Ex 8.1

Solution of Simultaneous Linear Equations Ex 8.1 Q1(i)

We have,

$$5x + 7y = -2$$
$$4x + 6y = -3$$

The above system of equations can be written in the matrix form as

$$\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

or $\quad AX = B$

where $A = \begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

Now, $\quad |A| = 30 - 28 = +2 \neq 0$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A, then

$$C_{11} = 6$$
$$C_{12} = -4$$
$$C_{21} = -7$$
$$C_{22} = 5$$

Also,

$$\text{adj } A = \begin{bmatrix} 6 & -4 \\ -7 & 5 \end{bmatrix}^T = \begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{adj}\,A = \frac{1}{+2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$$

$\therefore$
$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{+1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{+1}{2}\begin{bmatrix} -12 & +21 \\ 8 & -15 \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ \frac{-7}{2} \end{bmatrix}$$

Hence, $x = \frac{9}{2}$, $y = \frac{-7}{2}$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(ii)

The above system can be written in matrix form as

$$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

or $\quad A\,X = B$

Where,

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Now, $\quad |A| = 10 - 6 = 4 \neq 0$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A, then

$$C_{11} = 2$$
$$C_{12} = -3$$
$$C_{21} = -2$$
$$C_{22} = 5$$

Also,

$$\text{Adj}\,A = \begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}^T = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|}\,\text{adj}\,A = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$= \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$
$$= \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Hence, $x = -1$

$y = 4$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(iii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

or $\quad A X = B$

Where,

$$A = \begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

Now, $\quad |A| = -7 \neq 0$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A, then

$$C_{11} = -1$$
$$C_{12} = -1$$
$$C_{21} = -4$$
$$C_{22} = 3$$

Now,

$$\text{Adj}\, A = \begin{bmatrix} -1 & -1 \\ -4 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|} \text{adj}\, A = \frac{1}{-7} \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{7} \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$= \frac{-1}{7} \begin{bmatrix} 7 \\ -14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Hence, $x = -1$

$y = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(iv)

The above system can be written in matrix form as:

$$\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

or $\quad A X = B$

Where,

$$A = \begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

Now, $\quad |A| = -6 \neq 0$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A, then

$$C_{11} = -1$$
$$C_{12} = -3$$
$$C_{21} = -1$$
$$C_{22} = 3$$

Now,

$$\operatorname{Adj} A = \begin{bmatrix} -1 & -3 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\operatorname{adj} A = \frac{1}{-6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

$$= \frac{-1}{6}\begin{bmatrix} -19 & -23 \\ -57 & +69 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

Hence, $x = 7$

$y = -2$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(v)

The above system can be written in matrix form as:

$$\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

or $\quad AX = B$

where $A = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

Now,

$$|A| = -1 \neq 0$$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Now, let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 2$$
$$C_{12} = -1$$
$$C_{21} = -7$$
$$C_{22} = 3$$

$$\text{Adj } A = \begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}.\text{adj } A = \frac{1}{(-1)}\begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1}\begin{bmatrix} 15 \\ -7 \end{bmatrix} = \begin{bmatrix} -15 \\ 7 \end{bmatrix}$$

Hence, $x = -15$

$y = 7$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(vi)

The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

or $\quad AX = B$

Where,

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

Since, $|A| = 4 \neq 0$, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 3$$
$$C_{12} = -5$$
$$C_{21} = -1$$
$$C_{22} = 3$$

$$\text{adj}\,A = \begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj}\,A = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 9 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ \frac{1}{4} \end{bmatrix}$$

Hence, $x = \frac{9}{4}$

$$y = \frac{1}{4}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(i)

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

or $\quad AX = B$

Where,

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

Now, $$|A| = 1\begin{bmatrix} 3 & 1 \\ -1 & -7 \end{bmatrix} - 1\begin{bmatrix} 2 & 1 \\ 3 & -7 \end{bmatrix} - 1\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}$$
$$= (-20) - 1(-17) - 1(-11)$$
$$= -20 + 17 + 11 = 8 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = -20 \qquad C_{21} = 8 \qquad C_{31} = 4$

$C_{12} = -(-17) = 17 \qquad C_{22} = -4 \qquad C_{32} = -3$

$C_{13} = -11 \qquad C_{23} = -(-4) = 4 \qquad C_{33} = 1$

$$\text{adj } A = \begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}^T = \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

Now, $$X = A^{-1}B = \frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $x = 3$

$y = 1$

$z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(ii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

or $\quad AX = B$

Where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

Since, $|A| = 14 \neq 0$, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = 2 \qquad C_{21} = 4 \qquad C_{31} = 2$

$C_{12} = 8 \qquad C_{22} = -5 \qquad C_{32} = 1$

$C_{13} = 4 \qquad C_{23} = 1 \qquad C_{33} = -3$

$$\text{Adj}\,A = \begin{bmatrix} 2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \frac{1}{|A|} \times \text{Adj}\,A \times B$

$$= \frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix} -16 \\ 20 \\ 38 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-8}{7} \\ \frac{10}{7} \\ \frac{19}{7} \end{bmatrix}$$

Hence, $x = \frac{-8}{7}$, $y = \frac{10}{7}$, $z = \frac{19}{7}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(iii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

or $\quad A X = B$

Where,

$$A = \begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

Now,

$$\begin{aligned} |A| &= 6(225+360)+12(60+40)+25(72-30) \\ &= 6(585)+1200+25(42) \\ &= 3510+1200+1050 \\ &= 5760 \neq 0 \end{aligned}$$

So, the above system will have a unique solution, given by

$$X = A^{-1}B$$

$$\begin{array}{lll} C_{11} = 585 & C_{21} = -(-180-450) = 630 & C_{31} = -135 \\ C_{12} = -100 & C_{22} = 40 & C_{32} = 220 \\ C_{13} = 42 & C_{23} = -132 & C_{33} = 138 \end{array}$$

$$X = A^{-1}B = \frac{1}{|A|}(\mathrm{Adj}\,A) \times B = \frac{1}{5760}\begin{bmatrix} 585 & 630 & -135 \\ -100 & 40 & 220 \\ 42 & -132 & 138 \end{bmatrix}\begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5760}\begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Hence, $x = \frac{1}{2}$

