Verify the Different Criteria For Congruency Of Triangles

OBJECTIVE

To verify experimentally the different criteria for congruency of triangles using triangle cut outs.

Materials Required

1. Cardboard
2. Scissors/cutter
3. White paper
4. Geometry box
5. Coloured glazed papers
6. Adhesive

Prerequisite Knowledge

1. Concept of congruency of figures.
2. Different criteria for congruency of two triangles.

Theory

1. **Congruent Figures** Two figures are said to be congruent, if they are of same shape and of same size ('congruent' means equal in all respects).
   e.g. Two circles of the same radii and two squares of the same sides are congruent.
   Congruency of Triangles Two triangles are congruent, if sides and angles of a triangle are equal to the corresponding sides and angles of the other triangle.
   Or
   If a triangle coincides or covers the other triangle completely, then the two triangles are congruent.

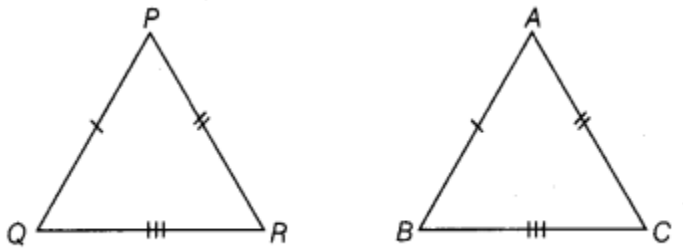

Fig. 14.1

If ΔPQR is congruent to ΔABC, then we write ΔPQR ≅ ΔABC. Here, '≅' is the sign of congruency.

In congruent triangles, corresponding parts are equal and we write it in short CPCT, i.e. corresponding parts of congruent triangles.

2. **Criterion for Congruency of Two Triangles**
   There are four different criteria for the two triangles to be congruent.

   1. **SSS (Side-Side-Side) criterion** If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

   2. **SAS (Side-Angle-Side) criterion** Two triangles are congruent, if two sides and the included angle of a triangle are equal to the two sides and the included angle of the other triangle.

   3. **ASA (Angle-Side-Angle) criterion** Two triangles are congruent, if two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle.

   4. **RHS (Right angle-Hypotenuse-Side) criterion** If in two right triangles, the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Procedure

1. Take a cardboard of suitable size and by using adhesive, paste a white paper on it.
2. Cut out a pair of ΔABC and ΔDEF from glazed paper such that AB = DE, BC = EF and AC = DF. (see Fig. 14.2)

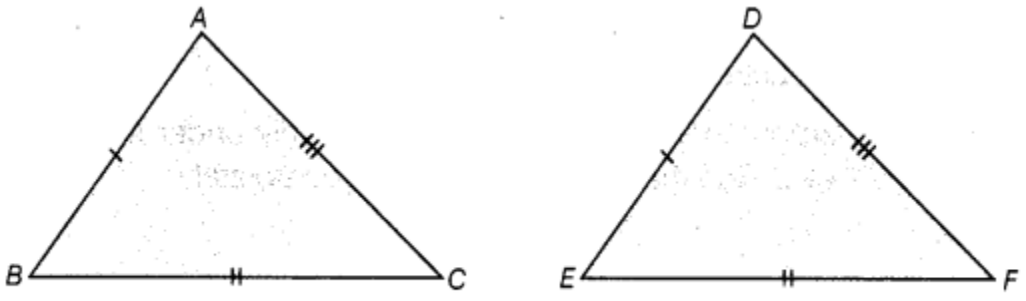

Fig. 14.2

3. Make a pair of ΔGHI and ΔJKL on glazed paper such that GH = JK, Gl = JL and ∠G = ∠J and cut them out. (see Fig. 14.3)

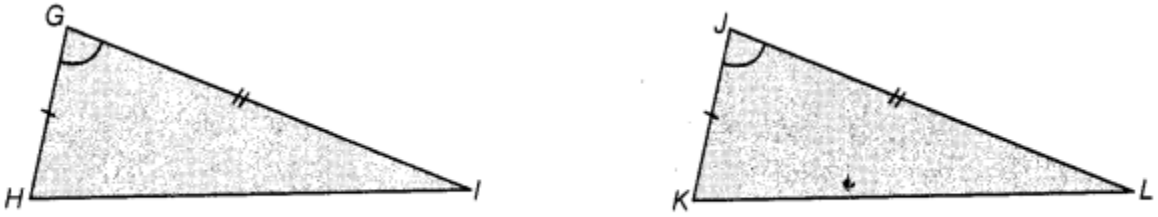

Fig. 14.3

4. Make a pair of ΔPQR and ΔSTU from glazed paper such that QR = TU, ∠Q = ∠T and ∠R = ∠U and cut them out. (see Fig. 14.4)

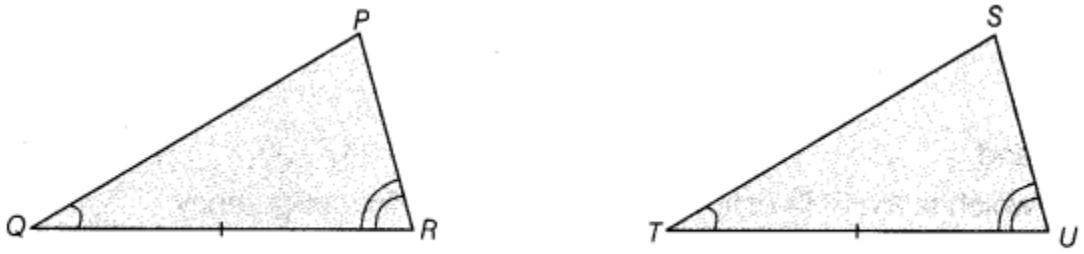

Fig. 14.4

5. Make two right angle triangles such that ΔXVZ and ΔLMN from glazed paper such that YZ = MN, XZ = LN and ∠X = ∠L = 90°. (see Fig. 14.5)

