Number Systems

Case Study Based Questions

Case Study 1

During revision hours, two students Vimal and Sunil were discussing with each other about the topic of rationalising the denominator.

Vimal explains that simplification of $\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ by rationalising the denominator is multiplying numerator and denominator by $\sqrt{5}-\sqrt{3}$. And Sunil explains the simplification of $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$ by using the identity $(a+b)\,(a-b)=a^2-b^2$.

On the basis of the above information, solve the following questions:

Q 1. The rationalising factor of $\sqrt{5}+\sqrt{3}$ is:

a. $\frac{1}{\sqrt{5}+\sqrt{3}}$ b. $(\sqrt{5}-\sqrt{3})$

c. $-\sqrt{5}-\sqrt{3}$ d. $(\sqrt{5}+\sqrt{3})$

Q 2. According to Vimal explanation, the simplification of $\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ is:

a. $5-\sqrt{15}$ b. $\frac{5+\sqrt{15}}{2}$

c. $\frac{\sqrt{15}-3}{2}$ d. $5+\sqrt{15}$

Q 3. According to Sunil explanation, the simplification of $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$ is:

a. 1 b. – 1 c. 2 d. 3

Q 4. Addition of two irrational numbers:

a. is always rational
b. is always irrational
c. may be rational or irrational
d. is always integer

Q 5. The square root of natural number is a/an:

a. rational b. irrational
c. rational or irrational d. None of these

Solutions

1. (b) The rationalising factor of $(\sqrt{5}+\sqrt{3})$ is $(\sqrt{5}-\sqrt{3})$.

So, option (b) is correct.

2. (c) We have, $\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

By rationalisation the denominator

$$\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{5-3}=\frac{\sqrt{15}-3}{2}$$

So, option (c) is correct.

3. (a) Using identity $(a+b)(a-b)=a^2-b^2$

$\therefore\ (\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})=(\sqrt{3})^2-(\sqrt{2})^2$

$=3-2=1$

So, option (a) is correct.

4. (c) Addition of two irrational numbers may be rational or irrational.

e.g. (i) $(2+\sqrt{5})+(1-\sqrt{5})=3$, which is rational.

(ii) $(2+\sqrt{5})+\sqrt{3}=2+\sqrt{5}+\sqrt{3}$,

which is irrational.

So, option (c) is correct.

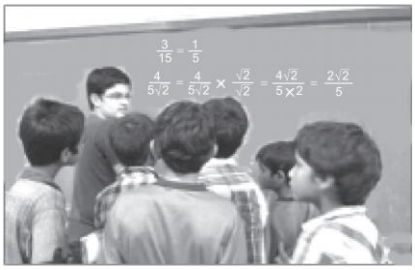
5. (c) The square root of a natural number is a rational or irrational number.

So, option (c) is correct.

Case Study 2

One day a math teacher taught students about the number system. She drew a number line on the black board and represented different types of numbers such as natural numbers, integers, rational numbers, etc.

A number of the form $\frac{p}{q}$ is said to be a rational number, if $q \neq 0$ and p and q are integers.



On the basis of the above information, solve the following questions:

Q 1. A rational number between $\frac{1}{3}$ and $\frac{1}{7}$ is:

a. $\frac{21}{5}$ b. $\frac{17}{21}$ c. $-\frac{5}{21}$ d. $\frac{5}{21}$

Q 2. An irrational number between $\sqrt{3}$ and $\sqrt{5}$ is:

a. 2.1 b. 2

c. $\sqrt{3.5}$ d. $\sqrt{7}$

Q 3. Decimal number $1.\overline{5}$ in the form of $\frac{p}{q}$ is:

a. $\frac{14}{3}$ b. $\frac{11}{9}$ c. $-\frac{14}{9}$ d. $\frac{14}{9}$

Q 4. The sum of two rational numbers is always:

a. integers b. naturals

c. rational d. irrational

Q 5. A terminating or repeating decimal number is a/an:

a. rational b. irrational

c. rational or irrational d. whole number

Solutions

1. (d) A rational number between $\frac{1}{3}$ and $\frac{1}{7}$ is

$$\frac{\frac{1}{3}+\frac{1}{7}}{2}=\frac{7+3}{2\times 21}=\frac{10}{2\times 21}=\frac{5}{21}$$

So, option (d) is correct.

