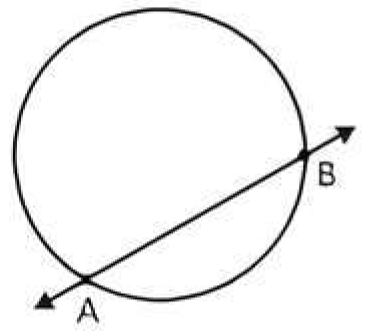
10 Circles

Fastrack« Revision

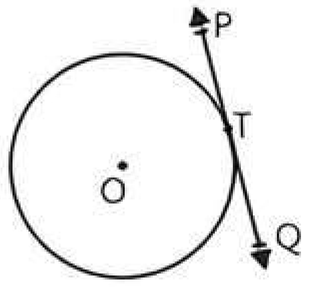
- **Circle:** A circle is a collection of all points in a plane which are at a constant distance from a fixed point. The fixed point is called **centre** and the constant distance is **radius**. The line joining two points on the circumference of the circle is **chord**.
- **Secant:** A line which intersects a circle in two distinct points is called a **secant** of the circle.



In figure, AB is the secant of the circle.

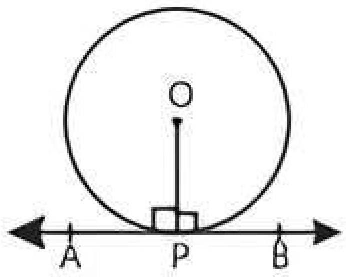
- **Tangent:** A line meets a circle only in one point is called a **tangent** to the circle at that point. The point at which the tangent line meets the circle is called the **point of contact**.

In figure, PQ is the tangent and T is the point of contact.



- The tangent at any point of a circle is perpendicular to the radius through the point of contact.

In figure, $\angle OPA = \angle OPB = 90°$



- **Number of Tangents to a Circle**
    1. No tangent can be drawn from a point inside the circle.
    2. Not more than one tangent can be drawn to a circle at a point on the circumference of the circle.
    3. Two tangents can be drawn to a circle from a point outside the circle. See figure (a).

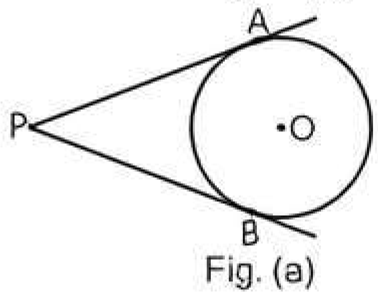


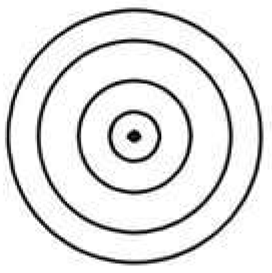
Fig. (a)

- **Length of a Tangent:** The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of a tangent from the point P to the circle. The length of two tangents drawn from the same external point to the circle are equal.

In Fig. (a), PA = PB

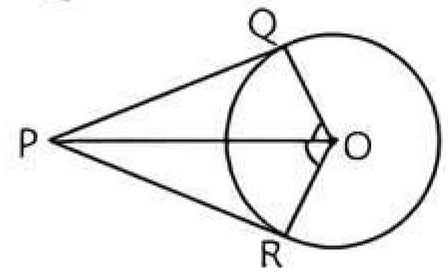
- **Concentric Circles:** Two or more circles having the same centre but different radii are called concentric circles.

In figure, circles are concentric.

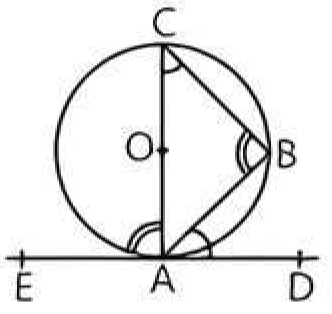


Knowledge BOOSTER

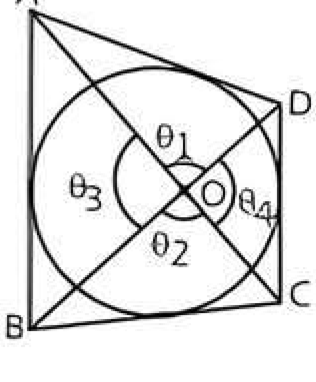
1. *A circle can have maximum two parallel tangents.*
2. *The distance between two parallel tangents to a circle is equal to the diameter of a circle.*
3. *The incentre of a triangle is the point where all the angle bisectors meet in the triangle.*
4. *If two tangents are drawn to a circle from an external point, then*
    (i) $\angle POQ = \angle POR$
    (ii) $\angle QPO = \angle RPO$
    (iii) $\angle QPR + \angle QOR = 180°$



5. *If a chord is drawn through a point of contact of a tangent to the circle then the angles formed by this chord from the tangent are equal to the angles of corresponding alternate segments.*
    i.e., $\angle BAD = \angle ACB$
    and $\angle EAC = \angle ABC$



6. *The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.*
    In figure, $\theta_1 + \theta_2 = 180°$
    and $\theta_3 + \theta_4 = 180°$



Practice Exercise

Multiple Choice Questions

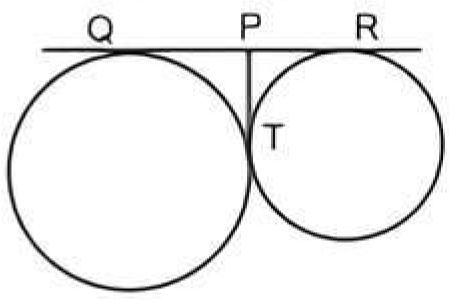
Q 1. **Two parallel tangents are drawn to a circle at a distance of 10 cm, then the radius of circle is:**

a. 3 cm b. 4 cm
c. 5 cm d. 7 cm

Q 2. **The length of tangent drawn to a circle of radius 9 cm from a point 41 cm from the centre is:** *[CBSE 2023]*

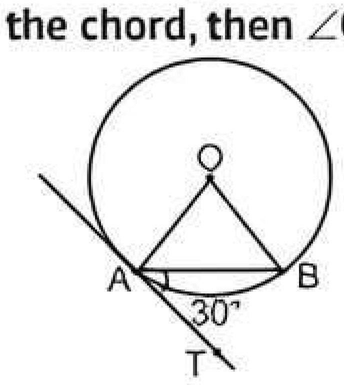
a. 40 cm b. 9 cm
c. 41 cm d. 50 cm

Q 3. **In the given figure, QR is a common tangent to given circle. Tangent at T meets QR at P. If PQ = 5.5 cm, then the length of QR is:**



a. 8 cm b. 10 cm c. 11 cm d. 7 cm

Q 4. **In the given figure, if O is the centre of a circle. AB is a chord and the tangent AT at A makes an angle of 30° with the chord, then ∠OAB is:**



a. 40° b. 30° c. 60° d. 50°

Q 5. **From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:** *[NCERT EXERCISE]*

a. 24.51 cm b. 12 cm c. 15 cm d. 7 cm

Q 6. **Two tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to:** *[NCERT EXERCISE]*

a. 80° b. 70° c. 60° d. 50°

Q 7. **At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is:** *[NCERT EXEMPLAR]*

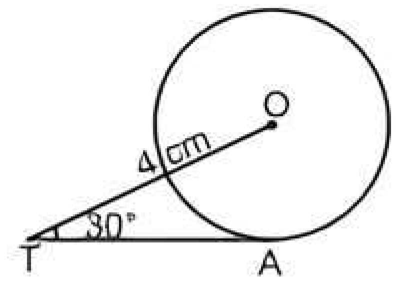
a. 4 cm b. 5 cm c. 6 cm d. 8 cm

Q 8. **If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to:** *[CBSE SQP 2022-23]*

a. $\frac{3\sqrt{3}}{2}$ cm b. 3 cm

c. 6 cm d. $3\sqrt{3}$ cm

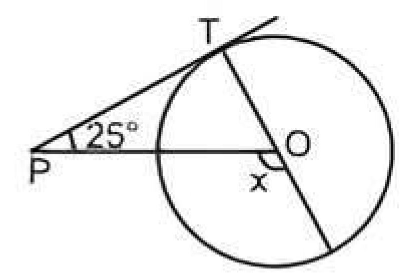
Q 9. **In the given figure, TA is a tangent to the circle with centre O such that OT = 4 cm, ∠OTA = 30°, then length of TA is:** *[CBSE 2023]*



a. $2\sqrt{3}$ cm b. 2 cm

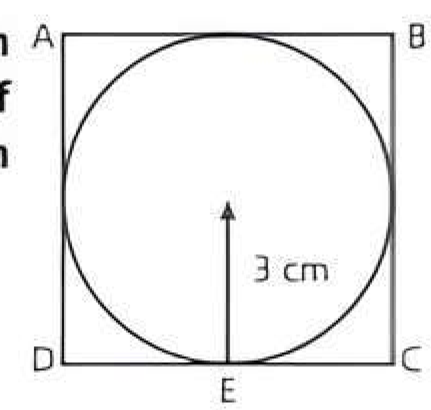
c. $2\sqrt{2}$ cm d. $\sqrt{3}$ cm

Q 10. **In the given figure, PT is a tangent at T to the circle with centre O. If ∠TPO = 25°, then x is equal to:** *[CBSE 2023]*



a. 25° b. 65° c. 90° d. 115°

Q 11. **A circle is inscribed in a square. The radius of inscribed circle is 3 cm, then the length of tangent is:**

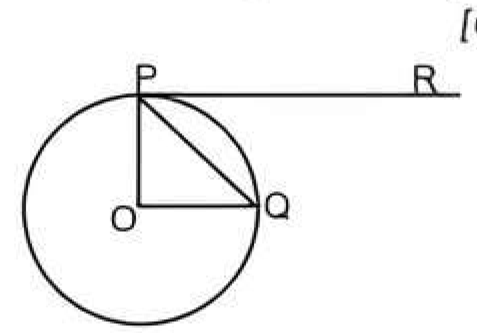


a. 3 cm
b. 9 cm
c. 6 cm
d. Can't be determined

Q 12. **If radii of two concentric circles are 6 cm and 4 cm, the length of chord touches the smaller circle is:**

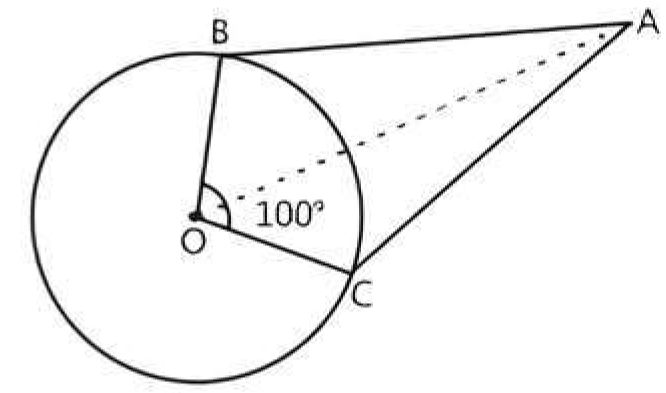
a. $\sqrt{5}$ cm b. $2\sqrt{5}$ cm
c. $3\sqrt{5}$ cm d. $4\sqrt{5}$ cm

Q 13. **If O is centre of a circle and chord PQ makes an angle 50° with the tangent PR at the point of contact P, find the angle made by the chord at the centre.** *[CBSE SQP 2023-24]*



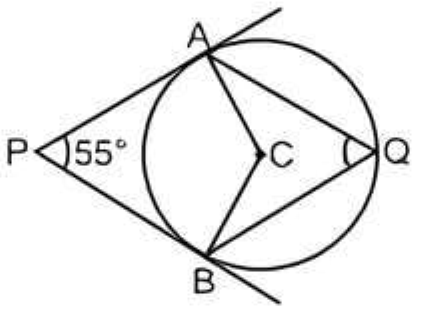
a. 130° b. 100° c. 50° d. 30°

Q 14. **In the given figure, if AB and AC are two tangents to a circle with centre O, so that ∠BOC = 100° then ∠ OAB is:**



a. 70° b. 40° c. 60° d. 50°

Q 15. **In the given figure, PA and PB are tangents from external point P to a circle with centre C and Q is any point on the circle. Then the measure of $\angle AQB$ is:**



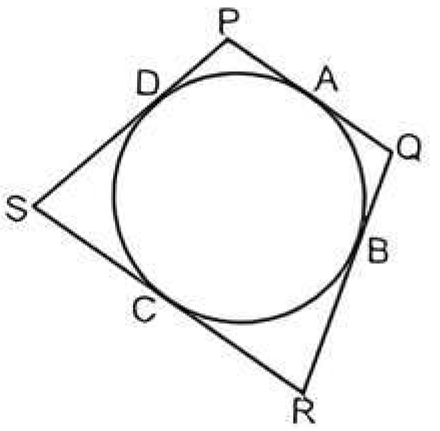
[CBSE 2023]

a. $66\frac{1}{2}°$ b. 125° c. 55° d. 90°

Q 16. **A quadrilateral PQRS is drawn to circumscribe a circle. If PQ = 12 cm, QR = 15 cm and RS = 14 cm, find the length of SP.** *[CBSE SQP 2023-24]*

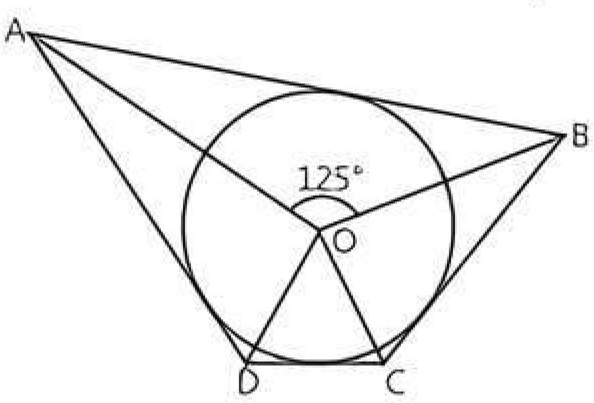
a. 15 cm b. 14 cm
c. 12 cm d. 11 cm

Q 17. **In the given figure, the quadrilateral PQRS circumscribes a circle. Here, PA + CS is equal to:** *[CBSE 2023]*



a. QR b. PR
c. PS d. PQ

Q 18. **In the given figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to:** *[NCERT EXEMPLAR]*



a. 62.5° b. 45° c. 35° d. 55°

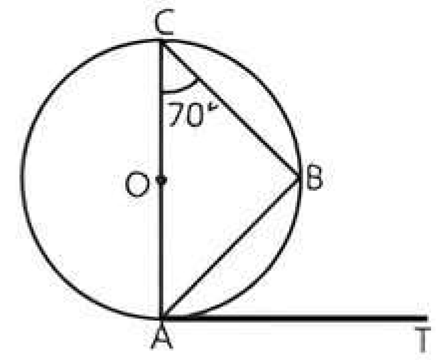
Assertion & Reason Type Questions

Directions (Q. Nos. 19-23): *In the following questions, a statement of assertion (A) is followed by a statement of a reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 19. **Assertion (A): PA and PB are two tangents to a circle with centre O. Such that $\angle AOB = 110°$, then $\angle APB = 90°$.**
Reason (R): The length of two tangents drawn from an external point are equal.

Q 20. **Assertion (A): In the given figure, O is the centre of a circle and AT is a tangent at point A, then $\angle BAT = 70°$.**



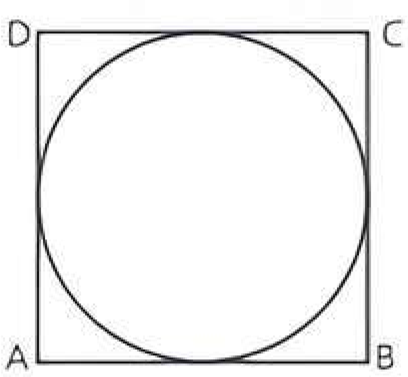
Reason (R): A straight line can intersect a circle at one point only.

Q 21. **Assertion (A): Suppose the distance between two parallel tangents of a circle is 16 cm, then radius of a circle is 10 cm.**
Reason (R): The distance between two parallel tangents of a circle is equal to the diameter of a circle.

Q 22. **Assertion (A): If PA and PB are tangents drawn from an external point P to a circle with centre O, then the quadrilateral AOBP is cyclic.**
Reason (R): The angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Q 23. **Assertion (A): In the given figure, a quadrilateral ABCD is drawn to circumscribe a given circle, as shown. Then**

AB + BC = AD + DC.



Reason (R): In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

Fill in the Blanks Type Questions

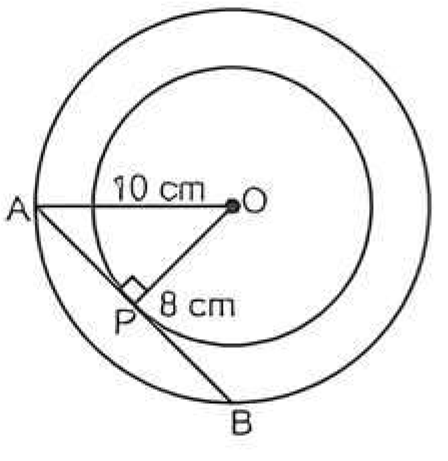
Q 24. **A line which intersects a circle in two distinct points is called a .... of the circle.** *[NCERT EXERCISE]*

Q 25. **A circle can have maximum .... parallel tangents.** *[NCERT EXERCISE]*

Q 26. **The common point of a tangent and the circle is called point of ....** *[NCERT EXERCISE]*

Q 27. **A tangent at a point P on a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 13 cm, then length of PQ is ....**

Q 28. **In the given figure, the length PB = ... cm.** *[CBSE 2020]*



True/False Type Questions

Q 29. If a point lies on a circle, then the number of tangents drawn from that point to the circle is 2.

Q 30. In two concentric circles, all chords of the outer circle, which touch the inner circle are of equal length.

Q 31. The tangent of a circle makes an angle of 90° with radius at point of contact.

Q 32. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠APO is equal to 70°.

