Chapter-5

SETS

Exercise 5.1

Question 1:

Find the value of each of the following :

(i) $(-3)^3 \times 5^2$

(ii) $(-1)^{501} \times [(27)^4 \div (9)^5]$

(iii) $\left(-3\frac{1}{2}\right)^3$

Solution:

(i) $(-3)^3 \times 5^2$

$= (-3) \times (-3) \times (-3) \times 5 \times 5$

$= -27 \times 25$

$= -675$

(ii) $(-1)^{501} \times [(27)^4 \div (9)^5]$

$= -1 \times [(3^3)^4 \div (3^2)^5]$

{∵ 501 is an odd numbers}

$= -1 \times [3^{12} \div 3^{10}]$

$= -1 \times [3^{12-10}]$

$= -1(3^2)$

$= -1 \times 3 \times 3$

$= -9$

(iii) $\left(-3\frac{1}{2}\right)^3 = \left(\frac{-7}{2}\right)^3$

$= \frac{-7}{2} \times \frac{-7}{2} \times \frac{-7}{2}$

$= \frac{-343}{8}$

$= -42\frac{7}{8}$

Question 2:

Simplify the following:

(i) (i) $\frac{7^3 \times 11^4 \times 13^0}{7^2 \times 11^2}$

(ii) $\frac{(-2)^3 \times (3x)^2 \times (-xy^3)}{3x^2y}$

(iii) $\frac{[(-5)^3]^4 \times 8^2}{4^3 \times (25)^5}$

Solution:

(i) $\frac{7^3 \times 11^4 \times 13^0}{7^2 \times 11^2}$

$= 7^{3-2} \times 11^{4-2} \times 13^0$

$= 7^1 \times 11^2 \times 1$

$= 7 \times 11 \times 11$

$= 847$

(ii)

$$\frac{(-2)^3\times(3x)^2\times(-xy^3)}{3x^2y}$$

$$= \frac{(-2)\times(-2)\times(-2)\times3^2\times x^2\times(-x)\times y^3}{3\times x^2\times y}$$

$$= \frac{-8\times9x^2\times(-1)x\times y^3}{3x^2\times y}$$

$$= \frac{72}{3}xy^3 - 1$$

$$= 24xy^2$$

(iii)

$$\frac{[(-5)^3]^4\times8^2}{4^3\times(25)^5}$$

$$= \frac{(-5)^{3\times4}\times(2^3)^2}{(2^2)^3\times(5^2)^5}$$

$$= \frac{(-5)^{12}\times2^6}{2^6\times5^{10}}$$

$$= \frac{(-1)^{12}(5)^{12}\times2^6}{2^6\times5^{10}}$$

$$= 1\times5^{12-10}\times2^{6-6}$$

$$= 5^2\times2^0$$

$$= 25\times1$$

$$= 25$$

Question 3:

Simplify and write the following in exponential form:

(i) $\frac{(-3)^5\times8^3\times2^5}{3^2\times4^4}$

(ii) $\frac{9^8\times(x^2)^5}{(27)^4\times(x^3)^2}$

(iii) $\frac{3^2\times7^8\times13^6}{21^2\times91^3}$

Solution :

(i) $\frac{(-3)^5\times8^3\times2^5}{3^2\times4^4}$

$= \frac{(-1)^5\times3^5\times(2^3)^3\times2^5}{3^2\times(2^2)^4}$

$= \frac{-1\times3^5\times2^9\times2^5}{3^2\times2^8}$

$= -1[3^{5-2}\times2^{9+5-8}]$

$= -1[3^3\times2^6]$

$= (-3)^3\times2^6$

$= (-27\times64)$

$= -1728$

(ii) $\frac{9^8\times(x^2)^5}{(27)^4\times(x^3)^2}$

$= \frac{(3^2)^8\times x^{2\times5}}{(3^3)^4\times x^{3\times2}}$

$= \frac{3^{16}\times x^{10}}{3^{12}\times x^6}$

$= 3^{16-12}\times x^{10-6}$

$= 3^4\times x^4$

$= (3x)^4$

(iii) $\frac{3^2 \times 7^8 \times 13^6}{21^2 \times 91^3} = \frac{3^2 \times 7^8 \times 13^6}{(3\times7)^2 \times (7\times13)^3}$

