CHAPTER

9.6

PROBABILITY AND STATISTICS

1. In a frequency distribution, the mid value of a class is 15 and the class interval is 4. The lower limit of the class is

(A) 14 (B) 13
(C) 12 (D) 10

2. The mid value of a class interval is 42. If the class size is 10, then the upper and lower limits of the class are

(A) 47 and 37 (B) 37 and 47
(C) 37.5 and 47.5 (D) 47.5 and 37.5

3. The following marks were obtained by the students in a test: 81, 72, 90, 90, 86, 85, 92, 70, 71, 83, 89, 95, 85,79, 62. The range of the marks is

(A) 9 (B) 17
(C) 27 (D) 33

4. The width of each of nine classes in a frequency distribution is 2.5 and the lower class boundary of the lowest class is 10.6. The upper class boundary of the highest class is

(A) 35.6 (B) 33.1
(C) 30.6 (D) 28.1

5. In a monthly test, the marks obtained in mathematics by 16 students of a class are as follows: 0, 0, 2, 2, 3, 3, 3, 4, 5, 5, 5, 5, 6, 6, 7, 8

The arithmetic mean of the marks obtained is

(A) 3 (B) 4
(C) 5 (D) 6

6. A distribution consists of three components with frequencies 45, 40 and 15 having their means 2, 2.5 and 2 respectively. The mean of the combined distribution is

(A) 2.1 (B) 2.2
(C) 2.3 (D) 2.4

7. Consider the table given below

Marks	Number of Students
0 – 10	12
10 – 20	18
20 – 30	27
30 – 40	20
40 – 50	17
50 – 60	6

The arithmetic mean of the marks given above, is

(A) 18 (B) 28
(C) 27 (D) 6

8. The following is the data of wages per day: 5, 4, 7, 5, 8, 8, 8, 5, 7, 9, 5, 7, 9, 10, 8 The mode of the data is

(A) 5 (B) 7
(C) 8 (D) 10

9. The mode of the given distribution is

Weight (in kg)	40	43	46	49	52	55
Number of Children	5	8	16	9	7	3

(A) 55 (B) 46
(C) 40 (D) None

10. If the geometric mean of x, 16, 50, be 20, then the value of x is

(A) 4 (B) 10

(C) 20 (D) 40

11. If the arithmetic mean of two numbers is 10 and their geometric mean is 8, the numbers are

(A) 12, 18 (B) 16, 4

(C) 15, 5 (D) 20, 5

12. The median of

0, 2, 2, 2, –3, 5, –1, 5, 5, –3, 6, 6, 5, 6 is

(A) 0 (B) –1.5

(C) 2 (D) 3.5

13. Consider the following table

Diameter of heart (in mm)	Number of persons
120	5
121	9
122	14
123	8
124	5
125	9

The median of the above frequency distribution is

(A) 122 mm (B) 123 mm

(C) 122.5 mm (D) 122.75 mm

14. The mode of the following frequency distribution, is

Class interval	Frequency
3–6	2
6–9	5
9–12	21
12–15	23
15–18	10
18–21	12
21–24	3

(A) 11.5 (B) 11.8

(C) 12 (D) 12.4

15. The mean-deviation of the data 3, 5, 6, 7, 8, 10, 11, 14 is

(A) 4 (B) 3.25

(C) 2.75 (D) 2.4

16. The mean deviation of the following distribution is

x	10	11	12	13	14
f	3	12	18	12	3

(A) 12 (B) 0.75

(C) 1.25 (D) 26

17. The standard deviation for the data 7, 9, 11, 13, 15 is

(A) 2.4 (B) 2.5

(C) 2.7 (D) 2.8

18. The standard deviation of 6, 8, 10, 12, 14 is

(A) 1 (B) 0

(C) 2.83 (D) 2.73

19. The probability that an event A occurs in one trial of an experiment is 0.4. Three independent trials of experiment are performed. The probability that A occurs at least once is

(A) 0.936 (B) 0.784

(C) 0.964 (D) None

20. Eight coins are tossed simultaneously. The probability of getting at least 6 heads is

(A) $\frac{7}{64}$ (B) $\frac{37}{256}$

(C) $\frac{57}{64}$ (D) $\frac{249}{256}$

21. A can solve 90% of the problems given in a book and B can solve 70%. What is the probability that at least one of them will solve a problem, selected at random from the book?

(A) 0.16 (B) 0.63

(C) 0.97 (D) 0.20

22. A speaks truth in 75% and B in 80% of the cases. In what percentage of cases are they likely to contradict each other narrating the same incident ?

(A) 5% (B) 45%

(C) 35% (D) 15%

23. The odds against a husband who is 45 years old, living till he is 70 are 7:5 and the odds against his wife who is 36, living till she is 61 are 5:3. The probability that at least one of them will be alive 25 years hence, is

(A) $\frac{61}{96}$ (B) $\frac{5}{32}$

(C) $\frac{13}{64}$ (D) None

24. The probability that a man who is x years old will die in a year is p. Then amongst n persons $A_1, A_2, \ldots, A_n$ each x years old now, the probability that A_1 will die in one year is

(A) $\frac{1}{n^2}$ (B) $1-(1-p)^n$

(C) $\frac{1}{n^2}[1-(1-p)^n]$ (D) $\frac{1}{n}[1-(1-p)^n]$

25. A bag contains 4 white and 2 black balls. Another bag contains 3 white and 5 black balls. If one ball is drawn from each bag, the probability that both are white is

(A) $\frac{1}{24}$ (B) $\frac{1}{4}$

(C) $\frac{5}{24}$ (D) None

26. A bag contains 5 white and 4 red balls. Another bag contains 4 white and 2 red balls. If one ball is drawn from each bag, the probability that one is white and one is red, is