$y = \frac{1}{3}$

$z = \frac{1}{5}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(iv)

The above system can be written as

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

or $\quad AX = B$

$$\begin{aligned} |A| &= 3(-3) - 4(-9) + 7(5) \\ &= -9 + 36 + 35 \\ &= 62 \neq 0 \end{aligned}$$

So, the above system will have a unique solution, given by

$$X = A^{-1}B$$

Now, $C_{11} = -3 \quad C_{21} = 26 \quad C_{31} = 19$

$C_{12} = 9 \quad C_{22} = -16 \quad C_{32} = 5$

$C_{13} = 5 \quad C_{23} = -2 \quad C_{33} = -11$

$$\text{adj } A = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj } A)B$$

$$= \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

$$= \frac{1}{62}\begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix} = \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $x = 1, y = 1, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(v)

The above system can be written as

$$\begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

Or $AX = B$

$$|A| = 2(-1) - 6(5) + 0(-3) = -32 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = -1 \quad C_{21} = -6 \quad C_{31} = -6$$
$$C_{12} = -5 \quad C_{22} = 2 \quad C_{32} = 2$$
$$C_{13} = -3 \quad C_{23} = 14 \quad C_{33} = -18$$

$$\text{adj}A = \begin{bmatrix} -1 & -5 & -3 \\ -6 & 2 & 14 \\ -6 & 2 & -18 \end{bmatrix}^T = \begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}$$

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj}A) \times B$$
$$= \frac{1}{-32}\begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}\begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-32}\begin{bmatrix} 64 \\ -32 \\ -64 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = -2, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(v)

Let $\frac{1}{x} = u, \frac{1}{y} = v, \frac{1}{z} = w$

$$2u - 3v + 3w = 10$$
$$u + v + w = 10$$
$$3u - v + 2w = 13$$

Which can be written as

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$$|A| = 2(3) + 3(-1) + 3(-4)$$
$$= 6 - 3 - 12 = -9 \neq 0$$

Hence, the system has a unique solution, given by

$$X = A^{-1} \times B$$

$$C_{11} = 3 \qquad C_{21} = 3 \qquad C_{31} = -6$$
$$C_{12} = 1 \qquad C_{22} = -5 \qquad C_{32} = 1$$
$$C_{13} = -4 \qquad C_{23} = -7 \qquad C_{33} = 5$$

$$X = \frac{1}{|A|}(\text{Adj}\,A) \times (B)$$
$$= \frac{1}{-9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$
$$= \frac{-1}{9} \begin{bmatrix} 30 + 30 - 78 \\ 10 - 50 + 13 \\ -40 - 70 + 65 \end{bmatrix}$$
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{-1}{9} \begin{bmatrix} -18 \\ -27 \\ -45 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Hence, $x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(vi)

$$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$
$$= 5(-2) - 3(5) + 1(3)$$
$$= -10 - 15 + 3 = -22 \neq 0$$

Hence, it has a unique solution, given by

$$X = A^{-1} \times B$$

$C_{11} = -2 \qquad C_{21} = -10 \qquad C_{31} = 8$
$C_{12} = -5 \qquad C_{22} = 19 \qquad C_{32} = -13$
$C_{13} = 3 \qquad C_{23} = -7 \qquad C_{33} = -1$

$$X = A^{-1} \times B = \frac{1}{|A|}(\text{Adj}\,A) \times B$$
$$= \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}\begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$
$$= \frac{-1}{22}\begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix}$$
$$= \frac{-1}{22}\begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 5$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(vii)

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

or $\quad AX = B$

$$\begin{aligned} |A| &= 3(6) - 4(3) + 2(-2) \\ &= 18 - 12 - 4 \\ &= 2 \neq 0 \end{aligned}$$

Hence, the system has a unique solution, given by

$$X = A^{-1}B$$

$$\begin{array}{lll} C_{11} = 6 & C_{21} = -28 & C_{31} = -16 \\ C_{12} = -3 & C_{22} = 16 & C_{32} = 9 \\ C_{13} = -2 & C_{23} = 10 & C_{33} = 6 \end{array}$$

Next, $\quad X = A^{-1}B = \dfrac{1}{|A|}(\text{Adj } A) \times B$

$$\begin{aligned} &= \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ 2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix} \\ &= \frac{1}{2}\begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

Hence, $x = -2, y = 3, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(viii)

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{aligned} |A| &= 2(-5) - 1(1) + 1(-8) \\ &= -10 - 1 - 8 = -19 \neq 0 \end{aligned}$$

Hence, the unique solution, given by

$$X = A^{-1} \times B$$

$$\begin{array}{lll} C_{11} = -5 & C_{21} = 3 & C_{31} = -4 \\ C_{12} = -1 & C_{22} = -7 & C_{32} = 3 \\ C_{13} = -8 & C_{23} = 1 & C_{33} = 5 \end{array}$$

Next, $\quad X = A^{-1} \times B$

$$\begin{aligned} &= \frac{1}{|A|}\begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \\ &= \frac{1}{-19}\begin{bmatrix} -10 + 15 - 24 \\ -2 - 35 + 18 \\ -16 + 5 + 30 \end{bmatrix} \\ &= \frac{-1}{19}\begin{bmatrix} -19 \\ -19 \\ 19 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Hence, $x = 1, y = 1, z = -1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(x)

The above system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 1(1) + 1(-2) + 1(4) = 1 - 2 + 4 = 3 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = 1 \qquad C_{21} = 1 \qquad C_{31} = +1$

$C_{12} = 2 \qquad C_{22} = -1 \qquad C_{32} = 2$

$C_{13} = 4 \qquad C_{23} = -2 \qquad C_{33} = 1$

$$\operatorname{adj} A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & -2 \\ +1 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & +1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(\operatorname{Adj} A) \times B$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & +1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xi)

The above system can be written as

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 8(-1) - 4(1) + 3(3) = -8 - 4 + 9 = -3 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = -1 \qquad C_{21} = 2 \qquad C_{31} = 1$