2. (c) Since, $\sqrt{3}=1.732$ and $\sqrt{5}=2.236$

But $\sqrt{3.5}=1.870$,which lies in the given interval.

Hence, irrational number $\sqrt{3.5}$ lies between $\sqrt{3}$ and $\sqrt{5}$.

So, option (c) is correct.

3. (d) Let $x=1.\overline{5}$

$\Rightarrow \quad x=1.555.....$ ...(1)

Multiplying both sides by 10, we get

$10\,x=15.55.....$ ...(2)

Subtracting eq. (1) from eq. (2), we get

$9x=14 \quad \Rightarrow \quad x=\frac{14}{9}$

So, option (d) is correct.

4. (c) The sum of two rational numbers is always a rational number.

So, option (c) is correct.

5. (a) A terminating or repeating decimal number is a rational number.

So, option (a) is correct.

Case Study 3

The secretary of Golf Course colony organised a free medical camp for the patients suffering from Corona Virus. During the medical camp, the pulse rate of many patients recorded in mathematical term was 150.35,

$\frac{174}{180}$, $(178)^{1/2}$, ...

But it was not easy to understand the mathematical number. So, one of the students of class IX asked the following questions.

On the basis of the above information, solve the following questions:

Q 1. What is the value of $2.\overline{13}$?

a. $\frac{203}{90}$ b. $\frac{211}{90}$

c. $\frac{211}{99}$ d. $-\frac{211}{99}$

Q 2. The value of $\sqrt{18} \times \sqrt{10} \times \sqrt{5}$ is:

a. 25 b. 35 c. 32 d. 30

Q 3. Rational number $\frac{22}{7}$ is equivalent to:

a. π b. 3.142857

c. $3.14\overline{2857}$ d. $3.\overline{142857}$

Q 4. If $4^{2x} = \frac{1}{8}$, then the value of x is:

a. $\frac{3}{4}$ b. $-\frac{3}{4}$ c. $-\frac{4}{3}$ d. $\frac{4}{3}$

Q 5. Which one of the following numbers is rational?

a. $\sqrt{5}$ b. π c. $\frac{7}{0}$ d. $\frac{0}{7}$

Solutions

1. (c) Let $x = 2.\overline{13}$

$\Rightarrow$ $x = 2.1313.....$ ...(1)

Multiplying both sides by 100, we get

$100x = 213.1313.....$ ...(2)

Subtracting eq. (1) from eq. (2), we get

$$99x = 211$$

$$\Rightarrow \quad x = \frac{211}{99}$$

So, option (c) is correct.

2. (d) $\sqrt{18} \times \sqrt{10} \times \sqrt{5}$

$$= \sqrt{2 \times 3 \times 3} \times \sqrt{2 \times 5} \times \sqrt{5}$$

$$= \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5} = 2 \times 3 \times 5 = 30$$

So, option (d) is correct.

3. (d) $\frac{22}{7} = 3.142857142.....$

$$= 3.\overline{142857}$$

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      7) 22(3.142857142
         21
         10
          7
          30
          28
           20
           14
            60
            56
             40
             35
              50
              49
               10
                7
                30
                28
                 20
                 14
                  6
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So, option (d) is correct.

4. (b) We have, $4^{2x} = \frac{1}{8}$

$$\therefore \quad (2^2)^{2x} = \frac{1}{2^3}$$

$\Rightarrow \quad 2^{4x} \times 2^3 = 1$

$\Rightarrow \quad 2^{4x+3} = 2^0$

Compare the exponent, we get

$$4x + 3 = 0 \quad \Rightarrow \quad x = -\frac{3}{4}$$

So, option (b) is correct.

5. (d) (a) $\sqrt{5}$ is an irrational number.

(b) π is an irrational number.

