PROBABILITY

CHAPTER – 13

PROBABILITY

There are various phenomena in nature, leading to an outcome, which cannot be predicted a priori e.g. in tossing of a coin, a head or a tail may result. Probability theory aims at measuring the uncertainties of such outcomes.

1. IMPORTANT TERMINOLOGY

(i) Random experiment:

It is a process which results in an outcome which is one of the various possible outcomes that are known to us before hand

Example

Throwing of a die is a random experiment as it leads to fall of one of the outcome from {1, 2, 3, 4, 5, 6}. Similarly taking a card from a pack of 52 cards is also a random experiment.

(ii) Sample space:

It is the set of all possible outcomes of a random experiment

Example

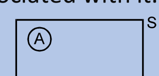
{H, T} is the sample space associated with tossing of a coin.
In set notation it can be interpreted as the universal set.

(iii) Event:

It is subset of sample space.

Example

Getting a head in tossing a coin or getting a prime number in throwing a die. In general, if a sample space consists 'n' elements, then a maximum of 2^n events can be associated with it.



(iv) Complement of event:

The complement of an event 'A' with respect to a sample space S is the set of all elements of 'S' which are not in A. It is usually denoted by $\bar{A}$ or A^C.

(v) Simple event:

If an event covers only one point of sample space, then it is called a simple event

Example

Getting a head followed by a tail in throwing of a coin 2 times is a simple event.

(vi) Compound event:
When two or more than two events occur simultaneously, the event is said to be a compound event. Symbolically $A \cap B$ or AB represent the occurrence of both A & B simultaneously.

Note

"$A \cup B$" or A + B represent the occurrence of either A or B.

(vii) Equally likely events:
If events have same chance of occurrence, then they are said to be equally likely.

Example

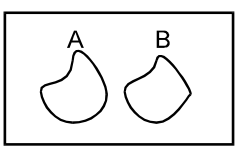
(i) In a single toss of a fair coin, the events {H} and {T} are equally likely.

(ii) In a single throw of an unbiased die the events {1}, {2}, {3} and {4}, are equally likely.

(iii) In tossing a biased coin the events {H} and {T} are not equally likely.

(viii) Mutually exclusive / disjoint / incompatible events:
Two events are said to be mutually exclusive if occurrence of one of them rejects the possibility of occurrence of the other i.e. both cannot occur simultaneously. In the vein diagram the events A and B are mutually exclusive. Mathematically, we write $A \cap B = \phi$

Events $A_1, A_2, A_3, \ldots\ldots A_n$ are said to be mutually exclusive events iff $A_i \cap A_j = \phi\ \forall\ i, j \in \{1, 2, \ldots, n\}$ where $i \neq j$



Note

If $A_i \cap A_j = \phi\ \forall\ i, j \in \{1, 2, \ldots, n\}$ where $i \neq j$, then $A_1 \cap A_2 \cap A_3 \ldots \cap A_n = \phi$ but converse need not to be true.

(ix) Exhaustive system of events:
If each outcome of an experiment is associated with at least one of the events $E_1, E_2, E_3, \ldots\ldots E_n$, then collectively the events are said to be exhaustive. Mathematically we write $E_1 \cup E_2 \cup E_3 \ldots\ldots E_n = S$. (Sample space)

2. CLASSICAL (A PRIORI) DEFINITION OF PROBABILITY

If an experiment results in a total of (m + n) outcomes which are equally likely and if 'm' outcomes are favorable to an event 'A' while 'n' are unfavorable, then the probability of occurrence of the event 'A', denoted by P((a), is defined by

$\frac{m}{m+n} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ i.e. $P(a) = \frac{m}{m+n}$. We say that odds in favour of 'A' are m : n, while odds against 'A' are n : m.

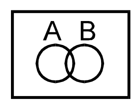
Note

$P(\bar{A})$ or $P(A')$ or $P(A^C)$, i.e. probability of non-occurrence of A = $\frac{n}{m+n} = 1 - P(A)$ In the above we shall denote the number of out-comes favorable to the event A by n(a) and the total number of out comes in the sample space S by n(S).

$\therefore P(a) = \frac{n(A)}{n(S)}$.

