L - X_C)^2 =$ minimum

$\Rightarrow \quad X_L - X_C = 0$

or $\quad X_L = X_C$

14. $\bar{A} + A$ should always be equal to one as addition of quantity like 1 to 0 or 0 to 1 always gives 1.

15. The given truth table is of NAND gate.

16. We have, $e = \left|\frac{\Delta\phi}{\Delta t}\right|$

$\therefore$ Current, $I = \frac{e}{R} = \frac{\Delta\phi}{R\Delta\phi}$

So, charge $Q = I \cdot \Delta t = \frac{\Delta\phi}{R\Delta\phi} \cdot \Delta t = \frac{\Delta\phi}{R}$

17. We have, $v^2 = u^2 + 2as$

$\Rightarrow \quad (80)^2 = 0 + 2(a)\left(4\pi \times \frac{20}{\pi}\right)$

[as two revolutions are completed]

$\Rightarrow \quad a = 40\,\text{m/s}^2$

18. The output of a OR gate is

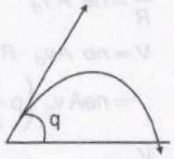
$Y = A + B$

19. In series, power, $P = \frac{V^2}{2R} = \frac{P}{2} = 50\ \text{W}$

In parallel, power, $P = \frac{V^2}{R/2} = 2P = 20\ \text{W}$

20. Barrier potential of a p-n junction diode does not depend on diode design.

21. $|(\mathbf{P}_f - \mathbf{P}_i)|$ = Change in momentum



$= \sqrt{P^2 + P^2 + 2P \cdot P \cos(\pi - 2\theta)}$

$= P\sqrt{2 + 2(-\cos\theta)}$

$= P\sqrt{2\ (2\sin^2\theta)}$

$= 2P \sin\theta$

22. Maximum tension $= \frac{mv^2}{r} + mg$

$3.7 \times 10 = \frac{0.5v^2}{4} + 0.5 \times 10$

$\Rightarrow \quad v = 16\ m/s$

$\therefore \quad \omega = \frac{v}{r} = \frac{16}{4} = 4\,\text{rad/s}$

23. Change in angular momentum = Torque × time

$\Rightarrow$ Torque $= \frac{4A - A}{4} = \frac{3}{4}A$

24. By the relation, $g = \frac{GM}{R^2}$ we have

$$\frac{g_P}{g_e} = \frac{M_p}{M_e} \times \frac{R_e^2}{R_p^2} = 2 \times (2)^2$$

$\therefore \quad g_p = 8 \times 10 = 80\,\text{m/s}^2$

25. Total volume is conserved.

$\therefore \quad \frac{4\pi}{3}(3)^3 + \frac{4\pi}{3}(4)^3 = \frac{4\pi}{3}R^3$

or $\quad R \cong 4.5\,\text{cm}$

26. (b) $E = 9 \times 10^9 \times \frac{q}{n}$

$$= 9 \times 10^9 \frac{1.6 \times 10^{-19}}{(1 \times 10^{-10})^2} = 1.44 \times 10^{11}\ \text{M/C}$$

27. For Q, closed pipe. $n = (2K + 1)\ C/4$

for $K = 0$, it gives

$L = 1 \times (10^2 \times 330) / 4 \times 260 = 31.7\ \text{cm}$

28. This is not possible as it is against the law of conservation of energy.

29. Only if the three forces are concurrent, they cannot produce a net clockwise or anticlockwise momentum, which is neccessary for rotation.

30. Moving beams of protons are like currents. Two parallel currents attaract each other.

31. We have, $\lambda_{Ph} = \frac{h}{P_{Ph}}$ and $\lambda_e = \frac{h}{P_e}$

Given, $\lambda_{Ph} = \lambda_e$

$\therefore$ We get, $P_{Ph} = P_e$

$$\frac{h}{\lambda_{Ph}} = mv$$

$\therefore \quad \frac{hc}{\lambda_{Ph}} = mcv = \frac{1}{2}mv^2\left(\frac{2c}{v}\right)$

or $\quad \frac{E_{Ph}}{E_e} = \frac{2c}{v}$

32. $Y = 1$, if $A = 1$, $B = 0$ and $C = 1$ because A and B are input of OR gate and the output of A and B and as well as C are the inputs of AND gate.

33. We have, $v \propto r$

$$\Rightarrow \quad v = Hr$$

where, H is Hubble's" constant.

34. Slope of displacement *versus* time graph gives the velocity. This is constant here only during the bc, as the graph is a straight line in it. Constant velocity means no acceleration,

35. For a long conductor, magnetic field at distance r is

$$B = \frac{\mu_0}{4\pi} \times \frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$$

$$\therefore \quad B \propto \frac{1}{r}$$

So, graph between B and r should be a parabola as given in option (c).

36. Time period is given by, $T = 2\pi\sqrt{\frac{I}{MH}}$

Hence, it does not depend upon the length of the suspension.

37. We have, $B = \mu H$

$$\therefore \quad \mu = \frac{B}{H}$$

So, the permeability μ is defined as the ratio of magnetic induction to the magnetising field.

38. As $\phi = \mathbf{B} \cdot \mathbf{A}$

$$\Rightarrow \quad \phi = 2 \times 0.5 \times \cos 60°$$
$$= 2 \times 0.5 \times \frac{1}{2} = 0.5 \text{ Wb}$$

39. In the diagram (b), the positive terminal is at lower potential and negative terminal is at higher potential, hence it is reversed biased.

40. Moment of inertia of the system

$$I = I_1 + I_2 + I_3$$
$$= 0 + m\left(\frac{l}{2}\right)^2 + ml^2$$
$$= \frac{ml^2}{4} + ml^2$$
$$= \frac{5}{4} ml^2$$

41. Given $R_g = 50\ \Omega$,

$$I_g = 25 \times 4 \times 10^{-4} \text{ A}$$
$$= 10^{-2} \text{ A}$$

Range of $V = 25$ volts

Resistance, $R = \frac{V}{I_g} - R_g$

$$= \left(\frac{25}{10^{-2}} - 50\right)\Omega$$
$$= 2450\ \Omega$$

Thus, a resistance of 2450 Ω is connected is series to convert it into a voltmeter.

42. We have, $\nu = \frac{v + v_s}{v - v_s}\nu$

or $\quad 2f = \frac{v + v_s}{v - v_s} f$

or $\quad 2(v - v_s) = v + v_s$

or $\quad 2v - 2v_s = v + v_s$

$\therefore \quad v = 3v_s$

or $\quad v_s = \frac{v}{3}$

43. Rise = Real depth-apparent depth

$$= R.D - \frac{R.D}{\mu}$$
$$= R.D\left(1 - \frac{1}{\mu}\right)$$

Thus, for least rise μ should be least. Refractive index is least for red colour.

44. We have, $\mu = \frac{\sin i}{\sin r}$ and also $\mu = \frac{c}{v}$

$$\therefore \quad \frac{c}{v} = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad v = c \cdot \left(\frac{\sin r}{\sin i}\right)$$

45. As the angle between **v** and **E** is neither zero nor π, a constant force $(-qE)$ will act on the electron. The motion of the electron, therefore will be along a parabola.

46. We know $r = \left(\frac{E}{V} - 1\right)R$

hence, $\quad R = 3.9\pi,\ E = 2.0\text{ V},\ r = 0.1\ \Omega$

hence, $\quad V = 1.95\text{ V}$

47. Heat required, $Q = 1000 \times 1 \times (100 - 20)$

$= 1000 \times 80$ cal

Heat produced in time t is, $H = VIt$

$= 220 \times 4 \times t$ J

$= \frac{220 \times 4 \times t}{4.18}$ cal

$\therefore \quad = \frac{220 \times 4 \times t}{4.18} = 1000 \times 80$

$\Rightarrow \quad t = \frac{1000 \times 80}{4.18 \times 220 \times 4} = 6.3$ min

48. (d) Pair production is just oppsite to mass annihilation. When suddenly γ-ray energy disappears to form a pair of a particle and an anti-particle, the process is called pair production.