(c) $\frac{7}{0}$ is not defined.

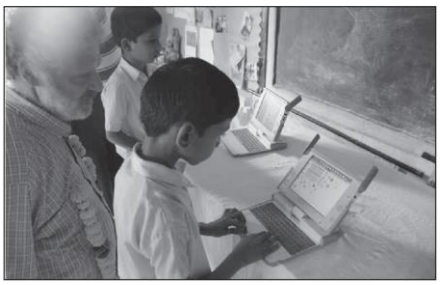
(d) $\frac{0}{7} = 0$, it is a rational number.

So, option (d) is correct.

Case Study 4

Mrs. Rakhi lives in an undeveloped area where there is no facility of proper education. But one thing is available in that area i.e, network. Since she was very keen to take education, so she decided to complete her education through e-learning.

One day she was studying number system, where she learnt about rational numbers, irrational numbers and decimal numbers, etc.



On the basis of the above information, solve the following questions:

Q 1. Convert the rational number $\frac{2}{15}$ into decimal number.

Q 2. Write one irrational number between 2.365 and 3.125.

Q 3. If $x+\sqrt{2}=3$, then find the value of $\frac{1}{x}$.

Q 4. Find the product of two irrational numbers $(7+3\sqrt{2})$ and $(7-3\sqrt{2})$.

Solutions

1. $\frac{2}{15} = 0.1333.....$

$= 0.1\overline{3}$

$$
\begin{array}{r}
15\overline{)2.0(}0.133 \\
15 \\
\hline
50 \\
45 \\
\hline
50 \\
45 \\
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5
\end{array}
$$

2. One irrational number between 2.365 and 3.125 is 2.6121121112..... .

3. We have, $x+\sqrt{2}=3$

$\Rightarrow \quad x = 3-\sqrt{2}$

$\because \quad \frac{1}{x} = \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$ [by rationalisation]

$= \frac{3+\sqrt{2}}{(3)^2-(\sqrt{2})^2}$

$= \frac{3+\sqrt{2}}{9-2} = \frac{3+\sqrt{2}}{7}$

4. $(7+3\sqrt{2})(7-3\sqrt{2}) = (7)^2-(3\sqrt{2})^2$

$= 49 - 18$

$= 31$

Case Study 5

In a coaching centre, one day a teacher told student about the laws of exponents, which was defined below:

Suppose $a > 0$, $b > 0$ be real numbers and let m and n be rational numbers. Then

(i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$ (iv) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(v) $(ab)^m = a^m b^m$

On the basis of the above information, solve the following questions:

Q1. Find the value of $2^3 \times 2^4$

Q2. If $8^{x+1} = 64$, then find the value of x.

Q 3. Find the value of $\left((81)^{1/2}\right)^{1/2}$.

Q 4. If $x = 0.000216$**, then find the value of** $(x)^{1/3}$.

Q 5. Simplify $(49)^{1/3} \times (7)^{1/3}$.

Solutions

1. $2^3 \times 2^4 = (2)^{3+4} = 2^7$

$= 128$

2. Given, $8^{x+1} = 64$

$\therefore \quad 8^{x+1} = 8^2$

Compare the exponents, we get

$x + 1 = 2$

$\Rightarrow \quad x = 1$

3. $\left((81)^{1/2}\right)^{1/2} = (3^4)^{\frac{1}{4}}$

$= 3^{4\times\frac{1}{4}} = 3^1$

$= 3$

4. Given, $x = 0.000216$

$$= \frac{216}{1000000} = \left(\frac{6}{100}\right)^3$$

$$\therefore \quad (x)^{1/3} = \left(\left(\frac{6}{100}\right)^3\right)^{1/3} = \left(\frac{6}{100}\right)^{3 \times \frac{1}{3}} = \frac{6}{100}$$

$$= 0.06$$

5. $(49)^{1/3} \times (7)^{1/3} = (7^2)^{1/3} \times (7)^{1/3}$

$$= 7^{\frac{2}{3}+\frac{1}{3}} = 7^{\frac{3}{3}} = 7^1 = 7$$