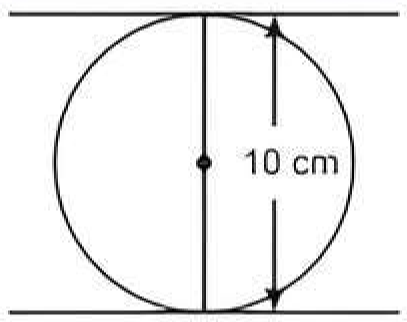
Q 33. If a chord AB subtends an angle of 60° at the centre of a circle, then angle between the tangents at A and B is also 60°.

Solutions

1. (c)

TiP

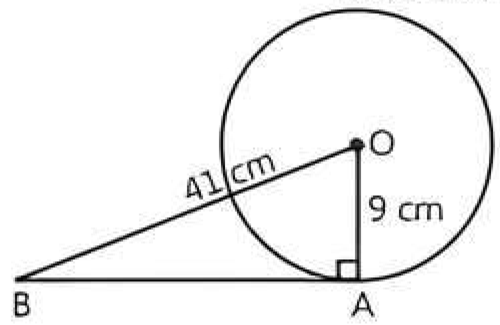
In two parallel tangents to a circle, the distance between two tangents is equal to the diameter of a circle.



Here, diameter of a circle, d = Distance between two parallel tangents = 10 cm

∴ Radius of circle, $r = \frac{d}{2} = \frac{10}{2} = 5$ cm

2. (a) Given radius of a circle, OA = 9 cm
and OB = 41 cm



TR!CK

In right-angled triangle,
$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$

Since, tangent at any point of a circle is perpendicular to the radius through the point of contact.

i.e., ∠OAB = 90°

In right-angled ΔOAB, use Pythagoras theorem

$$(OB)^2 = (OA)^2 + (AB)^2$$

$$\therefore \quad (41)^2 = (9)^2 + (AB)^2$$

$$\Rightarrow \quad AB = \sqrt{1681 - 81} = \sqrt{1600} = 40 \text{ cm}$$

3. (c) Given PQ = 5.5 cm

The lengths of the tangents drawn from an external point to a circle are equal.

∴ PT = QP = 5.5 cm

Again P is an external point to a smaller circle. Therefore,

PR = PT = 5.5 cm

Now, length of tangent QR = PQ + PR
= 5.5 + 5.5 = 11 cm

4. (c)

TiP

Radius is perpendicular to the point of contact of tangent.

In the given figure,

$$\angle OAT = 90°$$

$$\Rightarrow \quad \angle OAB + \angle BAT = 90°$$

$$\Rightarrow \quad \angle OAB + 30° = 90°$$

$$\Rightarrow \quad \angle OAB = 60°$$

5. (d) In right-angled triangle OPQ,

$$OP^2 + QP^2 = OQ^2$$

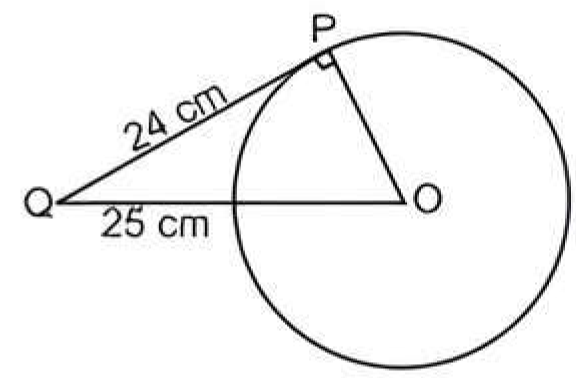
(by Pythagoras theorem)

$$\Rightarrow \quad (OP)^2 + (24)^2 = (25)^2$$

$$\Rightarrow \quad (OP)^2 = 625 - 576$$

$$\Rightarrow \quad (OP)^2 = 49$$

$$\Rightarrow \quad OP = 7 \text{ cm}$$



6. (d) Since OP line bisect ∠P. Therefore

$$\angle APO = \angle BPO = 40°$$

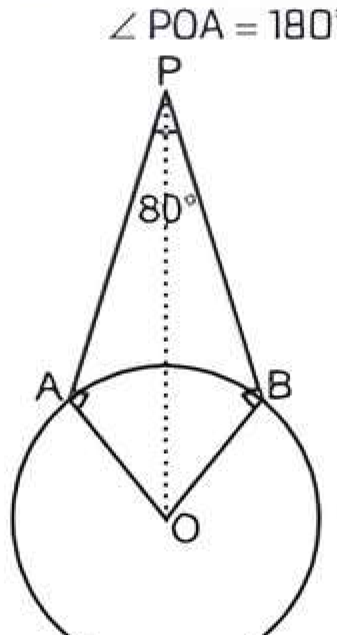
In right-angled ΔPAO,

$$\angle PAO + \angle APO + \angle POA = 180°$$

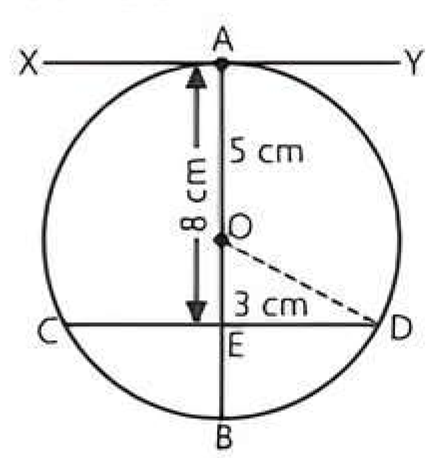
(angle sum property)

$$\Rightarrow \quad 90° + 40° + \angle POA = 180°$$

$$\Rightarrow \quad \angle POA = 180° - 130° = 50°$$



7. (d) Given, AB is a diameter of a circle and radius of a circle, AO = OD = 5 cm.



Suppose chord CD || XY, intersect AB at point E.

TR!CK

In right-angled triangle,
$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$

In right-angled ΔOED,

$$(OD)^2 = (ED)^2 + (OE)^2$$
(by Pythagoras theorem)

$$(5)^2 = (ED)^2 + (3)^2$$
$(\because OE = AE - OA = 8 - 5 = 3 \text{ cm})$

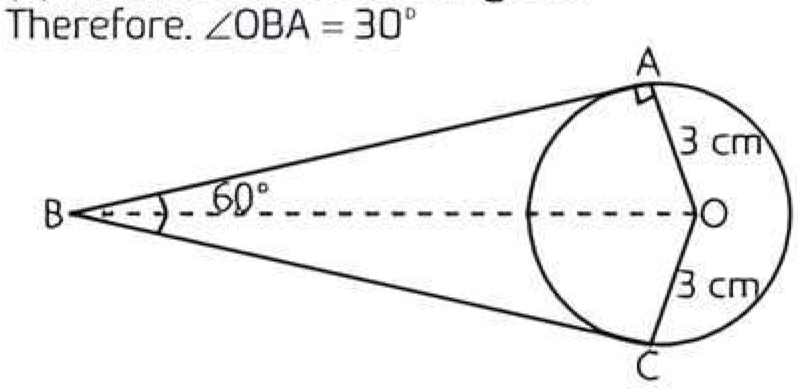
$$\Rightarrow \quad (ED)^2 = 25 - 9$$
$$\Rightarrow \quad (ED)^2 = 16$$
$$\Rightarrow \quad ED = 4 \text{ cm}$$

As we know that line drawn from centre O to the chord CD, it bisects the chord.

i.e., $CE = ED$

Now, length of chord = $CD = 2ED = 2 \times 4 = 8$ cm

8. (d) Here, OB bisects the angle B.
Therefore, $\angle OBA = 30^\circ$



In right-angled ΔBAO,

$$\tan 30^\circ = \frac{OA}{AB}$$
$$\frac{1}{\sqrt{3}} = \frac{3}{AB}$$
$$\Rightarrow \quad AB = 3\sqrt{3} \text{ cm}$$

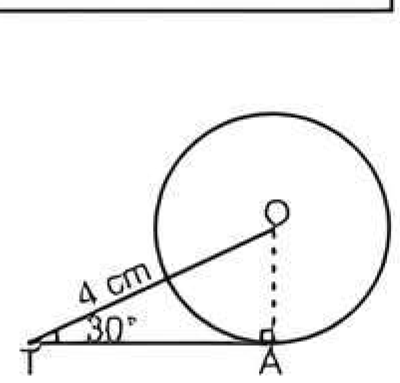
TiP
If two tangents are drawn from an external point, then the line joining from external point to the centre bisects the angle formed by the pair of tangents.

9. (a) Given, $OT = 4$ cm and $\angle OTA = 30^\circ$

TiP
Tangent at any point of a circle is perpendicular to the radius through the point of contact.

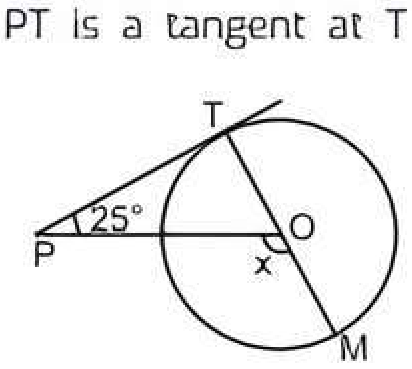
In right-angled ΔOAT,

$$\cos 30^\circ = \frac{AT}{OT}$$
$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AT}{4}$$
$$\Rightarrow \quad TA = 2\sqrt{3} \text{ cm}$$



So, length of TA is $2\sqrt{3}$ cm.

10. (d) Given, PT is a tangent at T to the circle with centre O.



$$\angle TPO = 25^\circ$$
Since, $\quad OT \perp PT$
$$\therefore \quad \angle OTP = 90^\circ$$

TiP
In a triangle, the exterior angle is equal to the sum of the two interior opposite angles.

In ΔPOT,

$$\text{ext. } \angle POM = \angle OPT + \angle PTO$$
$$\Rightarrow \quad x = 25^\circ + 90^\circ = 115^\circ$$

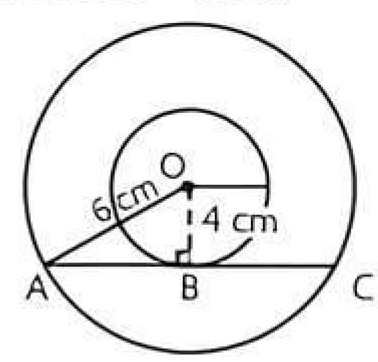
11. (c) Given radius of a circle, $r = 3$ cm. Therefore, diameter of a circle, $d = 2r = 2 \times 3 = 6$ cm.

Since, diameter of circle is equal to the side of a square.

$\therefore$ Side of a square = Diameter of a circle
$= 6$ cm.

$\therefore$ The length of a tangent line = side of a square
$= 6$ cm.

12. (d) Given radius of bigger circle, $OA = 6$ cm and radius of smaller circle, $OB = 4$ cm.



TiP
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In right-angled ΔOBA,

$$OA^2 = OB^2 + AB^2 \quad \text{(by Pythagoras theorem)}$$
$$(6)^2 = (4)^2 + (AB)^2 \quad (\because OB = 4 \text{ cm})$$
$$\Rightarrow \quad (AB)^2 = \sqrt{36 - 16}$$
$$\Rightarrow \quad AB = \sqrt{20} = 2\sqrt{5} \text{ cm}$$
$$\therefore \quad AC = 2\,AB$$
$$= 2 \times 2\sqrt{5} = 4\sqrt{5} \text{ cm}$$

13. (b)

TiP
Tangent is perpendicular to the radius through the point of contact.

Here, $\quad \angle OPR = 90^\circ$

$$\Rightarrow \quad \angle OPQ + \angle RPQ = 90^\circ$$
$$\Rightarrow \quad \angle OPQ + 50^\circ = 90^\circ \quad (\because \angle RPQ = 50^\circ, \text{ given})$$
$$\Rightarrow \quad \angle OPQ = 40^\circ$$

In Δ OPQ,

$$OP = OQ \quad \text{(radii of circle)}$$
$$\Rightarrow \angle OQP = \angle OPQ = 40^\circ$$
(angles opposite to equal sides of a triangle are equal)

Now, in ΔPOQ using angle sum property of a triangle,

$$\angle POQ + \angle OQP + \angle OPQ = 180^\circ$$
$$\Rightarrow \quad \angle POQ + 40^\circ + 40^\circ = 180^\circ$$
$$\Rightarrow \quad \angle POQ = 100^\circ$$

14. (b)

TiP

Angle between radii and pair of tangent is supplementary.

Here, $\angle BOC + \angle BAC = 180^\circ \Rightarrow \angle BAC = 180^\circ - 100^\circ$

$\Rightarrow \qquad \angle BAC = 80^\circ$

Also, line OA is bisector of $\angle A$.

$\therefore \qquad \angle OAB = \frac{80^\circ}{2} = 40^\circ$

15. (a) PA and PB are tangents from external point P to a circle.

Given, $\angle APB = 55^\circ$

TiPS

- *Angle between radii and pair of tangent is supplementary.*
- *The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

Here, $\angle APB + \angle ACB = 180^\circ$

$\Rightarrow \qquad 55^\circ + \angle ACB = 180^\circ$

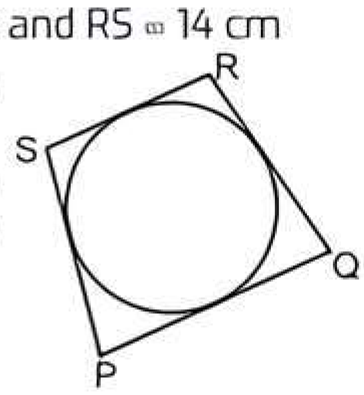
$\Rightarrow \qquad \angle ACB = 180^\circ - 55^\circ = 125^\circ$

Now, $\qquad \angle AQB = \frac{1}{2}\angle ACB = \frac{1}{2} \times 125^\circ$

$\therefore \qquad \angle AQB = 62\frac{1}{2}^\circ$

16. (d) Given, PQ = 12 cm, QR = 15 cm and RS = 14 cm

Also, a quadrilateral PQRS is drawn to circumscribe a circle.



We know that, when a quadrilateral PQRS is drawn to circumscribe a given circle then,

$PQ + RS = SP + QR$

$\therefore \qquad SP = PQ + RS - QR$

$= 12 + 14 - 15 = 26 - 15 = 11$

So, length of SP is 11 cm.

17. (c) Given, the quadrilateral PQRS circumscribes a circle.

TiP

The length of two tangents drawn from an external point are equal.

We know that, if a quadrilateral PQRS is drawn to circumscribe a given circle then,

$PQ + RS = SP + QR$

$\Rightarrow (PA + AQ) + (SC + CR) = (PD + SD) + (RB + BQ)$

$\Rightarrow \quad PA + SC + (AQ + CR) = (PD + SD) + (CR + AQ)$

$(\because AQ = BQ \text{ and } CR = RB)$

$\Rightarrow \qquad PA + SC = PS \qquad (\because PS = PD + SD)$

18. (d)

TR!CK

The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Given $\qquad \angle AOB = 125^\circ$

Here, $\angle AOB + \angle COD = 180^\circ$

$\therefore \qquad 125^\circ + \angle COD = 180^\circ$

$\Rightarrow \qquad \angle COD = 55^\circ$.

19. (d) **Assertion (A):** We have,

$OA \perp AP$

and $\qquad OB \perp PB$

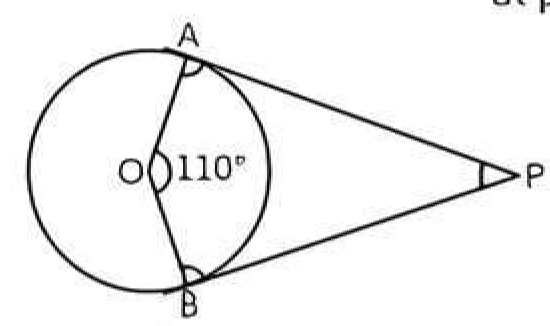
In quadrilateral OAPB, we have

$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360^\circ$

$\Rightarrow \qquad 90^\circ + \angle APB + 90^\circ + 110^\circ = 360^\circ$

$\Rightarrow \qquad \angle APB = 70^\circ$

($\because$ radius is perpendicular to the tangent at point of contact)



So, Assertion (A) is false.

Reason (R): It is true that the length of two tangents drawn from an external point are equal.

Hence, Assertion (A) is false but Reason (R) is true.

20. (c) **Assertion (A):**

TR!CK

If a chord is drawn through a point of contact of a tangent to the circle then the angles formed by this chord from the tangents are equal to the angles of corresponding alternate segments.

Here, $\qquad \angle BAT = \angle ACB$

(by alternate segment theorem)

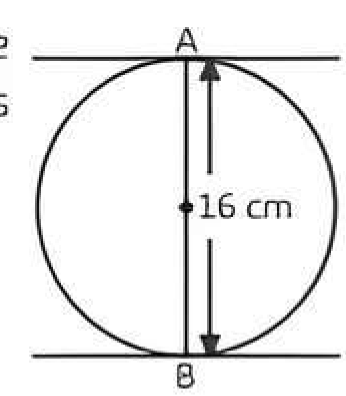
$\therefore \qquad \angle BAT = 70^\circ \qquad (\because \angle ACB = 70^\circ, \text{ given})$

So, Assertion (A) is true.

Reason (R): Any straight line can intersect a circle at two points.

So, Reason (R) is false.

21. (d) **Assertion (A):** The distance between two parallel tangents is equal to the diameter of a circle.



$\therefore \qquad d = AB = 16 \text{ cm}$

Now, radius of a circle, $r = \frac{d}{2} = \frac{16}{2}$

$= 8 \text{ cm}$

So, Assertion (A) is false.

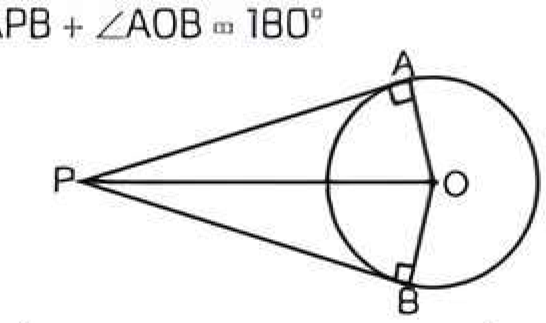
Reason (R): It is also true that the distance between two parallel tangents is equal to the diameter of a circle.

So, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

22. (a) **Assertion (A):** We know that, the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

i.e., $\angle APB + \angle AOB = 180^\circ \qquad$ ...(1)



Also, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

i.e., $PA \perp OA \Rightarrow \angle OAP = 90^\circ$

and $PB \perp OB \Rightarrow \angle OBP = 90^\circ$

$\therefore \angle OAP + \angle OBP = 90^\circ + 90^\circ = 180^\circ$ ...(2)

If the sum of a pair of opposite angles of a quadrilateral is 180° then quadrilateral is cyclic.

From eqs. (1) and (2), we get

Quadrilateral AOBP is cyclic.

So, Assertion (A) is true.

Reason (R): It is a true statement also.

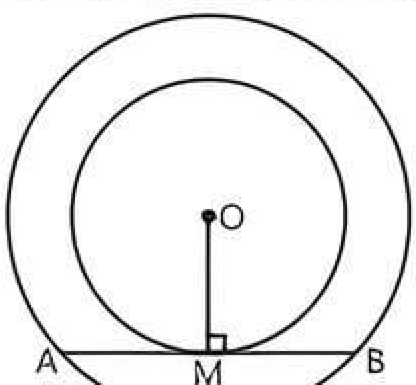
Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

23. (d) **Assertion (A):** If a quadrilateral ABCD is drawn to circumscribe a circle, then

$$AB + CD = AD + BC$$

So, Assertion (A) is false.

Reason (R): We have two concentric circles with O is the centre of concentric circles and AB is the tangent.



Since, $OM \perp AB$

$\therefore AM = MB$

($\because$ perpendicular drawn from centre O to the chord AB bisect the chord AB)

So, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

24. secant

25. two

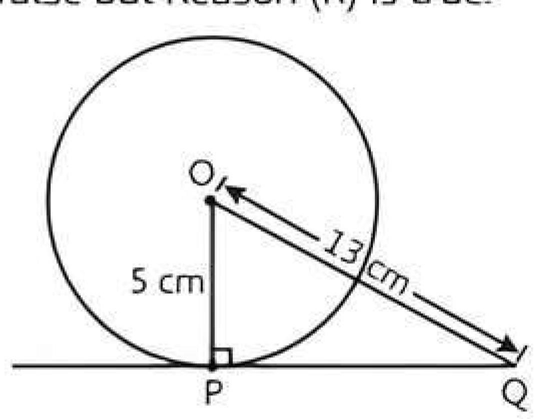
26. contact

27. Given, PQ is a tangent to the circle at point P and radius of circle

$OP = 5$ cm

Also, it is given

$OQ = 13$ cm.



TiP

A line drawn from centre O to the point of contact at point P is perpendicular.

In right-angled Δ OPQ, use Pythagoras theorem,

$$PQ = \sqrt{(OQ)^2 - (OP)^2} = \sqrt{(13)^2 - (5)^2} = \sqrt{169 - 25}$$

$$= \sqrt{144} = 12 \text{ cm}$$

Hence, length of PQ is 12 cm.

28. In right-angled ΔAPO, use Pythagoras theorem

$$AP = \sqrt{(OA)^2 - (OP)^2} = \sqrt{(10)^2 - (8)^2}$$

$$= \sqrt{100 - 64} = \sqrt{36} = 6 \text{ cm}$$

TR!CK

A line drawn from centre of circle to the chord, it bisects the chord.

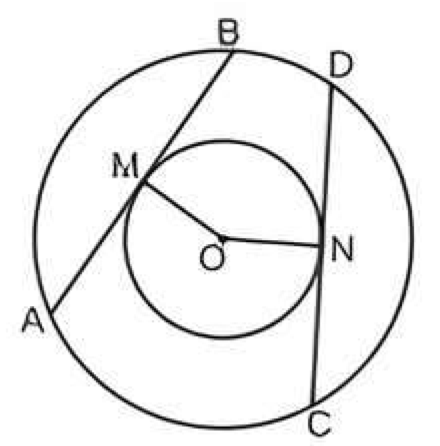
Since, $OP \perp AB$

$\therefore PB = AP$

$\Rightarrow PB = 6$ cm.

29. False, because not more one tangent can be drawn to a circle at a point on the circumference of the circle.

30. Suppose, AB and CD are two chords of larger circle touches the inner circle at M and N.



Here, $OM = ON$ (radii of circle)

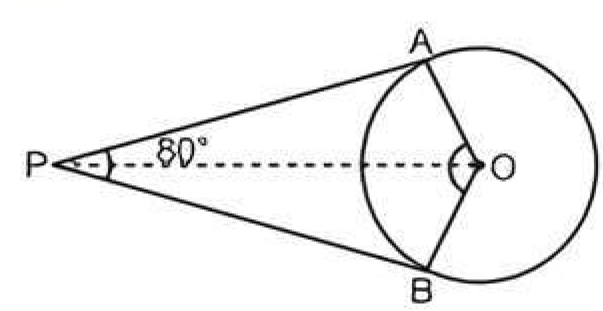
Since, AB and CD are two chords of a bigger circle and are equidistant from the centre.

So, $AB = CD$

Similarly, we can say that all chords of outer circle touch the inner circle are of equal length.

31. True

32. Given, tangents PA and PB are inclined an angle 80° i.e., $\angle APB = 80^\circ$.



As point joining the external point of pair of tangent P to the centre O, it bisects the angle.

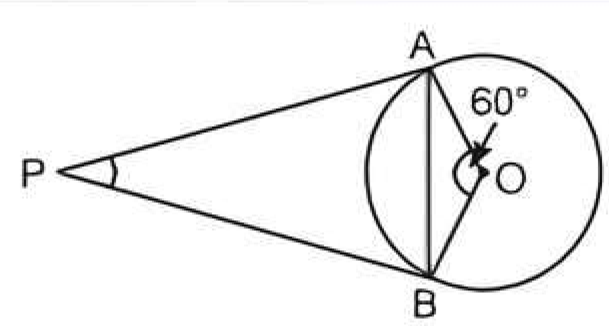
$$\therefore \angle APO = \frac{1}{2} \angle APB = \frac{1}{2} \times 80^\circ = 40^\circ$$

Hence, given statement is false.

33.

TR!CK

Angle subtended by the pair of tangents to the centre of circle is supplementary.



Here, $\angle APB + \angle AOB = 180^\circ$

$\therefore \angle APB + 60^\circ = 180^\circ$

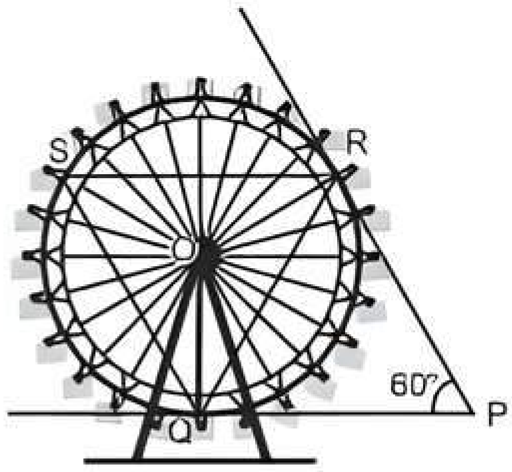
$\Rightarrow \angle APB = 120^\circ$

Hence, given statement is false.

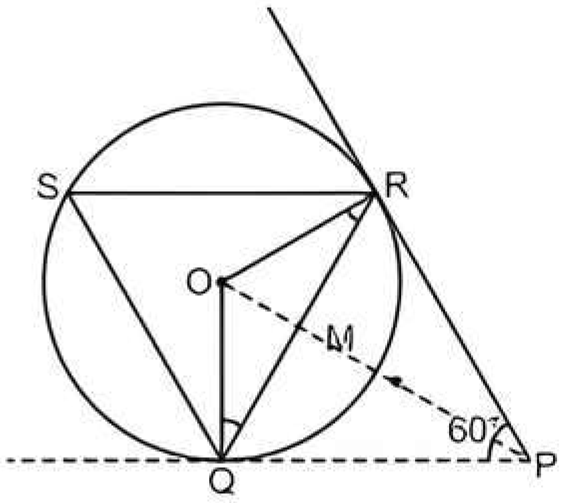
Case Study Based Questions

Case Study 1

A Ferris Wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger–carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.



After taking a ride in Ferris Wheel, Sanjeev came out from the crowd and was observing his friends who were enjoying the ride. He was curious about the different angles and measures that the wheel will form. He forms the figure as given below.



Based on the above information, solve the following questions:

Q 1. In the given figure, find $\angle ROQ$.

a. $60°$ b. $100°$ c. $120°$ d. $90°$

Q 2. Find $\angle ORQ$.

a. $15°$ b. $25°$ c. $30°$ d. $50°$

Q 3. Find $\angle RQP$.

a. $60°$ b. $75°$ c. $30°$ d. $90°$

Q 4. If the length of tangent PR = 4 cm and radius of the circle is 3 cm, then length of OP is:

a. 9 cm b. 16 cm c. 5 cm d. 25 cm

Q 5. If the length of chord RQ = 4 cm, then PM is:

a. 2 cm b. 4 cm c. $2\sqrt{3}$ cm d. $\sqrt{3}$ cm

Solutions

1. It is given that PR and PQ are tangents.

Thus, $OR \perp PR$ and $OQ \perp PQ$.

$\therefore$ $\angle ORP = 90°$ and $\angle OQP = 90°$

In quadrilateral ROQP,

Sum of all interior angles = $360°$

$\angle ORP + \angle OQP + \angle RPQ + \angle ROQ = 360°$

$\Rightarrow$ $90° + 90° + 60° + \angle ROQ = 360°$

$\therefore$ $\angle ROQ = 360° - 240° = 120°$

So, option (c) is correct.

COMMON ERROR

Some students are not familiar with the circle theorem e.g., angle between radius and tangent.

2. In ΔORQ,

$OR = OQ$ (radii)

TR!CK

Angles opposite to equal sides of a triangle are also equal.

$\angle OQR = \angle ORQ$

and $\angle ROQ = 120°$

$\because$ Sum of all interior angles in a triangle = $180°$

$\therefore$ $\angle ORQ + \angle OQR + \angle ROQ = 180°$

$\Rightarrow$ $\angle ORQ + \angle ORQ + 120° = 180°$

$\Rightarrow$ $2\angle ORQ = 180° - 120° = 60°$

$\Rightarrow$ $\angle ORQ = \frac{60°}{2} = 30°$

So, option (c) is correct.

3.

TiP

Equal radii subtends equal angles.

Here, $\angle OQR = \angle ORQ$

$\therefore$ $\angle OQR = 30°$ $(\because \angle ORQ = 30°)$

From part (1):

$\angle OQP = 90°$

$\Rightarrow$ $\angle OQR + \angle RQP = 90°$ $(\because \angle OQP = \angle OQR + \angle RQP)$

$\Rightarrow$ $30° + \angle RQP = 90°$

$\therefore$ $\angle RQP = 90° - 30° = 60°$

So, option (a) is correct.

4.

TiP

Tangent is perpendicular to the radius through the point of contact.

$\because$ RP is a tangent to the circle.

$\therefore$ $OR \perp PR \Rightarrow \angle ORP = 90°$

Given, PR = 4 cm and radius OR = 3 cm.

Now, in right-angled ΔORP,

$OP^2 = OR^2 + PR^2$ (by Pythagoras theorem)

$= (4)^2 + (3)^2 = 16 + 9 = 25$

$\therefore$ $OP = 5$ cm

So, option (c) is correct.

5.

TiP

A perpendicular drawn from the centre to the chord, bisect the chord.

$\therefore$ $PM \perp RQ \Rightarrow \angle RMP = 90°$

$\Rightarrow RM = QM = \frac{RQ}{2} = \frac{4}{2} = 2$ cm $(\because$ given, RQ = 4 cm)

In right-angled ΔRMP,

$RP^2 = RM^2 + PM^2$ (by Pythagoras theorem)

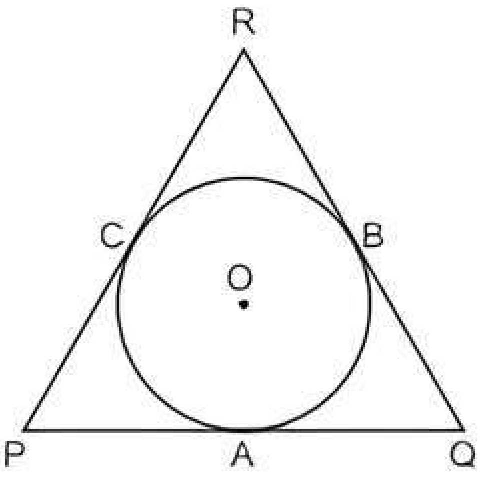
$\Rightarrow$ $PM = \sqrt{RP^2 - RM^2}$

$= \sqrt{(4)^2 - (2)^2} = \sqrt{16-4} = \sqrt{12} = 2\sqrt{3}$ cm.

So, option (c) is correct.

Case Study 2

Priyanshi has been selected by her school to design logo for Sports Day T-shirts for students and staff. The logo design is as given in the figure and she is working on the fonts and different colours according to the theme.



In the given figure, a circle with centre O is inscribed in a ΔPQR, such that it touches the sides PQ, QR and RP at points A, B and C respectively. The lengths of sides PQ, QR and RP are 10 cm, 14 cm and 18 cm respectively.

Based on the above information, solve the following questions:

Q 1. The length of PA is:

a. 7 cm b. 8 cm c. 5 cm d. 9 cm

Q 2. The length of QB is:

a. 8 cm b. 3 cm c. 2 cm d. 9 cm

Q 3. The length of RC is:

a. 9 cm b. 5 cm c. 2 cm d. 11 cm

Q 4. If radius of the circle is 6 cm, find the area of ΔOPQ.

a. 60 sq. cm b. 15 sq. cm

c. 30 sq. cm d. 30 sq. cm

Q 5. The ratio, area of ΔPQR : area of ΔOPQ is:

a. $10:7\sqrt{11}$ b. $7\sqrt{11}:10$

c. $5\sqrt{11}:7$ d. $7:5\sqrt{11}$

Solutions

1.

TiP

The length of two tangents drawn from the same external point of a circle are equal.

Let $RC = RB = x$, $PC = PA = y$ and $QA = QB = z$

$\therefore \quad RP = RC + CP \Rightarrow x + y = 18 \quad ...(1)$

$PQ = PA + QA \Rightarrow y + z = 10 \quad ...(2)$

and $\quad RQ = RB + QB \Rightarrow x + z = 14 \quad ...(3)$

Adding eqs. (1), (2) and (3), we get

$(x + y) + (y + z) + (x + z) = 18 + 10 + 14$

$\Rightarrow \quad 2(x + y + z) = 42$

$\Rightarrow \quad x + y + z = \frac{42}{2} = 21 \quad ...(4)$

Substituting the value of $(x + z)$ in eq. (4), we get

$y + 14 = 21$

$\Rightarrow \quad y = 21 - 14 = 7$

$\therefore$ Length of $PA = y = 7$ cm

So, option (a) is correct.

2. Substituting the value of $(x + y)$ in eq. (4), we get

$18 + z = 21$

$\Rightarrow \quad z = 21 - 18 = 3$

$\therefore$ Length of $QB = z = 3$ cm

So, option (b) is correct.

3. Substituting the value of $(y + z)$ in eq. (4), we get

$x + 10 = 21$

$x = 21 - 10 = 11$

$\therefore$ Length of $RC = x = 11$ cm

So, option (d) is correct.

4. Given that

radius of the circle (OA) = 6 cm

$\therefore$ Height of the triangle OPQ = 6 cm

and length of PQ = 10 cm

TR!CK

Area of the triangle $= \frac{1}{2} \times$ *base* $\times$ *height*

$\therefore \quad \text{Area of } \Delta OPQ = \frac{1}{2} \times PQ \times OA = \frac{1}{2} \times 10 \times 6$

$= 30 \text{ cm}^2$

So, option (c) is correct.

5. Given that in ΔPQR,

PQ = 10 cm, QR = 14 cm and RP = 18 cm

$\therefore$ Semi-perimeter $(s) = \frac{PQ + QR + RP}{2}$

$= \frac{10 + 14 + 18}{2} = \frac{42}{2} = 21$ cm

Now, area of $\Delta PQR = \sqrt{s(s - PQ)(s - QR)(s - RP)}$

(by Heron's formula)

$= \sqrt{21(21 - 10)(21 - 14)(21 - 18)}$

$= \sqrt{21 \times 11 \times 7 \times 3} = 21\sqrt{11}$ sq. cm

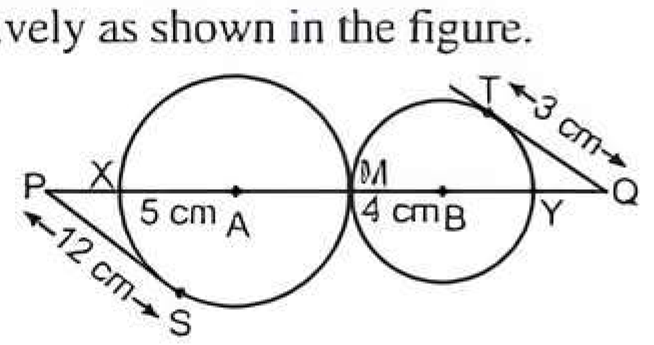
$\therefore$ Required ratio = Area of ΔPQR : Area of ΔOPQ

$= 21\sqrt{11} : 30 = 7\sqrt{11} : 10$

So, option (b) is correct.

Case Study 3

In a math class–IX, the teacher draws two circles that touch each other externally at point M with centres A and B and radii 5 cm and 4 cm respectively as shown in the figure.



Based on the given information, solve the following questions:

Q 1. Find the value of PX.

Q 2. Find the value of QY.

Q 3. Show that $PS^2 = PM \cdot PX$.

Solutions

1. Here, AS = 5 cm and BT = 4 cm (∵ radii of circles)

Since, radius at point of contact is perpendicular to tangent.