(A) $\frac{13}{27}$ (B) $\frac{5}{27}$

(C) $\frac{8}{27}$ (D) None

27. An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. The probability that the gun hits the plane is

(A) 0.76 (B) 0.4096

(C) 0.6976 (D) None of these

28. If the probabilities that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is

(A) pq (B) $p(1-q)$

(C) $q(1-p)$ (D) $p+1-2pq$

29. In a binomial distribution, the mean is 4 and variance is 3. Then, its mode is

(A) 5 (B) 6

(C) 4 (D) None

30. If 3 is the mean and (3/2) is the standard deviation of a binomial distribution, then the distribution is

(A) $\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$ (B) $\left(\frac{1}{2}+\frac{3}{2}\right)^{12}$

(C) $\left(\frac{4}{5}+\frac{1}{5}\right)^{60}$ (D) $\left(\frac{1}{5}+\frac{4}{5}\right)^{5}$

31. The sum and product of the mean and variance of a binomial distribution are 24 and 18 respectively. Then, the distribution is

(A) $\left(\frac{1}{7}+\frac{1}{8}\right)^{12}$ (B) $\left(\frac{1}{4}+\frac{3}{4}\right)^{16}$

(C) $\left(\frac{1}{6}+\frac{5}{6}\right)^{24}$ (D) $\left(\frac{1}{2}+\frac{1}{2}\right)^{32}$

32. A die is thrown 100 times. Getting an even number is considered a success. The variance of the number of successes is

(A) 50 (B) 25

(C) 10 (D) None

33. A die is thrown thrice. Getting 1 or 6 is taken as a success. The mean of the number of successes is

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$

(C) 1 (D) None

34. If the sum of mean and variance of a binomial distribution is 4.8 for five trials, the distribution is

(A) $\left(\frac{1}{5}+\frac{4}{5}\right)^{5}$ (B) $\left(\frac{1}{3}+\frac{2}{3}\right)^{5}$

(C) $\left(\frac{2}{5}+\frac{3}{5}\right)^{5}$ (D) None of these

35. A variable has Poission distribution with mean m. The probability that the variable takes any of the values 0 or 2 is

(A) $e^{-m}\left(1+m+\frac{m^2}{2!}\right)$ (B) $e^{m}(1+m)^{-3/2}$

(C) $e^{3/2}(1+m^2)^{-1/2}$ (D) $e^{-m}\left(1+\frac{m^2}{2!}\right)$

36. If X is a Poission variate such that $P(2)=9P(4)+90P(6)$, then the mean of X is

(A) ± 1 (B) ± 2

(C) ± 3 (D) None

37. When the correlation coefficient $r = \pm 1$, then the two regression lines

(A) are perpendicular to each other
(B) coincide
(C) are parallel to each other
(D) do not exist

38. If $r = 0$, then

(A) there is a perfect correlation between x and y
(B) x and y are not correlated.
(C) there is a positive correlation between x and y
(D) there is a negative correlation between x and y

39. If $\Sigma x_i = 15$, $\Sigma y_i = 36$, $\Sigma x_i y_i = 110$ and $n = 5$, then cov (x, y) is equal to

(A) 0.6 (B) 0.5
(C) 0.4 (D) 0.225

40. If cov $(x, y) = -16.5$, var $(x) = 2.89$ and var $(y) = 100$, then the coefficient of correlation r is equal to

(A) 0.36 (B) –0.64
(C) 0.97 (D) –0.97

41. The ranks obtained by 10 students in Mathematics and Physics in a class test are as follows

Rank in Maths	Rank in Chem.
1	3
2	10
3	5
4	1
5	2
6	9
7	4
8	8
9	7
10	6

The coefficient of correlation between their ranks is

(A) 0.15 (B) 0.224
(C) 0.625 (D) None

42. If $\Sigma x_i = 24$, $\sum y_i = 44$, $\Sigma x_i y_i = 306$, $\sum x_i^2 = 164$, $\sum y_i^2 = 574$ and $n = 4$, then the regression coefficient b_{yx} is equal to

(A) 2.1 (B) 1.6
(C) 1.225 (D) 1.75

43. If $\Sigma x_i = 30$, $\sum y_i = 42$, $\sum x_i y_i = 199$, $\sum x_i^2 = 184$, $\sum y_i^2 = 318$ and $n = 6$, then the regression coefficient b_{xy} is

(A) –0.36 (B) –0.46
(C) 0.26 (D) None

44. Let r be the correlation coefficient between x and y and b_{yx}, b_{xy} be the regression coefficients of y on x and x on y respectively then

(A) $r = b_{xy} + b_{yx}$ (B) $r = b_{xy} \times b_{yx}$
(C) $r = \sqrt{b_{xy} \times b_{yx}}$ (D) $r = \frac{1}{2}(b_{xy} + b_{yx})$

45. Which one of the following is a true statement.

(A) $\frac{1}{2}(b_{xy} + b_{yx}) = r$ (B) $\frac{1}{2}(b_{xy} + b_{yx}) < r$
(C) $\frac{1}{2}(b_{xy} + b_{yx}) > r$ (D) None of these

46. If $b_{yx} = 1.6$ and $b_{xy} = 0.4$ and θ is the angle between two regression lines, then $\tan\theta$ is equal to

(A) 0.18 (B) 0.24
(C) 0.16 (D) 0.3

47. The equations of the two lines of regression are : $4x + 3y + 7 = 0$ and $3x + 4y = 8 = 0$. The correlation coefficient between x and y is

(A) 1.25 (B) 0.25
(C) –0.75 (D) 0.92

48. If $\text{cov}(X, Y) = 10$, var $(X) = 6.25$ and $\text{var}(Y) = 31.36$, then $\rho(X, Y)$ is

(A) $\frac{5}{7}$ (B) $\frac{4}{5}$
(C) $\frac{3}{4}$ (D) 0.256

49. If $\sum x = \sum y = 15$, $\sum x^2 = \sum y^2 = 49$, $\sum xy = 44$ and $n = 5$, then $b_{xy} = ?$

(A) $-\frac{1}{3}$ (B) $-\frac{2}{3}$
(C) $-\frac{1}{4}$ (D) $-\frac{1}{2}$

50. If $\sum x = 125$, $\sum y = 100$, $\sum x^2 = 1650$, $\sum y^2 = 1500$, $\sum xy = 50$ and $n = 25$, then the line of regression of x on y is

(A) $22x + 9y = 146$ (B) $22x - 9y = 74$
(C) $22x - 9y = 146$ (D) $22x + 9y = 74$

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SOLUTION

1. (B) Let the lower limit be x. Then, upper limit is $x+4$. $\frac{x+(x+4)}{2}=15 \Rightarrow x=13$.

