Assignment

Significant Digit and Rounding off Numbers, Fundamental concepts

Basic Level

1. The number 3.14150 rounded to 3 decimals is [MP PET 2000]

(a) 3.14 (b) 3.141 (c) 3.142 (d) None of these

2. The number of significant digits in 0.003050 is

(a) 7 (b) 6 (c) 4 (d) None of these

3. The number of significant digits in 20.035 is

(a) 3 (b) 5 (c) 4 (d) None of these

4. The number of significant digits in 20340 is

(a) 4 (b) 5 (c) 3 (d) None of these

5. The number 0.0008857 when rounded off to three significant digits yields

(a) 0.001 (b) 0.000886 (c) 0.000885 (d) None of these

6. The number 3.68451 when rounded off to three decimal places becomes

(a) 3.68 (b) 3.684 (c) 3.685 (d) None of these

7. The number of significant digits in the number 0.00452000 is

(a) 3 (b) 5 (c) 8 (d) None of these

8. When a number is approximated to n decimal places by chopping off the extra digits, then the absolute value of the relative error does not exceed

(a) 10^{-n} (b) 10^{-n+1} (c) $0.5\times 10^{-n+1}$ (d) None of these

9. When the number 6.878652 is rounded off to five significant figures, then the round off error is

(a) – 0.000048 (b) –0.00048 (c) 0.000048 (d) 0.00048

10. The number 0.0009845 when rounded off to three significant digits yields [DCE 1998]

(a) 0.001 (b) 0.000987 (c) 0.000985 (d) None of these

11. A decimal number is chopped off to four decimal places, then the absolute value of the relative error is not greater than [DCE 1996]

(a) 10^{-2} (b) 10^{-3} (c) 10^{-4} (d) None of these

12. If e_1 and e_2 are absolute errors in two numbers n_1 and n_2 respectively due to rounding or truncation, then $\left|\frac{e_1}{n_1}+\frac{e_2}{n_2}\right|$

(a) Is equal to e_1+e_2 (b) Is less then e_1+e_2

(c) Is less then or equal to e_1+e_2 (d) Is greater then or equal to e_1+e_2

13. In general the ratio of the truncation error to that of round off error is

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) None of these

14. The equation $e^{-2x}-\sin x+1=0$ is of the form

(a) Algebraic (b) Linear (c) Quadratic (d) Transcendental

15. The root of the equation $x^3-6x+1=0$ lies in the interval

(a) (2, 3) (b) (3, 4) (c) (3, 5) (d) (4, 6)

16. The root of the equation $x^3-3x-5=0$ in the interval (1, 2) is

(a) 1.13 (b) 1.98

(c) 1.54 (d) No root lies in the interval (1, 2)

17. The equation $f(x)=0$ has repeated root $a\in(x_1,x_2)$, if

(a) $f'(a)<0$ (b) $f'(a)>0$ (c) $f'(a)=0$ (d) None of these

18. The root of the equation $2x-\log_{10}x=7$ lies between

(a) 3 and 3.5 (b) 2 and 3 (c) 3.5 and 4 (d) None of these

19. For the equation $f(x)=0$, if $f(a)<0,\ f(b)>0,\ f(c)>0$ and $b>c$ then we will discard the value of the function $f(x)$ at the point

(a) *a* (b) *b* (c) *c* (d) Anyone out of *a, b, c*

20. The positive root of the equation $e^x+x-3=0$ lies in the interval

(a) (0, 1) (b) (1, 2) (c) (2, 3) (d) (2, 4)

21. The positive root of the equation $x^3-2x-5=0$ lies in the interval

(a) (0, 1) (b) (1, 2) (c) (2, 3) (d) (3, 4)

22. One real root of the equation x^3-5x+1 = 0 must lie in the interval

(a) (0, 1) (b) (1, 2) (c) (–1, 0) (d) (–2, 0)

23. The number of positive roots of the equation $x^3-3x+5=0$ is [MP PET 1998]

(a) 1 (b) 2 (c) 3 (d) None of these

24. Let $f(x)=0$ be an equation and x_1, x_2 be two real numbers such that $f(x_1)f(x_2)<0$, then [MP PET 1989, 1997]

(a) At least one root of the equation lies in the interval (x_1, x_2)