∴ $AS \perp PS$

⇒ $\angle ASP = 90°$

In right-angled ΔASP,

$PA^2 = PS^2 + AS^2$ (by Pythagoras theorem)

⇒ $PA^2 = (12)^2 + (5)^2$

⇒ $PA = \sqrt{144 + 25} = \sqrt{169} = 13$ cm

∵ $PX = PA - XA$

∴ $PX = 13 - 5 = 8$ cm (∵ radius, XA = 5 cm)

2. ∵ $BT \perp TQ$

⇒ $\angle BTQ = 90°$

In right-angled ΔBTQ,

$BQ^2 = TQ^2 + BT^2$ (by Pythagoras theorem)

⇒ $BQ^2 = (3)^2 + (4)^2$

⇒ $BQ = \sqrt{9 + 16} = \sqrt{25} = 5$ cm

∵ $QY = BQ - BY$

∴ $QY = 5 - 4 = 1$ cm [∵ radius, BY = 4 cm]

3. In right-angled ΔASP,

$PS^2 = PA^2 - AS^2$

∴ $= PA^2 - AM^2$ [∵ AS = AM (radii)]

$= (PA + AM)(PA - AM)$

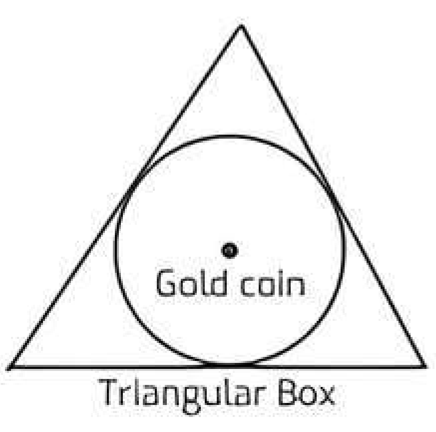
$= (PA + AM)(PA - AX)$

$= PM \cdot PX$ [∵ AM = AX (radii)]

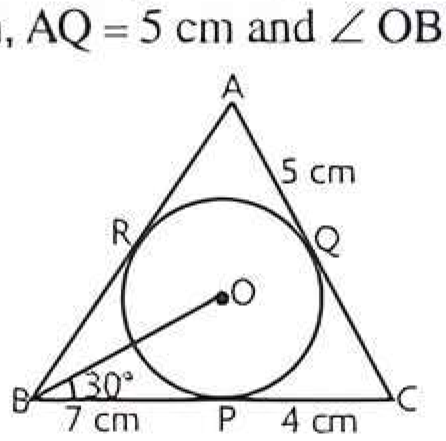
Hence proved.

Case Study 4

Arun recently bought a gold coin from a Jewellery shop. To protect it, he placed the gold coin in a triangular box. The edge of the triangle touch the gold coin.



In mathematical form, the given statement is defined with the given figure such that BP = 7 cm, CP = 4 cm, AQ = 5 cm and $\angle OBP = 30°$.



Based on the given information, solve the following questions:

Q 1. Find the length of tangent line AB.

Q 2. Find the radius of the gold coin.

Q 3. Find the perimeter of a ΔABC.

Solutions

1. Given, O is the centre of circle, BP = 7 cm, CP = 4 cm, AQ = 5 cm and $\angle OBP = 30°$.

TR!CK

The length of two tangents drawn from the same external point to the circle are equal.

Here, BR = BP = 7 cm

and AR = AQ = 5 cm

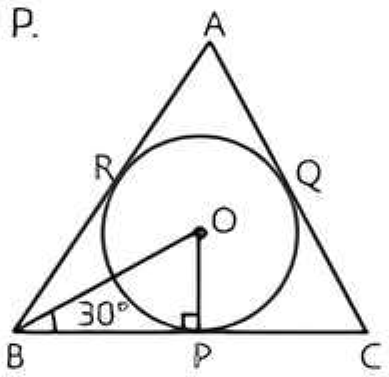
Now, AB = AR + BR

= 5 + 7 = 12 cm

Hence, length of tangent line AB is 12 cm.

2. In given figure, join points O and P.

In right-angled ΔOPB,



$\tan 30° = \frac{OP}{BP}$

⇒ $\frac{1}{\sqrt{3}} = \frac{OP}{7}$

⇒ $OP = \frac{7}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{7}{3}\sqrt{3}$ cm

Hence, radius of the gold coin is $\frac{7}{3}\sqrt{3}$ cm.

3.

TiP

Tangents are drawn from an external point to the circle are equal in lengths.

∴ QC = CP = 4 cm.

AR = AQ = 5 cm.

BR = BP = 7 cm

Now, length of different tangents in the given figure are

BC = BP + PC = 7 + 4 = 11 cm

AC = AQ + QC = 5 + 4 = 9 cm

From part 1, AB = 12 cm

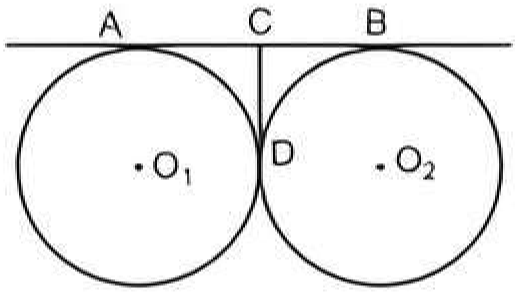
∴ The perimeter of ΔABC = AB + BC + AC

= 12 + 11 + 9 = 32 cm.

Case Study 5

In a bike, both tyres touches the ground. And when those points where they touch the ground are joined, we get a straight line. The straight line formed can be considered a common tangent for both the circles.

Mathematically, in the figure, AB and CD are common tangents of two equal circles which touch each other at D and AB = 8 cm and CD = 4 cm.



Based on the given information, solve the following questions:

Q 1. In the given figure, find the length AC.

Q 2. Find the area of the quadrilateral O_1 ACD.

Q 3. Find the number of common tangents in the given figure.

Solutions

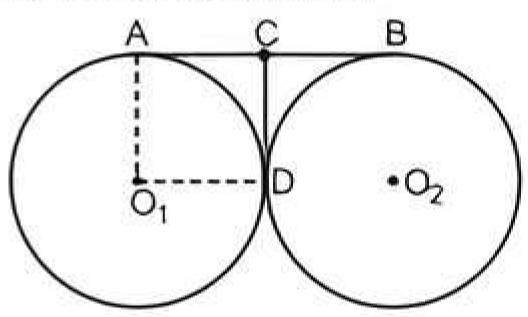
1.

TiP

As we know that tangents drawn from an external point to a circle are always equal in lengths.

$\therefore \quad AC = CD = 4$ cm.

2. Given both circles are of same radius. So, line DC is perpendicular to the line AB.



Also, tangent is perpendicular to the radius through the point of contact of circle.

$\therefore \quad \angle O_1AC = \angle O_1DC = 90^\circ$

Since, both circles are equal.

So, the line joining centres O_1 and O_2 are parallel to the common tangent AB.

$\therefore \quad \angle O_1AC = \angle AO_1D = 180^\circ$

$\Rightarrow \quad \angle AO_1D = 180^\circ - 90^\circ = 90^\circ$

In quadrilateral O_1DCA,

$\angle AO_1D + \angle O_1DC + \angle DCA + \angle CAO_1 = 360^\circ$

$\Rightarrow \quad 90^\circ + 90^\circ + 90^\circ + \angle DCA = 360^\circ$

$\therefore \quad \angle DCA = 360^\circ - 270^\circ = 90^\circ$

So, in quadrilateral

$\angle AO_1D = \angle O_1DC = \angle DCA = \angle CAO_1 = 90^\circ$

and $\quad O_1A = O_1D \quad$ (radii of circle)

Therefore quadrilateral O_1ACD formed a square.

$\therefore$ Area of quadrilateral O_1ACD = Area of square

$= (\text{side})^2 = (CD)^2 = (4)^2 = 16 \text{ cm}^2$

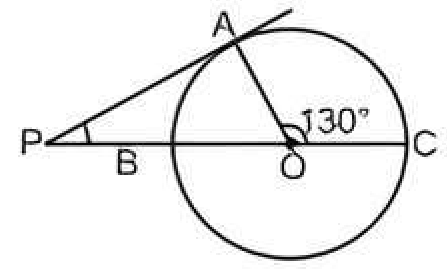
Hence, area of quadrilateral O_1ACD is 16 cm^2.

3. In the given figure, there are two common tangents i.e., AB and CD.

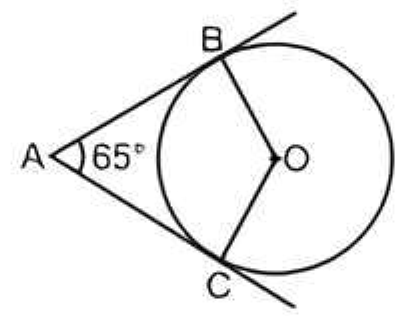
Very Short Answer Type Questions

Q 1. If the angle between two tangents drawn from an external point P to a circle of radius *a* and centre O, is 60°, then find the length of OP. *[CBSE 2017]*

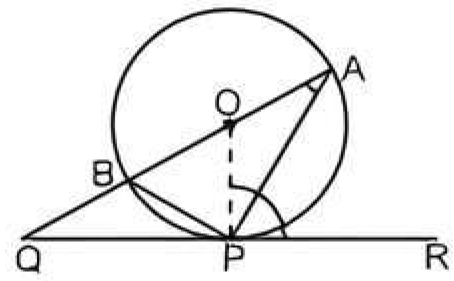
Q 2. In the given figure, PA is a tangent to the circle drawn from the external point P and PBC is the secant to the circle with BC as diameter. If $\angle AOC = 130^\circ$, then find the measure of $\angle APB$, where O is the centre of the circle. *[CBSE 2023]*



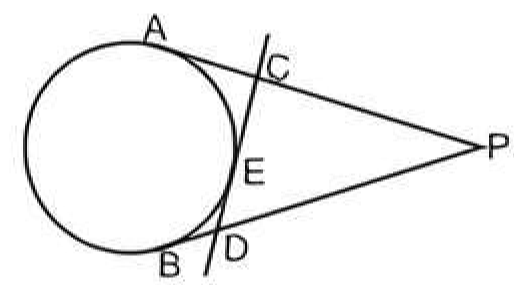
Q 3. In the given figure, O is the centre of the circle. AB and AC are tangents drawn to the circle from point A. If $\angle BAC = 65^\circ$, then find the measure of $\angle BOC$. *[CBSE 2023]*



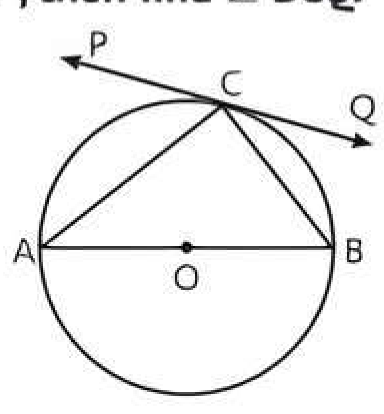
Q 4. In the given figure, O is the centre of the circle and QPR is a tangent to it at P. Prove that $\angle QAP + \angle APR = 90^\circ$. *[CBSE 2023]*



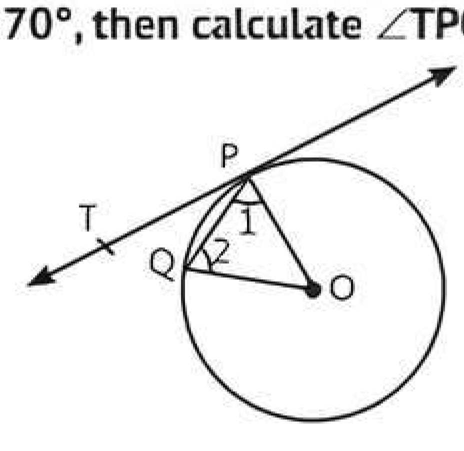
Q 5. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle, a tangent is drawn to intersect PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of ΔPCD. *[CBSE SQP 2023-24]*



Q 6. In the given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle PCA = 30^\circ$, then find $\angle BCQ$.



Q 7. In the given figure, O is the centre of a circle, PQ is a chord and PT is the tangent at P. If $\angle POQ = 70^\circ$, then calculate $\angle TPQ$. *[V. Imp.]*



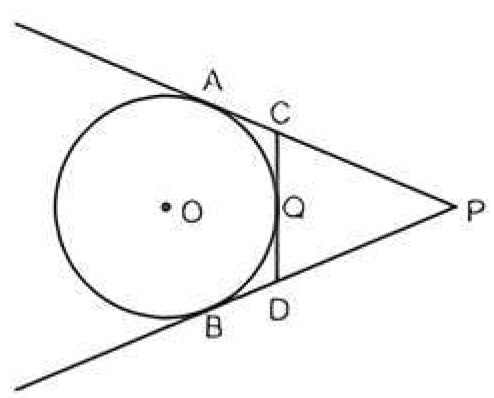
Q 8. From an external point P, tangents PA and PB are drawn to a circle with centre O. $\angle PAB = 50°$, then find $\angle AOB$. *[CBSE 2016]*

Short Answer Type-I Questions

Q 1. Prove that tangents drawn at the ends of a diameter of a circle are parallel to each other.

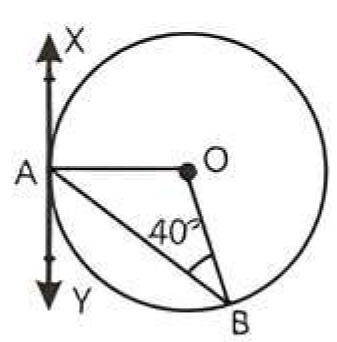
[NCERT EXERCISE; CBSE 2019, 17]

Q 2. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + PD. *[CBSE 2017]*

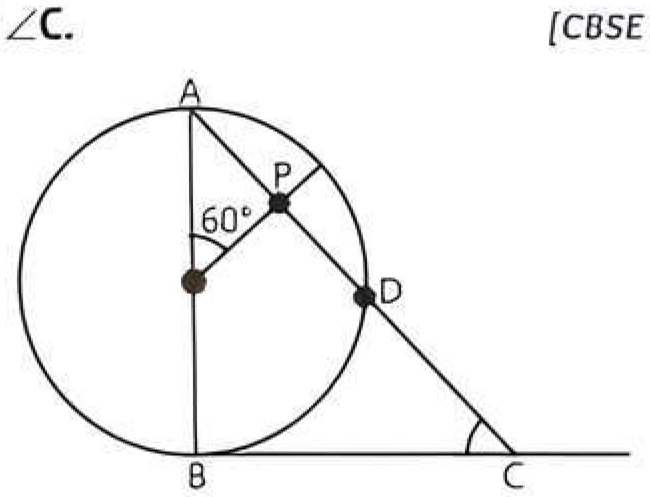


Q 3. In the given figure, XAY is a tangent to the circle centered at O. If $\angle ABO = 40°$, then find $\angle BAY$ and $\angle AOB$.

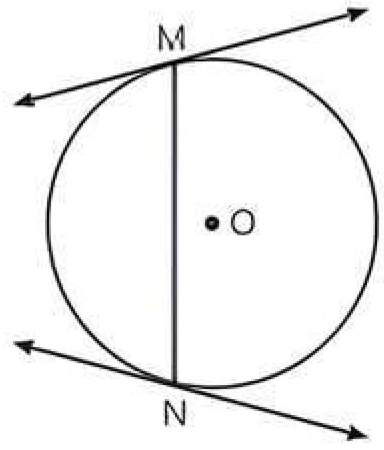
[CBSE 2022, Term-II]



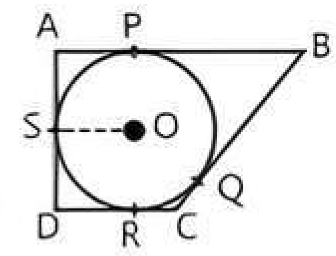
Q 4. In the given figure, AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60°$, then find $\angle C$. *[CBSE 2022 Term-II]*



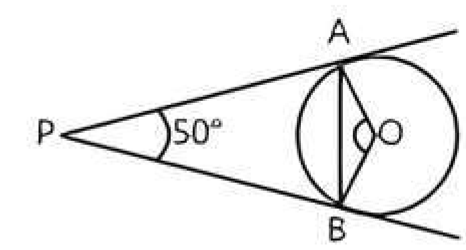
Q 5. Prove that tangents drawn at the ends of a chord of a circle make equal angles with the chord.



Q 6. In the adjoining figure, a circle is inscribed in a quadrilateral ABCD touching its sides AB, BC, CD and AD at P, Q, R and S respectively. If the radius of the circle is 10 cm, BC = 38 cm, PB = 27 cm and AD ⊥ CD, then calculate the length of CD.