2. (A) Let the lower limit be x. Then, upper limit $x+10$.

$\frac{x+(x+10)}{2}=42 \Rightarrow x=37$.

Lower limit $=37$ and upper limit $=47$.

3. (D) Range = Difference between the largest value $=(95-62)=33$.

4. (B) Upper class boundary $=10.6+(2.5\times 9)=33.1$.

5. (B)

Marks	Frequency f	$f\times 1$
0	2	0
2	2	4
3	3	9
4	1	4
5	4	20
6	2	12
7	1	7
8	1	8
	$\Sigma f=16$	$\Sigma(f\times x)=64$

A.M. $=\frac{\Sigma(f\times x)}{\Sigma f}=\frac{64}{16}=4$.

6. (B) Mean $=\frac{45\times 2+40\times 2.5+15\times 2}{100}=\frac{220}{100}=2.2$.

7. (B)

Class	Mid value x	Frequency f	Deviation $d=x-A$	$f\times d$
0–10	5	12	–20	–240
10–20	15	18	–10	–180
20–30	$25=A$	27	0	0
30–40	35	20	10	200
40–50	45	17	20	320
50–60	55	6	30	180
		$\Sigma f=100$		$\Sigma(f\times d)=390$

A.M. $=A+\frac{\Sigma(fd)}{\Sigma f}=\left(25+\frac{300}{100}\right)=28$.

8. (C) Since 8 occurs most often, mode =8.

9. (B) Clearly, 46 occurs most often. So, mode =46.

10. (B) $(x\times 16\times 50)^{1/3}=20 \Rightarrow x\times 16\times 50=(20)^3$

$\Rightarrow x=\left(\frac{20\times 20\times 20}{16\times 50}\right)=10$.

11. (B) Let the numbers be a and b Then,

$\frac{a+b}{2}=10 \Rightarrow (a+b)=20$ and

$\sqrt{ab}=8 \Rightarrow ab=64$

$a-b=\sqrt{(a+b)^2-4ab}=\sqrt{44-256}=\sqrt{144}=12$.

Solving $a+b=20$ and $a-b=12$ we get $a=16$ and $b=4$.

12. (D) Observations in ascending order are

–3, –3, –1, 0, 2, 2, 2, 5, 5, 5, 5 6, 6, 6

Number of observations is 14, which is even.

Median $=\frac{1}{2}$[7 the term +8 the term] $=\frac{1}{2}(2+5)=3.5$.

13. (A) The given Table may be presented as

Diameter of heart (in mm)	Number of persons	Cumulative frequency
120	5	5
121	9	14
122	14	28
123	8	36
124	5	41
125	9	50

Here $n=50$. So, $\frac{n}{2}=25$ and $\frac{n}{2}+1=26$.

Medium $=\frac{1}{2}$(25th term +26 th term) $=\frac{122+122}{2}=122$.

[... Both lie in that column whose c.f. is 28]

14. (B) Maximum frequency is 23. So, modal class is 12–15.

$L_1=12$, $L_2=15$, $f=23$, $f_1=21$ and $f_2=10$.

Thus Mode $=L_1+\frac{f-f_1}{2f-f_1-f_2}(L_2-L_1)$

$= 12 + \frac{(23-21)}{(46-21-10)}(15-12) = 12.4.$

15. (C) Mean $= \left(\frac{3+5+6+7+8+10+11+14}{8}\right) = 8.$

$\Sigma\delta = |3-8| + |5-8| + |8-8| + |10-8| + |11-8| + |14-8|$
$= 22$

Thus Mean deviation $= \frac{\Sigma\delta}{n} = \frac{22}{8} = 2.75.$

16. (B)

x	f	$f \times x$	$\delta = \|x - M\|$	$f \times \delta$
10	3	30	2	6
11	12	132	1	12
12	18	216	0	0
13	12	156	1	12
14	3	42	2	6
	$\Sigma f = 48$	$\Sigma fx = 576$		$\Sigma f\delta = 36$

Thus $M = \frac{576}{48} = 12.$

So, Mean deviation $= \frac{\Sigma f\delta}{n} = \frac{36}{48} = 0.75$

17. (D) $m = \frac{7+9+11+13+15}{5} = \frac{55}{5} = 11.$

$\Sigma\delta^2 = |7-11|^2 + |9-11|^2 + |11-11|^2 + |13-11|^2 + |15-11|^2 = 40$

$\sigma = \sqrt{\frac{\Sigma\delta^2}{n}} = \sqrt{\frac{40}{5}} = \sqrt{8} = 2\sqrt{2} = 2 \times 1.41 = 2.8.$

18. (C) $M = \frac{6+8+10+12+14}{5} = \frac{50}{5} = 10.$

$\Sigma\delta^2 = |6-10|^2 + |8-10|^2 + |10-10|^2 + |12-10|^2 + |14-10|^2 = 40$

$6 = \sqrt{\frac{\Sigma\delta^2}{n}} = \sqrt{\frac{40}{5}}$

$= \sqrt{8} = 2\sqrt{2} = 2 \times 1.414 = 2.83$ (app.)

19. (B) Here $p = 0.4$, $q = 0.6$ and $n = 3$.

