1 CHAPTER

Number System

DIVISIBILITY

1990

1. If n is any positive integer, then $n^3 - n$ is divisible

(a) Always by 12 (b) Never by 12

(c) Always by 6 (d) Never by 6

Directions for Questions 2 and 3: Each of the following questions is followed by two statements. MARK,

(a) if the question can be answered with the help of statement I alone,

(b) if the question can be answered with the help of statement II alone,

(c) if both, statement I and statement II are needed to answer the question, and

(d) if the statement cannot be answered even with the help of both the statements.

2. If R is an integer between 1 & 9, P – R = 2370, what is the value of R?

I. P is divisible by 4.

II. P is divisible by 9.

3. x, y and z are three positive odd integers, is x + z divisible by 4?

I. y – x = 2.

II. z – y = 2.

1991

4. Let k be a positive integer such that k + 4 is divisible by 7. Then the smallest positive integer n, greater than 2, such that k + 2n is divisible by 7 equals

(a) 9 (b) 7

(c) 5 (d) 3

5. If x is a positive integer such that 2x + 12 is perfectly divisible by x, then the number of possible values of x is

(a) 2 (b) 5

(c) 6 (d) 12

6. A positive integer is said to be a prime number if it is not divisible by any positive integer other than itself and 1. Let p be a prime number greater than 5. Then $(p^2 - 1)$ is

(a) never divisible by 6

(b) always divisible by 6, and may or may not be divisible by 12.

(c) always divisible by 12, and may or may not be divisible by 24.

(d) always divisible by 24.

1993

7. The number of positive integers not greater than 100, which are not divisible by 2, 3 or 5 is

(a) 26 (b) 18

(c) 31 (d) None

1995

8. For the product n(n + 1)(2n + 1), $n \in N$, which one of the following is not necessarily true?

(a) It is even

(b) Divisible by 3

(c) Divisible by the sum of the square of first n natural numbers

(d) Never divisible by 237

1996

9. If a number 774958A96B is to be divisible by 8 and 9, the respective values of A and B will be

(a) 7 and 8

(b) 8 and 0

(c) 5 and 8

(d) None of these

10. If n is any odd number greater than 1, then $n(n^2 - 1)$ is

(a) divisible by 96 always

(b) divisible by 48 always

(c) divisible by 24 always

(d) None of these

1997

11. If m and n are integers divisible by 5, which of the following is not necessarily true?

(a) m – n is divisible by 5

(b) $m^2 - n^2$ is divisible by 25

(c) m + n is divisible by 10

(d) None of these

Direction for Question 12: The question is followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of one statement alone.

Mark (b) if the question can be answered with the help of any one statement independently.

Mark (c) if the question can be answered with the help of both statements together.

Mark (d) if the question cannot be answered even with the help of both statements together.

12. Is the number completely divisible by 99?

I. The number is divisible by 9 and 11 simultaneously.

II. If the digits of the number are reversed, the number is divisible by 9 and 11.

1998

13. How many five-digit numbers can be formed using the digits 2, 3, 8, 7, 5 exactly once such that the number is divisible by 125?

(a) 0 (b) 1
(c) 4 (d) 3

2000

14. Let $N = 55^3 + 17^3 - 72^3$. N is divisible by

(a) both 7 and 13 (b) both 3 and 13
(c) both 17 and 7 (d) both 3 and 17

15. Let D be a recurring decimal of the form $D = 0.a_1 a_2 a_1 a_2 a_1 a_2 ...$, where digits a_1 and a_2 lie between 0 and 9. Further, at most one of them is zero. Which of the following numbers necessarily produces an integer, when multiplied by D?

(a) 18 (b) 108
(c) 198 (d) 288

16. Let S be the set of integers x such that

I. $100 \le x \le 200$,

II. x is odd and

III. x is divisible by 3 but not by 7.

How many elements does S contain?

(a) 16 (b) 12
(c) 11 (d) 13

2001

17. Let b be a positive integer and $a = b^2 - b$. If $b \ge 4$, then $a^2 - 2a$ is divisible by

(a) 15 (b) 20
(c) 24 (d) All of these

2002

18. $7^{6n} - 6^{6n}$, where n is an integer > 0, is divisible by

(a) 13 (b) 127
(c) 559 (d) All of these

2003(R)

19. Let n (>1) be a composite integer such that $\sqrt{n}$ is not an integer. Consider the following statements:

A : n has a perfect integer-valued divisor which is greater than 1 and less than $\sqrt{n}$.

B : n has a perfect integer-valued divisor which is greater than $\sqrt{n}$ but less than n.

(a) Both A and B are false
(b) A is true but B is false
(c) A is false but B is true
(d) Both A and B are true

20. If a, a + 2 and a + 4 are prime numbers, then the number of possible solutions for a is

(a) one (b) two
(c) three (d) more than three

2003(L)

21. How many even integers n, where $100 \le n \le 200$, are divisible neither by seven nor by nine?

(a) 40 (b) 37
(c) 39 (d) 38

22. The number of positive integers n in the range $12 \le n \le 40$ such that the product (n–1)(n–2)...3.2.1 is not divisible by n is

(a) 5 (b) 7
(c) 13 (d) 14

2005

23. The digits of a three-digit number A are written in the reverse order to form another three-digit number B. If B > A and B – A is perfectly divisible by 7, then which of the following is necessarily true?

(a) 100 < A < 299
(b) 106 < A < 305
(c) 112 < A < 311
(d) 118< A < 317

24. Let S be a set of positive integers such that every element n of S satisfies the conditions

I. $1000 \le n \le 1200$

II. Every digit in n is odd

Then how many elements of S are divisible by 3?

(a) 9 (b) 10
(c) 11 (d) 12

MEMORY BASED QUESTIONS

2009

25. The digits of all the two digit multiples of 8 are reversed. How many of the resulting numbers would be divisible by 4 but not by 8?

(a) 3 (b) 4
(c) 2 (d) 1

2015

26. Which of the following will completely divide $(106^{90} - 49^{90})$?

(a) 589 (b) 186
(c) 124 (d) None of these

2016

27. Find the smallest number which when divided by 3, 5 and 7 leaves remainders 2, 4 and 6 respectively.

(a) 104 (b) 105
(c) 209 (d) None of these

PROPERTY

Direction for Question 1: The question is followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of statement I alone,

Mark (b) if the question can be answered with the help of statement II alone,

Mark (c) if both the statement I and statement II are needed to answer the question, and

Mark (d) if the question cannot be answered even with the help of both the statements.

1991

1. What is the value of prime number x?

I. $x^2 + x$ is a two digit number greater than 50.

II. x^3 is a three digit number.

2. Three times the first of three consecutive odd integers is 3 more than twice the third. What is the third integer?

(a) 15 (b) 9
(c) 11 (d) 5

3. $2^{73} - 2^{72} - 2^{71}$ is the same as

(a) 2^{69} (b) 2^{70}
(c) 2^{71} (d) 2^{72}

4. The sum of two integers is 10 and the sum of their reciprocals is 5/12. Then the larger of these integers is

(a) 2 (b) 4
(c) 6 (d) 8

5. x, y and z are three positive integers such that x > y > z. Which of the following is closest to the product xyz?

(a) $(x - 1)yz$ (b) $x(y - 1)z$
(c) $xy(z - 1)$ (d) $x(y + 1)z$

6. A third standard teacher gave a simple multiplication exercise to the kids. But one kid reversed the digits of both the numbers and carried out the multiplication and found that the product was exactly the same as the one expected by the teacher. Only one of the following pairs of numbers will fit in the description of the exercise. Which one is that?

(a) 14, 22 (b) 13, 62
(c) 19, 33 (d) 42, 28

7. To decide whether a n digits number is divisible by 7, we can define a process by which its magnitude is reduced as follows: ($i_1, i_2, i_3, \dots$, are the digits of the number, starting from the most significant digit). $i_1 i_2 \dots i_n \Rightarrow i_1 . 3^{n-1} + 1_2 . 3^{n-2} + \dots + i_n . 3^0$.

E.g. 259 $\Rightarrow 2.3^2 + 5.3^1 + 9.3^0 = 18 + 15 + 9 = 42$

Ultimately the resulting number will be seven after repeating the above process a certain number of times. After how many such stages, does the number 203 reduce to 7?

(a) 2 (b) 3
(c) 4 (d) 1

8. If 8 + 12 = 2, 7 + 14 = 3, then 10 + 18 = ?

(a) 10 (b) 4
(c) 6 (d) 18

Directions for Questions 9 and 10 : Each of these items has a question followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of statement I alone,

Mark (b) if the question can be answered with the help of statement II, alone,

Mark (c) if both, statement I and statement II are needed to answer the question, and

Mark (d) if the question cannot be answered even with the help of both the statements.

1993

9. What are the values of 3 integers a, b and c?

I. ab = 8

II. bc = 9

10. What are the ages of the three brothers?

I. The product of their ages is 21.

II. The sum of their ages is not divisible by 3.

11. Suppose one wishes to find distinct positive integers x, y such that $\frac{x + y}{xy}$ is also a positive integer. Identify the correct alternative.
    (a) This is never possible.
    (b) This is possible and the pair (x, y) satisfying the stated condition is unique.
    (c) This is possible and there exist more than one but a finite number of ways of choosing the pair (x, y).
    (d) This is possible and the pair (x, y) can be chosen in infinite ways.

12. Given odd positive integers x, y and z, which of the following is not necessarily true?
    (a) $x^2 y^2 z^2$ is odd
    (b) $3(x^2 + y^3)z^2$ is even.
    (c) $5x + y + z^4$ is odd
    (d) $z^2 (x^4 + y^4)/2$ is even

13. Let x < 0.50, 0 < y < 1, z > 1. Given a set of numbers, the middle number, when they are arranged in ascending order, is called the median. So the median of the numbers x, y and z would be
    (a) less than one
    (b) between 0 and 1
    (c) greater than 1
    (d) cannot say

14. Let x < 0, 0 < y < 1, z > 1. Which of the following may be false?
    (a) $(x^2 – z^2)$ has to be positive.
    (b) yz can be less than one.
    (c) xy can never be zero.
    (d) $(y^2 – z^2)$ is always negative.

15. A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, little finger 5, then reversed direction, calling the ring finger 6, middle finger 7, index finger 8, thumb 9, then back to the index finger for 10, middle finger for 11, and so on. She counted up to 1994. She ended on her
    (a) thumb (b) index finger
    (c) middle finger (d) ring finger

16. Let x, y and z be distinct positive integers satisfying x < y < z and x + y + z = k. What is the smallest value of k that does not determine x, y, z uniquely?
    (a) 9 (b) 6
    (c) 7 (d) 8

1995

17. 72 hens cost Rs.__ 96.7__. Then what does each hen cost, where two digits in place of '__' are not visible or are written in illegible hand?
    (a) Rs.3.23 (b) Rs.5.11
    (c) Rs.5.51 (d) Rs.7.22

18. The value of $\frac{55^3 + 45^3}{55^2 - 55 \times 45 + 45^2}$ is
    (a) 100 (b) 105
    (c) 125 (d) 75

19. Three consecutive positive even numbers are such that thrice the first number exceeds double the third by 2, then the third number is
    (a) 10 (b) 14
    (c) 16 (d) 12

Directions for Questions 20 and 21: Each of these questions is followed by two statements, I and II. Mark the answer as:
(a) if the question can be answered with the help of statement I alone.
(b) if the question can be answered with the help of statement II alone.
(c) if both statement I and statement II are needed to answer the question.
(d) if the question cannot be answered even with the help of both the statements.

20. If x, y and z are real numbers, is z – x even or odd?
    I. xyz is odd.
    II. xy + yz + zx is even.

21. Is x + y – z + t even?
    I. x + y + t is even.
    II. t and z are odd.

DirectionS for Questions 22 and 23: Each of these questions is followed by two statements, I and II. Mark the answer as:
(a) if the question cannot be answered even with the help of both the statements taken together.
(b) if the question can be answered by any one of the two statements.
(c) if each statement alone is sufficient to answer the question, but not the other one (e.g. statement I alone is required to answer the question, but not statement II and vice versa).
(d) if both statements I and II together are needed to answer the question.

1995

22. If a, b and c are integers, is

(a – b + c) > (a + b – c)?

I. b is negative.

II. c is positive.

23. How old is Sachin in 1997?

I. Sachin is 11 years younger than Anil whose age will be a prime number in 1998.

II. Anil's age was a prime number in 1996.

1997

24. If n is an integer, how many values of n will give an integral value of $\frac{(16n^2 + 7n + 6)}{n}$?

(a) 2 (b) 3

(c) 4 (d) None of these

25. P and Q are two positive integers such that PQ = 64. Which of the following cannot be the value of P + Q?

(a) 20 (b) 65

(c) 16 (d) 35

26. Which of the following is true?

(a) $7^{3^2} = (7^3)^2$ (b) $7^{3^2} > (7^3)^2$

(c) $7^{3^2} < (7^3)^2$ (d) None of these

27. P, Q and R are three consecutive odd numbers in ascending order. If the value of three times P is 3 less than two times R, find the value of R.

(a) 5 (b) 7

(c) 9 (d) 11

Direction for Question 28: The question is followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of one statement alone.