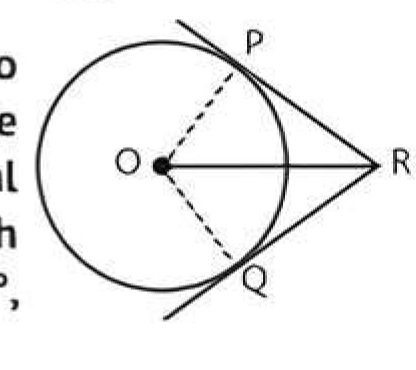


Q 7. In the given figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$. Write the measure of $\angle OAB$. *[CBSE 2015]*



Q 8. In the adjoining figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120°$, then prove that

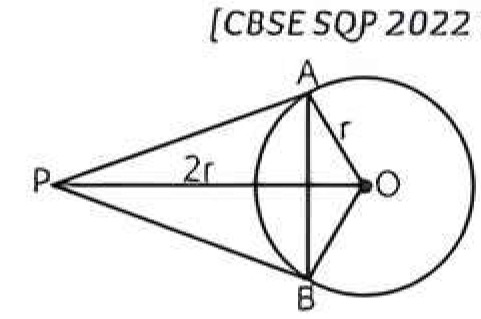
OR = PR + RQ *[CBSE 2015]*



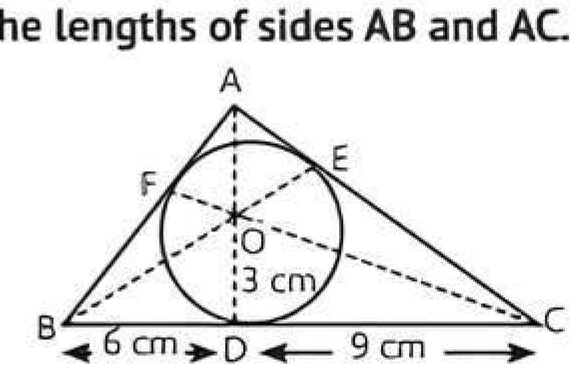
Q 9. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. *[CBSE 2022 Term-II, CBSE 2023]*

Q 10. From an external point, two tangents are drawn to a circle. Prove that the line joining the external point to the centre of the circle bisects the angle between the two tangents. *[CBSE 2023]*

Q 11. From a point P, two tangents PA and PB are drawn to a circle C (0, r). If OP = $2r$, then find $\angle APB$ and $\angle OAB$. What type of triangle is APB? *[CBSE SQP 2022 Term-II, CBSE 2016]*



Q 12. In the given figure, a $\triangle ABC$ is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of $\triangle ABC$ is 54 cm^2, then find the lengths of sides AB and AC. *[CBSE 2015]*

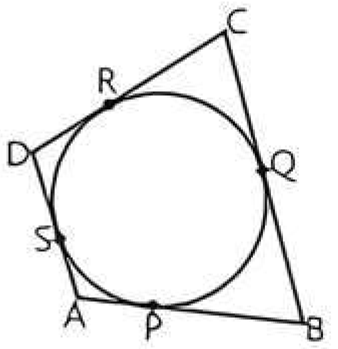


Short Answer Type-II Questions

Q 1. A quadrilateral ABCD is drawn to circumscribe a circle (see given figure). Prove that

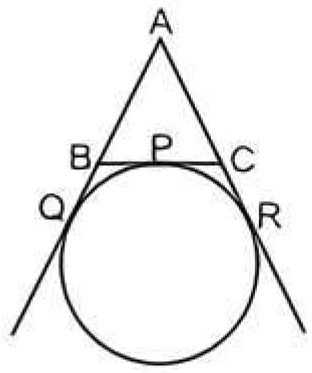
AB + CD = AD + BC.

[NCERT EXERCISE; CBSE 2017,16]

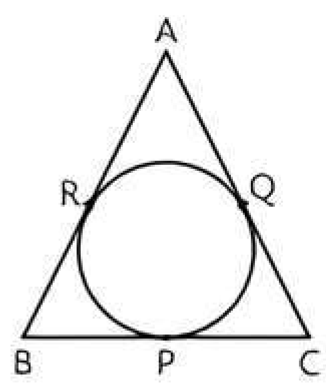


Q 2. A circle touches the side BC of a $\triangle ABC$ at a point P and touches AB and AC when produced at Q and R respectively. Show that $AQ = \frac{1}{2}$ (Perimeter of $\triangle ABC) = \frac{1}{2}$ (BC + CA + AB).

[NCERT EXEMPLAR; CBSE 2023, 20]



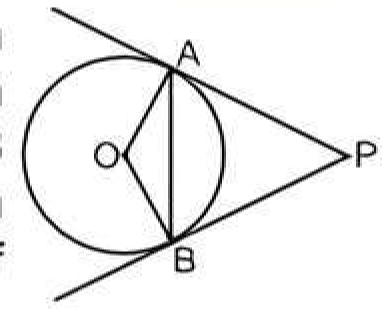
Q 3. In the given figure, the incircle of $\triangle ABC$ touches the sides BC, CA and AB at P, Q and R respectively. Prove that $(AR + BP + CQ) = (AQ + BR + CP) = \frac{1}{2}$ (Perimeter of $\triangle ABC$).



Q 4. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

[CBSE 2023]

Q 5. PA and PB are tangents drawn to a circle of centre O from an external Point P. Chord AB makes an angle of 30° with the radius at the point of contact.

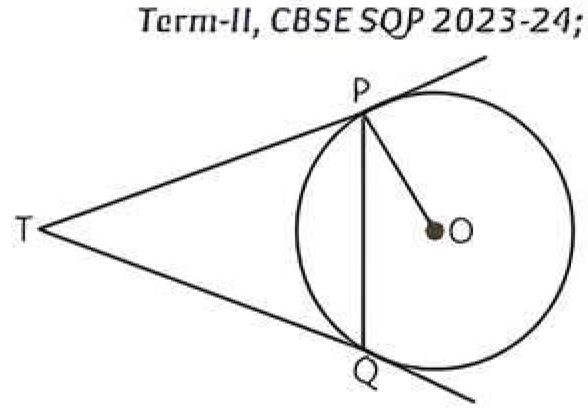


If length of the chord is 6 cm, find the length of the tangent PA and the length of the radius OA.

[CBSE SQP 2023-24]

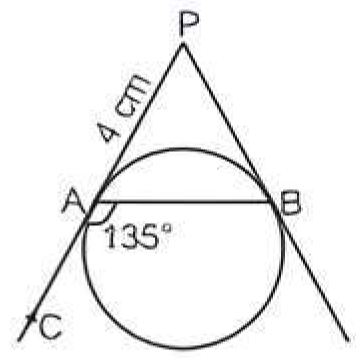
Q 6. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

[NCERT EXERCISE; CBSE SQP 2022 Term-II, CBSE SQP 2023-24; CBSE 2023, 17]



Q 7. In the given figure, PA and PB are tangents to a circle from an external point P such that PA = 4 cm and $\angle BAC = 135°$. Find the length of chord AB.

[CBSE 2017]

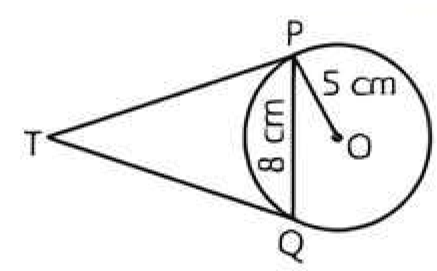


Q 8. ABC is a right triangle in which $\angle B = 90°$. If AB = 8 cm and BC = 6 cm, find the diameter of the circle inscribed in the triangle.

[NCERT EXERCISE; CBSE 2019, V. Imp]

Q 9. In the given figure, PQ is a chord of length 8 cm of a circle with radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.

[NCERT EXERCISE; CBSE 2019]

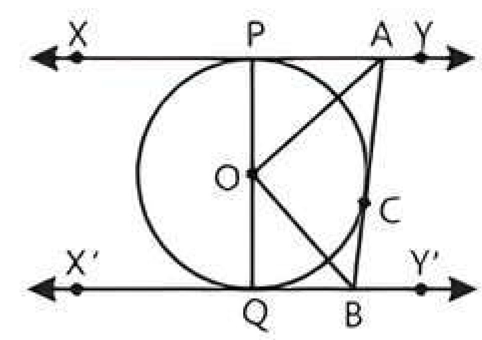


Q 10. In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B.

Prove that $\angle AOB = 90°$. *[NCERT EXERCISE; CBSE 2017]*

Or

What is the measure of $\angle AOB$? *[CBSE SQP 2022-23]*



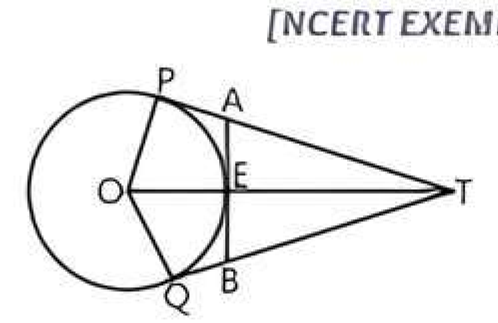
Long Answer Type Questions

Q 1. Prove that the parallelogram circumscribing a circle is a rhombus.

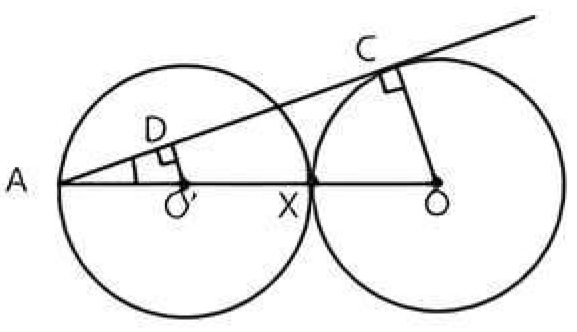
[NCERT EXERCISE; CBSE 2022 Term-II, CBSE SQP 2022-23]

Q 2. In the given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.

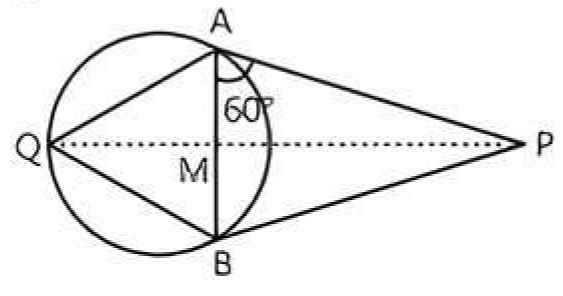
[NCERT EXEMPLAR; CBSE 2016]



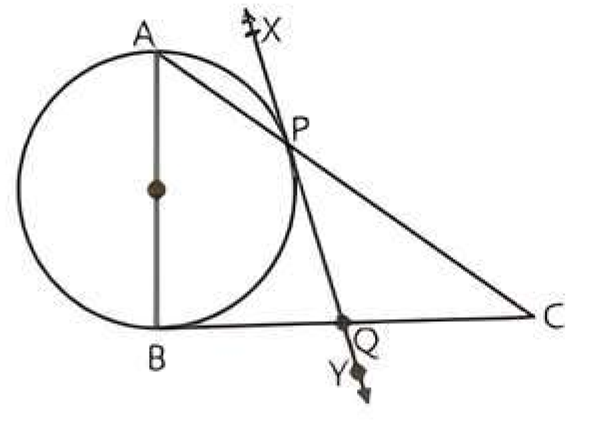
Q 3. In the adjoining figure, two equal circles with centres O and O′, touch each other at X, produce OO′ to meet the circle with centre O′ at A. AC is tangent to the circle with centre O, at the point C. O′D is perpendicular to AC. Find the ratio of $\frac{DO'}{CO}$. *[CBSE 2016]*



Q 4. PA and PB are the tangents to a circle which circumscribes an equilateral $\triangle$ABQ. If $\angle PAB = 60°$, as shown in the figure, prove that QP bisects AB at right angle. *[CBSE 2015]*



Q 5. In the given figure, a triangle ABC with $\angle B = 90°$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point P. Prove that the tangent drawn at point P bisects BC. *[CBSE 2022 Term-II]*



Q 6. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. *[NCERT EXERCISE; CBSE 2017]*

Solutions

Very Short Answer Type Questions

1. Two tangents *PA* and *PB* are drawn from an external point *P* to the circle.

Given, $\angle APB = 60°$

TR!CK

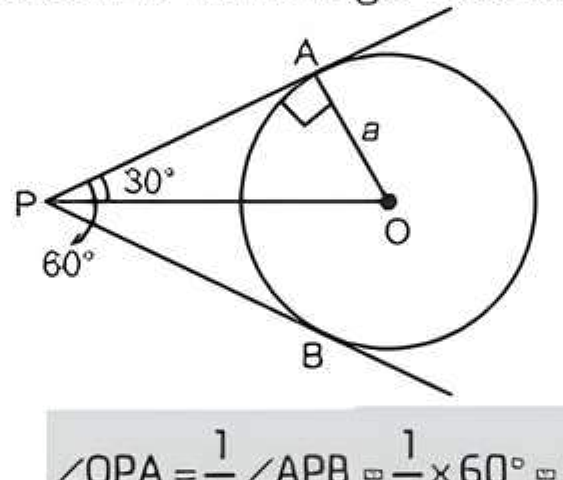
If two tangents are drawn from an external point to a circle, then the line joining that external point to the centre of circle bisect the angle between the tangents.

Join OA and OP.

$\because$ PA is a tangent, therefore

$OA \perp PA$

We know that OP is the angle bisector of $\angle APB$.



$\therefore \quad \angle OPA = \frac{1}{2}\angle APB = \frac{1}{2} \times 60° = 30°$

In right-angled ΔOAP,

$\sin 30° = \frac{OA}{OP} \Rightarrow \frac{1}{2} = \frac{a}{OP}$

$\therefore \quad OP = 2a$

2. Given, PA is a tangent to the circle drawn from the external point P and PBC is the secant to the circle with BC as diameter.

Also, $\angle AOC = 130°$.

TiP

Tangent at any point of a circle is perpendicular to the radius through the point of contact.

In ΔPOA, $\angle PAO = 90°$

TR!CK

Exterior angle of a triangle = Sum of interior opposite angles.

Now, $\angle APO + \angle PAO = \angle AOC$

$\therefore \quad \angle APO = 130° - 90° = 40°$

So, $\angle APB = \angle APO = 40°$

3. Since, tangent is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OBA = \angle OCA = 90°$

Now, $\angle OBA + \angle BAC + \angle OCA + \angle BOC = 360°$

(angle sum property of quadrilateral)

$\Rightarrow \quad 90° + 65° + 90° + \angle BOC = 360°$

$\Rightarrow \quad \angle BOC = 360° - 245° = 115°$

COMMON ERR(!)R

Some students are not known with the circle properties, i.e., they could not well identify $\angle OBA = \angle OCA = 90°$ (angle between radius and tangents)

4.

TiP

If a chord is drawn through a point of contact of a tangent to the circle then the angle formed by this chord from the tangent are equal to the angles of corresponding alternate segments.

Here, $\angle BPQ = \angle QAP$ ...(1)

(by alternate segment theorem)

Since, AOB is a diameter of the circle.

$\therefore \quad \angle APB = 90°$ ...(2)

(angle of semicircle)

From figure, QPR is a tangent *i.e.*, a straight line.

$\therefore \quad \angle BPQ + \angle APB + \angle APR = 180°$

$\Rightarrow \quad \angle BPQ + 90° + \angle APR = 180°$ [from eq. (1)]

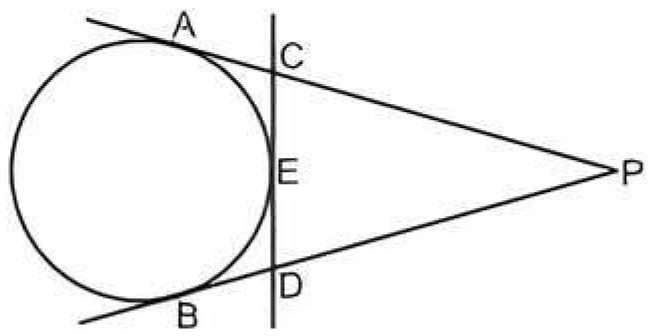
$\Rightarrow \quad \angle QAP + \angle APR = 180° - 90° = 90°$

[from eq. (2)] **Hence proved.**

5. Since BC touches the circle at R.

TiP

Perimeter of any triangle is equal to sum of all its three sides.



$\therefore$ Perimeter of $\Delta PCD = PC + CD + PD$

$= PC + (CE + ED) + PD$

TR!CK

Tangents are drawn from an external point to a circle are equal in lengths.

$= (PC + CE) + (ED + PD)$

$= (PC + CA) + (DB + PD)$

($\because$ CE = CA and ED = DB)

$= PA + PB$ ($\because$ PA = PC + CA and PB = PD + DB)

$= PA + PA$ ($\because$ AP = BP)

$= 2PA = 2 \times 10$ ($\because$ PA = 10 cm, given)

$= 20$ cm

6. Given, $\angle PCA = 30^\circ$

TR!CK

Diameter of a circle subtends right angled to the circumference of a circle.

Here, AB is a diameter of a circle.

Therefore $\angle ACB = 90^\circ$.

Since PQ is a straight line. Therefore sum of all angles of one side is equal to 180°.

i.e., $\angle PCA + \angle ACB + \angle BCQ = 180^\circ$

$\Rightarrow \quad 30^\circ + 90^\circ + \angle BCQ = 180^\circ$

$\Rightarrow \quad \angle BCQ = 60^\circ$

7. In ΔOPQ,

$\angle 1 + \angle 2 + \angle POQ = 180^\circ$

(angle sum property of triangle)

$\Rightarrow \quad \angle 1 + \angle 2 + 70^\circ = 180^\circ$

$\Rightarrow \quad \angle 1 + \angle 2 = 110^\circ$

TiP

Angles opposite to equal sides of a triangle are also equal.

$\Rightarrow \quad \angle 1 + \angle 1 = 110^\circ$ ($\because$ OP = OQ, $\therefore \angle 2 = \angle 1$)

$\Rightarrow \quad 2\angle 1 = 110^\circ$

$\therefore \quad \angle 1 = 55^\circ$

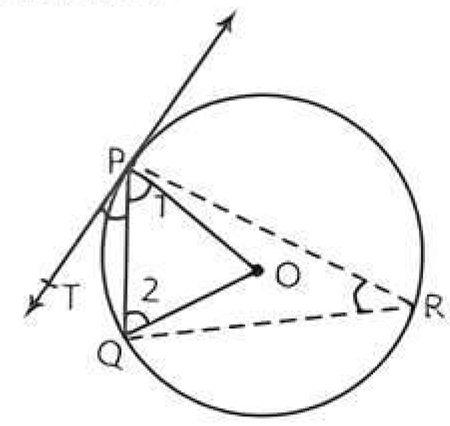
TiP

Tangent is perpendicular to the radius through the point of contact of circle.

