

Chapter - 7

Algebraic Expressions, Identities and Factorization

Exercise

In questions 1 to 33, there are four options out of which one is correct. Write the correct answer.

1. The product of a monomial and a binomial is a

- (a) Monomial (b) Binomial (c) Trinomial (d) None of these

Solution:

Let binomial = $x + y$ and monomial = $2x$

$$\begin{aligned}\text{So, the product of a monomial and a binomial} &= 2x \times (x + y) \\ &= 2x^2 + 2xy\end{aligned}$$

Hence, the product of a monomial and a binomial is a binomial.

2. In a polynomial, the exponents of the variables are always

- (a) Integers (b) Positive integers (c) Non-negative integers (d) Non-positive integers

Solution:

As we know that in a polynomial, the exponents of the variables are always positive integers.

3. Which of the following is correct?

- (a) $(a - b)^2 = a^2 + 2ab - b^2$ (b) $(a - b)^2 = a^2 - 2ab + b^2$
(c) $(a - b)^2 = a^2 - b^2$ (d) $(a + b)^2 = a^2 + 2ab - b^2$

Solution:

According to the question:

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b) \\ &= a(a - b) - b(a - b) \\ &= a \times a - a \times b - b \times a + b \times b \\ &= a^2 - ab - ab + b^2 && [\text{As } a \times b = b \times a] \\ &= a^2 - 2ab + b^2\end{aligned}$$

And:

$$\begin{aligned}
(a+b)^2 &= (a+b)(a+b) \\
&= a(a+b) + b(a+b) \\
&= a \times a + a \times b + b \times a + b \times b \\
&= a^2 + ab + ab + b^2 & [\text{As } a \times b = b \times a] \\
&= a^2 + 2ab + b^2
\end{aligned}$$

Hence, the correct option is (b).

4. The sum of $-7pq$ and $2pq$ is
(a) $-9pq$ (b) $9pq$ (c) $5pq$ (d) $-5pq$

Solution:

The sum of $-7pq$ and $2pq$ is calculated as:

$$\begin{aligned}
&= -7pq + 2pq \\
&= pq(-7 + 2) \\
&= -5pq
\end{aligned}$$

Hence, the correct option is (d).

5. If we subtract $-3x^2y^2$ from x^2y^2 , then we get
(a) $-4x^2y^2$ (b) $-2x^2y^2$ (c) $2x^2y^2$ (d) $4x^2y^2$

Solution:

To subtract $-3x^2y^2$ from x^2y^2 as follows:

$$\begin{aligned}
x^2y^2 - (-3x^2y^2) &= x^2y^2 + 3x^2y^2 \\
&= (1+3) x^2y^2 \\
&= 4x^2y^2
\end{aligned}$$

Hence, the correct option is (d).

6. Like term as $4m^3n^2$ is
(a) $4m^2n^2$ (b) $-6m^3n^2$ (c) $6pm^3n^2$ (d) $4m^3n$

Solution:

The like term as $4m^3n^2$ is $-6m^3n^2$ because it contains the same literal factor m^3n^2 .

7. Which of the following is a binomial?
(a) $7 \times a + a$ (b) $6a^2 + 7b + 2c$ (c) $4a \times 3b \times 2c$ (d) $6(a^2 + b)$

Solution:

As we know that binomials are algebraic expressions consisting of two unlike terms.
 From option (d);

$$6(a^2 + b) = 6a^2 + 6b$$

Hence, the correct option is (d).

8. Sum of $a - b + ab$, $b + c - bc$ and $c - a - ac$ is

(a) $2c + ab - ac - bc$ (b) $2c - ab - ac - bc$ (c) $2c + ab + ac + bc$ (d) $2c - ab + ac + bc$

Solution:

Sum of $a - b + ab$, $b + c - bc$ and $c - a - ac$ is calculated as follows:

$$= (a - b + ab) + (b + c - bc) + (c - a - ac)$$

$$= a - b + ab + b + c - bc + c - a - ac$$

$$= 2c + ab - ac - bc$$

Hence, the correct option is (a)

9. Product of the following monomials $4p$, $-7q^3$, $-7pq$ is

(a) $196 p^2 q^4$ (b) $196 pq^4$ (c) $-196 p^2 q^4$ (d) $196 p^2 q^3$

Solution:

Product of the following monomials $4p$, $-7q^3$, $-7pq$ is calculated as follows:

$$= 4p \times (-7q^3) \times (-7pq)$$

$$= 4 \times (-7) \times (-7) \times p \times q^3 \times pq$$

$$= 196 p^2 q^4$$

Hence, the correct option is (a).

10. Area of a rectangle with length $4ab$ and breadth $6b^2$ is

(a) $24a^2 b^2$ (b) $24ab^3$ (c) $24ab^2$ (d) $24ab$

Solution:

The formula of area of a rectangle = Length $\times$ Breadth

$$= 4ab \times 6b^2$$

$$= 24ab^3$$

Hence, the correct option is (b).

11. Volume of a rectangular box (cuboid) with length = $2ab$, breadth = $3ac$ and height = $2ac$ is

(a) $12a^3 bc^2$ (b) $12a^3 bc$ (c) $12a^2 bc$ (d) $2ab + 3ac + 2ac$

Solution:

The formula of volume of a cuboid = Length $\times$ Breadth $\times$ Height

$$= 2ab \times 3ac \times 2ab$$

$$= (2 \times 3 \times 2) \times ab \times ac \times ac$$

$$= 12a^3bc^2$$

Hence, the correct option is (a).

12. Product of $6a^2 - 7b + 5ab$ and $2ab$ is

(a) $12a^3b - 14ab^2 + 10ab$

(b) $12a^3b - 14ab^2 + 10a^2b^2$

(c) $6a^2 - 7b + 7ab$

(d) $12a^2b - 7ab^2 + 10ab$

Solution:

Product of $6a^2 - 7b + 5ab$ and $2ab$ is calculated as follows:

$$\text{Required product} = 2ab \times (6a^2 - 7b + 5ab)$$

$$= 2ab \times (6a^2 - 7b + 5ab)$$

$$= 2ab \times 6a^2 + 2ab \times (-7b) + 2ab \times 5ab$$

$$= 12a^3b - 14ab^2 + 10a^2b^2$$

Hence, the correct option is (b).

13. Square of $3x - 4y$ is

(a) $9x^2 - 16y^2$

(b) $6x^2 - 8y^2$

(c) $9x^2 + 16y^2 + 24xy$

(d) $9x^2 + 16y^2 - 24xy$

Solution:

Square of $3x - 4y$ is:

$$3x - 4y = (3x - 4y)^2$$

$$[\text{Now, use the identity: } (a - b)^2 = a^2 - 2ab + b^2]$$

So,

$$(3x - 4y) = (3x)^2 - 2 \times 3x \times 4y + (4y)^2$$

$$= 9x^2 - 24xy + 16y^2$$

Hence, the correct option is (d).

14. Which of the following are like terms?

(a) $5xyz^2, -3xy^2z$

(b) $-5xyz^2, 7xyz^2$

(c) $5xyz^2, 5x^2yz$

(d) $5xyz^2, x^2y^2z^2$

Solution:

As we know that the terms having same algebraic (literal) factors are called like term.

So, the term $-5xyz^2, 7xyz^2$ are like terms.

Hence, the correct option is (b).

15. Coefficient of y in the term $\frac{-y}{3}$ is

- (a) -1 (b) -3 (c) $\frac{-1}{3}$ (d) $\frac{1}{3}$

Solution:

The term $-\frac{y}{3}$ can be written as $-\frac{1}{3} \times y$.

Therefore, the coefficient of y is $-\frac{1}{3}$.

Hence, the correct option is (c).

16. $a^2 - b^2$ is equal to

- (a) $(a - b)^2$ (b) $(a - b)(a - b)$ (c) $(a + b)(a - b)$ (d) $(a + b)(a + b)$

Solution:

The standard identity of $a^2 - b^2$ is equal to:

$$a^2 - b^2 = (a + b)(a - b)$$

Hence, the correct option is (b).

17. Common factor of $17abc$, $34ab^2$, $51a^2b$ is

- (a) $17abc$ (b) $17ab$ (c) $17ac$ (d) $17a^2b^2c$

Solution:

Common factor of $17abc$, $34ab^2$, $51a^2b$ is calculated as follows:

$$17abc = 17 \times a \times b \times c$$

$$34ab^2 = 2 \times 17 \times a \times b \times b$$

$$51a^2b = 3 \times 17 \times a \times b \times c$$

So, common factor is $17 \times a \times b = 17ab$.

Hence, the correct option is (b).

18. Square of $9x - 7xy$ is

- (a) $81x^2 + 49x^2y^2$ (b) $81x^2 - 49x^2y^2$
(c) $81x^2 + 49x^2y^2 - 126x^2y$ (d) $81x^2 + 49x^2y^2 - 63x^2y$

Solution:

Square of $9x - 7xy$ is $(9x - 7xy)^2$.

Now, $(9x - 7xy)^2 = (9x)^2 - 2 \times 9x \times 7xy + (7xy)^2$ [Using the identity: $(a - b)^2 = a^2 - 2ab + b^2$]

$$= 81x^2 - 126x^2y + 49x^2y^2$$

$$= 81x^2 + 49x^2y^2 - 126x^2y$$

Hence, the correct option is (c).

19. Factorised form of $23xy - 46x + 54y - 108$ is

- (a) $(23x + 54)(y - 2)$ (b) $(23x + 54y)(y - 2)$
(c) $(23xy + 54y)(-46x - 108)$ (d) $(23x + 54)(y + 2)$

Solution:

Consider the expression:

Now, factorised $23xy - 46x + 54y - 108$ as follows:

$$= 23x(y - 2) + 54(y - 2) \quad [\text{Taking common out in I and II expressions}]$$

$$= (y - 2)(23x + 54) \quad [\text{Taking (y-2) common}]$$

$$= (23x + 54)(y - 2)$$

Hence, the correct option is (a).

20. Factorised form of $r^2 - 10r + 21$ is

- (a) $(r-1)(r-4)$ (b) $(r-7)(r-3)$ (c) $(r-7)(r+3)$ (d) $(r+7)(r+3)$

Solution:

Consider the expression:

$$r^2 - 10r + 21$$

Now, factorised the above expression as follows:

$$r^2 - 10r + 21 = r^2 - 7r - 3r + 21 \quad [\text{By splitting the middle term, So that the product of their numerical coefficient is equal constant term}]$$

$$= r(r - 7) - 3(r - 7)$$

$$= (r - 7)(r - 3)$$

Hence, the correct option is (b).

21. Factorised form of $p^2 - 17p - 38$ is

- (a) $(p-19)(p+2)$ (b) $(p-19)(p-2)$ (c) $(p+19)(p+2)$ (d) $(p+19)(p-2)$

Solution:

Consider the expression:

$$p^2 - 17p - 38$$

Now, factorised the above expression as follows:

$$p^2 - 17p - 38 = p^2 - 19p + 2p - 38 \quad [\text{By splitting the middle term, So that the product of their numerical coefficient is equal constant term}]$$

$$= p(p - 19) + 2(p - 19)$$

$$= (p - 19)(p + 2)$$

Hence, the correct option is (a).

22. On dividing $57p^2qr$ by $114pq$, we get

- (a) $\frac{1}{4}pr$ (b) $\frac{3}{4}pr$ (c) $\frac{1}{2}pr$ (d) $2pr$**

Solution:

According to the question:

$$\begin{aligned}\frac{57p^2qr}{114pq} &= \frac{57 \times p \times p \times q \times r}{114 \times p \times q} \\ &= \frac{57}{114} pr \\ &= \frac{1}{2} pr\end{aligned}$$

Hence, the correct option is (c).

23. On dividing $p(4p^2 - 16)$ by $4p(p - 2)$, we get

- (a) $2p + 4$ (b) $2p - 4$ (c) $p + 2$ (d) $p - 2$**

Solution:

According to the question:

$$\begin{aligned}\frac{p(4p^2 - 16)}{4p(p - 2)} &= \frac{4p(p^2 - 4)}{4p(p - 2)} \\ &= \frac{(p^2 - 2^2)}{(p - 2)} \\ &= \frac{(p - 2)(p + 2)}{(p - 2)} \quad \left[\text{As } (a^2 - b^2) = (a + b)(a - b) \right] \\ &= p + 2\end{aligned}$$

Hence, the correct option is (c).

24. The common factor of $3ab$ and $2cd$ is

- (a) 1 (b) -1 (c) a (d) c**

Solution:

There is no common factor of $3ab$ and $2cd$ except 1.

Hence, the correct option is (a).

25. An irreducible factor of $24x^2y^2$ is

- (a) x^2 (b) y^2 (c) x (d) $24x$**

Solution:

As we know that an irreducible factor is a factor which can't be expressed further as a product of factors. So,

$$24x^2y^2 = 2 \times 2 \times 2 \times 2 \times 3 \times x \times x \times y \times y$$

Therefore, an irreducible factor is x .

Hence, the correct option is (c).

26. Number of factors of $(a + b)^2$ is

- (a) 4 (b) 3 (c) 2 (d) 1

Solution:

$(a + b)^2$ can be written as $(a+b)(a+b)$.

Therefore, the number of factor is 2.

Hence, the correct option is (c).

27. The factorised form of $3x - 24$ is

- (a) $3x \times 24$ (b) $3(x - 8)$ (c) $24(x - 3)$ (d) $3(x - 12)$

Solution:

The factorised form of $3x - 24$ is $= 3(x-8)$

Hence, the correct option is (b).

28. The factors of $x^2 - 4$ are

- (a) $(x-2), (x-2)$ (b) $(x+2), (x-2)$ (c) $(x+2), (x+2)$ (d) $(x-4), (x-4)$

Solution:

The factors of $x^2 - 4$ are:

$$x^2 - 4 = x^2 - 2^2$$

$$= (x + 2)(x - 2) \quad [\text{As } (a^2 - b^2) = (a + b)(a - b)]$$

Hence, the correct option is (b).

29. The value of $(-27x^2y) \div (-9xy)$ is

- (a) $3xy$ (b) $-3xy$ (c) $-3x$ (d) $3x$

Solution:

The value of $(-27x^2y) \div (-9xy)$ is calculated as follows:

$$\frac{-27x^2y}{-9xy} = \frac{-3 \times 9 \times x \times x \times y}{-9xy}$$
$$= 3x$$

Hence, the correct option is (d).

30. The value of $(2x^2 + 4) \div 2$ is

(a) $2x^2 + 2$ (b) $x^2 + 2$ (c) $x^2 + 4$ (d) $2x^2 + 4$

Solution:

$$\begin{aligned}\text{The value of } (2x^2 + 4) \div 2 \text{ is } &= \frac{(2x^2 + 4)}{2} \\ &= \frac{2(x^2 + 2)}{2} \\ &= x^2 + 2\end{aligned}$$

Hence, the correct option is (b).

31. The value of $(3x^3 + 9x^2 + 27x) \div 3x$ is

(a) $x^2 + 9 + 27x$ (b) $3x^3 + 3x^2 + 27x$ (c) $3x^3 + 9x^2 + 9$ (d) $x^2 + 3x + 9$

Solution:

$$\begin{aligned}\text{The value of } (3x^3 + 9x^2 + 27x) \div 3x \text{ is } &= \frac{3x^3 + 9x^2 + 27x}{3x} \\ &= \frac{3x^3 + 9x^2 + 27x}{3x} \\ &= \frac{3x(x^2 + 3x + 9)}{3x} \\ &= x^2 + 3x + 9\end{aligned}$$

Hence, the correct option is (d).

32. The value of $(a + b)^2 + (a - b)^2$ is

(a) $2a + 2b$ (b) $2a - 2b$ (c) $2a^2 + 2b^2$ (d) $2a^2 - 2b^2$

Solution:

The value of $(a + b)^2 + (a - b)^2$ is calculated as follows:

$$\begin{aligned}(a + b)^2 + (a - b)^2 &= a^2 + b^2 + 2ab + a^2 + b^2 - 2ab \\ &= 2a^2 + 2b^2\end{aligned}$$

Hence, the correct option is (c).

33. The value of $(a + b)^2 - (a - b)^2$ is

(a) $4ab$ (b) $-4ab$ (c) $2a^2 + 2b^2$ (d) $2a^2 - 2b^2$

Solution:

The value of $(a + b)^2 - (a - b)^2$ is calculated as follows:

$$\begin{aligned}
 (a + b)^2 - (a - b)^2 &= a^2 + b^2 + 2ab - (a^2 + b^2 - 2ab) \\
 &= a^2 + b^2 + 2ab - a^2 - b^2 + 2ab \\
 &= 4ab
 \end{aligned}$$

Hence, the correct option is (a).

In questions 34 to 58, fill in the blanks to make the statements true:

34. The product of two terms with like signs is a _____ term.

Solution:

The product of two terms with like signs is a positive term.

For example: the product of $2x$ is $3y$ is,

$$= 2x \times 3y$$

$$= 6xy$$

35. The product of two terms with unlike signs is a _____ term.

Solution:

The product of two terms with unlike signs is a negative term.

For example: the product of $-2x$ is $3y$ is,

$$= -2x \times 3y$$

$$= -6xy$$

36. $a(b + c) = ax \text{ ______ } \times ax \text{ ______ }.$

Solution:

$$a(b + c) = a \times \underline{b} + a \times \underline{c}$$

[By using left distribution law]

$$= ab + ac$$

37. $(a - b) \text{ ______ } = a^2 - 2ab + b^2$

Solution:

$$\text{As we know that: } (a - b) \underline{(a - b)} = (a - b)^2 = a^2 - 2ab + b^2$$

38. $a^2 - b^2 = (a + b) \text{ ______ }.$

Solution:

$$\text{As we know that: } a^2 - b^2 = (a + b) \underline{(a - b)}$$

39. $(a - b)^2 + \underline{\hspace{2cm}} = a^2 - b^2$

Solution:

As we know that: $(a - b)^2 = a^2 + b^2 - 2ab$

Now, adding $2ab - 2b^2$ both sides in the above identity, get:

$$(a - b)^2 + 2ab - 2b^2 = a^2 + b^2 - 2ab + 2ab - 2b^2$$

$$(a - b)^2 + \underline{2ab - 2b^2} = a^2 - b^2$$

40. $(a + b)^2 - 2ab = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$

Solution:

$$(a + b)^2 - 2ab = a^2 + b^2 + 2ab - 2ab \quad [\text{Using the identity: } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= \underline{a^2} + \underline{b^2}$$

41. $(x + a)(x + b) = x^2 + (a + b)x + \underline{\hspace{2cm}}$.

Solution:

$$(x + a)(x + b) = x^2 + bx + ax + ab$$

$$= x^2 + (a + b)x + \underline{ab}$$

42. The product of two polynomials is a polynomial.

Solution:

As the product of two polynomials is again a polynomial.

