Chapter 7
Ratio and Proportion

Exercise 7.1

1. An alloy consists of $27\frac{1}{2}$ kg of copper and $2\frac{3}{4}$ kg of tin. Find the ratio by weight of tin to the alloy.

Solution:

It is give that

Copper = $27\frac{1}{2}$ kg = $\frac{55}{2}$kg

Tin = $2\frac{3}{4}$ kg = $\frac{11}{4}$ kg

We know that

Total alloy = $\frac{55}{2} + \frac{11}{4}$

Taking LCM

= $\frac{(110+11)}{4}$

= $\frac{121}{4}$ kg

Here

Ratio between tin and alloy = $\frac{11}{4}$ kg : $\frac{121}{4}$ kg

So we get

= 11 : 121

= 1 : 11

2. Find the compounded ratio of :

(i) 2 : 3 and 4 : 9

(ii) 4 : 5, 5 : 7 and 9 : 11

(iii) $(a-b):(a+b), (a+b)^2 : (a^2+b^2)$ and $(a^4-b^4):(a^2-b^2)^2$

Solution:

(i) 2 : 3 and 4 : 9

We know that

Compound ratio = $\frac{2}{3} \times \frac{4}{9}$

$= \frac{8}{27}$

(ii) 4 : 5, 5 : 7 and 9 : 11

We know that

Compound ratio = $\frac{4}{5} \times \frac{5}{7} \times \frac{9}{11}$

$= \frac{36}{77}$

= 36 : 77

(iii) $(a-b):(a+b),(a+b)^2 : (a^2+b^2)$ and $(a^4-b^4):(a^2-b^2)^2$

We know that

Compound ratio =

$(a-b)/(a+b),(a+b)^2 / (a^2+b^2)$ and $(a^4-b^4)/ (a^2-b^2)^2$

By further calculation

= $(a-b)/(a+b) \times [(a+b)(a+b)]/(a^2+b^2) \times [(a^2+b^2)(a+b)(a-b)]/[(a+b)^2 (a-b)^2]$

So we get

= $\frac{1}{1}$

= 1 : 1

3. Find the duplicate ratio of

(i) 2 : 3

(ii) $\sqrt{5}$: 7

(iii) $5a : 6b$

Solution:

(i) 2 : 3

We know that

Duplicate ratio of 2 : 3 = $2^2 : 3^2 = 4 : 9$

(ii) $\sqrt{5}$: 7

We know that

Duplicate ratio of $\sqrt{5}$: 7 = $\sqrt{5^2}$: 7^2 = 5 : 49

(iii) $5a : 6b$

We know that

Duplicate ratio of $5a : 6b$ = $(5a)^2 : (6b)^2 = 25a^2 : 36b^2$

4. Find the triplicate ratio of

(i) 3 : 4

(ii) $\frac{1}{2} : \frac{1}{3}$

(iii) 13 : 23

Solution:

(i) 3 : 4

We know that

Triplicate ratio of 3 : 4 = $3^3 : 4^3$ = 27 : 64

(ii) $\frac{1}{2} : \frac{1}{3}$

We know that

Triplicate ratio of $\frac{1}{2} : \frac{1}{3} = \left(\frac{1}{2}\right)^2 : \left(\frac{1}{3}\right)^2 = \frac{1}{8} : \frac{1}{27}$ = 27 : 8

(iii) $1^3 : 2^3$

We know that

Triplicate ratio of $1^3 : 2^3 = (1^3)^3 : (2^3)^3 = 1^3 : 8^3 = 1 : 512$

5. Find the sub-duplicate ratio of

(i) 9 : 16

(ii) $\frac{1}{4} : \frac{1}{9}$

(iii) $9a^2 : 49b^2$

(iii) $9a^2 : 49b^2$

Solution:

(i) 9: 16

We know that

Sub-duplicate ratio of 9: 16 = $\sqrt{9} : \sqrt{16}$ = 3 : 4

(ii) $\frac{1}{4} : \frac{1}{9}$

We know that

Sub-duplicate ratio of $\frac{1}{4} : \frac{1}{9} = \sqrt{\frac{1}{4}} : \sqrt{\frac{1}{9}}$

So we get

= $\frac{1}{2} : \frac{1}{3}$

= 3 : 2

(iii) $9a^2 : 49b^2$

We know that

Sub-duplicate ratio of $9a^2 : 49b^2$

$= \sqrt{9a^2} : \sqrt{49b^2}$

$= 3a : 7b$

6. Find the sub-triplicate ratio of

(i) 1 : 216

(ii) $\frac{1}{8} : \frac{1}{125}$

(iii) $27a^3 : 64b^3$

Solution:

(i) 1 : 216

We know that

Sub-triplicate ratio of 1 : 216 = $\sqrt[3]{1} : \sqrt[3]{216}$

By further calculation

$= (1^3)^{1/3} : (6^3)^{1/3}$

= 1 : 6

(ii) $\frac{1}{8} : \frac{1}{125}$

We know that

Sub-triplicate ratio of $\frac{1}{8} : \frac{1}{125}$

$= \left(\frac{1}{8}\right)^{1/3} : \left(\frac{1}{125}\right)^{1/3}$

it can be written as

$= \left[\left(\frac{1}{2}\right)^3\right]^{1/3} : \left[\left(\frac{1}{5}\right)^3\right]^{1/3}$

So we get

$= \frac{1}{2} : \frac{1}{5}$

= 5 : 2

(iii) $27a^3 : 64b^3$

We know that

Sub-triplicate ratio of $27a^3 : 64b^3$ = $[(3a)^3]^{1/3} : [(4b\ )^3]^{1/3}$

So we get

= 3a : 4b

7. Find the reciprocal ratio of

(i) 4 : 7

(ii) $3^2 : 4^2$

(iii) $\frac{1}{9} : 2$

Solution:

(i) 4 : 7

We know that

Reciprocal ratio of 4 : 7 = 7 : 4

(ii) $3^2 : 4^2$

We know that

Reciprocal ratio of $3^2 : 4^2 = 4^2 : 3^2$ = 16 : 9

(iii) $\frac{1}{9}$: 2

We know that

Reciprocal ratio of $\frac{1}{9}$: 2 = 2 : $\frac{1}{9}$ = 18 : 1

8. Arrange the following ratios in ascending order of magnitude ;

2 : 3, 17 : 21, 11 : 14 and 5 : 7

Solution:

It is given that

2 : 3, 17 : 21, 11 : 14 and 5 : 7

We can write it in fractions as

$\frac{2}{3}, \frac{17}{21}, \frac{11}{14}, \frac{5}{7}$

Here the LCM of 3, 21, 14 and 7 is 42

By converting the ratio as equivalent

$\frac{2}{3} = \frac{(2\times14)}{(3\times14)} = \frac{28}{42}$

$\frac{17}{21} = \frac{(17\times2)}{(21\times2)} = \frac{34}{42}$

$\frac{11}{14} = \frac{(11\times3)}{(14\times3)} = \frac{33}{42}$

$\frac{5}{7} = \frac{(5\times6)}{(7\times6)} = \frac{30}{42}$

Now writing it in ascending order

$\frac{28}{42}, \frac{30}{42}, \frac{33}{42}, \frac{34}{42}$

By further simplification

$\frac{2}{3}, \frac{5}{7}, \frac{11}{14}, \frac{17}{21}$

So we get

2 : 3, 5 : 7, 11: 14 and 17 : 21

9. (i) If A : B = 2 : 3, B : C = 4 : 5 and C : D = 6 :7, find A : D.

(ii) If $x : y = 2 : 3$ and y : z = 4 : 7, find x : y : z.

Solution;

(i) It is given that

A : B = 2 : 3, B : C = 4 : 5 and C : D = 6 :7

We can write it as

$\frac{A}{B} = \frac{2}{3}, \frac{B}{C} = \frac{4}{5}, \frac{C}{D} = \frac{6}{7}$

By multiplication

$\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7}$

So we get

$\frac{A}{D} = \frac{16}{35}$

A : D = 16 : 35

(ii) We know that the LCM of y terms 3 and 4 is 12

Now making equals of y as 12

$\frac{x}{y} = \frac{2}{3} = \frac{(2 \times 4)}{(3 \times 4)} = \frac{8}{12} =$ 8 : 12

$\frac{y}{z} = \frac{4}{7} \times \frac{3}{3} = \frac{12}{21} =$ 12 : 21

So, x : y : z = 8 : 12 : 21

10. (i) If A : B = $\frac{1}{4}$: $\frac{1}{5}$ and B : C = $\frac{1}{7}$: $\frac{1}{6}$, find A: B : C.

(ii) If 3A = 4B = 6C, find A : B : C

Solution:

(i) We know that

A : B = $\frac{1}{4} : \frac{1}{5} = \frac{5}{4}$

B : C = $\frac{1}{7} \times \frac{6}{1} = \frac{6}{7}$

Here the LCM of B terms 4 and 6 is 12

Now making terms of B as 12

$\frac{A}{B} = \frac{(5 \times 3)}{(4 \times 3)} = \frac{15}{12} =$ 15 : 12

$\frac{B}{C} = \frac{(6\times2)}{(7\times2)} = \frac{12}{14} = 12 : 14$

So A : B : C = 15 : 12 : 14

(ii) It is given that

3A = 4B

We can write it as

$\frac{A}{B} = \frac{4}{3}$

Similarly, 4B = 6C

We can write it as

$\frac{B}{C} = \frac{6}{4} = \frac{3}{2}$

B : C = 3 : 2

So we get

A : B : C = 4 : 3 : 2

11. (i) If $\frac{3x+5y}{3x-5y} = \frac{7}{3}$, Find $x : y$.

(ii) If $a : b = 3 : 11$, find $(15a - 3b) : (9a + 5b)$.

Solution:

(i) $\frac{3x+5y}{3x-5y} = \frac{7}{3}$

By cross multiplication

$9x + 15y = 21x - 35y$

By further simplification

$21x - 9x = 15y + 35y$

$12x = 50y$

So we get

$\frac{x}{y} = \frac{50}{12} = \frac{25}{6}$

Therefore, $x : y = 25 : 6$

(ii) It is given that

$a : b = 3 : 11$

$\frac{a}{b} = \frac{3}{11}$

It is given that

$\frac{(15a-3b)}{(9a+5b)}$

Now dividing both numerator and denominator by b

$= \frac{\left[\frac{15a}{b} - \frac{3b}{b}\right]}{\left[\frac{9a}{b} + \frac{5b}{b}\right]}$

By further calculation

$= \frac{\left[\frac{15a}{b} - 3\right]}{\left[\frac{9a}{b} + 5\right]}$

Substituting the value of $\frac{a}{b}$

$= \frac{\left[15 \times \frac{3}{11} - 3\right]}{\left[9 \times \frac{3}{11} + 5\right]}$

So we get

$= \frac{\left[\frac{45}{11} - 3\right]}{\left[\frac{27}{11} + 5\right]}$

Taking LCM

$= \frac{[(45-33)/11]}{[(27+55)/11]}$

$= \frac{12/11}{82/11}$

We can write it as

$= \frac{12}{11} \times \frac{11}{82}$

$= \frac{12}{82}$

$= \frac{6}{41}$

Hence, $(15a - 3b):(9a + 5b) = 6:41.$

12. (i) If $(4x^2 + xy):(3xy - y^2) = 12 : 5$, find $(x + 2y):(2x + y)$.

(ii) If $y(3x - y):x(4x + y) = 5:12.$ $find$ $(x^2 + y^2):(x + y)^2$.

Solution:

(i) $(4x^2 + xy):(3xy - y^2) = 12 : 5$

We can write it as

$\frac{(4x^2+xy)}{(3xy-y^2)} = \frac{12}{5}$

By cross multiplication

$20x^2 + 5xy = 36xy - 12y^2$

$20x^2 + 5xy - 36xy + 12y^2 = 0$

$20x^2 - 31xy + 12y^2 = 0$

Now divide the entire equation by y^2

$\frac{20x^2}{y^2} - \frac{31xy}{y^2} + \frac{12y^2}{y^2} = 0$

So we get

$20\left(\frac{x}{y}\right)^2 - 31\left(\frac{x}{y}\right) + 12 = 0$

$20\left(\frac{x}{y}\right)^2 - 15\left(\frac{x}{y}\right) - 16\left(\frac{x}{y}\right) + 12 = 0$

Taking common terms

$5\left(\frac{x}{y}\right)\left[4\left(\frac{x}{y}\right) - 3\right] - 4\left[4\left(\frac{x}{y}\right) - 3\right] = 0$

$\left[4\left(\frac{x}{y}\right) - 3\right]\left[5\left(\frac{x}{y}\right) - 4\right] = 0$

Here $4\left(\frac{x}{y}\right) - 3 = 0$

$4\left(\frac{x}{y}\right) = 3$

so we get $\frac{x}{y} = \frac{3}{4}$

Similarly $5\left(\frac{x}{y}\right) - 4 = 0$

$5\left(\frac{x}{y}\right) = 4$

So we get $\frac{x}{y} = \frac{4}{5}$

Now dividing by y

$$\frac{(x+2y)}{(2x+y)} = \frac{\left(\frac{x}{y}+2\right)}{\left(2\frac{x}{y}+1\right)}$$

(a) If $\frac{x}{y} = \frac{3}{4}$, then

$$= \frac{\left(\frac{x}{y}+2\right)}{\left(\frac{2x}{y}+1\right)}$$

Substituting the values

$$= \frac{\left(\frac{3}{4}+2\right)}{\left(\frac{2\times3}{4}+1\right)}$$

By further calculation

$$= \frac{\left(\frac{11}{4}\right)}{\left(\frac{3}{2}+1\right)}$$

$$= \frac{\frac{11}{4}}{\frac{5}{2}}$$

$$= \frac{11}{4} \times \frac{2}{5}$$

$$= \frac{11}{10}$$

So we get

$(x + 2y) : (2x + y) = 11 : 10$

(b) If $\frac{x}{y} = \frac{4}{5}$ then

$$\frac{(x+2y)}{(2x+y)} = \frac{\left[\frac{x}{y}+2\right]}{\left[\frac{2x}{y}+1\right]}$$

Substituting the value of $\frac{x}{y}$

$$= \frac{\left[\frac{4}{5}+2\right]}{\left[2 \times \frac{4}{5}+1\right]}$$

So we get

$$= \frac{\left[\frac{14}{5}\right]}{\left[\frac{8}{5}+1\right]}$$

$$= \frac{14}{5} \times \frac{13}{5}$$

$$= \frac{14}{5} \times \frac{5}{13}$$

$$= \frac{14}{13}$$

We get

$$\frac{(x+2y)}{(2x+y)} = \frac{11}{10} \; or \; \frac{14}{13}$$

$$(x + 2y) : (2x + y) = 11 : 10 \; or \; 14 : 13$$

(ii) $y(3x - y) : x(4x + y) = 5 : 12$

It can be written as

$$\frac{(3xy - y^2)}{(4x^2 + xy)} = \frac{5}{12}$$

By cross multiplication

$$36xy - 12y^2 = 20x^2 + 5xy$$

$$20x^2 + 5xy - 36xy + 12y^2 = 0$$

$$20x^2 - 31xy + 12y^2 = 0$$

Divide the entire equation by y^2

$$\frac{20x^2}{y^2} - \frac{31xy}{y^2} + \frac{12y^2}{y^2} = 0$$

$$20\left(\frac{x^2}{y^2}\right) - 31\left(\frac{xy}{y^2}\right) + 12 = 0$$

We can write it as

$$20\left(\frac{x^2}{y^2}\right) - 15\left(\frac{x}{y}\right) - 16\left(\frac{x}{y}\right) + 12 = 0$$

Taking common terms

$$5\left(\frac{x}{y}\right)\left[4\left(\frac{x}{y}\right) - 3\right] - 4\left[4\left(\frac{x}{y}\right) - 3\right] = 0$$

$$\left[4\left(\frac{x}{y}\right) - 3\right]\left[5\left(\frac{x}{y}\right) - 4\right] = 0$$

Here

$$4\left(\frac{x}{y}\right) - 3 = 0$$

So we get

$$4\left(\frac{x}{y}\right) = 3$$

$$\frac{x}{y} = \frac{3}{4}$$

Similarly,

$$5\left(\frac{x}{y}\right) - 4 = 0$$

So we get

$$5\left(\frac{x}{y}\right) = 4$$

$$\frac{x}{y} = \frac{4}{5}$$

(a) $\frac{x}{y} = \frac{3}{4}$

We know that

$$(x^2 + y^2) : (x + y)^2 = \frac{(x^2+y^2)}{(x+y)^2}$$

Dividing both numerator and denominator by y^2

$$= \frac{\left(\frac{x^2}{y^2}+\frac{y^2}{y^2}\right)}{\left[\frac{1}{y^2}(x+y)^2\right]}$$

$$= \left(\frac{x^2}{y^2} + 1\right)\left(\frac{x}{y} + 1\right)^2$$

Substituting the value of $\frac{x}{y}$

$$= \frac{\left[\left(\frac{3}{4}\right)^2+1\right]}{\left[\frac{3}{4}+1\right]^2}$$

By further calculation

$$= \frac{\left(\frac{9}{16}+1\right)}{\left(\frac{7}{4}\right)^2}$$

So we get

$$= \frac{25/16}{49/16}$$

$= \frac{25}{16} \times \frac{16}{49}$

$= \frac{25}{49}$

So we get

$(x^2 + y^2): (x + y)^2 = 25 : 49$

(b) $\frac{x}{y} = \frac{4}{5}$

We know that

$(x^2 + y^2): (x + y)^2 = \frac{(x^2+y^2)}{(x+y)^2}$

Dividing both numerator and denominator by y^2

$= \frac{\left(\frac{x^2}{y^2} + \frac{y^2}{y^2}\right)}{\left[\frac{1}{y^2}(x+y)^2\right]}$

$= \left(\frac{x^2}{y^2} + 1\right) / \left(\frac{x}{y} + 1\right)^2$

Substituting the value of $\frac{x}{y}$

$= \left[\left(\frac{x^2}{y^2}\right) + 1\right] / \left(\frac{4}{5} + 1\right)^2$

By further calculation

$= \frac{\left(\frac{16}{25}+1\right)}{\left(\frac{9}{5}\right)^2}$

So we get

$= \frac{\frac{41}{25}}{\frac{81}{25}}$

$= \frac{41}{25} \times \frac{25}{81}$

$= \frac{41}{81}$

So we get

$(x^2 + y^2): (x + y)^2 = 41 : 81$

13. (i) If $(x - 9): (3x + 6)$ is the duplicate ratio of 4 : 9, find the value of x.