Mark (b) if the question can be answered with the help of any one statement independently.

Mark (c) if the question can be answered with the help of both statements together.

Mark (d) if the question cannot be answered even with the help of both statements together.

28. What is the value of $a^3 + b^3$?

I. $a^2 + b^2 = 22$ II. ab = 3

1998

29. n^3 is odd. Which of the following statement(s) is(are) true?

I. n is odd. II. n^2 is odd.

III. n^2 is even.

(a) I only (b) II only

(c) I and II (d) I and III

30. $(BE)^2$ = MPB, where B, E, M and P are distinct integers. Then M =

(a) 2 (b) 3

(c) 9 (d) None of these

31. Five-digit numbers are formed using only 0, 1, 2, 3, 4 exactly once. What is the difference between the maximum and minimum number that can be formed?

(a) 19800 (b) 41976

(c) 32976 (d) None of these

32. I started climbing up the hill at 6 a.m. and reached the top of the temple at 6 p.m. Next day I started coming down at 6 a.m. and reached the foothill at 6 p.m. I walked on the same road. The road is so short that only one person can walk on it. Although I varied my pace on my way, I never stopped on my way. Then which of the following must be true?

(a) My average speed downhill was greater than that of uphill

(b) At noon, I was at the same spot on both the days.

(c) There must be a point where I reached at the same time on both the days.

(d) There cannot be a spot where I reached at the same time on both the days.

Direction for Question 33: The question is followed by two statements. As the answer,

Mark (a) if the question can be answered with the help of any one statement alone but not by the other statement.

Mark (b) if the question can be answered with the help of either of the statements taken individually.

Mark (c) if the question can be answered with the help of both statements together.

Mark (d) if the question cannot be answered even with the help of both statements together.

33. Is n odd?

I. n is divisible by 3, 5, 7 and 9.

II. 0 < n < 400

1999

34. Let a, b, c be distinct digits. Consider a two-digit number ‘ab’ and a three-digit number ‘ccb’, both defined under the usual decimal number system, if $(ab)^2 = ccb > 300$, then the value of b is

(a) 1 (b) 0

(c) 5 (d) 6

35. If $n = 1 + x$ where x is the product of four consecutive positive integers, then which of the following is/are true?

A. n is odd

B. n is prime

C. n is a perfect square

(a) A and C only

(b) A and B only

(c) A only

(d) None of these

36. If $n^2 = 12345678987654321$, what is n?

(a) 12344321 (b) 1235789

(c) 111111111 (d) 11111111

2000

37. Let x, y and z be distinct integers, that are odd and positive. Which one of the following statements cannot be true?

(a) xyz^2 is odd

(b) $(x - y)^2 z$ is even

(c) $(x + y - z)^2 (x + y)$ is even

(d) $(x - y)(y + z)(x + y - z)$ is odd

38. Let S be the set of prime numbers greater than or equal to 2 and less than 100. Multiply all elements of S. With how many consecutive zeros will the product end?

(a) 1 (b) 4

(c) 5 (d) 10

39. Convert the number 1982 from base 10 to base 12. The result is

(a) 1182 (b) 1912

(c) 1192 (d) 1292

2001

40. Let x, y and z be distinct integers. x and y are odd and positive, and z is even and positive. Which one of the following statements cannot be true?

(a) $y(x - z)^2$ is even

(b) $y^2(x - z)$ is odd

(c) $y(x - z)$ is odd

(d) $z(x - y)^2$ is even

41. In a number system, the product of 44 and 11 is 3414. The number 3111 of this system, when converted to the decimal number system, becomes

(a) 406 (b) 1086

(c) 213 (d) 691

Directions for Questions 42 and 43: Each question is followed by two statements, I and II.

Mark

(a) if the question can be answered by one of the statements alone and not by the other.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both statements together.

42. What are the values of m and n?

I. n is an even integer, m is an odd integer, and m is greater than n.

II. Product of m and n is 30.

43. What is the value of X?

I. X and Y are unequal even integers, less than 10, and $\frac{X}{Y}$ is an odd integer.

II. X and Y are even integers, each less than 10, and product of X and Y is 12.

2003(R)

44. Let x and y be positive integers such that x is prime and y is composite. Then,

(a) $y - x$ cannot be an even integer

(b) xy cannot be an even integer

(c) $\frac{(x + y)}{x}$ cannot be an even integer

(d) None of these

2005

45. For a positive integer n, let p_n denote the product of the digits of n and s_n denote the sum of the digits of n. The number of integers between 10 and 1000 for which $p_n + s_n = n$ is

(a) 81 (b) 16

(c) 18 (d) 9

2007

46. Consider four-digit numbers for which the first two digits are equal and the last two digits are also equal. How many such numbers are perfect squares?

(a) 3 (b) 2

(c) 4 (d) 0

(e) 1

MEMORY BASED QUESTIONS

2009

47. $N = 4^7 \times 5^9 + 2^4 \times 7 + 3 \times 5^3 + 2^6 \times 5^8$

How many distinct digits are there in the number N?

(a) 8 (b) 7

(c) 5 (d) 6

48. $a^6 + b^6$ is a prime number. If a and b are distinct integers, then what is their sum?

(a) 0 (b) 2

(c) 1 (d) Data Inconsistent

49. If $a = \frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}}$ and $b = \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}}$, then what is the value of $\frac{\frac{1}{a}+\frac{1}{b}}{1+\frac{1}{ab}}$?

(a) 15 (b) 113

(c) 23 (d) None of these

50. If $x = \sqrt{15} - \sqrt{11}$, $y = \sqrt{27} - \sqrt{23}$, $z = 2\left(\sqrt{6} - \sqrt{5}\right)$, then which of the following is correct?

(a) $x > y > z$

(b) $x < y < z$

(c) $x < z < y$

(d) $x > z > y$

2010

51. A 3-digit natural number 'abc', where a, b and c are distinct digits, when increased by 33.33% becomes 'cab'. When 'cab' is increased by 33.33% it becomes 'bca'. How many such numbers are there?

(a) 0 (b) 1

(c) 2 (d) 5

52. If a and b are real numbers such that $a^{a^b} = b$ and $a \neq b$, then what is the value of $a^b - b$?

(a) –1 (b) 0

(c) 1 (d) 2

53. $M = \sqrt{3 - \sqrt{5} + \sqrt{9 - 4\sqrt{5}}}$ and $N = \sqrt{\sqrt{7} - 1 - \sqrt{11 - 4\sqrt{7}}}$. What is the value of $\frac{M-N}{M+N}$?

(a) 0 (b) 1

(c) –1 (d) None of these

54. N is a five-digit perfect square whose unit digit is same as the tens digit. How many such N are there?

(a) 31 (b) 30

(c) 33 (d) 32

2011

55. Which of the following number(s) is/are not prime?

(i) $2^{5001} + 1$

(ii) $2^{5002} + 1$

(iii) $2^{5003} + 1$

(a) (i) and (ii) (b) (i) and (iii)

(c) (ii) and (iii) (d) (i), (ii) and (iii)

56. How many positive integer pairs (x, y) satisfy $\sqrt{x} + \sqrt{y} = \sqrt{2003}$?

(a) 0 (b) 1

(c) 2 (d) 3

57. The sum of thirty-two consecutive natural numbers is a perfect square. What is the least possible sum of the smallest and the largest of the thirty-two numbers?

(a) 81 (b) 36

(c) 49 (d) 64

2012

58. The number 44 is written as a product of 5 distinct integers. If 'n' is the sum of these five integers then what is the sum of all the possible values of n?

(a) 11 (b) 23

(c) 26 (d) 32

2013

59. Arrange the numbers $2^{\frac{7}{6}}, 3^{\frac{3}{4}}$ and $5^{\frac{2}{3}}$ in ascending order.

(a) $2^{\frac{7}{6}} > 3^{\frac{3}{4}} > 5^{\frac{2}{3}}$ (b) $3^{\frac{3}{4}} > 2^{\frac{7}{6}} > 5^{\frac{2}{3}}$

(c) $5^{\frac{2}{3}} > 3^{\frac{3}{4}} > 2^{\frac{7}{6}}$ (d) None of these

2014

60. If $7^a = 26$ and $343^b = 676$ then what is the relation between a and b?

(a) a = b

(b) a = 2b

(c) 2a = 3b

(d) 3a = 2b

2015

61. A four-digit number is divisible by the sum of its digits. Also, the sum of these four digits equals the product of the digits. What could be the product of the digits of such a number?

(a) 6 (b) 8

(c) 10 (d) 12

62. Let P be the set of all odd positive integers such that every element in P satisfies the following conditions.

I. $100 \le n < 1000$

II. The digit at the hundred's place is never greater than the digit at tens place and also never less than the digit at units place.

How many elements are there in P?

(a) 93 (b) 94

(c) 95 (d) 96

63. If $3x + y + 4 = 2xy$, where x and y are natural numbers, then find the ratio of the sum of all possible values of x to the sum of all possible values of y.

(a) $\frac{2}{3}$ (b) $\frac{15}{19}$

(c) $\frac{17}{21}$ (d) $\frac{7}{9}$

2016

64. How many natural numbers less than or equal to 15 have 4 factors each?

2017

65. Let a_1, a_2, a_3, a_4, a_5 be a sequence of five consecutive odd numbers. Consider a new sequence of five consecutive even numbers ending with $2a_3$.

If the sum of the numbers in the new sequence is 450, then a_5 is

2018 Slot 1

66. While multiplying three real numbers, Ashok took one of the numbers as 73 instead of 37. As a result, the product went up by 720. Then the minimum possible value of the sum of squares of the other two numbers is

2018 Slot 2

67. If the sum of squares of two numbers is 97, then which one of the following cannot be their product?

(1) 48 (2) –32

(3) 16 (4) 64

68. The smallest integer n for which $4^n > 17^{19}$ holds, is closest to

(1) 35 (2) 37

(3) 33 (4) 39

FACTORIAL AND ITS APPLICATION

1991

1. What is the greatest power of 5 which can divide 80! exactly?

(a) 16 (b) 20

(c) 19 (d) None of these

1993

2. The product of all integers from 1 to 100 will have the following numbers of zeros at the end.

(a) 20 (b) 24

(c) 19 (d) 22

1997

3. ABC is a three-digit number in which A > 0. The value of ABC is equal to the sum of the factorials of its three digits. What is the value of B?

(a) 9 (b) 7

(c) 4 (d) 2

2005

4. Let $n! = 1 \times 2 \times 3 \times \ldots \times n$ for integer $n \ge 1$.

If $p = 1! + (2 \times 2!) + (3 \times 3!) + \ldots + (10 \times 10!)$, then $p + 2$ when divided by 11! leaves a remainder of

(a) 10 (b) 0

(c) 7 (d) 1

MEMORY BASED QUESTIONS

2009

5. N is a natural number which has 4 factors. If $10 \le N \le 70$, then how many values are possible for N?

(a) 19 (b) 20

(c) 21 (d) 22

2010

6. $N = 70! \times 69! \times 68! \times \ldots 3! \times 2! \times 1!$

Which of the following represents the 147th digit from the right end of N?

(a) 2 (b) 0

(c) 5 (d) 7

2012

7. 500! + 505! + 510! + 515! is completely divisible by 5^n, where n is a natural number. How many distinct values of n are possible?

(a) 120

(b) 121

(c) 124

(d) 125

2013

8. 'ab' is a two-digit prime number such that one of its digits is 3. If the absolute difference between the digits of the number is not a factor of 2, then how many values can 'ab' assume?

(a) 5 (b) 3
(c) 6 (d) 8

9. The number of factors of the square of a natural number is 105. The number of factors of the cube of the same number is 'F'. Find the maximum possible value of 'F'.

(a) 208 (b) 217
(c) 157 (d) 280

10. Let $R(x) = mx^3 - 100x^2 + 3n$, where m and n are positive integers. For how many ordered pairs (m, n) will $(x - 2)$ be a factor of $R(x)$?

(a) 16 (b) 17
(c) 18 (d) 15

11. How many natural numbers divide exactly one out of 1080 and1800, but not both?

(a) 20 (b) 42
(c) 24 (d) 36

2014

12. 'P' is the product of ten consecutive two-digit natural numbers. If 2^a is one of the factors of P, then the maximum value that 'a' can assume is

(a) 11 (b) 12
(c) 13 (d) 14

2015

13. P is the product of the first 100 multiples of 15 and Q is the product of the first 50 multiples of 25^{20}. Find the number of consecutive zeroes at the end of $\frac{P^2}{Q} \times 10^{1767}$.

(a) 1968 (b) 1914
(c) 3 (d) 2024

2016

14. $N = 5^{1!+2!+3!+4!}$

The digits of the number N are added to get another number. Then the digits of the number obtained are added to get yet another number. The process is repeated till a single digit number is obtained. What is that single digit number?