Here, $\angle OPT = \angle 1 + \angle TPQ = 90^\circ$ ($\because OP \perp TP$)

$\Rightarrow \quad \angle TPQ = 90^\circ - 55^\circ = 35^\circ$

Alternate Method:



$\angle POQ = 2\angle PRQ$

(angle subtended by an arc at the centre is twice the angle subtended by it at any point on the remaining part of the circle)

$\Rightarrow \quad \frac{70^\circ}{2} = \angle PRQ$

$\therefore \quad \angle PRQ = 35^\circ$

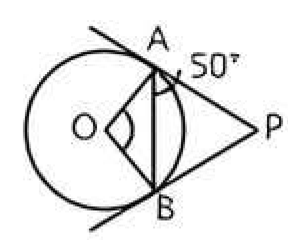
$\angle TPQ = \angle PRQ$

$= 35^\circ$ (alternate segment theorem)

8. Since, tangents drawn from external point are equal

$\therefore \quad PA = PB$

$\therefore \quad \angle PBA = \angle PAB = 50^\circ$ (angles opposite to equal sides are equal)



In ΔAPB,

$\angle APB + \angle PBA + \angle PAB = 180^\circ$

(by angle sum property of a triangle)

$\Rightarrow \quad \angle APB = 180^\circ - 50^\circ - 50^\circ = 80^\circ$

In cyclic quadrilateral OAPB,

$\angle AOB + \angle APB = 180^\circ$ ($\because$ sum of opposite angles of a cyclic quadrilateral is 180°)

$\Rightarrow \angle AOB + 80^\circ = 180^\circ$

$\therefore \quad \angle AOB = 180^\circ - 80^\circ = 100^\circ$

COMMON ERR(!)R

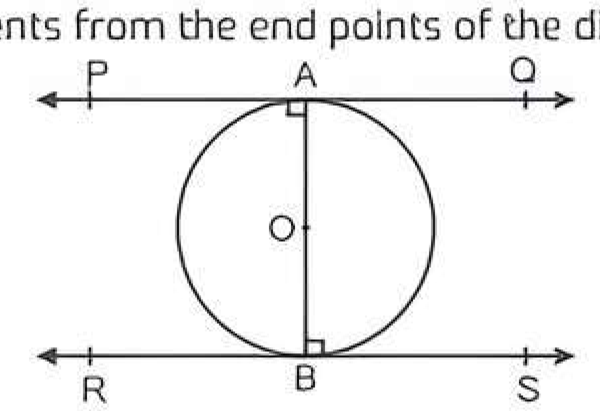
Some students could not apply the appropriate theorem of circle to find out the unknown angles.

Short Answer Type-I Questions

1. Given: AB is the diameter of a circle with centre O. Tangents are PAQ and RBS from the end points of the diameter on the circle.

To Prove: PQ || RS

Proof: $\because$ AB is the diameter and PAQ and RBS are tangents from the end points of the diameter.



TiP

Tangent is perpendicular to the radius through the point of contact of circle.

Here, $\angle PAB = 90^\circ$ and $\angle ABS = 90^\circ$

But $\angle PAB$ and $\angle ABS$ are the same alternate angle made by cutting the transverse line AB to lines PQ and RS.

So, PQ || RS **Hence proved.**

2.

TiP

Tangents drawn from an external point to a circle are equal in lengths.

Here, $PA = PB = 12$ cm ...(1)

$QC = AC = 3$ cm ...(2)

and $QD = BD = 3$ cm ...(3)

($\because$ PA = 12 cm, QC = QD = 3 cm)

Now, $PC + PD = (PA - AC) + (PB - BD)$ (from figure)

$= (12 - 3) + (12 - 3)$

(from eqs. (1), (2) and (3))

$= 9 + 9 = 18$ cm

3. In the given figure,

$OB = OA$ (radii of a circle)

$\Rightarrow$ $\angle BAO = \angle ABO$ (angles opposite to equal sides of a triangle are equal)

$\Rightarrow$ $\angle BAO = 40^\circ$

Since tangent is perpendicular to the radius through the point of contact of circle.

i.e., $OA \perp XY$

$\therefore$ $\angle OAY = 90^\circ$

$\Rightarrow$ $\angle OAB + \angle BAY = 90^\circ$

$\Rightarrow$ $40^\circ + \angle BAY = 90^\circ$

$\therefore$ $\angle BAY = 90^\circ - 40^\circ = 50^\circ$

In ΔOAB, use angle sum property of a triangle.

$\angle AOB + \angle BAO + \angle ABO = 180^\circ$

$\Rightarrow$ $\angle AOB + 40^\circ + 40^\circ = 180^\circ$

$\Rightarrow$ $\angle AOB = 100^\circ$

4. Given, OP bisect AD.

Since, the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

$\therefore$ $OP \perp AD$

i.e., $\angle OPA = 90^\circ$

In ΔOPA,

$\angle AOP + \angle OPA + \angle OAP = 180^\circ$

(by angle sum property of triangle)

$\Rightarrow$ $60^\circ + 90^\circ + \angle OAP = 180^\circ$

$\Rightarrow$ $\angle OAP = 30^\circ$

$\Rightarrow$ $\angle BAC = \angle OAP = 30^\circ$

Again in ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

(by angle sum property of triangle)

$\Rightarrow$ $30^\circ + 90^\circ + \angle ACB = 180^\circ$

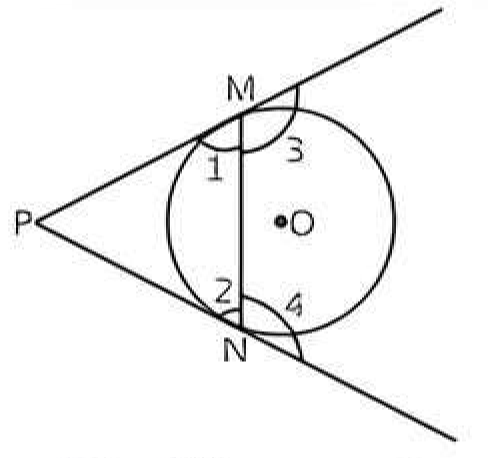
($\because$ radius is perpendicular to the tangent $\therefore \angle OBC = \angle ABC = 90^\circ$)

$\Rightarrow$ $\angle ACB = 60^\circ$

5. Let tangents at points M and N extends it and intersect these tangents at point P.

TiP

Tangents drawn from an external point to the circle are equal in lengths.



$PM = PN$ $\Rightarrow$ $\angle 2 = \angle 1$

($\because$ angles opposite to equal sides of a triangle are equal)

$\Rightarrow$ $180^\circ - \angle 4 = 180^\circ - \angle 3$ (by linear pair)

$\Rightarrow$ $\angle 3 = \angle 4$

Hence, tangents drawn at the ends of a chord of a circle make equal angles, with the chord.

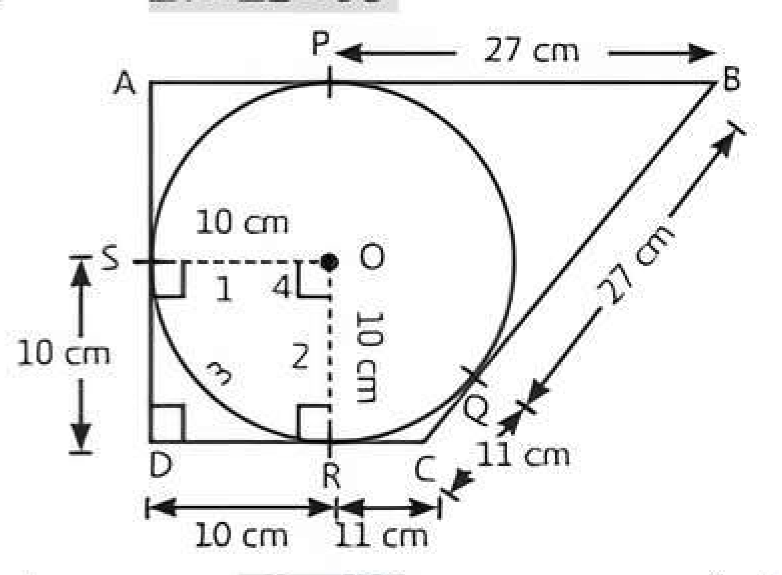
Hence proved.

6. Join OR.

TiP

Tangent is perpendicular to the radius through the point of contact of circle.

$\therefore$ $\angle 1 = \angle 2 = 90^\circ$



and $\angle 3 = 90^\circ$ ($\because AD \perp CD$)

It implies that $\angle 4 = 90^\circ$

$\therefore$ ORDS is a square.

$\Rightarrow$ $OR = DR = SD = OS$

$\because$ Tangents drawn from an external point are equal.

$\therefore$ $DR = DS = 10$ cm ($\because OS = RD$) ...(1)

and $BP = BQ = 27$ cm

Now, $CQ = BC - BQ = 38 - 27 = 11$ cm

$RC = CQ = 11$ cm ...(2)

($\because$ tangents drawn from an external point are equal)

So, $DC = DR + RC = 10 + 11 = 21$ cm

[from eqs. (1) and (2)]

7. Since, tangents drawn from an external point to a circle are equal in lengths.

$\therefore$ $PA = PB$

TiP

Tangent is perpendicular to the radius through the point of contact of circle.

Here, $\angle OAP = \angle OBP = 90^\circ$

$$OB = OA \qquad \text{(radii)}$$

$$\angle OAB = \angle OBA \qquad ...(1)$$

(angles opposite to equal sides of a triangle are equal)

By using angle sum property of quadrilateral, OAPB,

$$\angle OAP + \angle AOB + \angle OBP + \angle APB = 360^{\circ}$$

$$\Rightarrow \quad 90^{\circ} + \angle AOB + 90^{\circ} + 50^{\circ} = 360^{\circ}$$

$$\Rightarrow \quad \angle AOB = 360^{\circ} - 230^{\circ} = 130^{\circ} \quad ...(2)$$

In ΔOAB,

$$\angle AOB + \angle OAB + \angle OBA = 180^{\circ}$$

(angle sum property of triangle)

$$\Rightarrow \quad 130^{\circ} + \angle OAB + \angle OAB = 180^{\circ} \qquad \text{(from eq. (1))}$$

$$\Rightarrow \quad 2\angle OAB = 180^{\circ} - 130^{\circ} = 50^{\circ}$$

$$\therefore \quad \angle OAB = 25^{\circ}$$

COMMON ERR(!)R

Some students could not apply the appropriate theorem of circle to find out the unknown angles.

8. **Given:** $\angle PRQ = 120^{\circ}$

To Prove: OR = PR + RQ

Construction: Join OP and OQ.

TiP

Tangent is perpendicular to the radius through the point of contact of circle.

Proof: In ΔOPR and ΔOQR

$$\angle OPR = \angle OQR = 90^{\circ} \quad \text{and}$$

PR = RQ (tangents drawn from an external point to the circle are equal)

$$\because \quad \angle PRQ = 120^{\circ} \qquad \text{(given)}$$

TR!CK

If two tangents are drawn from an external point to a circle, then the line joining that external point to the centre of circle makes equal angles from two tangents.

$$\therefore \quad \angle PRO = \frac{1}{2}\angle PRQ = \frac{1}{2} \times 120^{\circ} = 60^{\circ}$$

Now, In ΔOPR,

$$\angle OPR + \angle POR + \angle PRO = 180^{\circ}$$

(angle sum property of triangle)

$$\Rightarrow \quad 90^{\circ} + \angle POR + 60^{\circ} = 180^{\circ}$$

$$\Rightarrow \quad \angle POR = 180^{\circ} - 150^{\circ} = 30^{\circ}$$

Now, In right-angled ΔOPR,

$$\sin 30^{\circ} = \frac{PR}{OR} = \frac{1}{2} \quad (\because \sin \angle POR = \sin 30^{\circ})$$

$$\Rightarrow \quad OR = 2PR = PR + PR$$

$$\therefore \quad PR = QR$$

$$\Rightarrow \quad OR = PR + QR$$

($\because$ tangents drawn from an external point of a circle are equal in length)

COMMON ERR(!)R

Some students are not familiar with the circle theorem e.g., could not well identify $\angle OTP = 90^{\circ}$ (angle between radius and tangent).

Hence proved.

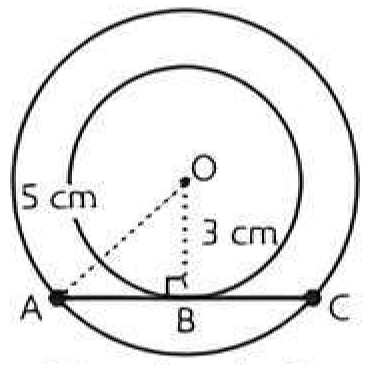
9. Given, radius of bigger circle OA = 5 cm and radius of smaller circle OB = 3 cm.

In right angled ΔOBA,

$$OA^2 = OB^2 + AB^2$$

$$(5)^2 = (3)^2 + AB^2$$

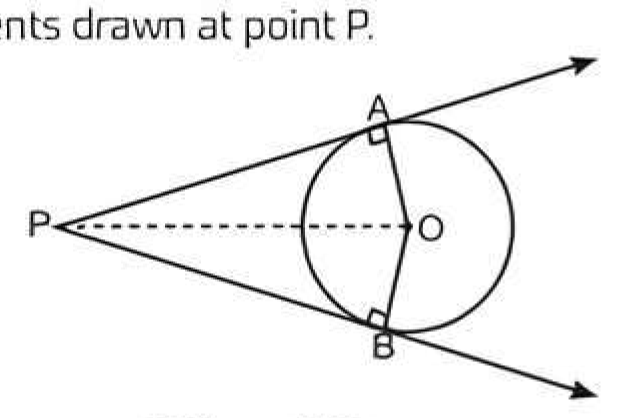
$$\Rightarrow \quad AB = \sqrt{25-9} = \sqrt{16} = 4$$



$\therefore$ The length of the chord of larger circle,

$$AC = 2\,AB = 2 \times 4 = 8 \text{ cm}.$$

10. **Given:** O is the centre of a circle and P is a point at a distance OP from the centre. PA and PB are two tangents drawn at point P.



To Prove: $\angle OPA = \angle OPB$

Construction: Join OA, OP and OB.

Proof: In ΔOAP and ΔOBP, we see that

$$OA = OB \qquad \text{(radii of a circle)}$$

$$OP = OP \qquad \text{(common arm)}$$

PA = PB (tangents drawn from an external point to a circle are of equal length)

$$\therefore \quad \Delta OAP \cong \Delta OPB \qquad \text{(from SSS congruency)}$$

$$\Rightarrow \quad \angle OPA = \angle OPB \qquad \text{(by CPCT)}$$

Which shows the line joining the external point to the centre of the circle bisects the angle between the two tangents. **Hence proved.**

11. Let $\angle APO = \theta$ and $\angle OAP = 90^{\circ}$

($\because$ tangent is perpendicular to radius through the point of contact)

In right angled triangle OAP,

$$\sin\theta = \frac{OA}{OP} = \frac{r}{2r} = \frac{1}{2}$$

$$\Rightarrow \quad \sin\theta = \sin 30^{\circ}$$

$$\Rightarrow \quad \theta = 30^{\circ}$$

Also, $\angle APB = 2\,\angle APO$ ($\because$ a line joining the centre of the circle to the intersection point of tangents, bisect the angle between the tangents)

$$= 2 \times 30^{\circ} = 60^{\circ}$$

In ΔOAP,

$$\angle OAP + \angle APO + \angle AOP = 180^{\circ}$$

(by angle sum property of triangle)

$$\Rightarrow \quad 90^{\circ} + 30^{\circ} + \angle AOP = 180^{\circ}$$

$$\Rightarrow \quad \angle AOP = 180^{\circ} - 120^{\circ} = 60^{\circ}$$

Also, $\angle AOB = 2 \times \angle AOP = 2 \times 60^{\circ} = 120^{\circ}$

Now, In ΔAOB

$$OA = OB \qquad \text{(radii of circle)}$$

$\therefore$ $\angle OAB = \angle OBA$

($\because$ angle opposite to equal sides of a triangle are also equal)

$\Rightarrow \angle OAB + \angle OBA + \angle AOB = 180°$

($\because$ angle sum property of a triangle)

$\Rightarrow \angle OAB + \angle OAB + 120° = 180°$

$\Rightarrow 2\angle OAB = 60° \Rightarrow \angle OAB = 30°$

In ΔAPB,

$PA = PB$ ($\because$ tangents drawn from external point to a circle are equal)

$\Rightarrow \angle PBA = \angle PAB$

Also, $\angle APB + \angle PAB + \angle PBA = 180°$

$\Rightarrow 60° + \angle PBA + \angle PBA = 180°$

$\Rightarrow 2\angle PBA = 120°$

$\Rightarrow \angle PBA = 60°$

$\therefore \angle APB = \angle PBA = \angle PAB = 60°$

Hence, ΔAPB is an equilateral triangle.

12. Given, $OD = 3$ cm

TiP

Tangents drawn from an external point of a circle are equal in length.

Let $AF = AE = x$ cm

$BD = BF = 6$ cm

$CD = CE = 9$ cm

Let $OF = OE = OD = 3$ cm (radii)

$\therefore$ $AB = AF + BF = x + 6$ ...(1)

$AC = AE + CE = x + 9$ ...(2)

$BC = BD + CD = 6 + 9 = 15$ cm ...(3)

$\because$ ar (ΔABC) = 54 cm^2 (given)

$\Rightarrow$ ar (ΔAOB) + ar (ΔAOC) + ar(ΔBOC) = 54

TR!CK

Area of triangle $= \frac{1}{2} \times$ *Base* $\times$ *Height*

$\Rightarrow \frac{1}{2} \times AB \times OF + \frac{1}{2} \times AC \times OE + \frac{1}{2} \times BC \times OD = 54$

$\Rightarrow \frac{1}{2} \times (6 + x) \times 3 + \frac{1}{2} \times (9 + x) \times 3 + \frac{1}{2} \times 15 \times 3 = 54$

$\Rightarrow \frac{1}{2}(6 + x + 9 + x + 15) \times 3 = 54$

$\Rightarrow 2x + 30 = 18 \times 2$

$\Rightarrow 2x = 36 - 30 = 6$

$\Rightarrow x = 3$

From eqs. (1) and (2),

$AB = x + 6 = 3 + 6 = 9$ cm

and $AC = x + 9 = 3 + 9 = 12$ cm

Short Answer Type-II Questions

1. Given : A quadrilateral ABCD circumscribes a circle.

To Prove : $AB + CD = AD + BC$

Proof: $\because$ The lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ $DR = DS$ ...(1)

$CR = CQ$ ...(2)

$BP = BQ$ ...(3)

$AP = AS$ ...(4)

On adding all these equations, we get

$DR + CR + BP + AP = DS + CQ + BQ + AS$

$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$

$\Rightarrow CD + AB = AD + BC$

Hence proved.