(b) No root of the equation lies in the interval (x_1, x_2)

(c) Either no root or more than one root of the equation lies the interval (x_1, x_2)

(d) None of these

25. Let $f(x)=0$ be an equation let x_1, x_2 be two real numbers such that $f(x_1)f(x_2)>0$, then

(a) At least one root of the equation lies in (x_1, x_2)

(b) No root of the equation lies in (x_1, x_2)

(c) Either no root or an even number of roots lie in (x_1, x_2)

(d) None of these

26. If for $f(x)=0$, $f(a)<0$ and $f(b)>0$, then one root of $f(x)=0$ is

(a) Between *a* and *b* (b) One of from *a* and *b*

(c) Less than *a* and greater than *b* (d) None of these

27. If $f(a)f(b)<0$, then an approximate value of a real root of $f(x)=0$ lying between *a* and *b* is given by

(a) $\frac{af(b)-bf(a)}{b-a}$ (b) $\frac{bf(a)-af(b)}{b-a}$

(c) $\frac{af(b)-bf(a)}{f(b)-f(a)}$ (d) None of these

Successive bisection method

Basic Level

28. One root of $x^3-x-4=0$ lies in (1, 2). In bisection method, after first iteration the root lies in the interval

(a) (1, 1.5) (b) (1.5, 2.0) (c) (1.25, 1.75) (d) (1.75, 2)

29. A root of the equation $x^3-x-1=0$ lies between 1 and 2. Its approximate value as obtained by applying bisection method 3 times is

[MP PET 1993]

(a) 1.375 (b) 1.625 (c) 1.125 (d) 1.25

30. A root of the equation $x^3-x-4=0$ lies between 1 and 2. Its approximate value, as obtained by applying bisection method 3 times, is

(a) 1.375 (b) 1.750 (c) 1.975 (d) 1.875

31. Performing 3 iterations of bisection method, the smallest positive approximate root of equation $x^3 - 5x + 1 = 0$ is [MP PET 1996]

(a) 0.25 (b) 0.125 (c) 0.50 (d) 0.1875

32. A root of the equation $x^3 - 3x - 5 = 0$ lies between 2 and 2.5. Its approximate value, by applying bisection method 3 times is

(a) 2.0625 (b) 2.3125 (c) 2.3725 (d) 2.4225

33. If for the function $f(x) = 0$, $f(a) < 0$ and $f(b) > 0$, then the value of x in first iteration is

(a) $\frac{a+b}{2}$ (b) $\frac{b-a}{2}$ (c) $\frac{2a-b}{2}$ (d) $\frac{2b-a}{2}$

34. Using successive bisection method, a root of the equation $x^3 - 4x + 1 = 0$ lies between 1 and 2, at the end of first interaction, it lies between [DCE 1996]

(a) 1.62 and 1.75 (b) 1.5 and 1.75 (c) 1.75 and 1.87 (d) None of these

35. The nearest real root of the equation $xe^x - 2 = 0$ correct to two decimal places, is

(a) 1.08 (b) 0.92 (c) 0.85 (d) 0.80

Regula-Falsi methoa

Basic Level

36. By the false position method, the root of the equation $x^3 - 9x + 1 = 0$ lies in interval (2, 4) after first iteration. It is

(a) 3 (b) 2.5 (c) 3.57 (d) 2.47

37. The formula [where $f(x_{n<1})$ and $f(x_n)$ have opposite sign at each step $n \geq 1$] of method of False position of successive approximation to find the approximate value of a root of the equation $f(x) = 0$ is [MP PET 1995, 97]

(a) $x_{n+1} = x_n - \frac{f(x_n) - f(x_{n-1})}{f(x_n)}(x_n - x_{n-1})$

(b) $x_{n+1} = x_n - \frac{f(x_n)}{f(x_n) - f(x_{n-1})}(x_n - x_{n-1})$

(c) $x_{n+1} = x_n + \frac{f(x_n) + f(x_{n-1})}{f(x_n)}(x_n - x_{n-1})$

(d) $x_{n+1} = x_n + \frac{f(x_n)}{f(x_n) + f(x_{n-1})}(x_n - x_{n-1})$

38. By false positioning, the second approximation of a root of equation $f(x) = 0$ is (where x_0, x_1 are initial and first approximations respectively) [MP PET 1996; DCE 2001]