$= \frac{3^2 \times 7^8 \times 13^6}{3^2 \times 7^2 \times 7^3 \times 13^2}$

$= 3^{2-2} \times 7^{8-2-3} \times 13^{6-3}$

$= 3^0 \times 7^3 \times 13^3 = 1 \times 7^3 \times 13^3$

$= (7 \times 13)^3 = (91)^3$

Question4:

If $\left(-\frac{3}{5}\right)^x = -\frac{27}{125}$ then find the value of x.

Solution :

$\left(-\frac{3}{5}\right)^x = -\frac{27}{125}$

$= \left(\frac{-3}{5}\right)^n = \left(\frac{-3}{5}\right)^3$

Comparing, we get

$x = 3$

Question 5:

Write the prime factorisation of the following numbers in the exponential form:

(i) 24000

(ii) 12600

(iii) 14157

Solution :

(i) 24000

2	24000
2	1200
2	6000
2	3000
2	1500
2	750
3	375
5	125
5	25
5	5
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$

$= 2^6 \times 3^1 \times 5^3$

(ii2) 12600

2	12600
2	6300
2	3150
3	1575
3	525
5	175
5	35

7	7
	1

$= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$
$= 2^3 \times 3^2 \times 5^2 \times 7^1$

(iii) 14157

3	14157
3	4789
11	1573
11	143
13	13
	1

$= 3 \times 3 \times 11 \times 11 \times 13$
$= 3^2 \times 11^2 \times 13^1$

Question 6:

Express the numbers appearing in the following statements in scientific notation:

(i) The earth has 1,353,000,000 cubic km of water.

(ii) The population of India was about 1,027,000,000 in march, 2001.

(iii) 60,230,000,000,000,000,000 molecules are contained in a drop of water.

Solution :

(i) The earth has 1,353,000,000 cubic km of water.

$= 1.353 \times 10^9$ cubic km

(ii) The population of India was about 1,027,000,000 in march, 2001.

$= 1.027 \times 10^9$

(iii) 60,230,000,000,000,000,000 molecules are contained in a drop of water.

6.023×10^{22} molecules

Question 7:

Compare the following numbers:

(i) 5.976×10^{24} ; 8.689×10^{23}

(ii) 3.7662×10^{17} ; 3.7671×10^{17}

Solution:

(i) 5.976×10^{24}; 8.689×10^{23}
Here in 5.976×10^{24}, 10^2 is greater than in 8.689×10^{23}
$5.976 \times 10^{24} > 8.689 \times 10^{23}$

(ii) 3.7662×10^{17}; 3.7671×10^{17}
Here 10^{17} is multiplied in both
and $76671 > 7662$
$3.76671 \times 10^{17} > 3.7662 \times 10^{17}$

Exercise 5.2

Question 1:

Classify the following sets into empty set , finite set and infinite set. In case of (non-empty) finite sets, mention the cardinal number.

(i) { all colours of a rainbow}

(ii) { x\x is a prime number between 7 and 11 }

(iii) { multiples of 5 }

(iv) { all straight lines drawn in a plane}

(v) { x\x is a digit in the numeral 550131527}

(vi) { x\x is a letter in word SUFFICIENT}

(vii) { x\x = 4n , n ∈ I and x < 10}

(viii) { x\x ∈ N , x is a prime factor of 180 }

(ix) { x:x is a vowel in the word WHY}

(x) { x:x = 5n ,n ∈ w and x < 60 }

Solution :

(i) It is a finite set having 7 elements.

(ii) it is an empty set.

(iii) It is an infinite set having unlimited elements

(iv) It is an infinite set having unlimited number of elements.

(v) It is a finite set having 6 elements i.e 0, 1, 2, 3, 5, 7,

(vi) It is a finite set having 6 elements i.e S, U, F, I, E, N, T.