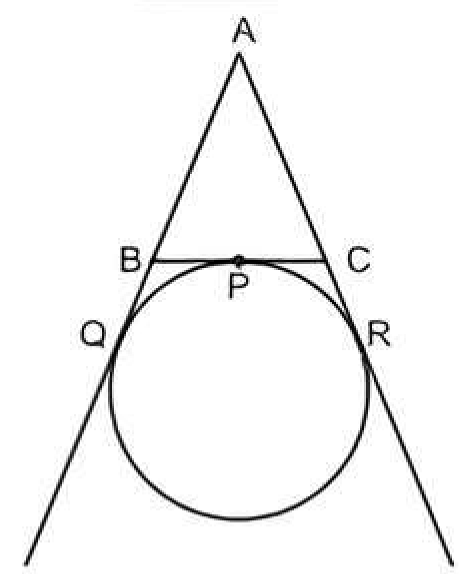
2. Given, a circle touches the side *BC* of a triangle *ABC* at *P* and extended sides *AB* and *AC* at *Q* and *R* respectively.

Since, the length of tangents drawn from an external point to a circle are equal.

$\therefore$ $AQ = AR$ ...(1)

$BQ = BP$ ...(2)

and $CR = CP$ ...(3)



TiP

Perimeter of any triangle is equal to sum of all its three sides.

So, perimeter of $\Delta ABC = AB + BC + AC$

$= AB + BP + PC + AC$ ($\because BC = BP + PC$)

$= (AB + BQ) + (CR + AC)$ [from eqs. (2) and (3)]

$= AQ + AR$ ($\because AQ = AB + BQ, AR = CR + AC$)

$= AQ + AQ$ [from eq. (1)]

$= 2AQ$

$\therefore AQ = \frac{1}{2} \times$ Perimeter of $\Delta ABC = \frac{1}{2}(BC + CA + AB)$

Hence proved.

3. Given: Incircle of DABC touches BC, CA and AB at P, Q and R respectively.

To Prove: $(AR + BP + CQ) = (AQ + BR + CP)$

$= \frac{1}{2}$(Perimeter of ΔABC)

TiP

The length of two tangents drawn from an external point of a circle are equal.

Proof:

In ΔABC, $AR = AQ$ ...(1)

$BP = BR$ ...(2)

and $CQ = CP$ ...(3)

On adding eqs. (1), (2) and (3), we get

$(AR + BP + CQ) = (AQ + BR + CP) = k$ (say)

Now, Perimeter of $\Delta ABC = AB + BC + CA$

$= (AR + BR) + (BP + CP) + (CQ + AQ)$

$= (AR + BP + CQ) + (AQ + BR + CP)$

$= k + k = 2k$

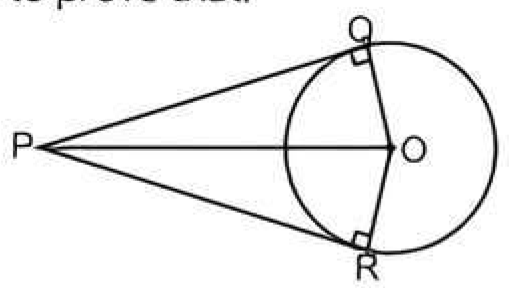
$\therefore \quad k = \frac{1}{2}$ (Perimeter of ΔABC)

Hence, $(AR + BP + CQ) = (AQ + BR + CP)$

$= \frac{1}{2}$ (Perimeter of Δ ABC) **Hence proved.**

4. Let PQ and PR be two tangents drawn from an external point P to a circle with centre O.

We have to prove that,



$$\angle QOR = 180^\circ - \angle QPR$$

or $\angle QOR + \angle QPR = 180^\circ$

In right ΔOQP and ΔORP

$$PQ = PR$$

(tangents drawn from an external point are equal)

$OQ = OR$ (radius of circle)

$OP = OP$ (common)

Therefore, by SSS criterion of congruence,

$$\Delta OQP \cong ORP$$

$\Rightarrow \angle QPO = \angle RPO$

and $\angle POQ = \angle POR$ (by CPCT)

$\Rightarrow \left.\begin{array}{l}\angle QPR = 2\angle OPQ \\ \angle QOR = 2\angle POQ\end{array}\right\}$ ...(1)

In ΔOPQ,

$\angle QPO + \angle QOP = 90^\circ$ ($\because \angle OQP = 90^\circ$)

$\Rightarrow QOP = 90^\circ - \angle QPO$

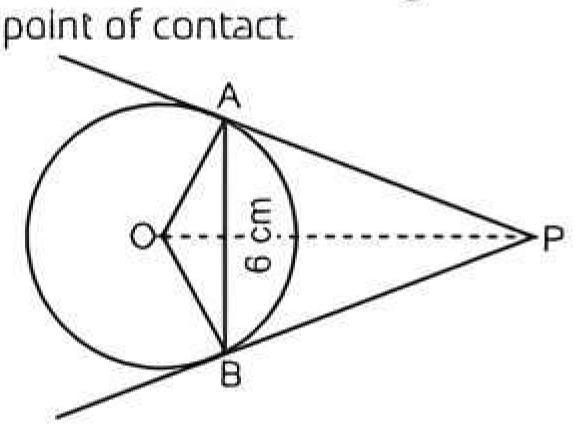
$\Rightarrow 2\angle QOP = 180^\circ - 2\angle QPO$

(multiplying both sides by 2)

$\Rightarrow \angle QOR = 180^\circ - \angle QPR$ [from eq. (1)]

$\Rightarrow \angle QOR + \angle QPR = 180^\circ$ **Hence proved.**

5. Given, chord AB makes an angle of 30° with the radius at the point of contact.



i.e., $\angle OAB = \angle OBA = 30^\circ$

Since, $OA \perp AP$ and $OB \perp BP$

$\therefore \angle OAP = \angle OBP = 90^\circ$

(angle between the tangent and the radius at the point of contact)

$$\angle PAB = \angle OAP - \angle OAB$$

$$= 90^\circ - 30^\circ = 60^\circ$$

$\Rightarrow \angle PAB = \angle PBA = 60^\circ$

Now, $AP = BP$

(tangents to a circle from an external point)

$\Rightarrow \angle PAB = \angle PBA$

(angles opposite to equal sides of a triangle)

In ΔABP,

$\angle PAB + \angle PBA + \angle APB = 180^\circ$

(by angle sum property of triangle)

$\Rightarrow 60^\circ + 60^\circ + \angle APB = 180^\circ$

$\Rightarrow \angle APB = 180^\circ - 120^\circ = 60^\circ$

$\therefore \Delta ABP$ is an equilateral triangle, where

$$AP = BP = AB$$

$\Rightarrow AP = BP = AB = 6$ cm

($\because$ AB = 6 cm, given)

Now, $\angle OPA = \angle OPB = \frac{1}{2}\angle APB$

$$= \frac{1}{2} \times 60^\circ = 30^\circ$$

($\therefore$ the line joining the external point to the centre of the circle bisects the angle between the two tangents)

In right ΔOAP,

$$\tan\angle OPA = \frac{OA}{AP} \Rightarrow \tan 30^\circ = \frac{OA}{6}$$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{OA}{6} \Rightarrow OA = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow OA = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$ cm

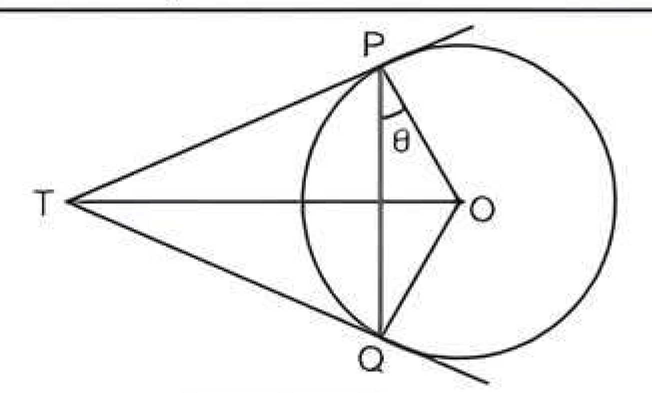
Hence, the length of PA and length of radius OA are 6 cm and $2\sqrt{3}$ cm respectively.

6. Let $\angle OPQ = \theta$, then

$TP \perp OP$ ($\because$ tangent $\perp$ radius)

TiPS

- *Tangent is perpendicular to the radius through the point of contact of circle.*
- *Angles opposite to equal sides of a triangle is also equal.*
- *Length of two tangents drawn from an external point of a circle are equal.*



$\therefore \angle OPT = 90^\circ$

$\Rightarrow \angle OPQ + \angle TPQ = 90^\circ$

$\Rightarrow \angle TPQ = 90^\circ - \theta$

$\because TP = TQ$

$\therefore \angle TQP = \angle TPQ = 90^\circ - \theta$

Now, In ΔTPQ,

$\angle PTQ + \angle TPQ + \angle TQP = 180^\circ$

($\because$ sum of internal angles in a triangle is 180°)

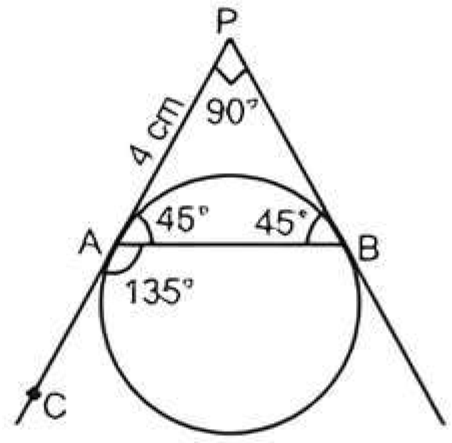
$\Rightarrow \angle PTQ + 90^\circ - \theta + 90^\circ - \theta = 180^\circ$

$\Rightarrow \angle PTQ = 2\theta = 2\angle OPQ$ **Hence proved.**

7. It is given that PA and PB are tangents drawn from an external point P to the circle.

$\therefore PA = PB = 4$ cm

($\because$ lengths of tangents drawn from an external point to a circle are equal)



Also, $\angle BAC = 135°$

Since, $\angle BAC + \angle PAB = 180°$ (linear pair)

$\therefore \quad \angle PAB = 180° - 135° = 45°$

In ΔPAB, $PA = PB$

TiP

Angles opposite to equal sides of a triangle is also equal.

$\therefore \quad \angle PBA = \angle PAB = 45°$

Also $\angle PBA + \angle PAB + \angle APB = 180°$

(angle sum property of triangle)

$\Rightarrow \quad 45° + 45° + \angle APB = 180°$

$\Rightarrow \quad \angle APB = 180° - 45° - 45° = 90°$

So, ΔAPB is a right triangle right angled at P. Using Pythagoras theorem, we have

TR!CK

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$AB^2 = PA^2 + PB^2 = \sqrt{(4)^2 + (4)^2}$$

$$= \sqrt{16 + 16} = \sqrt{32}$$

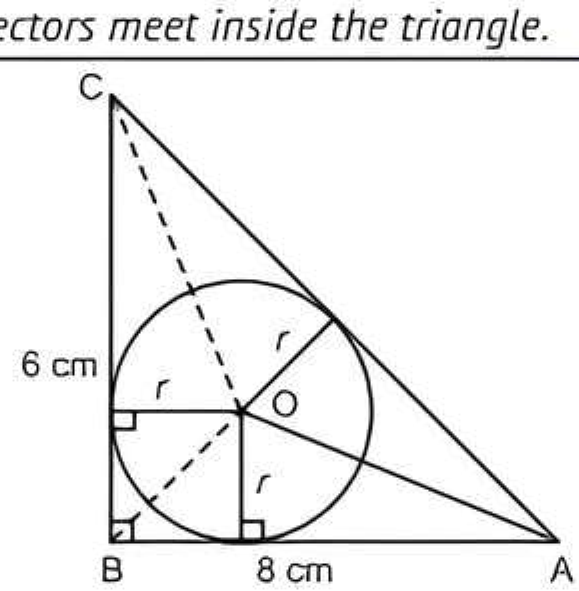
$$\therefore \quad AB = 4\sqrt{2} \text{ cm}$$

Hence, the length of the chord AB is $4\sqrt{2}$ cm.

8. Let r be the radius of the incircle.

TiP

The incentre of a triangle is the point where all of the angle bisectors meet inside the triangle.



TR!CK

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In right-angled ΔABC,

$$CA^2 = AB^2 + BC^2 \text{ (by Pythagoras theorem)}$$

$$= 8^2 + 6^2 = 64 + 36 = 100$$

$$\therefore \quad CA = \sqrt{100} = 10 \text{ cm}$$

The incentre O makes three smaller triangles BCO, ACO and ABO, whose areas add up to the area equal to ABC. Each of the smaller triangles has an altitude equal to the inradius 'r' and base of that triangles is equal to the side of the original triangle ABC.

i.e., ar (ΔABC) = ar(ΔABO) + ar(ΔACO) + ar(ΔBCO)

TR!CK

Area of the triangle $= \frac{1}{2} \times Base \times Height$

$$\Rightarrow \quad \frac{1}{2} \times AB \times BC = \frac{1}{2} \times AB \times r + \frac{1}{2} \times CA \times r + \frac{1}{2} \times BC \times r$$

$$\Rightarrow \quad 8 \times 6 = 8r + 10r + 6r$$

$$\Rightarrow \quad 48 = 24r$$

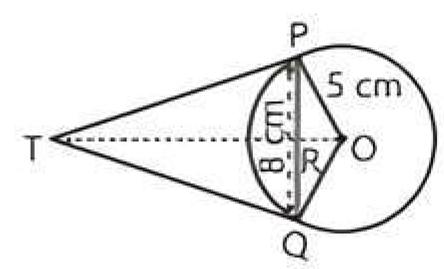
$$\therefore \quad r = \frac{48}{24} = 2 \text{ cm}$$

So, diameter $= 2r$

$= 2 \times 2 = 4$ cm

Hence, the diameter of the circle inscribed in the triangle is 4 cm.

9. In the given figure, join points O and T, which intersect chord PQ at point R.



TR!CK

Perpendicular drawn from centre of circle bisects the chord.

Since $\Delta ORP \sim \Delta ORQ$

$\therefore \quad \angle PRO = \angle QRO = 90°$

$\Rightarrow \quad PR = QR = \frac{8}{2} = 4$ cm

TiP

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In right-angled ΔORP, use Pythagoras theorem,

$$OP^2 = PR^2 + OR^2$$

$$\Rightarrow \quad OR^2 = OP^2 - PR^2$$

$$= (5)^2 - (4)^2$$

$$= 25 - 16 = 9 \text{ cm}$$

$$\Rightarrow \quad OR = 3 \text{ cm}$$

In right-angled triangles PRT and OPT, use Pythagoras theorem, we have

$$TP^2 = TR^2 + PR^2 \quad \text{...(1)}$$

and $$OT^2 = TP^2 + OP^2 \quad \text{...(2)}$$

$$\Rightarrow \quad OT^2 = (TR^2 + PR^2) + OP^2 \quad (\because \text{ from eq. (1)})$$

$$\Rightarrow \quad (TR + 3)^2 = TR^2 + PR^2 + OP^2$$

$$(\because OT = TR + OR = TR + 3)$$

$$\Rightarrow \quad TR^2 + 9 + 6TR = TR^2 + 4^2 + 5^2$$

$\Rightarrow \quad 6TR = 16 + 25 - 9$

$\Rightarrow \quad TR = \frac{16}{3}$ cm

Now from eq. (1).

$$TP^2 = TR^2 + PR^2 = \left(\frac{16}{3}\right)^2 + 4^2$$

$$= \frac{256}{9} + 16 = \frac{256 + 144}{9}$$

$$= \frac{400}{9}$$

$\Rightarrow \quad TP = \frac{20}{3}$ cm

10.

TiP

Know about the circle and related angle theorem, cyclic theorem, tangent and secant theorem thoroughly.

Given: XY and X'Y' are two parallel tangents to a circle with centre O. AB is another tangent at C meeting XY and X'Y' at A and B respectively.

To Prove: $\angle AOB = 90^\circ$

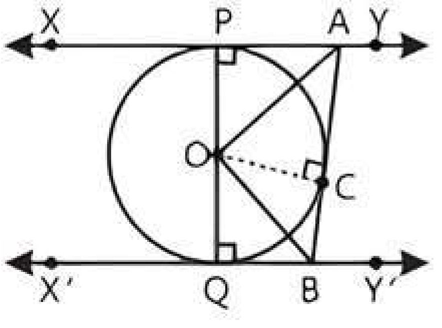
Construction: Join point O to C.

Proof: In ΔOPA and ΔOCA.

$OP = OC$ (radii of the same circle)

$AP = AC$

(tangents drawn from an external point A to the circle are equal)



$AO = AO$ (common side)

$\therefore \quad \Delta OPA \sim \Delta OCA$ (by SSS similarity)

$\Rightarrow \quad \angle POA = \angle COA$ (by CPCT) ...(1)

Similarly, $\Delta OQB \sim \Delta OCB$

$\Rightarrow \quad \angle QOB = \angle COB$ (by CPCT) ...(2)

Since, POQ is a diameter of the circle. So, sum of all adjacent angles lie on this line is 180°.

$\therefore \quad \angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$

From eqs. (1) and (2), it can be observed that

$2\angle COA + 2\angle COB = 180^\circ$

$\Rightarrow \quad \angle COA + \angle COB = 90^\circ$

$\Rightarrow \quad \angle AOB = 90^\circ$ **Hence proved.**

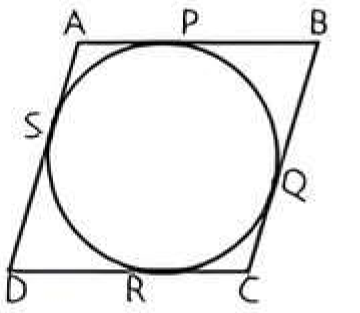
COMMON ERROR

Some candidates could not apply the appropriate theorems of circle to find out the unknown angles.

Long Answer Type Questions

1. **Given:** ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.



Proof: In parallelogram ABCD.

TiP

In a parallelogram, opposite sides are equal in length.