(ii) If $(3x + 1) : (2x - y)$is equal to the duplicate ratio of 3: 2, find $x : y$.

Solution:

(i) $\frac{(x-9)}{(3x+6)} = \left(\frac{4}{9}\right)^2$

So we get

$\frac{x-9}{3x+6} = \frac{16}{81}$

By cross multiplication

$81x - 729 = 48\,x + 96$

So we get

$33x = 825$

$x = \frac{825}{33} = 25$

(ii) $\frac{(3x+1)}{(5x+3)} = \frac{3^3}{4^3}$

So we get

$\frac{(3x+1)}{(5x+3)} = \frac{27}{64}$

By cross multiplication

$64\ (\ 3x + 1) = 27\ (5x + 3)$

$192\ x + 64 = 135\ x + 81$

$192x - 135x = 81 - 64$

$57x = 17$

So we get

$x = \frac{17}{57}$

(iii) $\frac{(x+2y)}{(2x-y)} = \frac{3^2}{2^2}$

So we get

$\frac{(x+2y)}{(2x-y)} = \frac{9}{4}$

By cross multiplication

$9(2x - y) = 4(x + 2y)$

$18x - 9y\ = 4x + 8y$

$18x = 4x = 8y + 9y$

So we get

$14x = 17y$

$\frac{x}{y} = \frac{17}{14}$

$x : y = 17 : 14$

14. (i) Find two numbers in the ratio of 8 : 7 such that when each isdecreased by $12\frac{1}{2}$, they are in the ratio 11 : 9.

(ii) The income of a man is increased in the ratio of 10 : 11. If the increase in his income is Rs 600 per month, find his new income.

Solution:

(i) Ratio = 8 : 7

Consider the numbers as $8x \ and \ 7x$

Using the condition

$$\frac{\left[8x-\frac{25}{2}\right]}{\left[7x-\frac{25}{2}\right]} = \frac{11}{9}$$

Taking LCM

$$\frac{\left[\frac{(16x-25)}{2}\right]}{\left[\frac{(14x-25)}{2}\right]} = \frac{11}{9}$$

By further calculation

$$\frac{[(16x-25)\times 2]}{[2\times (14x-25)]} = \frac{11}{9}$$

$$\frac{[(16x-25)]}{[\,(14x-25)]} = \frac{11}{9}$$

By cross multiplication

$154x - 275 = 144x - 225$

$154x - 144x = 275 - 225$

$10x = 50$

$x = \frac{50}{10} = 5$

So the numbers are

$8x = 8 \times 5 = 40$

$7x = 7 \times 5 = 35$

(ii) Consider the present income = *10x*

Increased income = $11x$

So the increase per month = $11x - 10x = x$

Here $x = Rs\ 600$

New income = $11x = 11 \times 600 = Rs\ 6600$

15. (i) A woman reduces her weight in the ratio 7 : 5. What does her weight become if originally it was 91 kg.

(ii) A school collected Rs 2100 for charity. It was decided to divide the money between an orphanage and a bline school in the ratio of 3 : 4. How much money did each receive ?

Solution:

(i) Ratio of original and reduced weight of woman = 7 : 5

Consider original weight = $7x$

Reduced weight = $5x$

Here original weight = 91kg

So the reduced weight = $\frac{(91 \times 5x)}{7x} = 65\ kg$

(ii) Amount collected for charity = Rs 2100

Here the ratio between orphanage and a bline school = 3 : 4

Sum of ratios = 3 + 4 = 7

We know that

Orphange schools share = $2100 \times \frac{3}{7}$ = Rs 900

Bline schools share = $2100 \times \frac{4}{7}$ = Rs 1200

16. (i) The sides of a triangle are in the ratio 7 : 5 :3 and its perimeter is 30 cm. Find the lengths of sides.

(ii) If the angles of a triangle are in the ratio 2: 3: 4, find the angles.

Solution:

(i) It is given that

Perimeter of triangle = 30 cm

Ratio among sides = 7: 5: 3

Here the sum of ratios = 7 + 5 + 3 = 15

We know that

Length of first side = $30 \times \frac{7}{15}$ = 14 cm

Length of seconds side = $30 \times \frac{5}{15}$ = cm

Therefore, the sides are 14cm, 10cm and 6 cm.

(ii) We know that

Sum of all the angles of a triangle = 180°

Here the ratio among angles = 2 : 3 : 4

Sum of ratios = 2 + 3 + 4= 9

So we get

First angle = $180 \times \frac{2}{9} = 40^\circ$

Second angle = $180 \times \frac{3}{9} = 60^\circ$

Third angle = $180 \times \frac{4}{9} = 80^\circ$

Hence, the angles are $40^\circ, 60^\circ$ and 80°.

17. Three numbers are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the sum of their squares is 244, find the numbers.

Solution:

It is given that

Ratio of three numbers = $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

$= \frac{(6:\ 4:3)}{12}$

= 6 : 4 : 3

Consider first number = 6 x

second number = $4x$

Third number = $3x$

So based on the condition

$(6x)^2 + (4x)^2 + (3x)^2 = 144$

$36x^2 + 16x^2 + 9x^2 = 244$

So we get

$61x^2 = 244$

$x^2 = \frac{244}{61} = 4 = 2^2$

$x = 2$

Here

First number = $6x = 6 \times 2 = 12$

Second number = $4x = 4 \times 2 = 8$

Third number = $3x = 3 \times 2 = 6$

18. (i) A certain sum was divided among A, B and C in the ratio 7 : 5 : 4. If B got Rs 500 more than C, find the total sum divided.

(ii) In a business, A invests Rs 50000 for 6 months, B Rs 60000 for 4 months and C Rs 80000 for 5 months. If they together earn Rs 18800 find the share of each.

Solution:

(i) It is given that

Ratio between A, B and C = 7 : 5 : 4

Consider A share = $7x$

B share = $5x$

C share = $4x$

So the total sum = $7x + 5x + 4x = 16x$

Based on the condition

$5x - 4x = 500$

$x = 500$

So the total sum = $16x = 16 \times 500 = Rs\ 8000$

(ii) 6 months investment of A = Rs 50000

1 month investment of A = $50000 \times 6 = Rs\ 300000$

4 months investment of B = Rs 60000

1 month investment of B = $60000 \times 4 = Rs\ 240000$

5 months investment of C = Rs 80000

1 month investment of C = $80000 \times 5 = Rs\ 400000$

Here the ratio between their investment = 300000 : 2400000 : 400000

= 30 : 24 : 40

sum of ratio = 30 = 24 + 40 = 94

Total earnings = Rs 18800

So we get

A share = $\frac{30}{94} \times 18800 = Rs\ 6000$

B share = $\frac{24}{94} \times 18800 = Rs\ 4800$

C share = $\frac{40}{94} = 18800 = Rs\ 8000$

19. (i) in a mixture of 45 litres, the ratio of milk to water is 13 : 2. How much water must be added to this mixture to make the ratio of milk to water as 3 : 1 ?

(ii) the ratio of the number of boys to the numbers of girls in a school of 560 pupils is 5 : 3 If 10 new boys are admitted, find how many new girls may be admitted so that the ratio of the number of boys to the number of girls may change to 3 : 2.

Solution:

(i) It is given that

Mixture of milk to water = 45 litres

Ratio of milk to water = 13 : 2

Sum of ratio = 13 + 2 = 15

Here the quantity of milk = $\frac{(45 \times 13)}{15} = 39\ litres$

Quantity of water = 45 × $\frac{2}{15}$ = 6 litres

Consider x litre of water to be added, then water = $(6 + x)$ litres

Here the new ratio = 3 : 1

39 : $(\ 6 + x) = 3 : 1$

We can write it as

$\frac{39}{(6+x)} = \frac{3}{1}$

By cross multiplication

39 = 18 + $3x$

$3x = 39 - 18 = 21$

$x = \frac{21}{3}$ = 7 litres

Hence, 7 litres of water is to be added to the mixture.

(ii) It is given that

Ratio between boys and girls = 5 : 3

Number of pupils = 560

so the sum of ratios = 5 + 3 = 8

We know that

Number of boys = $\frac{5}{8} \times 560 = 350$

Number of girls = $\frac{3}{8} \times 560 = 210$

Number of new boys admitted = 10

So the total number of boys = 350 + 10 = 360

Consider x as the number of girls admitted

Total number of girls = $210 + x$

Based on the condition

360 : 210 + x = 3 : 2

we can write it as

$\frac{360}{210} + x = \frac{3}{2}$

By cross multiplication

$630 + 3x = 720$

$3x = 720 - 630 = 90$

So we get

$x = \frac{90}{3} = 30$

Hence, 30 new girls are to be admitted.

20. (i) The monthly pocket money of Ravi and Sanjeev are in the ratio 5 : 7. Their expenditures are in the ratio 3 : 5. If each saves Rs 80 per month, find their monthly pocket money.

(ii) In class X of a school, the ratio of the number of boys to that of the girls is 4 :3. If there were 20 more boys and 12 less girls, then the ratio would have been 2 : 1. How many students were there in the class ?

Solution:

(i) Consider the monthly pocket money of Ravi and Sanjeev as 5x and 7x.

Their expenditure is 3y and 5y respectively.

$5x - 3y = 80 \ldots\ldots\ldots\ldots (i)$

$7x - 5y = 80$………...(ii)

Now multiply equation (1) by 7 ans (2) by 5

Subtracting both the equations

$35x - 21y = 560$

$35x - 25y = 400$

So we get

$4y = 160$

$y = 40$

In equation (1)

$5x = 80 + 3 \times 40 = 200$

$x = 40$

Here the monthly pocket money of Ravi = $5 \times 40 = 200$

(ii) Consider x as the number of students in class

Ratio of boys and girls = 4 : 3

Number of boys = $\frac{4x}{7}$

Number of girls = $\frac{3x}{7}$

Based on the problem

$\left(\frac{4x}{7} + 20\right) : \left(\frac{3x}{7} - 12\right) = 2 : 1$

We can write it as

$\frac{(4x+140)}{7} : \frac{(3x-84)}{7} = 2 : 1$

So we get

$\frac{(4x+140)}{7} \times \frac{7}{(3x-84)} = \frac{2}{1}$

$\frac{(4x+140)}{(3x-84)} = \frac{2}{1}$

$6x - 168 = 4x + 140$

$6x - 4x = 140 + 168$

$2x = 308$

$x = \frac{308}{2}$

= 154

Therefore, 154 students were there in the class.

21. In an examination, the ratio of passes to failures was 4 : 1. If 30 less had appeared and 20 less passed, the ratio of passes to failures would have been 5 : 1. How many students appeared for the examination.

Solution:

Consider number of passes = $4x$

Number of failures = x

Total number of students appeared = $4x + x = 5x$

In case 2

Number of students appeared = $5x - 30$

Number of passes = $4x - 20$

So the number of failures = $(5x - 30) - (4x - 20)$

By further calculation

= $5x - 30 - 4x + 20$

= $x - 10$

Based on the condition

$\frac{(4x-20)}{(x-10)} = \frac{5}{1}$

By cross multiplication

$5x - 50 = 4x - 20$

$5x - 4x = -20 + 50$

$x = 30$

Number of students appeared = $5x = 5 \times 30 = 150$

Exercise 7.2

1. Find the value of x in the following proportions :

(i) 10 : 35 = $x : 42$

(ii) $3 : x = 24 : 2$

(iii) $2.5 : 1.5 = x : 3$

(iv) $x : 50$: : 3 : 2

Solution:

(i) 10 : 35 = $x : 42$

We can write it as

$35 \times x = 10 \times 42$

So we get

$x = \frac{(10 \times 42)}{35}$

$x = 2 \times 6$

$x = 12$

(ii) $3 : x = 24 : 2$

We can write it as

$x \times 24 = 3 \times 2$

So we get

$x = \frac{(3 \times 2)}{24}$

$x = \frac{1}{4}$

(iii) $2.5 : 1.5 = x : 3$

We can write it as

$1.5 \times x = 2.5 \times 3$

So we get

$x = \frac{(2.5 \times 3)}{1.5}$

$x = 5.0$

(iv) $x : 50 :: 3 : 2$

We can write it as

$x \times 2 = 50 \times 3$

So we get

$x = \frac{(50 \times 3)}{2}$

$x = 75$

2. Find the fourth proportional to

(i) 3, 12, 15

(ii) $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

(iii) 9.6kg, 7.2 kg, 28.8 kg

Solution:

(i) 3, 12, 15

Consider x as the fourth proportional to 3, 12 and 15

(i) 3, 12, 15

Consider x as the fourth proportional to 3, 12 and 15

3: 12 :: 15: x

We can write it as

$3 \times x = 12 \times 15$

So we get

$x = \frac{(12\times15)}{3}$

$x = 60$

(ii) $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

Consider x as the fourth proportional to $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$

$\frac{1}{3} : \frac{1}{4} :: \frac{1}{5} : x$

We can write it as

$\frac{1}{3} \times x = \frac{1}{4} \times \frac{1}{5}$

So we get

$x = \frac{1}{4} \times \frac{1}{5} \times \frac{3}{1}$

$x = \frac{3}{20}$

(iii) 1.5, 2.5, 4.5

Consider x as the fourth proportional to 1, 5, 2.5 and 4.5

1. 5 : 2. 5 :: 4.5 : x

We can write it as

So we get

$x = \frac{(2.5\times4.5)}{1.5}$

x = 7.5

(iv) 9.6 kg, 7.2 kg, 28.8kg

Consider x as the fourth proportional to 9.6, 7.2 and 28.8

9.6: 7.2 :: 28.8: x

We can write it as

$9.6 \times x = 6.2 = 7.2 = \ 287$

9.6 × x = 7.2 × 28.8

So we get

x = (7.2 × 28.8)/ 9.6

x = 21.6

3. Find the third proportional to

(i) 5, 10

(ii) 0.24, 0.6

(iii) Rs. 3, Rs. 12

(iv) $5\frac{1}{4}$ and 7.