HCF AND LCM

1993

1. The smallest number which when divided by 4, 6 or 7 leaves a remainder of 2, is

(a) 44 (b) 62
(c) 80 (d) 86

1994

2. What is the smallest number which when increased by 5 is completely divisible by 8, 11 and 24?

(a) 264 (b) 259
(c) 269 (d) None of these

3. Which is the least number that must be subtracted from 1856, so that the remainder when divided by 7, 12 and 16 is 4.

(a) 137 (b) 1361
(c) 140 (d) 172

1995

4. Three bells chime at an interval of 18 min, 24 min and 32 min. At a certain time they begin to chime together. What length of time will elapse before they chime together again?

(a) 2 hr and 24 min (b) 4 hr and 48 min
(c) 1 hr and 36 min (d) 5 hr

Direction for Question 5: The question is followed by two statements, I and II. Mark the answer as:

(a) if the question can be answered with the help of statement I alone.
(b) if the question can be answered with the help of statement II alone.
(c) if both statement I and statement II are needed to answer the question.
(d) if the question cannot be answered even with the help of both the statements.

5. What is the number x?

I. The LCM of x and 18 is 36.
II. The HCF of x and 18 is 2.

1998

6. Three wheels can complete 60, 36 and 24 revolutions per minute. There is a red spot on each wheel that touches the ground at time zero. After how much time, all these spots will simultaneously touch the ground again?

(a) $\frac{5}{2}$ s (b) $\frac{5}{3}$ s
(c) 6 s (d) 7.5 s

7. A is the set of positive integers such that when divided by 2, 3, 4, 5, 6 leaves the remainders 1, 2, 3, 4, 5 respectively. How many integers between 0 and 100 belong to set A?

(a) 0 (b) 1
(c) 2 (d) None of these

8. Number of students who have opted for subjects A, B and C are 60, 84 and 108 respectively. The examination is to be conducted for these students such that only the students of the same subject are allowed in one room. Also the number of students in each room must be same. What is the minimum number of rooms that should be arranged to meet all these conditions?

(a) 28 (b) 60
(c) 12 (d) 21

1999

9. For two positive integers a and b define the function **h**(a, b) as the greatest common factor (G.C.F) of a, b. Let A be a set of n positive integers. G(A), the G.C.F of the elements of set A is computed by repeatedly using the function **h**. The minimum number of times **h** is required to be used to compute G is

(a) $\frac{1}{2}$ n (b) (n – 1)
(c) n (d) None of these

2001

10. A red light flashes three times per minute and a green light flashes five times in 2 min at regular intervals. If both lights start flashing at the same time, how many times do they flash together in each hour?

(a) 30 (b) 24
(c) 20 (d) 60

11. Of 128 boxes of oranges, each box contains at least 120 and at most 144 oranges. The number of boxes containing the same number of oranges is at least

(a) 5 (b) 103
(c) 6 (d) Cannot be determined

2002

12. At a bookstore, 'MODERN BOOK STORE' is flashed using neon lights. The words are individually flashed at the intervals of $2\frac{1}{2}$ s, $4\frac{1}{4}$ s and $5\frac{1}{8}$ s respectively, and each word is put off after a second. The least time after which the full name of the bookstore can be read again is

(a) 49.5 s (b) 73.5 s
(c) 1744.5 s (d) 855 s

13. Three pieces of cakes of weights $4\frac{1}{2}$ lb, $6\frac{3}{4}$ lb and $7\frac{1}{5}$ lb respectively are to be divided into parts of equal weight. Further, each part must be as heavy as possible. If one such part is served to each guest, then what is the maximum number of guests that could be entertained?

(a) 54 (b) 72
(c) 20 (d) None of these

2006

14. Which among $2^{\frac{1}{2}}$, $3^{\frac{1}{3}}$, $4^{\frac{1}{4}}$, $6^{\frac{1}{6}}$ and $12^{\frac{1}{12}}$ is the largest?

(a) $2^{1/2}$ (b) $3^{1/3}$
(c) $4^{1/4}$ (d) $6^{1/6}$
(e) $12^{1/12}$

MEMORY BASED QUESTIONS

2014

15. The ratio of two numbers whose is 600 is 7 : 8. What is the LCM of the given two numbers?

(a) 1120 (b) 560
(c) 2240 (d) 1152

2015

16. Out of 4 numbers a, b, c, and d, each pair of numbers has the same highest common factor. Find the highest common factor of all the four numbers if the least common multiple of a and b is 310 and that of c and d is 651.

17. How many ordered triplets (a, b, c) exist such that LCM (a, b) = 1000, LCM(b, c) = 2000, LCM (c, a) = 2000 and HCF (a, b) = k × 125?

(a) 32 (b) 28
(c) 24 (d) 20

REMAINDER

1990

1. The remainder when 2^{60} is divided by 5 equals

(a) 0 (b) 1
(c) 2 (d) None of these

1991

2. Find the minimum integral value of n such that the division 55n/124 leaves no remainder.

(a) 124 (b) 123
(c) 31 (d) 62

1995

3. $5^6 - 1$ is divisible by

(a) 13 (b) 31
(c) 5 (d) None of these

4. The remainder obtained when a prime number greater than 6 is divided by 6 is

(a) 1 or 3 (b) 1 or 5
(c) 3 or 5 (d) 4 or 5

1998

5. A certain number, when divided by 899, leaves a remainder 63. Find the remainder when the same number is divided by 29.

(a) 5
(b) 4
(c) 1
(d) Cannot be determined

6. What is the digit in the unit's place of 2^{51}?

(a) 2 (b) 8
(c) 1 (d) 4

7. A number is formed by writing first 54 natural numbers in front of each other as 12345678910111213 ... Find the remainder when this number is divided by 8.

(a) 1 (b) 7
(c) 2 (d) 0

1999

8. The remainder when 7^{84} is divided by 342 is

(a) 0 (b) 1
(c) 49 (d) 341

2000

9. Let N = 1421 × 1423 × 1425. What is the remainder when N is divided by 12?

(a) 0 (b) 9
(c) 3 (d) 6

10. The integers 34041 and 32506, when divided by a three-digit integer n, leave the same remainder. What is the value of n?

(a) 289 (b) 367
(c) 453 (d) 307

11. When 2^{256} is divided by 17, the remainder would be

(a) 1 (b) 16
(c) 14 (d) None of these

2002

12. After the division of a number successively by 3, 4 and 7, the remainders obtained are 2, 1 and 4 respectively. What will be the remainder if 84 divides the same number?

(a) 80 (b) 75
(c) 41 (d) 53

Directions for Question 13: The question is followed by two statements, A and B. Answer the question using the following instructions.

Choose 1 if the question can be answered by one of the statements alone but not by the other.

Choose 2 if the question can be answered by using either statement alone.

Choose 3 if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose 4 if the question cannot be answered even by using both statements together.

13. Four students were added to a dance class. Would the teacher be able to divide her students evenly into a dance team (or teams) of 8?

A. If 12 students were added, the teacher could put everyone in teams of 8 without any leftovers.

B. The number of students in the class is currently not divisible by 8.

(a) 1 (b) 2
(c) 3 (d) 4

2003(R)

14. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?

(a) 666 (b) 676
(c) 683 (d) 777

15. What is the remainder when 4^{96} is divided by 6?

(a) 0 (b) 2
(c) 3 (d) 4

Directions for Question 16: The question is followed by two statements, A and B. Answer each question using the following instructions.

Choose (a) if the question can be answered by one of the statements alone but not by the other.

Choose (b) if the question can be answered by using either statement alone.

Choose (c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

Choose (d) if the question cannot be answered even by using both the statements together.

2003(L)

16. Is $a^{44} < b^{11}$, given that a = 2 and b is an integer?

A. b is even

B. b is greater than 16

2004

17. The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is

(a) 4 (b) 15

(c) 0 (d) 18

2005

18. If $x = (16^3 + 17^3 + 18^3 + 19^3)$, then x divided by 70 leaves a remainder of

(a) 0 (b) 1

(c) 69 (d) 35

19. The rightmost non-zero digit of the number 30^{2720} is

(a) 1 (b) 3

(c) 7 (d) 9

2008

20. What are the last two digits of 7^{2008}?

(a) 21 (b) 61

(c) 01 (d) 41

(e) 81

MEMORY BASED QUESTIONS

2015

21. There are 40 students in a class. A student is allowed to shake hand only once with a student who is taller than him or equal in height to him. He can't shake hand with anyone who is shorter than him. Average height of the class is 5 feet. What is the difference between the maximum and minimum number of handshakes that can take place in the class?

(a) $\left(\frac{^{40}C_2}{2} - 20\right)$ (b) 361

(c) $^{40}C_2 - 40$ (d) $^{40}C_2$

MISCELLANEOUS

2000

Direction for Question 1: The question is followed by two statements, I and II. Answer the question using the following instructions.

Mark the answer as:

(a) if the question can be answered by one of the statements alone, but cannot be answered by using the other statement alone.

(b) if the question can be answered by using either statement alone.

(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.

(d) if the question cannot be answered even by using both statements together.

1. What are the ages of two individuals, X and Y?

I. The age difference between them is 6 years.

II. The product of their ages is divisible by 6.

2002

2. Number S is obtained by squaring the sum of digits of a two-digit number D. If difference between S and D is 27, then the two-digit number D is

(a) 24 (b) 54

(c) 34 (d) 45

3. If u, v, w and m are natural numbers such that $u^m + v^m = w^m$, then which one of the following is true?

(a) m $\geq$ min(u, v, w) (b) m $\geq$ max(u, v, w)

(c) m < min(u, v, w) (d) None of these

Directions for Questions 4 to 6: Answer the questions on the basis of the information given below.

The seven basic symbols in a certain numeral system and their respective values are as follows:

I = 1, V = 5, X = 10, L = 50, C = 100, D = 500 and M = 1000

In general, the symbols in the numeral system are read from left to right, starting with the symbol representing the largest value; the same symbol cannot occur continuously more than three times; the value of the numeral is the sum of the values of the symbols. For example, XXVII = 10 + 10 + 5 + 1 + 1 = 27. An exception to the left-to-right reading occurs when a symbol is followed immediately by a symbol of greater value; then the smaller value is subtracted from the larger.

For example, XLVI = (50 – 10) + 5 + 1 = 46.

2003(R)

4. The value of the numeral MDCCLXXXVII is

(a) 1687 (b) 1787

(c) 1887 (d) 1987

5. The value of the numeral MCMXCIX is

(a) 1999 (b) 1899

(c) 1989 (d) 1889

6. Which of the following represent the numeral for 1995?

I. MCMLXXV II. MCMXCV

III. MVD IV. MVM

(a) Only I and II (b) Only III and IV

(c) Only II and IV (d) Only IV

2003(L)

7. A positive whole number M less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit is 1. Then M equals

(a) 31 (b) 63
(c) 75 (d) 91

2004

8. Suppose n is an integer such that the sum of digits of n is 2, and $10^{10} < n < 10^{11}$. The number of different values of n is

(a) 11 (b) 10
(c) 9 (d) 8

2006

9. The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square.

Which of the following can possibly be one of these four numbers?

(a) 21 (b) 25
(c) 41 (d) 67
(e) 73

2008

10. Suppose, the seed of any positive integer n is defined as follows:

seed(n) = n, if n < 10
= seed(s(n)), otherwise,

where s(n) indicates the sum of digits of n. For example, seed(7) = 7, seed(248) = seed(2 + 4 + 8) = seed(14) = seed(1 + 4) = seed(5) = 5 etc. How many positive integers n, such that n < 500, will have seed (n) = 9?

(a) 39 (b) 72
(c) 81 (d) 108
(e) 55

MEMORY BASED QUESTIONS

2011

11. In a three-digit number, the unit digit is twice the tens digit and the tens digit is twice the hundreds digit. The same number is written as 1XY and 1YX in base 8 and base 9 respectively. Find the sum of X and Y in the decimal system.

(a) 15
(b) 7
(c) 11
(d) Cannot be determined

2015

12. If $x^2 + (x + 1)(x + 2)(x + 3)(x + 6) = 0$, where x is a real number, then one value of x that satisfies this equation is

(a) $3-\sqrt{3}$
(b) $3+\sqrt{3}$
(c) $(-3+\sqrt{3})$
(d) 0

13. If $x^4 - y^4 = 15$, where x and y are natural numbers, then find the value of the expression $x^4 + y^4$.

2016

14. The absolute difference between the average of first N1 natural numbers and that of the first N2 natural numbers is 10. What is the absolute difference between N1 and N2?

15. A rational number $\frac{A}{B}$, where A and B are co-prime, is converted into a decimal number. If both A and B are less than 100, then for how many values of B will $\frac{A}{B}$ always be a terminating decimal?

(a) 40 (b) 60
(c) 14 (d) 15

16. How many terms of the sequence given below are integers?

$$21, 7, \frac{21}{5}, \ldots, \frac{21}{2n-1}, n \in N$$

(a) 2
(b) 3
(c) 4
(d) 5

17. A florist sells only two kinds of flowers – Rose and Tulip. On a particular day, he sold 70 Roses and 90 Tulips. If none of his customers bought more than one flower of each type, what is the minimum number of customers that must have visited his shop on that day?

2017

18. The numbers 1, 2, ... 9 are arranged in a 3 × 3 square grid in such a way that each number occurs once and the entries along each column, each row, and each of the two diagonals add up to the same value.