3. ADDITION THEOREM OF PROBABILITY

If 'A' and 'B' are any two events associated with an experiment, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



De Morgan's laws: If A & B are two subsets of a universal set U, then

(a) $(A \cup B)^C = A^C \cap B^C$

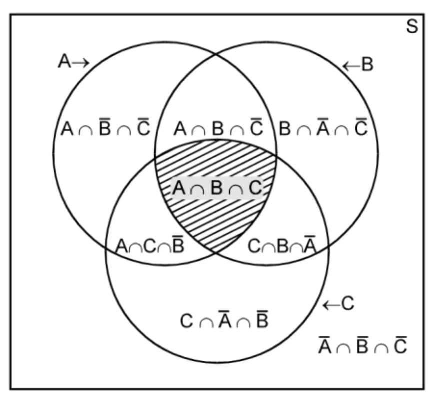
(b) $(A \cap B)^C = A^C \cup B^C$

Distributive laws:

(a) $A \cup B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap B \cup C) = (A \cap B) \cup (A \cap (c))$

For any three events A, B and C we have the figure



(i) $P(A \text{ or } B \text{ or } C) = P((A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(ii) P (at least two of A, B, C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 2P(A \cap B \cap C)$

(iii) P(exactly two of A, B, C occur) = $P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$

(iv) P(exactly one of A, B, C occur) = $P(A) + P(B) + P(C) - 2P(B \cap (c) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

4.CONDITIONAL PROBABILITY

If A and B are two events, then $P(A/B) = \frac{P(A \cap B)}{P(B)}$

Note

For mutually exclusive events P(A/B) = 0.

5. INDEPENDENT AND DEPENDENT EVENTS

If two events are such that occurence or non-occurence of one does not affect the chances of occurence or non-occurence of the other event, then the events are said to be independent. Mathematically: if $P(A \cap B) = P(A)\ P(B)$, then A and B are independent.

Note

(i) If A and B are independent, then

(a) A′ and B′ are independent,

(b) A and B′ are independent and

(c) A′ and B are independent.

(ii) If A and B are independent, then P(A / B) = P(A).

If events are not independent, then they are said to be dependent.

6. INDEPENDENCY OF THREE OR MORE EVENTS

Three events A, B & C are independent if & only if all the following conditions hold:

$P(A \cap B) = P(A) . P(B)$; $P(B \cap C) = P(B) . P(C)$

$P(C \cap A) = P(C) . P(A)$; $P(A \cap B \cap C) = P(A) . P(B) . P(C)$

7. BINOMIAL PROBABILITY THEOREM

The probability of getting exactly r success in n trials of such an experiment is ${}^nC_r\ p^r\ q^{n-r}$, where 'p' is the probability of a success and q is the probability of a failure in one particular experiment.

Note

p + q = 1.

8. TOTAL PROBABILITY THEOREM

If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots, B_n$ and the probabilities $P(A/B_1), P(A/B_2) \ldots P(A/B_n)$ are known, then

$$P(a) = \sum_{i=1}^{n} P(B_i) . P(A/B_i)$$

Proof:

The event A occurs with one of the n mutually exclusive and exhaustive events $B_1, B_2, B_3, \ldots, B_n$

$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \ldots\ldots \cup (A \cap B_n)$

$P(a) = P(A \cap B_1) + P(A \cap B_2) + \ldots\ldots + P(A \cap B_n) = \sum_{i=1}^{n} P(A \cap B_i)$ $P(A \cap B_i) = P(a) . P(B_i/A) = P(B_i) . P(A/B_i)$

$\therefore P(a) = \sum_{i=1}^{n} P(B_i) . P(A/B_i)$

9. BAYE'S THEOREM :

If an event A can occur with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots, B_n$ and the probabilities $P(A/B_1), P(A/B_2) \ldots P(A/B_n)$ are known, then

$$P(B_i / (a) = \frac{P(B_i) . P(A/B_i)}{\sum_{i=1}^{n} P(B_i) . P(A/B_i)}$$

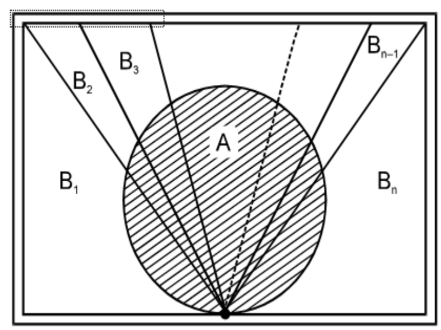
Proof:

The event A occurs with one of the n mutually exclusive and exhaustive events $B_1, B_2, B_3, \ldots, B_n$

$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \ldots\ldots \cup (A \cap B_n)$

$P(a) = P(A \cap B_1) + P(A \cap B_2) + \ldots\ldots + P(A \cap B_n) = \sum_{i=1}^{n} P(A \cap B_i)$