Required probability $= P$(A occurring at least once)

$= {}^3C_1 \cdot (0.4) \times (0.6)^2 + {}^3C_2 \cdot (0.4)^2 \times (0.6) + {}^3C_3 \cdot (0.4)^3$

$= \left(3 \times \frac{4}{10} \times \frac{36}{100} + 3 \times \frac{16}{100} \times \frac{6}{10} + \frac{64}{1000}\right) = \frac{784}{1000} = 0.784.$

20. (B) $p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 8$. Required probability

$= P$ (6 heads or 7 heads or 8 heads)

$= {}^8C_6 \cdot \left(\frac{1}{2}\right)^6 \cdot \left(\frac{1}{2}\right)^2 + {}^8C_7 \cdot \left(\frac{1}{2}\right)^7 \cdot \frac{1}{2} + {}^8C_8 \cdot \left(\frac{1}{2}\right)^8$

$= \frac{8 \times 7}{2 \times 1} \times \frac{1}{256} + 8 \times \frac{1}{256} + \frac{1}{256} = \frac{37}{256}$

21. (C) Let E = the event that A solves the problem. and F = the event that B solves the problem.

Clearly E and F are independent events.

$P(E) = \frac{90}{100} = 0.9$, $P(F) = \frac{70}{100} = 0.7$,

$P(E \equiv F) = P(E) \cdot P(F) = 0.9 \times 0.7 = 0.63$

Required probability $= P(E \cup F)$

$= P(E) + P(F) - P(E \equiv F) = (0.9 + 0.7 - 0.63) = 0.97.$

22. (C) Let E =event that A speaks the truth.

F =event that B speaks the truth.

Then, $P(E) = \frac{75}{100} = \frac{3}{4}$, $P(F) = \frac{80}{100} = \frac{4}{5}$

$P(\overline{E}) = \left(1 - \frac{3}{4}\right) = \frac{1}{4}$, $P(\overline{F}) = \left(1 - \frac{4}{5}\right) = \frac{1}{5}$

P (A and B contradict each other).

$= P$[(A speaks truth and B tells a lie) or (A tells a lie and B speaks the truth)]

$= P(E \text{ and } \overline{F}) + P(\overline{E} \text{ and } F)$

$= P(E) \cdot P(\overline{F}) + P(\overline{E}) \cdot P(F)$

$= \frac{3}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{4}{5} = \frac{3}{20} + \frac{1}{5} = \frac{7}{20} = \left(\frac{7}{20} \times 100\right)\% = 35\%.$

23. (A) Let E = event that the husband will be alive 25 years hence and F =event that the wife will be alive 25 years hence.

Then, $P(E) = \frac{5}{12}$ and $P(F) = \frac{3}{8}$

Thus $P(\overline{E}) = \left(1 - \frac{5}{12}\right) = \frac{7}{12}$ and $P(F) = \left(1 - \frac{3}{8}\right) = \frac{5}{8}$.

Clearly, E and F are independent events.

So, $\overline{E}$ and $\overline{F}$ are independent events.

P(at least one of them will be alive 25 years hence)

$= 1 - P$(none will be alive 24 years hence)

$= 1 - P(\overline{E} \equiv \overline{F}) = 1 - P(\overline{E}) \cdot P(\overline{F}) = \left(1 - \frac{7}{12} \times \frac{5}{8}\right) = \frac{61}{96}$

24. (D) P(none dies)

$= (1-p)(1-p)\ldots.n$ times $= (1-p)^n$

P(at least one dies) $= 1 - (1-p)^n$.

$P(A_1 \text{ dies}) = \frac{1}{n}\{1 - (1-p)^n\}.$

39. (C) $\bar{x} = \frac{\Sigma x_i}{n} = \frac{15}{5} = 3$, $\bar{y} = \frac{\Sigma y_i}{n} = \frac{36}{5} = 7.2$

$$\text{cov}(x, y) = \left(\frac{\Sigma x_i y_i}{n} - \bar{x}\,\bar{y}\right) = \left(\frac{110}{5} - 3 \times 7.2\right) = 0.4$$

40. (D) $r = \frac{\text{cov}(x, y)}{\sqrt{\text{var}(x) \cdot var(y)}} = \frac{-16.5}{\sqrt{2.89 \times 100}} = -0.97.$

41. (B) $D_i = -2, -8, -2, 3, 3, -3, 3, 0, 2, 4.$

$\Sigma D_i^2 = (4 + 64 + 4 + 9 + 9 + 9 + 9 + 0 + 4 + 16) = 128.$

$$R = \left[1 - \frac{6(\Sigma D_i^2)}{n(n^2 - 1)}\right] = \left(1 - \frac{6 \times 128}{10 \times 99}\right) = \frac{37}{165} = 0.224.$$

42. (A) $b_{yx} = \dfrac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\left[\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right]}$

$$= \frac{\left(306 - \frac{24 \times 44}{4}\right)}{\left[164 - \frac{(24)^2}{4}\right]} = \frac{(306 - 264)}{(164 - 144)} = \frac{42}{20} = 2.1$$

43. (B) $b_{yx} = \dfrac{\left[\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}\right]}{\left[\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right]} = \dfrac{\left(199 - \frac{30 \times 42}{6}\right)}{\left[318 - \frac{42 \times 42}{6}\right]}$

$$= \frac{(199 - 210)}{(318 - 294)} = \frac{-11}{24} = -0.46.$$

44. (C) $b_{yx} = r \cdot \frac{\sigma y}{\sigma x}$ and $b_{xy} = r \cdot \frac{\sigma x}{\sigma y}$

$r^2 = b_{xy} \times b_{yx} \Rightarrow r = \sqrt{b_{xy} \times b_{yx}}$.

45. (C) $\frac{1}{2}(b_{xy} + b_{yx}) > r$ is true if $\frac{1}{2}\left[r \cdot \frac{\sigma y}{\sigma x} + r \cdot \frac{\sigma x}{\sigma y}\right] > r$

i.e. if $\sigma_y^2 + \sigma_x^2 > 2\,\sigma_x\,\sigma_y$

i.e. if $(\sigma_y - \sigma_x)^2 > 0$, which is true.