If the top left and the top right entries of the grid are 6 and 2, respectively, then the bottom middle entry is

19. If the product of three consecutive positive integers is 15600 then the sum of the squares of these integers is
(a) 1777 (b) 1785
(c) 1875 (d) 1877

20. How many different pairs (a, b) of positive integers are there such that a ≤ b and $\frac{1}{a}+\frac{1}{b}=\frac{1}{9}$?

2018 Slot 2

21. The value of the sum 7 x 11 + 11 x 15 + 15 x 19 + ...+ 95 x 99 is
(a) 80730 (b) 80773
(c) 80707 (d) 80751

ANSWERS

Divisibility

1. (c)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (d)	**9.** (b)	**10.** (c)
11. (c)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (c)	**16.** (d)	**17.** (d)	**18.** (d)	**19.** (d)	**20.** (a)
21. (c)	**22.** (b)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (a)			

Property

1. (a)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (d)
11. (a)	**12.** (d)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (d)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (a)
21. (c)	**22.** (d)	**23.** (a)	**24.** (d)	**25.** (d)	**26.** (b)	**27.** (c)	**28.** (d)	**29.** (c)	**30.** (b)
31. (c)	**32.** (c)	**33.** (c)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (d)	**38.** (a)	**39.** (c)	**40.** (a)
41. (a)	**42.** (c)	**43.** (a)	**44.** (d)	**45.** (d)	**46.** (e)	**47.** (b)	**48.** (a)	**49.** (a)	**50.** (d)
51. (c)	**52.** (b)	**53.** (a)	**54.** (a)	**55.** (d)	**56.** (a)	**57.** (c)	**58.** (a)	**59.** (c)	**60.** (c)
61. (b)	**62.** (c)	**63.** (d)	**64.** (e)	**65.** (51)	**66.** (40)	**67.** (d)	**68.** (d)		

Factorial and its Application

1. (c)	**2.** (b)	**3.** (*)	**4.** (d)	**5.** (b)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (d)	**10.** (a)
11. (a)	**12.** (c)	**13.** (b)	**14.** (8)						

HCF and LCM

1. (d)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (*)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (a)
11. (c)	**12.** (b)	**13.** (d)	**14.** (b)	**15.** (c)	**16.** 31	**17.** (b)			

Remainder

1. (b)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (d)
11. (a)	**12.** (d)	**13.** (a)	**14.** (b)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (a)	**20.** (c)
21. (d)									

Miscellaneous

1. (d)	**2.** (b)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (c)	**10.** (e)
11. (c)	**12.** (c)	**13.** 17	**14.** (20)	**15.** (c)	**16.** (c)	**17.** (90)	**18.** (c)	**19.** (d)	**20.** (c)
21. (c)									

EXPLANATIONS

Divisibility

1. c $n^3 - n = n(n^2 - 1) = (n - 1)n(n + 1)$

Above expression is the product of three consecutive numbers. So at least one number is even and one number is a multiple of 3. So the product is always divisible by 6.

2. b From the question we can figure out that P lies between 2371 and 2379.

Using statement I: P = 2372 or 2376

Using statement II: P = 2376

Hence, statement II alone is sufficient to get a unique value of R.

3. c As none of the statement gives information about all three – x, y and z, they alone are not sufficient to answer the question.

Using both statements together: x, y and z are three consecutive odd integers.

Therefore, x + z is not divisible by 4.

4. a a = k + 4 is divisible by 7

b = k + 2n is divisible by 7

$\Rightarrow$ b – a = 2n – 4 is divisible by 7

$\Rightarrow$ 2n – 4 = 0 or 7 or 14 ...

For minimum integral value of n,

2n – 4 = 14

$\Rightarrow$ n = 9.

5. c If (2x + 12) is perfectly divisible by x, then (2x + 12)/x has to be an integer as x is an integer. Now if we divide, the expression simplifies to (2 + 12/x). The only way in which this expression would be an integer is when 12/x is an integer or if 12 is perfectly divisible by x. This is possible if x takes either of these values : 1, 2, 3, 4, 6, 12.

Hence, the answer is 6 values.

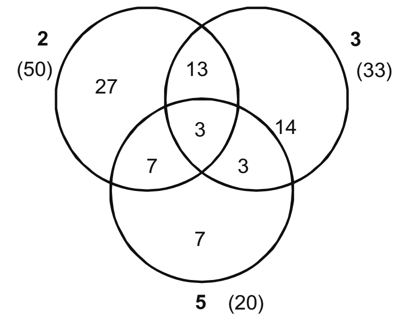
6. d As each prime number greater than or equal to 3 is of the form $6k \pm 1$, where k is a natural number,

$p = 6k \pm 1$.

$p^2 - 1 = (6k \pm 1)^2 - 1 = 12k(3k \pm 1)$

Above number is always divisible by 24 irrespective of value of k.

7. a The following Venn diagram shows the distribution of numbers between 1 and 100 that are divisible by 2, 3, 5 or a combination of two or more of them.



50 numbers are divisible by 2, 33 numbers are divisible by 3 and 20 numbers are divisible by 5.

3 numbers are divisible by all 2, 3 and 5.

16 numbers are divisible by both 2 and 3, therefore

13 numbers are divisible by 2 and 3 but not by 5.

10 numbers are divisible by both 2 and 5, therefore

7 numbers are divisible by 2 and 5 but not by 3.

6 numbers are divisible by both 3 and 5, therefore

3 numbers are divisible by 3 and 5 but not by 2.

Total number of numbers that are divisible by one or more among 2, 3 and 5 = 27 + 14 + 7 + 13 + 3 + 7 + 3 = 74

Hence, the required number = 100 – 74 = 26.

8. d Since n(n + 1) are two consecutive integers, one of them will be even and thus their product will always be even.

Also, sum of the squares of first ‘n’ natural numbers is given by $\frac{n(n+1)(2n+1)}{6}$.

Hence, our product will always be divisible by this.

Also you will find that the product is always divisible by 3 (you can use any value of n to verify this).

However, we can find that option (d) is not necessarily true. **E.g.** If n = 118, (2n + 1) = 237 or if n = 236, then (n + 1) = 237 or if n itself is 237, etc.

9. b For the number to be divisible by 9, the sum of the digits should be a multiple of 9.

We find that the sum of all the digits (excluding A and B) = (7 + 7 + 4 + 9 + 5 + 8 + 9 + 6) = 55. The next higher multiple of 9 is 63 or 72.

Hence, the sum of A and B should either be 8 or 17. We find that (a) and (c) cannot be the answer.

For a number to be divisible by 8, the number formed by its last three digits should be divisible by 8. The last three digits are 96B. The multiples of 8 beginning with 96 are 960 and 968. Hence, B can either be 0 or 8. Both of which satisfy our requirement of the number being divisible by 9 as well. Therefore, A and B could either be 0 and 8 or 8 and 0 respectively.

10. c $n(n^2 - 1) = (n - 1)n(n + 1)$. If you observe, this is the product of three consecutive integers with middle one being an odd integer. Since there are two consecutive even numbers, one of them will be a multiple of 4 and the other one will be a multiple of 2. Hence, the product will be a multiple of 8. Also since they are three consecutive integers, one of them will definitely be a multiple of 3. Hence, this product will always be divisible by $(3 \times 8) = 24$.

Hint: Students, please note if a number is divisible by 96, it will also be divisible by 48 and 24. Similarly, if a number is divisible by 48, it will always divisible by 24. Since there cannot be more than one right answers, we can safely eliminate options (a) and (b).

11. c The best way to solve this is the method of simulation, E.g. let m = 10 and n = 5. Therefore, m – n = 5, which is divisible by 5.

$m^2 - n^2 = 100 - 25 = 75$, divisible by 25. m + n = 10 + 5 = 15 is not divisible by 10.

Hence, the answer is (c).

Note that for the sum of two multiples of 5 to be divisible by 10, either both of them should be odd (i.e. ending in 5) or both of them should be even (i.e. ending in 0).

12. b 11 and 9 are coprimes of 99, and hence the number divisible by 99 must be divisible by 9 and 11.

Certainly statement I alone is sufficient to answer the question. Statement II says when the digits of the number are reversed, the new number formed is divisible by 9 and 11. The best way to handle this particular case is by simulation. Let us select any number which is divisible by 9 and 11. Let us select 1386 which is divisible by 9 and 11. Hence, the original number will be 6831 which, in turn, is also divisible by 99. Hence, statement II also is independently sufficient to answer the question. Since both the statements are independently sufficient to answer the question, the answer is (b).

13. c Let us find some of the smaller multiples of 125. They are 125, 250, 375, 500, 625, 750, 875, 1000, ...

A five-digit number is divisible by 125, if the last three digits are divisible by 125. So the possibilities are 375 and 875, 5 should come in unit's place, and 7 should come in ten's place. Thousand's place should contain 3 or 8. We can do it in 2! ways. Remaining first two digits, we can arrange in 2! ways. So we can have 2! × 2! = 4 such numbers.

There are: 23875, 32875, 28375, 82375.

14. d N can be written either as $(54 + 1)^3 + (18 - 1)^3 - 72^3$ or $(51 + 4)^3 + 17^3 - (68 + 4)^3$.

The first form is divisible by 3 and the second by 17.

15. c $99 \times D = a_1a_2$. Thus, $D = \frac{a_1 a_2}{99}$. Hence, D must be multiplied by 198 as 198 is a multiple of 99.

16. d There are 33 numbers between 100 and 200 which are divisible by 3.

Out of these, 17 are even and 16 are odd.

There are 5 numbers between 100 and 200 which are divisible by 21 (LCM of 3 and 7).

Out of these, 3 are odd.

Hence, the number of odd numbers divisible by 3, but not by 7 is (16 – 3) = 13.

17. d $a = b^2 - b,\ b \geq 4$

$a^2 - 2a = (b^2 - b)^2 - 2(b^2 - b) = (b - 2)(b - 1)\,b\,(b + 1)$

Using different values of $b \geq 4$, we will find that $a^2 - 2a$ is divisible by 15, 20 and 24.

Hence, all of these is the right answer.

18. d $7^{6n} - 6^{6n}$

Putting n = 1.

$7^6 - 6^6 = (7^3 - 6^3)(7^3 + 6^3)$

This is a multiple of $7^3 - 6^3 = 127$ and $7^3 + 6^3 = 559$ and 7 + 6 = 13. Hence, all of these is the right answer.

19. d Let n = 6.

Therefore, $\sqrt{n} = \sqrt{6} \approx 2.4$.

Therefore, divisors of 6 are 1, 2 and 3.

If we take 2 as divisor, then, $\sqrt{n} > 2 > 1$.

Therefore, statement A is true.

If we take 3 as divisor, then 6 > 3 > 2.4, i.e., $n > 3 > \sqrt{n}$.

Therefore, statement B is also true.

20. a Any prime number greater than 3 can be expressed in the form of $6n \pm 1$ and minimum difference between three consecutive prime numbers is 2 and 4. The values that satisfy the given conditions are only 3, 5 and 7, i.e., only one set is possible.

21. c There are 101 integers between 100 and 200, of which 51 are even.

Between 100 and 200, there are 14 multiples of 7, of which 7 are even.

There are 11 multiples of 9, of which 6 are even.

But there is one integer (i.e., 126) that is a multiple of both 7 and 9 and also even.

Hence, the answer is (51 – 7 – 6 + 1) = 39.

22. b From 12 to 40, there are 7 prime numbers, i.e., 13, 17, 19, 23, 29, 31 and 37 such that (n – 1)! is not divisible by any of them.

23. b Let A = abc. Then, B = cba.

Given, B > A $\Rightarrow$ c > a

As B – A = (100c + 10b + a) – (100a + 10b + c)

$\Rightarrow$ B – A = 100 (c – a) + (a – c)

$\Rightarrow$ B – A = 99 (c – a). Also, (B – A) is divisible by 7.

But, 99 is not divisible by 7 (no factor like 7 or 7^2). Therefore, (c – a) must be divisible by 7 {i.e., (c – a) must be 7, 7^2, etc.}. Since c and a are single digits, value of (c – a) must be 7. The possible values of (c, a) {with c > a} are (9, 2) and (8, 1). Thus, we can write A as:

A : abc $\equiv$ 1b8 or 2b9

As b can take values from 0 to 9, the smallest and largest possible value of A are:

A_{min} = 108 and A_{max} = 299

Only option (b) satisfies this.

Hence, (b) is the correct option.

24. a The 100^{th} and 1000^{th} position values will be only 1.

Different possibilities of unit and tens digits are

(1, 3), (1, 9), (3, 1), (3, 7), (5, 5), (7, 3), (7, 9), (9, 1) and (9, 7).

Hence, there are 9 elements in S.

25. c Two digit multiples of 8 are 16, 24, 32, 40, 48, 56, 64, 72, 80, 88 and 96. Out of these only two numbers - 40 and 48 satisfy the given condition. So the answer is 2.

26. a x = $106^{90} - 49^{90}$

$(x^n - a^n)$ is divisible by both (x – a) and (x + a) whenever n is even

$\Rightarrow$ $(106^{90} - 49^{90})$ is divisible by both 57 and 155

57 = 19 × 3

155 = 31 × 5

Therefore, $(106^{90} - 49^{90})$ will be divisible by (19 × 31) = 589 as well.

