Chapter - 2

Polynomials

Introduction to polynomial

Polynomials

A polynomial is an expression that contains one or more terms.
Example: 3, a + b, $x^2 + y + 3$, $u^6 + 3u^4 + 2u + 7$

Here, 3 is a Monomial (polynomial having one term). It is also a constant polynomial.

a + b is a binomial, that is a polynomial having two terms.

$x^2 + y + 3$ is a trinomial, that is a polynomial having three terms.

$3u^4 + 2u + 7$ is a polynomial having four terms.

Therefore, a polynomial can have any number of terms.

A polynomial in one variable x, is an algebraic expression of the form

$p(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2}+...+a_2x^2 + a_1x+a_0$

Where n is a positive integer and a0, a_1, a_2, a_3, ... , a_n are constants. They are also known as coefficients of polynomials.

The following are not polynomials

i) $\sqrt{x}$ + $3x^4$ + 7 as the power of x is $\frac{1}{2}$

ii) $3x^5 + 3x^4 + \frac{7}{x}$ as the power of x is -1 which is a negative integer.

If p(x) is a polynomial in x, the highest power of x in p(x) is called the degree of polynomial p(x).

Let us consider some examples here, x + 3

Here x + 3 has 2 terms, x and 3.

Now, the highest power of x is 1. So, x + 3 is a polynomial having degree 1.

$y^7 + 3y^3 + 2y + 9$ is a polynomial in y having 4 terms.

Now, the highest power of y is 7. So, the degree of $y^7 + 3y^3 + 2y + 9$ is 7.

Types of Polynomials according to their degree

Type of Polynomial	Degree	Example
Constant	0	3
Linear	1	$5y + 7$
Quadratic	2	$2y^2 + 5y + 7$
Cubic	3	$3x^3 + 2x^2 + x + 8$
Biquadratic Polynomial	4	$2x^4 + 3x^3 + 8$

Example: Write the degree of the following Polynomials.

i) $p(x) = 2x^3 - 3x + \frac{1}{\sqrt{5}}$

As the highest power of x in p(x)is 3. The degree of polynomial, $2x^3 - 3x + \frac{1}{\sqrt{5}}$ is 3.

ii) $p(u) = 9u^5 - \frac{2}{5} u^2 + u - \frac{1}{3}$

As the highest power of u in p(u)is5. The degree of the polynomial, $9u^5 - \frac{2}{5} u^2 + u - \frac{1}{3}$ is 5.

Example: Identify the type of the Polynomials given below (on the basis of degree)

i) $2y + 6$

Here, the highest power of y in the given polynomial is 1, so it is a Linear Polynomial

ii) $\sqrt{2} + y^2 + y$

Here, the highest power of y in the given polynomial is 2, so it is a Quadratic Polynomial.

iii) $y^3 + 4y^2 + 2y + 1$

Here, the highest power of y in the given polynomial is 3, so it is a Cubic Polynomial.

Zero of a polynomial

If p(x) is a polynomial in x and k is any real number, then the value obtained by replacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).

Let us consider the polynomial $p(x) = 2x^3 - 3x^2 + 1$

If we put x = 2 in the polynomial, that is, we replace x by 2 in the given polynomial, we get,

$$p(2) = 2(2)^3 - 3(2)^2 + 1 = 16 - 12 + 1 = 5$$

Therefore, the value of the polynomial p(x) at x = 2 is 5, that is, p(2) = 5

Similarly, we can find the value of p(x)at x = 0,

$$p(0) = 2(0)^3 - 3(0)^2 + 1 = 0 - 0 + 1 = 1$$

The value of p(x) at x = 0 is 1, that is, p(0) = 1

Now consider another case where we find the value of p(x)
at x = 1

$$p(1) = 2(1)^3 - 3(1)^2 + 1 = 2 - 3 + 1 = 3 - 3 = 0$$

Here, we see that the value of the polynomial p(x) is 0 at x = 1

As p(1) = 0, 1 is called a zero of the polynomial

$$p(x) = 2x^3 - 3x^2 + 1$$

A real number k is said to be a zero of the polynomial p(x) if p(k) = 0.

Now consider a linear polynomial, $p(x) = 2x + 1$,

If k is a zero of p(x), then $p(k) = 0$

$$2k + 1 = 0$$

$$2k = -1$$

$$k = \frac{-1}{2}$$

If k is a zero of a polynomial, $p(x) = ax + b$, then,

$$p(k) = ak + b = 0$$

$$k = \frac{-b}{a}$$

Therefore, the zero of the polynomial, $ax + b = \frac{-b}{a}$

$$= \frac{-Constant\ term}{Coefficient\ of\ x}$$

Example: If 3 is a zero of polynomial, $p(x) = 2x^2 + 3x - 9a$, then find the value of a.

If 3 is a zero of the polynomial, $p(x) = 2x^2 + 3x - 9a$ then $p(3) = 0$

Now, $p(x) = 2x^2 + 3x - 9a$

$$p(3) = 2(3)^2 + 3(3) - 9a$$

Now we will equate p(3) to 0.

$$2(3)^2 + 3(3) - 9a = 0$$

$$18 + 9 - 9a = 0$$

$$27 - 9a = 0$$

$$27 = 9a$$

$$a = 3$$

$\therefore$ The value of $a = 3$.

Example: For what value of k, -2 is a zero of the polynomial $3x^2 + 4x + 2k$.

If -2 is a zero of the polynomial, $p(x) = 3x^2 + 4x + 2k$ then $p(-2) = 0$

Now, $p(x) = 3x^2 + 4x + 2k$

As -2 is a zero of p(x), we will replace x by -2 in the given polynomial and equate it to 0.

$$p(-2) = 3(-2)^2 + 4(-2) + 2k = 0$$

$$12 - 8 + 2k = 0$$

$$2k = -4$$

$$k = -2$$

Example: If 1 and 2 are zeroes of polynomial,
$p(x) = 2x^2 - kx + 2m$, then find the value of k and m.