43. Common factor of $ax^2 + bx$ is $x(ax + b)$.

Solution:

Common factor of $ax^2 + bx$ is $x(ax + b)$. [Taking x as a common]

44. Factorised form of $18mn + 10mnp$ is $2mn(9 + 5p)$.

Solution:

Factorised form of $18mn + 10mnp$ is $2mn(9 + 5p)$. [Taking 2mn as common]

45. Factorised form of $4y^2 - 12y + 9$ is $(2y - 3)^2$.

Solution:

Consider the expression:

$$4y^2 - 12y + 9$$

Now, factorised form of $4y^2 - 12y + 9$ will be calculated by using the splitting the middle term as follows:

$$\begin{aligned} 4y^2 - 12y + 9 &= (2y)^2 - 2 \times 2y \times 3 + 3^2 \\ &= (2y - 3)^2 \quad [As (a - b)^2 = a^2 - 2ab + b^2] \\ &= (2y - 3)(2y - 3) \end{aligned}$$

Hence, Factorised form of $4y^2 - 12y + 9$ is $(2y - 3)(2y - 3)$.

46. $38x^3y^2z \div 19xy^2$ is equal to _____.

Solution:

Consider the expression:

$$38x^3y^2z \div 19xy^2$$

Now, simplify the above expression as follows:

$$\begin{aligned} \frac{38x^3y^2z}{19xy^2} &= \frac{2 \times 19 \times x \times x^2y^2z}{19xy^2} \\ &= 2x^2z \end{aligned}$$

Hence, $38x^3y^2z \div 19xy^2$ is equal to $2x^2z$.

47. Volume of a rectangular box with length $2x$, breadth $3y$ and height $4z$ is _____.

Solution:

The formula of the volume of the rectangular box = Length x Breadth x Height

So, the volume of the rectangular box is:

$$\begin{aligned} &= 2x \times 3y \times 4z \\ &= (2 \times 3 \times 4)xyz \\ &= 24xyz \end{aligned}$$

Hence, volume of a rectangular box with length $2x$, breadth $3y$ and height $4z$ is $24xyz$.

48. $67^2 - 37^2 = (67 - 37) \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$.

Solution:

$$\begin{aligned} 67^2 - 37^2 &= (67 - 37)(67 + 37) \quad [As, a^2 - b^2 = (a - b)(a + b)] \\ &= 30 \times 104 \\ &= 3120 \end{aligned}$$

Hence, $67^2 - 37^2 = (67 - 37)(\underline{67 + 37}) = \underline{3120}$.

49. $103^2 - 102^2 = \underline{\hspace{2cm}} \times (103 - 102) = \underline{\hspace{2cm}}$.

Solution:

$$\begin{aligned}
 103^2 - 102^2 &= (103+102)(103-102) & [\text{As, } a^2 - b^2 &= (a-b)(a+b)] \\
 &= 205 \times 1 \\
 &= 205
 \end{aligned}$$

Hence, $103^2 - 102^2 = (103+102)(103-102) = 205$.

50. Area of a rectangular plot with sides $4x^2$ and $3y^2$ is _____.

Solution:

The formula of the area of rectangle = Length $\times$ Breadth

$$\begin{aligned}
 \text{So, area of a rectangular plot} &= 4x^2 \times 3y^2 \\
 &= 4 \times 3x^2y^2 \\
 &= 12x^2y^2
 \end{aligned}$$

Hence, area of a rectangular plot with sides $4x^2$ and $3y^2$ is $12x^2y^2$.

51. Volume of a rectangular box with $l = b = h = 2x$ is _____.

Solution:

$$\begin{aligned}
 \text{The formula of the volume of the rectangular box is} &= l \times b \times h \\
 &= 2x \times 2x \times 2x \\
 &= 8x^3
 \end{aligned}$$

Hence, volume of a rectangular box with $l = b = h = 2x$ is $8x^3$.

52. The coefficient in $-37abc$ is _____.

Solution:

As we know that the constant term involved in term of an algebraic expression is called the numerical coefficient of that term.

Hence, the coefficient in $-37abc$ is -37 .

53. Number of terms in the expression $a^2 + bc \times d$ is _____.

Solution:

The expression $a^2 + bc \times d$ can be written as $a^2 + bcd$.

Hence, number of terms in the expression $a^2 + bc \times d$ is 2.

54. The sum of areas of two squares with sides $4a$ and $4b$ is _____.

Solution:

As we know that: Area of a square = (Side)²

So, area of the square whose one side is $4a = (4a)^2 = 16a^2$

And area of the square with side $4b = (4b)^2 = 16b^2$

Hence, the sum of areas = $16a^2 + 16b^2 = 16(a^2 + b^2)$.

55. The common factor method of factorisation for a polynomial is based on _____ property.

Solution:

The common factor method of factorisation for a polynomial is based on Distributive property.

56. The side of the square of area $9y^2$ is _____.

Solution:

As we know that: Area of a square = (Side)²

$$9y^2 = (\text{Side})^2$$

$$\begin{aligned}\text{Side} &= \sqrt{9y^2} \\ &= 3y\end{aligned}$$

Hence, the side of the square of area $9y^2$ is $3y$.

57. On simplification $\frac{3x+3}{3} =$ _____.

Solution:

$$\text{On simplification } \frac{3x+3}{3} = \frac{3x}{3} + \frac{3}{3} = \underline{x+1}.$$

58. The factorisation of $2x + 4y$ is _____.

Solution:

The factorisation of $2x + 4y$ is $2(x + 2y)$.

In questions 59 to 80, state whether the statements are True (T) or False (F):

59. $(a + b)^2 = a^2 + b^2$

Solution:

As we know that $(a + b)^2 = a^2 + b^2 + 2ab$.

Hence, the given statement is false.

60. $(a - b)^2 = a^2 - b^2$

Solution:

As we know that $(a - b)^2 = a^2 + b^2 - 2ab$.

Hence, the given statement is false.

61. $(a + b)(a - b) = a^2 - b^2$

Solution:

As we know that: $(a + b)(a - b) = a^2 - b^2$.

Hence, the given statement is false.

62. The product of two negative terms is a negative term.

Solution:

As we know that the product of two negative terms is always a positive term, i.e. $(-) \times (-) = (+)$.

Hence, the given statement is false.

63. The product of one negative and one positive term is a negative term.

Solution:

As we know that when we multiply a negative term by a positive term, the resultant will be a negative term, i.e. $(-) \times (+) = (-)$.

Hence, the given statement is true.

64. The coefficient of the term $-6x^2y^2$ is -6 .

Solution:

As we can see that the coefficient of the term $-6x^2y^2$ is -6 .

Hence, the given statement is true.

65. $p^2q + q^2r + r^2q$ is a binomial.

Solution:

As we can see that the given expression contains three unlike terms, so it is a trinomial.

Hence, the given statement is false.

66. The factors of $a^2 - 2ab + b^2$ are $(a + b)$ and $(a + b)$.

Solution:

As we know that: $(a - b)^2 = a^2 - 2ab + b^2 = (a + b)(a - b)$.

Hence, the given statement is false.

67. h is a factor of $2\pi(h + r)$.

Solution:

As we can see that the given expression has only two factor 2π and $(h + r)$.

Hence, the given statement is false.

68. Some of the factors of $\frac{n^2}{2} + \frac{n}{2}$ are $\frac{1}{2}n$ and $(n + 1)$.

Solution:

The factor of $\frac{n^2}{2} + \frac{n}{2}$ is calculated as:

$$\frac{n^2}{2} + \frac{n}{2} = \frac{1}{2}n(n + 1)$$

So, the factors of $\frac{n^2}{2} + \frac{n}{2}$ are $\frac{1}{2}n$ and $(n + 1)$.

Hence, the given statement is false.

69. An equation is true for all values of its variables.

Solution:

As equation is true only for some values of its variables, that is $2x - 4 = 0$ is true, only for $x = 2$.

Hence, the given statement is false.

70. $x^2 + (a + b)x + ab = (a + b)(x + ab)$

Solution:

$x^2 + (a + b)x + ab$ can be written as $(x + a)(x + b)$.

Hence, the given statement is false.

71. Common factor of $11pq^2$, $121p^2q^3$, $1331p^2q$ is $11p^2q^2$.

Solution:

Common factor of following term is calculated as follows:

$$11pq^2 = 11 \times p \times q \times q$$

$$121p^2q^3 = 11 \times 11 \times p \times p \times q \times q \times q$$

$$1331p^2q = 11 \times 11 \times 11 \times p \times p \times q$$

So, the common factor of the following term is $11p^2q^2$.

Hence, the given statement is false.

72. Common factor of $12a^2b^2 + 4ab^2 - 32$ is 4.**Solution:**

Common factor of following expression is calculated as follows:

$$12a^2b^2 + 4ab^2 - 32 = 4(3a^2b^2 + ab^2 - 8)$$

So, the common factor is 4.

Hence, the given statement is true.

73. Factorisation of $-3a^2 + 3ab + 3ac$ is $3a(-a - b - c)$.**Solution:**

Factorisation of $-3a^2 + 3ab + 3ac$ is calculated as follows:

$$-3a^2 + 3ab + 3ac = 3a(-a + b + c)$$

So, the factor of $-3a^2 + 3ab + 3ac$ is $3a(-a + b + c)$.

Hence, the given statement is false.

74. Factorised form of $p^2 + 30p + 216$ is $(p + 18)(p - 12)$.**Solution:**

Factorised form of $p^2 + 30p + 216$ by using the splitting the middle term is calculated as follows:

$$\begin{aligned} p^2 + 30p + 216 &= p^2 + 12p + 18p + 216 \\ &= p(p + 12) + 18(p + 12) \\ &= (p + 18)(p + 12) \end{aligned}$$

So, the factorised form of $p^2 + 30p + 216$ is $(p + 18)(p + 12)$.

Hence the given statement is false.

75. The difference of the squares of two consecutive numbers is their sum.**Solution:**

Suppose n and $n+1$ be any two consecutive numbers.

So, their sum $= n + n + 1 = 2n + 1$

Now, the difference of their square:

$$\begin{aligned}(n+1)^2 - n^2 &= n^2 + 1 + 2n - n^2 \\ &= 2n + 1\end{aligned}$$

$$[\text{As, } (a+b)^2 = a^2 + 2ab + b^2]$$

Hence, the given statement is true.

76. $abc + bca + cab$ is a monomial.

Solution:

Since, $abc + bca + cab = abc + abc + abc = 3abc$

So, the given expression is monomial.

Hence, the given statement is true.

77. On dividing $\frac{p}{3}$ by $\frac{3}{p}$, the quotient is 9.

Solution:

When $\frac{p}{3}$ dividing by $\frac{3}{p}$, get:

$$\begin{aligned}\frac{\frac{p}{3}}{\frac{3}{p}} &= \frac{p}{3} \times \frac{p}{3} \\ &= \frac{p^2}{9}\end{aligned}$$

So, the quotient is $\frac{1}{9}p^2$.

Hence, the given statement is false.

78. The value of p for $51^2 - 49^2 = 100p$ is 2.

Solution:

The value of p is calculated as follows:

$$51^2 - 49^2 = 100p$$

$$(51+49)(51-49) = 110p$$

$$[\text{As } a^2 - b^2 = (a+b)(a-b)]$$

$$100 \times 2 = 100p$$

$$p = 2$$

So, the value of p is 2.

Hence, the given statement is true.

79. $(9x - 51) \div 9$ is $x - 51$.

Solution:

Consider the expression:

$$(9x - 51) \div 9$$

Now, simplify the above expression as follows:

$$\begin{aligned}\frac{9x - 51}{9} &= \frac{9x}{9} - \frac{51}{9} \\ &= x - \frac{51}{9}\end{aligned}$$

Hence, the given statement is false.

80. The value of $(a + 1)(a - 1)(a^2 + 1)$ is $a^4 - 1$.

Solution:

The value of $(a + 1)(a - 1)(a^2 + 1)$ is calculated as follows:

$$\begin{aligned}(a + 1)(a - 1)(a^2 + 1) &= (a^2 - 1)(a^2 + 1) \quad [\text{Using the identity: } (a+b)(a-b)=a^2 - b^2] \\ &= (a^2)^2 - 1^2 \quad [\text{Again using the same identity}] \\ &= a^4 - 1\end{aligned}$$

81. Add:

(i) $7a^2bc, -3abc^2, 3a^2bc, 2abc^2$

(ii) $9ax, +3by - cz, -5by + ax + 3cz$

(iii) $xy^2z^2 + 3x^2y^2z - 4x^2yz^2, -9x^2y^2z + 3xy^2z^2 + x^2yz^2$

(iv) $5x^2 - 3xy + 4y^2 - 9, 7y^2 + 5xy - 2x^2 + 13$

(v) $2p^4 - 3p^3 + p^2 - 5p + 7, -3p^4 - 7p^3 - 3p^2 - p - 12$

(vi) $3a(a - b + c), 2b(a - b + c)$

(vii) $3a(2b + 5c), 3c(2a + 2b)$

Solution:

(i) Adding $7a^2bc, -3abc^2, 3a^2bc, 2abc^2$ as follows:

$$\begin{aligned}7a^2bc + (-3abc^2) + 3a^2bc + 2abc^2 &= 7a^2bc - 3abc^2 + 3a^2bc + 2abc^2 \\ &= (7a^2bc + 3a^2bc) + (-3abc^2 + 2abc^2) \quad [\text{Grouping like terms}] \\ &= 10a^2bc + (-abc^2) \\ &= 10a^2bc - abc^2\end{aligned}$$

(ii) Adding $9ax, +3by - cz, -5by + ax + 3cz$ as follows:

$$\begin{aligned}(9ax + 3by - cz) + (-5by + ax + 3cz) &= 9ax + 3by - cz - 5by + ax + 3cz \\ &= (9ax + ax) + (3by - 5by) + (-cz + 3cz) \quad [\text{Grouping like terms}] \\ &= 10ax - 2by + 2cz\end{aligned}$$

(iii) Adding $xy^2z^2 + 3x^2y^2z - 4x^2yz^2, -9x^2y^2z + 3xy^2z^2 + x^2yz^2$ as follows:

$$\begin{aligned}
 & xy^2z^2 + 3x^2y^2z - 4x^2yz^2 + (-9x^2y^2z + 3xy^2z^2 + x^2yz^2) = xy^2z^2 + 3x^2y^2z - 4x^2yz^2 - 9x^2y^2z + 3xy^2z^2 + x^2yz^2 \\
 & = (xy^2z^2 + 3xy^2z^2) + (3x^2y^2z - 9x^2y^2z) + (-4x^2yz^2 + x^2yz^2) \text{ [Grouping like terms]} \\
 & = 4xy^2z^2 - 6x^2y^2z - 3x^2yz^2
 \end{aligned}$$

(iv) Adding $5x^2 - 3xy + 4y^2 - 9$, $7y^2 + 5xy - 2x^2 + 13$ as follows:

$$\begin{aligned}
 5x^2 - 3xy + 4y^2 - 9 + 7y^2 + 5xy - 2x^2 + 13 &= 5x^2 - 3xy + 4y^2 - 9 + 7y^2 + 5xy - 2x^2 + 13 \\
 &= (5x^2 - 2x^2) + (-3xy + 5xy) + (4y^2 + 7y^2) + (-9 + 13) \\
 &\text{ [Grouping like terms]} \\
 &= 3x^2 - 2xy - 11y^2 + 4
 \end{aligned}$$

(v) Adding $2p^4 - 3p^3 + p^2 - 5p + 7$, $-3p^4 - 7p^3 - 3p^2 - p - 12$ as follows:

$$\begin{aligned}
 2p^4 - 3p^3 + p^2 - 5p + 7 + (-3p^4 - 7p^3 - 3p^2 - p - 12) &= 2p^4 - 3p^3 + p^2 - 5p + 7 - 3p^4 - 7p^3 - 3p^2 - p - 12 \\
 &= (2p^4 - 3p^4) + (-3p^3 - 7p^3) + (p^2 - 3p^2) + (-5p - p) + (7 - 12) \text{ [Grouping like terms]} \\
 &= -p^4 - 10p^3 - 2p^2 - 6p - 5
 \end{aligned}$$

(vi) Adding $3a(a - b + c)$, $2b(a - b + c)$ as follows:

$$\begin{aligned}
 3a(a - b + c) + 2b(a - b + c) &= (3a^2 - 3ab + 3ac) + (2ab - 2b^2 + 2bc) \\
 &= 3a^2 - 3ab + 2ab + 3ac + 2bc - 2b^2 \text{ [Grouping like terms]} \\
 &= 3a^2 - ab + 3ac + 2bc - 2b^2
 \end{aligned}$$

(vii) Adding $3a(2b + 5c)$, $3c(2a + 2b)$ as follows:

$$\begin{aligned}
 3a(2b + 5c) + 3c(2a + 2b) &= (6ab + 15ac) + (6ac + 6bc) \\
 &= 6ab + 15ac + 6ac + 6bc \\
 &= 6ab + 21ac + 6bc
 \end{aligned}$$

82. Subtract:

(i) $5a^2b^2c^2$ from $-7a^2b^2c^2$

(ii) $6x^2 - 4xy + 5y^2$ from $8y^2 + 6xy - 3x^2$

(iii) $2ab^2c^2 + 4a^2b^2c - 5a^2bc^2$ from $-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2$

(iv) $3t^4 - 4t^3 + 2t^2 - 6t + 6$ from $-4t^4 + 8t^3 - 4t^2 - 2t + 11$

(v) $2ab + 5bc - 7ac$ from $5ab - 2bc - 2ac + 10abc$

(vi) $7p(3q + 7p)$ from $8p(2p - 7q)$

(vii) $-3p^2 + 3pq + 3px$ from $3p(-p - a - r)$

Solution:

(i) Subtracting $5a^2b^2c^2$ from $-7a^2b^2c^2$ as follows:

$$-7a^2b^2c^2 - 5a^2b^2c^2 = -12a^2b^2c^2$$

(ii) Subtracting $6x^2 - 4xy + 5y^2$ from $8y^2 + 6xy - 3x^2$ as follows:

$$\begin{aligned}8y^2 + 6xy - 3x^2 - (6x^2 - 4xy + 5y^2) &= 8y^2 + 6xy - 3x^2 - 6x^2 + 4xy - 5y^2 \\&= (8y^2 - 5y^2) + (6xy + 4xy) - (3x^2 + 6x^2) \\&= 3y^2 + 10xy - 9x^2\end{aligned}$$