Also, note that $(106^{90} - 49^{90})$ will be odd and options (b) and (c) are even. Hence, they can be rejected.

27. a The form of a number which when divided by 3, 5 and 7 leaves remainders 2, 4 and 6 respectively

= k × (LCM of 3, 5 and 7) – 1,

where k is a natural number

= 105k – 1

For the number to be the smallest, k has to be 1.

Hence, the smallest such number = 105 × 1 – 1 = 104.

Property

1. a From statement I, $x^2 + x > 50$ and $x^2 + x < 100$

$\Rightarrow x(x+1) > 50$

Only prime number 7 satisfies the above equation.

So the question can be answered from statement I alone.

2. a Let the 3 odd numbers be (x – 2), x and (x + 2). It is given that 3(x – 2) = 3 + 2(x + 2)

$\Rightarrow$ x = 13. Hence, the third integer is (x + 2) = 15.

3. c $2^{73} - 2^{72} - 2^{71} = 2^{71}(2^2 - 2 - 1) = 2^{71}(4 - 2 - 1) = 2^{71}$.

4. c Since the sum of reciprocals is $\frac{5}{12}$. The two numbers must have their LCM as 12 and their sum as given will be 10.

Possible numbers are 4 and 6 only.

5. a Going by the options:

(a) $(x-1)yz = xyz - yz$ (b) $x(y-1)z = xyz - xz$
(c) $xy(z-1) = xyz - xy$ (d) $x(y+1)z = xyz + xz$

Here, yz will be minimum out of yz, xz, xy, xz as x > y > z

Hence, the correct answer is option (a).

6. b The last digits obtained by multiplying the units place digits should be the same as that obtained by multiplying the tens place digits.

Hence, option (b) is the correct answer.

7. a $203 = 2.3^2 + 0.3^1 + 3.3^0 = 18 + 0 + 1 = 21$

$21 = 2.3^1 + 1.3^0 = 6 + 1 = 7$.

Therefore, we can reduce 203 to 7 in 2 steps.

8. a Here logic is:

A + B = (A + B) – 18

Hence, 10 + 18 = {(10 + 18) – 18} = 10.

9. c **From statement I:**

(a, b) can be (1, 8), (2, 4), (4, 2) and (8, 1).

Therefore, statement I alone cannot give the value of a, b and c.

From statement II:

(b, c) can be (1, 9), (3, 3) and (9, 1).

On combining statements I and II:

b = 1, a = 8 and c = 9

Hence, the answer is (c).

10. d From statement I, the ages could be either (1, 3, 7) or (1, 1, 21). Statement II doesn't simplify this further as none of the above combinations when added is divisible by 3.

Hence, the answer is (d).

11. a It can be very easy to figure out that (x + y) will always be greater than xy, only if one of them is 1.

E.g. If x = 1 and y = 2, then (x + y) = 3 and xy = 2.

Hence, (x + y) > xy.

Other than this, for all other values of x & y, (x + y) will always be less than xy, and hence, the ratio of $\frac{(x+y)}{xy} < 1$, and hence, cannot be an integer.

Also, even if one of the values is 1, $\frac{(x+y)}{xy}$ will never be an integer. Hence, the answer is (a).

12. d You can do this by the method of simulation. For eg. Let the three numbers be 1, 3 and 5. So option (a) is $1^2.3^2.5^2$ = 225, which is odd. (b) is $3(1^2 + 3^3)5^2$ = 2100, which is even.

$5 + 3 + 5^4 = 633$, which is odd. (d) is $\frac{5^2(1^4 + 3^4)}{2} = 1025$, which is not even and hence, the answer is (d).

13. b Since there are two numbers which are less than 1 (viz. x and y), it is obvious that the median will be less than 1. Hence, (c) cannot be the answer.

Since x < 0.5 and 0 < y < 1, the median will not be less than 0. Hence, the answer is (b).

14. a Let us evaluate each option.

Option (a):

As x < 0 and z > 1, let x = – 1 and z = 3, then $(x^2 - z^2) = -8$. Hence, this option is not true.

Option (b):

As 0 < y < 1 and z > 1, let $y = \frac{1}{4}$ and z = 2,

then $yz = \frac{1}{4} \times 2 = \frac{1}{2}$.

Therefore, yz can be less than 1.

Option (c):

Since none of the x and y is equal to zero, therefore xy can never be zero.

Option (d):

0 < y < 1 and z > 1, therefore $(y^2 - z^2)$ is always negative.

Hence, answer is (a).

15. b She counted thumb on 1, 9, 17, 25 and so on. So it forms an arithmetic progression.

She counted thumb closest of 1994 on

(1 + 1992 (multiple of 8)) = 1993

Hence, she would have counted 1994 on the index finger.

16. d In this case since x, y and z are distinct positive integers, our aim is figure out which of the answer choices cannot be expressed as the sum of 3 integers uniquely. **E.g.** 6 can only be expressed as (1 + 2 + 3). 7 can only be expressed as (1 + 2 + 4). But 8 can be expressed as either (1, 2, 5) or (1, 3, 4).

17. c **Hint:** The best way to solve this question is to multiply the alternatives by 72 and find which one gives you the middle three digits as 96.7.

To save time, you can multiply 72 by integer values only.

E.g. 72 × 3 = 216, 72 × 5 = 360 and 72 × 7 = 504.

It is to be noted that when the decimal part of the answer will be multiplied by 72, the actual answer will increase.

Let us now roughly multiply the decimal values of the options also by 72.

E.g. 72 x 0.2 = 14.4, 72 × 0.1 = 7.2 and 72 × 0.5 = 36.

So option (a) will yield (216 + 14) = 230 (approximately), (b) will yield (360 + 7) = 367 (approximately), (c) will yield (360 + 36) = 396 (approximately) and (d) will yield (504 + 14) = 528 (approximately).

Of these, only option (c) satisfies our requirement of 2nd and 3rd digits being 96.

18. a The given expression is of the form $\frac{[x^3 + y^3]}{[x^2 - xy + y^2]}$.

We know, $\mathbf{x^3} + \mathbf{y^3} = (x + y)(x^2 - xy + y^2)$.

Thus, required value = (x + y) = (55 + 45) = 100.

19. b If the numbers are (x – 2), x and (x + 2), then 3(x – 2) – 2 = 2(x + 2) $\Rightarrow$ x = 12

$\therefore$ x + 2 = 14.

20. a Statement I suggests that xyz is odd. This is only possible if all three of them are odd.

Hence, z – x is even.

So only statement I is required to answer the question.

21. c This can be answered using both the statements. Statement II suggests that both t and z are odd.

Statement I suggests that (x + y + t) is even.

Since the difference between an even and an odd number is always odd, (x + y + t) – z will be odd.

22. d This question can be answered by using the two statements.

Given (a – b + c) > (a + b – c).

It is nothing but is (–b + c) > (b – c).

Since b is negative and c is positive,

$\Rightarrow$ c > b

Using both statements:

$c > 0$

$b < 0$

$c > b$

So always $(a - b + c) > (a + b - c)$.

23. a We cannot answer the question using both the statements.

Given that Anil's ages are prime numbers in 1998 and 1996. It is of difference 2. There are so many prime numbers with difference 2. They are (17, 19), (41, 43), . . . so on.

So we cannot find out exact age of Sachin.

24. d The given expression can be written as

$$\left(\frac{16n^2}{n}\right)+\left(\frac{7n}{n}\right)+\left(\frac{6}{n}\right)=16n+7+\left(\frac{6}{n}\right)$$

Since n is an integer, the expression $(16n + 7)$ will always be an integer. Hence, for the entire expression to be an integer, the part $\left(\frac{6}{n}\right)$ should also be an integer. This can be possible only if n is a factor of 6, viz.

$n = 1, 2, 3, 6, -1, -2, -3$ and -6.

Hence, n can have eight values.

25. d If we were to express 64 as product of two positive integers, we can get the following combinations:

$(64 \times 1), (32 \times 2), (16 \times 4), (8 \times 8)$.

Thus, we find that $P + Q$ cannot be 35.

26. b $7^{3^2} = 7^9$ and $(7^3)^2 = 7^6$. Since $7^9 > 7^6 \Rightarrow 7^{3^2} > (7^3)^2$.

27. c As P, Q and R are consecutive odd numbers, $Q = P + 2$ and $R = P + 4$. Now $3P = 2(P + 4) - 3$. On solving this equation, we get $P = 5$.

Therefore, $R = 5 + 4 = 9$

28. d $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$. Combining statements I and II, we get the value of $(a + b) = \sqrt{28}$ or $-\sqrt{28}$. Since we do not have the unique value of $(a + b)$, we cannot get the unique answer $a^3 + b^3$.

29. c If n^3 is odd, then n should also be odd. Hence, n^2 should also be odd, not even. So only I and II are true.

30. b Since MPB is a three-digit number, and also the square of a two-digit number, it can have a maximum value of 961 viz. 31^2. This means that the number BE should be less than or equal to 31. $\Rightarrow$ B can be 0, 1, 2 or 3.

Since the last digit of MPB is also B, it can only be 0 or 1 (as none of the squares end in 2 or 3). The only squares that end in 0 are 100, 400 and 900. But for this to occur the last digit of BE also has to be 0. Since E and B are distinct integers, both of them cannot be 0. Hence, B has to be 1. BE can be a number between 11 and 19 (as we have also ruled out 10), with its square also ending in 1.

Hence, the number BE can only be 11 or 19. $11^2 = 121$. This is not possible as this will mean that M is also equal to 1. Hence, our actual number is $19^2 = 361$. Hence, M = 3.

31. c The maximum and the minimum five-digit numbers that can be formed using only 0, 1, 2, 3, 4 exactly once are 43210 and 10234 respectively. The difference between them is $43210 - 10234 = 32976$.

32. c Since the distance travelled was the same both ways and also it was covered in the same time, the average speed will be the same uphill and downhill. Hence, statement (a) is false. Statement (b) need not be true. It would be true and had the speeds (and not average speed) been the same both ways. But it is clearly indicated that he varied his pace throughout. Now it has to be noted that the journey uphill and the journey downhill started at the same time, i.e. 6 a.m. and also ended at the same time 6 p.m. (though on different days). So if we were to assume a hypothetical case in which one person starts downhill at 6 a.m. and other one starts uphill at 6 a.m., along the same path, then there would be a point on the path where they would meet (i.e. they would reach at the same time), irrespective of their speeds. Our case is similar to that, except for the fact that here, we have only one person moving both ways. So there has to be a point on the path, where he reached at the same time on both days.

33. c LCM of 3, 5, 7, 9 = 315. Hence, all the multiples of 315 will be divisible by 3, 5, 7 and 9. These may be even or odd. Hence, the statement I in itself is not sufficient to answer the question.

The statement II however suggests that the number is 315 itself (as it is the only multiple that lies between 0 and 400). Hence, n is indeed odd. We require both the statements together to answer this.

34. a $(ab)^2 = ccb$, the greatest possible value of 'ab' to be 31.

Since $31^2 = 961$ and since $ccb > 300$, $300 < ccb < 961$, so $18 < ab < 31$. So the possible value of ab which satisfies $(ab)^2 = ccb$ is 21. So $21^2 = 441$,

$\therefore$ a = 2, b = 1, c = 4.

35. a Use the method of simulation, viz. take any sample values of x and verify that n is both odd as well as a perfect square.

Alternate solution:

Let $x = (a-1)\,a\,(a+1)\,(a+2)$

$= (a^2-1)\,a(a+2)$

$= a^4 + 2a^3 - a^2 - 2a$

$n = x + 1 = a^4 + 2a^3 - a^2 - 2a + 1$

$= (a^2 - a + 1)^2$

36. c The square root is 111111111.

37. d Take any three odd and positive integers and check all the options.

38. a There is only one 5 and one 2 in the set of prime numbers between 2 and 100. Hence, there would be only one zero at the end of the resultant product.

39. c

12	1982
12	165 – 2
12	13 – 9
	1 – 1

The answer is 1192.

40. a Check the answer choices basis the fact that:

Odd × Odd = Odd

Odd × Even = Even

Even × Even = Even

41. a The product of 44 and 11 is 484.

If base is x, then,

$3414 = 3x^3 + 4x^2 + 1x^1 + 4 \times x^0 = 484$

$\Rightarrow 3x^3 + 4x^2 + x = 480$

This equation is satisfied only when x = 5.

So base is 5.

In decimal system, the number 3111 can be written as 406.

42. c From statement II, m, n could be (2, 15) (5, 6), (3, 10) and (1, 30) but from statement I, we get m, n as (2, 15).

Hence, the question can be answered by using both the statements together.

43. a From statement I, unequal even integers less than 10 are 2, 4, 6 and 8.

$\frac{X}{Y}$ is an odd integer is possible only if X = 6 and Y = 2

From statement II, even integers less than 10 are 2, 4, 6 and 8.

XY = 12 $\Rightarrow$ X = 6, Y = 2 or X = 2, Y = 6

Hence, question can be answered by using statement I alone but not by statement II.

44. d In option (a), take x = 7, y = 9. Then, 9 – 7 = 2, which is even.