$AB = CD$ ...(1)

and $BC = AD$ ...(2)

Since, the length of tangents drawn from an external point to a circle are equal

$\therefore \quad DR = DS,$ ...(3)

$CR = CQ,$ ...(4)

$BP = BQ$ ...(5)

and $AP = AS$ ...(6)

Adding eqs. (3), (4), (5) and (6), we get

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$$CD + AB = AD + BC \quad ...(7)$$

On putting the values of eqs. (1) and (2) in eq. (7), we get

$$2AB = 2BC$$

$$AB = BC \quad ...(8)$$

From eqs. (1), (2) and (3), we get

$$AB = BC = CD = DA$$

Hence, ABCD is a rhombus. **Hence proved.**

2. Given, $OE = OP = OQ = 5$ cm (radii) and $OT = 13$ cm.

Since $OP \perp PT$

$\therefore \quad \angle OPT = 90^\circ$

In right-angled ΔOPT,

$OT^2 = OP^2 + PT^2$ (by Pythagoras theorem)

TR!CK

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$PT^2 = OT^2 - OP^2$

$PT^2 = 13^2 - 5^2 = 169 - 25 = 144$

$\Rightarrow \quad PT = \sqrt{144} = 12$ cm

$\therefore \quad ET = OT - OE = 13 - 5 = 8$ cm

Let $AP = AE = x$, then $AT = 12 - x$

($\because$ the length of two tangents drawn from an external point are equal)

In right-angled ΔAET,

$AT^2 = AE^2 + ET^2$

(by Pythagoras theorem)

$\Rightarrow \quad (12 - x)^2 = x^2 + 8^2$

$\Rightarrow \quad 144 + x^2 - 24x = x^2 + 64$

$\Rightarrow \quad 144 - 64 = 24x$

$\Rightarrow \quad 80 = 24x$

$\Rightarrow \qquad x = \frac{80}{24} = 3.3\text{ cm}$

Since, perpendicular drawn from the centre of the circle bisect the chord.

Hence, $AB = 2AE = 2x = 2 \times 3.3 = 6.6\text{ cm}$

COMMON ERR(!)R

Some students are not familiar with the circle theorems, i.e., could not well identify $\angle OPT = 90°$ (Angle between radius and tangent).

3. Given, two equal circles with centres O and O', touch each other at point X. OO' is produced to meet the circle with centre O' at the point A.

$\because \qquad O'D \perp AC$ and $OC \perp AC$

$\therefore \qquad \angle ACO = 90°$ and $\angle ADO' = 90°$

TiP

Tangent is perpendicular to the radius through the point of contact of circle.

Now, In $\Delta AO'D$ and ΔAOC,

$\angle O'AD = \angle OAC$ (common angle)

$\angle ADO = \angle ACO$ (each $90°$)

$\Delta AO'D \sim \Delta AOC$ (by AA similarity)

$\frac{AO'}{AO} = \frac{DO'}{CO}$

(corresponding sides are proportional)

Let $\quad AO' = O'X = XO = r$

Then $\quad AO = AO' + O'X + XO = r + r + r = 3r$

$\therefore \quad \frac{AO'}{AO} = \frac{DO'}{CO} = \frac{r}{3r}$

Hence, $\frac{DO'}{CO} = \frac{1}{3}$

4. Given: PA and PB are two tangents to a circle circumscribing ΔABQ and $\angle PAB = 60°$.

To Prove: QP bisects AB at right angle.

Proof: $\because$ ΔABQ is an equilateral triangle.

TiP

In an equilateral triangle, each angle of adjacent sides is equal to 60° and all sides are equal in length.

$\therefore \quad \angle QAB = 60°$ and $\angle QBA = 60°$

So, $\angle PAQ = \angle PAB + \angle QAB = 60° + 60° = 120°$

Since angles opposite to equal sides in a triangle is also equal.

Similarly, $\angle PBQ = 120°$ ...(1)

($\because \angle PAB = \angle PBA$, as $PB = PA$)

Now, in ΔPAQ and ΔPBQ,

$PA = PB$

(tangents drawn from an external point are equal in length)

$AQ = BQ$

($\because$ Δ ABQ is an equilateral triangle)

$\angle PAQ = \angle PBQ$ (proved above)

So, $\quad \Delta PAQ \cong \Delta PBQ$ (by SAS Congruence)

$\Rightarrow \quad \angle APQ = \angle BPQ$ (by CPCT) ...(2)

and $\quad \angle APM = \angle BPM$

Let QP intersect AB at M.

Now, in ΔPAM and ΔPBM,

$\angle APM = \angle BPM$ [from eq. (2)]

$PA = PB$ (tangents drawn from an external point are equal in length)

and $\quad PM = PM$ (common side)

So, $\quad \Delta PAM \cong \Delta PBM$ (by SAS congruence)

$\Rightarrow \quad AM = BM$

and $\quad \angle AMP = \angle BMP$ (by CPCT) ...(3)

But $\quad \angle AMP + \angle BMP = 180°$ (linear pair)

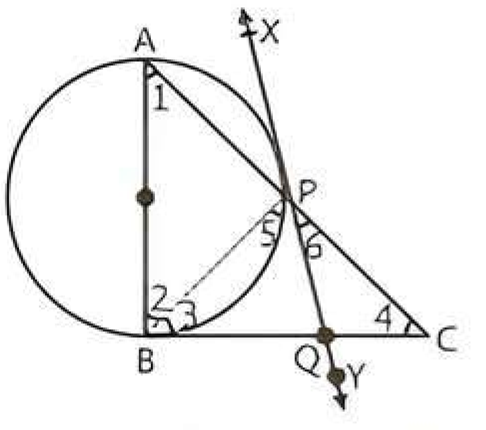
$\Rightarrow \quad \angle AMP + \angle AMP = 180°$ [from eq. (3)]

$\Rightarrow \quad 2\angle AMP = 180°$

$\Rightarrow \quad \angle AMP = 90°$ ...(4)

From eqs. (3) and (4), we can say that QP bisects AB at right angle. **Hence proved.**

5. Given: In ΔABC, $\angle B = 90°$ and AB is a diameter of a circle.



To Prove: PQ bisect BC., *i.e.*, BQ = QC.

Construction: Join BP

Proof: Since AB is a diameter of circle.

$\therefore \quad \angle APB = 90°$ (angle in a semicircle)

$\because$ APC is a straight line.

$\therefore \quad \angle APB + \angle BPC = 180°$ (linear pair)

$\Rightarrow \quad \angle BPC = 180° - 90° = 90°$

$\Rightarrow \quad \angle 5 + \angle 6 = 90°$ ...(1)

In ΔAPB,

$\angle APB + \angle BAP + \angle ABP = 180°$

(angle sum property of triangle)

$\Rightarrow \quad 90° + \angle 1 + \angle 2 = 180°$

$\Rightarrow \quad \angle 1 + \angle 2 = 90°$ ...(2)

Since ABC is a right-angled triangle.

$\therefore \quad \angle ABC = 90°$

$\Rightarrow \quad \angle 2 + \angle 3 = 90°$ ...(3)

From eqs. (2) and (3), we get

$\angle 1 + \angle 2 = \angle 2 + \angle 3 = 90°$

$\Rightarrow \quad \angle 1 = \angle 3$ ...(4)

Since, the length of two tangents drawn from an external point of a circle are equal.

$\therefore \quad BQ = PQ$ ...(5)

$\Rightarrow \quad \angle 3 = \angle 5$

(angle opposite to equal sides are equal)

Substituting this value in eq. (1), we get

$\angle 3 + \angle 6 = 90°$

$\Rightarrow \quad \angle 1 + \angle 6 = 90°$ [from eq. (4)] ...(6)

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180°$

(by angle sum property of triangle)

$\Rightarrow \quad 90° + \angle 1 + \angle 4 = 180°$

$\Rightarrow \quad \angle 1 + \angle 4 = 90°$ ...(7)

From eqs. (6) and (7), we get

$\Rightarrow \qquad \angle 4 = \angle 6$

$\Rightarrow \qquad CQ = PQ$ ...(8)

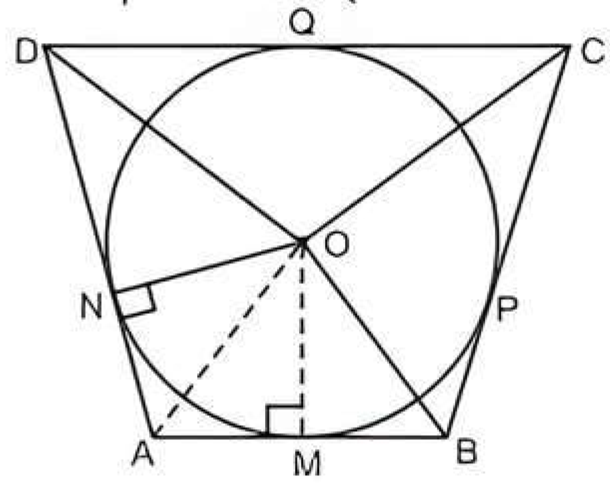
($\because$ opposite sides of equal angles in a triangle are equal)

From eqs. (5) and (8), we get

$BQ = CQ$

So, PQ bisect BC. **Hence proved.**

6. Given: A quadrilateral ABCD is circumscribed in a circle with centre O whose sides AB, BC, CD and DA touch the circle at points M, P, Q and N.



To Prove: $\angle AOB + \angle COD = 90^\circ$

Construction: Join the points of contact M and N to O.

Proof: Let $\angle A = 2\alpha$, $\angle B = 2\beta$,

$\angle C = 2\gamma$, $\angle D = 2\delta$

In ΔOAM and ΔOAN,

$\angle OMA = \angle ONA$ (each right angle)

$OM = ON$ (radii of a circle)

$OA = OA$ (common side)

$\therefore$ Both triangles are congruent.

i.e., $\Delta OAM \cong \Delta OAN$ (from RHS congruency)

$\Rightarrow \angle OAM = \angle OAN = \frac{1}{2}(\angle A) = \frac{1}{2}(2\alpha) = \alpha$ (from CPCT)

$\Rightarrow \qquad \angle OAB = \angle OAD = \alpha$

Similarly, $\angle OBA = \angle OBC = \beta$

$\angle OCB = \angle OCD = \gamma$

and $\qquad \angle ODA = \angle ODC = \delta$

TiP

The sum of interior angles of a triangle is 180°.

In ΔAOB,

$\angle AOB = 180^\circ - \angle OAB - \angle OBA$

(by angle sum property of a triangle)

$= 180^\circ - \alpha - \beta = 180^\circ - (\alpha + \beta)$ ...(1)

and $\qquad \angle COD = 180^\circ - \angle OCD - \angle ODC$

$= 180^\circ - \gamma - \delta = 180^\circ - (\gamma + \delta)$ ...(2)

Adding eqs. (1) and (2),

$\angle AOB + \angle COD = \{180^\circ - (\alpha + \beta)\} + \{180^\circ - (\gamma + \delta)\}$

$= 360^\circ - (\alpha + \beta + \gamma + \delta)$ ...(3)

But $\angle A + \angle B + \angle C + \angle D = 360^\circ$

TiP

The sum of interior angles of a quadrilateral is 360°.

$\Rightarrow \qquad 2\alpha + 2\beta + 2\gamma + 2\delta = 360^\circ$

$\Rightarrow \qquad \alpha + \beta + \gamma + \delta = 180^\circ$

Therefore from eq. (3),

$\angle AOB + \angle COD = 360^\circ - 180^\circ = 180^\circ$

Hence proved.

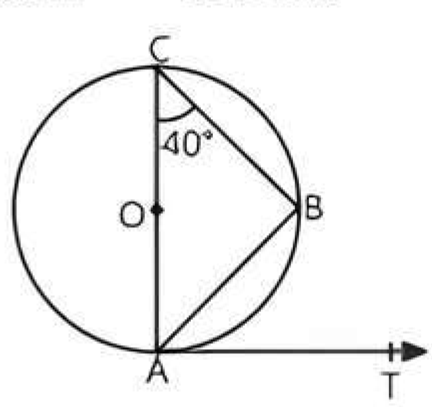
Chapter Test

Multiple Choice Questions

Q 1. If radii of two concentric circles are 8 cm and 10 cm, the length of the chord touches the smaller circle is:

a. 5 cm b. 7 cm c. 6 cm d. 12 cm

Q 2. In the figure, AB is a chord of the circle and AOC is its diameter such that $\angle$ ACB = 40°. If AT is tangent to the circle at the point A, then $\angle$ BAT is equal to:



a. 60° b. 20°

c. 40° d. 30°

Assertion and Reason Type Questions

Directions: (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

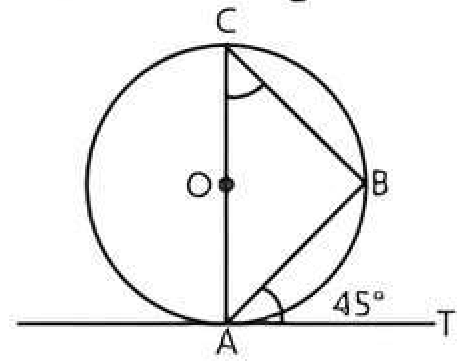
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): In the given figure, O is the centre of a circle and AT is a tangent at point A, then $\angle$ ACB is 45°.

Reason (R): Diameter of a circle is always perpendicular to the tangent line.



Q 4. Assertion (A): If the distance between two parallel tangents of a circle is 24 cm, then radius of a circle is 12 cm.

Reason (R): The distance between two parallel tangents of a circle is equal to twice the diameter of a circle.

Fill in the Blanks

Q 5. There are exactly two tangents to be drawn on a circle, if a point lying .... the circle.

Q 6. A line intersecting a circle at two points is said to be a ....

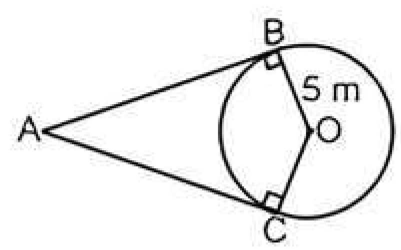
True/False

Q 7. If angle between two tangents drawn from a point P to a circle of radius a and centre O is 90°, then $OP = a\sqrt{2}$.

Q 8. The centre of the circle lies on the bisector of the angle between the two tangents.

Case Study Based Question

Q 9. There is a circular field of radius 5 m. A person was starting a walk along the tangents of the circular field. Two paths are connected by the tangents of circle AB and AC which is shown in the figure.



The distance of the point from where tangents are drawn, *i.e.* A to O is 13 m. A person running along path BA and AC, *i.e.*, person starts from B and stops at C.

Based on the above information, solve the following questions:

(i) Find the length of AB.

(ii) Find the total distance travelled by the person.

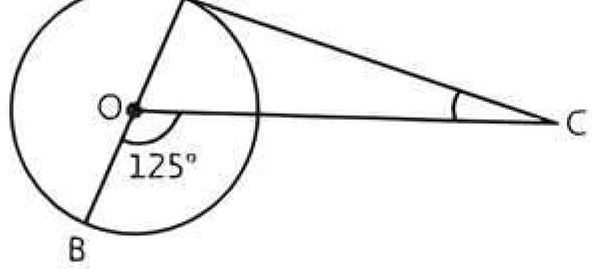
(iii) If $\angle OAB = 60°$, then find the value of $\angle BOA$ is:

Or

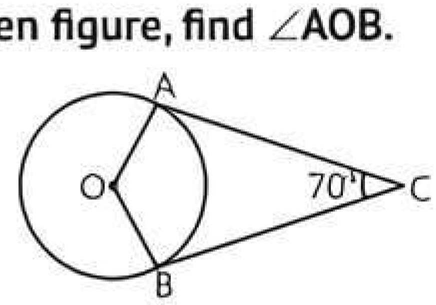
Find the measure of $\angle BOC$.

Very Short Answer Type Questions

Q 10. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 125°$, then find $\angle ACO$.



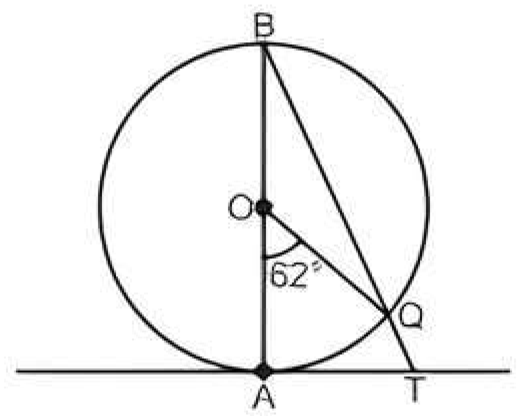
Q 11. In the given figure, find $\angle AOB$.



Short Answer Type-I Questions

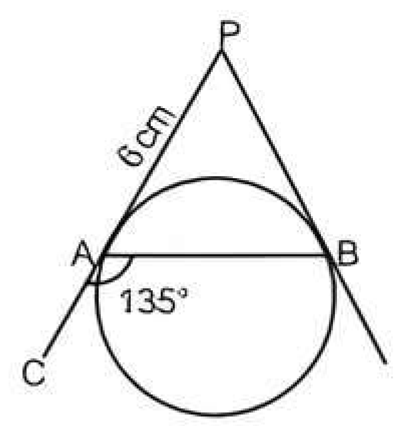
Q 12. If two tangents are inclined at an angle 120° are drawn to a circle of radius 6 cm, then find the length of each tangent.

Q 13. In the given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $AOQ = 62°$, find $\angle ATQ$.

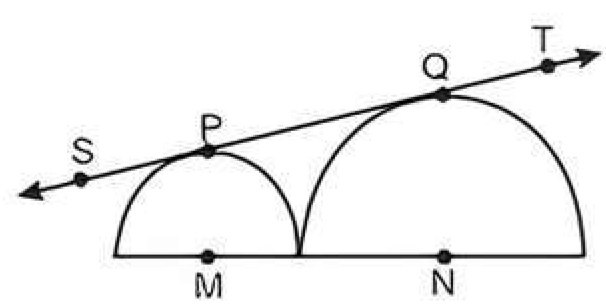


Short Answer Type-II Questions

Q 14. In the given figure, PA and PB are tangents to a circle from an external point P such that PA = 6 cm and $\angle BAC = 135°$. Find the length of the chord AB.



Q 15. In the figure below, M and N are the centres of two semi-circles having radii 9 cm and 16 cm respectively. ST is a common tangent. Find the length of PQ.



Long Answer Type Question

Q 16. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + CD.