Now,



$P(A \cap B_i) = P((a) . P(B_i/(a) = P(B_i) . P(A/B_i)$

$$P(B_i/(a) = \frac{P(B_i) . P(A/B_i)}{P(A)} = \frac{P(B_i) . P(A/B_i)}{\sum_{i=1}^{n} P(A \cap B_i)}$$

$$P(B_i/(a) = \frac{P(B_i) . P(A/B_i)}{\sum P(B_i) . P(A/B_i)}$$

10. PROBABILITY DISTRIBUTION

(i) A probability distribution spells out how a total probability of 1 is distributed over several values of a random variable (i.e. how .... possibilities)

(ii) Mean of any probability distribution of a random variable is given by :

$$\mu = \frac{\Sigma p_i x_i}{\Sigma p_i} = \sum_{i=1}^{n}(x_i p_i) \text{ (Since } \Sigma p_i = 1)$$

(iii) Variance of a random variable is given by, $\sigma^2 = \sum_{i=1}^{n}(x_i - \mu)^2 p(x_i)$

Note

(Note that SD = $+\sqrt{\sigma^2}$)

(iv) The probability distribution for a binomial variate 'X' is given by :

$P(X = r) = {}^nC_r\, p^r\, q^{n-r}$ where P(X = r) is the probability of r successes. (recurrence The recurrence formula $\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1} \cdot \frac{p}{q}$ is very helpful for quickly computing P(a) . P(b) . P(c) etc. if P(0) is known.

Mean of Binomial Probability Distribution = np; variance of Binomial Probability Distribution = npq.

(v) If p represents a person's chance of success in any venture and 'M' the sum of money which he will receive in case of success, then his expectations or probable value = pM

QUESTIONS

MCQ

Q1. Let A and B be the events such that $2\mathrm{P(A)} = \mathrm{P(B)} = \frac{5}{13}$ and $\mathrm{P}\left(\frac{\mathrm{A}}{\mathrm{B}}\right) = \frac{2}{5}$. Find $P(A \cap B)$.

(a) $\frac{11}{13}$ (b) $\frac{2}{13}$

(c) $\frac{1}{13}$ (d) None

Q2. A biased die is thrown. The probability of a particular outcome is $\frac{1}{6} + y$ and the probability of opposite face outcome is $\frac{1}{6} - y$ and the probability of each remaining outcome is $\frac{1}{6}$. The die is thrown again for 2 times and the probability of getting the sum 7 is $\frac{13}{96}$. Then value of $64y^2 - 1$ is: -

(a) 5 (b) 4

(c) 1 (d) 0

Q3. Let A and B be the events such that $P(A) = \frac{7}{13}, P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$ Find $P(A/B)$.

(a) $\frac{4}{9}$ (b) $\frac{3}{7}$

(c) $\frac{1}{7}$ (d) $\frac{5}{7}$

Q4. If E_1 and E_2 are independent events such that $P(E_1)=0.3$ and $P(E_2) = 0.4$, find $P(\bar{E}_1 \cap \bar{E}_2)$

(a) 0.42 (b) 0.14

(c) 0.41 (d) 0.35

Q5. A company manufactures scooters at two plants, A and B. plant A produces 80% and Plant B produces 20% of the total product. 85% of the scooters produced at plant A and 65% of the scooters produced at plant B are of standard quality. A scooter produced by the company is selected at random, and it is found to be of standard quality. What is the probability that it was manufactured at plant A?

(a) $\frac{65}{81}$ (b) $\frac{63}{81}$

(c) $\frac{62}{81}$ (d) $\frac{68}{81}$

Q6. Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find P(A or B), if A and B are mutually exclusive events.

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

(c) $\frac{2}{5}$ (d) None

Q7. A and B are two events such that P(A) = 0.54, P(B) = 0.69 and $P(A \cap B)$= 0.35.
Find $P(B \cap A')$.

(a) 0.35 (b) 0.32

(c) 0.34 (d) 0.31

Q8. If A and B are events such that $\mathrm{P}(A \mid B) = P(B \mid A)$ then which of the following is true?

(a) $A \subset B$ but $A \neq$B(b) $A = B$

(c) $A \cap B = \varphi$ (d) $P(A) = P(B)$

Q9. There are 6% defective items in a large bulk of times. Find the probability that a sample of 8 items will include not more than one detective item.