49. (b) Resolving power $\propto \frac{1}{\lambda}$ and $\propto \frac{1}{\sqrt{V}}$

$\therefore \quad R \cdot P \propto \sqrt{V}$

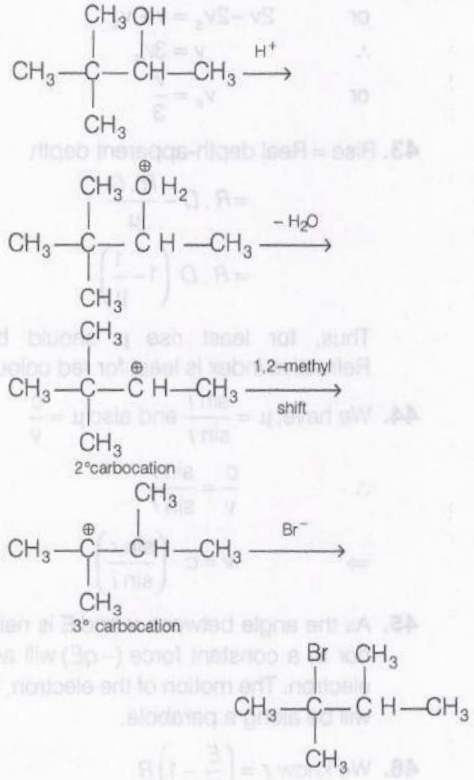
$\Rightarrow \quad (R.P)_2 = (R.P)_1 \times \sqrt{\frac{80\text{kV}}{20\text{kV}}}$

$= 2(R.P)_1$

50. (c) Electric field between the oppositely charged plates of a capacitor is twice that due to one plate Hence, when one plate is removed, the electric force reduces to half of its earlier value.

Chemistry

1.



2. A β-hydroxy carbonyl compound is formed by condensation of carbonyl compound with at least one hydrogen atom at α-carbon.

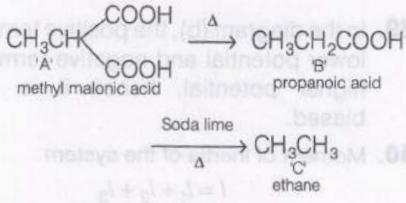
3. Positive inductive effect (+ *I* effect) is arises due to electron releasing group (*y*). It develops negative charge on chain and exerts a positive inductive effect.

$$\overset{\delta-}{C} \longleftarrow \overset{\delta+}{y}$$

Therefore, + *I* effect tends to decrease positive charge and tends to increase stability.

4. $\underset{A}{C_4H_6O_4} \xrightarrow{\Delta} \underset{B}{C_3H_6O_2}$

From above equation, it is clear that CO_2 is lost due to heating. Thus, '*A*' is a dibasic acid with two —COOH groups on same carbon atom (*gem* position). Thus, '*A*' is



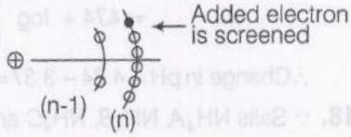
For 1 mole of '*C*' again, CH_3CH_3 (mol *wt* = 30)

∵ 30 g of *C* occupies volume = 22.42

$\therefore$ 1 g of *C* with occupy volume $= \frac{22.4}{30}$

$\simeq 0.75$ L

5. When an extra electron is added in nitrogen atom, added electron will be screened by five electrons in second orbit ($2s^2 2p^3$) and two electrons in first orbit ($1s^2$).



Now screening constant, $\sigma = 5 \times 0.35$ in nth orbit + 2×0.85 in $(n-1)$th orbit

$$= 1.75 + 1.70 = 3.45$$

$\therefore$ Effective nuclear charge $= 7 - 3.45 = 3.55$

6. $MS_2 \equiv 2D$

100 40.06

$$\therefore \quad \frac{M + 64}{100} = \frac{64}{40.06}$$

$$\text{or} \quad 40.06\,(M + 64) = 100 \times 64$$

$$\text{or} \quad M + 64 = \frac{6400}{40.06}$$

$$\text{or} \quad M + 64 \approx 160$$

$$\text{or} \quad M \approx 160 - 64$$

$$\text{or} \quad M \simeq 96$$

7. Formal charge, $F = v - \frac{s}{2} - u$

Where, v = valency electrons

s = shared electrons

u = unshared electrons

$$\underset{\text{2 shared electrons}}{H—N} \equiv \underset{\text{6 unshared electrons}}{C :\leftarrow 2}$$

unshared electrons

	H	N	C
v	1	5	4
s	2	8	6
u	0	0	2
F	0	1	−1

8. $R = 0.0821$ L atm mol^{-1} K^{-1}

$$= 8.314 \times 10^7 \text{ ergs mol}^{-1}\text{ K}^{-1}$$

$$= 8.314 \text{ J mol}^{-1}\text{K}^{-1}$$

$$= 0.002 \text{ kcal mol}^{-1}\text{ K}^{-1}$$

$$\therefore \quad 1 \text{ L atm} = \frac{R}{0.0821} \text{ mol K}$$

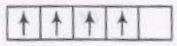
$$1 \text{ erg} = \frac{R}{8.314 \times 10^7} \text{ mol K}$$

$$1 \text{ J} = \frac{R}{8.314} \text{ mol K}$$

$$1 \text{ kcal} = \frac{R}{0.002} \text{ mol K}$$

Thus, II < III < I < IV

9. Fe^{2+} $(z = 26)$ = Ar $3d^6$ 

Cr^{2+} $(z = 24)$ = Ar $3d^4$ 

10. $2Cu + H_2O + SO_3 + O_2 \rightarrow$

$$Cu_2\,(OH)_2\,SO_4$$

$$2Cu_2 + H_2O + CO_2 + O_2 \longrightarrow$$

$$Cu_2\,(OH)_2\,CO_3$$

11. Mischmetal is an alloy of lanthanide metal (95%) and iron (about 5%) with traces of sulphur, carbon, calcium and aluminium. It is used in the production of bullets, shells and lighter flint.

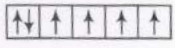
12. $[Fe(H_2O)_5NO]^{2+}$

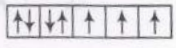
This is a typical case of complex formed by electron exchange. NO changes to NO^+ by loss of electron and this electron is gained by Fe^{2+} which changes to Fe^+

$$NO \longrightarrow NO^+ + e^-$$

$$Fe^{2+} + e^- \longrightarrow Fe^+$$

Thus, oxidation state of iron = + 1

Fe^{2+} = Ar 4s $3d^6$ 

Fe^+ = Ar 4s $3d^7$ 

Three unpaired electrons + 1 oxidation state is conirmed by magnetic moment of iron $= \sqrt{3(3+2)}$

$= \sqrt{15}$ BM. and also by diamagnetic character of NO (which is only possible when it has no unpaired electron).

13. $3BCl_3 + 3NH_4Cl \xrightarrow[C_6H_5Cl]{140°C} B_3N_3H_3Cl_3$

$$\xrightarrow{NaBH_4} \underset{\text{borazine or inorganic benzene}}{B_3N_3H_6}$$

14. Let atomicity of sulphur is n, then it exists as S_n.

$\therefore$ Molar mass, $m = 32n$

Now from, $\Delta T_f = \dfrac{1000\, w\, K_f}{m\, w}$

$$0.010 = \frac{1000 \times 2.56 \times 0.1}{m \times 100}$$

or $$m = \frac{1000 \times 2.56 \times 0.1}{0.010 \times 100} = 256$$

or $$32n = 256$$

or, $$n = \frac{256}{32} = 8$$

15. By set I, half-life is 2 min. In set-II, number of moles have been doubled, thus half-life is also doubled, *i.e.*, now it is 4 min. Thus,

8 mol change to 4 mol (half) in = 4 min and 4 mol change to 2 mol (half) in = 2 min.

Thus, total time = 4 + 2 = 6 min.

16. $NH_4OH + H^+ \rightleftharpoons NH_4^+ + H_2O$

$$\therefore \quad K = \frac{[NH_4^+]\,[H_2O]}{[NH_4OH]\,[H^+]} = 1.8 \times 10^9$$

Again,

$NH_3 + H_2O \longrightarrow NH_4OH \rightleftharpoons NH_4^+ + OH^-$

$$\therefore \quad K' = \frac{[NH_4^+]\,[OH^-]}{[NH_4OH]}$$

Now $$\frac{K'}{K} = \frac{[NH_4^+]\,[OH^-]}{[NH_4OH]} \times \frac{[NH_4OH]\,[H^+]}{[NH_4^+]\,[H_2O]}$$

$$= [OH^-]\,[H^+] \quad (\because H_2O \text{ is in excess})$$

$$= K_w = 1 \times 10^{-14}$$

$$\therefore \quad K' = K \times 1 \times 10^{-14}$$

$$= 1.8 \times 10^9 \times 10^{-14} = 1.8 \times 10^{-5}$$

17. pH of 0.01 M CH_3COOH

$$pH = \frac{1}{2}[pK_a - \log C]$$

$$= \frac{1}{2}(4.74 - \log 0.01)$$

$$= \frac{1}{2}(4.74 + 2)$$

$$= 3.37$$

When 0.01 mol CH_3COONa is added to it, it is now a buffer and [[CH_3COONa] = 0.01] M.