In option (b), take x = 2, y = 9. Then, 2 × 9 = 18, which is even.

In option (c), take x = 3, y = 9. Then, $\frac{3+9}{3} = 4$, which is even.

Hence, option (d) is the correct choice.

45. d It is given that 10 < n < 1000.

Let n be a two digit number. Then,

$n = 10a + b \quad \Rightarrow \quad p_n = ab,\ s_n = a + b$

Then, ab + a + b = 10a + b

$\Rightarrow ab = 9a \quad \Rightarrow \quad b = 9$

$\therefore$ There are 9 such numbers 19, 29, 39, …, 99.

Now, let n be a three digit number.

$\Rightarrow n = 100a + 10b + c \Rightarrow p_n = abc,\ s_n = a + b + c$

Then, abc + a + b + c = 100a + 10b + c

$\Rightarrow abc = 99a + 9b$

$\Rightarrow bc = 99 + 9\frac{b}{a}$

But the maximum value of bc = 81 (when both b & c are 9)

and RHS is more than 99. Hence, no such number is possible.

Hence, there are 9 such integers.

46. e Let the four-digit number be denoted by aabb = 11 × (100a + b).

Now since aabb is a perfect square, 100a + b should be a multiple of 11.

The only pairs of values of a and b that satisfy the above mentioned condition is a = 7 and b = 4. Clearly 7744 is a perfect square.

47. b $4^7 \times 5^9 = 2^{14} \times 5^9 = 2^5 \times (10)^9$

$= 32000000000$

$2^4 \times 7 = 112$

$3 \times 5^3 = 375$

$2^6 \times 5^8 = 5^2 \times (10)^6 = 25000000$

Adding the four numbers we get:

N = 32025000487

The distinct digits in N are 3, 2, 0, 5, 4, 8 and 7.

48. a $a^6 + b^6 = (a^2 + b^2)(a^4 + b^4 - a^2b^2)$

Since $a^6 + b^6$ is divisible by $a^2 + b^2$, it can be prime only in two cases:

(i) $a^2 + b^2$ is 1: This is not possible since both a and b are integers.

(ii) $a^6 + b^6 = a^2 + b^2$: If a and b are distinct integers and $a^6 + b^6$ is prime, then this is possible only if

$\Rightarrow \quad a = 1, b = -1$

$\Rightarrow \quad a = -1, b = 1$

The sum of a and b is 0 in both the cases.

49. a Let $y=\dfrac{\frac{1}{a}+\frac{1}{b}}{1+\frac{1}{ab}}=\dfrac{a+b}{ab+1}$

$$ab=\left(\frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}}\right)\left(\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}}\right)=1$$

$\Rightarrow$ $y=\dfrac{a+b}{1+1}=\dfrac{a+b}{2}$

Also, $a=\dfrac{\sqrt{8}-\sqrt{7}}{\sqrt{8}+\sqrt{7}}=\left(\sqrt{8}-\sqrt{7}\right)^2=15-2\sqrt{8}\sqrt{7}$

and $b=\dfrac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}}=\left(\sqrt{8}+\sqrt{7}\right)^2=15+2\sqrt{8}\sqrt{7}$

$$y=\frac{a+b}{2}=15$$

50. d $x=\sqrt{15}-\sqrt{11}$

$y=\sqrt{27}-\sqrt{23}$

$z=\sqrt{24}-\sqrt{20}$

Let $f(x)=\sqrt{(x+4)}-\sqrt{x}$ (where $x > 0$).

It can be observed that f(x) decreases as x increases.

So $x > z > y$.

51. c $abc\times 1.33=\dfrac{4}{3}abc=cab$ ...(i)

$cab\times 1.33=\dfrac{4}{3}cab=\dfrac{16}{9}abc=bca$ ...(ii)

From equation (ii), we can conclude that the resultant number is a multiple of 16 and the initial number is a multiple of 9. Hence, we can say that the resultant number should be a multiple of 16 as well as 9 i.e. a multiple of 144.

There are two multiples of 144 which satisfy the condition i.e. 432 and 864.

Thus the number ‘abc’ could be either 243 or 486.

52. b It is given that $a^{a^b}=b$

Putting the value of b in left-hand side, we get

$$a^{a^{a^{a^b}}}=b$$

On repeating the same step n times, we get

$$a^{a^{a^{\cdot^{\cdot^{\cdot^b}}}}}=b$$

When n tends to infinity, we get

$$a^{a^{a^{\cdot^{\cdot^{\cdot^b}}}}}=a^b=b$$

Hence $a^b - b = 0$.

Alternate Method:

$a^2=2$, then $a=\sqrt{2}$

$a^{a^2}=2$, then $a=\sqrt{2}$

Similarly, if $a^{a^b}=b$, then $a=\sqrt[b]{b}=b^{\frac{1}{b}}$

Hence, $a^b - b = 0$.

53. a $M=\sqrt{3-\sqrt{5}+\sqrt{9-4\sqrt{5}}}$

$=\sqrt{3-\sqrt{5}+\sqrt{((\sqrt{5})^2-2\times 2\times\sqrt{5}+2^2)}}$

$=\sqrt{3-\sqrt{5}+\sqrt{(\sqrt{5}-2)^2}}$

$=\sqrt{3-\sqrt{5}+\sqrt{5}-2}=\sqrt{1}=1$

$N=\sqrt{\sqrt{7}-1-\sqrt{11-4\sqrt{7}}}$

$=\sqrt{\sqrt{7}-1-\sqrt{\left(\sqrt{7}\right)^2-\left(2\times 2\times\sqrt{7}\right)+(2)^2}}$

$=\sqrt{\sqrt{7}-1-\sqrt{\left(\sqrt{7}-2\right)^2}}$

$=\sqrt{\sqrt{7}-1-\sqrt{7}+2}=\sqrt{1}=1$

Hence, $\dfrac{M-N}{M+N}=\dfrac{1-1}{1+1}=0.$

54. a The last two digits of such perfect squares could be either 00 or 44.

Also, the largest five digit perfect square is $(316)^2$ i.e. 99856. So the number must be less than 316.

Case I: The perfect square ends with 00.

Any multiple of 10 will always end up with two zeros. 100 would be the first such number and 310 the last. There are 22 such numbers.

Case II: The perfect square ends with 44.

For the square of a number to end with 4, its unit digit must be either 2 or 8.

(i) Let the number be ‘ab2’, where ‘a’ and ‘b’ are the digits at hundreds and unit place respectively.

$(ab2)^2=(ab\times 10+2)^2$

$=\left(a^2b^2\times 100\right)+\left(4\times ab\times 10\right)+4$

The last digit will always be 4, but for the second last digit to be 4, ‘4 × b’ must end with 4. Thus ‘b’ could be either 1 or 6.

Possible numbers are 112, 162, 212, 262 and 312.

(ii) Let the number be ‘ab8’, where ‘a’ and ‘b’ are the digits at hundred and unit place respectively.

$(ab8)^2 = (ab \times 10 + 8)^2$

$= \left(a^2b^2 \times 100\right) + \left(16 \times ab \times 10\right) + 64$

The last digit will always be 4, but for the second last digit to be 4, '6 × b + 6' must end with 4. Thus 'b' could be either 3 or 8.

Possible numbers are 138, 188, 238 and 288.

Total possible numbers = 22 + 5 + 4 = 31.

Alternate Method:

The last two digits of such perfect squares could be either 00 or 44.

Perfect squares ending with 00 are always of the form $N^2 \times 10^2$, where N is a natural number.

Total such numbers would be 22 i.e. 10000 to 96100.

Perfect squares ending with 44 are the squares of numbers of the form $50k \pm 12$, where k is whole number.

Total such numbers would be 9 i.e. 112^2, 138^2, 162^2, 188^2, 212^2, 238^2, 262^2, 288^2 and 312^2.

Total possible numbers = 22 + 9 = 31.

55. d $\frac{a^n + b^n}{a + b} \Rightarrow \text{Remainder} = 0$, if n is odd.

$\therefore$ $2^{5001} + 1$ and $2^{5003} + 1$ are divisible by 3.

Now, $2^{5002} + 1$ can be written as $4^{2501} + 1$.

$\therefore$ $4^{2501} + 1$ is divisible by 5.

Hence, none of the given numbers is a prime number.

56. a $\sqrt{x} = \sqrt{2003} - \sqrt{y}$

Squaring both the sides, we get

$$x = 2003 + y - 2\sqrt{2003 \times y}$$

For x to be an integer, y must be of the form $2003a^2$, where 'a' is a natural number.

Similarly, we can say that x must be of the form $2003a^2$, where 'b' is a natural number.

$\therefore$ $\sqrt{2003a^2} + \sqrt{2003b^2} = \sqrt{2003}$

$\Rightarrow$ $a + b = 1$

Hence, no solution is possible.

57. c Let the numbers be a, a + 1, a + 2, ……, a + 31.

Sum of these numbers

$$= 32a + \frac{31 \times 32}{2} = 16(2a + 31)$$

As 16 is a perfect square, the least possible value of 2a + 31 = 49.

Therefore, a = 9 and a + 31 = 40.

The least possible sum = 49.

58.a Prime factorization of 44 leads to:

44 = 2 × 2 × 11

To express 44 as product of five distinct integers we'll have to introduce 1 and –1.

The only possible way comes out to be:

44 = 2 × (–2) × 11 × 1 × (–1)

In this case the value of n would be 11 which is also the only possible value.

59. c LCM of 6, 4 and 3 = 12

$\therefore$ $2^{\frac{7}{6}} = (2^{14})^{\frac{1}{12}}$

$3^{\frac{3}{4}} = (3^9)^{\frac{1}{12}}$

$5^{\frac{3}{4}} = (5^8)^{\frac{1}{12}}$

It is clear from the above numbers that $5^8 > 3^9 > 2^{14}$.

60. c $343^b = 676$

$\Rightarrow$ $7^{3b} = 26^2$

Now, $7^a = 26$

$\Rightarrow$ $7^{3b} = (7^a)^2$

$\Rightarrow$ $2a = 3b$.

61. b Check the options.

Option (a): If the product of the digits is 6, then the factors of 6 are 1, 2, 3 and 6. There is no combination of digits which satisfies the given conditions. So it is not the answer.

Option (b): If the product of the digits is 8, then the factors of 8 are 1, 2, 4 and 8. So only possible combination is 1, 1, 2, 4.

Hence, the number is 4112.

Similarly, we can check options (c) and (d).

62. c Let n be xyz and since n is odd z can take only odd values i.e. 1, 3, 5 and 9 Now, $x \le y$ and $x \ge z$

Possible values			
x	y	z	n
1	1, 2, 3, 4, ...9	1	9
2	2, 3, 4, ...9	1	8
3	3, 4, ...9	1, 3	14
4	4, 5, 6, ...9	1, 3	12
5	5, 6, ...9	1, 3, 5	15
6	6, 7,...9	1, 3, 5	12
7	7, 8, 9	1, 3, 5, 7	12
8	8, 9	1, 3, 5, 7	8
9	9	1, 3, 5, 7, 9	5

$\therefore$ Total number of elements in P = 95.

63. d 3x + y + 4 = 2x y $\Rightarrow$ 3x + 4 = y (2x – 1)

$\Rightarrow y = \frac{(3x + 4)}{(2x - 1)}$

When x = 6 ⇒ y = 2

When x = 1 ⇒ y = 7

These two are the only possible pairs of values of x and y. Where x and y are natural numbers.

$\therefore \text{Required ratio} = \frac{(6+1)}{(2+7)} = \frac{7}{9}$.

64. e If a number which is of the form $a^p \times b^q \times \ldots$, where a, b, … are prime numbers and p, q, … are natural numbers, then the number of factors of the number is given by (p + 1) × (q + 1) × … .

According to the question,

(p + 1) × (q + 1) × … = 4 = 1 × 4 = 2 × 2

From the above equation, it can be concluded that the number in the given case can be of the form either a^3 or a × b.

There is only one number of the form a^3 which is less than or equal to 15 i.e. 2^3. There are 4 numbers of the form a × b which are less than or equal to 15 and these numbers are 2 × 3, 2 × 5, 2 × 7 and 3 × 5.

Hence, the number of required numbers is 5.

65. 51 Let five consecutive odd numbers be

x – 4, x – 2, x, x + 2, x + 4.

Now new sequence in revenue order

2x, 2x – 2, 2x – 4; 2x – 6, 2x – 8

So, sum = 450

10x – 20 = 450

10x = 470

x = 47

a_5 = x + 4

= 47 + 4 = 51.

66. 40 Let the other two numbers are x and y

∴ 73 xy – 37 xy = 720

⇒ xy = 20

$x = \sqrt{20}$ and $y = \sqrt{20}$ will satisfy the above the equation and give the minimum value of $x^2 + y^2$.

So, $x^2 + y^2 = (\sqrt{20})^2 + (\sqrt{20})^2 = 40$

67. d Let the two numbers be x and y

Since, AP ≥ GP

$\Rightarrow \frac{x+y}{2} \geq \sqrt{xy}$

Squaring both sides we get,

$$x^2 + y^2 \geq 2xy$$

We know that $x^2 + y^2 = 97$

⇒ 97 ≥ 2xy

⇒ 48.5 ≥ xy

Hence, the value of xy cannot be 64.