Solution:

(i) Consider x as the third proportional to 5, 10

5 : 10 : : 10 : x

It can be written as

$5 \times x = 10 \times 10$

$x = \frac{(10\times10)}{5}$ = 20

Hence, the third proportional to 5, 10 is 20.

(ii) Consider x as the third proportional to 0.24, 0.6

0.24 : 0.6 : : 0.6 : x

It can be written as

$0.24 \times x = 0.6 \times 0.6$

$x = \frac{(0.6\times0.6)}{0.24}$

= 1.5

Hence, the third proportional to 0.24, 0.6 is 1.5.

(iii) Consider x as the third proportional to Rs. 3 and Rs. 12

3 : 12 : : 12 : x

It can be written as

$3 \times x = 12 \times 12$

$x = \frac{(12\times12)}{3}$ = 48

Hence, the third proportional to Rs. 3 and Rs. 12 is Rs. 48

(iv) Consider x as the third proportional to $5\frac{1}{4}$ and 7

$5\frac{1}{4} : 7 :: 7 : x$

It can be written as

$\frac{21}{4} \times x = 7 \times 7$

$x = \frac{(7 \times 7 \times 4)}{21}$

$= \frac{28}{3}$

$= 9\frac{1}{3}$

Hence, the third proportional to $5\frac{1}{4}$ and 7 is $9\frac{1}{3}$.

4. Find the mean proportion of :

(i) 5 and 80

(ii) $\frac{1}{12}$ and $\frac{1}{75}$

(iii) 8.1 and 2.5

(iv) $(a - b)$ and $(a^3 - a^2 b)$, $a > b$

Solution:

(i) Consider x as the mean proportion of 5 and 80

$5 : x$ :: x : 80

It can be written as

$x^2 = 5 \times 80 = 400$

$x = \sqrt{400}$ = 20

Therefore, mean proportion of 5 and 80 is 20.

(ii) Consider x as the mean proportion of $\frac{1}{12}$ and $\frac{1}{75}$

$\frac{1}{12} : x$ :: $x : \frac{1}{75}$

It can be written as

$x^2 = \frac{1}{12} \times \frac{1}{75} = \frac{1}{900}$

$x = \frac{\sqrt{1}}{900}$ = $\frac{1}{30}$

Therefore, mean proportion of $\frac{1}{12}$ and $\frac{1}{75}$ is $\frac{1}{30}$.

(iii) Consider x as the mean proportion of 8.1 and 2.5

8.1 : x :: x : 2.5

It can be written as

$x^2 = 8.1 \times 2.5 = 20.25$

$x = \sqrt{20.25}$ = 4.5

Therefore, mean proportion of 8.1 and 2.5 is 4.5.

(iv) Consider $\boldsymbol{x}$ as the mean proportion of $(\boldsymbol{a}-\boldsymbol{b})$ and $(\boldsymbol{a}^3 - \boldsymbol{a}^2\boldsymbol{b}), \boldsymbol{a} > \boldsymbol{b}$ (a—b) : x :: $(\boldsymbol{a}^3 - \boldsymbol{a}^2\boldsymbol{b})$

It can be written as

$x^2 = (a-b)(a^3 - a^2b)$

So we get

$x^2 = (a-b)a^2(a^3 - b)$

$x^2 = a^2(a-b)^2$

Here

$x = a(a-b)$

Therefore, mean proportion of $(a-b)$ and $(a^3 - a^2b), a > b$ is a $a(a-b)$.

5. If a, 12, 16 and b are in continues proportion find a and b.

Solution:

It is given that

a, 12, 16 and b are in continued proportion

$\frac{a}{12} = \frac{12}{16} = \frac{16}{b}$

We know that

$\frac{a}{12} = \frac{12}{16}$

By cross multiplication

$16a = 144$

$a = \frac{144}{16}$ =9

Similarly,

$\frac{12}{16} = \frac{16}{b}$

By cross multiplication

$12b = 16 \times 16 = 256$

$b = \frac{256}{12} = \frac{64}{3} = 21\frac{1}{3}$

Therefore , a = 9 and b = $\frac{64}{3}$ or $21\frac{1}{3}$.

6. What number must be added to each of the numbers 5, 11, 19 and 37 so that they are in proportion ?

Solution:

Consider x to be added to 5, 11, 19 and 37 to make them in proportion

$5 + x : 11 + x :: 19 + x : 37 + x$

It can be written as

$(5 + x)(37 + x) = (11 + x)(19 + x)$

By further calculation

$185 + 5x + 37x + x^2 = 209 + 11x + 19x + x^2$

$185 + 42x + x^2 = 209 + 30x + x^2$

So we get

$42x - 30x + x^2 - x^2 = 209 - 185$

$12x = 24$

$x = 2$

Hence, the least number to be added is 2.

7. What number should be substracted from each of the numbers 23, 30, 57 and 78 so that the remainders are in proportion ?

Solution;

Consider *x* be subtracted from each term

$23 - x\ , 30 - x, 57 - x\ and\ 78 - x$ are proportional

It can be written as

$23 - x$: $30 - x$ :: 57 – x : 78 - x

$\frac{(23-x)}{(30-x)} = \frac{(57-x)}{(78-x)}$

By cross multiplication

$(23 - x)(78 - x) = (30 - x)(57 - x)$

By further calculation

$1794 - 23x - 78x + x^2 = 1710 - 30x - 57x + x^2$

$x^2 - 101x + 1794 - x^2 + 87x - 1710 = 0$

So we get

$-14x + 84 = 0$

$14x = 84$

$x = \frac{84}{14}$

= 6

Therefore, 6 is the number to be subtracted from each of the numbers.

8. If $k+3, k+2, 3k-7$ *and* $2k-3$ are in proportion, find k.

Solution:

It is given that

$k+3, k+2, 3k-7$ and 2k – 3 are in proportion

We can write it as

$(k+3)(2k-3) = (k+2)\ (3k-7)$

By further calculation

$2k^2 - 3k + 6k - 9 = 3k^2 - 7k + 6k - 14$

$3k^2 - 7k + 6k - 14 - 2k^2 + 3k - 6k + 9 = 0$

$k^2 - 4k - 5 = 0$

$k^2 - 5k + k - 5 = 0$

$k(k-5) + 1\ (k-5) = 0$

$(k+1)(k-5) = 0$

So,

$k+1 = 0$ or $k-5 = 0$

$k = -1\ or\ k = 5$

Therefor, the value of k is -1, 5.

9. If $x+5$ is the mean proportion between $x+2$ and $x+9$, find the value of x.

Solution :

It is given that

$x + 5$ is the mean proportion between $x + 2$ and $x + 9$

We can write it as

$(x + 5)^2 = (x + 2)(x + 9)$

By further calculation

$x^2 + 10x + 25 = x^2 + 11x + 18$

$x^2 + 10x - x^2 - 11x = 18 - 25$

So we get

$-x = -7$

$x = 7$

Hence, the value of x *is* 7.

10. What number must be added to each of the numbers 16, 26 and 40 so that the resulting numbers may be in continued proportion ?

Solution:

Consider x be added to each number

16 + x, 26 + x and 40 + x are in continues proportion

It can be written as

$\frac{(16+x)}{(26+x)} = \frac{(26+x)}{(40+x)}$

By cross multiplication

$(16 + x)(40 + x) = (26 + x)(26 + x)$

On further calculation

$640 + 16x + 40x + x^2 = 676 + 26x + 26x + x^2$

$640 + 56x + x^2 = 676 + 52x + x^2$

$56x + x^2 - 52x - x^2 = 676 - 640$

So we get

$4x = 36$

$x = \frac{36}{4} = 9$

Hence, 9 is the number to be added to each of the numbers.

11. Find two numbers such that the mean proportional between them is 28 and the third proportional to them is 224.

Solution:

Consider a and b as the two numbers

It is given that 28 is the mean proportional

$a : 28$ :: 28 : b

We get

$ab = 28^2 = 784$

Here a = $\frac{784}{b}$ ……….(1)

We know that 224 is the third proportional

$a : b$ :: b : 224

so we get

$b^2 = 224a$ ……..(2)

Now by substituting the value of a in equation (2)

$b^2 = 224 \times \frac{784}{b}$

So we get

$b^3 = 224 \times 784$

$b^3 = 175616 = 56^3$

$b = 56$

By substituting the value of b in equation (1)

$a = \frac{784}{56}$

= 14

Therefore, 14 and 56 are the two numbers.

12. If b is the mean proportional between a and c, prove that a, c, $a^2 + b^2$ and $b^2 + c^2$ are proportional.

Solution:

It is given that

b is the mean proportional between a and c

We can write it as

$b^2 = a \times c$

$b^2 = ac$ ……(1)

We know that

a, c, $a^2 + b^2$ and $b^2 + c^2$ are in proportion

It can be written as

$\frac{a}{c} = \frac{(a^2+b^2)}{(b^2+c^2)}$

By Cross multiplication

$a(b^2 + c^2) = c(a^2 + b^2)$

Using equation (1)

$a(ac + c^2) = c\,(a^2 + ac)$

So we get

$ac(a + c) = a^2c + ac^2$

Here $ac(a + c) = ac(a + c)$ which is true.

Therefore, it is proved.

13. If b is the mean proportional between a and c, prove that $(ab + bc)$ is the mean proportional between $(a^2 + b^2)$ and $(b^2 + c^2)$.

Solution:

It is given that

b is the mean proportional between a and c

$b^2 = ac$ ………(1)

Here $(ab + bc)$ is the mean proportional between $(a^2 + b^2)$ and $(b^2 + c^2)$

$(ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$

Consider LHS = $(ab + bc)^2$

Expanding using formula

= $a^2b^2 + b^2c^2 + 2ab^2c$

Using equation (1)

= $a^2(ac) + ac\,(c)^2 + 2a.\,ac\,.\,c$

= $a^3c + ac^3 + 2a^2c^2$

Taking ac as common

= $ac\,(a^2 + c^2 + 2ac)$

$= ac(a+c)^2$

RHS = $(a^2+b^2)(b^2+c^2)$

Using equation (1)

$= (a^2+ac)(ac+c^2)$

Taking common terms out

$= a(a+c)c(a+c)$

$= ac(a+c)^2$

Hence, LHS = RHS.

14. If y is mean proportional between x and z, prove that $xyz\ (x+y+z)^3 = (xy+yz+zx)^3$.

Solution:

It is given that

y is mean proportional between x and z

We can write is as

$y^2 = xz$ …….(1)

Consider

LHS = xyz $(x+y+z)^3$

It can be written as

$= xz.y\ (x+y+z)^3$

Using equation (1)

$= y^2.y(x+y+z)^3$

$= y^3.(x+y+z)^3$

So we get

$= [y(x+y+z)]^3$

By further calculation

$= (xy + y^2 + yz)^3$

Using equation (1)

$= (xy+yz+zx)^3$

= RHS

Hence, it is proved.

15. $If\ \boldsymbol{a+c=mb}$ **and** $\boldsymbol{\frac{1}{b}+\frac{1}{d}=\frac{m}{c}}$**, Prove that a, b, c and d are in proportion.**

Solution:

It is given that

$a+c=mb$ and $\boldsymbol{\frac{1}{b}+\frac{1}{d}=\frac{m}{c}}$

$a+c=mb$

Dividing the equation by b

$\frac{a}{b}+\frac{c}{d}=m$ …………(1)

$\frac{1}{b}+\frac{1}{d}=\frac{m}{c}$

Multiplying the equation by c

$\frac{a}{b}+\frac{c}{b} = \frac{c}{b}+\frac{c}{d}$

So we get

$\frac{a}{b} = \frac{c}{d}$

Therfore, it is proved that, a,b, c and d are in proportion.

16. $If\ \frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, prove that

(i) $\frac{x^3}{a^2} + \frac{y^3}{b^2} + \frac{z^3}{c^2} = \frac{(x+y+z)^3}{(a+b+c)^2}$

(ii) $\left[\frac{a^2x^2+b^2y^2+c^2z^2}{a^3x+b^3y+c^3z}\right]^3 = \frac{xyz}{abc}$

(iii) $\frac{ax-by}{(a+b)(x-y)} + \frac{by-cz}{(b+c)(y-z)} + \frac{cz-ax}{(c+a)(z-x)} = 3$

Solution:

It is given that

$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

We can write it as

$x = ak, y = bk$ and z = ck

(i) LHS $= \frac{x^3}{a^2} + \frac{y^3}{b^2} + \frac{z^3}{c^2}$

It can be written as

$= \frac{a^3k^3}{a^2} + \frac{b^3k^3}{b^2} + \frac{c^3k^3}{c^2}$

So we get

$= ak^3 + bk^3 + ck^3$

Taking common terms

$= k^3 (a + b + c)$

$\text{RHS} = \frac{(x+y+z)^3}{(a+b+c)^2}$

It can be written as

$= \frac{(ak+bk+ck)^3}{(a+b+c)^2}$

So we get

$= \frac{k^3 (a+b+c)^3}{(a+b+c)^2}$

$= k^3(a + b + c)$

Therefore, LHS = RHS.

(ii) $\text{LHS} = \left[\frac{a^2x^2+b^2y^2+c^2z^2}{a^3x+b^3y+c^3z}\right]^3$

It can be written as

$= \left[\frac{a^2a^2k^2+b^2b^2k^2+c^2c^2k^2}{a^3.ak+b^3.bk+c^3.ck}\right]^3$

By further calculation

$= \left[\frac{a^4k^2+b^4k^2+c^4k^2}{a^4k+b^4k+c^4k}\right]^3$

So we get

$= \left[\frac{k^2(a^4+b^4+c^4)}{k(a^4+b^4+c^4)}\right]^3$

$= k^3$

$RHS = \frac{xyz}{abc}$

We can write it as

$= \frac{ak.bk.ck}{abc}$

$= k^3$

Therefore, LHS = RHS.

(iii) LHS $= \frac{ax-by}{(a+b)(x-y)} + \frac{by-cz}{(b+c)(y-z)} + \frac{cz-ax}{(c+a)(z-x)}$

It can be written as

$= \frac{ax-by}{(a+b)(ak-bk)} + \frac{by-cz}{(b+c)(bk-ck)} + \frac{cz-ax}{(c+a)(ck-ak)}$

By further calculation

$= \frac{a^2k-b^2k}{(a+b)k(a-b)} + \frac{b^2k-c^2k}{(b+c)k(b-c)} + \frac{c^2k-a^2k}{(c+a)k(c-a)}$

Taking common terms

$= \frac{k(a^2-b^2)}{k(a^2-b^2)} + \frac{k(b^2-c^2)}{k(b^2-c^2)} + \frac{k(c^2-a^2)}{k(c^2-a^2)}$

So we get

$= 1 + 1 + 1$

= 3

$= RHS$

Therefore, LHS = RHS.

17. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ prove that:

(i) $(b^2 + d^2 + f^2)(a^2 + c^2 + e^2) = (ab + cd + ef)^2$

(ii) $\frac{(a^3+c^3)^2}{(b^3+d^3)^2} = \frac{e^6}{f^6}$

(iii) $\frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2} = \frac{ac}{bd} + \frac{ce}{df} + \frac{ae}{df}$

(iv) bdf $\left[\frac{a+b}{b} + \frac{c+d}{d} + \frac{c+f}{f}\right]^3 = 27(a + b)(c + d)(e + f)$

Solution:

Consider

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

So we get

$a = bk, c = dk, e = fk$

(i) LHS $= (b^2 + d^2 + f^2)(a^2 + c^2 + e^2)$

We can write it as

$= (b^2 + d^2 + f^2)(b^2k^2 + d^2k^2 + f^2k^2)$

Taking out the common terms

$= (b^2 + d^2 + f^2)k^2(b^2 + d^2 + f^2)$

So we get

$= k^2(b^2 + d^2 + f^2)$

RHS $= (ab + cd + ef)^2$

We can write it as

$= (b.kb + dk.d + fk.f)^2$

So we get

$= (kb^2 + kd^2 + kf^2)$

Taking out common terms

$= k^2\ (b^2 + d^2 + f^2)^2$

Therefore, LHS = RHS.