(iii) Subtracting $2ab^2c^2 + 4a^2b^2c - 5a^2bc^2$ from $-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2$ as follows:

$$\begin{aligned}&\left[-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2 - (2ab^2c^2 + 4a^2b^2c - 5a^2bc^2) \right] \\&= (-10a^2b^2c - 4a^2b^2c) + (4ab^2c^2 - 2ab^2c^2) + (2a^2bc^2 + 5a^2bc^2) \\&= -14a^2b^2c + 2ab^2c^2 + 7a^2bc^2\end{aligned}$$

(iv) Subtracting $3t^4 - 4t^3 + 2t^2 - 6t + 6$ from $-4t^4 + 8t^3 - 4t^2 - 2t + 11$ as follows:

$$\begin{aligned}&-4t^4 + 8t^3 - 4t^2 - 2t + 11 - (3t^4 - 4t^3 + 2t^2 - 6t + 6) = \begin{bmatrix} -4t^4 + 8t^3 - 4t^2 - 2t \\ +11 - 3t^4 + 4t^3 - 2t^2 + 6t - 6 \end{bmatrix} \\&= (-4t^4 - 3t^4) + (8t^3 + 4t^3) + (-4t^2 - 2t^2) + (-2t + 6t) + (11 - 6) \\&= 7t^4 + 12t^3 - 6t^2 + 4t + 5\end{aligned}$$

(v) Subtracting $2ab + 5bc - 7ac$ from $5ab - 2bc - 2ac + 10abc$ as follows:

$$\begin{aligned}5ab - 2bc - 2ac + 10abc - (2ab + 5bc - 7ac) &= 5ab - 2bc - 2ac + 10abc - 2ab - 5bc + 7ac \\&= (5ab - 2ab) + (-2bc - 5bc) + (-2ac + 7ac) + 10abc \quad \text{[Grouping like terms]} \\&= 3ab - 7bc + 5ac + 10abc\end{aligned}$$

(vi) Subtracting $7p(3q + 7p)$ from $8p(2p - 7q)$ as follows:

$$\begin{aligned}8p(2p - 7q) - 7p(3q + 7p) &= 16p^2 - 56pq - 21pq - 49p^2 \\&= (16p^2 - 49p^2) + (-56pq - 21pq) \quad \text{[Grouping like terms]} \\&= -33p^2 - 77pq\end{aligned}$$

(vii) Subtracting $-3p^2 + 3pq + 3px$ from $3p(-p - a - r)$ as follows:

$$\begin{aligned}3p(-p - a - r) - (-3p^2 + 3pq + 3px) &= -3p^2 - 3ap - 3pr + 3p^2 - 3pq - 3px \\&= (-3p^2 + 3p^2) - 3ap - 3pr - 3pq - 3px \quad \text{[Grouping like terms]} \\&= -3ap - 3pr - 3pq - 3px\end{aligned}$$

83. Multiply the following:

(i) $-7pq^2r^3$, $-13p^3q^2r$

- (ii) $3x^2y^2z^2$, $17xyz$
- (iii) $15xy^2$, $17yz^2$
- (iv) $-5a^2bc$, $11ab$, $13abc^2$
- (v) $-3x^2y$, $(5y - xy)$
- (vi) abc , $(bc + ca)$
- (vii) $7pqr$, $(p - q + r)$
- (viii) $x^2y^2z^2$, $(xy - yz + zx)$
- (ix) $(p + 6)$, $(q - 7)$
- (x) $6mn$, $0mn$
- (xi) a , a^5 , a^6
- (xii) $-7st$, -1 , $-13st^2$
- (xiii) b^3 , $3b^2$, $7ab^5$
- (xiv) $-\frac{100}{9}rs$; $\frac{3}{4}r^3s^2$
- (xv) $(a^2 - b^2)$, $(a^2 + b^2)$
- (xvi) $(ab + c)$, $(ab + c)$
- (xvii) $(pq - 2r)$, $(pq - 2r)$
- (xviii) $\left(\frac{3}{4}x - \frac{4}{3}y\right)$, $\left(\frac{2}{3}x + \frac{3}{2}y\right)$
- (xix) $\frac{3}{2}p^2 + \frac{2}{3}q^2$, $(2p^2 - 3q^2)$
- (xx) $(x^2 - 5x + 6)$, $(2x + 7)$
- (xxi) $(3x^2 + 4x - 8)$, $(2x^2 - 4x + 3)$
- (xxii) $(2x - 2y - 3)$, $(x + y + 5)$

Solution:

- (i) Multiplying $-7pq^2r^3$ to $13p^3q^2r$ as follows:

$$-7pq^2r^3 \times 13p^3q^2r = (-7) \times (-13) p^4q^4r^4$$

$$= 91p^4q^4r^4$$
- (ii) Multiplying $3x^2y^2z^2$ to $17xyz$ as follows:

$$3x^2y^2z^2 \times 17xyz = (3 \times 17) x^2y^2z^2 \times xyz$$

$$= 51x^3y^3z^3$$
- (iii) Multiplying $15xy^2$ to $17yz^2$ as follows:

$$15xy^2 \times 17yz^2 = 15xy^2 \times 17yz^2$$

$$= (15 \times 17) xy^2 \times yz^2$$

$$= 255xy^3z^2$$
- (iv) Multiplying $-5a^2bc$, $11ab$, and $13abc^2$ as follows:

$$\begin{aligned} -5a^2bc \times 11ab \times 13abc^2 &= (-5 \times 11 \times 13)a^2bc \times ab \times abc^2 \\ &= -715a^4b^3c^3 \end{aligned}$$

(v) Multiplying $-3x^2y$ to $(5y - xy)$ as follows:

$$\begin{aligned} -3x^2y \times (5y - xy) &= -3x^2y \times 5y + 3x^2y \times xy \\ &= -15x^2y^2 + 3x^3y^2 \end{aligned}$$

(vi) Multiplying abc to $(bc + ca)$ as follows:

$$\begin{aligned} abc \times (bc + ca) &= abc \times bc + abc \times ca \\ &= ab^2c^2 + a^2bc^2 \end{aligned}$$

(vii) Multiplying $7pqr$ to $(p - q + r)$ as follows:

$$\begin{aligned} 7pqr \times (p - q + r) &= 7pqr \times p - 7pqr \times q + 7pqr \times r \\ &= 7p^2qr - 7pq^2r + 7pqr^2 \end{aligned}$$

(viii) Multiplying $x^2y^2z^2$ to $(xy - yz + zx)$ as follows:

$$\begin{aligned} x^2y^2z^2 \times (xy - yz + zx) &= x^2y^2z^2 \times xy - x^2y^2z^2 \times yz + x^2y^2z^2 \times zx \\ &= x^3y^3z^2 - x^2y^3z^3 + x^3y^2z^3 \end{aligned}$$

(ix) Multiplying $(p + 6)$ to $(q - 7)$ as follows:

$$\begin{aligned} (p + 6) \times (q - 7) &= p(q - 7) + 6(q - 7) \\ &= pq - 7p + 6q - 42 \end{aligned}$$

(x) Multiplying $6mn$ to $0mn$ as follows:

$$\begin{aligned} 6mn \times 0mn &= (6 \times 0)mn \\ &= 0m^2n^2 \\ &= 0 \end{aligned}$$

(xi) Multiplying a, a^5 and a^6 as follows:

$$\begin{aligned} a \times a^5 \times a^6 &= a^{1+5+6} \\ &= a^{12} \end{aligned}$$

(xii) Multiplying $-7st, -1$ and $-13st^2$ as follows:

$$\begin{aligned} -7st \times -1 \times -13st^2 &= [-7 \times (-1) \times (-13)]st \times (st^2) \\ &= -91s^2t^3 \end{aligned}$$

(xiii) Multiplying $b^3, 3b^2$ and $7ab^5$ as follows:

$$b^3 \times 3b^2 \times 7ab^5 = (1 \times 3 \times 7)b^3 \times b^2 \times ab^5 \\ = 21ab^{10}$$

(xiv) Multiplying $-\frac{100}{9}rs$ to $\frac{3}{4}r^3s^2$ as follows:

$$-\frac{100}{9}rs \times \frac{3}{4}r^3s^2 = \left(-\frac{100}{9} \times \frac{3}{4}\right)rs \times r^3s^2 \\ = -\frac{25}{3}r^4s^3$$

(xv) Multiplying $(a^2 - b^2)$ to $(a^2 + b^2)$ as follows:

$$(a^2 - b^2)(a^2 + b^2) = a^2(a^2 + b^2) - b^2(a^2 + b^2) \\ = a^4 + a^2b^2 - b^2a^2 - b^4 \\ = a^4 - b^4$$

(xvi) Multiplying $(ab + c)$ to $(ab + c)$ as follows:

$$(ab + c)(ab + c) = ab(ab + c) + c(ab + c) \\ = a^2b^2 + abc + cab + c^2 \\ = a^2b^2 + 2abc + c^2$$

(xvii) Multiplying $(pq - 2r)$ to $(pq - 2r)$ as follows:

$$(pq - 2r)(pq - 2r) = pq(pq - 2r) - 2r(pq - 2r) \\ = p^2q^2 - 2pqr - 2rpq + 4r^2 \\ = p^2q^2 - 4pqr + 4r^2$$

(xviii) Multiplying $\left(\frac{3}{4}x - \frac{4}{3}y\right)$ to $\left(\frac{2}{3}x + \frac{3}{2}y\right)$ as follows:

$$\begin{aligned}
\left(\frac{3}{4}x - \frac{4}{3}y\right) \text{ to } \left(\frac{2}{3}x + \frac{3}{2}y\right) &= \frac{3}{4}x\left(\frac{2}{3}x + \frac{3}{2}y\right) - \frac{4}{3}y\left(\frac{2}{3}x + \frac{3}{2}y\right) \\
&= \frac{3}{4} \times \frac{2}{3}x^2 + \frac{3}{4} \times \frac{3}{2}xy - \frac{4}{3} \times \frac{2}{3}yx - \frac{4}{3} \times \frac{3}{2}y^2 \\
&= \frac{1}{2}x^2 + \frac{9}{8}xy - \frac{8}{9}xy - 2y^2 \\
&= \frac{1}{2}x^2 + \left(\frac{9}{8} - \frac{8}{9}\right)xy - 2y^2 \\
&= \frac{1}{2}x^2 + \left(\frac{81-64}{72}\right)xy - 2y^2 \\
&= \frac{1}{2}x^2 + \frac{17}{72}xy - 2y^2
\end{aligned}$$

(xix) Multiplying $\left(\frac{3}{2}p^2 + \frac{2}{3}q^2\right)$ to $(2p^2 - 3q^2)$ as follows:

$$\begin{aligned}
\left(\frac{3}{2}p^2 + \frac{2}{3}q^2\right) \times (2p^2 - 3q^2) &= \frac{3}{2}p^2(2p^2 - 3q^2) + \frac{2}{3}q^2(2p^2 - 3q^2) \\
&= \frac{3}{2}p^2 \times 2p^2 - \frac{9}{2}p^2q^2 + \frac{4}{3}q^2p^2 - 2q^4 \\
&= 3p^4 + \left(\frac{4}{3} - \frac{9}{2}\right)p^2q^2 - 2q^4 \\
&= 3p^4 + \left(\frac{8-27}{6}\right)p^2q^2 - 2q^4 \\
&= 3p^4 - \frac{19}{6}p^2q^2 - 2q^4
\end{aligned}$$

(xx) Multiplying $(x^2 - 5x + 6)$ to $(2x + 7)$ as follows:

$$\begin{aligned}
(x^2 - 5x + 6)(2x + 7) &= x^2(2x + 7) - 5x(2x + 7) + 6(2x + 7) \\
&= 2x^3 + 7x^2 - 10x^2 - 35x + 12x + 42 \\
&= 2x^3 - 3x^2 - 23x + 42
\end{aligned}$$

(xxi) Multiplying $(3x^2 + 4x - 8)$ to $(2x^2 - 4x + 3)$ as follows:

$$\begin{aligned}
(3x^2 + 4x - 8) \times (2x^2 - 4x + 3) &= 3x^2(2x^2 - 4x + 3) + 4x(2x^2 - 4x + 3) - 8(2x^2 - 4x + 3) \\
&= 6x^4 - 12x^3 + 9x^2 + 8x^3 - 16x^2 + 12x - 16x^2 + 32x - 24 \\
&= 6x^4 - 12x^3 + 8x^3 + 9x^2 - 16x^2 - 16x^2 + 12x + 32x - 24 \\
&= 6x^4 - 4x^3 - 23x^2 + 44x - 24
\end{aligned}$$

(xxii) Multiplying $(2x - 2y - 3)$ to $(x + y + 5)$ as follows:

$$\begin{aligned}(2x - 2y - 3) \times (x + y + 5) &= 2x(x + y + 5) - 2y(x + y + 5) - 3(x + y + 5) \\&= 2x^2 + 2xy + 10x - 2yx - 2y^2 - 10y - 3x - 3y - 15 \\&= 2x^2 + 2xy - 2yx + 10x - 3x - 2y^2 - 10y - 3y - 15 \\&= 2x^2 + 7x - 13y - 2y^2 - 15\end{aligned}$$

84. Simplify

(i) $(3x + 2y)^2 + (3x - 2y)^2$

(ii) $(3x + 2y)^2 - (3x - 2y)^2$

(iii) $\left(\frac{7}{9}a + \frac{9}{7}b\right)^2 - ab$

(iv) $\left(\frac{3}{4}x - \frac{4}{3}y\right)^2 + 2xy$

(v) $(1.5p + 1.2q)^2 - (1.5p - 1.2q)^2$

(vi) $(2.5m + 1.5q)^2 + (2.5m - 1.5q)^2$

(vii) $(x^2 - 4) + (x^2 + 4) + 16$

(viii) $(ab - c)^2 + 2abc$

(ix) $(a - b)(a^2 + b^2 + ab) - (a + b)(a^2 + b^2 - ab)$

(x) $(b^2 - 49)(b + 7) + 343$

(xi) $(4.5a + 1.5b)^2 + (4.5b + 1.5a)^2$

(xii) $(pq - qr)^2 + 4pq^2r$

(xiii) $(s^2t + tq^2)^2 - (2stq)^2$

Solution:

(i) Consider the expression:

$$(3x + 2y)^2 + (3x - 2y)^2$$

Now, simplify the above expression as follows:

$$(3x + 2y)^2 + (3x - 2y)^2 = (3x)^2 + (2y)^2 + 2 \times 3x \times 2y + (3x)^2 + (2y)^2 - 2 \times 3x \times 2y$$

$$[\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab \text{ and } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= 9x^2 + 4y^2 + 12xy + 9x^2 + 4y^2 - 12xy$$

$$= (9x^2 + 9x^2) + (4y^2 + 4y^2) + 12xy - 12xy$$

$$= 18x^2 + 8y^2$$

(ii) Consider the expression:

$$(3x + 2y)^2 - (3x - 2y)^2$$

Now, simplify the above expression as follows:

$$(3x + 2y)^2 - (3x - 2y)^2 = [(3x + 2y) + (3x - 2y)][(3x + 2y) - (3x - 2y)]$$

$$\begin{aligned}
& \text{[Using the identity: } a^2 - b^2 = (a - b)(a + b) \text{]} \\
& = (3x + 2y + 3x - 2y)(3x + 2y - 3x + 2y) \\
& = 6x \times 4y \\
& = 24xy
\end{aligned}$$

(iii) Consider the expression:

$$\left(\frac{7}{9}a + \frac{9}{7}b\right)^2 - ab$$

Now, simplify the above expression as follows:

$$\begin{aligned}
& \left(\frac{7}{9}a\right)^2 + \left(\frac{9}{7}b\right)^2 + 2 \times \frac{7}{9}a \times \frac{9}{7}b - ab \quad \text{[Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab \text{]} \\
& = \frac{49}{81}a^2 + \frac{81}{49}b^2 + 2ab - ab \\
& = \frac{49}{81}a^2 + ab + \frac{81}{49}b^2
\end{aligned}$$

(iv) Consider the expression:

$$\left(\frac{3}{4}x - \frac{4}{3}y\right)^2 + 2xy$$

Now, simplify the above expression as follows:

$$\begin{aligned}
& \left(\frac{3}{4}x - \frac{4}{3}y\right)^2 + 2xy = \left(\frac{3}{4}x\right)^2 + \left(\frac{4}{3}y\right)^2 - 2 \times \frac{3}{4}x \times \frac{4}{3}y + 2xy \quad \text{[Using the identity: } (a - b)^2 = a^2 + b^2 - 2ab \text{]} \\
& = \frac{9}{16}x^2 + \frac{16}{9}y^2 - 2xy + 2xy \\
& = \frac{9}{16}x^2 + \frac{16}{9}y^2
\end{aligned}$$

(v) Consider the expression:

$$(1.5p + 1.2q)^2 - (1.5p - 1.2q)^2$$

Now, simplify the above expression as follows:

$$\begin{aligned}
& (1.5p + 1.2q)^2 - (1.5p - 1.2q)^2 = [(1.5p + 1.2q) + (1.5p - 1.2q)][(1.5p + 1.2q) - (1.5p - 1.2q)] \\
& \text{[Using the identity: } a^2 - b^2 = (a + b)(a - b) \text{]} \\
& = [(1.5p + 1.5p) + (1.2q - 1.2q)][(1.5p - 1.5p) + (1.2q + 1.2q)] \\
& = 3p \times 2.4q \\
& = 7.2pq
\end{aligned}$$

(vi) Consider the expression:

$$(2.5m + 1.5q)^2 + (2.5m - 1.5q)^2$$

Now, simplify the above expression as follows:

$$\begin{aligned}(2.5m+1.5q)^2 + (2.5m-1.5q)^2 &= (2.5m)^2 + (1.5q)^2 + 2 \times 2.5m \times 1.5q + (2.5m)^2 + (1.5q)^2 - 2 \times (2.5m) \times (1.5q) \\ \text{[Using the identity: } (a+b)^2 &= a^2 + b^2 + 2ab \text{ and } (a-b)^2 = a^2 + b^2 - 2ab \text{]} \\ &= 6.25m^2 + 2.25q^2 + 6.25m^2 + 2.25q^2 \\ &= (6.25 + 6.25)m^2 + (2.25 + 2.25)q^2 \\ &= 12.5m^2 + 4.5q^2\end{aligned}$$

(vii) Consider the expression:

$$(x^2 - 4) + (x^2 + 4) + 16$$

Now, simplify the above expression as follows:

$$\begin{aligned}(x^2 - 4) + (x^2 + 4) + 16 &= x^2 - 4 + x^2 + 4 + 16 \\ &= 2x^2 + 16\end{aligned}$$