68. d Given that: $4^n > 17^{19}$

At n = 38, LHS will become 16^{19} which is just less than 17^{19}

Hence, smallest possible required value of n will be 39.

Factorial and its Application

1. c Number of powers of 5 in 80!

$= \left(\frac{80}{5} = 16\right) + \left(\frac{80}{5^2} \Rightarrow 3\right) = 19.$

2. b Number of 2 in the product of all integers from

$1 \text{ to } 100 = \frac{100}{2} + \frac{100}{4} + \frac{100}{8} + \frac{100}{16} + \frac{100}{32} + \frac{100}{64}$

$= 50 + 25 + 12 + 6 + 3 + 1 = 97$ and number of 5 in the product of all integers from 1 to 100

$= \frac{100}{5} + \frac{100}{25} = 20 + 4 = 24$

Hence, number of zeros at the end

= Lowest of the (number of 2, number of 5) = 24.

3. The value of ABC, three-digit number should satisfy the condition 100A + 10B + C = A! + B! + C! ....(i)

The maximum value of three digit number is 999 and minimum is 100.

We observe that 7! = 5040 > 999

So the 3-digits of the number must be

	6 as 6! = 720
and/or	5 as 5! = 120
and/or	4 as 4! = 24
and/or	3 as 3! = 6
and/or	2 as 2! = 4
and/or	1 as 1! = 1

If we consider 6 at the hundred's place digit we see that condition (i) is not satisfied as 600 < 720 (6!)

So we conclude that 6 cannot occupy any position in the number.

If we place '5' at the hundred's place then the number should lie between the range of 500 and 600. Considering the RHS of equation (i) by putting A + B = C = 5 we get the sum as 360 which is less than 500.

Similarly, putting 4, 3, 2 at the hundred's place does not satisfies the given condition (i).

Only 1 can be placed at hundred place and 5 should be one of the digit at other two position in order to make it a three digit number.

Thus, only combination we satisfies the given condition (i) is (1, 4, 5) i.e. 145 = 1! + 4! + 5! = 145.

4. d If p = 1! = 1, then

p + 2 = 3 when divided by 2! will give a remainder of 1.

If p = 1! + 2 × 2! = 5, then

p + 2 = 7 when divided by 3! will give a remainder of 1.

Hence, p = 1! + (2 × 2!) + (3 × 3!) + … + (10 × 10!) when divided by 11! leaves a remainder 1.

Alternative method:

P = 1 + 2.2! + 3.3! + … + 10.10!

= (2 – 1)1! + (3 – 1)2! + (4 – 1)3! + … + (11 – 1)10!

= 2! – 1! + 3! – 2! + …+ 11! –10! = 1 + 11!

Hence, the remainder is 1.

5. b Since N has 4 factors, N must be of the form P_1^3 or $P_2 \times P_3$ where P_1, P_2, P_3 are prime numbers.

If N takes the form P_1^3 then P_1 can only be 3.

If N takes the form $P_2 \times P_3$ then the possibilities are:

Nine possible values where 2 is the smaller factor: 2 × 5, 2 × 7, 2 × 11, 2 × 13, 2 × 17, 2 × 19, 2 × 23, 2 × 29, 2 × 31

Seven possible values where 3 is the smaller factor: 3 × 5, 3 × 7, 3 × 11, 3 × 13, 3 × 17, 3 × 19, 3 × 23

Three possible values where 5 is the smaller factor: 5 × 7, 5 × 11, 5 × 13

So N can take 1 + 9 + 7 + 3 = 20 values in all.

6. b We have to calculate the number of zeroes starting from the right end of the number N.

The number of zeroes from:

1! to 4! = 0

5! to 9! = 1× 5 = 5

10! to 14! = 2 × 5 = 10

15! to 19! = 3 × 5 = 15

20! to 24! = 4 × 5 = 20

25! to 29! = 6 × 5 = 30

30! to 34! = 7 × 5 = 35

35! to 39! = 8 × 5 = 40

So we get 155 zeroes till 39! only. From this we can easily conclude that the 147th digit from the right end of N will be zero.

7. c 500! + 505! + 510! + 515!

= 500!(1 + 5k) (where k is a natural number)

It can be seen that 5k + 1 won't be a multiple of 5.

Minimum value of n for which 500! Is divisible by 5^n = 1.

Maximum value of n for which 500! Is divisible by 5^n

$$\left[\frac{500}{5}\right]+\left[\frac{500}{5^2}\right]+\left[\frac{500}{5^3}\right]+\left[\frac{500}{5^4}\right]= 100+20+4=124$$

Hence, there are 124 possible values of n.

8. b Since 'ab' is a two-digit prime number and one of its digit is 3, it can assume any of the values among 13, 23, 31, 37 ,43, 53, 73 and 83.

As the absolute difference between the digits of the number is not a factor of 2, the number among the obtained numbers that satisfy the aforementioned condition are 37, 73 and 83. Hence, the number of values that 'ab' can assume is 3.

9. d Let the number be N.

In order to maximize the number of factors of N^3, N^2 must be expressed as a product of as many prime factors as possible.

No. of factors of $N^2 = 105 = 3 \times 5 \times 7$

$= (2+1)(4+1)(6+1)$

$\therefore$ $N^2 = (a)^2(b)^4(c)^6$, where a, b and c are prime numbers.

$\therefore$ $N^3 = (a)^3(b)^6(c)^9$

Hence, the number of factors of N^3

$= (3+1)\times(6+1)\times(9+1) = 4\times7\times10 = 280.$

10. a (x – 2) is a factor of R(x).

$\therefore$ R(2) = 0

$\Rightarrow$ $m(2)^3 - 100(2)^2 + 3n = 0$

$\Rightarrow$ 8m – 400 + 3n = 0

$\Rightarrow$ $m = \frac{400-3n}{8} = 50 - \frac{3n}{8}$

As m and n are positive integers, so n must be a multiple of 8. For n = 8, 16, 24, …, 128, we get m = 47, 44, 41, …, 2 respectively.

Hence, the number of possible values of ordered pair (m, n) is 16.

11. a $1080 = 2^3 \times 3^3 \times 5^1$

$1800 = 2^3 \times 3^2 \times 5^2$

HCF of 1080 and 1800 = $2^3 \times 3^2 \times 5$

Number of factors of 1080 = 4 × 4 × 2 = 32

Number of factors of 1800 = 4 × 3 × 3 = 36

Number of factors of HCF of the two numbers

= 4 × 3 × 2

= 24

Hence, the required number of divisiors

= (32 + 36) – 2 × 24

= 20.

12. c In order to maximize the power of 2 in the product, one of the ten numbers has to be 64 as this is the highest two-digit number of the form 2^k, where k is a natural number.

There has to be maximum number of multiples of 8 among the ten numbers. In a set of ten consecutive natural numbers, there can be a maximum of two numbers that will be a multiple of 8.

The possible sets of ten consecutive natural numbers that satisfy the aforementioned conditions are 55 to 64, 56 to 65, 63 to 72 and 64 to 73. The highest power of 2 in the product of any of these sets of ten numbers will be 13.

13. b $P = 15^{100} (1 \times 2 \times 3 \times ... \times 100)$

$= 15^{100} \times 100!$

Highest power of 2 in P = 97 (2 will be deciding factor for number of zeroes because number of fives will be greater than number of zeroes in this number)

$Q = 25^{20\times50} (1 \times 2 \times 3x...\times 50) = 5^{2000} \times 50!$

Highest power of 2 in Q = 47

So Highest power of 2 in

$$\frac{P^2}{Q}\times 10^{1767} = 2 \times 97 + 1767 - 47$$

$$= 1914.$$

Hence, number of zeroes = 1914.

14. 8 The number finally obtained will be the "digit sum" of the original number. It can be directly obtained by dividing the original number by 9 and finding the remainder.

$$\text{Rem}\left[\frac{5^{1!+2!+3!+4!}}{9}\right] = \text{Rem}\left[\frac{5^{33}}{9}\right]$$

$$= \text{Rem}\left[\frac{(5^3)^{11}}{9}\right]$$

$$= \text{Rem}\left[\frac{(-1)^{11}}{9}\right]$$

$$= -1 = 8.$$

HCF and LCM

1. d Required number = LCM (4, 6, 7) + 2 = 86.

2. b Required number = LCM (8, 11, 24) – 5 = 259.

3. d The LCM of 7, 12 and 16 is 336. The closest multiple of 336 to 1856 is 1680. So 1684 when divided by 7, 12 and 16 leaves a remainder of 4. This is the closest such number, less than 1856. Hence, the number to be subtracted from 1856 to get 1684, must be the least such number.

So the answer is (1856 – 1684) = 172.

Alternate method:

Subtract options from 1856 and check.

4. b The bells will chime together after a time that is equal to the LCM of 18, 24 and 32 = 288 min = 4 hr and 48 min.

5. c We know that product of two numbers = LCM × HCF

$= 36 \times 2 = 72.$

So $x = \frac{72}{18} = 4.$

Hence, both the statements are required to answer the question.

6. * The time taken by the red spots on all three wheels to simultaneously touch the ground again will be equal to the LCM of the times taken by the three wheels to complete one revolution. The first wheel completes 60 revolutions per minute i.e., to complete one revolution, it takes $\left(\frac{60}{60}\right) = 1$ sec

The second wheel completes 36 revolutions per minute i.e., to complete one revolution, it takes $\left(\frac{60}{36}\right) = \frac{5}{3}$ sec.

Similarly, the third wheel takes $\left(\frac{60}{24}\right) = \frac{5}{2}$ sec to complete one revolution.

$$\therefore \text{ LCM of } 1, \frac{5}{3}, \frac{5}{2} = \frac{\text{LCM}(1, 5, 5)}{\text{HCF}(1, 3, 2)} = \frac{5}{1} = 5 \text{ sec}$$

*** The correct answer is not available in the given options.**

7. b Note that the difference between the divisors and the remainders is constant.

2 – 1 = 3 – 2 = 4 – 3 = 5 – 4 = 6 – 5 = 1

In such a case, the required number will always be

[a multiple of LCM of (2, 3, 4, 5, 6) – (The constant difference)].

LCM of (2, 3, 4, 5, 6) = 60

Hence, the required number will be 60n – 1.

Thus, we can see that the smallest such number is (60 × 1) – 1 = 59

The second smallest is (60 × 2) – 1 = 119

So between 1 and 100, there is only one such number, viz. 59.

8. d HCF of 60, 84 and 108 is 12. Hence, 12 students should be seated in each room. So for subject A we would require $\left(\frac{60}{12}\right)$ = 5 rooms, for subject B we would require $\left(\frac{84}{12}\right)$ = 7 rooms and for subject C we would require $\left(\frac{108}{12}\right)$ = 9 rooms. Hence, minimum number of rooms required to satisfy our condition = 5 + 7 + 9 = 21 rooms.

9. b If there are n numbers, the function h has to be performed one time less.

10. a First light blinks after 20 s.

Second light blinks after 24 s.

They blink together after LCM (20 and 24) = 120 s = 2 min. Hence, the number of times they blink together in an hour = 30.

11. c We can put a minimum of 120 oranges and a maximum of 144 oranges, i.e., 25 oranges need to be filled in 128 boxes.

There are 25 different possibilities if there are 26 boxes.

In such a case, at least 2 boxes contain the same number of oranges. (i.e., even if each of the 25 boxes contain a different number of oranges, the 26th must contain one of these numbers).

Similarly, if there are 51 boxes, at least 3 boxes contain the same number of oranges.

Hence, at least 6 boxes have the same number of oranges in case of 128 boxes.

12. b Since each word is lit for a second, least time after which the full name of the bookstore can be read again

$$= \text{LCM}\left(\frac{5}{2}+1,\ \frac{17}{4}+1,\ \frac{41}{8}+1\right)$$

$$= \text{LCM}\left(\frac{7}{2},\ \frac{21}{4},\ \frac{49}{8}\right)$$

$$= \frac{\text{LCM}(7,21,49)}{\text{HCF}(2,4,8)} = \frac{49\times 3}{2} = 73.5\,\text{s}.$$

13. d $\text{HCF}\left(\frac{9}{2},\ \frac{27}{4},\ \frac{36}{5}\right) = \frac{\text{HCF}(9,\ 27,\ 36)}{\text{LCM}(2,\ 4,\ 5)} = \frac{9}{20}$ lb

= Weight of each piece

Also, total weight of three pieces of cakes = 18.45 lb

$\therefore$ Maximum number of guests that could be entertained = $\frac{18.45 \times 20}{9} = 41$.

14. b LCM of 2, 3, 4, 6, 12 = 12

We can rewrite the given surds as

$\sqrt[12]{2^6}$, $\sqrt[12]{3^4}$, $\sqrt[12]{4^3}$, $\sqrt[12]{6^2}$, $\sqrt[12]{12^1}$

$\therefore$ 3^4 is the greatest.

Note: $n^{1/n}$ is maximum when n = e (2.718). Among the given options, n = 3 is closest to the value of e.

15. c Let the numbers be a and b.