46. (A) $r = \sqrt{1.6 \times 0.4} = \sqrt{.64} = 0.8$

$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} \Rightarrow \frac{\sigma_y}{\sigma_x} = \frac{b_{yx}}{r} = \frac{1.6}{0.8} = 2$

$m_1 = \frac{1}{r} \cdot \frac{\sigma_y}{\sigma_x} = \frac{1}{0.8} \times 2 = \frac{5}{2}$, $m_2 = r \cdot \frac{\sigma_y}{\sigma_x} = 0.8 \times 2 = 1.6.$

$$\tan\theta = \left(\frac{m_1 - m_2}{1 + m_1 m_2}\right) = \left(\frac{2.5 - 1.6}{1 + 2.5 \times 1.6}\right) = \frac{0.9}{5} = 0.18.$$

47. (C) Given lines are : $y = -2 - \frac{3}{4}x$

and $x = \left(-\frac{7}{4} - \frac{3}{4}y\right)$

$b_{yx} = \frac{-3}{4}$ and $b_{xy} = \frac{-3}{4}$.

So, $r^2 = \left(\frac{-3}{4} \times \frac{-3}{4}\right) = \frac{9}{16}$ or $r = -\frac{3}{4} = -0.75.$

[$\because$ b_{yx} and b_{xy} are both negative $\rightarrow$ r is negative]

48. (A) $\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\,\text{var}(Y)}} = \frac{10}{\sqrt{6.25 \times 31.36}} = \frac{5}{7}$

49. (C) $b_{yx} = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}$

$$= \left(\frac{5 \times 44 - 15 \times 15}{5 \times 49 - 15 \times 15}\right) = -\frac{1}{4}$$

50. (B) $b_{xy} = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma y^2 - (\Sigma y)^2}$

$$= \frac{25 \times 50 - 125 \times 100}{25 \times 1500 - 100 \times 100} = \frac{9}{22}$$

Also, $\bar{x} = \frac{125}{25} = 5$, $\bar{y} = \frac{100}{25} = 4.$

Required line is $x = \bar{x} + b_{xy}(y - \bar{y})$

$\Rightarrow x = 5 + \frac{9}{22}(y - 4) \Rightarrow 22x - 9y = 74.$

(B) $\frac{x^2}{2}+\frac{x^5}{20}+\frac{x^8}{160}+\frac{x^{11}}{4400}$

(C) $\frac{x^2}{2}+\frac{x^5}{20}+\frac{x^8}{160}+\frac{x^{11}}{2400}$

(D) $\frac{x^2}{2}+\frac{x^5}{40}+\frac{x^8}{480}+\frac{x^{11}}{2400}$

12. For $dy/dx = xy$ given that $y=1$ at $x=0$. Using Euler method taking the step size 0.1, the y at $x=0.4$ is

(A) 1.0611 (B) 2.4680

(C) 1.6321 (D) 2.4189

Statement for Q. 13–15.

For $dy/dx = x^2 + y^2$ given that $y=1$ at $x=0$. Determine the value of y at given x in question using modified method of Euler. Take the step size 0.02.

13. y at $x=0.02$ is

(A) 1.0468 (B) 1.0204

(C) 1.0346 (D) 1.0348

14. y at $x=0.04$ is

(A) 1.0316 (B) 1.0301

(C) 1.403 (D) 1.0416

15. y at $x=0.06$ is

(A) 1.0348 (B) 1.0539

(C) 1.0638 (D) 1.0796

16. For $dy/dx = x + y$ given that $y=1$ at $x=0$. Using modified Euler's method taking step size 0.2, the value of y at $x=1$ is

(A) 3.401638 (B) 3.405417

(C) 9.164396 (D) 9.168238

17. For the differential equation $dy/dx = x - y^2$ given that

x:	0	0.2	0.4	0.6
y:	0	0.02	0.0795	0.1762

Using Milne predictor–correction method, the y at next value of x is

(A) 0.2498 (B) 0.3046

(C) 0.4648 (D) 0.5114

Statement for Q. 18–19:

For $\frac{dy}{dx}=1+y^2$ given that

x:	0	0.2	0.4	0.6
y:	0	0.2027	0.4228	0.6841

Using Milne's method determine the value of y for x given in question.

18. $y(0.8)=?$

(A) 1.0293 (B) 0.4228

(C) 0.6065 (D) 1.4396

19. $y(1.0)=?$

(A) 1.9428 (B) 1.3428

(C) 1.5555 (D) 2.168

Statement for Q.20–22:

Apply Runge Kutta fourth order method to obtain $y(0.2)$, $y(0.4)$ and $y(0.6)$ from $dy/dx = 1 + y^2$, with $y=0$ at $x=0$. Take step size $h=0.2$.

20. $y(0.2)=?$

(A) 0.2027 (B) 0.4396

(C) 0.3846 (D) 0.9341

21. $y(0.4)=?$

(A) 0.1649 (B) 0.8397

(C) 0.4227 (D) 0.1934

22. $y(0.6)=?$

(A) 0.9348 (B) 0.2935

(C) 0.6841 (D) 0.563

23. For $dy/dx = x + y^2$, given that $y=1$ at $x=0$. Using Runge Kutta fourth order method the value of y at $x=0.2$ is $(h=0.2)$

(A) 1.2735 (B) 2.1635

(C) 1.9356 (D) 2.9468

24. For $dy/dx = x + y$ given that $y=1$ at $x=0$. Using Runge Kutta fourth order method the value of y at $x=0.2$ is $(h=0.2)$

(A) 1.1384 (B) 1.9438

(C) 1.2428 (D) 1.6389

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SOLUTIONS

1. (B) Let $f(x) = x^3 - 4x - 9$

Since $f(2)$ is negative and $f(3)$ is positive, a root lies between 2 and 3.

First approximation to the root is

$x_1 = \frac{1}{2}(2 + 3) = 2.5.$

Then $f(x_1) = 2.5^3 - 4(2.5) - 9 = -3.375$

i.e. negative∴ The root lies between x_1 and 3. Thus the second approximation to the root is

$x_2 = \frac{1}{2}(x_1 + 3) = 2.75.$

Then $f(x_2) = (2.75)^3 - 4(2.75) - 9 = 0.7969$ i.e. positive.