(a) $x_0 - \frac{f(x_0)}{f(x_1) - f(x_0)}$ (b) $\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$ (c) $\frac{x_0 f(x_0) - x_1 f(x_1)}{f(x_1) - f(x_0)}$ (d) $x_0 - \frac{f(x_0)}{f(x_1) - f(x_0)}$

39. A root of the equation $x^3 - 18 = 0$ lies between 2 and 3. The value of the root by the method of false position is

(a) 2.526 (b) 2.536 (c) 2.546 (d) 2.556

40. The equation $x^3 - 3x + 4 = 0$ has only one real root. What is its first approximate value as obtained by the method of false position in (–3, –2) **[MP PET 1999]**

(a) –2.125 (b) 2.125 (c) –2.812 (d) 2.812

41. A root of equation $x^3 + 2x - 5 = 0$ lies between 1 and 1.5. its value as obtained by applying the method of false position only once is

[MP PET 1993]

(a) $\frac{4}{3}$ (b) $\frac{35}{27}$ (c) $\frac{23}{25}$ (d) $\frac{5}{4}$

Newton-Raphson method

Basic Level

42. If successive approximations are given by $x_1, x_2, x_3, ... x_n, x_{n+1}$, then Newton-Raphson formula is given as **[MP PET 1993, 95]**

(a) $x_{n+1} = x_n + \frac{f(x_{n+1})}{f'(x)}$ (b) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$

(c) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ (d) $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$

43. Newton-Raphson method is applicable only when

(a) $f(x) \neq 0$ in the neighbourhood of actual root $x = \alpha$ (b) $f'(x) \neq 0$ in the neighbourhood of actual root $x = \alpha$

(c) $f''(x) \neq 0$ in the neighbourhood of actual root $x = \alpha$ (d) None of these

44. Newton-Raphson processes has a

(a) Linear convergence (b) Quadratic convergence (c) Cubic convergence (d) None of these

45. The condition for convergence of the Newton-Raphson method to a root α is **[MP PET 2001]**

(a) $\frac{1}{2}\frac{f'(\alpha)}{f''(\alpha)} < 1$ (b) $\frac{f'(\alpha)}{f''(\alpha)} < 1$

(c) $\frac{1}{2}\frac{f'(\alpha)}{f''(\alpha)} > 1$ (d) None of these

46. The real root of the equation $x^3 - x - 5 = 0$ lying between –1 and 2 after first iteration by Newton-Raphson method is

(a) 1.909 (b) 1.904 (c) 1.921 (d) 1.940

47. A root of the equation $x^3 - 4x + 1 = 0$ lies between 1 and 2. Its value as obtained by using Newton-Raphson method is

(a) 1.775 (b) 1.850 (c) 1.875 (d) 1.950

48. The value of x_0 (the initial value of x) to get the solution in interval (0.5, 0.75) of the equation $x^3 - 5x + 3 = 0$ by Newton-Raphson method, is

(a) 0.5 (b) 0.75 (c) 0.625 (d) None of these

49. If *a* and *a* + *h* are two consecutive approximate roots of the equation $f(x)=0$ as obtained by Newtons method, then *h* is equal to [MP PET 1999]

(a) $f(a)/f'(a)$ (b) $f'(a)/f(a)$ (c) $-f'(a)/f(a)$ (d) $-f(a)/f'(a)$

50. The Newton-Raphson method converges fast if $f'(\alpha)$ is (α is the exact value of the root) [DCE 1998]

(a) Small (b) Large (c) 0 (d) None of these

Advance Level

51. If one root of the equation $f(x)=0$ is near to x_0 then the first approximation of this root as calculated by Newton-Raphson method is the abscissa of the point where the following straight line intersects the *x*-axis [MP PET 1998]

(a) Normal to the curve $y=f(x)$ at the point $(x_0, f(x_0))$

(b) Tangent to the curve $y=f(x)$ at the point $(x_0, f(x_0))$

(c) The straight line through the point $(x_0, f(x_0))$ having the gradient $\frac{1}{f'(x_0)}$