(ii) LHS $= \frac{(a^3+c^3)^2}{(b^3+d^3)^2}$

It can be written as

$= \frac{(b^3k^3+d^3k^3)^2}{(b^3+d^3)^2}$

Taking out the common terms

$= \frac{[k^3(b^3+d^3)]^2}{(b^3+d^3)^2}$

So we get

$= \frac{k^6(b^3+d^3)^2}{(b^3+d^3)^2}$

$= k^6$

$RHS = \frac{e^6}{f^6}$

We get

$= \frac{f^6k^6}{f^6}$

$= k^6$

Therefore, LHS = RHS.

(iii) $LHS = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

It can be written as

$= \frac{\mathrm{b^2k^2}}{\mathrm{b^2}} + \frac{\mathrm{d^2k^2}}{\mathrm{d^2}} + \frac{\mathrm{f^2k^2}}{\mathrm{f^2}}$

So we get

$= \mathrm{k^2 + k^2 + k^2}$

$= \mathrm{3k^2}$

$RHS = \frac{ac}{bd} + \frac{ce}{df} + \frac{ae}{bf}$

It can be written as

$= \left[\frac{bk.dk}{bd} + \frac{dk.fk}{df} + \frac{bk.fk}{bf}\right]$

So we get

$= k^2 + k^2 + k^2$

$= 3k^2$

Therefore, LHS = RHS.

(iv) $LHS = bdf\left[\frac{a+b}{b} + \frac{c+d}{d} + \frac{e+f}{f}\right]^3$

It can be written as

$= bdf\left[\frac{bk+b}{b} + \frac{dk+d}{d} + \frac{fk+f}{f}\right]^3$

Taking out the common terms

$= bdf\ \left[\frac{b(k+1)}{b} + \frac{d(k+1)}{d} + \frac{f(k+1)}{f}\right]^3$

So we get

$= \mathrm{bdf}\ (\mathrm{k} + 1 + \mathrm{k} + 1 + \mathrm{k} + 1)^3$

By further calculation

$= \mathrm{bdf}\ (3\mathrm{k} + 3)^3$

$= 27\ \mathrm{bdf}\ (\mathrm{k} + 1)^3$

RHS = 27 $(\mathrm{a} + \mathrm{b})(\mathrm{c} + \mathrm{d})(\mathrm{e} + \mathrm{f})$

It can be written as

$= 27\ (\mathrm{bk} + \mathrm{b})(\mathrm{dk} + \mathrm{d})(\mathrm{fk} + \mathrm{f})$

Taking out the common terms

$= 27\ \mathrm{b}\ (\mathrm{k} + 1)\ \mathrm{d}\ (\mathrm{k} + 1)\ \mathrm{f}\ (\mathrm{k} + 1)$

So we get

$= 27\ \mathrm{bdf}\ (\mathrm{k} + 1)^3$

Therefore, LHS = RHS.

18. If ax = by = cz; prove that

$$\frac{\mathbf{x^2}}{\mathbf{yz}} + \frac{\mathbf{y^2}}{\mathbf{zx}} + \frac{\mathbf{z^2}}{\mathbf{xy}} = \frac{\mathbf{bc}}{\mathbf{a^2}} + \frac{\mathbf{ca}}{\mathbf{b^2}} + \frac{\mathbf{ab}}{\mathbf{c^2}}$$

Solution:

Consider ax = by = cz = k

It can be written as

$\mathrm{x} = \frac{\mathrm{k}}{\mathrm{a}}, \mathrm{y} = \frac{\mathrm{k}}{\mathrm{b}}, \mathrm{z} = \frac{\mathrm{k}}{\mathrm{c}}$

$LHS = \frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy}$

Substituting the values

$$= \frac{\frac{k^2}{a^2}}{\frac{k}{b} \cdot \frac{k}{c}} + \frac{\frac{k^2}{b^2}}{\frac{k}{c} \cdot \frac{k}{a}} + \frac{\frac{k^2}{c^2}}{\frac{k}{a} \cdot \frac{k}{b}}$$

By further calculation

$$= \frac{\frac{k^2}{a^2}}{\frac{k^2}{bc}} + \frac{\frac{k^2}{b^2}}{\frac{k^2}{ca}} + \frac{\frac{k^2}{c^2}}{\frac{k^2}{ab}}$$

It can be written as

$$= \frac{k^2}{a^2} \times \frac{bc}{k^2} + \frac{k^2}{b^2} \times \frac{ca}{k^2} + \frac{k^2}{c^2} \times \frac{ab}{k^2}$$

So we get

$$= \frac{bc}{a^2} + \frac{ca}{b^2} + \frac{ab}{c^2}$$

$= RHS$

19. If a, b, c and d are in proportion, prove that:

(i) $\mathbf{(5a + 7v)(2c - 3d) = (5c + 7d)(2a - 3b)}$

(ii) $\mathbf{(ma + nb) : b = (mc + nd) : d}$

(iii) $\mathbf{(a^4 + c^4) : (b^4 + d^4) = a^2c^2 : b^2d^2}$

(iv) $\mathbf{\frac{a^2+ab}{c^2+cd} = \frac{b^2-2ab}{d^2-2cd}}$

(v) $\mathbf{\frac{(a+c)^3}{(b+d)^3} = \frac{a(a-c)^2}{b(b-d)^2}}$

(vi) $\mathbf{\frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{c^2+cd+d^2}{c^2-cd+d^2}}$

(vii) $\frac{a^2+b^2}{c^2+d^2} = \frac{ab+ad-bc}{bc+cd-ad}$

(viii) $abcd\left[\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{d^2}\right] = a^2+b^2+c^2+d^2$

Solution:

It is given that

a, b, c, d are in proportion

Consider $\frac{a}{b} = \frac{c}{d} = k$

$a = b, c = dk$

(i) LHS = $(5a + 7b)(2c - 3d)$

Substituting the values

= $(5bk + 7b)(2dk - 3d)$

Taking out the common terms

= $k\,(5b + 7b)k\,(2d - 3d)$

So we get

$= k^2\,(12b)(-d)$

= $-12\,bd\,k^2$

RHS = $(5c + 7d)(2a - 3b)$

Substituting the values

= $(5dk + 7d)(2kb - 3b)$

Taking out the common terms

= $k\,(5d + 7d)\,k\,(2b - 3b)$

So we get

$= k^2\ (12d)(-b)$

$= -12\ bd\ k^2$

Therefore, LHS = RHS.

(ii) $(ma + nb): b = (mc + nd): d$

We can write it as

$\frac{ma+nb}{b} = \frac{mc+nd}{d}$

Consider

$LHS = \frac{mbk+nb}{b}$

Taking out the common terms

$= \frac{b(mk+n)}{b}$

$= mk + n$

$RHS = \frac{mc+nd}{d}$

It can be written as

$= \frac{mdk + nd}{d}$

Taking out the common terms

$= \frac{d(mk+n)}{d}$

$= mk + n$

Therefore, LHS = RHS.

(iii) $(a^4 + c^4):(b^4 + d^4) = a^2c^2:b^2d^2$

We can write it as

$$\frac{a^4+c^4}{b^4+d^4} = \frac{a^2c^2}{b^2d^2}$$

Consider

$$LHS = \frac{a^4+c^4}{b^4+d^4}$$

Substituting the values

$$= \frac{b^4k^4+d^4k^4}{b^4+d^4}$$

Taking out the common terms

$$= \frac{k^4(b^4+d^4)}{(b^4+d^4)}$$

$$= k^4$$

$$RHS = \frac{a^2c^2}{b^2d^2}$$

We can write it as

$$= \frac{k^2b^2.k^2d^2}{b^2d^2}$$

$$= k^4$$

Therefore, LHS = RHS.

(iv) $LHS = \frac{a^2+ab}{c^2+cd}$

It can be written as

$= \frac{k^2b^2+bk.b}{k^2d^2+dk.d}$

Taking out the common terms

$= \frac{\mathrm{kb}^2(\mathrm{k}+1)}{\mathrm{d}^2\mathrm{k}(\mathrm{k}+1)}$

$= \frac{b^2}{d^2}$

$RHS = \frac{b^2-2ab}{d^2-2cd}$

It can be written as

$= \frac{b^2-2bkb}{d^2-2dkd}$

Taking out the common terms

$= \frac{b^2(1-2k)}{d^2(1-2k)}$

$= \frac{b^2}{d^2}$

Therefore, LHS = RHS.

(v) $LHS = \frac{(a+c)^3}{(b+d)^3}$

We can write it as

$= \frac{(\mathrm{bk}+\mathrm{dk})^3}{(\mathrm{b}+\mathrm{d})^3}$

$= k^3$

$RHS = \frac{a(a-c)^2}{b(b-d)^2}$

We can write it as

$= \frac{bk(bk-dk)^2}{b(b-d)^2}$

Taking out the common terms

$= \frac{bk.k^2(b-d)^2}{b(b-d)^2}$

$= k^3$

Therefore, LHS = RHS.

(vi) $LHS = \frac{a^2+ab+b^2}{a^2-ab+b^2}$

We can write it as

$= \frac{b^2k^2+bk.b+b^2}{b^2k^2-bk.b+b^2}$

Taking out the common terms

$= \frac{b^2(k^2+k+1)}{b^2(k^2-k+1)}$

So we get

$= \frac{(k^2+k+1)}{(k^2-k+1)}$

$\text{RHS} = \frac{\text{c}^2+\text{cd}+\text{d}^2}{\text{c}^2-\text{cd}+\text{d}^2}$

We can write it as

$= \frac{d^2k^2+dkd+d^2}{d^2k^2-dkd+d^2}$

Taking out the common terms

$$= \frac{d^2(k^2+k+1)}{d^2(k^2-k+1)}$$

So we get

$$= \frac{(k^2+k+1)}{(k^2-k+1)}$$

Therefore, LHS = RHS.

(vii) LHS $= \frac{a^2+b^2}{c^2+d^2}$

We can write it as

$$= \frac{b^2k^2+b^2}{d^2k^2+d^2}$$

Taking out the common terms

$$= \frac{b^2(k^2+1)}{d^2(k^2+1)}$$

$$= \frac{b^2}{d^2}$$

$$RHS = \frac{ab+ad-bc}{bc+cd-ad}$$

We can write it as

$$= \frac{bk.b+bk.d-b.dk}{bk.d+dk.d-bk.d}$$

Taking out the common terms

$$= \frac{k(b^2+bd-bd)}{k(bd+d^2-bd)}$$

$$= \frac{b^2}{d^2}$$

Therefore, LHS = RHS.

(viii) $LHS = abcd\left[\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}\right]$

We can write it as

$= bk.b.dk.d\left[\frac{1}{b^2k^2} + \frac{1}{b^2} + \frac{1}{d^2k^2} + \frac{1}{d^2}\right]$

By further calculation

$= k^2b^2d^2\left[\frac{d^2+d^2k^2+b^2+b^2k^2}{b^2d^2k^2}\right]$

So we get

$= d^2\ (1 + k^2) + b^2\ (1 + k^2)$

$= (1 + k^2)(b^2 + d^2)$

RHS = $a^2 + b^2 + c^2 + d^2$

We can write it as

$= b^2k^2 + b^2 + d^2k^2 + d^2$

Taking out the common terms

$= b^2\ (k^2 + 1) + d^2(k^2 + 1)$

$= (k^2 + 1)(b^2 + d^2)$

Therefore, LHS = RHS.

(viii) LHS = $aabcd\ \left[\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}\right]$

We can write it as

= $bk. b. dk. d\left[\frac{1}{b^2k^2} + \frac{1}{b^2} + \frac{1}{d^2k^2} + \frac{1}{d^2}\right]$

By further calculation

= $k^2b^2d^2\left[\frac{d^2+d^2k^2+b^2+b^2k^2}{b^2d^2k^2}\right]$

= So we get

= $d^2(1 + k^2)$ +$b^2(1 + k^2)$

= $(1 + k^2)(b^2 + d^2)$

RHS = $a^2 + b^2 + c^2 + d^2$

We can write it as

= $b^2k^2 + b^2 + d^2(k^2 + 1)$

= $(k^2 + 1)(b^2 + d^2)$

Therefore, LHS = RHS.

20. If x, y, z are in continued proportion, prove that :

$\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$

Solution:

It is given that

x, y, z are in continued proportion

Consider $\frac{x}{y} = \frac{y}{z} = k$

So we get

$y = kz$

$x = yk = kz \times k = k^2z$

LHS = $\frac{(x+y)^2}{(y+z)^2}$

We can write it as

= $\frac{\left(k^2z+kz\right)^2}{(kz+z)^2}$

Taking out the common terms

= $\frac{[kz\ (\ k+1)]^2}{[z\ (k+1)]^2}$

So we get

= $\frac{k^2z^2(k+1)^2}{z^2\ (k+1)^2}$

=k^2

RHS = $\frac{x}{z}$

We can write it as

= $\frac{k^2z}{z}$

= k^2

Therefore, LHS = RHS.

21. If a, b, c are in continued proportion, prove that :

$\frac{pa^2+qab+rb^2}{pb^2+qbc+rc^2} = \frac{a}{c}$

Solution:

It is given that

a, b, c are in continued proportion

$\frac{pa^2+qab+rb^2}{pb^2+qbc+rc^2} = \frac{a}{c}$

Consider $\frac{a}{b} = \frac{b}{c} = k$

So we get

$a = bk$ and $b = ck \dots\dots (1)$

From equation (1)

$a = (ck)k = ck^2$ and $b = ck$

we know that

LHS = $\frac{pa^2+qab+rb^2}{pb^2+qbc+rc^2}$

We can write it as

$= \frac{p\,(ck^2)^2+q(ck^2)ck+r(ck)^2}{p(ck)^2+q(ck)c+rc^2}$

By further calculation

$= \frac{pc^2k^4+qc^2k^3+rc^2k^2}{pc^2k^2+qc^2k+rc^2}$

By further calculation

$= \frac{pc^2k^4+qc^2k^3+rc^2k^2}{pc^2k^2+qc^2k+rc^2}$

Taking out the common terms

$= \frac{c^2k^2}{c^2}\left[\frac{pk^2+qk+r}{pk^2+qk+r}\right]$

$= k^2$

RHS = $\frac{a}{c}$

We can write it as

$= \frac{ck^2}{c}$

$= k^2$

Therefore, LHS = RHS.

22. If a, b, c are in coninued proportion, prove that :

(i) $\frac{a+b}{b+c} = \frac{a^2(b-c)}{b^2(a-b)}$

(ii) $\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} = \frac{a}{a^2} + \frac{b}{c^2a^2} + \frac{c}{a^2b^2}$

(iii) $a : c = (a^2+b^2) : (b^2+c^2)$

(iv) $a^2b^2c^2(a^{-4}+b^{-4}+c^{-4}) = b^{-2}(a^4+b^4+c^4)$

(v) $abc\,(a+b+c)^3 = (ab+bc+ca)^3$

(vi) $(a+b+c)(a-b+c) = a^2+b^2+c^2$

Solution;

It is given that

a, b, c are in continued proportion

So we get

$\frac{a}{b} = \frac{b}{c} = k$

(i) $LHS = \frac{a+b}{b+c}$

We can write it as

$= \frac{ck^2+ck}{ck+c}$

$= k$

RHS = $\frac{a^2(b-c)}{b^2\ (a-b)}$

We can write it as

$= \frac{\left(ck^2\right)^2(ck-c)}{(ck)^2\ (ck^2-ck)}$

Taking out the common terms

$= \frac{c^2k^4c(k-1)}{c^2k^2ck(k-1)}$

so we get

$= \frac{c^3k^4(k-1)}{c^3k^3(k-1)}$

$= k$

Taking out the common terms

$= \frac{ck(k+1)}{c(k+1)}$

$= k$

RHS = $\frac{a^2(b-c)}{b^2(a-b)}$

We can write it as

$= \frac{\left(ck^2\right)^2(ck-c)}{(ck)^2(ck^2-ck)}$

Taking out the common terms

$= \frac{c^2k^4c(k-1)}{c^2k^2ck(k-1)}$

So we get

$= \frac{c^3k^4(k-1)}{c^3k^3(k-1)}$

$= k$

Therefore, LHS = RHS.