$C_{12} = -1 \qquad C_{22} = 5 \qquad C_{32} = -2$

$C_{13} = 3 \qquad C_{23} = -12 \qquad C_{33} = 0$

$$\operatorname{adj} A = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T = \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \frac{1}{|A|}(\operatorname{Adj} A) \times B$

$$= \frac{-1}{3}\begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}\begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{3}\begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = 1, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xii)

This system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 1(-2) - 1(-5) + 1(1) = -2 + 5 + 1 = 4 \neq 0$$

So, $AX = B$ has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = -2 & C_{21} = 0 & C_{31} = 2 \\ C_{12} = +5 & C_{22} = -2 & C_{32} = -1 \\ C_{13} = 1 & C_{23} = 2 & C_{33} = -1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}^T = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$X = A^{-1} \times B = \frac{1}{|A|}(\text{Adj } A) \times B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = -3, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xiii)

Let

$$\frac{1}{x}=u, \frac{1}{y}=v, \frac{1}{z}=w$$

The above system can be written as

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix}=\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Or $\quad AX = B$

$$|A|=2(75)-3(-110)+10(72)=1200 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11}=75 \quad C_{21}=150 \quad C_{31}=75$$
$$C_{12}=110 \quad C_{22}=-100 \quad C_{32}=30$$
$$C_{13}=72 \quad C_{23}=0 \quad C_{33}=-24$$

$$\text{adj}A=\begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T=\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now,

$$X = A^{-1}B=\frac{1}{|A|}(\text{Adj}A)\times B$$

$$=\frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}=\frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}=\begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Hence, $x=2, y=3, z=5$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xiv)

The above system can be written as

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Or $AX = B$

$$|A| = 1(7) + 1(19) + 2(-11) = 4 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 7 & C_{21} = 1 & C_{31} = -3 \\ C_{12} = -19 & C_{22} = -1 & C_{32} = 11 \\ C_{13} = -11 & C_{23} = -1 & C_{33} = 7 \end{array}$$

$$\text{adj}A = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj}A) \times B$$

$$= \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2, y = 1, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(i)

The above system can be written as

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

or $AX = B$

$$|A| = 36 - 36 = 0$$

So, A is singular. Now, X will be consistent if $(\text{adj}A) \times B = 0$

$$C_{11} = 6$$
$$C_{12} = -9$$
$$C_{21} = -4$$
$$C_{22} = 6$$

$$\text{adj } A = \begin{bmatrix} 6 & -9 \\ -4 & 6 \end{bmatrix}^T = \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}$$

$$(\text{Adj } A) \times B = \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 12 - 12 \\ -18 + 18 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ will have infinite solutions.

Let $y = k$

Hence, $6x = 2 - 4k$ or $9x = 3 - 6k$

$$x = \frac{1 - 2k}{3} \quad \text{or} \quad x = \frac{1 - 2k}{3}$$

Hence, $x = \dfrac{1 - 2k}{3}, y = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(ii)

The system can be written as

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 18 - 18 = 0$$

So, A is singular. Now the system will be inconsistent if $(\text{adj}\,A) \times B \neq 0$

$$C_{11} = 9 \qquad C_{21} = -3$$
$$C_{12} = -6 \qquad C_{22} = 2$$

$$\text{adj}\,A = \begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$$

$$(\text{Adj}\,A) \times B = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 15 \end{bmatrix}$$
$$= \begin{bmatrix} 45 - 45 \\ -30 + 30 \end{bmatrix} = [0]$$

Since, $(\text{Adj}\,A \times B) = 0$, the system will have infinite solutions.

Now,

Let $y = k$

$$2x = 5 - 3k \quad \text{or} \quad x = \frac{5 - 3k}{2}$$
$$x = 15 - 9k \quad \text{or} \quad x = \frac{5 - 3k}{2}$$

Hence, $x = \dfrac{5 - 3k}{2}, y = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(iii)

This can be written as

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 5(256) - 3(16) + 7(6 - 182)$$
$$= 0$$

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solutions according as

$$(\text{Adj}\,A) \times B \neq 0 \quad \text{or} \quad (\text{Adj}\,A) \times B = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 256 \qquad C_{21} = -16 \qquad C_{31} = -176$$
$$C_{12} = -16 \qquad C_{22} = 1 \qquad C_{32} = 11$$
$$C_{13} = -176 \qquad C_{23} = 11 \qquad C_{33} = 121$$

$$\text{adj}\,A = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}^T = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$$

$$\text{adj}\,A \times B = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ has infinite many solutions.

Now, $\quad$ let $z = k$

then, $\quad 5x + 3y = 4 - 7k$

$\quad 3x + 26y = 9 - 2k$

Which can be written as

$$\begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 2$$

$$\text{adj}\,A = \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \dfrac{1}{|A|} \times \text{adj}\,A \times B$

$$= \frac{1}{121}\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$$

$$= \frac{1}{121}\begin{bmatrix} 77 - 176k \\ 11k + 33 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{7 - 16k}{11} \\ \dfrac{k + 3}{11} \end{bmatrix}$$

There values of x, y, z satisfies the third eq.

Hence, $x = \dfrac{7 - 16k}{11}, y = \dfrac{k + 3}{11}, z = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(iv)

This above system can be written as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

or $\quad A X = B$

$$\begin{aligned} |A| &= 1(2-2) + 1(4-1) + 1(-3) \\ &= 0 + 3 - 3 \\ &= 0 \end{aligned}$$

So, A is singular. Thus, the given system is either inconsistent or consistent with infinitely many solutions according as

$$(\text{Adj } A) \times (B) \neq 0 \quad \text{or} \quad (\text{Adj } A) \times B = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 0 & C_{21} = 0 & C_{31} = 0 \\ C_{12} = -3 & C_{22} = 3 & C_{32} = 3 \\ C_{13} = -3 & C_{23} = -3 & C_{33} = 3 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 0 & -3 & -3 \\ 0 & 3 & 3 \\ 0 & 3 & 3 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ -3 & 3 & 3 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ -3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ has infinite many solutions.