Now from

$$pH = pK_a + \log \frac{[CH_3COO]}{[CH_3COOH]}$$

$$= 4.74 + \log \frac{0.01}{0.01} = 4.74$$

$\therefore$ Change in pH = 4.74 – 3.37 = 1.37

18. $\because$ Salts NH_4A, NH_4B, NH_4C are of weak acid and weak base.

$$\therefore \quad pH = 7 + \frac{pK_a}{2} - \frac{pK_a}{2}$$

Thus, greater the value of pK_a of HA, greater the pH.

pK_b (A^-, B^-, C^-) | $\underset{4}{A^-} < \underset{5}{B^-} < \underset{6}{C^-}$

$\therefore pKa$ (HA, HB, HC) | $\underset{8}{HC} < \underset{9}{HB} < \underset{10}{HA}$

Thus, order of pH values is

($NH_4C < NH_4B < NH_4A$).

19. $p = 0.0072$ atm

$= 0.0072 \times 76$ cm of Hg

$= 0.0072 \times 76 \times 13.6$ cm of mercury column

Also, $p = h \times 1$ cm of water column

$\therefore \quad h \times 1 = 0.0072 \times 76 \times 13.6$ cm

or $h = 7.4$ cm

20. $\because \Delta H°$ and $\Delta S°$ remain constant in the given temperature range.

$$\therefore \quad \Delta S° = -\frac{\Delta G° - \Delta H°}{T}$$

$$= -\left(\frac{-66.9 + 41.8}{300}\right)$$

$$= 0.08367 \text{ kJ K}^{-1} \text{ mol}^{-1}$$

$$\therefore \quad \Delta G°_{330} = \Delta H° - T\Delta S°$$

$$= -41.8 - 330 \times 0.08367$$

$$= -69.4 \text{ kJ mol}^{-1}$$

21. From Kirchhoff's equation

$$\Delta H_2\,(1000\text{ K}) = \Delta H_1\,(300\text{ K}) + \Delta C_p\,(1000 - 300)$$

Here, ΔH_2 (1000 K) = – 123.77 kJ mol^{-1}

ΔH_1 (300 K) = ?

$$\Delta C_p = 2C_p\ (NH_3) - [C_p(N_2), +3C_p\ (H_2)]$$
$$= -6\ R$$
$$= -6 \times 8.314 \times 10^{-3}\ kJ$$
$$\therefore \quad -123.77 = \Delta H_1\ (300\ K) - 6 \times 8.314 \times 10^{-3} \times 700$$
or $\Delta H_1\ (300\ K) = -88.85\ kJ$

For two moles of NH_3

$$\therefore \quad \Delta H_f\ (NH_3) = \frac{\Delta H_1 (300\ K)}{2}$$
$$= -\frac{88.85}{2}$$
$$= -44.42\ kJ\ mol^{-1}$$

22.

	Electrode	General Name
(A)	Calomel	Reference
(B)	Glass	Memberane
(C)	Hydrogen	Gas
(D)	Quinhydrone	Redox

23. $E_{cell} = E^\circ_{cell} = -\frac{0.0591}{2} \log \frac{[Zn^{2+}]}{[Cu^{2+}]}$

$$= 1.1 - \frac{0.0591}{2} \log \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

$E_{cell} < 1.1$ if $\frac{[Zn^{2+}]}{[Cu^{2+}]} > 1$

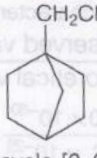
24. $2H^+ + 2e^- \longrightarrow H_2$

$Al^{3+} + 3e^- \longrightarrow Al$

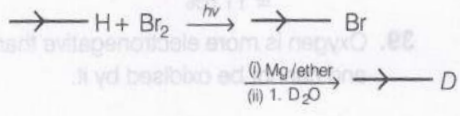
$$\frac{w(H_2)}{w(Al)} = \frac{E(H_2)}{E\ (Al)} = \frac{1}{9}$$

$$w(H_2) = 4.5 \times \frac{1}{9} = 0.5\ g = \frac{1}{4}\ mol\ H_2$$
$$= \frac{22.4}{4} = 5.6\ L\ H_2\ at\ STP$$

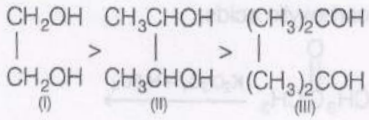
25.

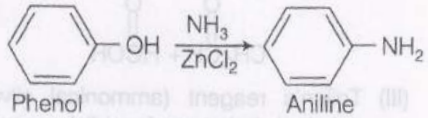


1 – ethylbicyclo [2, 2, 1] heptane

26. 

27. Rate of oxidation depends upon steric hinderance. Greater the steric hinderance the slower is the oxidation. Thus, the correct order is

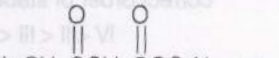


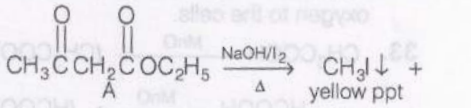
28. 

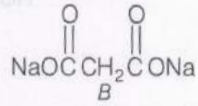
Phenol — Aniline (Primany amine)

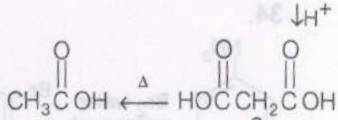
29. '*A*' gives iodoform test thus *A* has group. '*C*' eliminates CO_2 on heating hence, *C* has two —COOH groups at same (*gem*) position.

Since, only (a) has  group, therefore,

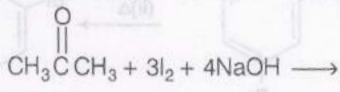
compound '*A*' is  .

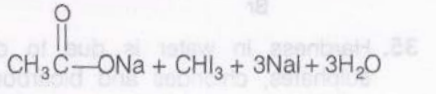






30. (I) Methyl ketones (CH_3CR) react rapidly with halogens, (Cl_2, Br_2, I_2) in presence of alkali to form haloform.





$$CH_3\overset{O}{\overset{\|}{C}}-ONa + H^+ \xrightarrow[-Na^+]{\Delta} CH_3\overset{O}{\overset{\|}{C}}OH$$

(II) Stronger oxidising agents oxidise ketones to carboxylic acids.

$$CH_3\overset{O}{\overset{\|}{C}}CH_3 \xrightarrow{K_2Cr_2O_7 + H_2SO_4} CH_3\overset{O}{\overset{\|}{C}}H + H\overset{O}{\overset{\|}{C}}OH$$

(III) Tollen's reagent (ammonical silver nitrate solution), being weak oxidising agent cannot reduce acetone into acetic acid.

$$CH_3\overset{O}{\overset{\|}{C}}CH_3 + [Ag(NH_3)_2^+] \longrightarrow \text{No reaction}$$

31. Benzyl carbocation is more stable then alkyl ($CH_3\overset{\oplus}{C}H_2$). CH_3O- and CH_3-, both stabilise benzyl carbocation and among them CH_3O- is more stable then CH_3-. Thus, correct order of stabilities will be

$$IV < II < III < I$$

32. Haemoglobin is transports protein. It transport oxygen to the cells.

33.
$$CH_3COOH \xrightarrow{MnO} (CH_3COO)_2Mn$$
$$HCOOH \xrightarrow{MnO} (HCOO)_2Mn$$
$$(CH_3COO)_2Mn + (HCOO)_2Mn \xrightarrow{\Delta} CH_3CHO$$

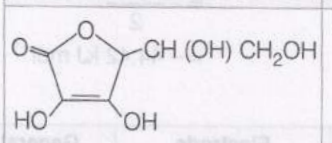
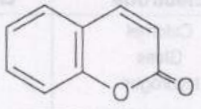
34.