Here, 1 and 2 are zeroes of the polynomial,

$$p(x) = 2x^2 - kx + 2m$$

So, $p(1) = 2(1)^2 - k(1) + 2m = 0$

$$2 - k + 2m = 0$$

$k = 2m + 2 \rightarrow$(i)

Now $p(2) = 2(2^2) - k(2) + 2m = 0$

$$8 - 2k + 2m = 0$$

Putting the value of k in the above equation, we get

$$8 - 2(2m + 2) + 2m = 0$$

$$8 - 4m - 4 + 2m = 0$$

$$4 - 2m = 0$$

$$2m = 4$$

$$m = 2$$

We will now find the value of k by putting $m = 2$ in equation (i)

$$k = 2(2) + 2$$

$$k = 4$$

$\therefore k = 4$ and $m = 2$

Geometrical meaning of zeroes of polynomial

We know that a real number k, is a zero of the polynomial $p(x)$ if $p(k) = 0$.

A linear polynomial is of the form $ax + b$ where $a \neq 0$.

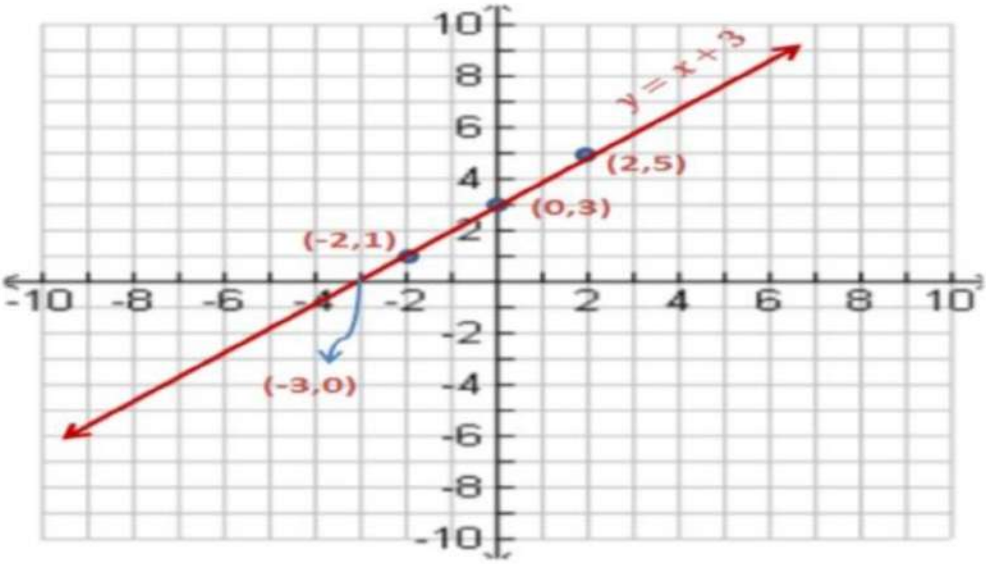
We will first study the graph of a linear polynomial, $y = x + 3$

If we put $x = 2$ in the above equation, we get

$y = 2 + 3 = 5$

Similarly, we can find more values of y by putting different values of x.

x	2	0	-2
$y = x + 3$	5	3	1



Here, we can see that the graph of a linear polynomial is a straight line.

The graph of $y = x + 3$ intersects the $x -$ axis at $x = -3$

Thus, -3 is the zero of the linear polynomial, $y = x + 3$.

Therefore, the zero of the polynomial, $x + 3$ is the $x -$coordinate of the point where the graph of $y = x + 3$ intersects the x-axis.

• For a linear polynomial $ax + b$, where $a \neq 0$, the graph of $y = ax + b$ is a straight line which intersects the x-axis at exactly one point, that is, $\left(\frac{-b}{a}, 0\right)$

• Therefore, the linear polynomial $ax + b$ has exactly one zero, namely the x $-$coordinate of the point where the graph of $y = ax + b$ intersects the $x -$ axis.

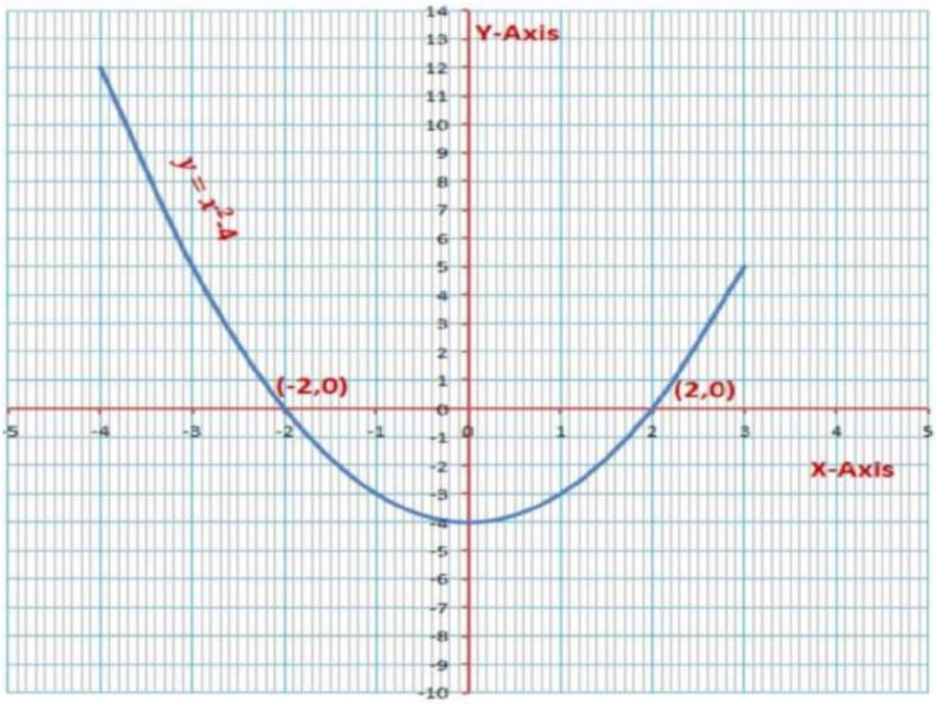
Next, we will study the geometrical meaning of a zero of a quadratic polynomial.

Consider a quadratic polynomial, $x^2 - 4$

First, we will find some values of $y = x^2 - 4$ corresponding to some values of x.

x	-4	-3	-2	-1	0	1	2	3
$y = x^2 - 4$	12	5	0	-3	-4	-3	0	5

If we plot these points on a graph, this is how the graph will look like.