Now, $\quad$ let $z = k$

So, $\quad x - y = 3 - k$

$\quad\quad 2x + y = 2 + k$

Which can be written as

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3-k \\ 2+k \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 1 + 2 = 3 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

and, $\quad X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3-5 \\ 2+k \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 - k + 2 + k \\ -6 + 2k + 2 + k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ \frac{3k-4}{3} \end{bmatrix}$$

Hence, $x = \frac{5}{3}, y = k - \frac{4}{3}, z = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(v)

This system can be written as

$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & 3\\1 & 4 & 7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\14\\30\end{bmatrix}$$

or $\quad A\,X = B$

$$\begin{aligned}|A| &= 1(2)-1(4)+1(2)\\&= 2-4+2\\&= 0\end{aligned}$$

So, A is singular. Thus, the given system has either no solution or infinite solutions depending on as

$$(\text{Adj } A)\times(B)\neq 0 \quad\text{or}\quad (\text{Adj } A)\times(B)=0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll}C_{11}=2 & c_{21}=-3 & C_{31}=1\\c_{12}=-4 & c_{22}=6 & C_{32}=-2\\c_{13}=2 & c_{23}=-3 & C_{33}=1\end{array}$$

$$\text{adj } A=\begin{bmatrix}2 & -4 & 2\\-3 & 6 & -3\\1 & -2 & 1\end{bmatrix}^T=\begin{bmatrix}2 & -3 & 1\\-4 & 6 & -2\\2 & -3 & 1\end{bmatrix}$$

$$(\text{adj } A)\times B=\begin{bmatrix}2 & -3 & 1\\-4 & 6 & -2\\2 & -3 & 1\end{bmatrix}\begin{bmatrix}6\\14\\30\end{bmatrix}=\begin{bmatrix}12-42+30\\-24+84-60\\12-42+30\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

So, $AX=B$ has infinite solutions.

Now, $\quad$ let $z=k$

So, $\quad x+y=6-k$

$\quad\quad x+2y=14-3k$

Which can be written as

$$\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}6-k\\14-3k\end{bmatrix}$$

or $\quad A\,X=B$

$$|A|=1\neq 0$$

$$\text{adj } A=\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}^T=\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}$$

$$X = A^{-1}B=\frac{1}{|A|}\text{adj } A\times B$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{1}\begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}6-k\\14-3k\end{bmatrix}$$

$$=\begin{bmatrix}12-2k-14+3k\\-6+k+14-3k\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2+k\\8-2k\end{bmatrix}$$

Hence, $x=k-2$

$\quad y=8-2k$

$\quad z=k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(vi)

This system can be written as

$$\begin{bmatrix} 2 & 2 & -2 \\ 4 & 4 & -1 \\ 6 & 6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 2(14) - 2(14) - 2(0) = 0$$

So, A is singular and the system has either no solution or infinite solutions according as

$$(\text{Adj } A) \times (B) \neq 0 \quad \text{or} \quad (\text{Adj } A) \times (B) = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 14 & C_{21} = -16 & C_{31} = 6 \\ C_{12} = -14 & C_{22} = 16 & C_{32} = -6 \\ C_{13} = 0 & C_{23} = 0 & C_{33} = 0 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 14 & -14 & 0 \\ -16 & 16 & 0 \\ 6 & -6 & 0 \end{bmatrix}^T = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 - 32 + 18 \\ -14 + 32 - 18 \\ 0 + 0 + 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So, $AX = B$ has infinite solutions.

Now, $\quad$ let $z = k$

So, $\quad 2x + 2y = 1 + 2k$

$\quad 4x + 4y = 2 + k$

Which can be written as

$$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 + 2k \\ 2 + k \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 0, \; z = 0$$

Again,

$$2x + 2y = 1$$
$$4x + 4y = 2$$

Solution of Simultaneous Linear Equations Ex 8.1 Q4(i)

The above system can be written as

$$\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 0$$

So, A is singular, and the above system will be inconsistent if

$$(\text{adj } A) \times B \neq 0$$

Now, $C_{11} = 15$

$C_{12} = -6$

$C_{21} = -5$

$C_{22} = 2$

$$\text{adj } A = \begin{bmatrix} 15 & -6 \\ -5 & 2 \end{bmatrix}^T = \begin{bmatrix} 15 & -5 \\ -6 & 2 \end{bmatrix}$$

$$(\text{adj } A) \times (B) = \begin{bmatrix} 15 & -5 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

$$= \begin{bmatrix} 105 - 65 \\ -42 + 26 \end{bmatrix}$$

$$= \begin{bmatrix} 40 \\ -16 \end{bmatrix}$$

$$\neq 0$$

Hence, the above system is inconsistent

Solution of Simultaneous Linear Equations Ex 8.1 Q4(ii)

This system can be written as

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 0$$

So, the above system will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$C_{11} = 9$

$C_{12} = -6$

$C_{21} = -3$

$C_{22} = 2$

$$\text{adj } A = \begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 45 - 30 \\ -30 + 20 \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ -10 \end{bmatrix}$$

$$\neq 0$$

Hence, the above system is inconsistent

Solution of Simultaneous Linear Equations Ex 8.1 Q4(iii)

This system can be written as

$$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = -12 + 12 = 0$$

So, A is singular. Now system will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$C_{11} = -3$
$C_{12} = -6$
$C_{21} = 2$
$C_{22} = 4$

$$\text{adj } A = \begin{bmatrix} -3 & -6 \\ 2 & 4 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$$

$$\begin{aligned}(\text{adj } A) \times (B) &= \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} -9 + 10 \\ -18 + 20 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &\neq 0\end{aligned}$$

Hence, the above system is inconsistent

Solution of Simultaneous Linear Equations Ex 8.1 Q4(iv)

The above system can be written as

$$\begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

or $\quad A\,X = B$

$$\begin{aligned}|A| &= 4(-36) + 5(36) - 2(18) \\ &= -144 + 180 - 36 \\ &= 0\end{aligned}$$

So, A is singular and the above system will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$C_{11} = -36 \qquad C_{21} = 36 \qquad C_{31} = -18$
$C_{12} = -36 \qquad C_{22} = 36 \qquad C_{32} = -18$
$C_{13} = 18 \qquad C_{23} = -18 \qquad C_{33} = 9$

$$(\text{adj } A) = \begin{bmatrix} -36 & -36 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9 \end{bmatrix}^T = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$$

$$(\text{adj } A) \times (B) = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} -72 - 72 + 18 \\ -72 - 72 + 18 \\ +36 + 36 - 9 \end{bmatrix} \neq 0$$

Hence, the above system is inconsistent.