(ii) LHS = $\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}$

We can write it as

$= \frac{1}{(ck^2)^3} + \frac{1}{(ck)^3} + \frac{1}{c^3}$

$= \frac{1}{c^3k^6} + \frac{1}{c^3k^3} + \frac{1}{c^3}$

Taking out the common terms

$= \frac{1}{c^3}\left[\frac{1}{k^6} + \frac{1}{k^3} + \frac{1}{1}\right]$

RHS = $\frac{a}{b^2c^2} + \frac{b}{c^2a^2} + \frac{c}{a^2b^2}$

We can write it as

$= \frac{ck^2}{(ck)^2c^2} + \frac{ck}{c^2(ck^2)^2} + \frac{c}{(ck^2)^2(ck)^2}$

$= \frac{ck^2}{c^4k^2} + \frac{ck}{c^4k^4} + \frac{c}{c^4k^6}$

So we get

$= \frac{1}{c^3} + \frac{1}{c^3k^3} + \frac{1}{c^3k^6}$

Taking out the common terms

$= \frac{1}{c^3}\left[\frac{1}{k^6} + \frac{1}{k^3} + 1\right]$

Therefore, LHS = RHS

(iii) a : c = $(a^2 + b^2) : (b^2 + c^2)$

We can write it as

$\frac{a}{c} = \frac{a^2 + b^2}{b^2 + c^2}$

We know that

LHS = $\frac{a}{c} = \frac{ck^2}{c} = k^2$

RHS = $\frac{a^2 + b^2}{b^2 + c^2}$

We can write it as

$= \frac{(ck^2)^2 + (ck)^2}{(ck)^2 + c^2}$

So we get

$= \frac{c^2\,k^4 + c^2k^2}{c^2k^2 + c^2}$

Taking out the common terms

$= \frac{c^2\,k^2(k^2+1)}{c^2(k^2+1)}$

$= k^2$

Therefore, LHS = RHS.

(iv) $a^2b^2c^2\ (a^{-4}+b^{-4}+c^{-4}) = b^{-2}(a^4+b^4+c^4)$

LHS = $a^2b^2c^2\ (a^{-4}b^{-4}c^{-4})$

We can write it as

$= a^2b^2c^2(a^{-4}b^{-4}c^{-4})$

$= \ a^2b^2c^2\left[\frac{1}{a^4}+\ \frac{1}{b^4}+\ \frac{1}{c^4}\right]$

So we get

$= \ \left[\frac{a^2b^2c^2}{a^4}+\ \frac{a^2b^2c^2}{b^4}+\ \frac{a^2b^2c^2}{c^4}\right]$

By further calculation

$= \frac{b^2c^2}{a^2}+\ \frac{c^2a^2}{b^2}+\ \frac{a^2b^2}{c^2}$

It can be written as

$= \frac{(ck)^2c^2}{(ck^2)^2}+\ \frac{c^2\left(ck^2\right)^2}{(ck)^2}+\ \frac{\left(ck^2\right)^2(ck)^2}{c^2}$

$= \ \ \frac{c^2k^2c^2}{c^2k^4}+\frac{c^2c^2k^4}{c^2\ k^2}+\frac{c^2k^4c^2k^2}{c^2}$

On further simplification

$= \frac{c^2}{k^2}+\frac{c^2\ k^2}{1}+\frac{c^2\ k^6}{1}$

Taking out the common terms

$= c^2\left[\frac{1}{k^2}+k^2+k^6\right]$

$= \frac{c^2}{k^2}\left[1+k^4+k^8\ \right]$

RHS = $b^{-2}[a^4 + b^4 + c^4]$

we can write it as

= $\frac{1}{b^2}$ $[a^4 + b^4 + c^4]$

So we get

= $\frac{1}{(ck)^2}[(ck^2)^4 + (ck)^4 + c^4]$

By further calculation

= $\frac{1}{c^2k^2}[c^4k^8 + \ c^4k^4 + \ c^4]$

Taking out the common terms

= $\frac{c^4}{c^2k^2}$ $[k^8 + k^4 + 1]$

= $\frac{c^2}{k^2}$ $[1 + k^4 + k^8]$

Therefore, LHS = RHS.

(v) LHS = abc $(a + b + c)^3$

We can write it as

= $ck^2 . ck . c[(ck^2 + ck + c)]^3$

Taking out the common terms

= $c^3k^3[(k^2 + k + 1)]^3$

So we get

= $c^3k^3 . c^3(k^2 + k + 1)^3$

= $c^6k^3(k^2 + k + 1)^3$

RHS = $(ab + bc + ca)^3$

We can write it as

= $(ck^2.ck + ck.c + c.ck^2)^3$

So we get

= $(c^2k^3 + c^2k + c^2k^2)^3$

= $(c^2k^3 + c^2k^2 + c^2k)^3$

Taking out the common terms

= $[c^2k(k^2 + k + 1)]^3$

= $c^6k^3(k^2 + k + 1)^3$

Therefore, LHS = RHS.

(vi) LHS = (a + b + c) (a – b + c)

We can write it as

= $(ck^2 + ck + c)\ (ck^2 - ck + c)$

Taking out the common terms

= $c\ (k^2 + k + 1)c(k^2 - k + 1)$

So we get

= $c^2(k^4 + k^2 + 1)$

RHS = $a^2 + b^2 + c^2$

We can write it as

= $(ck^2)^2 + (ck)^2 + (c)^2$

So we get

= $c^2k^4 + c^2k^2 + c^2$

Taking out the common terms

= $c^2\ (k^4 + k^2 + 1)$

Therefore, LHS = RHS.

23. If a, b, c, d are in continued proportion, prove that:

(i) $\frac{\mathbf{a^3+b^3+c^3}}{\mathbf{b^3+c^3+d^3}} = \frac{\mathbf{a}}{\mathbf{d}}$

(ii) $(\mathbf{a^2 - b^2})\ (\mathbf{c^2 - d^2}) = (\mathbf{b^2 - c^2})^2$

(iii) $(\mathbf{a + d})\ (\mathbf{b + c}) - (\mathbf{a + c})(\mathbf{b + d}) = (\mathbf{b - c})^2$

(iv) $\mathbf{a}:\mathbf{d}$ **=triplicate ratio of** $(\mathbf{a - b}):(\mathbf{b - c})$

(v) $\left(\frac{a-b}{c} + \frac{a-c}{b}\right)^2 - \left(\frac{d-b}{c} + \frac{d-c}{b}\right)^2 = (a-d)^2\left(\frac{1}{c^2} - \frac{1}{b^2}\right)$

Solution:

It is given that

a, b, c, d are in continued proportion

Here we get

$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$

$c = dk, b = ck = dk\,.\,k = dk^2$

$a = bk = dk^2.\,k = dk^3$

(i) $LHS = \frac{a^3+b^3+c^3}{b^3+c^3+d^3}$

We can write it as

$= \frac{(dk^3)^3+(dk^2)^3+(dk)^3}{(dk^2)^3+(dk)^3+d^3}$

So we get

$= \frac{d^3k^9+d^3k^6+d^3k^3}{d^3k^6+d^3k^3+d^3}$

Taking out the common terms

$= \frac{d^3k^3(k^6+k^3+1)}{d^3(k^6+k^3+1)}$

$= k^3$

$RHS = \frac{a}{d} = \frac{dk^3}{d} = k^3$

Therefore, LHS = RHS.

(ii) LHS = $(a^2 - b^2)(c^2 - d^2)$

We can write it as

$= [(dk^3)^2 - (dk^2)^2][(dk)^2 - d^2]$

By further calculation

$= (d^2k^6 - d^2k^4)\ (d^2k^2 - d^2)$

Taking out the common terms

$= d^2k^4\ (k^2 - 1)\ d^2\ (k^2 - 1)$

$= d^4k^4\ (k^2 - 1)^2$

RHS = $(b^2 - c^2)^2$

We can write it as

$= [(dk^2)^2 - (dk)^2]^2$

By further calculation

$= [d^2k^4 - d^2k^2]^2$

Taking out the common terms

$= [d^2k^2\ (k^2 - 1)]^2$

$= d^4\ k^4\ (k^2 - 1)^2$

Therefore, LHS = RHS.

(iii) LHS = $(a + d)(b + c) - (a + c)(b + d)$

We can write it as

$= (dk^3 + d)(dk^2 + dk) - (dk^3 + dk)(dk^2 + d)$

Taking out the common terms

$= d(k^3 + 1)\, dk(k + 1) - dk(k^2 + 1)\, d(k^2 + 1)$

By further simplification

$= d^2k(k + 1)(k^3 + 1) - d^2k(k^2 + 1)(k^2 + 1)$

So we get

$= d^2k(k^4 + k^3 + k + 1 - k^4 - 2k^2 - 1)$

$= d^2k(k^3 - 2k^2 + k)$

Taking K as common

$= d^2k^2(k^2 - 2k + 1)$

$= d^2k^2(k - 1)^2$

RHS $= (b - c)^2$

We can write it as

$= (dk^2 - dk)^2$

Taking out the common terms

$= d^2k^2(k - 1)^2$

Therefore, LHS $=$ RHS.

(iv) a: d = triplicate ratio of $(a-b):(b-c) = (a-b)^3:(b-c)^3$

We know that

$LHS = a : d = \frac{a}{d}$

It can be written as

$= \frac{dk^3}{d}$

$= k^3$

$RHS = \frac{(a-b)^3}{(b-c)^3}$

We can write it as

$= \frac{\left(dk^3 - dk^2\right)^3}{(dk^2 - dk)^3}$

Taking out the common terms

$= \frac{d^3k^6(k-1)^3}{d^3k^3\ (k-1)^3}$

$= k^3$

Therefore, LHS = RHS.

(v)

$LHS = \left(\frac{a-b}{c} + \frac{a-c}{b}\right)^2 - \left(\frac{d-b}{c} + \frac{d-c}{b}\right)^2$

We can write it as

$= \left(\frac{dk^3 - dk^2}{dk} + \frac{dk^3 - dk}{dk^2}\right)^2 - \left(\frac{d - dk^2}{dk} + \frac{d - dk}{dk^2}\right)^2$

Taking out the common terms

$$= \left(\frac{dk^2(k-1)}{dk} + \frac{dk\left(k^2-1\right)}{dk^2}\right)^2 - \left(\frac{d\left(1-k^2\right)}{dk} + \frac{d(1-k)}{dk^2}\right)^2$$

By further calculation

$$= \left(k(k-1) + \frac{(k^2-1)}{k}\right)^2 - \left(\frac{1-k^2}{k} + \frac{(1-k)}{k^2}\right)^2$$

Taking LCM we get

$$= \left(\frac{k^2(k-1)+(k^2-1)}{k}\right)^2 - \left(\frac{k\left(1-k^2\right)+1-k}{k^2}\right)^2$$

So we get

$$= \left(\frac{k^3-k^2+k^2-1}{k}\right)^2 - \left(\frac{k-k^3+1-k}{k^2}\right)^2$$

$$= \frac{\left(k^3-1\right)^2}{k^2} - \frac{\left(-k^3+1\right)^2}{k^4}$$

$$= \frac{\left(k^3-1\right)^2}{k^2} - \frac{\left(1-k^3\right)^2}{k^4}$$

On further simplification

$$= \left(\frac{k^3-1}{k^2}\right)^2 \left(1 - \frac{1}{k^2}\right)$$

We get

$$= \frac{\left(k^3-1\right)^2\left(k^2-1\right)}{k^4}$$

$$RHS = (a-d)^2 \left(\frac{1}{c^2} - \frac{1}{b^2}\right)$$

We can write it as

$$= (dk^3 - d)^2 \left(\frac{1}{d^2k^2} - \frac{1}{d^2k^4}\right)$$

So we get

$$= d^2\ (k^3 - 1)^2\ \left(\frac{k^2-1}{d^2k^4}\right)$$

$$= \frac{\left(k^3-1\right)^2\left(k^2-1\right)}{k^4}$$

Therefore, LHS = RHS.

Exercise 7.3

1. If a : b :: c : d, prove that

(i) $\frac{\mathbf{2a + 5b}}{\mathbf{2a - 5b}} = \frac{\mathbf{2c + 5d}}{\mathbf{2c - 5d}}$

(ii) $\frac{\mathbf{5a+11b}}{\mathbf{5c+11d}} = \frac{\mathbf{5a-11b}}{\mathbf{5c-11d}}$

(iii) $(\mathbf{2a + 3b})(\mathbf{2c - 3d}) = (\mathbf{2a - 3b})(\mathbf{2c + 3d})$

(iv) $(\mathbf{la + mb}) : (\mathbf{lc + mb}) :: (\mathbf{la - mb}) : (\mathbf{lc - mb})$

Solution:

(i) We know that :

If a: b :: c: d we get $\frac{a}{b} = \frac{c}{d}$

By multiplying $\frac{2}{5}$

$\frac{2a}{5b} = \frac{2c}{5d}$

By applying componendo and dividendo

$\frac{(2a+5b)}{(2a-5b)} = \frac{(2c+5d)}{(2c-5d)}$

(ii) We know that

If a: b :: c: d we get $\frac{a}{b} = \frac{c}{d}$

By multiplying $\frac{5}{11}$

$\frac{5a}{11b} = \frac{5c}{11d}$

By applying componendo and dividendo

$\frac{(5a+11b)}{(5a-11b)} = \frac{(5c+11d)}{(5c-11d)}$

Now by applying alternendo

$\frac{(5a+11b)}{(5c+11d)} = \frac{(5a-11b)}{(5c-11d)}$

(iii) We know that

If $a: b :: c: d$ we get $\frac{a}{b} = \frac{c}{d}$

By multiplying $\frac{2}{3}$

$\frac{2a}{3b} = \frac{2c}{3d}$

By applying componendo and dividendo

$\frac{(2a+3b)}{(2a-3b)} = \frac{(2c+3d)}{(2c-3d)}$

By cross multiplication

$(2a + 3b)(2c - 3d) = (2a - 3b)(2c + 3d)$

(iv) We know that

If $a: b :: c: d$ we get $\frac{a}{b} = \frac{c}{d}$

By multiplying $\frac{I}{m}$

$\frac{la}{mb} = \frac{lc}{md}$

By applying componendo and dividendo

$\frac{(la+mb)}{(la-mb)} = \frac{(lc+md)}{(lc-md)}$

Now by applying alternendo

$\frac{(la+mb)}{(lc+md)} = \frac{(la-mb)}{(lc-md)}$

So we get

$(la + mb): (lc + md) :: (la - mb): (lc - md)$

2.

(i) If $\frac{5x + 7y}{5u + 7v} = \frac{5x - 7y}{5u - 7v}$, show that $\frac{x}{y} = \frac{u}{v}$.

(ii) $\frac{8a - 5b}{8c - 5d} = \frac{8a + 5b}{8c + 5d}$, prove that $\frac{a}{b} = \frac{c}{d}$.

Solution:

(i) $\frac{5x + 7y}{5u + 7v} = \frac{5x - 7y}{5u - 7v}$

By applying alternendo

$\frac{5x+7y}{5x-7y} = \frac{5u + v}{5u-7v}$

Now by applying componendo and dividendo

$\frac{5x+7y+5x-7y}{5x+7y-5x+7y} = \frac{5u+7v+5u-7v}{5u+7v-5u+7v}$

By further calculation

$\frac{10x}{14y} = \frac{10u}{14v}$

Dividing by $\frac{10}{14}$

$\frac{x}{y} = \frac{u}{v}$

Therefore, it is proved.

(ii) $\frac{8a - 5b}{8c - 5d} = \frac{8a+5b}{8c+5d}$

By applying alternendo

$\frac{8a+5b}{8a-5b} = \frac{8c+5d}{8c-5d}$

Now by applying componendo and dividendo

$\frac{8a+5b+8a-5b}{8a+5b-8a+5b} = \frac{8c+5d+8c-5d}{8c+5d-8c+8d}$

By further calculation

$\frac{16a}{10b} = \frac{16c}{10d}$

Dividing by $\frac{16}{10}$

$\frac{a}{b} = \frac{c}{d}$

Therefore, it is proved.