(vii) It is an infinite set having the set of integers i.e unlimited number of elements.

(viii) It is a finite set having 3 elements.

(ix) { x:x is a vowel in the word WHY}

It is an empty set as there is no vowel in the word why

(x) { x:x = 5n n ∈ W and x < 60 }

= { 5,10,15,20,25,30,35,40,45,50,55}

It is finite set and it has 12 elements.

Question 2:

which of the following describe the same sets:

(i) {vowels of English alphabet } and { e,a,u,i,o}

(ii) { a,b,d} and { d,a,b,b}

(iii) { letters of PUPPET } and {E,T,P,U}

(iv) {1,2,3 } and {2,3,4,)

(v) {1,2,3,4,5 } and {x\x ∈ N, x≤ 5 }

Solution:

(i) the given sets are the same sets.

(ii) These are the same sets.

(iii) the given sets are the same sets.

(iv) These are not the same sets.

(v) The given sets are the same sets.

Question 3:

Find pairs \groups of equal sets from the following sets:

A = { 0, 1, 2, 3}

B = { x:x^2 < 10, x ∈ w}

C = {letters of word FOLLOW}

D = { days of a week }

E = { x\x ∈ W , x < 4 }

F = { Letters of word FLOW }

G = { Monday, Tuesday, ..., Sunday }

H = { letters of word WOLF }

Solution :

A = B = E because if we write B and E in tabular form, we get the same elements.

C = F = H because the elements in a set can be rearranged as each set can be written as { F,O,L,W} form.

D = G because if we write D in tabular we get the same elements.

Question 4:

Find pairs \groups of equivalent sets from the following sets.

A = { colours of a rainbow }

B = { letters of word GOOD }

C = {letters of word TOM }

E = { x:x ∈ I , $x^2 < 10$ }

F = {months of a year }

G = { days of a week }

H = { x\x = 3n ,

I = { all even numbers between 1 and 53 }

J = { all letters of English alphabets }

Solution :

A,C,E and G are equivalent sets as these all have same number of elements i.e 7 elements.

B ⇔ as n (B) = 3 = n (D)

F ⇔ H as n (F) = 12 = n (H)

I ⇔ J as n (I) = 26 = n (j)

Question 5:

In the following, find whether A ⊂ B or B ⊂ A or none of these:

(i) A = {1,2,3} B = { 2,3,3,3,1,3}

(ii) A = { 2,4,6,....} B = { all natural numbers }

(iii) A = { x\x ∈ I, $x^2 < 20$ } , B = { 0,1,2,3,4}

(iv) A = {letters of king}

B = { letters of Queen}

Solution :

In the following find whether A ⊂ B or B ⊂ A or none of these

(i) $A = \{ 1,2,3 \}$ $B = \{ 2,3,3,3,1,3\} = \{2,3,1\}$

$A \subset B$ and $B \subset A$: i,e $A = B$

(ii) $A = \{ 2,4,6, ... \}$ $B = \{$ all natural numbers $\}$

$= \{ 1,2,3,4,5,6,7.....\}$

$A \subset B$ but $B \not\subset A$

(iii) $A = \{ x \backslash x \in I , x^2 < 20 \}$, $B = \{ 0,1,2,3,4\}$

$= \{ 0,1,4,9,16\}$

$= \{ (0)^2,(1)^2,(2)^2,(3)^2,(4)^2\}$

$B = \{ 0,1,2,3,4\}$

$B \subset A$ but $A \not\subset B$

(iv) $A = \{$ letters of king $\} = \{K,I,N,G\}$

$B = \{$ letters of QUEEN $\} = \{ Q,U,E,N\}$

Here , $A \not\subset B$ and $B \not\subset C$

Neither $A \subset B$ nor $B \subset A$

Question 6:

State whether each of the following statement is true or false for the sets A and B where

$A = \{$ letters of CLOUD$\}$ and $B = \{$ letters of KOLKATA$\}$

(i) $A \subset B$

(ii) $B \subset A$

(iii) $A \leftrightarrow B$

Solution :

$A = \{\text{letters of CLOUD }\} = \{ C, L, O, U, D\}$

$B = \{\text{letters of Kolkata }\} = \{ K, O, L, A, T \}$

(i) $A \subset B$: It is false because some elements of A are not the element of B .