35. Hardness in water is due to dissolved sulphates, chlorides and bicarbonates of calcium and magnesium. When such type of water is used for cleaning clothes with soap (*i.e.*, sodium stearate) a curdy precipitate of calcium and magnesium salts are formed.

$$\underbrace{RCOONa}_{\text{soap}} + \underbrace{Ca^{2+} \text{ or } Mg^{2+}}_{\text{in hard water}} \longrightarrow (RCOO)_2Ca \text{ or } Mg$$

Cardy precipitate

36.

	Structure of compound	Name of compound
(a)		Ascorbic acid
(b)		Coumarin

37.

S.No.	Heterocyclic amine base	Abbreviation	Occurrence
(i)	Adenine	A	DNA, RNA
(ii)	Guanine	G	DNA, RNA
(iii)	Cytosine	C	DNA, RNA
(iv)	Thymine	T	DNA
(v)	Uracil	U	RNA

38. Theoretical value of dipole moment of 100% ionic character $= e \times d$

$$= (1.6 \times 10^{-19}C)\,(1.41 \times 10^{-10}m)$$
$$= 2.26 \times 10^{-29}\ Cm$$

Observed value of dipole moment
$$= 2.60 \times 10^{-30} Cm$$

$\therefore$ Percent ionic character
$$= \frac{\text{observed value}}{\text{theoretical value}} \times 100$$
$$= \frac{2.60 \times 10^{-30}}{2.26 \times 10^{-29}} \times 100$$
$$= 11.5\%$$

39. Oxygen is more electronegative than chlorine and cannot be oxidised by it.

40. In Goldschmidt's alumino thermite process, is used when a high temperature is needed for carbon to reduce an oxide. In this process a large amount of energy (1675 kJ mol^{-1}) is liberated on oxidation to Al_2O_3.

$$3Mn_3O_4 + 8Al \longrightarrow 9Mn + 4Al_2O_3$$
$$B_2O_3 + 2Al \longrightarrow 2B + Al_2O_3$$
$$Cr_2O_3 + 2Al \longrightarrow 2Cr + Al_2O_3$$

41. In Nelson's cell, H_2 gas is liberated at cathode wheres Cl_2 gas is liberated at anode, on electrolysis of solution of NaCl.

$$NaCl \rightleftharpoons Na^+ + Cl^-$$
$$H_2O \rightleftharpoons H^+ + OH^-$$

at cathode

$$H^+ + e^- \longrightarrow H$$
$$H + H \longrightarrow H_2 \uparrow$$

at anode

$$Cl^- \longrightarrow Cl + e^-$$
$$Cl + Cl \longrightarrow Cl_2 \uparrow$$

42. $SnCl_2 + 2NaOH \longrightarrow \underset{\text{white ppt}}{Sn(OH)_2 \downarrow} + 2NaCl$

$$Sn(OH)_2 + 2NaOH \longrightarrow \underset{\text{sodium stannite}}{Na_2SnO_2} + 2H_2O$$

43. $N_3^-(aq) + 3H_2O(l) \longrightarrow NH_3(g) + 3OH^-(aq)$

N_3^- accepts proton, hence, it is strong Bronsted Lowry base and reacts with water to produce NH_3 and OH^- ions.

44. $\bar{\nu}$ (wave number) $= \bar{R}_H Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$\bar{\nu}_1 (He^+, Z = 2) = \bar{R}_H (2)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\bar{\nu}_2 (Be^{3+}, Z = 4) = \bar{R}_H = (4)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\therefore \quad \frac{\nu_2}{\nu_1} = \frac{(4)^2}{(2)^2}$$

$$= \frac{16}{4} = 4$$

$$\therefore \quad \bar{\nu}_2 = 4\bar{\nu}_1$$

$$\bar{\nu}_2 = 4x \text{ cm}^{-1}$$

45.

$$\overset{-2}{H_2O_2} + \overset{+4}{MnO_2} \longrightarrow \overset{+7}{Mn}O_4^- + \overset{-4}{2H_2O}$$

$-3e^-$, $+2e^-$

(Oxidation number two O atoms taken)

On balancing of oxidation number

$$3H_2O_2 + 2MnO_2 \longrightarrow 2MnO_4^- + 6H_2O$$

Thus, $H_2O_2 : MnO_2 = 3 : 2$

46. $\frac{KCl}{M_1V_1} = \frac{AsNO_3}{M_2V_2}$

$$M_1 = \frac{1 \times 20}{25}$$
$$= 0.8\,M$$
$$i = (1 + x)$$
$$= 1 + 1 = 2$$
$$\therefore \quad \Delta T_f = \text{molality} \times K_f \times i$$
$$= 0.8 \times 2 \times 2$$
$$= 3.2°$$

47. According to Lux- Flood concept, acid is defined as a substance which can accept oxide and base as a substance which can donate oxide.

$$\underset{\text{base}}{CaO} + \underset{\text{acid}}{CO_2} \longrightarrow Ca^{2+}CO_3^{2-}$$

48. Negatively charged colloidal sol is formed when $AgNO_3$ is completely precipitated as AgI and extra KI is absorbed on AgI.

$$Ag^+ + I^- \longrightarrow AgI$$
$$AgI + I^- \longrightarrow [AgI]I^-$$

Thus, $[Ag^+] < [I^-]$

49. $\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log p$

Given, $\log k = 0.3010 =$ intercept on y-axis

and $\frac{1}{n} = \tan 45° = 1 =$ slope

$$\therefore \quad \log\left(\frac{x}{m}\right) = 0.3010 + 1x \log 0.2$$
$$= \log 2 + \log 0.2$$
$$= \log 0.4$$
$$\therefore \quad \frac{x}{m} = 0.4$$

50. We have to calculate ΔH° for the following reaction in which one mole of $CaC_2(s)$ is formed form its elements.

$$Ca(s) + 2C(s) \longrightarrow CaC_2(s)$$

This is obtained when we operate.

$$(B) + (C) + z(D) - (E) - a$$

$$\therefore \Delta H = (-635.1) + (-65.2) + 2(-393.5) - (-1299.58) - (-127.9)\ kJ\ mol^{-1}$$

$$= -59.82\ kJ\ mol^{-1}$$

Mathematics

1. Since, $x_r = \cos\left(\frac{\pi}{3^r}\right) - i\sin\left(\frac{\pi}{3^r}\right)$

$$x_1 \cdot x_2 \cdot x_3 \ldots \infty = \cos\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \ldots \infty\right) - i\sin\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \ldots \infty\right)$$

$$= \cos\left(\frac{\frac{\pi}{3}}{1-\frac{1}{3}}\right) - i\sin\left(\frac{\frac{\pi}{3}}{1-\frac{1}{3}}\right)$$

$$= \cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right) = -i$$

2. Common terms are 5, 11, 17, ...

$\therefore$ term of common sequence,

$$t_n = 5 + (n-1)6 = (6n-1)$$

Also, 100th term of first sequence

$$= 2 + (100-1)3 = 299$$

and 100th term of the second sequence

$$= 3 + (100-1)2 = 201$$

$$\Rightarrow \quad t_n \le 201$$

$$\Rightarrow \quad 6n - 1 \le 201$$

$$\Rightarrow \quad n \le 33\frac{2}{3}$$

$$\therefore \quad n = 33 \qquad (\because n \in N)$$

3. $\text{LHS} = \frac{1(1-\lambda^{n+1})}{(1-\lambda)} = \left(\frac{1-\lambda^{n+1}}{1-\lambda}\right)$

and $\text{RHS} = (1+\lambda)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})$

$$= \frac{\left[(1-\lambda)(1+\lambda)(1+\lambda^2)(1+\lambda^4)(1+\lambda^8)(1+\lambda^{16})\right]}{(1-\lambda)}$$

$$= \frac{(1-\lambda^{32})}{(1-\lambda)}$$

$$\Rightarrow \quad \frac{1-\lambda^{n+1}}{1-\lambda} = \frac{1-\lambda^{32}}{1-\lambda}$$

$$\Rightarrow \quad 1-\lambda^{n+1} = 1-\lambda^{32}$$

$$\Rightarrow \quad n+1 = 32$$

$$\therefore \quad n = 31$$

4. We know that, only even prime is 2.

Then, $(2)^2 - \lambda(2) + 12 = 0$

$$\Rightarrow \quad \lambda = 8 \qquad \ldots(i)$$

and $x^2 + \lambda x + \mu = 0$

has equal roots.

$$\therefore \quad \lambda^2 - 4\mu = 0$$

or $(8)^2 - 4\mu = 0$ [from Eq. (i)]

$$\therefore \quad \mu = 16$$

5. $ax^2 - bx + c = 0$

$$\alpha + \beta = \frac{b}{a}, \alpha\beta = \frac{c}{a}$$

Also, $(a+cy)^2 = b^2 y$

$$\Rightarrow \quad c^2y^2 - (b^2 - 2ac)y + a^2 = 0$$

$$\Rightarrow \quad \left(\frac{c}{a}\right)^2 y^2 - \left\{\left(\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)\right\}y + 1 = 0$$

$$\Rightarrow \quad (\alpha\beta)^2 y^2 - (\alpha^2 + \beta^2)y + 1 = 0$$

$$\Rightarrow \quad y^2 - (\alpha^{-2} + \beta^{-2})y + \alpha^{-2}\beta^{-2} = 0$$

$$\Rightarrow \quad (y - \alpha^{-2})(y - \beta^{-2}) = 0$$

So, the roots are α^{-2} and β^{-2}.