(d) The ordinate through the point $(x_0, f(x_0))$

52. A root of the equation $x^3-3x-5=0$ lies between 2 and 2.5. Its value as obtained by using Newton-Raphson method, is

(a) 2.25 (b) 2.33 (c) 2.35 (d) 2.45

53. After second iteration of Newton-Raphson method, the positive root of equation $x^2=3$ is (taking initial approximation $\frac{3}{2}$) [MP PET 1996]

(a) $\frac{3}{2}$ (b) $\frac{7}{4}$ (c) $\frac{97}{56}$ (d) $\frac{347}{200}$

54. If one root of the equation $x^3+x^2-1=0$ is near to 1.0, then by Newton-Raphson method the first calculated approximate value of this root is [MP PET 1998]

(a) 0.9 (b) 0.6 (c) 1.2 (d) 0.8

55. The approximate value of a root of the equation $x^3-3x-5=0$ at the end of the second iteration by taking the initial value of the roots as 2, and by using Newton-Raphson method, is [AI CBSE 1990]

(a) 2.2806 (b) 2.2701 (c) 2.3333 (d) None of these

56. Newton-Raphson method is used to calculate $\sqrt[3]{65}$ by solving $x^3=65$. If $x_0=4$ is taken as initial approximation then the first approximation x_1 is [AMU 1999]

(a) 65/16 (b) 131/32 (c) 191/48 (d) 193/48

57. Starting with $x_0=1$, the next approximation x_1 to $2^{1/3}$ obtained by Newton's method is [DCE 1997]

(a) $\frac{5}{3}$ (b) $\frac{4}{3}$ (c) $\frac{5}{4}$ (d) $\frac{5}{6}$

Trapezoidal rule

Basic Level

58. Approximate value of $\int_{x_0}^{x_0+nh} y\,dx$ by Trapezoidal rule, is **[MP PET 1993, 97]**

[Where $y(x_i)=y_i,\ \ x_{i+1}-x_i=h, i=0,1,2,.....n$]

(a) $\frac{h}{2}[y_0+y_n+2(y_1+y_2+y_3+.....+y_{n-1})]$

(b) $\frac{h}{3}[y_0+y_n+4(y_1+y_3+y_5+.....+y_{n-1})+2(y_2+y_4+....+y_{n-2})]$

(c) $\frac{h}{4}[y_0+y_n+2(y_1+y_3+y_5+.....+y_{n-1})+4(y_2+y_4+....+y_{n-2})]$

(d) $\frac{h}{2}[y_0+y_2+y_4+.....+y_n)+2(y_1+y_3+y_5+....+y_{n-1})]$

59. Trapezoidal rule for evaluation of $\int_a^b f(x)\,dx$ requires the interval (*a*, *b*) to be divided into **[DCE 1994; MP PET 1996]**

(a) 2*n* sub-intervals of equal width (b) 2*n* + 1 sub-intervals of equal width

(c) Any number of sub-intervals of equal width (d) 3*n* sub-intervals of equal width

60. The value of *f*(*x*) is given only at $x=0,\frac{1}{3},\frac{2}{3},1$. Which of the following can be used to evaluate $\int_0^1 f(x)\,dx$ approximately **[MP PET 1999]**

(a) Trapezoidal rule (b) Simpson rule

(c) Trapezoidal as well as Simpson rule (d) None of these

61. A river is 80 *metre* wide. Its depth *d metre* and corresponding distance *x metre* from one bank is given below in table

x :	0	10	20	30	40	50	60	70	80
y :	0	4	7	9	12	15	14	8	3

Then the approximate area of cross-section of river by Trapezoidal rule, is **[MP PET 1994]**

(a) 710 *sq.m* (b) 730 *sq.m* (c) 705 *sq.m* (d) 750 *sq.m*

62. A curve passes through the points given by the following table

x :	1	2	3	4	5
y :	10	50	70	80	100

By Trapezoidal rule, the area bounded by the curve, the *x*-axis and the lines *x* = 1, *x* = 5, is