(ii) $B \subset A$: It is false because some element of B is not a member

(iii) $A \leftrightarrow B$. It is true as $n(A) = 5\ n(B)$.

Question. 7:

Write all the subsets of the following sets:

(i) Φ

(ii) {3,5}

(iii) {2,4,6}

Solution:

(i) Subset of Φ is Φ

(ii) Empty set is a subset of every set so, the subsets are Φ {3} , {5} {3 , 5}

(iii) Empty set is a subset of every set. So the subsets are Φ {2},{4},{6},{2,4} ,{4,6} ,(2,6) ,(2,4,6)

Question 8:

If $A = \{ x:x = 2n, n < 5\}$ then find A when

(i) $\xi = N$

(ii) $\xi = W$

(iii) $\xi = I$

Solution :

(i) Natural numbers less than 5 are 1,2,3,4

Given $x = 2n$, putting $n = 1,2,3,4$ we get,

$X = 2 \times 1, 2\times2, 2\times3, 2\times 4$

$= 2,4,6,8$

The given set can be written as {2,4,6,8}

(Every set is a subset of universal set i.e. $A \subset \xi$)

(ii) whole numbers less than 5 are 0, 1, 2, 3, 4,

Given 2n i.e. $2 \times 0, 2 \times1, 2 \times 2, 2 \times 3, 2 \times 4,$

i.e. 0,2,4,6,8

The given set i.e A can be written as (0,2,4,6,8)

(every set is a subset of universal set i.e $A \subset \xi$)

(iii) Integers less than 5 are ...., -4,-3,-2,-1 ,0,1,2,3,4

Given 2n i.e ..... $2 \times -2, 2 \times -1, 2\times 0, 2 \times 1, 2 \times 2, 2 \times 3, 2 \times4,$

i.e ..... -4 ,-2 0 2,4, 6,8

The given set i.e A can be written as { ....., -4, -2, 0 , 2, 4, 6, 8 }

(Every set is a subset of universal set i. e $A \subset \xi$)

Objective type Questions

Question 1: Fill in the blanks:

(i) If x is not a member of the set A , then symbolically we write it as ...

(ii) Each element of a set is listed once and only ... repetitions are removed.

(iii) A set that contains a limited number of different elements is called a ..... set.

(iv) Two finite sets are called equivalent if and only if they have ... a number of elements.

Solution:

(i) If x is not a member of the set, A then symbolically we write it as $x \notin A$

(ii) Each element of a set is listed once and only once repetitions are removed.

(iii) A set that contains a limited number of different elements is called a finite set.

(iv) Two finite sets are called equivalent if and only if they have equal number of elements.

Question 2:

State whether the following statements are true (T) or false (F)

(i) A collection of books is a set.

(ii) If x = { letters of the word PRINCIPAL }then the cardinal number of set X is 9 .

(iii) if P = {letters of the word AHMEDABAD}, then n(p) = 6

(iv) If A is any set, then A ⊂ A.

(v) An empty set is a subset of every set.

(vi) If set A = {0} , then n{A} = 0 .

(vii) If A and B are two sets such that A ⇔ B , then A = B .

Solution:

(i) A collection of books is a set. (False)

Correct:

A collection of different books is a set.

(ii) If X = {letters of the word PRINCIPAL}, then the cardinal number of set X is 9 (false)

Correct:

N(x) = P,R,I,N,C,A,L = 7 not 9

(iii) If P = { letters of the word AHMEDABAD },then n(P) = 6, (true) {A, H, M , E, D, B}

(iv) If A is any set, then A ⊂ A. (True)

(v) An empty set is a subset of every set. (True)

(vi) If A and B are two sets such that A ⇔ B , then A = B. (False)

Correct: A = B is A ⇔ B but is A ⇔ B is not always A = B

Multiple Choice Questions

Choose the correct answer from the given four options (3to9) :

Question: 3

Which of the following collection forms a set?