For that matter, any quadratic polynomial $y = ax^2 + bx + c$, where $a \neq 0$ the graph will have either one of the two shapes depending on the value of a

i) If $a > 0$, then the shape is open upwards.

ii) If $a < 0$, then the shape is open downwards

These curves are called parabolas.

If we see the graph, then -2 and 2 are the points on the x- axis where the graph of $y = x^2 - 4$ intersects the x-axis.

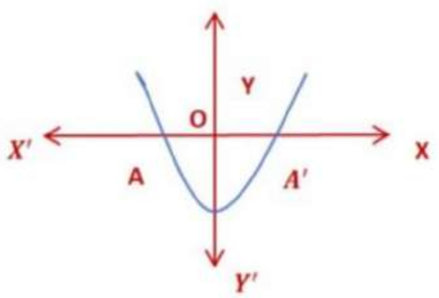
Therefore -2 and 2 are the zeroes of the polynomial $x^2 - 4$.

The zeroes of a quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ are the x – coordinates of the point, where the parabola representing $y = ax^2 + bx + c$ intersects the x – axis.

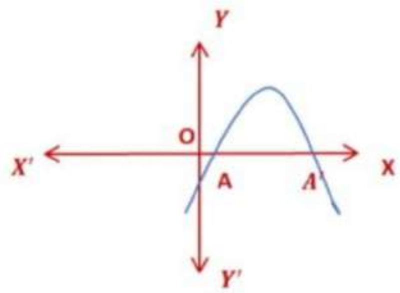
According to the shape of the graph of $y = ax^2 + bx + c$, the following cases may arise.

Case 1:

In this case, the graph intersects the x-axis at two distinct points A and A′, then the x- coordinates of A and A′are the two zeroes of the quadratic polynomial ax^2 bx + c.



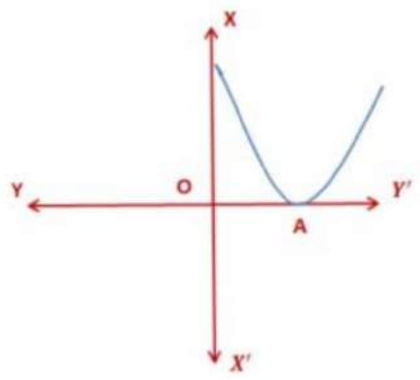
$a > 0$



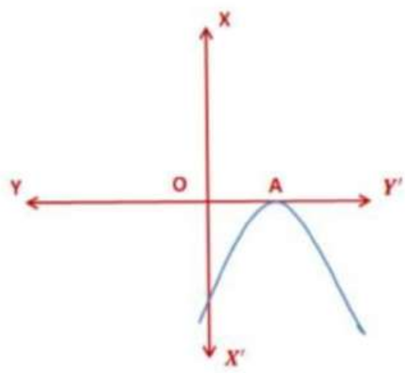
$a < 0$

Case 2:

In this case, the graph cuts the x − axis at exactly one point. Therefore, the two points A and A′of Case 1 coincide here to become one point A.



a > 0

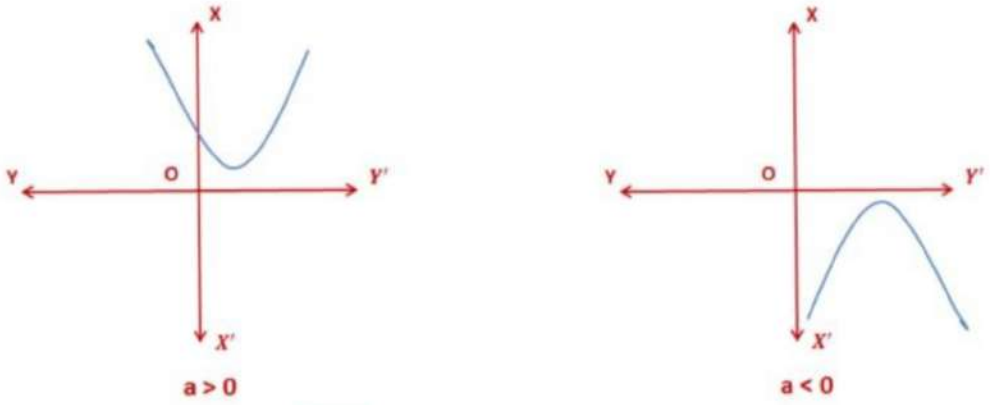


a < 0

Here, the x- coordinate of A is the only zero for the quadratic polynomial, $ax^2 + bx + c$.

Case 3:

In the third case, the graph is either completely above x-axis or completely below x-axis. Therefore, the graph does not intersect the x-axis at any point. Thus, the quadratic polynomial has no zero.



So, after studying all the cases we can see that a quadratic polynomial can have,

- Two distinct zeroes (shown in case 1)
- Two equal zeroes or one zero (shown in case 2)
- No zero (shown in case 3)

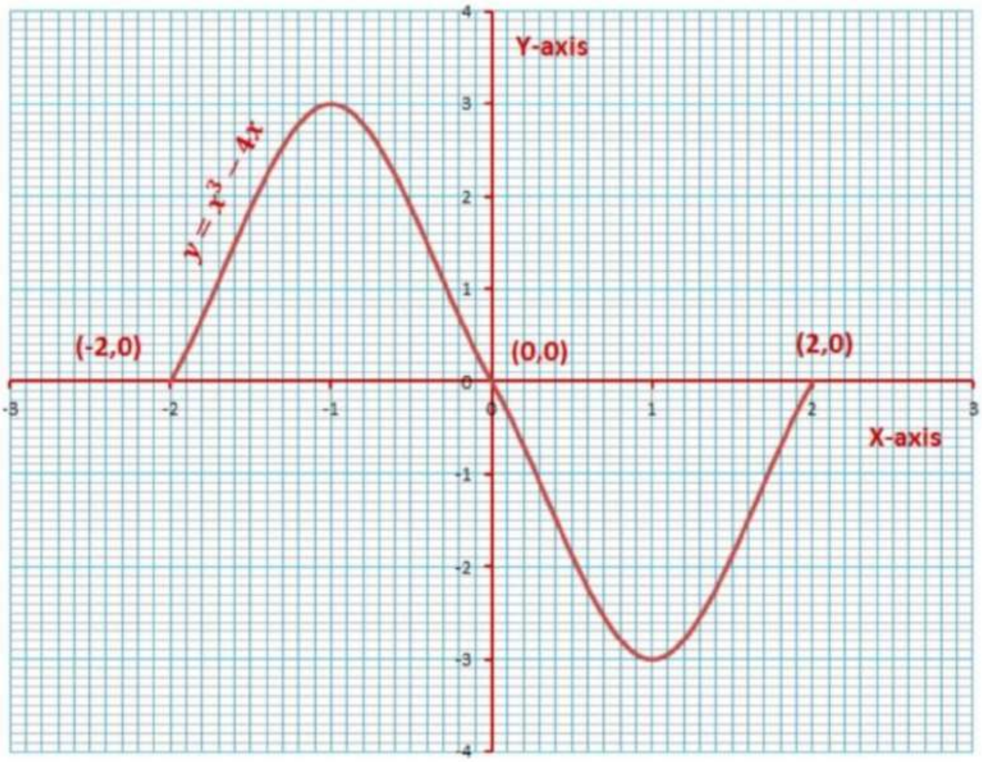
Therefore, geometrically we can see that a quadratic polynomial can have either two distinct zeroes or two equal zeroes that are one zero or no zero. Thus a quadratic polynomial of degree 2 has at most 2 zeroes.