The root lies between x_1 and x_2. Thus the third approximation to the root is $x_3 = \frac{1}{2}(x_1 + x_2) = 2.625.$

Then $f(x_3) = (2.625)^3 - 4(2.625) - 9 = -1.4121$ i.e. negative.

The root lies between x_2 and x_3. Thus the fourth approximation to the root is $x_4 = \frac{1}{2}(x_2 + x_3) = 2.6875.$

Hence the root is 2.6875 approximately.

2. (B) Let $f(x) = x^3 - 2x - 5$

So that $f(2) = -1$ and $f(3) = 16$

i.e. a root lies between 2 and 3.

Taking $x_0 = 2, x_1 = 3, f(x_0) = -1, f(x_1) = 16,$ in the method of false position, we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) = 2 + \frac{1}{17} = 2.0588$$

Now, $f(x_2) = f(2.0588) = -0.3908$ i.e., that root lies between 2.0588 and 3.

Taking $x_0 = 2.0588, x_1 = 3, f(x_0)$

$= -0.3908,$ $f(x_1) = 16$ in (i), we get

$$x_3 = 2.0588 - \frac{0.9412}{16.3908}(-0.3908) = 2.0813$$

Repeating this process, the successive approxima- tions are

$x_4 = 2.0862, x_5 = 2.0915, x_6 = 2.0934, x_7 = 2.0941,$

$x_8 = 2.0943$ etc.

Hence the root is 2.094 correct to 3 decimal places.

3. (C) Let $f(x)2x - \log_{10} x - 7$

Taking $x_0 = 3.5,$ $x_1 = 4,$ in the method of false position, we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

$$= 3.5 - \frac{0.5}{0.3979 + 0.5441}(-0.5441) = 3.7888$$

Since $f(3.7888) = -0.0009$ and $f(4) = 0.3979,$ therefore the root lies between 3.7888 and 4.

Taking $x_0 = 3.7888, x_1 = 4,$ we obtain

$$x_3 = 3.7888 - \frac{0.2112}{0.3988}(-.009) = 3.7893$$

Hence the required root correct to three places of decimal is 3.789.

4. (D) Let $f(x) = xe^x - 2,$ Then $f(0) = -2,$ and

$f(1) = e - 2 = 0.7183$

So a root of (i) lies between 0 and 1. It is nearer to 1. Let us take $x_0 = 1.$

Also $f'(x) = xe^x + e^x$ and $f'(1) = e + e = 5.4366$

By Newton's rule, the first approximation x_1 is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{0.7183}{5.4366} = 0.8679$$

$f(x_1) = 0.0672,$ $f'(x_1) = 4.4491.$

Thus the second approximation x_2 is

$$x_2 = x_1 - \frac{f(x_1)}{f(x_1)} = 0.8679 - \frac{0.0672}{4.4491} = 0.8528$$

Hence the required root is 0.853 correct to 3 decimal places.

5. (B) Let $y = x + \log_{10} x - 3.375$

To obtain a rough estimate of its root, we draw the graph of (i) with the help of the following table :

x	1	2	3	4
y	-2.375	-1.074	0.102	1.227

Taking 1 unit along either axis $= 0.1$, The curve crosses the x–axis at $x_0 = 2.9$, which we take as the initial approximation to the root.

Now let us apply Newton–Raphson method to

$f(x) = x + \log_{10} x - 3.375$

$$f'(x) = 1 + \frac{1}{x}\log_{10} e$$

$f(2.9) = 2.9 + \log_{10} 2.9 - 3.375 = -0.0126$

$$f'(2.9) = 1 + \frac{1}{2.9}\log_{10} e = 1.1497$$

The first approximation x_1 to the root is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.9 + \frac{0.0126}{1.1497} = 2.9109$$

$f(x_1) = -0.0001,\ f'(x_1) = 1.1492$

Thus the second approximation x_2 is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.9109 + \frac{0.0001}{1.1492} = 2.91099$$

Hence the desired root, correct to four significant figures, is 2.911

6. (B) Let $x = \sqrt{28}$ so that $x^2 - 28 = 0$

Taking $f(x) = x^2 - 28$, Newton's iterative method gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 28}{2x_n} = \frac{1}{2}\left(x_n + \frac{28}{x_n}\right)$$

Now since $f(5) = -3,\ f(6) = 8$, a root lies between 5 and 6.

Taking $x_0 = 5.5$,

$$x_1 = \frac{1}{2}\left(x_0 + \frac{28}{x_0}\right) = \frac{1}{2}\left(5.5 + \frac{28}{5.5}\right) = 5.29545$$

$$x_2 = \frac{1}{2}\left(x_1 + \frac{28}{x_1}\right) = \frac{1}{2}\left(5.29545 + \frac{28}{5.29545}\right) = 5.2915$$

$$x_3 = \frac{1}{2}\left(x_2 + \frac{28}{x_2}\right) = \frac{1}{2}\left(5.2915 + \frac{28}{5.2915}\right) = 5.2915$$

Since $x_2 = x_3$ upto 4 decimal places, so we take $\sqrt{28} = 5.2915$.