3. If $(4a + 5b)(4c - 5d) = (4a - 5d)(4c + 5d)$, prove that a, b, c, d are in proportion.

Solution:

It is given that

$(4a + 5b)(4c - 5d) = (4a - 5d)(4c + 5d)$

We can write it as

$\frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$

By applying componendo and dividendo

$$\frac{4a+5b+4a-5b}{4a+5b-4a+5b} = \frac{4c+5d+4c-5d}{4c+5d-4c+5d}$$

So we get

$$\frac{8a}{10b} = \frac{8a}{10b}$$

Dividing by $\frac{8}{10}$.

$$\frac{a}{b} = \frac{c}{d}$$

Therefore, it is proved that a, b, c, d are in proportion.

4. If $(\mathbf{pa + qb}) : (\mathbf{pc + qd}) :: (\mathbf{pa - qb}) : (\mathbf{pc - qd})$ prove that $\mathbf{a : b :: c : d}$.

Solution:

It is given that

$(\mathrm{pa + qb}) : (\mathrm{pc + qd}) :: (\mathrm{pa - qb}) : (\mathrm{pc - qd})$

We can write it as

$$\frac{pa+qb}{pc+qd} = \frac{pa-qb}{pc-qd}$$

$$\frac{pa+qb}{pa-qb} = \frac{pc+qd}{pc-qd}$$

By applying componendo and dividendo

$$\frac{pa+qb+pa-qb}{pa+qb-pa+qb} = \frac{pc+qd+pc-qd}{pc+qd-pc+qd}$$

So we get

$$\frac{2pa}{2qb} = \frac{2pc}{2qd}$$

Dividing by $\frac{2p}{2q}$

$\frac{a}{b} = \frac{c}{d}$

Therefore, it is proved that a: b :: c: d.

5. If $(\mathbf{ma + nb})$: b :: $(\mathbf{mc + nd})$: d, prove that a, b, c, d are in proportion.

Solution:

It is given that

(ma + nb): b :: (mc + nd): d

We can write it as

$\frac{(ma+nb)}{b} = \frac{(mc+nd)}{d}$

By cross multiplication

mad + nbd = mbc + nbd

Here mad = mbc

ad = bc

By further calculation

$\frac{a}{b} = \frac{c}{d}$

Therefore, it is proved that a, b, c, d are in proportion.

6. If $(11a^2 + 3b^2)\ (11c^2 - 13d^2) = (11a^2 - 13b^2)\ (11c^2 + 13d^2)$, Prove that a : b : : c : d.

Solution;

It is given that

$$(11a^2 + 13b^2)\ (11c^2 - 13d^2) = (11a^2 - 13b^2)\ (11c^2 + 13d^2)$$

We can write it as

$$\frac{11a^2+13b^2}{11a^2-13b^2} = \frac{11c^2+13d^2}{11c^2-13d^2}$$

By applying componendo and dividendo

$$\frac{11a^2+13b^2+11a^2-13b^2}{11a^2+13b^2-11a^2+13b^2} = \frac{11c^2+13d^2+11c^2-13d^2}{11c^2+13d^2-11c^2+13d^2}$$

So we get

$$\frac{22a^2}{26b^2} = \frac{22c^2}{26d^2}$$

Dividing by $\frac{22}{26}$

$$= \frac{a^2}{b^2} = \frac{c^2}{d^2}$$

$$= \frac{a}{b} = \frac{c}{d}$$

Therefore, it is proved that a : b :: c : d.

7. If x = $\frac{2ab}{a+b}$ find the value of $\frac{x+a}{x-a}+\frac{x+b}{x-b}$.

Solution:

It is given that

$$x=\frac{2ab}{a+b}$$

$$\frac{x}{a}=\frac{2b}{a+b}$$

By applying componendo and dividendo

$$\frac{x+a}{x-a}=\frac{2b+a+b}{2b-a-b}=\frac{3b+a}{b-a} \;\ldots\ldots\; (1)$$

Similarly

$$\frac{x}{b}=\frac{2a}{a+b}$$

By applying componendo and dividendo

$$\frac{x+b}{x-b}=\frac{2a+a+b}{2a-a-b}=\frac{3a+b}{a-b} \;\ldots\ldots\; (2)$$

Now adding both the equations

$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$$

By further calculation

$$=-\frac{a+3b}{a-b}+\frac{3a+b}{a-b}$$

So we get

$$=\frac{-a-3b+3a+b}{a-b}$$

$$=\frac{2a-2b}{a-b}$$

Taking 2 as common

$= \frac{2(a-b)}{(a-b)}$

$= 2$

8. If $x = \frac{8ab}{a+b}$ find the value of $\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$

Solution:

It is given that

$x = \frac{8ab}{a+b}$

$\frac{x}{4a} = \frac{2b}{a+b}$

By applying componendo and dividendo

$\frac{x+4a}{x-4a} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a}$ …… (1)

Similarly

$\frac{x}{4b} = \frac{2a}{a+b}$

By applying componendo and dividendo

$\frac{x+b}{x-b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b}$ …… (2)

Now adding both the equations

$\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$

By further calculation

$= -\frac{a+3b}{a-b} + \frac{3a+b}{a-b}$

So we get

$= \frac{-a-3b+3a+b}{a-b}$

$= \frac{2a-2b}{a-b}$

Taking 2 as common

$= \frac{2(a-b)}{(a-b)}$

= 2

9. If $x = \frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ find the value of $\frac{x+2\sqrt{2}}{x-2\sqrt{2}} + \frac{x+2\sqrt{3}}{x-2\sqrt{3}}$

Solution:

It is given that

$x = \frac{4\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

We can write it as

$x = \frac{4\sqrt{2}.\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

Here

$\frac{x}{2\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

By applying componendo and dividendo

$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} = \frac{2\sqrt{3}+\sqrt{2}+\sqrt{3}}{2\sqrt{3}-\sqrt{2}-\sqrt{3}}$

By further calculation

$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ …… (1)

Similarly

$\frac{x}{2\sqrt{3}} = \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{3}}$

By applying componendo and dividendo

$$\frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{2\sqrt{2}+\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{2}-\sqrt{3}}$$

By further calculation

$$\frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \quad \ldots\ldots. \quad (2)$$

By adding both the equations

$$\frac{x+2\sqrt{2}}{x-2\sqrt{2}} + \frac{x+2\sqrt{3}}{x-2\sqrt{3}} = \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

We can write it as

$$= \frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{3\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

So we get

$$= \frac{3\sqrt{3}+\sqrt{2}-3\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

Taking out 2 as common

$$= \frac{2(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2}}$$

= 2

10. Using properties of properties, find x from the following equations:

(i) $\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}} = 3$

(ii) $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}} = \frac{5}{2}$

(iii) $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=\frac{a}{b}$

(iv) $\frac{\sqrt{12x+1}+\sqrt{2x-3}}{\sqrt{12x+1}-\sqrt{2x-3}}=\frac{3}{2}$

(v) $\frac{3x+\sqrt{9x^2+5}}{3x-\sqrt{9x^2+5}}=5$

(vi) $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}=\frac{c}{d}$

Solution:

(i) $\frac{\sqrt{2-x}+\sqrt{2+x}}{\sqrt{2-x}-\sqrt{2+x}}=3$

By applying componendo and dividendo

$$\frac{\sqrt{2-x}+\sqrt{2+x}+\sqrt{2-x}-\sqrt{2+x}}{\sqrt{2-x}+\sqrt{2+x}-\sqrt{2-x}+\sqrt{2+x}}=\frac{3+1}{3-1}$$

On further calculation

$$\frac{2\sqrt{2-x}}{2\sqrt{2+x}}=\frac{4}{2}$$

$$\frac{\sqrt{2-x}}{\sqrt{2+x}}=\frac{2}{1}$$

By squaring on both sides

$$\frac{2-x}{2+x}=\frac{4}{1}$$

By cross multiplication

8 + 4x = 2 – x

So we get

4x + x = 2 – 8

5x = – 6

x = $-\frac{6}{5}$

(ii) $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}} = \frac{5}{2}$

By applying componendo and dividendo

$$\frac{\sqrt{x+4}+\sqrt{x-10}+\sqrt{x+4}-\sqrt{x-10}}{\sqrt{x+4}+\sqrt{x-10}-\sqrt{x+4}+\sqrt{x-10}} = \frac{5+2}{5-2}$$

On further calculation

$$\frac{2\sqrt{x+4}}{2\sqrt{x-10}} = \frac{7}{3}$$

$$\frac{\sqrt{x+4}}{\sqrt{x-10}} = \frac{7}{3}$$

By squaring on both sides

$$\frac{x+4}{x-10} = \frac{49}{9}$$

By cross multiplication

49x – 490 = 9x + 36

49x – 9x = 36 + 490

So we get

40x = 526

$x = \frac{526}{40}$

$x = \frac{263}{20}$

(iii) $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}} - \sqrt{1-x} = \frac{a}{b}$

By applying componendo and dividendo

$$\frac{\sqrt{1+x}+\sqrt{1-x}+\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}-\sqrt{1+x}+\sqrt{1-x}} = \frac{a+b}{a-b}$$

On further calculation

$$\frac{2\sqrt{1+x}}{2\sqrt{1-x}} = \frac{a+b}{a-b}$$

$$\frac{\sqrt{1+x}}{\sqrt{1-x}} = \frac{a+b}{a-b}$$

By squaring on both sides

$$\frac{1+x}{1-x} = \frac{(a+b)^2}{(a-b)^2}$$

By applying componendo and dividendo

$$\frac{1+x+1-x}{1+x-1+x} = \frac{(a+b)^2+(a-b)^2}{(a+b)^2-(a-b)^2}$$

By further calculation

$$\frac{2}{2x} = \frac{2\left(a^2+b^2\right)}{4ab}$$

$$\frac{1}{x} = \frac{a^2+b^2}{2ab}$$

So we get

$$x = \frac{2ab}{a^2+b^2}$$

(iv) $\frac{\sqrt{12x+1}+\sqrt{2x-3}}{\sqrt{12x+1}-\sqrt{2x-3}} = \frac{3}{2}$

By applying componendo and dividendo

$$\frac{\sqrt{12x+1}+\sqrt{2x-3}+\sqrt{12x+1}-\sqrt{2x-3}}{\sqrt{12x+1}+\sqrt{2x-3}-\sqrt{12x+1}+\sqrt{2x-3}} = \frac{3+2}{3-2}$$

On further calculation

$$\frac{2\sqrt{12x+1}}{2\sqrt{2x-3}} = \frac{5}{1}$$

$$\frac{\sqrt{12x+1}}{\sqrt{2x-3}} = \frac{5}{1}$$

By squaring on both sides

$\frac{12x+1}{2x-3} = \frac{25}{1}$

By cross multiplication

$50x - 75 = 12x + 1$

$50x - 12x = 1 + 75$

So we get

$38x = 76$

$x = \frac{76}{38} = 2$

(v) $\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}} = \frac{5}{1}$

By applying componendo and dividendo

$\frac{3x+\sqrt{9x^2-5}+3x-\sqrt{9x^2-5}}{3x+\sqrt{9x^2-5}-3x+\sqrt{9x^2-5}} = \frac{5+1}{5-1}$

On further calculation

$\frac{6x}{2\sqrt{9x^2-5}} = \frac{6}{4}$

$\frac{3x}{\sqrt{9x^2-5}} = \frac{3}{2}$

By squaring on both sides

$\frac{9x^2}{9x^2-5} = \frac{9}{4}$

By cross multiplication

$81x^2 - 45 = 36x^2$

$81x^2 - 36x^2 = 45$

So we get

$45x^2 = 45$

$x^2 = 1$

$x = \pm 1$

$x = 1, -1$

Verification:

(i) If $x = 1$

$\frac{3\times1+\sqrt{9\times1-5}}{3\times1-\sqrt{9\times1-5}} = \frac{3+\sqrt{4}}{3-\sqrt{4}}$

So we get

$= \frac{3+2}{3-2}$

$= \frac{5}{1}$

Hence, $x = 1$.

(ii) If $x = -1$

$\frac{3\times(-1)+\sqrt{9\times(-1)^2-5}}{3\times(-1)-\sqrt{9\times(-1)^2-5}} = \frac{-3+\sqrt{9-5}}{-3-\sqrt{9-5}}$

By further calculation

$= \frac{-3+\sqrt{4}}{-3-\sqrt{4}}$

So we get

$= \frac{-3+2}{-3-2}$

$= \frac{-1}{-5}$

$= \frac{1}{5}$

Here $\frac{1}{5} \neq \frac{5}{1}$

$x = -1$ is not the solution

Therefore, x = 1.

(vi) $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}} = \frac{c}{d}$

By applying componendo and dividendo

$$\frac{\sqrt{a+x}+\sqrt{a-x}+\sqrt{a+x}-\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}-\sqrt{a+x}+\sqrt{a-x}} = \frac{c+d}{c-d}$$

On further calculation

$$\frac{2\sqrt{a+x}}{2\sqrt{a-x}} = \frac{c+d}{c-d}$$

$$\frac{\sqrt{a+x}}{\sqrt{a-x}} = \frac{c+d}{c-d}$$

By squaring on both sides

$$\frac{a+x}{a-x} = \frac{(c+d)^2}{(c-d)^2}$$

By applying componendo and dividendo

$$\frac{a+x+a-x}{a+x-a+x} = \frac{(c+d)^2+(c-d)^2}{(c+d)^2-(c-d)^2}$$

By further calculation

$$\frac{2a}{2x} = \frac{2(c^2+d^2)}{4cd}$$

$$\frac{a}{x} = c^2 + \frac{d^2}{2cd}$$

By cross multiplication

$$x(c^2 + d^2) = 2acd$$

$$x = \frac{2acd}{c^2 + d^2}$$

11. Using properties of proportion solve for x. Given that x is positive.

(i) $\frac{\mathbf{3x+\sqrt{9x^2-5}}}{\mathbf{3x-\sqrt{9x^2-5}}} = \mathbf{5}$

(ii) $\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = 4$

Solution:

(i) $\frac{3x+\sqrt{9x^2-5}}{3x-\sqrt{9x^2-5}} = \frac{5}{1}$

By applying componendo and dividendo

$$\frac{3x+\sqrt{9x^2-5}+3x-\sqrt{9x^2-5}}{3x+\sqrt{9x^2-5}-3x+\sqrt{9x^2-5}} = \frac{5+1}{5-1}$$

On further calculation

$$\frac{6x}{2\sqrt{9x^2-5}} = \frac{6}{4}$$

$$\frac{3x}{\sqrt{9x^2-5}} = \frac{3}{2}$$

By squaring on both sides

$$\frac{9x^2}{9x^2-5} = \frac{9}{4}$$

By cross multiplication

$81x^2 - 45 = 36x^2$

$81x^2 - 36x^2 = 45$

So we get

$45x^2 = 45$

$x^2 = 1$

$x = \pm 1$

$x = 1, -1$

Verification:

(i) If x = 1

$\frac{3\times1+\sqrt{9\times1-5}}{3\times1-\sqrt{9\times1-5}} = \frac{3+\sqrt{4}}{3-\sqrt{4}}$

So we get

$= \frac{3+2}{3-2}$

$= \frac{5}{1}$

Hence, x $= 1$.

(ii) $\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = 4$

Solution;

$0.\overline{35}$

Upon cross multiplication,

$36x^2 = 25\ (4x^2 - 1)$

$36x^2 = 100x^2 - 25$

$64x^2 = 25$

$x^2 = \frac{25}{64}$

Taking square root on both sides,

$x = \sqrt{\frac{25}{64}}$

$x = \pm\frac{5}{8}$

Given that, x is positive

Thus the value of $x = \frac{5}{8}$.