(a) 310 (b) 255 (c) 305 (d) 275

63. From the following table, using Trapezoidal rule, the area bounded by the curve, the *x*-axis and the lines *x* = 7.47, *x* = 7.52, is

x :	7.47	7.48	7.49	7.50	7.51	7.52

$f(x)$:	1.93	1.95	1.98	2.01	2.03	2.06

(a) 0.0996 (b) 0.0896 (c) 0.1096 (d) 0.0776

64. Let $f(0) = 1$, $f(1) = 2.72$, then the trapezoidal rule gives approximate value of $\int_0^1 f(x)\,dx$ [MP PET 1999; DCE 2001]

(a) 3.72 (b) 1.86 (c) 1.72 (d) 0.86

65. By Trapezoidal rule, the value of $\int_0^1 x^3\,dx$ considering five sub-intervals, is

(a) 0.21 (b) 0.23 (c) 0.24 (d) 0.26

Advance Level

66. The approximate value of $\int_1^9 x^2 dx$ by using Trapezoidal rule with 4 equal intervals is [EAMCET 2002]

(a) 243 (b) 248 (c) 242.8 (d) 242.5

67. Taking $n = 4$, by trapezoidal rule, the value of $\int_0^2 \frac{dx}{1+x}$ is [DCE 1999, 2000]

(a) 1.1125 (b) 1.1176 (c) 1.118 (d) None of these

68. With the help of trapezoidal rule for numerical integration and the following table

x :	0	0.25	0.50	0.75	1
$f(x)$:	0	0.0625	0.2500	0.5625	1

The value of $\int_0^1 f(x)\,dx$ is [MP PET 1996]

(a) 0.35342 (b) 0.34375 (c) 0.34457 (d) 0.33334

69. If for $n = 3$, the integral $\int_1^{10} x^3 dx$ is approximately evaluated by Trapezoidal rule $\int_1^{10} x^3 dx = 3\left[\frac{1+10^3}{2} + \alpha + 7^3\right]$, then $\alpha =$

[MP PET 2000]

(a) 3^3 (b) 4^3 (c) 5^3 (d) 6^3

70. By trapezoidal rule, the value of $\int_1^2 \frac{1}{x}\,dx$, (using five ordinates) is nearly [DCE 1994]

(a) 0.216 (b) 0.697 (c) 0.921 (d) None of these

Simpson's one third rule

Basic Level

71. The value of $\int_{x_0}^{x_0+nh} y\,dx$, n is even number, by Simpson's one-third rule is [MP PET 1995]

(a) $\frac{h}{3}[(y_0+y_n)+2(y_1+y_3+.....+y_{n-1})+4(y_2+y_4+......+y_{n-2})]$ (b) $\frac{h}{3}[(y_0+y_n)+4(y_1+y_3+.....+y_{n-1})+2(y_2+y_4+......+y_{n-2})]$

(c) $\frac{h}{3}[(y_0+y_n)-2(y_1+y_3+.....+y_{n-1})+4(y_2+y_4+......+y_{n-2})]$ (d) None of these

72. Simpson's one-third rule for evaluation $\int_a^b f(x)dx$ requires the interval [a, b] to be divided into [DCE 1999]

(a) An even number of sub-intervals of equal width (b) Any number of sub-intervals

(c) Any number of sub-intervals of equal width (d) An odd number of sub-intervals of equal width

73. Simpson rule for evaluation of $\int_a^b f(x)dx$ requires the interval (a, b) to be divided into [Haryana CEE 1993; DCE 1994]

(a) $3n$ intervals (b) $2n + 1$ intervals (c) $2n$ intervals (d) Any number of intervals

74. To calculate approximate value of π by Simpson's rule, the approximate formula is [MP PET 2000]

(a) $\int_0^1 \left(\frac{1}{1+x^2}\right)dx$, $n=16$ (b) $\int_0^1 \left(\frac{1}{1+x^2}\right)dx$, $n=9$ (c) $\int_0^1 \left(\frac{1}{1+x}\right)dx$, $n=11$ (d) $\int_0^1 \left(\frac{1}{1+x}\right)dx$, $n=9$