(viii) Consider the expression:

$$(ab - c)^2 + 2abc$$

Now, simplify the above expression as follows:

$$\begin{aligned}&= (ab)^2 + c^2 - 2abc + 2abc \quad \text{[Using the identity: } (a-b)^2 = a^2 + b^2 - 2ab \text{]} \\ &= a^2b^2 + c^2\end{aligned}$$

(ix) Consider the expression:

$$(a-b)(a^2 + b^2 + ab) - (a+b)(a^2 + b^2 - ab)$$

Now, solve the above expression as follows:

$$\begin{aligned}&\left\{ (a-b)(a^2 + b^2 + ab) - (a+b)(a^2 + b^2 - ab) \right\} \\ &\left\{ -a(a^2 + b^2 - ab) - b(a^2 + b^2 - ab) \right\} \\ &= a^3 + ab^2 + a^2b - ba^2 - b^3 - ab^2 - a^3 - ab^2 + a^2b - ba^2 - b^3 + ab^2 \\ &= (a^3 - a^3) + (-b^3 - b^3) + (ab^2 - ab^2) + (a^2b - a^2b + a^2b - a^2b) \\ &= 0 - 2b^3 + 0 + 0 + 0 \\ &= -2b^3\end{aligned}$$

(x) Consider the expression:

$$(b^2 - 49)(b + 7) + 343$$

Now, solve the above expression as follows:

$$\begin{aligned}(b^2 - 49)(b + 7) + 343 &= b^2(b + 7) - 49(b + 7) + 343 \\ &= b^3 + 7b^2 - 49b - 343 + 343 \\ &= b^3 - 49b + 7b^2\end{aligned}$$

(xi) Consider the expression:

$$(4.5a + 1.5b)^2 + (4.5b + 1.5a)^2$$

Now, simplify the above expression as follows:

$$\left\{ \begin{aligned} &(4.5a + 1.5b)^2 + (4.5b + 1.5a)^2 = (4.5a)^2 + (1.5b)^2 + 2 \times 4.5a \times 1.5b \\ &+ (4.5b)^2 + (1.5a)^2 + 2 \times 4.5b \times 1.5a \end{aligned} \right\}$$

$$[\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab]$$

$$= 20.25a^2 + 2.25b^2 + 13.5ab + 20.25b^2 + 2.25a^2 + 13.5ab$$

$$= 40.5a^2 + 4.5b^2 + 27ab$$

(xii) Consider the expression:

$$(pq - qr)^2 + 4pq^2r$$

Now, simplify the above expression as follows:

$$(pq - qr)^2 + 4pq^2r = p^2q^2 + q^2r^2 - 2pq^2r + 4pq^2r$$

$$[\text{Using the identity: } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= p^2q^2 + q^2r^2 + 2pqr$$

(xiii) Consider the expression:

$$(s^2t + tq^2)^2 - (2stq)^2$$

Now, solve the above expression as follows:

$$(s^2t + tq^2)^2 - (2stq)^2 = (s^2t)^2 + (tq^2)^2 + 2 \times s^2t \times tq^2 - 4s^2t^2q^2$$

$$[\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab]$$

$$= s^4t^2 + t^2q^4 + 2s^2t^2q^2 - 4s^2t^2q^2$$

$$= s^4t^2 + t^2q^4 - 2s^2t^2q^2$$

85. Expand the following, using suitable identities.

(i) $(xy + yz)^2$

(ii) $(x^2y - xy^2)^2$

(iii) $\left(\frac{4}{5}a + \frac{5}{4}b\right)^2$

(iv) $\left(\frac{2}{4}x - \frac{3}{2}y\right)^2$

(v) $\left(\frac{4}{5}p + \frac{5}{3}\right)^2$

(vi) $(x + 3)(x + 7)$

(vii) $(2x + 9)(2x - 7)$

(viii) $\left(\frac{4x}{5} + \frac{y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$

$$\text{(ix)} \left(\frac{2x}{3} - \frac{2}{3} \right) \left(\frac{2x}{3} + \frac{2a}{3} \right)$$

$$\text{(x)} (2x - 5y) (2x - 5y)$$

$$\text{(xi)} \left(\frac{2a}{3} + \frac{b}{3} \right) \left(\frac{2a}{3} - \frac{b}{3} \right)$$

$$\text{(xii)} (x^2 + y^2) (x^2 - y^2)$$

$$\text{(xiii)} (a^2 + b^2)^2$$

$$\text{(xiv)} (7x + 5)^2$$

$$\text{(xv)} (0.9p - 0.5q)^2$$

$$\text{(xvi)} x^2y^2 = (xy)^2$$

Solution:

(i) Consider the expression:

$$(xy + yz)^2$$

Now, simplify the above expression as follows:

$$\begin{aligned} (xy + yz)^2 &= (xy)^2 + (yz)^2 + 2 \times xy \times yz \quad [\text{Using the identity: } (a+b)^2 = a^2 + b^2 + 2ab] \\ &= x^2y^2 + y^2z^2 + 2xy^2z \end{aligned}$$

(ii) Consider the expression:

$$(x^2y - xy^2)^2$$

Now, simplify the above expression as follows:

$$\begin{aligned} (x^2y - xy^2)^2 &= (x^2y)^2 + (xy^2)^2 + 2 \times x^2y \times xy^2 \quad [\text{Using the identity: } (a-b)^2 = a^2 + b^2 - 2ab] \\ &= x^4y^2 + y^4x^2 + 2x^3y^3 \end{aligned}$$

(iii) Consider the expression:

$$\left(\frac{4}{5}a + \frac{5}{4}b \right)^2$$

Now, simplify the above expression as follows:

$$\begin{aligned} \left(\frac{4}{5}a + \frac{5}{4}b \right)^2 &= \left(\frac{4}{5}a \right)^2 + \left(\frac{5}{4}b \right)^2 + 2 \times \frac{4}{5}a \times \frac{5}{4}b \\ &= \frac{16}{25}a^2 + \frac{25}{16}b^2 + 2ab \end{aligned}$$

(iv) Consider the expression:

$$\left(\frac{2}{4}x - \frac{3}{2}y \right)^2$$

Now, simplify the above expression as follows:

$$\left(\frac{2}{4}x - \frac{3}{2}y\right)^2 = \left(\frac{2}{3}x\right)^2 + \left(\frac{3}{2}y\right)^2 - 2 \times \frac{2}{3}x \times \frac{3}{2}y$$

[Using the identity: $(a-b)^2 = a^2 + b^2 - 2ab$]

$$= \frac{4}{9}x^2 + \frac{9}{4}y^2 - 2xy$$

(v) Consider the expression:

$$\left(\frac{4}{5}p + \frac{5}{3}q\right)^2$$

Now, simplify the above expression as follows:

$$\left(\frac{4}{5}p + \frac{5}{3}q\right)^2 = \left(\frac{4}{5}p\right)^2 + \left(\frac{5}{3}q\right)^2 + 2 \times \frac{4}{5}p \times \frac{5}{3}q$$

[Using the identity: $(a+b)^2 = a^2 + b^2 + 2ab$]

$$= \frac{16}{25}p^2 + \frac{25}{9}q^2 + \frac{8}{3}pq$$

(vi) Consider the expression:

$$(x+3)(x+7)$$

Now, simplify the above expression as follows:

$$(x+3)(x+7) = x^2 + (3+7)x + 3 \times 7$$

[Using the identity: $(x+a)(x+b) = x^2 + (a+b)x + ab$]

$$= x^2 + 10x + 21$$

(vii) Consider the expression:

$$(2x+9)(2x-7)$$

Now, simplify the above expression as follows:

$$(2x+9)(2x-7) = (2x+9)[2x+(-7)]$$

$$= (2x^2) + [9+(-7)]2x + 9 \times (-7)$$

[Using the identity: $(x+a)(x+b) = x^2 + (a+b)x + ab$]

$$= 4x^2 + 4x - 63$$

(viii) Consider the expression:

$$\left(\frac{4x}{5} + \frac{y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$$

Now, simplify the above expression as follows:

$$\left(\frac{4x}{5} + \frac{y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right) = \left(\frac{4x}{5}\right)^2 + \left(\frac{y}{4} + \frac{3y}{4}\right)\frac{4x}{5} + \frac{y}{4} \times \frac{3y}{4}$$

[Using the identity: $(x+a)(x+b) = x^2 + (a+b)x + ab$]

$$= \frac{16}{25}x^2 + \frac{4xy}{5} + \frac{3y^2}{16}$$

(ix) Consider the expression:

$$\left(\frac{2x}{3} - \frac{2}{3}\right)\left(\frac{2x}{3} + \frac{2a}{3}\right)$$

Now, simplify the above expression as follows:

$$\left(\frac{2x}{3} - \frac{2}{3}\right)\left(\frac{2x}{3} + \frac{2a}{3}\right) = \left(\frac{2x}{3}\right)^2 + \left(\frac{-2}{3} + \frac{2a}{3}\right)\frac{2x}{3} + \left(\frac{-2}{3} \times \frac{2a}{3}\right)$$

[Using the identity: $(x+a)(x+b) = x^2 + (a+b)x + ab$]

$$\begin{aligned} &= \frac{4x^2}{9} + \frac{2a-2}{3} \times \frac{2}{3}x - \frac{4}{9}a \\ &= \frac{4x^2}{9} + \frac{4}{9}(a-1)x - \frac{4}{9}a \end{aligned}$$

(x) Consider the expression:

$$(2x-5y)(2x-5y)$$

Now, simplify the above expression as follows:

$$\begin{aligned} (2x-5y)(2x-5y) &= (2x-5y)^2 \\ &= (4x)^2 + (5y)^2 - 2 \times 2x \times 5y \quad [\text{Using the identity: } (a-b)^2 = a^2 + b^2 - 2ab] \\ &= 16x^2 + 25y^2 - 20xy \end{aligned}$$

(xi) Consider the expression:

$$\left(\frac{2a}{3} + \frac{b}{3}\right)\left(\frac{2a}{3} - \frac{b}{3}\right)$$

Now, simplify the above expression as follows:

$$\begin{aligned} \left(\frac{2a}{3} + \frac{b}{3}\right)\left(\frac{2a}{3} - \frac{b}{3}\right) &= \left(\frac{2a}{3}\right)^2 - \left(\frac{b}{3}\right)^2 \\ &= \frac{4}{9}a^2 - \frac{1}{9}b^2 \quad [\text{Using the identity: } (a+b)(a-b) = a^2 - b^2] \end{aligned}$$

(xii) Consider the expression:

$$(x^2 + y^2)(x^2 - y^2)$$

Now, simplify the above expression as follows:

$$\begin{aligned} (x^2 + y^2)(x^2 - y^2) &= (x^2)^2 - (y^2)^2 \quad [\text{Using the identity: } (a+b)(a-b) = a^2 - b^2] \\ &= x^4 - y^4 \end{aligned}$$

(xiii) Consider the expression:

$$(a^2 + b^2)^2$$

Now, simplify the above expression as follows:

$$\begin{aligned}(a^2 + b^2)^2 &= (a^2)^2 + (b^2)^2 + 2a^2 \times b^2 \\ &= a^4 + b^4 + 2a^2b^2 \quad [\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab]\end{aligned}$$

(xiv) Consider the expression:

$$(7x + 5)^2$$

Now, simplify the above expression as follows:

$$\begin{aligned}(7x + 5)^2 &= (7x)^2 + 5^2 + 2 \times 7x \times 5 \\ &= 49x^2 + 25 + 70x \quad [\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab]\end{aligned}$$

(xv) Consider the expression:

$$(0.9p - 0.5q)^2$$

Now, simplify the above expression as follows:

$$\begin{aligned}(0.9p - 0.5q)^2 &= (0.9p)^2 + (0.5q)^2 - 2 \times 0.9p \times 0.5q \\ [\text{Using the identity: } (a - b)^2 &= a^2 + b^2 - 2ab] \\ &= 0.81p^2 + 0.25q^2 - 0.9pq\end{aligned}$$

(xvi) It is equation not exponents.

86. Using suitable identities, evaluate the following.

(i) $(52)^2$

(ii) $(49)^2$

(iii) $(103)^2$

(iv) $(98)^2$

(v) $(1005)^2$

(vi) $(995)^2$

(vii) 47×53

(viii) 52×53

(ix) 105×95

(x) 104×97

(xi) 101×103

(xii) 98×103

(xiii) $(9.9)^2$

(xiv) 9.8×10.2

(xv) 10.1×10.2

(xvi) $(35.4)^2 - (14.6)^2$

(xvii) $(69.3)^2 - (30.7)^2$

(xviii) $(9.7)^2 - (0.3)^2$

(xix) $(132)^2 - (68)^2$

(xx) $(339)^2 - (161)^2$

(xxi) $(729)^2 - (271)^2$

Solution:

(i) $(52)^2 = (50 + 2)^2$

$$= (50)^2 + (2)^2 + 2 \times 50 \times 2 \quad [\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab]$$

$$= 2500 + 4 + 200$$

$$= 2704$$

$$(ii) \quad (49)^2 = (50 - 1)^2$$

$$= (50)^2 + (1)^2 - 2 \times 50 \times 1 \quad [\text{Using the identity: } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= 2500 + 1 - 100$$

$$= 2401$$

$$(iii) \quad (103)^2 = (100 + 3)^2$$

$$= (100)^2 + (3)^2 + 2 \times 100 \times 3 \quad [\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab]$$

$$= 10000 + 9 + 600$$

$$= 10609$$

$$(iv) \quad (98)^2 = (100 - 2)^2$$

$$= (100)^2 + (2)^2 - 2 \times 100 \times 2 \quad [\text{Using the identity: } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= 10000 + 4 - 400$$

$$= 9604$$

$$(v) \quad (1005)^2 = (1000 + 5)^2$$

$$= (1000)^2 + (5)^2 + 2 \times 1000 \times 5 \quad [\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab]$$

$$= 1000000 + 25 + 10000$$

$$= 1010025$$

$$(vi) \quad (995)^2 = (1000 - 5)^2$$

$$= (1000)^2 + (5)^2 - 2 \times 1000 \times 5 \quad [\text{Using the identity: } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= 1000000 + 25 - 10000$$

$$= 990025$$

$$(vii) \quad 47 \times 53 = (50 - 3)(50 + 3)$$

$$= (50)^2 - (3)^2 \quad [\text{Using the identity: } (a - b)(a + b) = a^2 - b^2]$$

$$= 2500 - 9$$

$$= 2491$$

$$(viii) \quad 52 \times 53 = (50 + 2)(50 + 3)$$

$$= (50)^2 + (2 + 3) \times 50 + 2 \times 3 \quad [\text{Using the identity: } (x + b)(x + b) = x^2 + (a + b)x + ab]$$

$$= 2500 + 250 + 6$$

$$= 2756$$

$$(ix) \quad 105 \times 95 = (100 + 5)(100 - 5)$$

$$= (100)^2 - (5)^2 \quad [\text{Using the identity: } (a + b)(a - b) = a^2 - b^2]$$

$$= 10000 - 25$$

$$= 9975$$

$$(x) \quad 104 \times 97 = (100 + 4)(100 - 3)$$

$$= (100)^2 + (4 - 3) \times 100 + 4 \times (-3) \quad [\text{Using the identity: } (x + b)(x + b) = x^2 + (a + b)x + ab]$$

$$= 10000 + 100 - 12$$

$$= 10088$$

$$(xi) \quad 101 \times 103 = (100 + 1)(100 + 3)$$

$$= (100)^2 + (1 + 3) \times 100 + 3 \times 1 \quad [\text{Using the identity: } (x + b)(x + b) = x^2 + (a + b)x + ab]$$

$$= 10000 + 400 + 3$$

$$= 10403$$

$$(xii) \quad 98 \times 103 = (100 - 2)(100 + 3)$$

$$= (100)^2 + (-2 + 3) \times 100 + (-2) \times 3 \quad [\text{Using the identity: } (x + b)(x + b) = x^2 + (a + b)x + ab]$$

$$= 10000 + 100 - 6$$

$$= 10094$$

$$(xiii) \quad (9.9)^2 = (10 - 0.1)^2$$

$$= (10)^2 + (0.1)^2 - 2 \times 10 \times 0.1 \quad [\text{Using the identity: } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= 100 + 0.01 - 2$$

$$= 98.01$$

$$(xiv) \quad (9.8) \times (10.2) = (10 - 0.2)(10 + 0.2)$$

$$= (10)^2 - (0.2)^2 \quad [\text{Using the identity: } (a + b)(a - b) = a^2 - b^2]$$

$$= 100 - 0.04$$

$$= 99.96$$

$$(xv) \quad (10.1) \times 10.2 = (10 + 0.1)(10 + 0.2)$$

$$= (10)^2 + (0.1 + 0.2) \times 10 + 0.1 \times 0.2 \quad [\text{Using the identity: } (x + b)(x + b) = x^2 + (a + b)x + ab]$$

$$= 100 + 0.3 \times 10 + 0.02$$

$$= 103.02$$

$$\begin{aligned} \text{(xvi)} \quad (35.4)^2 - (14.6)^2 &= (35.4 + 14.6)(35.4 - 14.6) \\ &= 50 \times 20.8 \\ &= 1040 \end{aligned}$$

[Using the identity: $(a + b)(a - b) = a^2 - b^2$]

$$\begin{aligned} \text{(xvii)} \quad (68.3)^2 - (30.7)^2 &= (68.3 + 30.7)(68.3 - 30.7) \\ &= 100 \times 38.6 \\ &= 3860 \end{aligned}$$

[Using the identity: $(a + b)(a - b) = a^2 - b^2$]

$$\begin{aligned} \text{(xviii)} \quad (9.7)^2 - (0.3)^2 &= (9.7 + 0.3)(9.7 - 0.3) \\ &= 10 \times 9.4 \\ &= 94 \end{aligned}$$

[Using the identity: $a^2 - b^2 = (a + b)(a - b)$]

$$\begin{aligned} \text{(xix)} \quad (132)^2 - (68)^2 &= (132 + 68)(132 - 68) \\ &= 200 \times 64 \\ &= 12800 \end{aligned}$$

[Using the identity: $a^2 - b^2 = (a + b)(a - b)$]

$$\begin{aligned} \text{(xx)} \quad (339)^2 - (161)^2 &= (339 + 161)(339 - 161) \\ &= 500 \times 178 \\ &= 89000 \end{aligned}$$

[Using the identity: $a^2 - b^2 = (a + b)(a - b)$]

87. Write the greatest common factor in each of the following terms.

(i) $-18a^2$, $108a$

(ii) $3x^2y$, $18xy^2$, $-6xy$

(iii) $2xy$, $-y^2$, $2x^2y$

(iv) l^2m^2n , lm^2n^2 , l^2mn^2

(v) $21pqr$, $-7p^2q^2r^2$, $49p^2qr$

(vi) qrx , $pryz$, $rxzy$

(vii) $3x^3y^2z$, $-6xy^3z^2$, $12x^2yz^3$

(viii) $63p^2a^2r^2s$, $-9pq^2r^2s^2$, $15p^2qr^2s^2$, $-60p^2a^2rs^2$

(ix) $13x^2y$, $169xy$

(x) $11x^2$, $12y^2$

Solution:

(i) Factor of $-18a^2$ and $108a$ will be:

$$-18a^2 = -8 \times a \times a$$

$$108a = 18 \times 10 \times a$$

So, the greatest common factor is $18a$.