Now we will study the geometrical meaning of the zeroes of a cubic polynomial. Consider a cubic polynomial

$x^3 - 4x$. First, we will find some values of y corresponding to a few values of x.

x	-2	-1	0	1	2
$y = x^3 - 4x$	0	3	0	-3	0

If we plot these points on the graph, the graph will look like this,

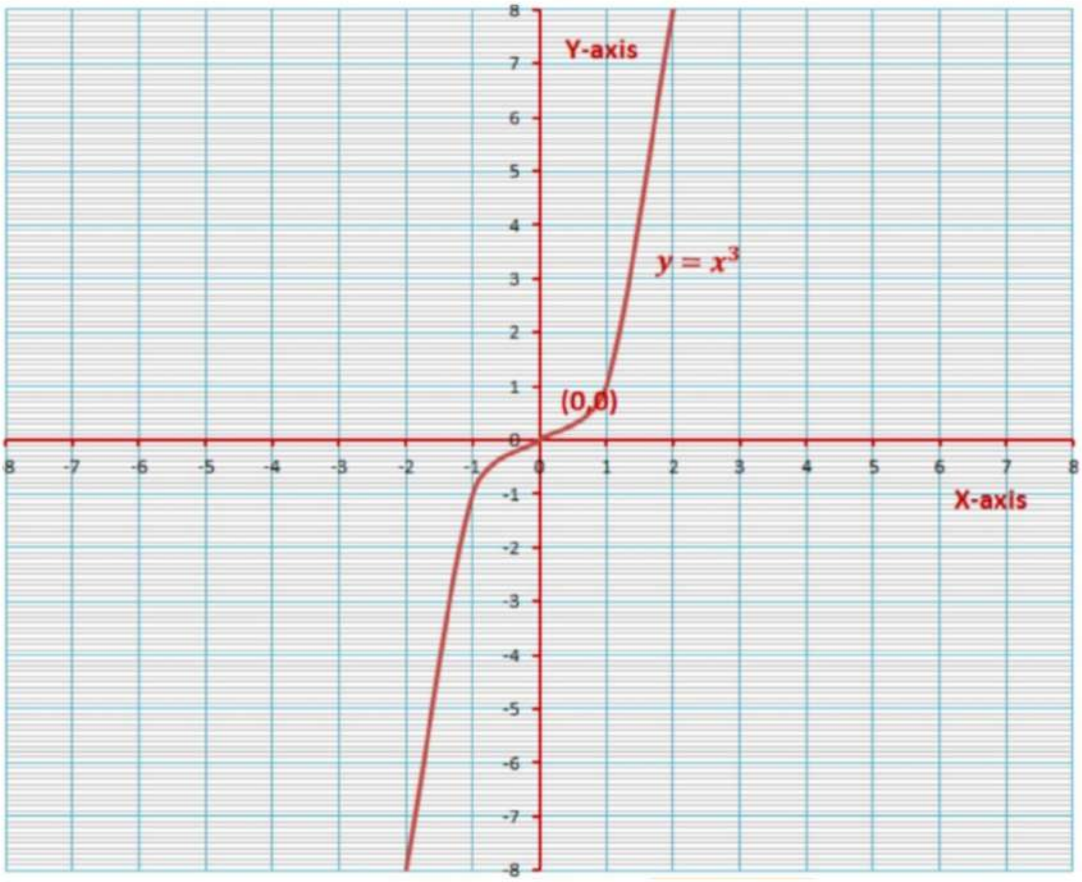


If we observe the graph, we see that -2, 0 and 2 are the x-coordinate of the points where the graph of $y = x^3 - 4x$ intersect the x-axis. Thus -2, 0 and 2 are the zeroes of the cubic polynomial, $y = x^3 - 4x$.

We will draw the graphs of a few more cubic polynomials.

Let us first consider the cubic polynomial, $y = x^3$. We will find a few values of x and y.