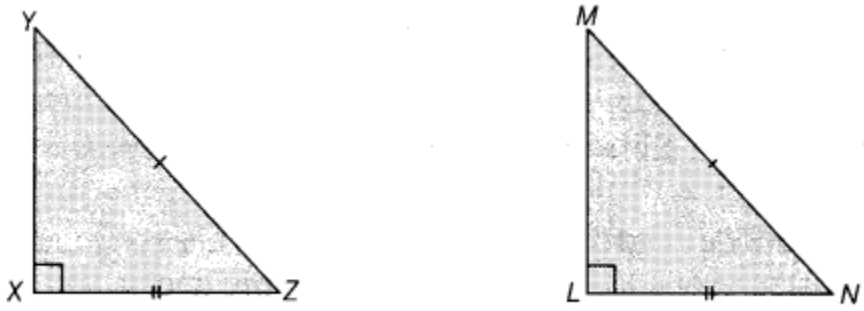

Fig. 14.5

Demonstration

1. Superpose ΔABC on ΔDEF completely only under the correspondence A ↔ D, B ↔ E and C ↔ F. See that ΔABC covers ΔDEF completely.
   Hence, ΔABC ≅ ΔDEF
   if AB = DE
   BC = EF
   and AC = DF
   which is the SSS criterion for congruency.
2. Similarly, superpose ΔGHI on ΔJKL completely only under the correspondence G ↔ J, H ↔ K and I ↔ L. See that ΔGHI covers ΔJKL completely.
   Hence, ΔGHI ≅ ΔJKL
   if GH = JK

∠G = ∠J
and GI = JL
which is the SAS criterion for congruency.

3. Similarly, superpose ΔPQR on ΔSTU only under the correspondence P ↔ S, Q ↔ T and R ↔ U. See that ΔPQR covers ΔSTU completely.
   Hence, ΔPQR ≅ ΔSTU
   if ∠Q = ∠T
   QR = TU
   and ∠R = ∠U
   which is the ASA criterion for congruency,
4. Similarly, superpose ΔYXZ on ΔMLN only under the correspondence Y ↔ M, X ↔ L and Z ↔ N. See that ΔYXZ covers ΔMLN completely.
   Hence, ΔYXZ ≅ ΔMLN
   if ∠X = ∠L = 90°
   YZ = MN
   and XZ = LN
   which is the RHS criterion of right triangles for congruency.

Observation

By actual measurement,

1. In the pair of ΔABC and ΔDEF,
   AB = DE = …… , BC = EF = …… , AC = DF = …… ,
   ∠A = …… , ∠B = …… ,
   ∠C = …… , ∠D = …… ,
   ∠E = …… , ∠F = …… .
   Hence, ΔABC ≅ ΔDEF
2. In the pair of ΔGFII and ΔJKL,
   GH =JK = …… , GI = JL = …… ,
   HI = …… , KL = …… ,
   ∠G = …… , ∠J = …… ,
   ∠H = …… , ∠K = …… ,
   ∠I= …… , ∠L = …… ,
   Hence, ΔGHI ≅ ΔJKL
3. In the pair of ΔPQR and ΔSTU,
   QR = TU = …… , PQ = …… ,
   ST = …… , PR = …… ,
   SU = …… , ∠S = …… ,
   ∠Q = ∠T= …… , ∠R = ∠U = …… ,
   ∠P = …… .
   Hence, ΔPQR ≅ ΔSTU
4. In the pair of ΔXYZ and ΔLMN, hypotenuse YZ = hypotenuse MN = …… ,
   XZ = LN =…… , XY = …… ,
   LM = …… , ∠X = ∠L = …… ,
   ∠Y= …… , ∠M = …… ,

$\angle Z$ = …… , $\angle N$ = …… .
Hence, $\Delta XYZ \cong \Delta LMN$

Result

Using triangle cut outs, we have verified experimentally the different criteria for congruence of triangles.

Application

These criteria are useful in

1. solving many problems in geometry.
2. practical problems such as finding width of a river without crossing it.

Viva Voce

Question 1:
What do you understand by congruent figures?
Answer:
Two figures are congruent, if they are of the same shape and same size.

Question 2:
What is the measure of each angle in an equilateral triangle?
Answer:
In equilateral triangle each angle measures 60°.

Question 3:
Is there any AAA congruency criterion for triangles?
Answer:
No

Question 4:
Are the sides opposite to equal angles of a triangle unequal?
Answer:
No, side opposite to equal angles in a triangle are always equal.

Question 5:
Are congruent triangles similar?
Answer:
Yes

Question 6:
What is the full form of CPCT?
Answer:
The full form of CPCT is Corresponding Parts of Congruent Triangles.

Question 7:
What do you mean by the RHS congruence rule for triangles?
Answer:
According to the RHS congruence rule, in two right triangles, the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two right. triangles are congruent.

Question 8:
If two triangles are congruent. Does they have different perimeters?
Answer:
No, both have equal perimeters.

Question 9:
If $\Delta ABC \cong \Delta DEF$ and AB = DE, BC = EF, then what is the third necessary condition to prove them congruent?
Answer:
$\angle B = \angle E$

Question 10:
If $\Delta ABC \cong \Delta DEF$ and $\angle A = \angle D$, $\angle C = \angle F$, then what is the third necessary condition to prove them congruent?
Answer:
AC = DF

Suggested Activity

To verify experimentally that, if any two angles and a non-included side of one triangle are equal to the corresponding angles and side of another triangle, then the two triangles are congruent.