- (ii) Factor of $3x^2y$, $18xy^2$, $-6xy$ will be:

$$3x^2y = -3 \times x \times x \times y$$

$$18xy^2 = 3 \times 6 \times x \times y \times y$$

$$-6xy = -1 \times 3 \times 2 \times x \times y$$

So, the greatest common factor is $3xy$.

- (iii) Factor of $2xy$, $-y^2$, $2x^2y$ will be:

$$2xy = 2 \times x \times y$$

$$-y^2 = -y \times y$$

$$2x^2y = 2 \times x \times x \times y$$

So, the greatest common factor is y .

- (iv) Factor of l^2m^2n , lm^2n^2 , l^2mn^2 will be:

$$l^2m^2n = l \times l \times m \times m \times n$$

$$lm^2n^2 = l \times m \times m \times n \times n$$

$$l^2mn^2 = l \times l \times m \times n \times n$$

So, the greatest common factor is lmn .

- (v) Factor of $21pqr$, $-7p^2q^2r^2$, $49p^2qr$ will be:

$$21pqr = 7 \times 3 \times p \times q \times r$$

$$-7p^2q^2r^2 = -7 \times p \times p \times q \times q \times r \times r$$

$$49p^2qr = 7 \times 7 \times p \times p \times q \times r$$

So, the greatest common factor is $7pqr$.

- (vi) Factor of $qrxy$, $pryz$, $rxzy$ will be:

$$qrxy = q \times r \times x \times y$$

$$pryz = p \times r \times y \times z$$

$$rxzy = r \times x \times y \times z$$

So, the greatest common factor is ry .

- (vii) Factor of $3x^3y^2z$, $-6xy^3z^2$, $12x^2yz^3$ will be:

$$3x^3y^2z = 3 \times x \times x \times x \times y \times y \times z$$

$$-6xy^3z^2 = -3 \times 2 \times x \times y \times y \times y \times z \times z$$

$$12x^2yz^3 = 3 \times 4 \times x \times x \times y \times z \times z \times z$$

So, the greatest common factor is $3xyz$.

(viii) Factor of $63p^2a^2r^2s$, $-9pq^2r^2s^2$, $15p^2qr^2s^2$, $-60p^2a^2rs^2$ will be:

$$63p^2a^2r^2s = 3 \times 3 \times 7 \times p \times p \times a \times a \times r \times r \times s$$

$$-9pq^2r^2s^2 = -3 \times 3 \times p \times q \times q \times r \times r \times s \times s$$

$$15p^2qr^2s^2 = 3 \times 5 \times p \times p \times q \times r \times r \times s \times s$$

$$-60p^2a^2rs^2 = -2 \times 2 \times 3 \times 5 \times p \times p \times a \times a \times r \times s \times s$$

So, the greatest common factor is $3prs$.

(ix) Factor of $13x^2y$, $169xy$ will be:

$$13x^2y = 13 \times x \times x \times y$$

$$169xy = 13 \times 13 \times x \times y$$

So, the greatest common factor is $13xy$.

(x) We have $11x^2, 12y^2$

There is no common factor between x^2 and y^2 .

So, the greatest common factor is 1.

88. Factorise the following expressions.

(i) $6ab + 12bc$

(ii) $-xy - ay$

(iii) $ax^3 - bx^2 + cx$

(iv) $l^2m^2n - lm^2n^2 - l^2mn^2$

(v) $3pqr - 6p^2q^2r^2 - 15r^2$

(vi) $x^3y^2 + x^2y^3 - xy^4 + xy$

(vii) $4xy^2 - 10x^2y + 16x^2y^2 + 2xy$

(viii) $2a^3 - 3a^2b + 5ab^2 - ab$

(ix) $63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2$

(x) $24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz$

(xi) $a^3 + a^2 + a + 1$

(xii) $lx + my + mx + ly$

(xiii) $a^3x - x^4 + a^2x^2 - ax^3$

(xiv) $2x^2 - 2y + 4xy - x$

(xv) $y^2 + 8zx - 2xy - 4yz$

(xvi) $ax^2y - bxyz - ax^2z + bxy^2$

(xvii) $a^2b + a^2c + ab + ac + b^2c + c^2b$

(xviii) $2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2$

Solution:

(i) Consider the expression:

$$6ab + 12bc$$

Now, simplify the above expression as follows:

$$\begin{aligned} 6ab + 12bc &= 6ab + 6 \times 2bc \\ &= 6b(a + 2c) \end{aligned}$$

- (ii) Consider the expression:

$$-xy - ay$$

Now, simplify the above expression as follows:

$$-xy - ay = -y(x + a)$$

- (iii) Consider the expression:

$$ax^3 - bx^2 + cx$$

Now, simplify the above expression as follows:

$$ax^3 - bx^2 + cx = x(ax^2 - bx + c)$$

- (iv) Consider the expression:

$$l^2m^2n - lm^2n^2 - l^2mn^2$$

Now, simplify the above expression as follows:

$$l^2m^2n - lm^2n^2 - l^2mn^2 = lmn(lm - mn - ln)$$

- (v) Consider the expression:

$$3pqr - 6p^2q^2r^2 - 15r^2$$

Now, simplify the above expression as follows:

$$\begin{aligned} 3pqr - 6p^2q^2r^2 - 15r^2 &= 3pqr - 3 \times 2p^2q^2r^2 - 3 \times 5r^2 \\ &= 3r(pq - 2p^2q^2r - 5r) \end{aligned}$$

- (vi) Consider the expression:

$$x^3y^2 + x^2y^3 - xy^4 + xy$$

Now, simplify the above expression as follows:

$$x^3y^2 + x^2y^3 - xy^4 + xy = xy(x^2y + xy^2 - y^3 + 1)$$

- (vii) Consider the expression:

$$4xy^2 - 10x^2y + 16x^2y^2 + 2xy$$

Now, simplify the above expression as follows:

$$\begin{aligned} 4xy^2 - 10x^2y + 16x^2y^2 + 2xy &= 2 \times 2xy^2 - 2 \times 5 \times x^2y + 2 \times 8 \times x^2y^2 + 2xy \\ &= 2xy(2y - 5x + 8xy + 1) \end{aligned}$$

- (viii) Consider the expression:

$$2a^3 - 3a^2b + 5ab^2 - ab$$

Now, simplify the above expression as follows:

$$2a^3 - 3a^2b + 5ab^2 - ab = a(2a^2 - 3ab + 5b^2 - b)$$

(ix) Consider the expression:

$$63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2$$

Now, simplify the above expression as follows:

$$\begin{aligned} & \left[63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2 = 3 \times 21p^2q^2r^2s \right. \\ & \qquad \qquad \qquad \left. - 3 \times 3pq^2r^2s^2 + 3 \times 5p^2qr^2s^2 - 3 \times 20p^2q^2rs^2 \right] \\ & = 3pqrs(21pqr - 3qrs + 5prs - 20pqs) \end{aligned}$$

(x) Consider the expression:

$$24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz$$

Now, simplify the above expression as follows:

$$24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz = xyz(24xz^2 - 6y^2z + 15xy - 5)$$

(xi) Consider the expression:

$$a^3 + a^2 + a + 1$$

Now, simplify the above expression as follows:

$$\begin{aligned} a^3 + a^2 + a + 1 &= a^2(a+1) + 1(a+1) \\ &= (a+1)(a^2+1) \end{aligned}$$

(xii) Consider the expression:

$$lx + my + mx + ly$$

Now, simplify the above expression as follows:

$$\begin{aligned} lx + my + mx + ly &= lx + mx + my + ly \\ &= x(l+m) + y(m+l) \\ &= (l+m)(x+y) \end{aligned}$$

(xiii) Consider the expression:

$$a^3x - x^4 + a^2x^2 - ax^3$$

Now, simplify the above expression as follows:

$$\begin{aligned} a^3x - x^4 + a^2x^2 - ax^3 &= x(a^3 - x^3 + a^2x - ax^2) \\ &= x(a^3 + a^2x - x^3 - ax^2) \\ &= x[a^2(a+x) - x^2(x+a)] \\ &= x[(x+a)(a^2 - x^2)] \\ &= x(a^2 - x^2)(a+x) \end{aligned}$$

(xiv) Consider the expression:

$$2x^2 - 2y + 4xy - x$$

Now, simplify the above expression as follows:

$$\begin{aligned}
2x^2 - 2y + 4xy - x &= 2x^2 - x - 2y + 4xy \\
&= x(2x - 1) - 2y(1 - 2x) \\
&= x(2x - 1) + 2y(2x - 1) \\
&= (2x - 1)(x + 2y)
\end{aligned}$$

(xv) Consider the expression:

$$y^2 + 8zx - 2xy - 4yz$$

Now, simplify the above expression as follows:

$$\begin{aligned}
y^2 + 8zx - 2xy - 4yz &= y^2 - 2xy + 8zx - 4yz \\
&= y(y - 2x) - 4z(y - 2x) \\
&= (y - 2x)(y - 4z)
\end{aligned}$$

(xvi) Consider the expression:

$$ax^2y - bxyz - ax^2z + bxy^2$$

Now, simplify the above expression as follows:

$$\begin{aligned}
ax^2y - bxyz - ax^2z + bxy^2 &= x(axy - byz - axz + by^2) \\
&= x(axy - axz - byz + by^2) \\
&= x[ax(y - z) + by(-z + y)] \\
&= x[(ax + by)(y - z)]
\end{aligned}$$

(xvii) Consider the expression:

$$a^2b + a^2c + ab + ac + b^2c + c^2b$$

Now, simplify the above expression as follows:

$$\begin{aligned}
a^2b + a^2c + ab + ac + b^2c + c^2b &= (a^2b + ab + b^2c) + (a^2c + ac + c^2b) \\
&= b(a^2 + a + bc) + c(a^2 + a + bc) \\
&= (a^2 + a + bc)(b + c)
\end{aligned}$$

(xviii) Consider the expression:

$$2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2$$

Now, simplify the above expression as follows:

$$\begin{aligned}
2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2 &= (2ax^2 + 2ay^2 + 4axy) + (3bx^2 + 3by^2 + 6bxy) \\
&= 2a(x^2 + y^2 + 2xy) + 3b(x^2 + y^2 + 2xy) \\
&= (2a + 3b)(x + y)^2
\end{aligned}$$

89. Factorise the following, using the identity $a^2 + 2ab + b^2 = (a + b)^2$

(i) $x^2 + 6x + 9$

- (ii) $x^2 + 12x + 36$
 (iii) $x^2 + 14x + 49$
 (iv) $x^2 + 2x + 1$
 (v) $4x^2 + 4x + 1$
 (vi) $a^2x^2 + 2ax + 1$
 (vii) $a^2x^2 + 2abx + b^2$
 (viii) $a^2x^2 + 2abxy + b^2y^2$
 (ix) $4x^2 + 12x + 9$
 (x) $16x^2 + 40x + 25$
 (xi) $9x^2 + 24x + 16$
 (xii) $9x^2 + 30x + 25$
 (xiii) $2x^3 + 24x^2 + 72x$
 (xiv) $a^2x^3 + 2abx^2 + b^2x$
 (xv) $4x^4 + 12x^3 + 9x^2$
 (xvi) $(x^2/4) + 2x + 4$
 (xvii) $9x^2 + 2xy + (y^2/9)$

Solution:

$$\begin{aligned}
 \text{(i)} \quad x^2 + 6x + 9 &= x^2 + 2 \times 3 \times x + 3^2 \quad [\text{As } a^2 + 2ab + b^2 = (a+b)^2] \\
 &= (x+3)^2 \\
 &= (x+3)(x+3)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad x^2 + 12x + 36 &= x^2 + 2 \times 6 \times x + 6^2 \quad [\text{As } a^2 + 2ab + b^2 = (a+b)^2] \\
 &= (x+6)^2 \\
 &= (x+6)(x+6)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad x^2 + 14x + 49 &= x^2 + 2 \times 7 \times x + 7^2 \quad [\text{As } a^2 + 2ab + b^2 = (a+b)^2] \\
 &= (x+7)^2 \\
 &= (x+7)(x+7)
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad x^2 + 2x + 1 &= x^2 + 2 \times 1 \times x + 1^2 \quad [\text{As } a^2 + 2ab + b^2 = (a+b)^2] \\
 &= (x+1)^2 \\
 &= (x+1)(x+1)
 \end{aligned}$$

$$\text{(v)} \quad 4x^2 + 4x + 1 = (2x)^2 + 2 \times 2x \times 1 + 1^2 \quad [\text{As } a^2 + 2ab + b^2 = (a+b)^2]$$

$$= (2x+1)^2$$

$$= (2x+1)(2x+1)$$

$$(vi) \quad a^2x^2 + 2ax + 1 = (ax)^2 + 2 \times ax \times 1 + 1^2 \quad [As \quad a^2 + 2ab + b^2 = (a+b)^2]$$

$$= (ax+1)^2$$

$$= (ax+1)(ax+1)$$

$$(vii) \quad a^2x^2 + 2abc + b^2 = (ax)^2 + 2 \times ax \times b + b^2 \quad [As \quad a^2 + 2ab + b^2 = (a+b)^2]$$

$$= (ax+b)^2$$

$$= (ax+b)(ax+b)$$

$$(viii) \quad a^2x^2 + 2abxy + b^2y^2 = (ax)^2 + 2 \times ax \times by + (by)^2$$

$$= (ax+by)^2$$

$$= (ax+by)(ax+by)$$

$$(ix) \quad 4x^2 + 12x + 9 = (2x)^2 + 2 \times 2x \times 3 + 3^2$$

$$= (2x+3)^2$$

$$= (2x+3)(2x+3)$$

$$(x) \quad 16x^2 + 40x + 25 = (4x)^2 + 2 \times 4x \times 5 + 5^2$$

$$= (4x+5)^2$$

$$= (4x+5)(4x+5)$$

$$(xi) \quad 9x^2 + 24x + 16 = (3x)^2 + 2 \times 3x \times 4 + 4^2$$

$$= (3x+4)^2$$

$$= (3x+4)(3x+4)$$

$$(xii) \quad 9x^2 + 30x + 25 = (3x)^2 + 2 \times 3x \times 5 + 5^2$$

$$= (3x+5)^2$$

$$= (3x+5)(3x+5)$$

$$(xiii) \quad 2x^3 + 24x^2 + 72x = 2x(x^2 + 12x + 36)$$

$$\begin{aligned}
 &= 2x(x^2 + 2 \times 6 \times x + 6^2) \\
 &= 2x(x+6)^2 \\
 &= 2x(x+6)(x+6)
 \end{aligned}$$

$$\begin{aligned}
 \text{(xiv)} \quad a^2x^3 + 2abx^2 + b^2x &= x(a^2x^2 + 2abx + b^2) \\
 &= x[(ax)^2 + 2 \times ax \times b + b^2] \\
 &= x[(ax)^2 + 2 \times ax \times b + b^2] \\
 &= x(ax+b)^2 \\
 &= x(ax+b)(ax+b)
 \end{aligned}$$

$$\begin{aligned}
 \text{(xv)} \quad 4x^4 + 12x^3 + 9x^2 &= x^2(4x^2 + 12x + 9) \\
 &= x^2[(2x)^2 + 2 \times 2x \times 3 + 3^2] \\
 &= x^2(2x+3)^2 \\
 &= x^2(2x+3)(2x+3)
 \end{aligned}$$

$$\begin{aligned}
 \text{(xvi)} \quad \frac{x^2}{4} + 2x + 4 &= \frac{x^2}{4} + 2 \times \frac{x}{2} \times 2 + 2^2 \\
 &= \left(\frac{x}{2} + 2\right)^2 \\
 &= \left(\frac{x}{2} + 2\right)\left(\frac{x}{2} + 2\right)
 \end{aligned}$$

$$\begin{aligned}
 \text{(xvii)} \quad 9x^2 + 2xy + \frac{y^2}{9} &= (3x)^2 + 2 \times 3x \times \frac{y}{3} + \left(\frac{y}{3}\right)^2 \\
 &= \left(3x + \frac{y}{3}\right)^2 \\
 &= \left(3x + \frac{y}{3}\right)\left(3x + \frac{y}{3}\right)
 \end{aligned}$$

90. Factorise the following, using the identity $a^2 - 2ab + b^2 = (a - b)^2$.

(i) $x^2 - 8x + 16$

(ii) $x^2 - 10x + 25$

(iii) $y^2 - 14y + 49$

(iv) $p^2 - 2p + 1$

(v) $4a^2 - 4ab + b^2$

(vi) $p^2y^2 - 2py + 1$

(vii) $a^2y^2 - 2aby + b^2$

(viii) $9x^2 - 12x + 4$

(ix) $4y^2 - 12y + 9$

(x) $(x^2/4) - 2x + 4$

(xi) $a^2y^3 - 2aby^2 + b^2y$

(xii) $9y^2 - 4xy + (4x^2/9)$

Solution:

(i) $x^2 - 8x + 16 = x^2 - 2 \times x \times 4 + 4^2$
 $= (x - 4)^2$
 $= (x - 4)(x - 4)$

(ii) $x^2 - 10x + 25 = x^2 - 2 \times x \times 5 + 5^2$
 $= (x - 5)^2$
 $= (x - 5)(x - 5)$

(iii) $y^2 - 14y + 49 = y^2 - 2 \times y \times 7 + 7^2$
 $= (y - 7)^2$
 $= (y - 7)(y - 7)$

(iv) $p^2 - 2p + 1 = p^2 - 2 \times p \times 1 + 1^2$
 $= (p - 1)^2$
 $= (p - 1)(p - 1)$

(v) $4a^2 - 4ab + b^2 = (2a)^2 - 2 \times 2a \times b + b^2$
 $= (2a - b)^2$
 $= (2a - b)(2a - b)$

(vi) $p^2y^2 - 2py + 1 = (py)^2 - 2 \times py \times 1 + 1^2$
 $= (py - 1)^2$
 $= (py - 1)(py - 1)$