Solution of Simultaneous Linear Equations Ex 8.1 Q4(v)

The above system can be written as

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 3(-5) + 1(3) - 2(-6) = -15 + 3 + 12 = 0$$

So, A is singular and the above system of equations will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$C_{11} = -5 \qquad C_{21} = +10 \qquad C_{31} = 5$

$C_{12} = 3 \qquad C_{22} = 6 \qquad C_{32} = 3$

$C_{13} = -6 \qquad C_{23} = 12 \qquad C_{33} = 6$

$$(\text{adj } A) = \begin{bmatrix} -5 & 3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6 \end{bmatrix}^T = \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(\text{adj } A) \times (B) = \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 - 10 + 15 \\ 6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} \neq 0$$

Hence, the given system of equations is inconsistent.

Solution of Simultaneous Linear Equations Ex 8.1 Q4(vi)

The above system can be written as

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 1(-3) - 1(3) - 2(-3) = -3 - 3 + 6 = 0$$

So, A is singular. Now the system can be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$C_{11} = -3 \qquad C_{21} = -3 \qquad C_{31} = -3$

$C_{12} = -3 \qquad C_{22} = -3 \qquad C_{32} = -3$

$C_{13} = -3 \qquad C_{23} = -3 \qquad C_{33} = -3$

$$(\text{adj } A) = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}^T = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}$$

$$(\text{adj } A) \times (B) = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -15 + 6 - 12 \\ -15 + 6 - 12 \\ -15 + 6 - 12 \end{bmatrix}$$

$$= \begin{bmatrix} -21 \\ -21 \\ -21 \end{bmatrix}$$

$$\neq 0$$

Hence, the given system is inconsistent.

Solution of Simultaneous Linear Equations Ex 8.1 Q5

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$AB = 6I$, where I is a 3×3 unit matrix

or $A^{-1} = \frac{1}{6}B$ [By def. of inverse]

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Now, the ginven system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

or $AX = B$

or $X = A^{-1}B$

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2, y = -1, z = 4$

Solution of Simultaneous Linear Equations Ex 8.1 Q6

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = 2(0) + 3(-2) + 5(1) = -1 \neq 0$$

Also, $C_{11} = 0$ $\quad C_{21} = -1$ $\quad C_{31} = 2$

$C_{12} = 2$ $\quad C_{22} = -9$ $\quad C_{32} = 23$

$C_{13} = 1$ $\quad C_{23} = -5$ $\quad C_{33} = 13$

$$(\text{adj } A) = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations can be written as

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ -1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

or $\quad A\,X = B$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 \\ -22+45+69 \\ -11-25+39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q7

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

$$|A| = 1(1+3) - 2(-1+2) + 5(5) = 4 - 2 + 25 = 27 \neq 0$$

$C_{11} = 4$ $\quad C_{21} = 17$ $\quad C_{31} = 3$

$C_{12} = -1$ $\quad C_{22} = -11$ $\quad C_{32} = 6$

$C_{13} = 5$ $\quad C_{23} = 1$ $\quad C_{33} = -3$

$$A^{-1} = \frac{1}{|A|} \times \text{adj } A = \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

Now, the given set of equations can be represented as

$x + 2y + 5z = 10$

$x - y - z = -2$

$2x + 3y - z = -11$

or $$\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

or $$X = A^{-1} \times B$$

$$= \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

$$= \frac{1}{27} \begin{bmatrix} 40 - 34 - 33 \\ -10 + 22 - 66 \\ 50 - 2 + 33 \end{bmatrix} = \frac{1}{27} \begin{bmatrix} -27 \\ -54 \\ 81 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$$

Hence, $x = -1, y = -2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q8

$$A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}$$

$$|A|=1(7)+2(2)=11$$

$$\begin{array}{lll} C_{11}=7 & C_{21}=2 & C_{31}=-6\\ C_{12}=-2 & C_{22}=1 & C_{32}=-3\\ C_{13}=-4 & C_{23}=2 & C_{33}=5\end{array}$$

$$A^{-1}=\frac{1}{|A|}\text{adj }A=\frac{1}{11}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

Now, $x-2y=10$

$2x+y+3z=8$

$-2y+z=7$

or $$\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}=\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$$

or $$X=A^{-1}\times B$$

$$=\frac{1}{11}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$$

$$=\frac{1}{11}\begin{bmatrix}70+16-42\\ -20+8-21\\ -40+16+35\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{11}\begin{bmatrix}44\\ -33\\ 11\end{bmatrix}=\begin{bmatrix}4\\ -3\\ 1\end{bmatrix}$$

Hence, $x=4, y=-3, z=1$

Solution of Simultaneous Linear Equations Ex 8.1 Q8(ii)

$$A=\begin{bmatrix}3 & -4 & 2\\ 2 & 3 & 5\\ 1 & 0 & 1\end{bmatrix}$$

$$|A|=3(3)+4(-3)+2(-3)=-9$$

$$\begin{array}{lll} C_{11}=3 & C_{21}=4 & C_{31}=-26\\ C_{12}=3 & C_{22}=1 & C_{32}=-11\\ C_{13}=-3 & C_{23}=-4 & C_{33}=17\end{array}$$

$$A^{-1}=\frac{1}{|A|}\text{adj}A=\frac{1}{-9}\begin{bmatrix}3 & 4 & -26\\ 3 & 1 & -11\\ -3 & -4 & 17\end{bmatrix}$$

Now,

$3x-4y+2z=-1$

$2x+3y+5z=7$

$x+z=2$

Or $$\begin{bmatrix}3 & -4 & 2\\ 2 & 3 & 5\\ 1 & 0 & 1\end{bmatrix}=\begin{bmatrix}-1\\ 7\\ 2\end{bmatrix}$$

$$X=A^{-1}\times B$$

Or $$=\frac{1}{-9}\begin{bmatrix}3 & 4 & -26\\ 3 & 1 & -11\\ -3 & -4 & 17\end{bmatrix}\begin{bmatrix}-1\\ 7\\ 2\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{-9}\begin{bmatrix}-27\\ -18\\ 9\end{bmatrix}$$