7. (B) Let $h = 0.1$, given $x_0 = 0$, $x_1 = x_0 + h = 0.1$

$$\frac{dy}{dx} = 1 + xy \Rightarrow \frac{d^2y}{dx^2} = x\frac{dy}{dx} + y$$

$$\frac{d^3y}{dx^3} = x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}, \quad \frac{d^4y}{dx^4} = x\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2}$$

given that $x = 0$, $y = 1$

$$\Rightarrow \frac{dy}{dx} = 1; \frac{d^2y}{dx^2} = 1, \frac{d^3y}{dx^3} = 2, \frac{d^4y}{dx^4} = 3 \text{ and so on}$$

The Taylor series expression gives :

$$y(x+h) = y(x) + h\frac{dy}{dx} + \frac{h^2}{2!}\frac{d^2y}{dx^2} + \frac{h^3}{3!}\frac{d^3y}{dx^3} +$$

$$\Rightarrow y(0.1) = 1 + 0.1 \times 1 + \frac{(0.1)^2}{2!}\cdot 1 + \frac{(0.1)^3}{3!}2 + \ldots$$

$$\Rightarrow y(0.1) = 1 + 0.1 + \frac{0.01}{2} + \frac{0.001}{3} + \ldots$$

$$= 1 + 0.1 + 0.005 + 0.000033\ldots\ldots\ldots = 1.1053$$

8. (B) Let $h = 0.1$, given $x_0 = 0$, $y_0 = 1$

$$x_1 = x_0 + h = 0.1, \quad \frac{dy}{dx} = x - y^2$$

at $x = 0$, $y = 1$, $\frac{dy}{dx} = -1$

$$\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$$

at $x = 0$, $y = 1$, $\frac{d^2y}{dx^2} = 1 + 2 = 3$

$$\frac{d^3y}{dx^3} = -2\left(\frac{dy}{dx}\right)^2 - 2y\frac{d^2y}{dx^2}$$

at $x = 0$, $y = 1$, $\frac{d^3y}{dx^3} = -8$

$$\frac{d^4y}{dx^4} = -2\left[3\frac{dy}{dx}\frac{d^2y}{dx^2} + y\frac{d^3y}{dx^3}\right]$$

at $x = 0$, $y = 1$ $\frac{d^4y}{dx^4} = 34$

The Taylor series expression gives

$$y(x+h) = y(x) + h\frac{dy}{dx} + \frac{h^2}{2!}\frac{d^2y}{dx^2} + \frac{h^3}{3!}\frac{d^3y}{dx^3} + \frac{h^4}{4!}\frac{d^4y}{dx^4} + \ldots$$

$$y(0.1) = 1 + 0.1(-1) + \frac{(0.1)^2}{2!}3 + \frac{(0.1)^3}{3!}(-8) + \frac{(0.1)^4}{4!}34 + \ldots\ldots$$

$$= 1 - 0.1 + 0.015 - 0.001333 + 0.0001417 = 0.9138$$

9. (C) Here $f(x, y) = x^2 + y^2,\ x_0 = 0 \quad y_0 = 0$

We have, by Picard's method

$$y = y_0 + \int_{x_0}^{x} f(x, y)dx \quad \ldots(1)$$

The first approximation to y is given by

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

Where $y_0 = 0 + \int_0^x f(x, 0)dx = \int_0^x x^2 dx.$ ...(2)

The second approximation to y is given by

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})dx = 0 + \int_0^x f\left(x, \frac{x^3}{3}\right)dx$$

$$= 0 + \int_0^x \left(x^2 + \frac{x^6}{9}\right)dx = \frac{x^3}{3} + \frac{x^7}{63}$$

Now, $y(0.4) = \frac{(0.4)^3}{3} + \frac{(0.4)^7}{63} = 0.02135$

10. (C) Here $f(x, y) = y - x$; $x_0 = 0,\ y_0 = 2$

We have by Picard's method

$$y = y_0 + \int_{x0}^{x} f(x, y)\ dx$$

The first approximation to y is given by

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)dx = 2 + \int_0^x f(x, 2)\,dx$$

$$=2+\int_0^x (2-x)dx \quad = 2+2x-\frac{x^2}{2} \qquad ....(1)$$

The second approximation to y is given by

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})dx$$

$$=2+\int_{x_0}^{x} f\left(x, 2+2x-\frac{x^2}{2}\right)dx$$

$$=2+\int_0^2 (2+2x-\frac{x^2}{2}-x)dx$$

$$=2+2x+\frac{x^2}{2}-\frac{x^3}{6} \qquad ....(2)$$

The third approximation to y is given by

$$y^{(3)} = y_0 + \int_{x_0}^{x} f(x, y^{(2)})dx$$

$$=2+\int_{x_0}^{x} f\left(x, 2+2x+\frac{x^2}{2}-\frac{x^3}{6}\right)dx$$

$$=2+\int_0^x \left(2+2x+\frac{x^2}{2}-\frac{x^3}{6}-\right)dx$$

$$=2+2x+\frac{x^2}{2}+\frac{x^3}{6}-\frac{x^4}{24}$$

11. (B) Here $f(x, y) = x + y^2, \quad x_0 = 0 \quad y_0 = 0$

We have, by Picard's method

$$y = y_0 + \int_{x_0}^{x} f(x, y_0)dx$$

The first approximation to y is given by

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)dx \quad = 0 + \int_0^x f(x, 0)dx$$

$$=0+\int_0^x xdx \quad = \frac{x^2}{2}$$

The second approximation to y is given by

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})dx \quad = 0 + \int_0^x f\left(x, \frac{x^2}{2}\right)dx$$

$$=\int_0^x \left(x+\frac{x^4}{4}\right)dx \quad = \frac{x^2}{2}+\frac{x^5}{50}$$

The third approximation is given by

$$y^{(3)} = y_0 + \int_{x_0}^{x} f(x, y^{(2)})dx$$

$$=0+\int_0^x f\left(x, \frac{x^2}{2}+\frac{x^5}{20}\right)dx$$

$$=\int_0^x \left(x+\frac{x^4}{4}+\frac{x^{10}}{400}+\frac{2x^7}{40}\right)dx = \frac{x^2}{2}+\frac{x^5}{20}+\frac{x^8}{160}+\frac{x^{11}}{4400}$$