12. Solve :

$$\frac{\mathbf{1+x+x^2}}{\mathbf{1-x+x^2}} = \frac{\mathbf{62(1+x)}}{\mathbf{63(1-x)}}$$

Solution;

$$\frac{1+x+x^2}{1-x+x^2} = \frac{62(1+x)}{63(1-x)}$$

We can write it as

$$\frac{(1-x)(1+x+x^2)}{(1+x)(1-x+x^2)} = \frac{62}{63}$$

$$\frac{(1+x)(1-x+x^2)}{(1-x)(1+x+x^2)} = \frac{63}{62}$$

$$\frac{1+x^3}{1-x^3} = \frac{63}{62}$$

By applying componendo and dividendo

$$\frac{1+x^3+1-x^3}{1+x^3-1+x^3} = \frac{63+62}{63-62}$$

On further calculation

$$\frac{2}{2x^3} = \frac{125}{1}$$

$$\frac{1}{x^3} = \frac{125}{1}$$

So we get

$$x^3 = \left(\frac{1}{5}\right)^3$$

$$\mathrm{x} = \frac{1}{5}$$

13. Solve for x:

$$16\left(\frac{\mathrm{a-x}}{\mathrm{a+x}}\right)^3 = \frac{\mathrm{a+x}}{\mathrm{a-x}}$$

Solution:

$$16\left(\frac{\mathrm{a-x}}{\mathrm{a+x}}\right)^3 = \frac{\mathrm{a+x}}{\mathrm{a-x}}$$

We can write it as

$$\left(\frac{a+x}{a-x}\right) \times \left(\frac{a+x}{a-x}\right)^3 = 16$$

$$\left(\frac{a+x}{a-x}\right)^4 = 16 = (\pm 2)^4$$

Here

$$\frac{a+x}{a-x} = \pm 2$$

If $\frac{a+x}{a-x} = \frac{2}{1}$

By applying componendo and dividendo

$$\frac{a+x+a-x}{a+x-a+x} = \frac{2+1}{2-1}$$

On further calculation

$$\frac{2a}{2x} = \frac{3}{1}$$

$$\frac{a}{x} = \frac{3}{1}$$

So we get

$$3\mathrm{x} = \mathrm{a}$$

$$\mathrm{x} = \frac{\mathrm{a}}{3}$$

If $\frac{a+x}{a-x} = \frac{-2}{1}$

By applying componendo and dividendo

$$\frac{a+x+a-x}{a+x-a+x} = \frac{-2+1}{-2-1}$$

On further calculation

$$\frac{2a}{2x} = \frac{-1}{-3}$$

$$\frac{a}{x} = \frac{1}{3}$$

So we get

x = 3a

Therefore, x = $\frac{a}{3}$, 3a.

14. If $x = \frac{\sqrt{a+x} + \sqrt{a-1}}{\sqrt{a+1} - \sqrt{a-1}}$, using properties of proportion, show that $x^2 - 2ax + 1 = 0$

Solution:

It is given that

$$x = \frac{\sqrt{a+x}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$$

We can write it as

$$\frac{x+1}{x-1} = \frac{2\sqrt{a+1}}{2\sqrt{a-1}}$$

By applying componendo and dividendo

$$\frac{(x+1)^2}{(x-1)^2} = \frac{a+1}{a-1}$$

On further calculation

$$\frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2} = \frac{2a}{2}$$

By applying componendo and dividendo

$$\frac{x^2+1+2x+x^2+1-2x}{x^2+1+2x-x^2-1+2x} = a$$

So we get

$$\frac{2x^2+2}{4x} = a$$

Taking out common terms

$$\frac{2(x^2+1)}{4x} = a$$

$$\frac{x^2+1}{2x} = a$$

We get

$2ax = x^2 + 1$

$x^2 - 2ax + 1 = 0$

Therefore, it is proved.

15.

Give $\boldsymbol{x = \frac{\sqrt{a^2} + \sqrt{b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}}$ use componendo and dividendo to prove that $\boldsymbol{b^2 = \frac{2a^2x}{x^2+1}}$

Solution:

$$\frac{x}{1} = \frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}$$

By applying componendo and dividendo

$$\frac{x+1}{x-1} = \frac{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}+\sqrt{a^2+b^2}-\sqrt{a^2-b^2}}{\sqrt{a^2+b^2}+\sqrt{a^2-b^2}-\sqrt{a^2+b^2}+\sqrt{a^2-b^2}}$$

On further calculation

$$\frac{(x+1)}{(x-1)} = \frac{2\sqrt{a^2+b^2}}{2\sqrt{a^2+b^2}}$$

$$\frac{(x+1)}{(x-1)} = \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

By squaring both sides

$$\frac{(x+1)^2}{(x-1)^2} = \frac{a^2+b^2}{a^2-b^2}$$

Expanding the equations

$$\frac{x^2+1+2x}{x^2+1-2x} = \frac{a^2+b^2}{a^2-b^2}$$

By applying componendo and dividendo

$$\frac{x^2+1+2x+x^2+1-2x}{x^2+1+2x-x^2-1+2x} = \frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}$$

So we get

$$\frac{2x^2+2}{4x} = \frac{2a^2}{2b^2}$$

Taking out common terms

$$\frac{2(x^2+1)}{4x} = \frac{2a^2}{2b^2}$$

$$\frac{x^2+1}{2x} = \frac{a^2}{b^2}$$

So we get

$$b^2 = \frac{2a^2x}{x^2+1}$$

16. Given that $\frac{a^3+3ab^2}{b^3+3a^2b} = \frac{63}{62}$. Using componendo and dividendo find a : b.

Solution:

It is given that

$$\frac{a^3+3ab^2}{b^3+3a^2b} = \frac{63}{62}$$

By applying componendo and dividendo

$$\frac{a^3+3ab^2+b^3+3a^2b}{a^3+3ab^2-b^3-3a^2b} = \frac{63+62}{63-62} = \frac{125}{1}$$

On further calculation

$$\frac{(a+b)^3}{(a-b)^3} = \left(\frac{5}{1}\right)^3$$

So we get

$$\frac{(a+b)}{(a-b)} = 5$$

By cross multiplication

a + b = 5a – 5b

We can write it as

5a – a – 5b – b = 0

4a – 6b = 0

4a = 6b

We get

$$\frac{a}{b} = \frac{6}{4}$$

$$\frac{a}{b} = \frac{3}{2}$$

$\therefore$ a: b = 3: 2

17. Give $\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$. Using componendo and dividendo find x : y.

Solution:

It is given that

$$\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$$

By applying componendo and dividendo

$$= \frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8} = \frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

On further calculation

$$\frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3}$$

We can write it as

$$\left(\frac{x+2}{x-2}\right)^2 = \left(\frac{y+3}{y-3}\right)^3$$

So we yet

$$\frac{x+2}{x-2} = \frac{y+3}{y-3}$$

By applying componendo and dividendo

$$\frac{x+2+x-2}{x-2-x+2} = \frac{y+3+y-3}{y-3-y+3}$$

By further calculation

$$\frac{2x}{4} = \frac{2y}{3}$$

$$\frac{x}{2} = \frac{y}{3}$$

By cross multiplication

$$\frac{x}{y} = \frac{2}{3}$$

Hence, the required ratio $x : y\ is\ 2:3$

$\frac{2x}{4} = \frac{2y}{3}$

$\frac{x}{2} = \frac{y}{3}$

By cross multiplication

$\frac{x}{y} = \frac{2}{3}$

Hence, the required ratio $x:y$ is 2 : 3.

18. Using the properties of proportion, solve the following equation for x; *given*

$\frac{x^3+3x}{3x^2+1} = \frac{341}{91}$

Solution:

It is given that

$\frac{x^3+3x}{3x^2+1} = \frac{341}{91}$

By applying componendo and dividendo

$\frac{x^3+3x+3x^2+1}{x^3+3x-3x^2-1} = \frac{341+91}{341-91}$

On further calculation

$\frac{x^3+3x+3x+1}{x^3-3x^2+3x-1} = \frac{432}{250} = \frac{216}{125}$

We can write it as

$\frac{(x+1)^3}{(x-1)^3} = \frac{216}{125} = \left(\frac{6}{5}\right)^3$

By cross multiplication

$6x - 6 = 5x + 5$

$6x - 5x = 5 + 6$

$x = 11$

19. If $\frac{x+y}{ax+by} = \frac{y+z}{ay+bz} = \frac{z+x}{az+bx}$, Prove that each of these ratio is equal to $\frac{2}{a+b}$ unless $x + y + z = 0$.

Solution :

It is given that

$\frac{x+y}{ax+by} = \frac{y+z}{ay+bz} = \frac{z+x}{az+bx}$

By addition

$= \frac{x+y+y+z+z+x}{ax+by+ay+bz+az+bx}$

By further calculation

$= \frac{2(x+y+z)}{x(a+b)+y\ (a+b)+z\ (a+b)}$

So we get

$= \frac{2(x+y+z)}{(a+b)(x+y+z)}$

$= \frac{2}{a+b}$

If $x + y + z \neq 0$

Therefore, it is proved.

Chapter Test

1. Find the compound ratio of

$(a+b)^2 : (a-b)^2, (a^2-b^2) : (a^2+b^2)$ and $(a^4-b^4) : (a+b)^4$

Solution;

$(a+b)^2 : (a-b)^2$

$(a^2-b^2) : (a^2+b^2)$

$(a^4-b^4) : (a+b)^4$

We can write it as

$= \frac{(a+b)^2}{(a-b)^2} \times \frac{a^2-b^2}{a^2+b^2} \times \frac{a^4-b^4}{(a+b)^4}$

By further calculation

$= \frac{(a+b)^2}{(a-b)^2} \times \frac{(a+b)\,(a-b)}{a^2+b^2} \times \frac{(a^2+b^2)(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{a}-\boldsymbol{b})}{(a+b)^4}$

So we get

$= \frac{1}{1}$

$= 1 : 1$

2. If $(7p+3q):(3p-2q) = 43:2$, find p : q.

Solution:

It is given that

$(7p+3q):(3p-2q) = 43:2$

we can writte it as

$\frac{(7p+3q)}{(3p-2q)} = \frac{43}{2}$

By cross multiplication

$129p - 86q = 14p + 6q$

$129\, p - 14p = 6q + 86q$

So we get

$115p = 92q$

By division

$\frac{p}{q} = \frac{92}{115} = \frac{4}{5}$

Hence, p : q = 4 : 5.

3. If $\boldsymbol{a : b = 3 : 5}$, find $\boldsymbol{(3a + 5b) : (7a - 2b)}$.

Solution:

It is given that

$a : b = 3 : 5$

Here

$(3a + 5b) : (7a - 2b)$

Now dividing the terms by b

Here

(3a + 5b) : (7a – 2b)

Noow dividing the terms by b

$3 \times \frac{a}{b} + 5 : 3 \times \frac{a}{b} - 2$

substituting the values of $\frac{a}{b}$

$3 \times \frac{3}{5} + 5 : 3 \times \frac{3}{5} - 2$

By further calculation

$\left(\frac{9}{5}+5\right):\left(\frac{21}{5}-2\right)$

Taking LCM

$\frac{9+25}{5}:\frac{21-10}{5}$

So we get

$\frac{34}{5}:\frac{11}{5}$

Here

$(3a+5b):(7a-2b)=34:11$

4. The ratio of the shorter sides of a right angled triangle is 5 : 12. If the perimeter of the triangle is 360 cm, find the length of the longest sides.

Solution:

Consider the two shorter sides of a right-angled triangle as $5x$ and $12x$

So the third longest side

$=\sqrt{(5x)^2+(12x)^2}$

$=\sqrt{25x^2+144x^2}$

$=\sqrt{169x^2}$

$= 13x$

It is given that

$5x+12x+13x=360cm$

by further calculation

$30x=360$

we get

$x = \frac{360}{30} = 12$

Here the length of the longest side = $13x$

Substituting the value of x

$= 13 \times 12$

= 156 cm

5. The ratio of the pocket money saved by Lokesh and his sister is 5 :6. If the sister saves Rs 30 more, how much more the brother should save in order to keep the ratio of their savings unchanged ?

Solution:

Consider $5x$ and $6x$ as the savings of Lokesh and his sister.

Lokesh should save Rs y more

Based on the problem

$\frac{(5x+y)}{(6x+30)} = \frac{5}{6}$

By cross multiplication

$30x + 6y = 30x + 150$

By further calculation

$30x + 6y - 30x = 150$

So we get

$6y = 150$

$y = \frac{150}{6} = 25$

Therefore, Lokesh should save Rs 25 more than his sister.

6. In an examination, the number of those who passed and the number of those who failed were in the ratio of 3:1. Has 8 more appeared, and 6 less passed, the ratio of passed to failures would have been 2:1. Find the number of candidates who appeared.

Solution:

Consider the number of passed = $3x$

Number of failed = x

So the total candidates appeared = $3x + x = 4x$

In the second case

Number of candidates appeared = $4x + 8$

Number of passed = $3x - 6$

Number of failed = $4x + 8 - 3x + 6 = x + 14$

Ratio = 2 : 1

Based on the condition

$\frac{(3x-6)}{(x+14)} = \frac{2}{1}$

By cross multiplication

$3x - 6 = 2x + 28$

$x = 34$

Here the number of candidates appeared = $4x = 4 \times 34 = 136$.

7. What number must be added to each of the numbers 15, 17, 334 and 38 to make them proportional ?

Solution:

Consider x be added to each number

So the numbers will be

$15 + x, 17 + x,\ \ 34 + x$ and $38 + x$

By cross multiplication

$(15 + x)(38 + x) = (34 + x)(17 + x)$

By further calculation

$570 + 53x + x^2 = 578 + 51x + x^2$

So we get

$x^2 + 53x - x^2 - 51x = 578 - 570$

$2x = 8$

$x = 4$

Hence, 4 must be added to each of the numbers.

$2x = 8$

$x = 4$

Hence, 4 must be added to each of the numbers.

8. If $(\boldsymbol{a} + \boldsymbol{2b} + \boldsymbol{c}), (\boldsymbol{a} - \boldsymbol{c})$ and $(\boldsymbol{a} - \boldsymbol{2b} + \boldsymbol{c})$ are in continued proportion, prove that b is the mean proportional between a and c.

Solution;

It is given that

$(a + 2b + c), (a - c)$ and $(a - 2b + c)$ are in continued proportion

we can write it as

$\frac{(a+2b+c)}{(a-c)} = \frac{(a-c)}{(a-2b+c)}$

By cross multiplication

$(a + 2b + c)(a - 2b + c) = (a - c)^2$

On further calculation

$a^2 - 2ab + ac + 2ab - 4b^2 + 2bc + ac - 2bc + c^2 = a^2 - 2ac + c^2$

So we get

$a^2 - 2ab + ac + 2ab - 4b^2 + 2bc + ac - 2bc + c^2 - a^2 + 2ac - c^2 = 0$

$4ac - 4b^2 = 0$

Dividing by 4

$ac - b^2 = 0$

$b^2 = ac$

Therefore, it is proved that b is the mean proportional between a and c.

9. If 2, 6, p, 54 and q are in continued proportion, find the values of p and q.

Solution;

It is given that

2, 6, p , 54 and q are in continued proportion

We can write it as

$\frac{2}{6} = \frac{6}{p} = \frac{p}{54} = \frac{54}{q}$

(i) We know that

$\frac{2}{6} = \frac{6}{p}$

By cross multiplication

$2p = 36$

$p = 18$

(ii) We know that

$\frac{p}{54} = \frac{54}{q}$

By cross multiplication

$pq = 54 \times 54$

Substituting the value of p

$q = \frac{(54 \times 54)}{18} = 162$

Therefore, the values of p and q are 18 and 62.

10. If a, b, c ,d, e are in continued proportion, prove that : a : e = $a^4 : b^4$.

Solution :

It is given that

a, b, c, d, e are in continued proportion

We can write it as

$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = k$

d = ek, c = ek^2, b = ek^3 and $a = ek^4$

Here

LHS = $\frac{a}{e}$

Substituting the values

$= \frac{ek^4}{e}$

$= k^4$

RHS = $\frac{a^4}{b^4}$

Substituting the values

= $\frac{ek^4}{e}$

= k^4

RHS = $\frac{a^4}{b^4}$

Substituting the values

= $\frac{(ek^4)^4}{(ek^3)^4}$

So we get

= $\frac{e^4k^{16}}{e^4k^{12}}$

= k^{16-12}

= k^4

Hence, it is proved that a :e = $a^4 : b^4$.

11. Find two numbers whose mean proportional is 16 and the third proportional is 128.

Solution :

Consider x and y as the two numbers

Mean proportion = 16

Third proportion = 128

$\sqrt{xy}$ = 16

$xy = 256$

Here

$x = \frac{256}{y}$ ……….(1)

$\frac{y^2}{x}$ = 128

Here

$x = \frac{y^2}{128}$ ……..(2)

Using both the equations

$\frac{256}{y} = \frac{y^3}{128}$

By cross multiplication

$y^3 = 256 \times 128 = 32768$

$y^3 = 32^3$

$y = 32$

Substituting the value of y in equation (1)

$x = \frac{256}{y}$

so we get

$x = \frac{256}{32} =$

8

Hence, the two numbers are 8 and 32.