Hence $x=3, y=2, z=-1$

Solution of Simultaneous Linear Equations Ex 8.1 Q8(iii)

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$AB = 11I$, where I is a 3×3 unit matrix

$$A^{-1} = \frac{1}{11}B \qquad \text{[By def. of inverse]}$$

Or

$$= \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

Now, the given system of equations can be written as

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

Or $\quad AX = B$

$$X = A^{-1}B$$

Or

$$= \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Hence, $x = 4, y = -3, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q9

Let the numbers are x, y, z.

$$x + y + z = 2 \qquad ---(1)$$

Also, $2y + (x + z) = 1$

$$x + 2y + z = 1 \qquad ---(2)$$

Again,

$$x + z + 5(x) = 6$$

$$5x + y + z = 6 \qquad ---(3)$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 1(1) - 1(-4) + 1(-9)$$
$$= 1 + 4 - 9 = -4 \neq 0$$

Hence, the unique solutions given by $x = A^{-1}B$

$C_{11} = 1 \quad C_{21} = 0 \quad C_{31} = -1$

$C_{12} = 4 \quad C_{22} = -4 \quad C_{32} = 0$

$C_{13} = -9 \quad C_{23} = 4 \quad C_{33} = 1$

or $$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A) \times B = \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$$= \frac{-1}{4}\begin{bmatrix} 2-6 \\ 8-4 \\ -18+4+6 \end{bmatrix} = \frac{-1}{4}\begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Hence, $x = 1, y = -1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q10

Let the three investments are x, y, z

$$x + y + z = 10,000 \qquad \text{...... (1)}$$

Also

$$\frac{10}{100}x + \frac{12}{100}y + \frac{15}{100}z = 1310$$
$$0.1x + 0.12y + 0.15z = 1310 \qquad \text{...... (2)}$$

Also

$$\frac{10}{100}x + \frac{12}{100}y = \frac{15}{100}z - 190$$
$$0.1x + 0.12y - 0.15z = -190 \qquad \text{...... (3)}$$

The above system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.12 & 0.15 \\ 0.1 & 0.12 & -0.15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

Or $\quad AX = B$

$$|A| = 1(-0.036) - 1(-0.03) + 1(0) = -0.006 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = -0.036 \qquad C_{21} = 0.27 \qquad C_{31} = 0.03$$
$$C_{12} = 0.03 \qquad C_{22} = -0.25 \qquad C_{32} = -0.05$$
$$C_{13} = 0 \qquad C_{23} = -0.02 \qquad C_{33} = 0.02$$

$$\text{adj}A = \begin{bmatrix} -0.036 & 0.03 & 0 \\ 0.27 & -0.25 & -0.02 \\ 0.03 & -0.05 & 0.02 \end{bmatrix}^T = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}$$

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj}A) \times B$$
$$= \frac{1}{-0.006} \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix} \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-0.006} \begin{bmatrix} -12 \\ -18 \\ -30 \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$$

Hence, $x = \text{Rs}\,2000, y = \text{Rs}\,3000, z = \text{Rs}\,5000$

Solution of Simultaneous Linear Equations Ex 8.1 Q11

$$x + y + z = 45 \quad ---(1)$$
$$z = x + 8 \quad ---(2)$$
$$x + z = 2y \quad ---(3)$$

or $$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

$$|A| = 1(2) - 1(-2) + 1(2)$$
$$= 2 + 2 + 2 = 6 \neq 0$$

$$C_{11} = 2 \qquad C_{21} = -3 \qquad C_{31} = 1$$
$$C_{12} = 2 \qquad C_{22} = 0 \qquad C_{32} = -2$$
$$C_{13} = 2 \qquad C_{23} = +3 \qquad C_{33} = 1$$

$$X = A^{-1} \times B = \frac{1}{|A|}(\text{adj } A) \times B$$
$$= \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$
$$= \frac{1}{6}\begin{bmatrix} 90 - 24 \\ 90 \\ 90 + 24 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 66 \\ 90 \\ 114 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$$

Hence, $x = 11, y = 15, z = 19$

Solution of Simultaneous Linear Equations Ex 8.1 Q12

The given problem can be modelled using the following system of equations

$$3x + 5y - 4z = 6000$$
$$2x - 3y + z = 5000$$
$$-x + 4y + 6z = 13000$$

Which can write as $Ax = B$,

Where

$$A = \begin{bmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

Now

$$|A| = 3(-18-4) - 2(30+16) - 1(5-12)$$
$$= 3(-22) - 2(46) + 7$$
$$= -66 - 92 + 7$$
$$= -151 \neq 0$$

$\therefore$ A^{-1} exists.

Now $Ax = B \Rightarrow \quad x = A^{-1}\text{B}$

$$A^{-1} = \frac{adj(A)}{|A|}$$

Cofators of A are

$C_{11} = -22 \quad C_{21} = -13 \quad C_{31} = 5$

$C_{12} = -46 \quad C_{22} = 14 \quad C_{32} = -17$

$C_{13} = -7 \quad C_{23} = -11 \quad C_{33} = -19$

$$\therefore \quad adj(A) = \begin{bmatrix} -22 & -46 & -7 \\ -13 & +14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

Hence,

$$X = \frac{1}{|A|} adj(A)(B)$$
$$= \frac{1}{-151} \begin{bmatrix} -22 & -46 & -7 \\ -13 & +14 & -11 \\ 5 & -17 & -19 \end{bmatrix} \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$
$$= \frac{1}{-151} \begin{bmatrix} -132000 & -23000 & -91000 \\ -78000 & +70000 & -143000 \\ -3000 & -85000 & -247000 \end{bmatrix}$$
$$= \begin{bmatrix} 3000 \\ 1000 \\ 2000 \end{bmatrix}$$

$\therefore \quad x = 3000, y = 1000$ and $z = 2000$.