(a) Collection of 5 odd prime numbers

(b) Collection of 3 most intelligent students of your class

(c) Collection of 4 vowels of the English alphabet

(d) Collection of first 6 months of a year.

Solution :

Collection of 5 odd prime number, collection of 4 vowels of English alphabet and collection of first 6 months of a year, all are sets but a

Collection of 3 most intelligent students of your class in not a set, because intelligence is not well defined (b)

Question 4:

The tabular form for the statement ‘ Days of the week starting with the letter T’ will be

(a) { days of week starting with letter T }

(b) { Tuesday, Thursday}

(c) { x\x is a day of the week starting with letter T }

(d) none of these

Solution :

Days of the week starting with the letter T in tabular form is { Tuesday, Thursday} (b)

Question 5: A set with a limited number of distinct elements is called

(a) a finite set

(b) an infinite set

(c) both finite as well as an infinite set

(d) none of these

Solution:

Set with a limited number of distinct elements is called a finite set.

(a)

Question 6: The symbol ↔ stands for

(a) belongs to

(b) is a subset of

(c) is equivalent to

(d) none of these

Solution:

The symbol ↔ stands for is equivalent to . (c)

Question 7: The empty set is denoted as

(a) {Φ }

(b) { }

(c) {0}

(d) 0

Solution:

Empty set is denoted as { }. (b)

Question 8:

The cardinal number n(A) = for A = { x:x is an odd prime number less than 20} is

(a) 8

(b) 7

(c) 9

(d) 10

Solution :

The cardinal number n(A) = {x:x is an odd prime number less than 20} = { 3, 5, 7, 11, 13, 17, 19 }

n(A) = 7 (b)

Question 9: if A = {x\x is a positive multiple of 3 less than 20 } and B = { x\x is a prime number less than 20}, then n(A) + n(B) is

(a) 6

(b) 8

(c) 13

(d) 14

Solution :

A = { x\x is a positive multiple of 3 less than 20 }

= { 3,6,12,15,18}

= n(A) = 6

B = {x\x is a prime number less than 20 }

= {2,3,5,11,13,17,19 }

= n(B) = 8

N(A) + n(B) = 6+8 = 14 (d)

Check your Progress

Question 1: Write the following sets in tabular form and also in set builder form :

(i) The set of even integers which lie between -6 and 10 .

(ii) The set of two digit numbers which are perfect square.

(iii) { factors of 42}

Solution :

(i) Given set = { -4, -2, 0, 2, 4, 6, 8) (tabular form)

Or { x:x = 2n, n∈I and -3 < n < 5 } (set builder form)

(ii) The set can be written as { 16,25,36,49,64,81} (tabular form

Or { x:x = n^2, n ∈ N and 4 ≤ n ≤ 9)

(iii) the set can be written as { 1,2,3,6,7,14,21,42) (tabular form)

Or { x:x is a factor of 42) (set builder form)

Question 2: Write the following sets in roster form:

(i) { x:x = 5n, n∈I and -3 < n ≤ 13 }

(ii) { x:x = n^2 , n ∈ W and n < 5 }

(iii) { x:x = n^2 – 2 , n∈w and n<4}

Solution :

The set can be written as

(i) Integers lie between -2 and 3 are -2 ,-1,0 1,2,3

Given x = 5n , putting n = -2, -1, 0 1,2,3 we get

X = 5 × -2, 5× -1, 5× 1 , 5 × 2, 5 × 3,

= -10, -5, 0, 5, 10, 15

Set = { -10, -5, 0 ,5, 10, 15 }

(ii) whole numbers less than 5 are 0 ,1,2,3,4

Given x = n^2, putting n 0, 1,2,3,4 we get

X = 0^2, 1^2, 2^2, 3^2, 4^2 = 0, 1, 4, 9, 16

Given set = { 0, 1, 4, 9, 16 } (roster form)