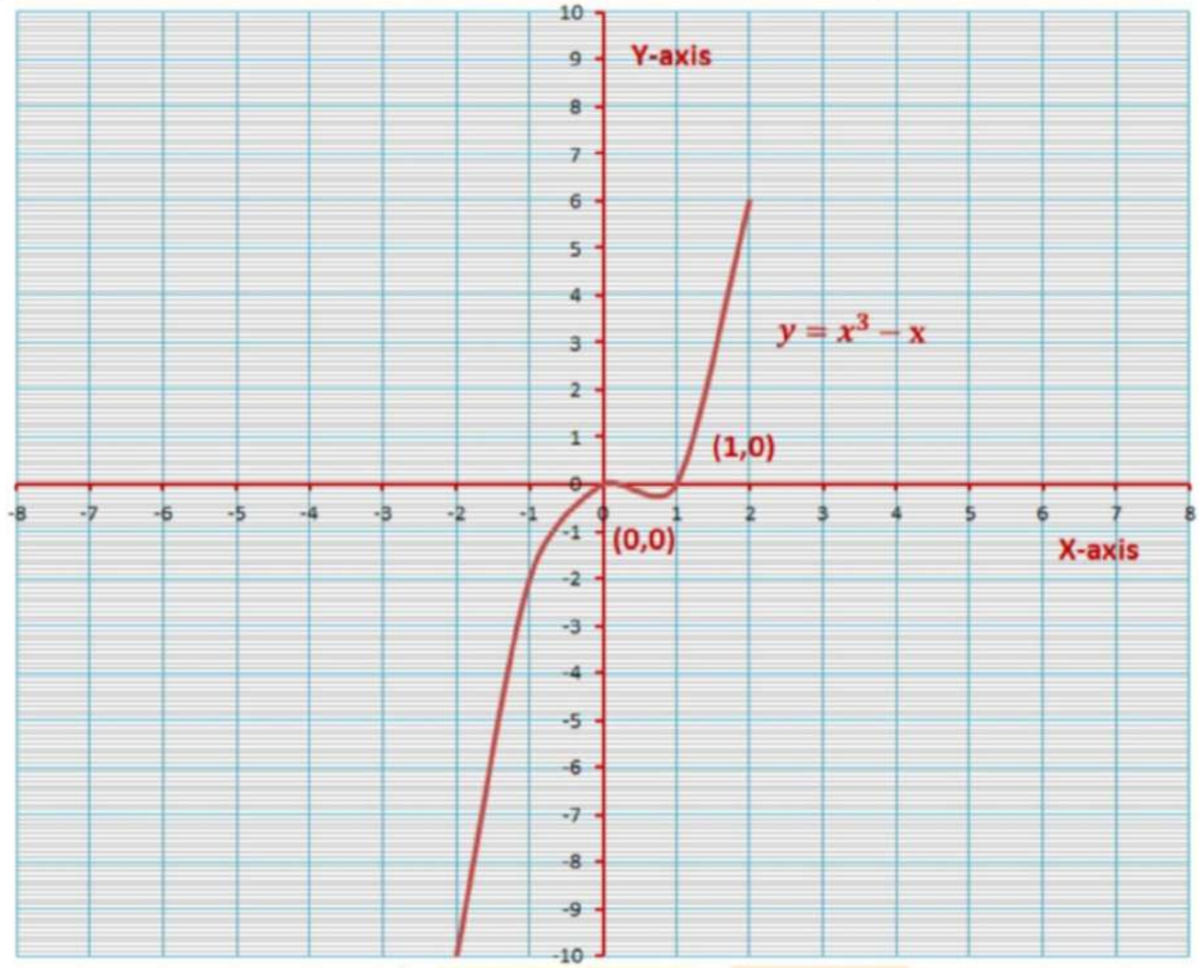
x	-2	-1	0	1	2
$y = x^3$	-8	-1	0	1	8



If we observe the graph we see that 0 is the only zero of the cubic polynomial, $y = x^3$ as its graph intersects the x-axis at the origin only.

Now consider one more cubic polynomial, $y = x^3 - x$. We will again find a few values of x and y.

x	-2	-1	0	0	2
$y = x^3 - x$	-10	-2	0	0	6

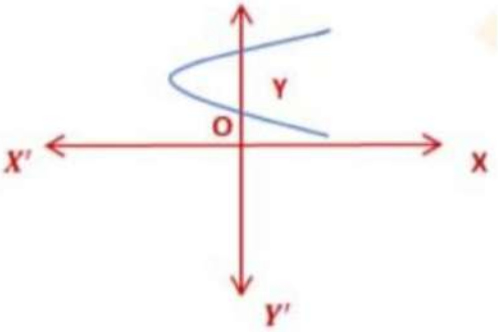


Now, we can see in the graph that 0 and 1 are the two zeroes of the cubic polynomial, $y = x^3 - x$ as its graph is intersecting the x-axis at (0, 0) and (1, 0).

Therefore, any cubic polynomial can have at most 3 zeroes.

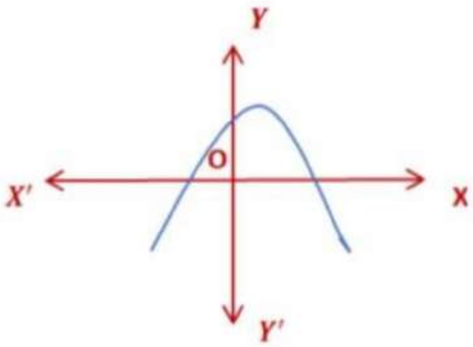
Example: The graph of $y = p(x)$ is given, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$ in each case.

a)

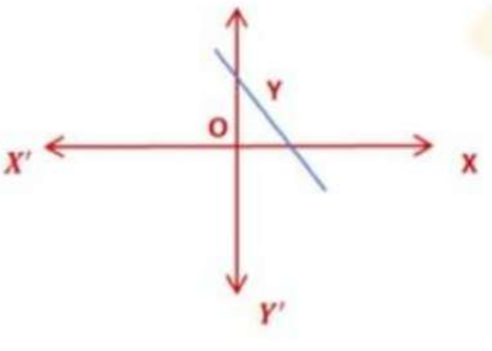


This is a graph of a quadratic polynomial. As the graph of y = p(x) does not intersect the x – axis at any point. Thus, it has no zero.

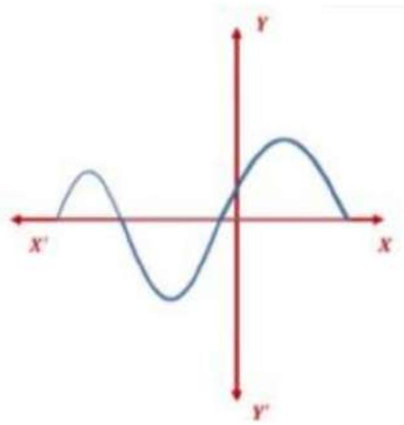
b)



This is again a graph of a quadratic polynomial. Here, the graph of y = p(x) intersects the x- axis at two points. Hence the number of zeroes is 2.
c)



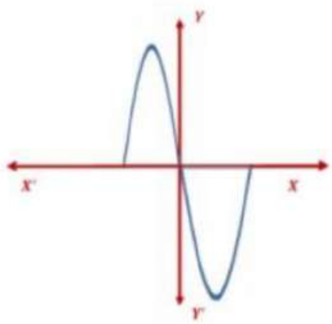
This is a graph of a linear polynomial. The number of zeroes is 1. As the graph of y = p(x) is intersecting the x-axis at one point only.
d)



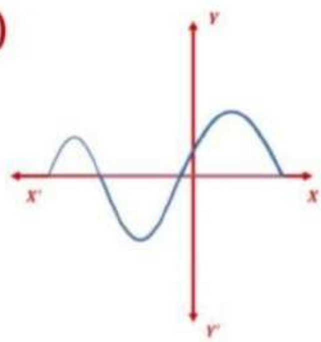
The given graph is of a cubic polynomial. Here the polynomial $y = p(x)$ is intersecting the x – axis at 3 points. Therefore, the number of zeroes is 3.