(vii) $a^2y^2 - 2aby + b^2 = (ay)^2 - 2 \times ay \times b + b^2$

$$= (ay - b)^2$$

$$= (ay - b)(ay - b)$$

$$\text{(viii) } 9x^2 - 12x + 4 = (3x)^2 - 2 \times 3x \times 2 + 2^2$$

$$= (3x - 2)^2$$

$$= (3x - 2)(3x - 2)$$

$$\text{(ix) } 4y^2 - 12y + 9 = (2y)^2 - 2 \times 2y \times 3 + 3^2$$

$$= (2y - 3)^2$$

$$= (2y - 3)(2y - 3)$$

$$\text{(x) } \frac{x^2}{4} - 2x + 4 = \left(\frac{x}{2}\right)^2 - 2 \times \frac{x}{2} \times 2 + 2^2$$

$$= \left(\frac{x}{2} - 2\right)^2$$

$$= \left(\frac{x}{2} - 2\right)\left(\frac{x}{2} - 2\right)$$

$$\text{(xi) } a^2y^3 - 2aby^2 + b^2y = y(a^2y^2 - 2aby + b^2)$$

$$= y[(ay)^2 - 2 \times ay \times b + b^2]$$

$$= y(ay - b)^2$$

$$= y(ay - b)(ay - b)$$

$$\text{(xii) } 9y^2 - 4xy + \frac{4x^2}{9} = (3y)^2 - 2 \times 3y \times \frac{2}{3}x + \left(\frac{2}{3}x\right)^2$$

$$= \left(3y - \frac{2}{3}x\right)^2$$

$$= \left(3y - \frac{2x}{3}\right)\left(3y - \frac{2x}{3}\right)$$

91. Factorise the following.

(i) $x^2 + 15x + 26$

(ii) $x^2 + 9x + 20$

(iii) $y^2 + 18x + 65$

(iv) $p^2 + 14p + 13$

(v) $y^2 + 4y - 21$

(vi) $y^2 - 2y - 15$

(vii) $18 + 11x + x^2$

(viii) $x^2 - 10x + 21$

(ix) $x^2 = 17x + 60$

(x) $x^2 + 4x - 77$

(xi) $y^2 + 7y + 12$

(xii) $p^2 - 13p - 30$

(xiii) $a^2 - 16p - 80$

Solution:

(i)
$$\begin{aligned}x^2 + 15x + 26 &= x^2 + 2x + 13x + 2 \times 13 \\&= x(x + 2) + 13(x + 2) \\&= (x + 2)(x + 13)\end{aligned}$$

(ii)
$$\begin{aligned}x^2 + 9x + 20 &= x^2 + 5x + 4x + 5 \times 4 \\&= x(x + 5) + 4(x + 5) \\&= (x + 5)(x + 4)\end{aligned}$$

(iii)
$$\begin{aligned}y^2 + 18y + 65 &= y^2 + 13y + 5y + 5 \times 13 \\&= y(y + 13) + 5(y + 13) \\&= (y + 13)(y + 5)\end{aligned}$$

(iv)
$$\begin{aligned}p^2 + 14p + 13 &= p^2 + 13p + p + 13 \times 1 \\&= p(p + 13) + 1(p + 13) \\&= (p + 13)(p + 1)\end{aligned}$$

(v)
$$\begin{aligned}y^2 + 4y - 21 &= y^2 + (7 - 3)y - 21 \\&= y^2 + 7y - 3y - 21 \\&= y(y + 7) - 3(y + 7) \\&= (y + 7)(y - 3)\end{aligned}$$

(vi)
$$y^2 - 2y - 15 = y^2 + (3 - 5)y - 15$$

$$\begin{aligned}
&= y^2 + 3y - 5y - 15 \\
&= y(y + 3) - 5(y + 3) \\
&= (y + 3)(y - 5)
\end{aligned}$$

$$\begin{aligned}
\text{(vii)} \quad 18 + 11x + x^2 &= x^2 + 11x + 18 \\
&= x^2 + (9 + 2)x + 18 \\
&= x^2 + 9x + 2x + 18 \\
&= x(x + 9) + 2(x + 9) \\
&= (x + 9)(x + 2)
\end{aligned}$$

$$\begin{aligned}
\text{(viii)} \quad x^2 - 10x + 21 &= x^2 - (7 + 3)x + 21 \\
&= x^2 - 7x - 3x + 21 \\
&= x(x - 7) - 3(x - 7) \\
&= (x - 7)(x - 3)
\end{aligned}$$

$$\begin{aligned}
\text{(ix)} \quad x^2 - 17x + 60 &= x^2 - (12 + 5)x + 60 \\
&= x^2 - 12x - 5x + 60 \\
&= x(x - 12) - 5(x - 12) \\
&= (x - 12)(x - 5)
\end{aligned}$$

$$\begin{aligned}
\text{(x)} \quad x^2 + 4x - 77 &= x^2 + (11 - 7)x - 77 \\
&= x^2 + 11x - 7x - 77 \\
&= x(x + 11) - 7(x + 11) \\
&= (x + 11)(x - 7)
\end{aligned}$$

$$\begin{aligned}
\text{(xi)} \quad y^2 + 7y + 12 &= y^2 + (4 + 3)y + 12 \\
&= y^2 + 4y + 3y + 12 \\
&= y(y + 4) + 3(y + 4) \\
&= (y + 4)(y + 3)
\end{aligned}$$

$$\begin{aligned}
\text{(xii)} \quad p^2 - 13p - 30 &= p^2 - (15 - 2)p - 30 \\
&= p^2 - 15p + 2p - 30 \\
&= p(p - 15) + 2(p - 15) \\
&= (p - 15)(p + 2)
\end{aligned}$$

$$\begin{aligned}
 \text{(xiii)} \quad p^2 - 16p - 80 &= p^2 - (20 - 4)p - 80 \\
 &= p^2 - 20p + 4p - 80 \\
 &= p(p - 20) + 4(p - 20) \\
 &= (p - 20)(p + 4)
 \end{aligned}$$

92. Factorise the following using the identity $a^2 - b^2 = (a + b)(a - b)$.

- (i) $x^2 - 9$**
- (ii) $4x^2 - 25y^2$**
- (iii) $4x^2 - 49y^2$**
- (iv) $3a^2b^3 - 27a^4b$**
- (v) $28ay^2 - 175ax^2$**
- (vi) $9x^2 - 1$**
- (vii) $25ax^2 - 25a$**
- (viii) $(x^2/9) - (y^2/25)$**
- (ix) $(2p^2/25) - 32q^2$**
- (x) $49x^2 - 36y^2$**
- (xi) $y^3 - \frac{y}{9}$**
- (xii) $(x^2/25) - 625$**
- (xiii) $(x^2/8) - (y^2/18)$**
- (xiv) $(4x^2/9) - (9y^2/16)$**
- (xv) $(x^3y/9) - (xy^3/16)$**
- (xvi) $1331x^3y - 11y^3x$**
- (xvii) $\frac{1}{36}a^2b^2 - \frac{16}{49}b^2c^2$**
- (xviii) $a^4 - (a - b)^4$**
- (xix) $x^4 - 1$**
- (xx) $y^4 - 625$**
- (xxi) $p^5 - 16p$**
- (xxii) $16x^4 - 81$**
- (xxiii) $x^4 - y^4$**
- (xxiv) $y^4 - 81$**
- (xxv) $16x^4 - 625y^4$**
- (xxvi) $(a - b)^2 - (b - c)^2$**
- (xxvii) $(x + y)^4 - (x - y)^4$**
- (xxviii) $x^4 - y^4 + x^2 - y^2$**
- (xxix) $8a^3 - 2a$**
- (xxx) $x^2 - (y^2/100)$**
- (xxxi) $9x^2 - (3y + z)^2$**

Solution:

$$\begin{aligned}\text{(i)} \quad x^2 - 9 &= x^2 - 3^2 \\ &= (x-3)(x+3)\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad 4x^2 - 25y^2 &= (2x)^2 - (5y)^2 \\ &= (2x-5y)(2x+5y)\end{aligned}$$

$$\begin{aligned}\text{(iii)} \quad 4x^2 - 49y^2 &= (2x)^2 - (7y)^2 \\ &= (2x-7y)(2x+7y)\end{aligned}$$

$$\begin{aligned}\text{(iv)} \quad 3a^2b^3 - 27a^4b &= 3a^2b(b^2 - 9a^2) \\ &= 3a^2b[b^2 - (3a)^2] \\ &= 3a^2b(b+3a)(b-3a)\end{aligned}$$

$$\begin{aligned}\text{(v)} \quad 28ay^2 - 175ax^2 &= 7a(4y^2 - 25x^2) \\ &= 7a[(2y)^2 - (5x)^2] \\ &= 7a(2y-5x)(2y+5x)\end{aligned}$$

$$\begin{aligned}\text{(vi)} \quad 9x^2 - 1 &= (3x)^2 - 1^2 \\ &= (3x-1)(3x+1)\end{aligned}$$

$$\begin{aligned}\text{(vii)} \quad 25ax^2 - 25a &= 25a(x^2 - 1^2) \\ &= 25a(x-1)(x+1)\end{aligned}$$

$$\begin{aligned}\text{(viii)} \quad \frac{x^2}{9} - \frac{y^2}{25} &= \left(\frac{x}{3}\right)^2 - \left(\frac{y}{5}\right)^2 \\ &= \left(\frac{x}{3} - \frac{y}{5}\right)\left(\frac{x}{3} + \frac{y}{5}\right)\end{aligned}$$

$$\text{(ix)} \quad \frac{2p^2}{25} - 32q^2 = 2\left(\frac{p^2}{25} - 16q^2\right)$$

$$\begin{aligned}
&= 2 \left[\left(\frac{p}{5} \right)^2 - (4q)^2 \right] \\
&= 2 \left(\frac{p}{5} + 4q \right) \left(\frac{p}{5} - 4q \right)
\end{aligned}$$

$$\begin{aligned}
\text{(x)} \quad 49x^2 - 36y^2 &= (7x)^2 - (6y)^2 \\
&= (7x - 6y)(7x + 6y)
\end{aligned}$$

$$\begin{aligned}
\text{(xi)} \quad y^3 - \frac{y}{9} &= y \left(y^2 - \frac{1}{9} \right) \\
&= y \left[y^2 - \left(\frac{1}{3} \right)^2 \right] \\
&= y \left(y + \frac{1}{3} \right) \left(y - \frac{1}{3} \right)
\end{aligned}$$

$$\begin{aligned}
\text{(xii)} \quad \frac{x^2}{25} - 625 &= \left(\frac{x}{5} \right)^2 - (25)^2 \\
&= \left(\frac{x}{5} - 25 \right) \left(\frac{x}{5} + 25 \right)
\end{aligned}$$

$$\begin{aligned}
\text{(xiii)} \quad \frac{x^2}{8} - \frac{y^2}{18} &= \frac{1}{2} \left(\frac{x^2}{4} - \frac{y^2}{9} \right) \\
&= \frac{1}{2} \left[\left(\frac{x}{2} \right)^2 - \left(\frac{y}{3} \right)^2 \right] \\
&= \frac{1}{2} \left(\frac{x}{2} + \frac{y}{3} \right) \left(\frac{x}{2} - \frac{y}{3} \right)
\end{aligned}$$

$$\begin{aligned}
\text{(xiv)} \quad \frac{4x^2}{9} - \frac{9y^2}{16} &= \left(\frac{2x}{3} \right)^2 - \left(\frac{3y}{4} \right)^2 \\
&= \left(\frac{2x}{3} + \frac{3y}{4} \right) \left(\frac{2x}{3} - \frac{3y}{4} \right)
\end{aligned}$$

$$\text{(xv)} \quad \frac{x^3y}{9} - \frac{xy^3}{16} = xy \left(\frac{x^2}{9} - \frac{y^2}{16} \right)$$

$$\begin{aligned}
&= xy \left[\left(\frac{x}{3} \right)^2 - \left(\frac{y}{4} \right)^2 \right] \\
&= xy \left(\frac{x}{3} + \frac{y}{4} \right) \left(\frac{x}{3} - \frac{y}{4} \right)
\end{aligned}$$

$$\begin{aligned}
\text{(xvi)} \quad 1331x^3y - 11y^3x &= (11)^3 x^3y - 11y^3x \\
&= 11xy(11^2x^2 - y^2) \\
&= 11xy[(11x)^2 - y^2] \\
&= 11xy(11x + y)(11x - y)
\end{aligned}$$

$$\begin{aligned}
\text{(xvii)} \quad \frac{1}{36}a^2b^2 - \frac{16}{49}b^2c^2 &= \left(\frac{ab}{6} \right)^2 - \left(\frac{4bc}{7} \right)^2 \\
&= \left(\frac{ab}{6} + \frac{4bc}{7} \right) \left(\frac{ab}{6} - \frac{4bc}{7} \right) \\
&= b^2 \left(\frac{a}{6} + \frac{4c}{7} \right) \left(\frac{a}{6} - \frac{4c}{7} \right)
\end{aligned}$$

$$\begin{aligned}
\text{(xviii)} \quad a^4 - (a-b)^4 &= (a^2)^2 - [(a-b)^2]^2 \\
&= [a^2 + (a-b)^2][a^2 - (a-b)^2] \\
&= [a^2 + a^2 + b^2 - 2ab][a^2 - (a^2 + b^2 - 2ab)] \\
&= [2a^2 + b^2 - 2ab][-b^2 + 2ab] \\
&= (2a^2 + b^2 - 2ab)(2ab - b^2)
\end{aligned}$$

$$\begin{aligned}
\text{(xix)} \quad x^4 - 1 &= (x^2)^2 - 1 \\
&= (x^2 + 1)(x^2 - 1) \\
&= (x^2 + 1)(x + 1)(x - 1)
\end{aligned}$$

$$\begin{aligned}
\text{(xx)} \quad y^4 - 625 &= (y^2)^2 - (25)^2 \\
&= (y^2 + 25)(y^2 - 25) \\
&= (y^2 + 25)(y^2 - 5^2) \\
&= (y^2 + 25)(y + 5)(y - 5)
\end{aligned}$$

$$\begin{aligned}
\text{(xxi)} \quad p^5 - 16p &= p(p^4 - 16) \\
&= p\left[(p^2)^2 - 4^2\right] \\
&= p(p^2 + 4)(p^2 - 4) \\
&= p(p^2 + 4)(p^2 - 2^2) \\
&= p(p^2 + 4)(p + 2)(p - 2)
\end{aligned}$$

$$\begin{aligned}
\text{(xxii)} \quad 16x^4 - 81 &= (4x^2)^2 - 9^2 \\
&= (4x^2 + 9)(4x^2 - 9) \\
&= (4x^2 + 9)\left[(2x)^2 - 3^2\right] \\
&= (4x^2 + 9)(2x + 3)(2x - 3)
\end{aligned}$$

$$\begin{aligned}
\text{(xxiii)} \quad x^4 - y^4 &= (x^2)^2 - (y^2)^2 \\
&= (x^2 + y^2)(x^2 - y^2) \\
&= (x^2 + y^2)(x + y)(x - y)
\end{aligned}$$

$$\begin{aligned}
\text{(xxiv)} \quad y^4 - 81 &= (y^2)^2 - (9)^2 \\
&= (y^2 + 9)\left[(y)^2 - (3)^2\right] \\
&= (y^2 + 9)(y + 3)(y - 3)
\end{aligned}$$

$$\begin{aligned}
\text{(xxv)} \quad 16x^4 - 625y^4 &= (4x^2)^2 - (25y^2)^2 \\
&= (4x^2 + 25y^2)(4x^2 - 25y^2) \\
&= (4x^2 + 25y^2)\left[(2x)^2 - (5y)^2\right] \\
&= (4x^2 + 25y^2)(2x + 5y)(2x - 5y)
\end{aligned}$$

$$\text{(xxvi)} \quad (a - b)^2 - (b - c)^2 = (a - b + b - c)(a - b - b + c)(a - c)(a - 2b + c)$$

$$\text{(xxvii)} \quad (x + y)^4 - (x - y)^4 = \left[(x + y)^2\right]^2 - \left[(x - y)^2\right]^2$$

$$\begin{aligned}
&= \left[(x+y)^2 + (x-y)^2 \right] \left[(x+y)^2 - (x-y)^2 \right] \\
&= (x^2 + y^2 + 2xy + x^2 + y^2 - 2xy)(x+y+x-y)(x+y-x+y) \\
&= (2x^2 + 2y^2)(2x)(2y) \\
&= 2(x^2 + y^2)(2x)(2y) \\
&= 8xy(x^2 + y^2)
\end{aligned}$$

$$\begin{aligned}
\text{(xxix)} \quad 8a^3 - 2a &= 2a(4a^2 - 1) \\
&= 2a \left[(2a)^2 - (1)^2 \right] \\
&= 2a(2a+1)(2a-1)
\end{aligned}$$

$$\begin{aligned}
\text{(xxx)} \quad x^2 - \frac{y^2}{100} &= x^2 - x - \left(\frac{y}{10} \right)^2 \\
&= \left(x + \frac{y}{10} \right) \left(x - \frac{y}{10} \right)
\end{aligned}$$

$$\begin{aligned}
\text{(xxxi)} \quad 9x^2 - (3y+z)^2 &= (3x)^2 - (3y+z) \\
&= (3x+3y+z)(3x-3y-z)
\end{aligned}$$

93. The following expressions are the areas of rectangles. Find the possible lengths and breadths of these rectangles.

(i) $x^2 - 6x + 8$

(ii) $x^2 - 3x + 2$

(iii) $x^2 - 7x + 10$

(iv) $x^2 + 19x - 20$

(v) $x^2 + 9x + 20$

Solution:

(i) Consider the expression:

$$x^2 - 6x + 8$$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$x^2 - 6x + 8 = x^2 - (4+2)x + 8$$

$$= x^2 - 4x - 2x + 8$$

$$= x(x-4) - 2(x-4)$$

$$= (x-4)(x-2)$$

So, area of rectangle = Length $\times$ Breadth.

Hence, the possible length and breadth are (x-4) and (x-2).

- (ii) Consider the expression:

$$x^2 - 3x + 2$$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$\begin{aligned}x^2 - 3x + 2 &= x^2 - (2+1)x + 2 \\&= x^2 - 2x - x + 2 \\&= x(x-2) - 1(x-2) \\&= (x-2)(x-1)\end{aligned}$$

So, area of rectangle = Length $\times$ Breadth.

Hence, the possible length and breadth are (x-2) and (x-1).

- (iii) Consider the expression:

$$x^2 - 7x + 10$$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$\begin{aligned}x^2 - 7x + 10 &= x^2 - (5+2)x + 10 \\&= x^2 - 5x - 2x + 10 \\&= x(x-5) - 2(x-5) \\&= (x-5)(x-2)\end{aligned}$$

So, area of rectangle = Length $\times$ Breadth.

Hence, the possible length and breadth are (x-5) and (x-2).