6. Using $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{45}{10} \quad \text{and} \quad \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{120}{45}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{9}{2}$$

and $\frac{n-r}{r+1} = \frac{8}{3}$

$$\Rightarrow \quad \frac{8}{3}(r+1)+1=\frac{9}{2}r$$
$$\Rightarrow \quad 16r+16+6=27r$$
$$\Rightarrow \quad 11r=22$$
$$\therefore \quad r=2$$

7. Terminal digits are the first and last digits.
Since, terminal digits are even.
∴ 1st place can be filled in 3 ways and last place can be filled in 2 ways and remaining places can be filled in
$${}^5P_4=120 \text{ ways}$$
Hence, the number of six digit number, the terminal digits are even, is $3\times120\times2=720$.

8. $\because \quad (1+x^2)^{40}\cdot\left(x^2+2+\frac{1}{x^2}\right)^{-5}$
$$=(1+x^2)^{20}\cdot x^{10}$$
$$=(1+x^2)^{30}\cdot x^{20}$$
∴ Coefficient of x^{10} in $(1+x^2)^{30}\cdot x^{10}$
⇒ Coefficient of x^{10} in $(1+x^2)^{30}$ is ${}^{30}C_5$ or ${}^{30}C_{25}$.

9. $\because \quad \Delta=\begin{vmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{vmatrix}$
$$=1(1+\sin^2\theta)-\sin\theta(0)+1(\sin^2\theta+1)$$
$$=2(1+\sin^2\theta)$$
$$\because \quad 0\le\sin^2\theta\le1$$
$$\Rightarrow \quad 1\le1+\sin^2\theta\le2$$
$$\Rightarrow \quad 2\le2(1+\sin^2\theta)\le4$$
$$\Rightarrow \quad 2\le\Delta\le4$$
$$\therefore \quad \Delta\in[2,4]$$

10. $\because \quad A^2\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}=\begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix}$
and $8A+KI_2=8\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}+K\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$
$$=\begin{bmatrix}8+K & 0\\ -8 & 56+K\end{bmatrix}$$
$$\therefore \quad A^2=8A+KI_2$$
$$\therefore \quad \begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix}=\begin{bmatrix}8+K & 0\\ -8 & 56+K\end{bmatrix}$$
On comparing, we get $8+K=1$
$$\therefore \quad K=-7$$

11. $\because x\in\left(\frac{3\pi}{2},2\pi\right)$
Now, $\cos^{-1}(\cos x)=2\pi-x$
and $\sin^{-1}(\sin x)=x-2\pi$
$\therefore \quad \cos^{-1}(\cos x)+\sin^{-1}(\sin x)=0$
Therefore,
$$\sin^{-1}[\cos\{\cos^{-1}(\cos x)+\sin^{-1}(\sin x)\}]$$
$$=\sin^{-1}\{\cos(0)\}=\sin^{-1}(1)=\frac{\pi}{2}$$

12. Here, $x^2-2x+2=(x-1)^2+1\ge1$
But $-1\le(x^2-2x+2)\le1$
which is possible only when
$$x^2-2x+2=1$$
$$\therefore \quad x=1$$
Then, $a(1)^2+\sin^{-1}(1)+\cos^{-1}(1)=0$
$$\Rightarrow \quad a+\frac{\pi}{2}+0=0$$
$$\therefore \quad a=-\frac{\pi}{2}$$

13. $\because \quad \frac{y+\frac{1}{y}}{2}\ge\sqrt{y\cdot\frac{1}{y}}$
$$\Rightarrow \quad \sqrt{\left(y+\frac{1}{y}\right)}\ge\sqrt{2}$$
but $|\sin x+\cos x|\le\sqrt{2}$
which is possible only when
$$y+\frac{1}{y}=2$$
$$\therefore \quad y=1$$
and $$x=\frac{\pi}{4}$$

14. $\sin\left(\frac{\pi}{2n}\right)+\cos\left(\frac{\pi}{2n}\right)=\frac{\sqrt{n}}{2}$
$$\Rightarrow \quad \sqrt{2}\left\{\frac{1}{\sqrt{2}}\cdot\cos\left(\frac{\pi}{2n}\right)+\frac{1}{\sqrt{2}}\cdot\sin\left(\frac{\pi}{2n}\right)\right\}=\frac{\sqrt{n}}{2}$$
$$\Rightarrow \quad \sqrt{2}\left\{\cos\left(\frac{\pi}{4}-\frac{\pi}{2n}\right)\right\}=\frac{\sqrt{n}}{2}$$
$$\Rightarrow \quad \cos\left(\frac{\pi}{4}-\frac{\pi}{2n}\right)=\frac{\pi}{2\sqrt{2}} \qquad \ldots(i)$$

$$\Rightarrow \sqrt{2}\left\{\cos\left(\frac{\pi}{4}-\frac{\pi}{2n}\right)\right\}=\frac{\sqrt{n}}{2}$$

$$\Rightarrow \cos\left(\frac{\pi}{4}-\frac{\pi}{2n}\right)=\frac{\pi}{2\sqrt{2}} \quad ...(i)$$

when $n=6$, $\text{LHS}=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

and $\text{RHS}=\frac{\sqrt{6}}{2\sqrt{2}}=\frac{\sqrt{3}}{2}$

Eq. (i) is not satisfied for $n = 1, 2, 3, 4, 5$.

15. $\because \tan x \cdot \tan y = a$

and $\tan(x+y)=\tan\left(\frac{\pi}{6}\right)$

$$\Rightarrow \frac{\tan x+\tan y}{1-\tan x\cdot\tan y}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan x+\tan y=\frac{1}{\sqrt{3}}(1-a)$$

Equation whose roots are $\tan x$ and $\tan y$ is

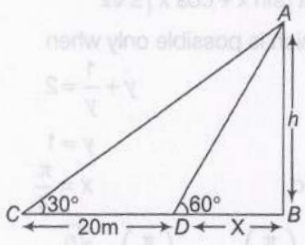
$$x^2-\frac{(1-a)}{\sqrt{3}}\cdot x+a=0$$

$$\Rightarrow \sqrt{3}x^2-(1-a)x+a\sqrt{3}=0$$

16. Let height of the tower AB is h m.

In ΔABD, $\tan 60°=\frac{h}{x}\Rightarrow\sqrt{3}=\frac{h}{x}$

$$\Rightarrow x=\frac{h}{\sqrt{3}}$$



In ΔABC, $\tan 30°=\frac{h}{20+x}$

$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{20+x}$$

$$\Rightarrow \sqrt{3}h=20+\frac{h}{\sqrt{3}} \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)h=20$$

$$\therefore h=20\times\frac{\sqrt{3}}{2}$$

$$=10\sqrt{3}\text{ m}$$

17. Since, the centroid divides the join of orthocentre and circumcentre in the ratio 2 : 1. Let the circumcentre of triangle is (α, β).