Solution of Simultaneous Linear Equations Ex 8.1 Q13

From the given data, we get
the following three equations:

$x + y + z = 12$

$2x + 3y + 3z = 33$

$x - 2y + z = 0$

This system of equations can be written in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} \ldots .(1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(9) - 1(-1) + 1(-7) = 3$$

$$cofA = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

$$adjA = [cofA]^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 11 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 36 - 33 + 0 \\ 4 + 0 + 0 \\ -28 + 33 + 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

An award for organising different festivals in the colony can be included by the management.

Solution of Simultaneous Linear Equations Ex 8.1 Q14

Let X, Y and Z be the cash awards for
Honesty, Regularity and Hard work respectively.
As per the data in the question, we get

$X + Y + Z = 6000$

$X + 3Z = 11000$

$X - 2Y + Z = 0$

The above three simulataneous equations
can be written in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(6) - 1(-2) + 1(-2) = 6$$

$$cofA = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q15

Let x, y and z be teh prize amount per person for
Resourcefulness, Competence and Determination respectively.
As per the data in the question, we get

$4x + 3y + 2z = 37000$

$5x + 3y + 4z = 47000$

$x + y + z = 12000$

The above three simulataneous equations
can be written in matrix form as

$$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 4(-1) - 3(1) + 2(2) = -3$$

$$cofA = \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 6 & -6 & -3 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{-3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{-3}\begin{bmatrix} -12000 \\ -15000 \\ -9000 \end{bmatrix} = \begin{bmatrix} 4000 \\ 5000 \\ 3000 \end{bmatrix}$$

The values x, y and z describe the amount of prizes per person for resourcefulness, competence and determination.

Solution of Simultaneous Linear Equations Ex 8.1 Q16

Let x, y and z be the prize amount per person for adaptibility, carefulness and calmness respectively.

As per the given data, we get

$2x + 4y + 3z = 29000$

$5x + 2y + 3z = 30500$

$x + y + z = 9500$

The above three simulataneous equations can be written in the matrix form as

$$\begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 2(-1) - 4(2) + 3(3) = -1$$

$$cofA = \begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 6 & 9 & -16 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{\begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}}{-1} = \begin{bmatrix} 1 & 1 & -6 \\ 2 & 1 & -9 \\ -3 & -2 & 16 \end{bmatrix}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -6 \\ 2 & 1 & -9 \\ -3 & -2 & 16 \end{bmatrix}\begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix} \ldots(1)$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2500 \\ 3000 \\ 4000 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q17

Let x, y and z be the prize amount per student for sincerity, truthfulness and helpfulness respectively.
As per the data in the question, we get
3x + 2y + z = 1600
4x + y + 3z = 2300
x + y + z = 900
The above three simulataneous equations can be written in matrix form as

$$\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1600\\2300\\900\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}^{-1}\begin{bmatrix}1600\\2300\\900\end{bmatrix}\ldots(1)$$

$$A=\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}$$

$$|A|=3(-2)-2(1)+1(3)=-5$$

$$cofA=\begin{bmatrix}-2 & -1 & 3\\-1 & 2 & -1\\5 & -5 & -5\end{bmatrix}$$

$$adjA=(cofA)^T=\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}$$

$$A^{-1}=\frac{adjA}{|A|}=\frac{\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\frac{\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}}{-5}\begin{bmatrix}1600\\2300\\900\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}\begin{bmatrix}-320\\-460\\-180\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}200\\300\\400\end{bmatrix}$$

Excellence in extra-curricular activities should be another value considered for an award.

Solution of Simultaneous Linear Equations Ex 8.1 Q18

x, y and z be prize amount per student for
Discipline, Politeness and Punctuality respectively.
As per the data in the question, we get
3x+2y+z=1000
4x+y+3z=1500
x+y+z=600
The above three simulataneous equations
can be written in matrix form as

$$\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1000\\1500\\600\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}^{-1}\begin{bmatrix}1000\\1500\\600\end{bmatrix}\ldots(1)$$

$$A=\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}$$

$$|A|=3(-2)-2(1)+1(3)=-5$$

$$cofA=\begin{bmatrix}-2 & -1 & 3\\-1 & 2 & -1\\5 & -5 & -5\end{bmatrix}$$

$$adjA=(cofA)^T=\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}$$

$$A^{-1}=\frac{adjA}{|A|}=\frac{\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\frac{\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}}{-5}\begin{bmatrix}1000\\1500\\600\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}-2 & -1 & 5\\-1 & 2 & -5\\3 & -1 & -5\end{bmatrix}\begin{bmatrix}-200\\-300\\-120\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}X\\Y\\Z\end{bmatrix}=\begin{bmatrix}100\\200\\300\end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q19

x, y and z be prize amount per student for
Tolerance, Kindness and Leadership respectively.
As per the data in the question, we get
3x+2y+z=2200
4x+y+3z=3100
x+y+z=1200
The above three simulataneous equations
can be written in matrix form as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 3(-2) - 2(1) + 1(3) = -5$$

$$cofA = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}\begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} -440 \\ -620 \\ -240 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q20

Let the amount deposited be x, y and z respectively.
As per the data in the question, we get

$x + y + z = 7000$

$5\%x + 8\%y + 8.5\%z = 550$

$\Rightarrow 5x + 8y + 8.5z = 55000$

$x - y = 0$

The above equations can be written in matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix}^{-1}\begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix} \ldots(1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix}$$

$$|A| = 1(8.5) - 1(-8.5) + 1(-13) = 4$$

$$cofA = \begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}^T$$

$$adjA = (cofA)^T = \begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{4}\begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}$$

From (1)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}\begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4500 \\ 4500 \\ 19000 \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \\ 4750 \end{bmatrix}$$

Hence, the amounts deposited in the three accounts are 1125, 1125 and 4750 respectively.

Ex 8.2

Solution of Simultaneous Linear Equations Ex 8.2 Q1

$2x - y + z = 0$
$3x + 2y - z = 0$
$x + 4y + 3z = 0$

The systm can be written as

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A \quad X = 0$$

Now $|A| = 2(10) + 1(10) + 1(10)$
$= 40$
$\neq 0$

Since $|A| \neq 0$, hence $x = y = z = 0$ is the only solution of this homogeneous system.