- (iv) Consider the expression:

$$x^2 + 19x - 20$$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$\begin{aligned}x^2 + 19x - 20 &= x^2 + (20-1)x - 20 \\&= x^2 + 20x - x - 20 \\&= x(x+20) - 1(x+20) \\&= (x+20)(x-1)\end{aligned}$$

So, area of rectangle = Length $\times$ Breadth.

Hence, the possible length and breadth are (x+20) and (x-1).

- (v) Consider the expression:

$$x^2 + 9x + 20$$

To find the possible length and breadth of the rectangle we have to factorise the given expression as follows:

$$\begin{aligned}
 x^2 + 9x + 20 &= x^2 + (5 + 4)x + 20 \\
 &= x^2 + 5x + 4x + 20 \\
 &= x(x + 5) + 4(x + 5) \\
 &= (x + 5)(x + 4)
 \end{aligned}$$

So, area of rectangle = Length $\times$ Breadth.

Hence, the possible length and breadth are $(x+5)$ and $(x+4)$.

94. Carry out the following divisions:

(i) $51x^3y^2z \div 17xyz$

(ii) $76x^3yz^3 \div 19x^2y^2$

(iii) $17ab^2c^3 \div (-abc^2)$

(iv) $-121p^3q^3r^3 \div (-11xy^2z^3)$

Solution:

(i) Consider the expression:

$$51x^3y^2z \div 17xyz$$

Simplify the above expression as follows:

$$\begin{aligned}
 \frac{51x^3y^2z}{17xyz} &= \frac{17 \times 3 \times x \times x \times x \times y \times y \times z}{17 \times x \times y \times z} \\
 &= 3x^2y
 \end{aligned}$$

(ii) Consider the expression:

$$76x^3yz^3 \div 19x^2y^2$$

Simplify the above expression as follows:

$$\begin{aligned}
 \frac{76x^3yz^3}{19x^2y^2} &= \frac{4 \times 19 \times x \times x \times x \times y \times z \times z \times z}{19 \times x \times x \times y \times y} \\
 &= \frac{4xz^3}{y}
 \end{aligned}$$

(iii) Consider the expression:

$$17ab^2c^3 \div (-abc^2)$$

Simplify the above expression as follows:

$$\begin{aligned}
 \frac{17ab^2c^3}{-abc^2} &= \frac{17 \times a \times b \times b \times c \times c \times c}{-a \times b \times c \times c} \\
 &= -17bc
 \end{aligned}$$

(iv) Consider the expression:

$$-121p^3q^3r^3 \div (-11xy^2z^3)$$

Simplify the above expression as follows:

$$\begin{aligned}\frac{-12p^3q^3r^3}{-11xy^2z^3} &= \frac{-11 \times 11 \times p \times p \times p \times q \times q \times q \times r \times r \times r}{-11 \times x \times y \times y \times z \times z \times z} \\ &= \frac{11p^3q^3r^3}{xy^2z^3}\end{aligned}$$

95. Perform the following divisions:

(i) $(3pqr - 6p^2q^2r^2) \div 3pq$

(ii) $(ax^3 - bx^2 + cx) \div (-dx)$

(iii) $(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$

(iv) $(-qrxy + pryz - rxyz) \div (-xyz)$

Solution:

(i) Consider the expression:

$$(3pqr - 6p^2q^2r^2) \div 3pq$$

Now, simplify the above expression as follows:

$$\begin{aligned}\frac{3pqr - 6p^2q^2r^2}{3pq} &= \frac{3pqr}{3pq} \\ &= \frac{3pqr}{3pq} - \frac{6p^2q^2r^2}{3pq} \\ &= r - \frac{2 \times 3 \times p \times p \times q \times q \times r \times r}{3 \times p \times q} \\ &= r - 2pqr^2\end{aligned}$$

(ii) Consider the expression:

$$(ax^3 - bx^2 + cx) \div (-dx)$$

Now, simplify the above expression as follows:

$$\begin{aligned}\frac{ax^3 - bx^2 + cx}{-dx} &= \frac{ax^3}{-dx} + \frac{bx^2}{dx} + \frac{cx}{-dx} \\ &= \frac{a \times x \times x \times x}{-d \times x} + \frac{b \times x \times x}{-d \times x} + \frac{c \times x}{-d \times x} \\ &= -\frac{a}{d}x^2 + \frac{d}{d}x - \frac{c}{d}\end{aligned}$$

(iii) Consider the expression:

$$(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$$

Now, simplify the above expression as follows:

$$\begin{aligned}\frac{x^3y^3 + x^2y^3 - xy^4 + xy}{xy} &= \frac{x^3y^3}{xy} + \frac{x^2y^3}{xy} - \frac{xy^4}{xy} + \frac{xy}{xy} \\ &= \frac{x \times x \times x \times y \times y \times y}{x \times y} + \frac{x \times x \times y \times y \times y}{x \times y} - \frac{x \times y \times y \times y \times y}{x \times y} + \frac{x \times y}{x \times y} \\ &= x^2y^2 + xy^2 - y^3 + 1\end{aligned}$$

(iv) Consider the expression:

$$(-qrxy + pryz - rxyz) \div (-xyz)$$

Now, simplify the above expression as follows:

$$\begin{aligned}\frac{-qrxy + pryz - rxyz}{-xyz} &= \frac{-qrxy}{-xyz} + \frac{pryz}{-xyz} - \frac{rxyz}{-xyz} \\ &= \frac{qr}{z} - \frac{pr}{x} + r\end{aligned}$$

96. Factorise the expressions and divide them as directed:

(i) $(x^2 - 22x + 117) \div (x - 13)$

(ii) $(x^3 + x^2 - 132x) \div x(x - 11)$

(iii) $(2x^3 - 12x^2 + 16x) \div (x - 2)(x - 4)$

(iv) $(9x^2 - 4) \div (3x + 2)$

(v) $(3x^2 - 48) \div (x - 4)$

(vi) $(x^4 - 16) \div x^3 + 2x^2 + 4x + 8$

(vii) $(3x^4 - 1875) \div (3x^2 - 75)$

Solution:

(i) Consider the expression:

$$(x^2 - 22x + 117) \div (x - 13)$$

Now, simplify the above expression as follows:

$$\begin{aligned}\frac{x^2 - 22x + 117}{x - 13} &= \frac{x^2 - 13x - 9x + 117}{x - 13} \\ &= \frac{x(x - 13) - 9(x - 13)}{x - 13} \\ &= \frac{(x - 13)(x - 9)}{x - 13} \\ &= x - 9\end{aligned}$$

(ii) Consider the expression:

$$(x^3 + x^2 - 132x) \div x(x - 11)$$

Now, simplify the above expression as follows:

$$\begin{aligned}\frac{x^3 + x^2 - 132x}{x(x - 11)} &= \frac{x(x^2 + x - 132)}{x(x - 11)} \\ &= \frac{x^2 + 12x - 11x - 132}{x - 11} \\ &= x + 12\end{aligned}$$

(iii) Consider the expression:

$$(2x^3 - 12x^2 + 16x) \div (x - 2)(x - 4)$$

Now, simplify the above expression as follows:

$$\begin{aligned} \frac{2x^3 - 12x^2 + 16x}{(x - 2)(x - 4)} &= \frac{2x(x^2 - 6x + 8)}{(x - 2)(x - 4)} \\ &= \frac{2x(x^2 - 4x - 2x + 8)}{(x - 2)(x - 4)} \\ &= \frac{2x[x(x - 4) - 2(x - 4)]}{(x - 2)(x - 4)} \\ &= \frac{2x(x - 4)(x - 2)}{(x - 2)(x - 4)} \\ &= 2x \end{aligned}$$

(iv) Consider the expression:

$$(9x^2 - 4) \div (3x + 2)$$

Now, simplify the above expression as follows:

$$\begin{aligned} \frac{9x^2 - 4}{3x + 2} &= \frac{(3x)^2 - (2)^2}{3x + 2} \\ &= \frac{(3x + 2)(3x - 2)}{3x + 2} \\ &= 3x - 2 \end{aligned}$$

(v) Consider the expression:

$$(3x^2 - 48) \div (x - 4)$$

Now, simplify the above expression as follows:

$$\begin{aligned} \frac{3x^2 - 48}{x - 4} &= \frac{3(x^2 - 16)}{x - 4} \\ &= \frac{3(x^2 - 4^2)}{x - 4} \\ &= \frac{3(x + 4)(x - 4)}{x - 4} \\ &= 3(x + 4) \end{aligned}$$

(vi) Consider the expression:

$$(x^4 - 16) \div x^3 + 2x^2 + 4x + 8$$

Now, simplify the above expression as follows:

$$\begin{aligned}
\frac{x^4 - 16}{x^3 + 2x^2 + 4x + 8} &= \frac{(x^2)^2 - 4^2}{x^2(x+2) + 4(x+2)} \\
&= \frac{(x^2 + 4)(x^2 - 4)}{(x^2 + 4)(x + 2)} \\
&= \frac{x^2 - 2^2}{x + 2} \\
&= \frac{(x + 2)(x - 2)}{x + 2} \\
&= x - 2
\end{aligned}$$

- (vii) Consider the expression:
 $(3x^4 - 1875) \div (3x^2 - 75)$
 Now, simplify the above expression as follows:

$$\begin{aligned}
\frac{3x^4 - 1875}{3x^2 - 75} &= \frac{x^4 - 625}{x^2 - 25} \\
&= \frac{(x^2)^2 - (25)^2}{x^2 - 25} \\
&= \frac{(x^2 + 25)(x^2 - 25)}{(x^2 - 25)} \\
&= x^2 + 25
\end{aligned}$$

97. The area of a square is given by $4x^2 + 12xy + 9y^2$. Find the side of the square.

Solution:

Given:

Area of square = $4x^2 + 12xy + 9y^2$

Now, the sides of square will be calculated as follows:

$4x^2 + 12xy + 9y^2 = \text{side}^2$ [As, area of square = side^2]

Side = $(2x)^2 + 2 \times 2x \times 3y + (3y)^2$

Side = $(2x + 3y)^2$

Side = $2x + 3y$

Hence, the side of the given square is $(2x + 3y)$.

98. The area of a square is $9x^2 + 24xy + 16y^2$. Find the side of the square.

Solution:

Given:

$$\text{Area of square} = 9x^2 + 24xy + 16y^2$$

Now, the sides of square will be calculated as follows:

$$9x^2 + 24xy + 16y^2 = \text{side}^2 \quad [\text{As, area of square} = \text{side}^2]$$

$$\text{Side}^2 = (3x)^2 + 2 \times 3x \times 4y + (4y)^2$$

$$\text{Side}^2 = (3x + 4y)^2$$

$$\text{Side} = 3x + 4y$$

Hence, the side of the given square is $(3x + 4y)$.

99. The area of a rectangle is $x^2 + 7x + 12$. If its breadth is $(x + 3)$, then find its length.

Solution:

Given:

$$\text{Area of rectangle} = x^2 + 7x + 12$$

Now, the length of rectangle will be calculated as follows:

$$\text{Length} \times \text{Breadth} = x^2 + 7x + 12 \quad [\text{As, area of rectangle} = \text{Length} \times \text{Breadth}]$$

$$\text{Length} \times (x + 3) = x^2 + 4x + 3x + 12$$

$$\text{Length} \times (x + 3) = x(x + 4) + 3(x + 4)$$

$$\text{Length} \times (x + 3) = (x + 3)(x + 4)$$

$$\text{Length} = x + 4$$

Hence, the length of the given rectangle is $x + 4$.

100. The curved surface area of a cylinder is $2\pi(y^2 - 7y + 12)$ and its radius is $(y - 3)$. Find the height of the cylinder (C.S.A. of cylinder = $2\pi rh$).

Solution:

Given:

$$\text{Curved surface area of a cylinder} = 2\pi(y^2 - 7y + 12) \text{ and radius of cylinder} = y - 3.$$

Now, the height of the cylinder will be calculated as follows:

$$\text{Curved surface area of a cylinder} = 2\pi(y^2 - 7y + 12) \quad [\text{As, Curved surface area of a cylinder} = 2\pi rh]$$

$$2\pi \times (y - 3) \times h = 2\pi(y^2 - 7y + 12)$$

$$2\pi \times (y - 3) \times h = 2\pi(y^2 - 4y - 3y + 12)$$

$$2\pi \times (y - 3) \times h = 2\pi(y(y - 4) - 3(y - 4))$$

$$2\pi \times (y - 3) \times h = 2\pi(y - 3)(y - 4)$$

$$h = y - 4$$

Hence, the height of the cylinder is $y - 4$.

101. The area of a circle is given by the expression $\pi x^2 + 6\pi x + 9\pi$. Find the radius of the circle.

Solution:

Given:

Area of a circle = $\pi x^2 + 6\pi x + 9\pi$.

Now, the radius of the circle(r) will be calculated as follows:

Area of a circle = $\pi x^2 + 6\pi x + 9\pi$ [As, area of a circle = πr^2]

$$\pi r^2 = \pi x^2 + 6\pi x + 9\pi$$

$$\pi r^2 = \pi(x^2 + 6x + 9)$$

$$\pi r^2 = \pi x(x + 3) + 3(x + 3)$$

$$\pi r^2 = \pi(x + 3)(x + 3)$$

$$\pi r^2 = \pi(x + 3)^2$$

$$r^2 = (x + 3)^2$$

$$r = x + 3$$

Hence, the radius of the circle is $x + 3$.

102. The sum of first n natural numbers is given by the expression $(n^2/2) + \frac{n}{2}$. Factorise this expression.

Solution:

As we know that the sum of first n natural numbers = $\frac{n^2}{2} + \frac{n}{2}$

$$\text{Factorisation of given expression} = \frac{1}{2}(n^2 + n) = \frac{1}{2}n(n + 1)$$

103. The sum of (x + 5) observations is $x^4 - 625$. Find the mean of the observations.

Solution:

Given: The sum of (x + 5) observations is $x^4 - 625$.

As we know that the mean of the n observations $x_1, x_2, \dots, x_n$ is $\frac{x_1 + x_2 + \dots + x_n}{n}$.

$$\text{So, the mean of (x+5) observations} = \frac{\text{Sum of (x+5) observations}}{x + 5}$$

$$\begin{aligned}
&= \frac{x^4 - 625}{x + 5} \\
&= \frac{(x^2)^2 - (25)^2}{x + 5} \\
&= \frac{(x^2 + 25)(x^2 - 25)}{x + 5} \\
&= \frac{(x^2 + 25)[(x)^2 - (5)^2]}{x + 5} \\
&= \frac{(x^2 + 25)(x + 5)(x - 5)}{x + 5} \\
&= (x^2 + 25)(x - 5)
\end{aligned}$$

104. The height of a triangle is $x^4 + y^4$ and its base is $14xy$. Find the area of the triangle.

Solution:

Given: The height of a triangle and its base are $x^4 + y^4$ and $14xy$, respectively.

$$\begin{aligned}
\text{As we know that the area of a triangle} &= \frac{1}{2} \times \text{Base} \times \text{Height} \\
&= \frac{1}{2} \times 14xy \times (x^4 + y^4) \\
&= 7xy(x^4 + y^4)
\end{aligned}$$

105. The cost of a chocolate is Rs $(x + y)$ and Rohit bought $(x + y)$ chocolates. Find the total amount paid by him in terms of x . If $x = 10$, find the amount paid by him.

Solution:

Given: cost of a chocolate = Rs. $x + 4$

Rohit bought $(x+4)$ chocolates.

$$\begin{aligned}
\text{So, the cost of } (x+4) \text{ chocolates} &= \text{Cost of one chocolate} \times \text{Number of chocolates} \\
&= (x + 4)(x + 4) \\
&= (x + 4)^2
\end{aligned}$$

The total amount paid by Rohit = Rs. $x^2 + 8x + 16$

Therefore, if $x = 10$. Then, the amount paid by Rohit =

$$10^2 + 8 \times 10 + 16 = 100 + 80 + 16 = \text{Rs. } 196.$$

106. The base of a parallelogram is $(2x + 3)$ units and the corresponding height is $(2x - 3)$ units. Find the area of the parallelogram in terms of x . What will be the area of parallelogram of $x = 30$ units?

Solution:

Given: the base and the corresponding height of a parallelogram are $(2x+3)$ units and $(2x-3)$ units, respectively.

$$\begin{aligned}\text{Area of a parallelogram} &= \text{Base} \times \text{Height} \\ &= (2x + 3) \times (2x - 3) \\ &= (2x)^2 - (3)^2 \\ &= (4x^2 - 9) \text{ sq units}\end{aligned}$$

Therefore, if $x = 10$. Then, the area of the parallelogram = $4 \times 10^2 - 9 = 400 - 9 = 391$ sq units

107. The radius of a circle is $7ab - 7bc - 14ac$. Find the circumference of the circle. ($\pi = \frac{22}{7}$)

Solution:

Given: Radius of the circle = $7ab - 7bc - 14ac = r$

As we know that the circumference of the circle = $2\pi r$

$$\begin{aligned}&= 2 \times \frac{22}{7} \times (7ab - 7bc - 14ac) \\ &= \frac{44}{7} \times 7(ab - bc - 2ac) \\ &= 44[ab - c(b + 2a)]\end{aligned}$$

108. If $p + q = 12$ and $pq = 22$, then find $p^2 + q^2$.

Solution:

Given: $p+q=12$ and $pq=22$

Now, the value of $p^2 + q^2$ will be calculated as follows:

$$\begin{aligned}(p + q)^2 &= p^2 + q^2 + 2pq && [\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab] \\ 12^2 &= p^2 + q^2 + 2 \times 22 \\ p^2 + q^2 &= 12^2 - 44 \\ p^2 + q^2 &= 144 - 44 \\ p^2 + q^2 &= 100\end{aligned}$$

109. If $a + b = 25$ and $a^2 + b^2 = 225$, then find ab .

Solution:

Given: $a + b = 25$ and $a^2 + b^2 + 2ab$.

$$(25)^2 = 225 + 2ab$$

$$2ab = 25^2 - 225$$

$$2ab = 625 - 225$$

$$2ab = 400$$

$$ab = \frac{400}{2}$$

$$ab = 200$$

110. If $x - y = 13$ and $xy = 28$, then find $x^2 + y^2$.

Solution:

Given: $x - y = 13$ and $xy = 28$

Since, $(x - y)^2 = x^2 + y^2 - 2xy$ [Using the identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$(13)^2 = x^2 + y^2 - 2 \times 28$$

$$x^2 + y^2 = 13^2 + 56$$

$$x^2 + y^2 = 169 + 56$$

$$x^2 + y^2 = 225$$

111. If $m - n = 16$ and $m^2 + n^2 = 400$, then find mn .

Solution:

Given: $m - n = 16$ and $m^2 + n^2 = 400$

Since, $(m - n)^2 = m^2 + n^2 - 2mn$ [Using the identity: $(a - b)^2 = a^2 + b^2 - 2ab$]

$$(16)^2 = 400 - 2mn$$

$$2mn = 400 - 256$$

$$2mn = 400 - (16)^2$$

$$2mn = 400 - 256$$

$$2mn = 144$$

$$mn = \frac{144}{2}$$

$$mn = 72$$

112. If $a^2 + b^2 = 74$ and $ab = 35$, then find $a + b$.

Solution:

Given: $a^2 + b^2 = 74$ and $ab = 35$.