Example: Which of the following is not the graph of a cubic polynomial?

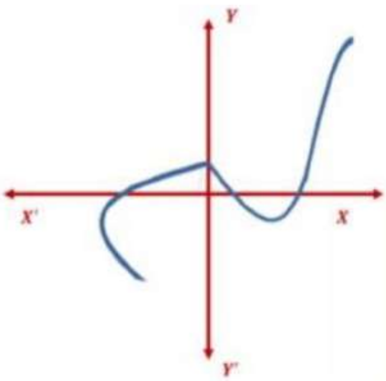
a)



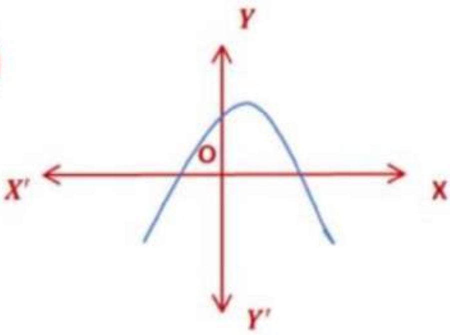
b)



c)



d)



Option d) is not the graph of a cubic polynomial. It is a graph of a quadratic polynomial, as the graph is in the shape of a parabola which is opening downwards.

Relationship between zeroes and coefficient of a Polynomial

Relationship between zeroes and coefficients of a Polynomial.

We know that a quadratic polynomial is of the form $ax^2 + bx + c$. A quadratic polynomial can have at the most two zeroes.

In general, if α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, then $x - \alpha$ and $x - \beta$ are the actors of the p(x). Therefore,

$$ax^2 + bx + c = k(x - a)(x - \beta)$$

$$= k(x^2 - x\beta - ax + a\beta)$$

$$= [x^2 - (\beta + a)x + a\beta]$$

$$= kx^2 - k(\beta + a)x + ka\beta$$

Now, comparing the coefficients of x^2, x and constant terms on both sides, we get

$$a = k, b = -k(\beta + a), c = ka\beta$$

$$(a + \beta) = \frac{-b}{k}$$

Now $k = a$

$$(a + \beta) = \frac{-b}{a}(a + \beta) = \frac{-b}{a}$$

$$a\beta = \frac{c}{k}$$

$$a\beta = \frac{c}{a}\,(k = a)$$

Sum of zeroes$(\alpha + \beta)$ = $\frac{-b}{a} = \frac{-Coefficient\ of\ x}{Coefficient\ of\ x^2}$

Product of zeroes$(\alpha\beta)$ = $\frac{c}{a} = \frac{Constant\ term}{Coefficient\ of\ x^2}$

Consider a quadratic polynomial, $2x^2 - 5x + 3$. Now we split the product of 2 and 3, that is 6, in such a way that the sum or difference of the numbers is equal to the middle

term.

Product of 2 and 3 = 6

$6 = 2 \times 3$ and $2 + 3 = 5$

$2x^2 - 5x + 3 = 2x^2 - (2+3)x + 3$

$= 2x^2 - 2x - 3x + 3$

$= 2x(x-1) - 3(x-1)$

$= (2x-3)(x-1)$

The value of polynomial, $2x^2 - 5x + 3$ is zero when $2x - 3 = 0$ and $x - 1 = 0$ or $x = \frac{3}{2}$ and $x = 1$

Therefore, the zeroes of $2x^2 - 5x + 3$ are $\frac{3}{2}$ and 1.

Sum of zeroes$(\alpha + \beta) = \frac{-b}{a} = \frac{-(5)}{2} = \frac{-Coefficient\ of\ x}{Coeficient\ of\ x^2}$

Product of zeroes$(\alpha\beta) = \frac{c}{a} = \frac{3}{2} = \frac{constant\ term}{Coefficient\ of\ x^2}$

Example: If one zero of $2x^2 - 3x + k$ is reciprocal to the other, then find the value of k.

Let one zero be α, then the other zero will be $\frac{1}{a}$

We know that, Product of zeroes$(\alpha\beta) = \frac{c}{a} = \frac{Constant\ term}{Coefficient\ of\ x^2}$

$(aX\frac{1}{a}) = \frac{c}{a} = \frac{k}{2}$

$1 = \frac{k}{2}$

K=2

Example: If α and β are zeroes of the polynomial

$x^2 - p(x + 1) + d$ such that $(\alpha + 1)(\beta + 1) = 0$, then find the value of d.

The given polynomial is $x^2 - p(x + 1) + d$

$x^2 - px - p + d$

Comparing the above equation with $ax^2 + bx + c$, we get

$a = 1, b = -p, c = -p + d$

α and β are the zeroes of the polynomial $x^2 - px - p + d$

Sum of zeroes$(\alpha + \beta) = \frac{-b}{a} = \frac{-Coefficient\ of\ x}{Coefficient\ of\ x^2} = \frac{-(-p)}{1} = p$

Product of zeroes$(\alpha\beta) = \frac{c}{a} = \frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{d-p}{1} = d - p$

$(\alpha + 1)(\beta + 1) = 0$ (given)

$$\alpha\beta + \alpha + \beta + 1 = 0$$

Putting $(\alpha + \beta) = p$ and $(\alpha\beta) = d - p$ in the above equation we get,

$d - p + p + 1 = 0$
$d + 1 = 0$
$d = -1$

Example: Form a quadratic polynomial, whose one zero is 7 and the product of zeroes is -56.

Let the zeroes be α and β.