75. In Simpson's one-third rule, the curve $y = f(x)$ is assumed to be a [MP PET 2001]

(a) Circle (b) Parabola (c) Hyperbola (d) None of these

76. A river is 80 feet wide. The depth d (in feet) of the river at a distance of x feet from one bank is given by the following table

x:	0	10	20	30	40	50	60	70	80
y:	0	4	7	9	12	15	14	8	3

By Simpson's rule, the area of the cross-section of the river is

(a) 705 sq. feet (b) 690 sq. feet (c) 710 sq. feet (d) 715 sq. feet

77. A curve passes through the points given by the following table

x:	1	1.5	2	2.5	3	3.5	4
y:	2	2.4	2.7	2.8	3	2.6	2.1

By Simpson's rule, the area bounded by the curve, the x-axis and the lines $x = 1$, $x = 4$, is

(a) 7.583 (b) 6.783

(c) 7.783 (d) 7.275

78. Using Simpson's $\frac{1}{3}$ rule, the value of $\int_1^3 f(x)dx$ for the following data, is

x:	1	1.5	2	2.5	3
$f(x)$:	2.1	2.4	2.2	2.8	3

[MP PET 1993]

(a) 55.5 (b) 11.1 (c) 5.05 (d) 4.975

79. By the application of Simpson's one-third rule for numerical integration, with two subintervals, the value of $\int_0^1 \frac{dx}{1+x}$ is [MP PET 1996]

(a) $\frac{17}{24}$ (b) $\frac{17}{36}$ (c) $\frac{25}{35}$ (d) $\frac{17}{25}$

80. By Simpson's rule, the value of $\int_{-3}^{3} x^4 dx$ by taking 6 sub-intervals, is

(a) 98 (b) 96 (c) 100 (d) 99

81. If $\int_a^b f(x)dx$ is numerically integrated by Simpson's rule, then in any pair of consecutive sub-intervals by which of the following curves, the curve $y = f(x)$ is approximated [MP PET 1998]

(a) Straight line (b) Parabola (c) Circle (d) Ellipse

82. If by Simpson's rule $\int_0^1 \frac{1}{1+x^2}dx = \frac{1}{12}[3.1 + 4(a+b)]$ when the interval [0, 1] is divided into 4 sub-intervals and a and b are the values of $\frac{1}{1+x^2}$ at two of its division points, then the values of a and b are the following [MP PET 1998]

(a) $a = \frac{1}{1.0625}, b = \frac{1}{1.25}$ (b) $a = \frac{1}{1.0625}, b = \frac{1}{1.5625}$ (c) $a = \frac{1}{1.25}, b = 1$ (d) $a = \frac{1}{1.5625}, b = \frac{1}{1.25}$

83. If $e^0 = 1, e^1 = 2.72, e^2 = 7.39, e^3 = 20.09$ and $e^4 = 54.60$, then by Simpson's rule, the value of $\int_0^4 e^x dx$ is

[MP PET 1994, 95, 2001, 02]

(a) 5.387 (b) 53.87 (c) 52.78 (d) 53.17

84. If (2, 6) is divided into four intervals of equal region, then the approximate value of $\int_2^6 \frac{1}{x^2 - x} dx$ using Simpson's rule, is

[EAMCET 2002]

(a) 0.3222 (b) 0.2333 (c) 0.5222 (d) 0.2555

85. If $h = 1$ in Simpson's rule, the value of $\int_1^5 \frac{dx}{x}$ is

(a) 1.62 (b) 1.43 (c) 1.48 (d) 1.56

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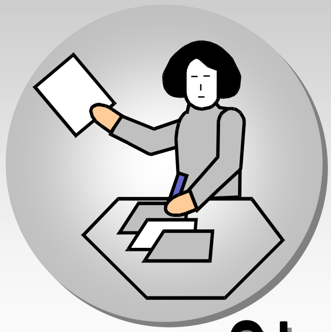

Answer Sheet

Numerical Methods ***Assignment (Basic and Advance Level)***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	c	b	a	b	c	d	b	a	c	b	c	b	d	a	d	c	c	b	a
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
c	a	d	a	c	a	c	b	a	d	d	b	a	d	c	d	b	b	a	a
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
b	c	b	b	c	a	c	b	d	b	a	b	c	d	a	d	b	a	c	a
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
c	b	a	b	d	b	a	b	b	b	b	a	c	a	b	c	c	c	c	a
81	**82**	**83**	**84**	**85**															
b	b	b	c	a															