(a) $\left(\frac{47}{50}\right)^6 \times \left(\frac{71}{50}\right)$ (b) $\left(\frac{47}{50}\right)^5 \times \left(\frac{71}{50}\right)$

(c) $\left(\frac{47}{50}\right)^7 \times \left(\frac{71}{50}\right)$ (d) $\left(\frac{47}{50}\right)^7 \times \left(\frac{71}{50}\right)^2$

Q10. Three cars participate in a race. The probability that any one of them has an accident is 0.1. Find the probability that all the cars reach the finishing line without any accident.

(a) 0.5 (b) 0.71

(c) 0.61 (d) 0.729

Q11. A man can hit a bird, once in 3 shots. On this assumption he fires 3 shots. What is the chance that at least one bird is hit?

(a) $\frac{19}{27}$ (b) $\frac{17}{27}$

(c) $\frac{14}{27}$ (d) None

Q12. An experiment fails twice as often as it succeeds. The probability of at least 5 failures in the six trials of this experiment is

(a) $\frac{192}{729}$ (b) $\frac{256}{729}$

(c) $\frac{240}{729}$ (d) $\frac{496}{729}$

Q13. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event " A. Then the events A and B are

(a) Independent but not equally likely

(b) Independent and equally likely

(c) Mutually exclusive and independent

(d) Equally likely but not independent

Q14. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is

(a) $\frac{235}{256}$ (b) $\frac{21}{256}$

(c) $\frac{3}{256}$ (d) $\frac{253}{256}$

Q15. Two marbles are drawn successively from a box containing 3 black and 4 white marbles. Find the Probability that both the marbles are black if the first marble is not replaced before the second draw.

(a) $\frac{1}{7}$ (b) $\frac{1}{5}$

(c) $\frac{1}{6}$ (d) $\frac{2}{7}$

Q16. There is a box containing 30 bulbs, of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs are chosen are defective?

(a) $\frac{2}{87}$ (b) $\frac{3}{87}$

(c) $\frac{5}{87}$ (d) $\frac{10}{87}$

Q17. Given the probability that A can solve a problem is 2/3, and the probability that B can solve the same problem is $\frac{3}{5}$, find the probability that at least one of A and B will solve the problem

(a) $\frac{11}{15}$ (b) $\frac{13}{15}$

(c) 1 (d) $\frac{9}{15}$

Q18. A problem is given to three students whose chances of solving it are 1/4, 1/5 and 1/6,respectively. Find the probability that the problem is solved.

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{1}$

Q19. An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shots are 0.4,0.3,0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

(a) 0.521 (b) 0.428

(c) 0.697 (d) 0.314

Q20. A bag A contains 1 white and 6 red balls. Another bag contains 4 white and 3 red balls. One Of the bags is selected at random, and a ball is drawn from it, which is found to be white.

Find the probability that the ball is drawn is from bag A.

(a) 1/6 (b) 1/7

(c) 1/5 (d) 1/3

Q21. A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of the cars, and plant Y manufactures 30%. At pant X, 80% of the cars are rated of standard quality, and at plant Y, 90% are rated of standard quality. A car is picked up at random and is found to be of standard quality. A car is picked up at random and is found to be of standard quality. Find the probability that it has come from plant X.

(a) $\frac{53}{83}$ (b) $\frac{43}{83}$

(c) $\frac{46}{83}$ (d) $\frac{56}{83}$

Q22. An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0.01, and that of motorcycles is 0.02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle.

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$

(c) $\frac{7}{8}$ (d) $\frac{3}{5}$

Q23. The probability of the safe arrival of one ship out of 5 is $\frac{1}{5}$. What is the probability of the safe arrival of at least 3 ships?

(a) $\frac{1}{31}$ (b) $\frac{3}{52}$

(c) $\frac{181}{3125}$ (d) $\frac{184}{3125}$

Q24. The probability that an event E occurs in one trial is 0.4, Three independent trials of the experiment are performed. What is the probability that E occurs at least once?

(a) 0.784

(b) 0.936

(c) 0.964

(d) None

Q25. A coin is tossed 5 times. What is the probability that the head appears an even number of times?

(a) $\frac{3}{5}$ (b) $\frac{2}{15}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Q26. An unbiased die is tossed twice. What is the probability of getting a 4, 5 or 6 on the first toss and 1, 2, 3 or 4 on the second toss?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{4}$ (d) $\frac{5}{6}$

Q27. A fair coin is tossed 6 times. What is the probability of getting at least 3 heads?