It is given that the value of one zero is 7, then let us assume that $\alpha = 7$

Product of zeroes $= \alpha\beta = 7\beta$

Now, $7\beta = -56$

$$\beta = -8$$

$\therefore \alpha = 7$ and $\beta = -8$

$$(\alpha + \beta) = 7 - 8 = -1$$

$$(\alpha\beta) = 7 \times (-8) = -56$$

We know that a quadratic polynomial is
$x^2 -$(sum of zeroes)$x +$ product of zeroes

$$x^2 - (\alpha + \beta)x + (\alpha\beta)$$

Putting $(\alpha + \beta) = -1$ and $(\alpha\beta) = -56$

$$x^2 + x - 56$$

Example: If the zeroes of the polynomial $x^2 + px + q$ are double in value of the zeroes of $2x^2 - 5x - 3$, then find the value of p and q.

Letα and β be the zeroes of 2x2 − 5x − 3.

Sum of zeroes$(\alpha + \beta) = -$ba$= \frac{-Coefficient\ of\ x}{Coefficient\ of\ x^2} = \frac{-(-5)}{2} = \frac{5}{2}$

Product of zeroes$(\alpha\beta) = \frac{c}{a} = \frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{-3}{2}$

As the zeroes of $x^2 + px + q$ are double in value. Therefore, 2α and 2β are the zeroes of $x^2 + px + q$

Sum of zeroes$(2\alpha + 2\beta) = 2(\alpha + \beta) = 2 \times \frac{5}{2} = 5$

Product of zeroes$(2\alpha \times 2\beta) = 4\alpha\beta = 4\left(\frac{-3}{2}\right) = -6$

We know that a quadratic polynomial is
$x^2 -$(sum of zeroes)$x +$ product of zeroes

$$\therefore x^2 - (2\alpha + 2\beta)x + (2\alpha \times 2\beta)$$

$x^2 - 5x - 6$

Comparing the above equation with $x^2 + px + q$, we get

$$p = -5, q = -6$$

Example: Find the zeroes of the quadratic polynomial, $4\sqrt{3x^2} + 5x - 2\sqrt{3}$ and verify the relationship between the zeroes and the coefficients.

Let $p(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

By splitting the middle term we get,

$p(x) = 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$

$p(x) = 4\sqrt{3}x^2 + (8 - 3)x - 2\sqrt{3}$

$p(x) = 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3}$

$p(x) = 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2)$

$p(x) = (\sqrt{3}x + 2)(4x - \sqrt{3})$

The value of $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ is zero, when $x = \frac{-2}{\sqrt{3}}$ and $x = \frac{\sqrt{3}}{4}$

Therefore, the zeroes of $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are $x = \frac{-2}{\sqrt{3}}$ and $x = \frac{\sqrt{3}}{4}$

Sum of zeroes$(\alpha + \beta) = \frac{-2}{\sqrt{3}} + \frac{\sqrt{3}}{4} = \frac{-8+3}{4\sqrt{3}} = \frac{-5}{4\sqrt{3}}$

$\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2} = \frac{-5}{4\sqrt{3}}$

Sum of zeroes$= \frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}$

Product of zeroes$(\alpha\beta) = \frac{-2}{\sqrt{3}}$ X $\frac{\sqrt{3}}{4} = \frac{-1}{2}$

Product of zeroes(αβ)= $\frac{Constant\ term}{Coefficient\ of\ x^2}$

Let α, β, γ be the zeroes of a cubic polynomial

$ax^3 + bx^2 + cx + d$

$\alpha + \beta + \gamma = \frac{-Coefficient\ of\ x^2}{Coefficient\ of\ x^3} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{Coeffiecient\ of\ x}{Coefficient\ of\ x^3} \frac{c}{a}$

$\alpha\beta\gamma = \frac{Constant\ Term}{Coefficient\ of\ x^3} = \frac{-d}{a}$

x^3 −(sum of zeroes)x^2 + (sum of the product of zeroes taking two at a time)x – product of zeroes

$x^3 +(\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

Example: If two zeroes of the polynomial,
$f(x) = x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.

Letα and β be the two roots of the polynomial,

$$x^3 - 4x^2 - 3x + 12$$

Then $\alpha = \sqrt{3}$ and $\beta = -\sqrt{3}$,

$\alpha + \beta + \gamma = \frac{-Coefficient\ of\ x^2}{Coefficient\ of\ x^3} = \frac{-b}{a}$

$\sqrt{3} - \sqrt{3} + y = \frac{-(4)}{1} = 4$

$\gamma = 4$

Therefore the third zero is 4.
Example: If zeroes of the polynomial,
$f(x) = x^3 - 3x^2 + x + 1$ are $a - b$, a and $a + b$, then find a and b.

Now, a − b, a, and a + b are zeroes of the cubic polynomial

Let $\alpha = a - b$, $\beta = a$ and $\gamma = a + b$

We know that, $\alpha + \beta + \gamma = \frac{-Coefficient\ of\ x^2}{Coefficient\ of\ x^3}$

$a - b + a + a + b = \frac{-(3)}{1} = 3$

$3a = 3$, $a = 1$

$\alpha\beta\gamma = \frac{Constant\ Term}{Coefficient\ of\ x^3}$

$(a - b)a(a + b) = \frac{-(1)}{1} = 1$

$a(a^2 - b^2) = -1$

Putting a = 1 in the above equation

$1(1 - b^2) = -1$

$(1 - b^2) = -1$
$b^2 = 2$
$b = \pm\sqrt{2}$

Division algorithm of Polynomials

Division Algorithm for Polynomials

If p(x) and g(x) are any two polynomials with $g(x) \neq 0$, then we can find the polynomial q(x) and r(x) such that,

$$p(x) = g(x) \times q(x) + r(x)$$

where(x) = 0 or degree of r(x)

This result is known as the Division Algorithm for polynomials.

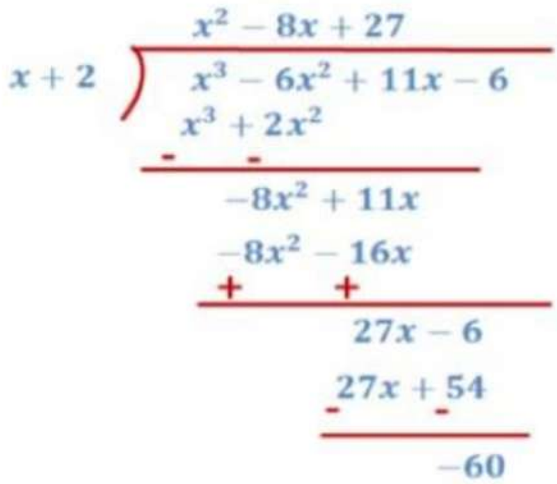
p(x) → Dividend
g(x) → Divisor
q(x) → Quotient
r(x) → Remainder

Thus , Dividend = Divisor × Quotient + Remainder

First, we will study the method of dividing one polynomial by another with the help of an example.