$$\therefore G(3,3)=G\left(\frac{2\alpha-3}{2+1},\frac{2\beta+5}{2+1}\right)$$

$$\Rightarrow G(3,3)=G\left(\frac{2\alpha-3}{3},\frac{2\beta+5}{3}\right)$$

$$\Rightarrow \frac{2\alpha-3}{3}=3 \text{ and } \frac{2\beta+5}{3}=3$$

$$\Rightarrow 2\alpha-3=9 \text{ and } 2\beta+5=9$$

$$\Rightarrow 2\alpha=12 \text{ and } 2\beta=4$$

$$\therefore \alpha=6 \text{ and } \beta=2$$

18. Let the equation of line AB is

$$\frac{x}{a}+\frac{y}{b}=1 \quad ...(i)$$

Let a point $R(h, k)$ divide line AB in the ratio 1 : 2. By using internally ratio,

$$R(h,k)=\left(\frac{1x_2+2x_1}{1+2},\frac{1y_2+2y_1}{1+2}\right)$$

$$\because h=\frac{1\times0+2\times a}{1+2},\ k=\frac{1\times b+2\times0}{1+2}$$

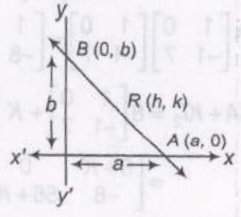
[$\because P(x, y)$ divide the line $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ internally]

$$\therefore P(x,y)=\left(\frac{nx_2+mx_2}{n+m},\frac{ny_2+my_1}{n+m}\right)$$

$$\Rightarrow h=\frac{2a}{3},\ k=\frac{b}{3}$$

$$\Rightarrow a=\frac{3h}{2},\ b=3k$$

On putting the values of a and b in Eq. (i), we get



$$\frac{x}{\left(\frac{3h}{2}\right)}+\frac{y}{3k}=1\Rightarrow\frac{2x}{3h}+\frac{y}{3k}=1$$

$$\Rightarrow \frac{2kx+hy}{3hk}=1$$

$$2kx+hy=3hk$$

19. Here, $m_1 = m_2^2 \Rightarrow m_2^2 + m_2 = \frac{-2h}{b}$ ...(i)

and $m_2^2\, m_2 = \frac{a}{b}$

$\Rightarrow \quad m_2 = \left(\frac{a}{b}\right)^{1/3}$ ... (ii)

Putting this value of m_2 in Eq. (i) we get

$$\left\{\left(\frac{a}{b}\right)^{1/3}\right\}^2 + \left(\frac{a}{b}\right)^{1/3} = \frac{-2h}{b}$$

On cubing both sides, we get

$$\left(\frac{a}{b}\right)^2 + \frac{a}{b} + 3\left(\frac{a}{b}\right)^{2/3}\cdot\left(\frac{a}{b}\right)^{1/3}\left\{\left(\frac{a}{b}\right)^{2/3} + \left(\frac{a}{b}\right)^{1/3}\right\} = \frac{-8h^3}{b^3}$$

$$\Rightarrow \quad \left(\frac{a}{b}\right)^2 + \frac{a}{b} - \frac{6ah}{b^2} = \frac{-8h^3}{b^3}$$

$$\left\{\therefore \left(\frac{a}{b}\right)^{2/3} + \left(\frac{a}{b}\right)^{1/3} = \frac{-2h}{b}\right\}$$

$$= ab(a+b) - 6abh + 8h^3 = 0$$

20. Given line is

$$(x-2)\cos\theta + (y-2)\sin\theta = 1$$

$$= \cos^2\theta + \sin^2\theta$$

On comparing we get,

$$x - 2\cos\theta \quad \text{...(i)}$$

$$y - 2 = \sin\theta \quad \text{...(ii)}$$

and On squaring and then adding Eqs. (i) and (ii), we get

$$(x-2)^2 + (y-2)^2 = \cos^2\theta + \sin^2\theta$$

$$\Rightarrow \quad (x-2)^2 + (y-2)^2 = 1$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 4y + 7 = 0$$

21. Equation of family of concentric circles to the circle $x^2 + y^2 + 6x + 8y - 5 = 0$

$$x^2 + y^2 + 6x + 8y + \lambda = 0$$

which is similar to $x^2 + y^2 + 2gx + 2fy + c = 0$.

Thus, the point $(-3, 2)$ lies on the circle.

$$x^2 + y^2 + 6x + 8y + c = 0$$

$$\Rightarrow (-3)^2 + (2)^2 + 6(-3) + 8(2) + c = 0$$

$$\Rightarrow \quad 9 + 4 - 18 + 16 + c = 0$$

$$\therefore \quad c = -11$$

22. Any line through origin is $y = mx$. Since, it is a tangent to $y^2 = 4a(x-a)$, it will cut it in two coincident points.

So, roots of $m^2x^2 - 4ax + 4a^2$ are equal.

$\therefore$ Product of slope $= -1$ *i.e.*, $b^2 - 4ac = 0$

$$\Rightarrow \quad 16a^2 - 16a^2m^2 = 0$$

$$\Rightarrow \quad m^2 = 1 \text{ or } m = 1, -1$$

Hence, required angle is right angle *i.e.*, $\frac{\pi}{2}$.

23. For the ellipse, $a = 5$ and $e = \sqrt{\frac{25-9}{25}} = \frac{4}{5}$

$$\therefore \quad ae = 4$$

Hence, the focii are $(-4,0)$ and $(4,0)$.

For the hyperbola, $ae = 4, e = 2$

$$\therefore \quad a = 2$$

$$b^2 = 4(4-1) = 12$$

$$b = \sqrt{12}$$

Length of latusrectum $= \frac{2b^2}{a} = 2 \times \frac{12}{2}$

$$= 12$$

24. $$\frac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\infty}{1 + \frac{1}{3!} + \frac{1}{5!} + \ldots\infty}$$

$$= \frac{2\left\{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\infty\right\}}{2\left\{1 + \frac{1}{3!} + \frac{1}{5!} + \ldots\infty\right\}} = \frac{(e + e^{-1}) - 2}{(e - e^{-1})}$$

$$= \frac{e + \frac{1}{e} - 2}{e - \frac{1}{e}}$$

$$= \frac{e^2 + 1 - 2e}{e^2 - 1} = \frac{(e-1)^2}{(e-1)(e+1)} = \frac{e-1}{e+1}$$

25. $$\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right)^2 = \left(\frac{e^x + e^{-x}}{2}\right)^2$$

$$= \frac{1}{4}(e^{2x} + e^{-2x} + 2)$$

$$\frac{1}{4}\left\{2\left(1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \ldots\right) + 2\right\}$$

So, coefficient of x^n (n even) $= \frac{1}{2}\left\{\frac{2^n}{n!}\right\} = \frac{2^{n-1}}{n!}$

26. Let θ be the angle between the line and the plane, we have

$$a=2,\ b=3,\ c=6$$

and $$a_1=10,\ b_2=2, c_1=-11$$

$$\therefore \quad \cos(90-\theta)=\sin\theta$$

$$=\left|\frac{a a_1+bb_1+cc_1}{\sqrt{a^2+b^2+c^2}\sqrt{a_1^2+b_1^2+c_1^2}}\right|$$

$$=\left|\frac{(2)(10)+(3)(2)+(6)(-11)}{\sqrt{2^2+3^2+6^2}\sqrt{10^2+2^2+(-11)^2}}\right|$$

$$=\left|\frac{20+6-66}{\sqrt{49}\cdot\sqrt{225}}\right|=\left|\frac{-40}{7\cdot 15}\right|$$

$$\theta=\sin^{-1}\left(\frac{8}{21}\right)$$

27. Let A_x A_y and A_z be the areas of projection of ΔABC on yz, zx and xy-planes. Then,

$$A_x=\frac{1}{2}\times\text{Absolute values of}\begin{vmatrix}2&3&1\\-1&1&1\\2&-4&1\end{vmatrix}=\frac{21}{2}$$

$$A_y=\frac{1}{2}\times\text{Absolute values of}\begin{vmatrix}3&1&1\\1&2&1\\-4&1&1\end{vmatrix}=\frac{7}{2}$$

and $$A_z=\frac{1}{2}\times\text{Absolute values of}\begin{vmatrix}1&2&1\\2&-1&1\\1&2&1\end{vmatrix}$$

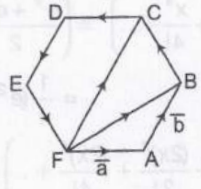
$$=0$$

$$\therefore \text{Area of } \Delta ABC=\sqrt{A_x^2+A_y^2+A_z^2}$$

$$=\sqrt{\frac{441}{4}+\frac{49}{4}+0}=\frac{7}{2}\sqrt{10}\text{ sq units}$$

28. $OABCDE$ is a regular hexagon.