Whole numbers less than 4 are 0, 1, 2, 3,

Given x = n^2-2 putting n = 0, 1, 2, 3, we get

X = 0^2 – 2, 1^2 -2, 2^2-2, 3^2-2

= -2, -1, 2 ,7

Given set = { -2, -1, 2, 7} (roster form)

Question 3:

Write the following sets in set builder form:

(i) { -14, -7, 0, 7, 14, 21, 28}

(ii) { 1,2,3,6,9,18}

Solution :

(i) {x\x = 7n n∈I and -2 ≤ n ≤ 4 }(set builder form)

(ii) given set = { x\x ∈ N, is a factor of 18}

(set builder form)

Question 4: Classify the following sets into the finite set, infinite set the empty set. In the case of a (non_- empty) finite set, mention the cardinal number.

(i) The set of even prime numbers.

(ii) {multiples of 9}

(iii) { x:x is a month of a year having less than 30 days }

(vi) {x\x is a month of a leap year having 28 days }

Solution:

(i) It is a finite set having 1 element. So, cardinal number = 1

(ii) It is an infinite set as it has unlimited number of different elements.

Because, if we write it in roster form, the given set = {9,18,27,36....}

(iii) Prime factors of 84 = 2,3,7.

The set can be written as = {2,3,7}

It is a finite set having 3 elements.

(iv) 2x +5 = 1

= 2x = 1-5

= 2x = -4

= x = -2

But x ∈ N and Natural numbers are { 1,2,3....}

It is an empty set.

(v) { x:x is a month of a year having less than 30 days = February

It is a finite set as it is one element.

(vi) {x\x is a month of a leap year having 28 days } = Φ it is an empty set as there is no month in the leap year which has 28 days.

Question 5: In the following, determine whether A and B are equivalent sets and if so, whether A = B.

(i) A = { 1,3,5) B = {Red, Blue , Green }

(ii) A = { Prime factors of 70 }, B = {Prime factors of 60 }

(iii) A = { even natural numbers less than 10 }, B = { odd natural numbers less than 10 }

Solution :

(i) A ↔ B as n (A) = 3 = n(B)

But A ≠ B because , they have different elements.

(ii) Prime factors of 70 = 2,5,7

A = (2,5,7)

Prime factors of 60 = 2,3,5

B = (2,3,5)

A ↔ B as n (A) = n(B)

But A ≠ B

They have not the same elements.

(iii) if we write A and B in tabular form, we get

A = { 2,4,6,8)

B = { 1,3,5,7,9}

So , n (A) ≠ n(B)

A is not equivalent to B.

Question 6:

Let p = { letters of SCHOOL} and Q = { letters of FALSE}, then state whether each of the following statement is true or false for the above sets:

(i) $P \subset Q$

(ii) $Q \subset P$

(iii) $P \leftrightarrow Q$

Solution :

If P = { letters of SCHOOL }

Q = { letters of FALSE }

P = { S, C, H, O, L} and Q = { F,A,L,S,E}

(i) $P \subset Q$ False.

(ii) $Q \subset P$ False.

(iii) $P \leftrightarrow Q$ True

{Both have equal number of elements}

Question 7: State whether each of the following statements is true or false for the sets, A,B and C where

A = {x\x ∈ N, X < 40 and x is a multiple of 6}

B = { x\x ∈ W, x ≤40, and x is a multiple of 8 }

C = { x\x is a factor of 28}.

(i) A ↔ B

(ii) B ↔ C

(iii) A ↔ C

Solution :

If we write A,B and C in tabular form, we get 32,

A = { 6, 12, 18, 24, 30, 36 }

B = { 0, 8, 16, 24, 40} and C = { 1, 2, 4, 7, 14, 18}

(i) A ↔ B True, because n (A) = 6 = n(B)

(ii) B ↔ C True, because n(B) = 6 = n(C)

(iii) A ↔ C True, because n(A) = 6 = n(C)