12. If q is the mean proportional between p and r, prove that:

$$\boldsymbol{p^2 - 3q^2 + r^2 = q^4\left(\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2}\right)}$$

Solution:

It is given that

q is the mean proportional between p and r

$q^2 = pr$

Here

LHS = $p^2 - 3q^2 + r^2$

we can write it as

= $p^2 - 3pr + r^2$

RHS = $q^4\left(\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2}\right)$

We can write it as

= $(q^2)^2\left(\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2}\right)$

= Substituting the values of q

= $(pr)^2\left(\frac{1}{p^2} - \frac{3}{q^2} + \frac{1}{r^2}\right)$

Taking LCM

= $p^2r^2\left(\frac{r^2-3pr+p^2}{p^2r^2}\right)$

So we get

= $r^2 - 3pr + p^2$

Here

LHS = RHS

Therefore, it is proved.

13. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that each ratio is

(i) $\sqrt{\frac{3a^2 - 5c^2 + 7e^2}{3b^2 - 5d^2 + 7f^2}}$

(ii) $\left[\frac{2a^3 + 5c^3 + 7e^3}{2b^3 + 5d^3 + 7f^3}\right] \frac{1}{3}$

Solution;

It is given that

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

So we get

$a = k, c = dk, e = fk$

(i) $\sqrt{\frac{3a^2 - 5c^2 + 7e^2}{3b^2 - 5d^2 + 7f^2}}$

Substituting the values

$= \sqrt{\frac{3b^2k^2 - 5d^2k^2 + 7f^2k^2}{3b^2 - 5d^2 + 7f^2}}$

Taking k as common

$= k\sqrt{\frac{3b^2 - 5d^2k^2 + 7f^2}{3b^2 - 5d^2 + 7f^2}}$

$= k$

Therefore, it is proved.

(ii) $\left[\frac{2a^3+5c^3+7e^3}{2b^3+5d^3+7f^3}\right]\frac{1}{3}$

Substituting the values

$= \left[\frac{2b^3k^3+5d^3k^3+7f^3k^3}{2b^3+5d^3+7f^3}\right]\frac{1}{3}$

Taking k as common

$= k\left[\frac{2b^3+5d^3+7f^3}{2b^3+5d^3+7f^3}\right]\frac{1}{3}$

$= k$

Therefore, it is proved.

14. if $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, Prove that

$\frac{3x^3-5y^3+4z^3}{3a^3-5b^3+4c^3} = \left(\frac{3x-5y+4z}{3a-5b+4c}\right)^3$

Solution:

It is given that

$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$

So we get

$x = ak, y = bk, z = ck$

Here

LHS = $\frac{3x^3-5y^3+4z^3}{3a^3-5b^3+4c^3}$

Substituting the values

$$= \frac{3a^3k^3 - 5b^3k^3 + 4c^3k^3}{3a^3 - 5b^3 + 4c^3}$$

Taking out the common terms

$$= \frac{k^3(3a^3 - 5b^3 + 4c^3)}{3a^3 - 5b^3 + 4c^3}$$

$$= k^3$$

RHS = $\left(\frac{3x - 5y + 4z}{3a - 5b + 4c}\right)^3$

Substituting the values

$$= \left(\frac{3ak - 5bk + 4ck}{3a - 5b + 4c}\right)^3$$

Taking out the common terms

$$= \left(\frac{k(3a - 5b + 4c)}{3a - 5b + 4}\right)^3$$

$$= k^3$$

Hence LHS = RHS

15. If x : a = y : b, prove that

$$\frac{x^4 + a^4}{x^3 + a^3} + \frac{y^4 + b^4}{y^3 + b^3} = \frac{(x+y)^4 + (a+b)^4}{(x+y)^3 + (a+b)^3}$$

Solution:

We know that

$$\frac{x}{a} = \frac{y}{b} = k$$

So we get

$x = ak,\ y = bk$

Here

LHS = $\frac{x^4 + a^4}{x^3 + a^3} + \frac{y^4 + b^4}{y^3 + b^3}$

Subtracting the values

$= \frac{a^4k^4 + a^4}{a^3k^3 + a^3} + \frac{b^4k^4 + b^4}{b^3k^3 + b^3}$

Taking out the common terms

$= \frac{a^4\left(k^4+1\right)}{a^3\left(k^3+1\right)} + \frac{b^4\left(k^4+1\right)}{b^3\left(k^3+1\right)}$

We get

$= \frac{a\left(k^4+1\right)}{\left(k^3+1\right)} + \frac{b^4\left(k^4+1\right)}{\left(k^3+1\right)}$

We can write it as

$= \frac{a\left(k^4+1\right)+b\left(k^4+1\right)}{\left(k^3+1\right)}$

$= \frac{\left(k^4+1\right)(a+b)}{k^3+1}$

RHS = $\frac{(x+y)^4+(a+b)^4}{(x+y)^3+(a+b)^3}$

Substituting the values

$= \frac{(ak+bk)^4 + (a+b)^4}{(ak+bk)^3 + (a+b)^3}$

Taking out the common terms

$= \frac{k^4(a+b)^4+(a+b)^4}{k^3(a+b)^3(a+b)^3}$

We get

$= \frac{(a+b)^4(k^4+1)}{(a+b)^3(k^3+1)}$

We can write it as

$= \frac{(a+b)(k^4+1)}{k^3+1}$

Here LHS = RHS

Therefore, it is proved.

16. If $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$ prove that each ratio's equal to : $\frac{x+y+z}{a+b+c}$

Solution:

Consider

$\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c} = k$

So we get

$x = k(b + c - a)$

$y = k(c + a - b)$

$z = k(a + b - a)$

Here

$\frac{x+y+z}{a+b+c} = \frac{k(b+C-a)+k(c+a-b)+k(a+b-c)}{a+b+c}$

By further calculation

$= \frac{k\ (b+c-a+c+a-b+a+b-c)}{a+b+c}$

So we get

$= \frac{k(a+b+c)}{a+b+c}$

$= \text{k}$

Therefore, it is proved.

17. If a : b = 9 : 10 find the value of

(i) $\frac{5a+3b}{5a-3b}$

(ii) $\frac{2a^2-3b^2}{2a^2+3b^2}$

Solution:

It is given that

$a: b = 9 : 10$

So we get

$\frac{a}{b} = \frac{9}{10}$

(i) $\boldsymbol{\frac{5a+3b}{5a-3b} = \frac{\frac{5a}{b}+\frac{3b}{b}}{\frac{5a}{b}-\frac{3b}{b}}}$

By further calculation

$= \frac{\frac{5a}{b}+3}{\frac{5a}{b}-3}$

Substituting the values of $\frac{a}{b}$

$= \frac{5 \times \frac{9}{10}+3}{5 \times \frac{9}{10}-3}$

So we get

$$= \frac{\frac{9}{2}+3}{\frac{9}{2}-3}$$

$$= \frac{\frac{15}{2}}{\frac{3}{2}}$$

By further simplification

$$= \frac{15}{2} \times \frac{2}{3}$$

$$= 5$$

(ii) $\boldsymbol{\frac{2a^2-3b^2}{2a^2+3b^2}}$

Dividing by b^2

$$= \frac{\frac{2a^2}{b^2}-\frac{3b^2}{b^2}}{\frac{2a^2}{b^2}+\frac{3b^2}{b^2}}$$

By further calculation

$$= \frac{2\left(\frac{a}{b}\right)^2-3}{2\left(\frac{a}{b}\right)^2+3}$$

Substituting the values of $\frac{a}{b}$

$$= \frac{2\left(\frac{9}{10}\right)^2-3}{2\left(\frac{9}{10}\right)^2+3}$$

So we get

$$= \frac{2 \times \frac{81}{100}-3}{2 \times \frac{81}{100}+3}$$

$$= \frac{\frac{81}{50} - 3}{\frac{81}{50} + 3}$$

By further simplification

$$= \frac{\frac{81-150}{50}}{\frac{81+150}{50}}$$

We get

$$= \frac{-69}{50} \times \frac{50}{231}$$

$$= \frac{-69}{231}$$

$$= \frac{-23}{77}$$

18. If $(3x^2 + 2y^2) : (3x^2 - 2y^2) = 11 : 9$, find the value of $\frac{3x^4+25y^4}{3x^4-25y^4}$

Solution:

It is given that

$$(3x^2 + 2y^2) : (3x^2 - 2y^2) = 11 : 9$$

We can write it as

$$\frac{(3x^2 + 2y^2)}{(3x^2 - 2y^2)} = \frac{11}{9}$$

By applying componendo and dividendo

$$\frac{3x^2 + 2y^2 + 3x^2 - 2y^2}{3x^2 + 2y^2 - 3x^2 + 2y^2} = \frac{11+9}{11-9}$$

By further calculation

$$\frac{6x^2}{4y^2} = \frac{20}{2}$$

$$\frac{3x^2}{2y^2} = 10$$

We can write it as

$$\frac{x^2}{y^2} = 10 \times \frac{2}{3} = \frac{20}{3}$$

Here

$$\frac{3x^4+25y^4}{3x^4-25y^4} = \frac{\frac{3x^4}{y^4}+\frac{25y^4}{y^4}}{\frac{3x^4}{y^4}-\frac{25y^4}{y^4}}$$

We can write it as

$$= \frac{3\left(\frac{x^2}{y^2}\right)^2+25}{3\left(\frac{x^2}{y^2}\right)^2-25}$$

By substituting the values

$$= \frac{3\left(\frac{20}{3}\right)^2+25}{3\left(\frac{20}{3}\right)^2-25}$$

By further calculation

$$= \frac{3 \times \frac{400}{9}+25}{3 \times \frac{400}{9}-25}$$

Taking LCM

$$= \frac{\frac{400+75}{3}}{\frac{400-75}{3}}$$

So we get

$= \frac{475}{3} \times \frac{3}{325}$

$= \frac{19}{13}$

19. If $x = \frac{2mab}{a+b}$, find the value of $\frac{x+ma}{x-ma} + \frac{x+mb}{x-mb}$

Solution:

It is given that

$$x = \frac{2mab}{a+b}$$

we can write it as

$= \frac{x}{ma} + \frac{2b}{a+b}$

By applying componendo and dividendo

$= \frac{x+ma}{x-ma} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a}$ …..(1)

Similarly

$= \frac{x}{mb} = \frac{2a}{a+b}$

By applying componendo and dividendo

$= \frac{x+mb}{x-mb} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b}$ …..(2)

Now adding both the equations

$= \frac{x+ma}{x-ma} - \frac{x+mb}{x-mb} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$

By further calculations

$= \frac{-3b+a}{a-b} + \frac{3a+b}{a-b}$

So we get

$= \frac{-3b-a+3a+b}{a-b}$

$= \frac{2a-2b}{a-b}$

Taking out 2 as common

$= \frac{2\,(a-b)}{a-b}$

= 2

20. If $x = \frac{pab}{a+b},\ prove\ that\ \frac{x+pa}{x-pa} - \frac{x+pb}{x-pb} = \frac{2(a^2-b^2)}{ab}$

Solution:

It is given that :

$x = \frac{pab}{a+b}$

we can write is as

$= \frac{x}{pa} + \frac{b}{a+b}$

By applying componendo and dividendo

$\frac{x+pa}{x-pa} - \frac{x+pb}{x-pb} = \frac{2(a^2-b^2)}{ab}$ ….(1)

Similarly,

$\frac{x}{pb} = \frac{a}{a+b}$

By applying componendo and dividendo

$\frac{x+pb}{x-pb} = \frac{a+a+b}{a-a-b} = \frac{2a+b}{-b}$ ….(2)

We know that

LHS = $\frac{x+pa}{x-pa} - \frac{x+pb}{x-pb}$

Using both the equations

$= \frac{a+2b}{-a} - \frac{2a+b}{-b}$

$= \frac{a+2b}{-a} + \frac{2a+b}{b}$

Taking LCM

$= \frac{ab+2b^2-2a^2-ab}{-ab}$

$= \frac{2b^2-2a^2}{-ab}$

So we get

$= \frac{-2a^2+2b^2}{-ab}$

Taking out 2 as common

$= \frac{-2\,(a^2-b^2)}{-ab}$

$= \frac{2\,(a^2-b^2)}{ab}$

= RHS.

21. Find x from the equation $\frac{a+x+\sqrt{a^2-x^2}}{a+x-\sqrt{a^2-x^2}} = \frac{b}{x}$

Solution:

It is given that

$$\frac{a+x+\sqrt{a^2-x^2}}{a+x-\sqrt{a^2-x^2}} = \frac{b}{x}$$

By applying componendo and dividendo

$$\frac{a+x+\sqrt{a^2-x^2}+a+x-\sqrt{a^2-x^2}}{a+x+\sqrt{a^2-x^2}-a-x+\sqrt{a^2-x^2}} = \frac{b+x}{b-x}$$

By further calculation

$$\frac{2(a+x)}{2\sqrt{a^2-x^2}} = \frac{b+x}{b-x}$$

Dividing by 2

$$\frac{(a+x)}{\sqrt{a^2-x^2}} = \frac{b+x}{b-x}$$

By squaring on both sides

$$\frac{(a+x)^2}{a^2-x^2} = \frac{(b+x)^2}{(b-x)^2}$$

We can write it as

$$\frac{(a+x)^2}{(a+x)(a-x)} = \frac{(b+x)^2}{(b-x)^2}$$

$$\frac{a+x}{a-x} = \frac{(b+x)^2}{(b-x)^2}$$

By applying componendo and dividendo

$$\frac{a+x+a-x}{a+x-a+x} = \frac{(b+x)^2-(b-x)^2}{(b+x)^2-(b-x)^2}$$

By further calculation

$$\frac{2a}{2x} = \frac{2\left(b^2+x^2\right)}{4bx}$$

Dividing by 2

$$\frac{a}{x} = \frac{\left(b^2+x^2\right)}{2bx}$$

By cross multiplication

$$2abx = x(b^2 + x^2)$$

$$2ab = (b^2 + x^2)$$

$$x^2 = 2ab - b^2$$

$$x = \sqrt{2ab - b^2}$$

22. If $\boldsymbol{x = \frac{\sqrt[3]{a+1}+\sqrt[3]{a-1}}{\sqrt[3]{a+1}+\sqrt[3]{a-1}}}$ $\boldsymbol{, Prove\ that: \ x^3 - 3ax^2 + 3x - a = 0}$.

Solution:

It is given that

$$x = \frac{\sqrt[3]{a+1}+\sqrt[3]{a-1}}{\sqrt[3]{a+1}+\sqrt[3]{a-1}}$$

By applying componendo and dividendo

$$\frac{x+1}{x-1} = \frac{\sqrt[3]{a+1}+\sqrt[3]{a-1}+\sqrt[3]{a+1}-\sqrt[3]{a-1}}{\sqrt[3]{a+1}-\sqrt[3]{a-1}-\sqrt[3]{a+1}+\sqrt[3]{a-1}}$$

On further calculation

$$\frac{x+1}{x-1} = \frac{\mathbf{2}\sqrt[3]{\boldsymbol{a}+\mathbf{1}}}{\mathbf{2}\sqrt[3]{\boldsymbol{a}+\mathbf{1}}}$$

$$\frac{x+1}{x-1} = \frac{\sqrt[3]{\boldsymbol{a}+\mathbf{1}}}{\sqrt[3]{\boldsymbol{a}+\mathbf{1}}}$$

By cubing on both sides

$$\frac{(x+1)^3}{(x-1)^3} = \frac{a+1}{a-1}$$

By applying componendo and dividendo

$$\frac{(x+1)^3+(x-1)^3}{(x+1)^3-(x-1)^3} = \frac{a+1+a-1}{a+1-a-1}$$

By further calculation

$$\frac{2(x^3-3x)}{2(3x^2+1)} = \frac{2a}{2}$$

Dividing by 2

$$\frac{(x^3+3x)}{(3x^2+1)} = \frac{a}{1}$$

By cross multiplication

$x^3 + 3x = 3ax^2 + a$

Therefore, it is proved.