Let $\overrightarrow{OA}=\vec{a}$ and $\mathbf{AB}=\mathbf{b}$, join OB and OC, we have



$$\mathbf{OB}=\mathbf{OA}+\mathbf{AB}=\mathbf{a}+\mathbf{b}$$

Since, OC is parallel to AB and double of AB.

$$\therefore \quad \mathbf{OC}=2\,\mathbf{AB}=2\mathbf{b}$$

Now, $$\mathbf{BC}=\mathbf{OC}-\mathbf{OB}$$

$$=2\mathbf{b}-(\mathbf{a}+\mathbf{b})=\mathbf{b}-\mathbf{a}$$

$$\mathbf{CD}=-\mathbf{OA}=-\mathbf{a}$$

and $$\mathbf{DE}=-\mathbf{AB}=-\mathbf{b}$$

Also, $$\mathbf{EO}=\mathbf{BC}=-(\mathbf{b}-\mathbf{a})=\mathbf{a}-\mathbf{b}$$

29. $\because$ $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are linearly dependent vectors.

$$\Rightarrow \quad [\mathbf{a}\,\mathbf{b}\,\mathbf{c}]=0$$

$$\Rightarrow \quad \begin{vmatrix}1&1&2\\4&3&4\\1&\alpha&\beta\end{vmatrix}=0$$

$$\Rightarrow 1(3\beta-4\alpha)-1(4\beta-4)+1(4\alpha-3)=0$$

$$\Rightarrow \quad -\beta+1=0 \quad \Rightarrow \beta=1$$

Now, $$|\mathbf{c}|=\sqrt{3}$$

$$\Rightarrow \quad \sqrt{1+\alpha^2+\beta^2}=\sqrt{3}$$

$$\Rightarrow \quad 1+1+\alpha^2=3\Rightarrow \alpha^2=1$$

$$\therefore \quad \alpha=\pm 1$$

30. We have, $\mathbf{u}\,.\,\hat{\mathbf{n}}=0$ and $\mathbf{v}\,.\,\hat{\mathbf{n}}=0$

$$\Rightarrow \quad \hat{\mathbf{n}}\perp\mathbf{u} \text{ and } \hat{\mathbf{n}}\perp\mathbf{v}$$

$$\Rightarrow \quad \hat{\mathbf{n}}=\pm\frac{\mathbf{u}\times\mathbf{v}}{|\mathbf{u}\times\mathbf{v}|}$$

Now, $$\mathbf{u}\times\mathbf{v}=(\mathbf{i}+\mathbf{j})\times(\mathbf{j}-\mathbf{k})=-2\mathbf{k}$$

$$\therefore \quad \hat{\mathbf{n}}=\pm\mathbf{k}$$

Hence, $|\vec{w}\cdot\hat{\mathbf{n}}|=|(\mathbf{i}+2\mathbf{j}+3\mathbf{k}).(\pm 3\mathbf{k})|=3$

31. Given, $g(x)=1+\sqrt{x}$

and $$f\{g(x)\}=3+2\sqrt{x}+x \qquad ...(i)$$

$$\Rightarrow \quad f(1+\sqrt{x})=3+2\sqrt{x}+x$$

Put $$1+\sqrt{x}=y \quad \Rightarrow \quad x=(y-1)^2$$

$$\therefore f(y)=3+2(y-1)+(y-1)^2=2+y^2$$

$$\therefore \quad f(x)=2+x^2$$

32. Here 'y' would assume real values if

$$\log_3\cos\{(\sin x)\}\geq 0 \quad \Rightarrow \quad \cos(\sin x)\geq 1$$

$$\Rightarrow \quad \cos(\sin x)=1 \text{ as } -1\leq\cos\theta\leq 1$$

$$\Rightarrow \quad \sin x=0$$

$$\Rightarrow \quad x=n\pi, n\in I$$

Hence, the domain consists of the multiples of π *i.e.*, domain $(y)=\{n\pi : n\in I\}$.

33. (b) $\lim_{x\to 1}(\log_2 2x\log_x^5)$

$$=\lim_{x\to 1}\left[(1+\log_2 x)^{\frac{1}{\log_2 x}}\right]^{\log_2 x.\log_x 5}$$

$$=e^{\frac{\lim_{x\to 1}\log_2 5}{x+1}}$$

$$=e^{\log_2 5}$$

34. Continuity at $x = 0$

LHL At $x=0$, $\lim_{x\to 0} f(x)=\lim_{x\to 0^-}(1)=1$

RHL At $x=0$, $\lim_{x\to 0^+} f(x)$

$$=\lim_{x\to 0}(1+\sin x)=1$$

$$f(0)=1+\sin 0=1$$

$= L = \text{RHL} = f(0)$, so f(x) is continuous at $x = 0$.

Continuity at $x=\frac{\pi}{2}$

LHL At $x=\frac{\pi}{2}$, $\lim_{x\to\frac{\pi}{2}} f(x)$

$$=\lim_{x\to\frac{\pi}{2}}(1+\sin x)=1+1=2$$

RHL At $x=\frac{\pi}{2}$, $\lim_{x\to\frac{\pi}{2}} f(x)=2+\left(\frac{\pi}{2}-\frac{\pi}{2}\right)^2=2$

$$f\left(\frac{\pi}{2}\right)=2+\left(\frac{\pi}{2}-\frac{\pi}{2}\right)^2=2$$

$\therefore$ LHL = RHL $= f\left(\frac{\pi}{2}\right)$

So, $f(x)$ is continuous at $x=\frac{\pi}{2}$.

Hence, $f(x)$ is continuous over the whole real number.

35. $f(x)=\sqrt{1-\sqrt{1-x^2}}$ clearly, $f(x)$ is continous on $[-1, 1]$.

The domain of $f(x)$ is $[-1,1]$.

For $x \neq 0$, $x \neq \pm 1$, we have

$$f'(x)=\frac{1}{2\sqrt{1-\sqrt{1-x^2}}}\cdot\frac{x}{\sqrt{1-x^2}}$$

Since, $f(x)$ is not defined on the right side of $x = 1$ and on the left side of $x = -1$. Also, $f'(x)\to\infty$ when $x\to 1$ or $x\to -1$. So, we check the differentiability at $x = 0$.

Now, $\angle f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$

$$\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=-\frac{1}{\sqrt{2}}$$

Similarly, $Rf'(0)=-\frac{1}{\sqrt{2}}$

Hence, $f(x)$ is not differentiable at $x = 0$.

36. Given, $x=\sqrt{a^{\sin^{-1}t}}$ ...(i)

and $y=\sqrt{a^{\cos^{-1}t}}$ ...(ii)

On multiplying Eqs. (i) and (ii), we get

$$xy=\sqrt{a^{\sin^{-1}t}}\times\sqrt{a^{\cos^{-1}t}}$$

$$\Rightarrow\quad xy=\sqrt{a^{\sin^{-1}t}.a^{\cos^{-1}t}}$$

$$=\sqrt{a^{(\sin^{-1}t+\cos^{-1}t)}}$$

$$\left(\because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\right)$$

$$\Rightarrow\quad xy=\sqrt{a^{\pi/12}}$$

On differentiating w.r.t. x, we get

$$x\frac{dy}{dx}+y=0\Rightarrow x\frac{dy}{dx}=-y\Rightarrow\frac{dy}{dx}=\frac{-y}{x}$$

$$\therefore\quad \frac{d}{dx}(\text{constant})=0$$

37. Since, $fog = I$

$$\Rightarrow\quad fog(x)=x, \text{ for all } x$$

$$\Rightarrow\quad f'\{g(x)\}\cdot g'(x)=1 \text{ for all } x.$$

$$\Rightarrow\quad f'\{g(a)\}=\frac{1}{g'(a)}=\frac{1}{2}$$

$$\Rightarrow\quad f'(b)=\frac{1}{2}$$

$[\because g(a)=b]$

38. We have, $x=at^2$ and $y=2at$

$$\Rightarrow\quad \frac{dx}{dt}=2at \text{ and } \frac{dy}{dt}=2a$$

$$\therefore\quad \frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{1}{t}$$

Now, $\frac{d^2y}{dx^2}=-\frac{1}{t^2}\cdot\frac{dt}{dx}=-\frac{1}{t^2}\cdot\frac{1}{2at}=-\frac{1}{2at^3}$

At $t=2$, $\frac{d^2y}{dx^2}=\frac{-1}{2a(2)^3}=-\frac{1}{16a}$

39. Tangent to the curve is parallel to the axis is when slope of the tangent is 0.

$\therefore$ Equation of the curve is

$$y = x^3 - 3x^2 - 9x + 7 \quad ...(i)$$

$$\therefore \quad \frac{dy}{dx} = 3x^2 - 6x - 9$$

Now, the tangent is parallel to x-axis, then slope the tangent is zero or we can say that $\frac{dy}{dx} = 0$.

$$\Rightarrow \quad 3x^2 - 6x - 9 = 0$$

$$\Rightarrow \quad 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow \quad (x-3)(x+1) = 0$$

$$\Rightarrow \quad x = 3, -1$$

When $x = 3$, then from Eq. (i), we get

$$y = (3)^3 - (3)\cdot(3)^2 - 9\cdot 3 + 7$$

$$= 27 - 27 - 27 + 7 = -20$$

When $x = -1$, then from Eq. (i), we get

$$y = (-1)^3 - 3(-1)^2 - 9(-1) + 7$$

$$= -1 - 3 + 9 + 7 = 12$$

Hence, the points at which the tangent is parallel to x-axis are $(3, -20)$ and $(-1, 12)$.