We know, a + b = 600 and $\frac{a}{b} = \frac{7}{8}$

Solving, the above two equations, we get

a = 280

b = 320

LCM (a, b) = LCM (280, 320)

= 2240.

16. 31 Let the four numbers be XK, XL, XM and XN, where X is the common factor of each pair possible pair of numbers and K, L, M, N are prime to each other.

310 = 2 × 5 × 31

651 = 31 × 21 = 3 × 7 × 31 $\therefore$ GCF (310, 651) = 31

which is the highest common factor of all.

17. b 1000 = $2^3 \times 5^3$ and 2000 = $2^4 \times 5^3$

Since LCM (c, a) and LCM (b, c) is $2^4 \times 5^3$ and LCM (a, b) = $2^3 \times 5^3$, so the factor 2^4 must be present in c.

Hence c = $2^4 \times 5^x$, where x ranges from 0 to 3

Therefore, there are four possible values of C.

Since, HCF of (a, b) = K × 5^3, it means

a = $2^y \times 5^3$

b = $2^z \times 5^3$

x = 0 to 3, y = 0, then z = 3 $\rightarrow$ 4 cases.

x = 0 to 3, y = 1, then z = 3 $\rightarrow$ 4 cases.

x = 0 to 3, y = 2, then z = 3 $\rightarrow$ 4 cases.

x = 0 to 3, y = 3, then z = 3 $\rightarrow$ 4 cases.

x = 0 to 3, y = 3, then z = 2 $\rightarrow$ 4 cases.

x = 0 to 3, y = 3, then z = 1 $\rightarrow$ 4 cases.

x = 0 to 3, y = 3, then z = 0 $\rightarrow$ 4 cases.

Hence, total cases = 28.

Remainder

1. b $\text{Rem}\left[\frac{2^{60}}{5}\right] = \text{Rem}\left[\frac{\left(2^4\right)^{15}}{5}\right] = \text{Rem}\left[\frac{1^{15}}{5}\right] = 1.$

2. a As 55 does not have factor common with 124, for 55n to be exactly divisible by 124, n should be a multiple of 124.

Hence, the minimum value that n can have is 124 itself.

3. b $5^6 - 1 = (5^3)^2 - 1 = (125)^2 - (1)^2 = (125 + 1)(125 - 1)$ $= 124 \times 126 = 31 \times 4 \times 126$.

So among the given answer choices, it is divisible by 31.

4. b The best way to solve this question is by the method of simulation. Choose any prime number greater than 6 and verify the result.

When 7 is divided by 6, it gives a remainder 1. So our answer could be (a) or (b). When 11 is divided by 6, it gives a remainder 5. Hence, our answer is (b).

5. a Since 899 is divisible by 29, so you can directly divide the remainder of 63 by 29, so $\frac{63}{29}$ will give 5 as a remainder, option (a).

6. b Since 2 has a cyclicity of 4,

i.e. $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, 2^6 = 64, \ldots$, the last digits (2, 4, 8, 6) are in four cycles.

$\therefore$ On dividing $\frac{51}{4}$, we get the remainder as 3.

$\therefore$ The last digit has to be $2^3 = 8$

Shortcut:

Since cyclicity of the power of 2 is 4, so 2^{51} can be written in $2^{4(12)+3}$ or unit digit will be $2^3 = 8$.

7. c The number formed by the last 3 digits of the main number is 354. The remainder is 2 if we divide 354 by 8. So the remainder of the main number is also 2 if we divide it by 8.

8. b **Note:** $342 = 7^3 - 1$. On further simplification we get,

$$= \frac{(7^3)^{28}}{342} = \frac{343^{28}}{342} = \frac{(342+1)^{28}}{342} = \frac{342\,N + 1}{342}$$

Hence, remainder = 1.

9. c N = 1421 × 1423 × 1425. When divided by 12, it shall look like $\frac{[(1416+5)\times(1416+7)\times(1416+9)]}{12}$.

Now the remainder will be governed by the term 5 × 7 × 9, which when divided by 12 leaves the remainder 3.

10. d Let r be the remainder. Then, 34041 – r and 32506 – r are perfectly divisible by n. Hence, their difference should also be divisible by the same number.

$\therefore$ (34041 – r) – (32506 – r) = 1535, which is divisible by only 307.

11. a $(2^4)^{64} = (17-1)^{64} = 17n + (-1)^{64} = 17n + 1$

Hence, remainder = 1.

12. d 3(4(7x + 4) + 1) + 2 = 84x + 53

Therefore, remainder is 53.

13. a From A, if by adding 12 students, the total number of students is divisible by 8. By adding 4 students, it will be divisible by 8.

14. b Such numbers are 10, 17, …, 94.

These numbers are in AP. There are 13 numbers.

$\therefore$ Sum $= \frac{10+94}{2} \times 13 = 52 \times 13 = 676$.

15. d To find the remainder when $\frac{4^{96}}{6}$, let us use the basic property of dividing the power of 4 by 6, i.e.,

$$\frac{4^1}{6} = 4$$

$$\frac{4^2}{6} = 4$$

$$\frac{4^3}{6} = 4$$

$$\frac{4^4}{6} = 4$$

Hence, any power of 4 when divided by 6 leaves a remainder of 4.

16. a Since b can take any even number 2, 4, 6,…,we cannot say anything from statement A.

Consider statement B.

If b > 16, say b = 17, then $2^{44} < (16 + 1)^{11}$.

$\Rightarrow$ $2^{44} < (2^4 + 1)^{11}$

Hence, we can answer the question using statement B alone.

17. c $15^{23} = (19-4)^{23} = 19x + (-4)^{23}$, where x is a natural number.

$23^{23} = (19+4)^{23} = 19y + (4)^{23}$, where y is a natural number.

$15^{23} + 23^{23} = 19(x+y) + 4^{23} + (-4)^{23} = 19(x+y)$.

Hence, remainder will be zero.

18. a $x = 16^3 + 17^3 + 18^3 + 19^3$ is an even number

Therefore, 2 divides x.

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$\Rightarrow$ a + b always divides $a^3 + b^3$.

Therefore, $16^3 + 19^3$ is divisible by 35.

$18^3 + 17^3$ is divisible by 35.

Thus, x is divisible by 70.

Hence, option (a) is the correct choice.

19. a $((30)^4)^{680} = (8100)^{680}$.

Hence, the right most non-zero digit is 1.

20. c The last two digits of any number in the form of 7^{4n} will always be equal to 01.

For example: $7^4 = 2401$ and $7^8 = 5764801$.

21. d If all are of equal height, number of handshakes $= {}^{40}C_2$.

If all are of different heights, number of handshakes = 0.

Difference $= {}^{40}C_2 - 0 = {}^{40}C_2$.

Miscellaneous

1. d Statement I implies X – Y = 6.
Statement II implies XY is divisible by 6.
You can see that many values of X and Y can satisfy statement I and II.

2. b Check choices.

Choice (b) 54 $\Rightarrow$ S = $(5 + 4)^2 = 81$

$\Rightarrow$ D – S = 81 – 54 = 27. Hence, the number = 54.

3. d $u^m + v^m = w^m$

$u^2 + v^2 = w^2$

Taking Pythagorean triplet 3, 4 and 5, we see that m < min (u, v, w).

Also, $1^1 + 2^1 = 3^1$ and hence, m $\le$ min (u, v, w).

4. b MDCCLXXXVII = 1000 + 500 + 100 + 100 + 50 + 10 + 10 + 10 + 5 + 1 + 1 = 1787

5. a MCMXCIX = 1000 + (1000 – 100) + (100 – 10) + (10 – 1) = 1000 + 900 + 90 + 9 = 1999

6. c (I) MCMLXXV = 1000 + (1000 – 100) + 50 + 10 + 10 + 5 = 1975

(II) MCMXCV = 1000 + (1000 – 100) + (100 – 10) + 5 = 1995

(III) MVD = 1000 + (500 – 5) = 1495

(IV) MVM = 1000 + (1000 – 5) = 1995

Therefore, the answer is (II) and (IV), i.e. option (c).

7. d Since the last digit in base 2, 3 and 5 is 1, the number should be such that on dividing by either 2, 3 or 5 we should get a remainder 1. The smallest such number is 31. The next set of numbers are 61, 91.

Among these only 31 and 91 are a part of the answer choices.

Among these, $(31)_{10} = (11111)_2 = (1011)_3 = (111)_5$

Thus, all three forms have leading digit 1.

Hence, the answer is 91.

8. a We have

(a) $10^{10} < n < 10^{11}$

(b) Sum of the digits for 'n' = 2

Clearly,

(n) min = 10000000001 (1 followed by 9 zeros and finally 1)

Obviously, we can form 10 such numbers by shifting '1' by one place from right to left again and again.

Again, there is another possibility for 'n'

n = 20000000000

So number of different values of n = 10 + 1 = 11.

9. c By option (c), if four consecutive odd numbers are 37, 39, 41 and 43, then sum of these 4 numbers is 160.

When divided by 10, we get 16, which is a perfect square.

$\therefore$ 41 is one of the odd numbers.

10. e seed(n) function will eventually give the digit-sum of any given number, n.

All the numbers 'n' for which seed(n) = 9 will give the remainder 0 when divided by 9.

For all positive integers n, n < 500, there are 55 multiples of 9.

11. c Let the hundreds digit be n.

The tens digit will be 2n.

The unit digit will be 4n.

The possible values of 'n' are 1 and 2 and hence the possible numbers are 124 and 248 respectively.

On converting 248 in base 8 and base 9, the given condition gets violated.

On converting 124 in base 8 and base 9, we get

$(174)_8 = (147)_9$.

Required sum = 4 + 7 = 11.

12. c $(x+1)(x+2)(x+3)(x+6) = -x^2$

$(x^2 + 7x + 6)(x^2 + 5x + 6) = -x^2$

$x^2\left(x + 7 + \frac{6}{x}\right)\left(x + 5 + \frac{6}{x}\right) = -x^2$

Put $x + \frac{6}{x} = y$

$(y+7)(y+5) = -1$

$y^2 + 12y + 36 = 0 \Rightarrow (y+6)^2 = 0 \Rightarrow y = -6$

$\therefore\ x + \frac{6}{x} = -6$

$x^2 + 6x + 6 = 0$

$x = \frac{-6 \pm \sqrt{36 - 4(6)}}{2} = \frac{-6 \pm 2\sqrt{3}}{2} = -3 \pm \sqrt{3}$

13. 17 $(x^2 - y^2)(x^2 + y^2) = 15$

$\Rightarrow (x^2 - y^2)(x^2 + y^2) = 1 \times 15 = 3 \times 5$

$\Rightarrow x^2 - y^2 = 3$ and $x^2 + y^2 = 5$ (because x and y are natural numbers)

$\therefore 2x^2 = 8$

$\Rightarrow x^2 = 4$

$\therefore y^2 = 1$

$\Rightarrow x^4 + y^4 = 4^2 + 1^2 = 17.$

14. 20 According to the question,

$$\frac{N1+1}{2} - \frac{N2+1}{2} = 10$$

$$\Rightarrow \quad N1 - N2 = 20$$

15. 3 If $\frac{A}{B}$ is always a terminating decimal, then B can either be 1 or can have only two prime factors i.e. 2 and 5. The possible values of B are 1, 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64 and 80, i.e. 14 values in all.

16. 3 $\frac{21}{2n-1} = m(\text{say})$, where m is an integer.

For m to be an integer, (2n – 1) should be 1, 3, 7 or 21 i.e. one of the factors of 21.

Hence, the number of integers in the sequence is 4.

17. 90 His shop must have been visited by at least 90 customers.

The number of customers who bought both Rose and Tulip = 70.

The rest 20 customers must have bought just 1 Tulip each.

18. c Numbers in the square grid is shown below.

6	7	2
1	5	9
8	3	4

19. d a(a + 1) (a + 2) = 15600.

Now factors of 15600

= 2 × 2 × 2 × 2 × 3 × 5 × 5 × 13

So, 15600 is the product of 24, 25, 26

So, sum of the squares of these integers is $24^2 + 25^2 + 26^2 = 1877$.

20. c $\frac{1}{a} + \frac{1}{b} = \frac{1}{9}$

ab = 9(a + b)

ab = 9a + 9b

9a = b(a – 9)

$$b = \frac{9a}{a-9} = \frac{9a-81+81}{a-9}$$

$$= \frac{9(a-9)}{a-9} + \frac{81}{a-9}$$

$$b = \frac{81}{a-9} + 9$$

For b to be integer and $a \le b$, the first term should be an integer. So factors of 81 are 1, 3, 9, 27, 81. Now put these values satisfying $a \le b$. So only 3 values (1, 3, 9) will satisfy the given condition.

So, answer will be 3.

21. c The n^{th} term of the given series

= [4(n + 1) – 1][4(n + 2) – 1]

= (4n + 3)(4n + 7)

= $16n^2 + 40n + 21$

$\Rightarrow$ Required sum

$$= \frac{16[n(n+1)(2n+1)]}{6} + \frac{40n(n+1)}{2} + 21n \quad ....(1)$$

As, n = 23 and all the individual terms are odd.

Hence, correct answer will be odd multiple of 23 which will be 80707.

■■