(a) $\frac{11}{16}$ (b) $\frac{21}{32}$

(c) $\frac{1}{18}$ (d) $\frac{3}{64}$

Q28. 5 cards are drawn successively from a well-shuffled pack of 52 cards with replacement. Determine the probability that , all the five cards should be spades?

(a) $\frac{1}{256}$

(b) $\frac{1}{1442}$

(c) $\frac{1}{1024}$

(d) None of these

Q29. Let P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$, and P (A$\cup B) = \frac{7}{11}$. Find P(B|A)

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) None of these

Q30. An fair die is thrown double times. Assume that the event A is “odd number on the first throw” and B the event “odd number on the second throw”.

(a) Two events A and B are independent events

(b) Two events A and B are dependent events

(c) Two events A and B are not independent events

(d) None of these

SUBJECTIVE QUESTIONS

Q1. In throwing of a fair die find the probability of the event of getting a prime numbers.

Q2. A four-digit number is formed using the digits 0, 1, 2, 3, 4 without repetition. Find the probability that it is divisible by 3.

Q3. A bag contains 4 white, 3red and 4 green balls. A ball is drawn at random. Find the probability of the event 'the ball drawn is white or green'.

Q4. If P(A/B) = 0.2 and P(b) = 0.5 and P(a) = 0.2. Find $P(\bar{A} \cap B)$.

Q5. A pair of dice is thrown 5 times. Find the probability of getting a doublet twice.

NUMERICAL TYPE QUESTIONS

Q1. Two dies are rolled simultaneously. The probability that the sum of the two numbers on the top faces will be at least 10 is_______.

Q2. A number is chosen at random among the set of first 120 natural numbers the probability of the number chosen from the set being a multiple of 5 or 15, is_______.

Q3. Thirteen persons take their places at a round table, then the probability that two particular persons sitting together are ____.

Q4. If A and B are events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(\bar{A}) = \frac{2}{3}$, then $P(\bar{A} \cap B)$ is_______.

Q5. If odds against solving a question independently by three students are 2 : 1 , 5 : 2 and 5 : 3 respectively, then probability that the question is solved only by one student is_______.

TRUE AND FALSE

Q1. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval $[0,1]$

Q2. Let A and B be two event such that $P\left(\overline{A \cup B}\right) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P\left(\bar{A}\right) = \frac{1}{4}$, where $\bar{A}$ stands for the complement of the event A. Then the events A and B are independent.

Q3. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then E_1, E_2 *and* E_3 are independent.

Q4. For three events A, B and C, P (Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P (Exactly one of C or A occurs) = $\frac{1}{4}$ and P (All the three events occur simultaneously) = $\frac{1}{6}$ .Then the probability that at least one of the events occurs, is $\frac{7}{18}$.

Q5. A coin is tossed 4 times. The probability that at least one head turns up, is $\frac{15}{16}$

ASSERTION AND REASONING

Directions: (Q1-Q5) : In the following questions , A statement of assertion (A) is followed by a statement of Reason (R) .Mark the correct choice as

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

Q1. **Assertion (A) :** Let A and B be two events such that P(A) $= \frac{1}{5}$, while P(A or B) $= \frac{1}{2}$.Let P (B) = P , then for P $= \frac{3}{8}$, A and B independent.

Reason (R): For independent events,

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\quad = P(A) + P(B) - P(A)P(B)$

Q2. **Assertion (A) :** If A and B are two mutually exclusive events with $P(\bar{A}) = \frac{5}{6}$ and $P(B) = \frac{1}{3}$ .Then $P(A/\bar{B})$ is equal to $\frac{1}{4}$.

Reason (R): If A and B are two events such that P(A) = 0.2 , P(B) = 0.6 and P(A|B)= 0.2 then the value of $P(A|\bar{B})$ is 0.2

Q3. **Assertion (A) :** If A is proper subset of B and B is proper subset of A $\Rightarrow P(A) = P(B)$

Reason (R): If A is proper subset of B then $P(\bar{A}) \leq P(\bar{B})$

Q4. **Assertion (A) :** The probability of an impossible event is 1

Reason (R): If A is a perfect subset of B and P(A) < P(B) , then P(B – A) is equal to P(B) – P(A)

Q5. **Assertion (A) :** Let A and B be two events such that the occurrence of A implies occurrence of B , but not vice-versa , then the correct relation between P(A) and P(B) is P(B)$\geq P(A)$.