40. $$\int \frac{dx}{\sin^2 x \cdot \cos^2 x} = \int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cdot \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cdot \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cdot \cos^2 x} dx$$

$$= \int \frac{dx}{\cos^2 x} dx + \int \frac{dx}{\sin^2 x}$$

$$= \int \sec^2 x\, dx + \int \operatorname{cosec}^2 x\, dx$$

$$= \tan x - \cot x + C$$

41. $$\int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{(e^x \cdot e^x - 1)}{(e^x \cdot e^x + 1)} dx$$

$$= \int \frac{e^x\left(e^x - \frac{1}{e^x}\right)}{e^x\left(e^x + \frac{1}{e^x}\right)} \cdot dx$$

$$= \int \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot dx$$

Let $\quad (e^x + e^{-x}) = t$

$$\Rightarrow \quad e^x - e^{-x} = \frac{dt}{dx}$$

$$\Rightarrow \quad dx = \frac{dt}{e^x - e^{-x}}$$

$$\int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{t} \cdot \frac{dt}{e^x - e^{-x}}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log|e^x + e^{-x}| + C$$

42. Let $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$

$$= \int_0^1 \tan^{-1}\left\{\frac{x + (x-1)}{1 - x(x-1)}\right\} dx$$

$$= \int_0^1 \{\tan^{-1} x + \tan^{-1}(x-1)\} dx$$

$$\left\{\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$$

$$\Rightarrow \quad I = \int_0^1 \{\tan^{-1} x - \tan^{-1}(1-x)\}\, dx \quad ...(i)$$

Also, $I = \int_0^1 \{\tan^{-1}(1-x) - \tan^{-1}\{1-(1-x)\}\} dx$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$I = \int_0^1 [\tan^{-1}(1-x) - \tan^{-1} x]\, dx \quad ...(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = 0$$

$$\Rightarrow \quad I = 0$$

43. Let $I = \int_{\log 2}^{x} \frac{1}{\sqrt{e^x - 1}} dx$

Put $\quad e^x - 1 = t^2$

$$\Rightarrow \quad e^x dx = 2t\, dt$$

$$\therefore \quad I = 2\int_1^{\sqrt{e^x - 1}} \frac{1}{1+t^2} dt$$

$$= 2\left[\tan^{-1}\sqrt{e^x - 1} - \tan^{-1} 1\right]$$

$$= 2\tan^{-1}\sqrt{e^x - 1} - \frac{\pi}{4}$$

But $\int_{\log_e 2}^{x} \frac{1}{\sqrt{e^x - 1}}\, dx = \frac{\pi}{6}$

$$= 2\tan^{-1}\sqrt{e^x - 1}$$

$$= \frac{2\pi}{3}$$

$\Rightarrow \quad \sqrt{e^x - 1} = \tan\frac{\pi}{3}$

$$= \sqrt{3}$$

$$= e^x - 1$$

$$= 3$$

$\Rightarrow \quad e^x = 4$

$\therefore \quad x = \log 4$

44. The graph of $y = \sin x$ can be drawn as

required area = Area of $OABO$ + Area $BCDB$

$$= \int_0^x |\sin x|\, dx + \int_{\pi}^{2\pi} |\sin x|\, dx$$

$$= \int_0^x \sin x\, dx + \int_{\pi}^{2\pi} (-\sin x)\, dx$$

$(\because \sin \geq 0 \text{ for } x \in [x, \pi]$

and $\sin x \leq 0$ for $x \in [\pi_1\, 2\pi])$

$$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$$

$$= -\cos\pi + \cos 0 + \cos 2\pi - \cos\pi$$

$$= -(-1) + 1 + 1 - (-1)$$

$$= 4 \text{ sq units}$$

45. Area bounded by curves $2^{kx} = y$ and $x = 0$ and $x = 2$ is given by

$$A = \int_0^2 2^{kx} dx$$

$$= \frac{1}{k}\left[\frac{2^{kx}}{\log 2}\right]_0^2 = \left[\frac{2^{2k} - 1}{k \log 2}\right]$$

But $\quad A = \frac{3}{\log 2}$

$\therefore \quad \frac{2^{2k} - 1}{k \log 2} = \frac{3}{\log 2}$

$\Rightarrow \quad 2^{2k} - 1 = 3k$

This relation is satisfied by only option (b).

46. Given, $\sec^2 y \cdot \tan y\, dx + \sec^2 y \cdot \tan x\, dy = 0$

On separating the variables, we get

$\Rightarrow \quad \sec^2 x \,.\, \tan y dx = -\sec^2 y \,.\, \tan x\, dy$

$\Rightarrow \quad \frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$

On integrating both the sides, we get

$$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 x}{\tan y} dy$$

Let $\quad \tan x = u$

$\Rightarrow \quad \sec^2 x = \frac{du}{dx}$

$\Rightarrow \quad dx = \frac{du}{\sec^2 y}$ and $\tan y = v$

$\Rightarrow \quad \sec^2 y = \frac{dv}{dy}$

$\Rightarrow \quad dy = \frac{du}{\sec^2 y}$

$$\int \frac{\sec^2 x}{u} \cdot \frac{du}{\sec^2 x} = -\int \frac{\sec^2 y}{v} \cdot \frac{dv}{\sec^2 y}$$

$\Rightarrow \quad \int \frac{du}{u} = -\int \frac{dv}{v}$

$\Rightarrow \quad \log|x| = -\log|v| + \log|c|$

$\Rightarrow \quad \log|\tan x| = -\log|\tan y| + \log|c|$

$\Rightarrow \quad \log|\tan x \cdot \tan y| = \log|c|$

$(\because \log m + \log n = \log mn)$

$\Rightarrow \quad (\because \log m = \log n \Rightarrow m = n)$

which is the required general solution.

47. Total number of cases, $n(s) = 55C_2$

$n(E)$ = The number of selection of 1 spade, 1 ace from 3 aces or selections of the ace of spade and 1 other spade.

$$13C_1 \times 13C_1 + 12C_1 \times 11C_1 = 51$$

$\therefore \quad P(E)\frac{51}{52C_2} = \frac{1}{26}$

48. Let $A = \{1,2,3,4,5,6\}$

A relation R is defined on set A is

$$R = \{(a,b): b = a+1\}, \text{ therefore}$$

$$R = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$$

Now, $6 \in A$ but $(6,6) \notin R$

Therefore, R is not reflexive.

It can be observed that $(1, 2) \in R$ but $(1, 2) \notin R$. Therefore, R is not symmetric.

Now, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R$. Therefore, R is not transitive.

Hence, R is neither reflexive nor symmetric nor transitive.

49. $\sim(p \Leftrightarrow q) \equiv \sim[(p \Rightarrow q) \wedge (q \Rightarrow p)]$

$$\equiv \sim(p \Rightarrow q) \vee (\sim(q \Rightarrow p))$$

($\because$ by De-Morgan's law)

$$\equiv (p \wedge \sim q) \vee (q \wedge \sim p).$$

50. Let the magnitude of each force be P. Then,

R_1 = Resultant of two equal forces each of magnitude P inclined at an angle $2\alpha = 2P\cos\alpha$, and

R_2 = Resultant of two equal forces each of magnitude P inclined at an angle $2\beta = 2P\cos\beta$.

It is given that, $R_1 = 2R_2$

$$\Rightarrow \quad 2P\cos\alpha = 4P\cos\beta$$

$$\Rightarrow \quad \cos\alpha = 2\cos\beta$$