12. (A) x: 0 0.1 0.2 0.3 0.4

Euler's method gives

$$y_{n+1} = y_n + h(x_n, y_n) \qquad ....(1)$$

$n = 0$ in (1) gives

$y_1 = y_0 + hf(x_0, y_0)$

Here $x_0 = 0, \quad y_0 = 1, \quad h = 0.1$

$y_1 = 1 + 0.1\ f(0, 1) \ = 1 + 0 \ = 1$

$n = 0$ in (1) gives $y_2 = y_1 + h\ f(x_1, y_1)$

$= 1 + 0.1\ f(0.1, 1) \ = 1 + 0.1(0.1) \ = 1 + 0.01$

Thus $y_2 = y_{(0.2)} = 1.01$

$n = 2$ in (1) gives

$y_3 = y_2 + hf(x_2, y_2) \ = 1.01 + 0.1\ f(0.2, 1.01)$

$y_3 = y_{(0.3)} = 1.01 + 0.0202 \ = 1.0302$

$n = 3$ in (1) gives

$y_4 = y_3 + hf(x_3, y_3) \ = 1.0302 + 0.1f(0.3, 1.0302)$

$= 1.0302 + 0.03090$

$y_4 = y_{(0.4)} = 1.0611$

Hence $y_{(0.4)} = 1.0611$

13. (B) The Euler's modified method gives

$y_1^* = y_0 + hf(x_0, y_0)$,

$$y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^*)]$$

Now, here $h = 0.02, \quad y_0 = 1, \quad x_0 = 0$

$y_1^* = 1 + 0.02f(0, 1), \quad y_1^* = 1 + 0.02 \ = 1.02$

Next $y_1 = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x, y_1^*)]$

$$= 1 + \frac{0.02}{2}[f(0, 1) + f(0.02, \ 1.02)]$$

$= 1 + 0.01[1 + 1.0204] = 1.0202$

So, $y_1 = y(0.02) = 1.0202$

14. (D) $y_2^* = y_1 + h\ f(x_1, y_1)$

$= 1.0202 + 0.02[f(0.02, \ 1.0202)]$

$= 1.0202 + 0.0204 \qquad = 1.0406$

Next $y_2 = y_1 + \frac{h}{2}[f(x, y) + f(x_2, y_2^*)]$

$$y_2 = 1.0202 + \frac{0.02}{2}[f(0.02, 1.0202) + f(0.04, 1.0406)]$$

$= 1.0202 + 0.01[1.0206 + 1.0422] = 1.0408$

$y_2 = y_{(0.04)} = 1.0408$

15. (C) $y_3^* = y_2 + hf(x_2, y_2)$

$= 1.0416 + 0.02f(0.04, 1.0416)$

$= 1.0416 + 0.0217 = 1.0633$

Next $y_3 = y_2 + \frac{h}{2}[f(x_2, y_2) + f(x_3, y_3^*)]$

$k_2 = hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1\right) = (0.2)f(0.1, 0.1) = 0.202$

$k_3 = hf\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2\right) = (0.2)f(0.1, 0.101) = 0.2020$

$k_4 = hf(x_0 + h, y_0 + k_3) = 0.2f(0.2, 0.2020) = 0.20816$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2 + 2(.202) + 2(.20204) + 0.20816],$

$k = 0.2027$

such that $y_1 = y(0.2) = y_0 + k = 0 + 0.2027 = 0.2027$

21. (C) We now to find $y_2 = y(0.4)$, $k_1 = hf(x_1, y_1)$

$= (0.2)f(0.2, 0.2027) = 0.2(1.0410) = .2082$

$k_2 = hf\left(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}k_1\right)$

$= (0.2)f(0.3, 0.3068) = 0.2188$

$k_3 = hf\left(x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}k_2\right)$

$= 0.2f(0.3, 0.3121) = .2194$

$k_4 = hf(x_1 + h, y_1 + k_3) = 0.2f(0.4, .4221) = 0.2356$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2082 + 2(.2188) + 2(.2194) + 0.356] = 0.2200$

$y_2 = y_{(0.4)} = y_1 + k = 0.2200 + .2027 = 0.4227$

22. (C) We now to find $y_3 = y_{(0.6)}$, $k_1 = hf(x_2, y_2)$

$= (0.2)f(0.4, 0.4228) = 0.2357$

$k_2 = hf\left(x_2 + \frac{1}{2}h, y_2 + \frac{1}{2}k_1\right)$

$= (0.2)f(0.5, 0.5406) = 0.2584$

$k_3 = hf\left(x_2 + \frac{1}{2}h, y_2 + \frac{1}{2}k_2\right)$

$= 0.2f(0.5, .5520) = 0.2609$

$k_4 = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2357 + 2(.2584) + 2(0.2609) + 0.2935]$

$= \frac{1}{6}[0.2357 + 0.5168 + 0.5218 + 0.2935] = 0.2613$

$y_3 = y_{(0.6)} = y_2 + k = .4228 + 0.2613 = 0.6841$

23. (A) Here given $x_0 = 0$ $y_0 = 1$, $h = 0.2$

$f(x, y) = x + y^2$

To find $y_1 = y_{(0.2)}$,

$k_1 = hf(x_0, y_0) = (0.2)f(0, 1) = (0.2) \times 1 = 0.2$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.2)f(0.1, 1.1) = 0.2(1.31) = 0.262$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.2f(0.1, 1.131) = 0.2758$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= (0.2)f(0.2, 1.2758) = 0.3655$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + 2k_4]$

$= \frac{1}{6}[0.2 + 2(0.262) + 2(0.2758) + 0.3655] = 0.2735$

Here $y_1 = y_{(0.2)} = y_0 + k = 1 + 0.2735 \Rightarrow 1.2735$

24. (C) Here $f(x, y) = x + y$ $h = 0.2$

To find $y_1 = y_{(0.2)}$,

$k_1 = hf(x_0, y_0) = 0.2f(0, 1) = 0.2$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = (0.2)f(0.1, 1.1) = 0.24$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = (0.2)f(0.1, 1.12) = 0.244$

$k_4 = hf(x_0 + h, y_0 + k_3) = (0.2)f(0.2, 1.244) = 0.2888$

$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.2 + 2(0.24) + 2(0.244) + 0.2888] = 0.2428$

$y_1 = y_{(0.2)} = y_0 + k = 1 + 0.2428 = 1.2428$

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