Solution of Simultaneous Linear Equations Ex 8.2 Q2

$2x - y + 2z = 0$
$5x + 3y - z = 0$
$x + 5y - 5z = 0$

$$\begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \\ 1 & 5 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\text{or } \ A \quad x = 0$$

$$\begin{aligned} |A| &= 2(-10) + 1(-24) + 2(22) \\ &= -20 - 24 + 44 \\ &= 0 \end{aligned}$$

Hence, the system has infinite solutions.

Let $\quad z = k$

$$2x - y = -2k$$
$$5x + 3y = k$$
$$\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ k \end{bmatrix}$$
$$A \quad x = B$$

$|A| = 6 + 5 = 11 \neq 0 \quad$ so A^{-1} exist

Now $adj\ A = \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}$

$$x = A^{-1}.B = \frac{1}{|A|}(adj\ A)B = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} -2k \\ k \end{bmatrix} = \begin{bmatrix} \frac{-5k}{11} \\ \frac{12k}{11} \end{bmatrix}$$

Hence, $x = \frac{-5k}{11}$, $y = \frac{12k}{11}$, $z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q3

$3x - y + 2z = 0$
$4x + 3y + 3z = 0$
$5x + 7y + 4z = 0$

$$|A| = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 3 \\ 5 & 7 & 4 \end{bmatrix}$$

$$\begin{aligned} &= B(-9) + 1(1) + 2(13) = -27 + 1 + 26 = -27 + 27 \\ &= 0 \end{aligned}$$

Hence, it has infinite solutions.

Let $\quad z = k$

$$3x - y = -2k$$
$$4x + 3y = -3k$$

or $\quad \begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ -3k \end{bmatrix}$

or $\quad A \quad x = B$

$|A| = 9 + 4 = 13 \neq 0 \quad$ hence A^{-1} exists

$adj\ A = \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}$

Now $x = A^{-1}B = \frac{1}{|A|}(adj\ A)B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -2k \\ -3k \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -9k \\ -k \end{bmatrix}$$

Hence, $x = \frac{-9k}{13}$, $y = \frac{-k}{13}$, $z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q4

$x + y - 6z = 0$
$x - y + 2z = 0$
$-3x + y + 2z = 0$

$$\text{Hence, } |A| = \begin{bmatrix} 1 & 1 & -6 \\ 1 & -1 & 2 \\ -3 & 1 & 2 \end{bmatrix}$$

$$\begin{aligned} &= 1(-4) - 1(8) - 6(-2) \\ &= -4 - 8 + 12 \\ &= 0 \end{aligned}$$

Hence, the system has infinite solutions.

Let $z = k$
$x + y = 6k$
$x - y = -2k$

or $\begin{bmatrix} 1 & +1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6k \\ -2k \end{bmatrix}$

or $A \quad x = B$

$|A| = -1 - 1 = -2 \neq 0$ hence A^{-1} exists.

$$adj\ A = \begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$x = A^{-1}B = \frac{1}{|A|}(adj\ A)B = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix}\begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -6k + 2k \\ -6k - 2k \end{bmatrix} = \left(\frac{1}{-2}\right)\begin{bmatrix} -4k \\ -8k \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -4k \\ -8k \end{bmatrix} = \begin{bmatrix} 2k \\ 4k \end{bmatrix}$$

Hence, $x = 2k,\ y = 4k, z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q5

$x + y + z = 0$
$x - y - 5z = 0$
$x + 2y + 4z = 0$

$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -5 \\ 1 & 2 & 4 \end{bmatrix}$$
$$= 1(6) - 1(9) + 1(3) = 9 - 9 = 0$$

Hence, the system has infinite solutions.

Let $z = k$
$x + y = -k$
$x - y = 5k$

$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -k \\ 5k \end{bmatrix}$

or $A \quad x = B$

$|A| = -2 \neq 0$, hence A^{-1} exists.

$$adj\ A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

so, $x = A^{-1}B = \frac{1}{|A|}(adj\ A)B = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -k \\ 5k \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left(\frac{1}{-2}\right)\begin{bmatrix} k - 5k \\ k + 5k \end{bmatrix} = \begin{bmatrix} 2k \\ -3k \end{bmatrix}$$

$x = 2k,\ y = -3k, z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q6

$x + y - z = 0$
$x - 2y + z = 0$
$3x + 6y - 5z = 0$

$$\text{Hence, } |A| = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{bmatrix}$$
$$= 1(4) - 1(-8) - 1(12)$$
$$= 4 + 8 - 12 = 0$$

Hence, the system will have infinite solutions.

Let $z = k$
$x + y = -k$
$x - 2y = -k$

$$\text{or} \quad \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ -k \end{bmatrix}$$
$$\text{or} \quad A \quad x = B$$

$|A| = -3 \neq 0$, hence A^{-1} exists.

$$\text{Now, } adj\ A = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}' = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}$$

Next $x = A^{-1}B$
$$= \frac{1}{|A|}(adj\ A)(B) = \frac{1}{-3}\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} k \\ -k \end{bmatrix}$$
$$= \frac{-1}{3}\begin{bmatrix} -2k + k \\ -2k \end{bmatrix}$$
$$= \frac{-1}{3}\begin{bmatrix} -k \\ -2k \end{bmatrix} = \begin{bmatrix} \frac{k}{3} \\ \frac{2k}{3} \end{bmatrix}$$

Hence, $x = \frac{k}{3}$, $y = \frac{2k}{3}$, $z = k$
or $x = k$, $y = 2k$, $z = 3k$

Solution of Simultaneous Linear Equations Ex 8.2 Q7

$3x + y - 2z = 0$
$x + y + z = 0$
$x - 2y + z = 0$

$$\text{Hence, } |A| = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$
$$|A| = B(1+2) - 1(1-1) - 2(-3)$$
$$= 9 - 0 + 6$$
$$= 15 \neq 0$$

Hence, the given system has only trivial solutions given by $x = y = z = 0$

Solution of Simultaneous Linear Equations Ex 8.2 Q8

$2x + 3y - z = 0$
$x - y - 2z = 0$
$3x + y + 3z = 0$

Hence, $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$

$$|A| = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= 2(-3+2) - 3(3+6) - 1(4)$$
$$= -2 - 27 - 4$$
$$\neq 0$$

Hence, the system has only trivial solutions given by $x = y = z = 0$