Since, $(a+b)^2 = a^2 + b^2 + 2ab$ [Using the identity, $(a+b)^2 = a^2 + b^2 + 2ab$]

$$(a+b)^2 = 74 + 2 \times 35$$

$$(a+b)^2 = 74 + 70$$

$$(a+b)^2 = 144$$

$$a+b = \sqrt{144}$$

$$a+b = 14$$

113. Verify the following:

(i) $(ab + bc)(ab - bc) + (bc + ca)(bc - ca) + (ca + ab)(ca - ab) = 0$

(ii) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$

(iii) $(p - q)(p^2 + pq + q^2) = p^3 - q^3$

(iv) $(m + n)(m^2 - mn + n^2) = m^3 + n^3$

(v) $(a + b)(a + b)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$

(vi) $(a - b)(a - b)(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$

(vii) $(a^2 - b^2)(a^2 + b^2) + (b^2 - c^2)(b^2 + c^2) + (c^2 - a^2)(c^2 + a^2) = 0$

(viii) $(5x + 8)^2 - 160x = (5x - 8)^2$

(ix) $(7p - 13q)^2 + 364pq = (7p + 13q)^2$

(x) $\left(\frac{3p}{7} + \frac{7}{6p}\right)^2 - \left(\frac{3}{7}p + \frac{7}{6p}\right)^2 = 2$

Solution:

(i) Taking LHS = $(ab + bc)(ab - bc) + (bc + ca)(bc - ca) + (ca + ab)(ca - ab)$

$$= \left[(ab)^2 - (bc)^2\right] + \left[(bc)^2 - (ca)^2\right] + \left[(ca)^2 - (ab)^2\right]$$

[Using the identity: $(a+b)(a-b) = a^2 - b^2$]

$$= a^2b^2 - b^2c^2 + b^2c^2 - c^2a^2 + c^2a^2 - a^2b^2$$

$$= 0$$

$$= RHS$$

Hence, verified.

(ii) Taking LHS = $(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$\begin{aligned}
&= a(a^2 + b^2 + c^2 - ab - bc - ca) + b(a^2 + b^2 + c^2 - ab - bc - ca) + c(a^2 + b^2 + c^2 - ab - bc - ca) \\
&= a^3 + ab^2 + ac^2 - a^2b - abc - a^2c + ba^2 + b^3 + bc^2 - b^2a - b^2c - bca + ca^2 + cb^2 + c^3 - cab - c^2b - c^2a \\
&= a^3 + b^3 + c^3 - 3abc \\
&= RHS \\
&\text{Hence, verified.}
\end{aligned}$$

(iii) Taking LHS = $(p - q)(p^2 + pq + q^2)$

$$\begin{aligned}
&= p(p^2 + pq + q^2) - q(p^2 + pq + q^2) \\
&= p^3 + p^2q + pq^2 - qp^2 - pq^2 - q^3 \\
&= p^3 - q^3 \\
&= RHS \\
&\text{Hence, verified.}
\end{aligned}$$

(iv) Taking LHS = $(m + n)(m^2 - mn + n^2)$

$$\begin{aligned}
&= m(m^2 - mn + n^2) + n(m^2 - mn + n^2) \\
&= m^3 - m^2n + mn^2 + nm^2 - mn^2 + n^3 \\
&= m^3 + n^3 \\
&= RHS \\
&\text{Hence, verified.}
\end{aligned}$$

(v) Taking LHS = $(a + b)(a + b)(a + b)$

$$\begin{aligned}
&= (a + b)(a + b)^2 \\
&= (a + b)(a^2 + b^2 + 2ab) \\
&= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2) \\
&= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3 \\
&= a^3 + 3a^2b + 3ab^2 + b^3 \\
&= RHS \\
&\text{Hence, verified.}
\end{aligned}$$

(vi) Taking LHS = $(a - b)(a - b)(a - b)$

$$\begin{aligned}
&= (a-b)(a-b)^2 \\
&= (a-b)(a^2 - 2ab + b^2) \\
&= a(a^2 - 2ab + b^2) + b(a^2 - 2ab + b^2) \\
&= a^3 - 2a^2b + ab^2 - ba^2 + 2ab^2 - b^3 \\
&= a^3 - 3a^2b + 3ab^2 - b^3 \\
&= RHS
\end{aligned}$$

Hence, verified.

(vii) Taking LHS $= (a^2 - b^2)(a^2 + b^2) + (b^2 - c^2)(b^2 + c^2) + (c^2 - a^2)(c^2 + a^2)$

$$\begin{aligned}
&= (a^4 - b^4 + b^4 - c^4 + c^4 - a^4) \\
&= 0 \\
&= RHS
\end{aligned}$$

Hence, verified.

(viii) Taking LHS $= (5x+8)^2 - 160x$

$$\begin{aligned}
&= (5x)^2 + 8^2 + 2 \times 5x \times 8 - 160x \\
&= (5x)^2 + (8)^2 + 80x - 160x \\
&= (5x)^2 + (8)^2 - 80x \\
&= (5x)^2 + (8)^2 - 2 \times 5x \times 8 \\
&= (5x-8)^2 \\
&= RHS
\end{aligned}$$

Hence, verified.

(ix) Taking LHS $= (7p-13q)^2 + 364pq$

$$\begin{aligned}
&= (7p)^2 + (13q)^2 - 2 \times 7p \times 13q + 364pq \\
&= (7p)^2 + (13q)^2 - 182pq + 364pq \\
&= (7p)^2 + (13q)^2 + 182pq \\
&= (7p)^2 + (13q)^2 + 2 \times 7p \times 13q \\
&= (7p+13q)^2 \\
&= RHS
\end{aligned}$$

Hence, verified.

(x) Taking LHS $= \left(\frac{3p}{7} + \frac{7}{6p}\right)^2 - \left(\frac{3p}{7} - \frac{7}{6p}\right)^2$

$$\begin{aligned}
&= \left[\left(\frac{3p}{7} + \frac{7}{6p} \right) + \left(\frac{3p}{7} - \frac{7}{6p} \right) \right] \left[\left(\frac{3p}{7} + \frac{7}{6p} \right) - \left(\frac{3p}{7} - \frac{7}{6p} \right) \right] \\
&= \left(\frac{3p}{7} + \frac{7}{6p} + \frac{3p}{7} - \frac{7}{6p} \right) \left(\frac{3p}{7} + \frac{7}{6p} - \frac{3p}{7} + \frac{7}{6p} \right) \\
&= \frac{6p}{7} \times \frac{14}{6p} \\
&= 2 \\
&= RHS \\
&\text{Hence, verified.}
\end{aligned}$$

114. Find the value of a, if

(i) $8a = 35^2 - 27^2$

(ii) $9a = 76^2 - 67^2$

(iii) $pqa = (3p + q)^2 - (3p - q)^2$

(iv) $pq^2a = (4pq + 3q)^2 - (4pq - 3q)^2$

Solution:

(i) Consider the equation:

$$8a = 35^2 - 27^2$$

Now, the value of a will be calculated as follows:

$$8a = (35 + 27)(35 - 27) \quad [\text{Using the identity: } a^2 - b^2 = (a + b)(a - b)]$$

$$8a = 62 \times 8$$

$$a = \frac{62 \times 8}{8}$$

$$a = 62$$

Hence, the value of a is 62.

(ii) Consider the equation:

$$9a = 76^2 - 67^2$$

Now, the value of a will be calculated as follows:

$$9a = (76 + 67)(76 - 67) \quad [\text{Using the identity: } a^2 - b^2 = (a + b)(a - b)]$$

$$9a = 143 \times 9$$

$$a = \frac{143 \times 9}{9}$$

$$a = 143$$

Hence, the value of a is 143.

(iii) Consider the equation:

$$pqa = (3p + q)^2 - (3p - q)^2$$

Now, the value of a will be calculated as follows:

$$\begin{aligned}
 pqa &= (3p+q)^2 - (3p-q)^2 \\
 pqa &= [(3p+q) + (3p-q)][(3p+q) - (3p-q)] \\
 pqa &= [(3p+q+3p-q)][3p+q-3p+q] \\
 pqa &= 6p \times 2q \\
 a &= \frac{6p \times 2q}{pq} \\
 a &= \frac{(6 \times 2) pq}{pq} \\
 a &= 12
 \end{aligned}$$

- (iv) Consider the equation:
 $pq^2a = (4pq + 3q)^2 - (4pq - 3q)^2$
 Now, the value of a will be calculated as follows:

$$\begin{aligned}
 pq^2a &= (4pq + 3q)^2 - (4pq - 3q)^2 \\
 &= [(4pq + 3q) + (4pq - 3q)][(4pq + 3q) - (4pq - 3q)] \\
 &= (4pq + 3q + 4pq - 3q)(4pq + 3q - 4pq + 3q) \\
 &= 8pq \times 6q \\
 pq^2a &= 48pq^2 \\
 a &= \frac{48pq^2}{pq^2} \\
 a &= 48
 \end{aligned}$$

115. What should be added to $4c(-a + b + c)$ to obtain $3a(a + b + c) - 2b(a - b + c)$?

Solution:

Let x be added to the given expression to $4c(-a + b + c)$ to obtain $3a(a + b + c) - 2b(a - b + c)$.

$$\begin{aligned}
 x + 4c(-a + b + c) &= 3a(a + b + c) - 2b(a - b + c) \\
 x &= 3a(a + b + c) - 2b(a - b + c) - 4c(-a + b + c) \\
 &= 3a^2 + 3ab + 3ac - 2ba + 2b^2 - 2bc + 4ca - 4cb - 4c^2 \\
 x &= 3a^2 + ab + 7ac + 2b^2 - 6bc - 4c^2
 \end{aligned}$$

116. Subtract $b(b^2 + b - 7) + 5$ from $3b^2 - 8$ and find the value of expression obtained for $b = -3$.

Solution:

According to the question:

$$\begin{aligned}\text{Required difference} &= (3b^2 - 8) - [b(b^2 + b - 7) + 5] \\ &= 3b^2 - 8 - b^3 - b^2 + 7b - 5 \\ &= -b^3 + 2b^2 + 7b - 13\end{aligned}$$

Now, if $b = -3$

$$\begin{aligned}\text{The value of above expression} &= -(-3)^3 + 2(-3)^2 + 7(-3) - 13 \\ &= -(-27) + 2 \times 9 - 21 - 13 \\ &= 27 + 18 - 21 - 13 \\ &= 45 - 34 \\ &= 11\end{aligned}$$

117. If $x - \frac{1}{x} = 7$ then find the value of $x^2 + \frac{1}{x^2}$

Solution:

$$\text{Given: } x - \frac{1}{x} = 7$$

Now, the value of $x^2 + \frac{1}{x^2}$ will be calculated as follows:

$$\begin{aligned}\left(x - \frac{1}{x}\right)^2 &= x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} \\ 7^2 &= x^2 + \frac{1}{x^2} - 2\end{aligned}$$

$$x^2 + \frac{1}{x^2} = 49 + 2$$

$$x^2 + \frac{1}{x^2} = 51$$

Hence, the value of $x^2 + \frac{1}{x^2}$ is 51.

118. Factorise $x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x}$

Solution:

Consider the expression:

$$x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x}$$

Now, factorise the above expression as follows:

$$\begin{aligned}
 x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x} &= x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} - 3 \left(x + \frac{1}{x} \right) \\
 &= \left(x + \frac{1}{x} \right)^2 - 3 \left(x + \frac{1}{x} \right) \\
 &= \left(x + \frac{1}{x} \right) \left(x + \frac{1}{x} - 3 \right)
 \end{aligned}$$

119. Factorise $p^4 + q^4 + p^2q^2$.

Solution:

Consider the expression:

$$p^4 + q^4 + p^2q^2$$

Now, factorise the above expression as follows:

$$\begin{aligned}
 p^4 + q^4 + p^2q^2 &= p^4 + q^4 + 2p^2q^2 - 2p^2q^2 + p^2q^2 \\
 &= p^4 + q^4 + 2p^2q^2 - p^2q^2 \\
 &= \left[(p^2)^2 + (q^2)^2 + 2p^2q^2 \right] - p^2q^2 \\
 &= (p^2 + q^2) - (pq)^2 \\
 &= (p^2 + q^2 + pq)(p^2 + q^2 - pq)
 \end{aligned}$$

120. Find the value of

(i) $\frac{6.25 \times 6.25 - 1.75 \times 1.75}{4.5}$

(ii) $\frac{198 \times 198 - 102 \times 102}{96}$

Solution:

$$\begin{aligned}
 \text{(i)} \quad \frac{6.25 \times 6.25 - 1.75 \times 1.75}{4.5} &= \frac{(6.25)^2 - (1.75)^2}{4.5} \\
 &= \frac{(6.25 + 1.75)(6.25 - 1.75)}{4.5} \\
 &= \frac{8 \times 4.5}{4.5} \\
 &= 8
 \end{aligned}$$

$$\text{(ii)} \quad \frac{198 \times 198 - 102 \times 102}{96} = \frac{(198)^2 - (102)^2}{96}$$

$$\begin{aligned}
 &= \frac{(198+102)(198-102)}{96} \\
 &= \frac{300 \times 96}{96} \\
 &= 300
 \end{aligned}$$

121. The product of two expressions is $x^5 + x^3 + x$. If one of them is $x^2 + x + 1$, find the other.

Solution:

Given: The expression $x^5 + x^3 + x$ has two product where one of them is $x^2 + x + 1$.

Let other expression is A. So, according to the question,

$$A \times (x^2 + x + 1) = x^5 + x^3 + x$$

$$A = \frac{x(x^4 + x^2 + 1)}{x^2 + x + 1}$$

$$A = \frac{x[x^4 + 2x^2 + 1 - x^2]}{x^2 + x + 1}$$

$$A = \frac{x(x^4 + 2x^2 + 1 - x^2)}{x^2 + x + 1}$$

$$A = \frac{x[(x^4 + 2x^2 + 1) - x^2]}{x^2 + x + 1}$$

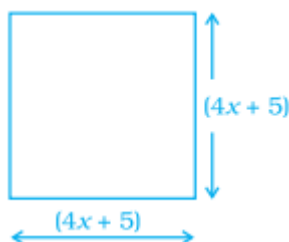
$$A = \frac{x[(x^2 + 1)^2 - x^2]}{x^2 + x + 1}$$

$$A = \frac{x(x^2 + 1 + x)(x^2 + 1 - x)}{x^2 + x + 1}$$

$$A = x(x^2 + 1 - x)$$

Hence, the another expression is $x(x^2 + 1 - x)$.

122. Find the length of the side of the given square if area of the square is 625 square units and then find the value of x.



Solution:

Given: A square having length of a side $(4x+5)$ units and area is 625 sq units.

As, area of a square = $(\text{Side})^2$

$$(4x+5)^2 = 625$$

$$(4x+5)^2 = 25^2$$

$$4x+5 = 25$$

$$4x = 25 - 5$$

$$4x = 20$$

$$x = 5$$

Hence, side = $4x+5 = 4 \times 5 + 5 = 25$ units.

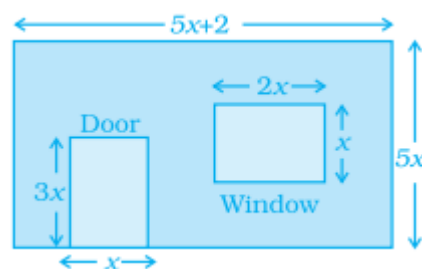
123. Take suitable number of cards given in the adjoining diagram [$G(x \times x)$ representing x^2 , $R(x \times 1)$ representing x and $Y(1 \times 1)$ representing 1] to factorise the following expressions, by arranging the cards in the form of rectangles: (i) $2x^2 + 6x + 4$ (ii) $x^2 + 4x + 4$. Factorise $2x^2 + 6x + 4$ by using the figure.

Calculate the area of figure.

Solution:

The given information is incomplete for solution of this question.

124. The figure shows the dimensions of a wall having a window and a door of a room. Write an algebraic expression for the area of the wall to be painted.

**Solution:**

Given: A wall of dimension $5x \times (5x+2)$ having a window and a door of dimension $(2x \times x)$ and $(3x \times x)$, respectively.

Now, area of the window = $2x \times x = 2x^2$ sq units

Area of the door = $3x \times x = 3x^2$ sq units

And area of wall = $(5x + 2) \times 5x = 25x^2 + 10x$ sq units

So, area of the required part of the wall to be painted = Area of the wall – (Area of the window + Area of the door)

$$= 25x^2 + 10x - (2x^2 + 3x^2)$$

$$= 25x^2 + 10x - 5x^2$$

$$= 20x^2 + 10x$$

$$= 2 \times 2 \times 5 \times x \times x + 2 \times 5 \times x$$

$$= 2 \times 5 \times x(2x + 1)$$

$$= 10x(2x + 1) \text{ sq units}$$

125. Match the expressions of column I with that of column II:

Column I	Column II
(1) $(21x + 13y)^2$	(a) $441x^2 - 169y^2$
(2) $(21x - 13y)^2$	(b) $441x^2 + 169y^2 + 546xy$
(3) $(21x - 13y)(21x + 13y)$	(c) $441x^2 + 169y^2 - 546xy$
	(d) $441x^2 - 169y^2 + 546xy$

Solution:

$$(i) \quad (21x + 13y)^2 = (21x)^2 + (13y)^2 + 2 \times 21x \times 13y$$

$$[\text{Using the identity: } (a + b)^2 = a^2 + b^2 + 2ab]$$

$$= 441x^2 + 169y^2 + 546xy$$

$$(ii) \quad (21x - 13y)^2 = (21x)^2 + (13y)^2 - 2 \times 21x \times 13y$$

$$[\text{Using the identity: } (a - b)^2 = a^2 + b^2 - 2ab]$$

$$= 441x^2 + 169y^2 - 546xy$$

$$(iii) \quad (21x - 13y)(21x + 13y) = (21x)^2 - (13y)^2$$

$$[\text{Using the identity: } (a - b)(a + b) = a^2 - b^2]$$

$$= 441x^2 - 169y^2$$

Hence, $(i) \rightarrow (b), (ii) \rightarrow (c), (iii) \rightarrow (a)$