Divide $x^3 - 6x^2 + 11x - 6$ by $x + 2$

i) Firstly, arrange the terms of the dividend and the divisor in descending order of the degrees (also known as writing the polynomial in standard form). In this case, the dividend and divisor are in the standard form.



ii) To obtain the first term of the quotient, we will divide the highest degree term of the dividend, x^3 by the highest degree term of the divisor, x. We get the first term of the quotient, x and carry out the division process. What remains is $-8x^2 + 11x$.

iii) To obtain the second term of the quotient divide the highest degree term of the new dividend ($-8x^2$) by the highest degree term of the divisor, x. We get $-8x$ and what remains is $27x - 6$. As the degree of $27x - 6$ is equal to the degree of the divisor, $(x + 2)$ We will continue the division process.

iv) The new dividend, $27x - 6$ is now divided by $(x + 2)$, the third term of the quotient will be 27 and the remainder is -60. As the degree of reminder is less than the divisor, we stop our division process.

Dividend $p(x) = x^3 - 6x^2 + 11x - 6$

Divisor $g(x) = x + 2$

Quotient $q(x) = x^2 - 8x + 27$

Remainder $r(x) = 60$

$$p(x) = g(x) \times q(x) + r(x)$$

Example: On dividing $x^3 - 3x^2 + 5x - 3$ by a polynomial $g(x)$, the quotient and remainder will be $x^2 + x - 3$ and 8 respectively.

Find $g(x)$.

Let Dividend $p(x) = x^3 - 3x^2 + 5x - 3$

Quotient $q(x) = x - 3$

Remainder $r(x) = 7x - 9$

We have to $g(x)$.

By using division Algorithm, we have

$$p(x) = g(x) \times q(x) + r(x)$$

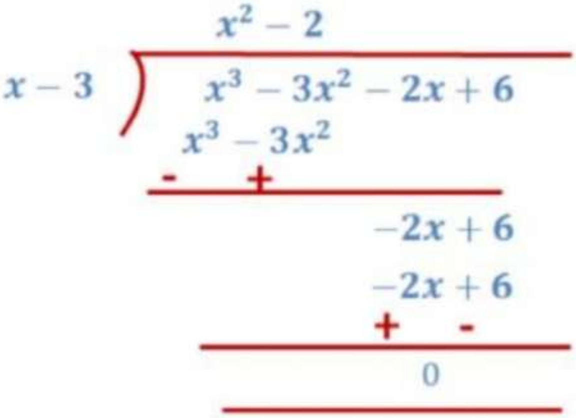
Putting the values of $p(x)$, $q(x)$ and $g(x)$ we get,

$x^3 - 3x^2 + 5x - 3 = g(x) \times x - 3 + 7x - 9$

$x^3 - 3x^2 + 5x - 3 - 7x + 9 = g(x) \times x - 3$

$x^3 - 3x^2 + 5x - 7x - 3 + 9 = g(x) \times x - 3$
$x^3 - 3x^2 - 2x + 6 = g(x) \times x^2 + x - 3$

$$g(x) = \frac{x^3 - 3x^2 - 2x + 6}{x - 3}$$



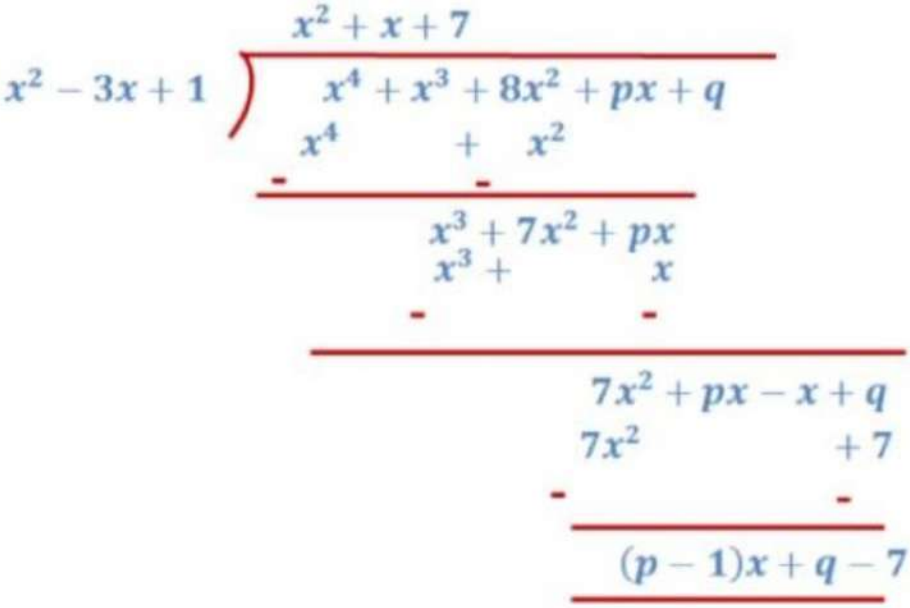
On dividing $x^3 - 3x^2 + 5x - 3$ by $x - 3$ we get,

Quotient $q(x) = x^2 - 2$
Example: If a polynomial, $x^4 + x^3 + 8x^2 + px + q$, is exactly divisible by $x^2 + 1$, then find the value of p and q.

Here, $p(x) = x^4 + x^3 + 8x^2 + px + q$

$$g(x) = x^2 + 1$$

On dividing p(x) by g(x) we get,



As $x^4 + x^3 + 8x^2 + px + q$ is exactly divisible by $x^2 + 1$, the remainder should be equal to zero.

$(p - 1)x + q - 7 = 0$
$(p - 1)x + q - 7 = 0x + 0$

Comparing the coefficient of x and constant term, we get

$(p - 1) = 0$
$p = 1$
$q - 7 = 0$
$q = 7$

$\therefore p = 1$ and $q = 7$