Reason (R): Here according to the given statement A is a subset of B

P(B) = $P(A \cup (A \cap B))$

P(B) = P(A) + $P(A \cap B)$

$\therefore P(B) \geq P(A)$

HOMEWORK

MCQ

Q1. 2 /3rd of the students in a class are boys & the rest girls. It is known that probability of a girl getting a first class is 0.25 & that of a boy is 0.28. The probability that a student chosen at random will get a first class is:

(a) 0.26 (b) 0.265
(c) 0.27 (d) 0.275

Q2. A basket contains 5 apples and 7 oranges and another basket contains 4 apples and 8 oranges. One fruits is picked out form each basket. Find the probability that both fruits are apples or both are oranges

(a) $\frac{24}{144}$ (b) $\frac{56}{144}$
(c) $\frac{68}{144}$ (d) $\frac{76}{144}$

Q3. Two cards are drawn successively from a well-shuffled ordinary deck of 52-playing cards without replacement and is noted that the second card is a king. The probability of the event 'first card is also a 'king' is

(a) $\frac{2}{19}$ (b) $\frac{1}{17}$
(c) $\frac{3}{49}$ (d) $\frac{4}{5}$

Q4. A bag contains (n + 1) coins. It is known that one of these coins has a head on both sides, whereas the other coins are normal. One of these coins is selected at random & tossed. If the probability that the toss results in head, is 7/12, then the value of n is.

(a) 5 (b) 6
(c) 4 (d) 3

Q5. A die is tossed thrice. A success is getting 1 or 6 on a toss. The mean and the variance of number of successes

(a) $\mu = 1, \sigma^2 = 2/3$ (b) $\mu = 2/3, \sigma^2 = 1$
(c) $\mu = 2, \sigma^2 = 2/3$ (d) none of these

Q6. In a series of 3 independent trials the probability of exactly 2 success is 12 times as large as the probability of 3 successes. The probability of a success in each trial is:

(a) 1/5 (b) 2/5
(c) 3/5 (d) 4/5

Q7. If on an average 1 vessel in every 10 is wrecked, then the chance that out of 5 vessels expected 4 at least will arrive safely is

(a) $\frac{45981}{50000}$ (b) $\frac{1}{10}$
(c) $\frac{45927}{50000}$ (d) 1

Q8. An unbiased coin is tossed n times. Let X denote the number of times head occurs. If P(X = 4), P (X = 5) and P(X = 6) are in AP, then the value of n can be

(a) 9 (b) 10
(c) 12 (d) 14

Q9. India decides to destroy one of the militants holdings. In the bombing attack there is 50% chance of a bomb hitting the target, only two direct bomb hits are required to destroy the target completely. Least number of bombes required to give 99% chance or better of completely destroying the target is

(a) 9 (b) 10
(c) 11 (d) 12

Q10. A fair die is tossed eight times. The probability that a third six is observed on the eight throw, is

(a) $\frac{{}^7C_2 \times 5^5}{6^7}$ (b) $\frac{{}^7C_2 \times 5^5}{6^8}$
(c) $\frac{{}^7C_2 \times 5^5}{6^6}$ (d) None of these

SUBJECTIVE QUESTIONS

Q1. Events A, B, C are mutually exclusive events such that P(a) = $\frac{3x+1}{3}$, P (b) = $\frac{1-x}{4}$ and P (c) = $\frac{1-2x}{2}$. The set of possible values of x are in the interval:

Q2. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then P(X = 1) is :

Q3. The probability that A speaks truth is 4/5 while this probability for B is 3/4. The probability that they contradict each other when asked to speak on a fact, is

Q4. A random variable X has the probability distribution:

X :	1	2	3	4	5	6	7	8
P(X) :	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {X < 4}, the probability P(E ∪ F) is :

Q5. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap (b)) = \frac{1}{4}$ and $P(\bar{A}) = \frac{1}{4}$, where $\bar{A}$ stands for complement of event A. Then events A and B are:

NUMERICAL TYPE QUESTIONS

Q1. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others, the probability that all the three apply for the same house, is_____.

Q2. It is given that the events A and B are such that P(a) = $\frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$. Then, P (b) is______.

Q3. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is________.

Q4. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, equal _______.

Q5. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is_____.

TRUE AND FALSE

Q1. There are 4 white and 4 black balls in a bag and 3 balls are drawn at random. If balls of same colour are identical, the probability that none of them is black, is $\frac{1}{2}$

Q2. A box contains 10 mangoes out of which 4 are rotten. 2 mangoes are taken out together. If one of them is found to be good, the probability that the other is also good is $\frac{8}{15}$

Q3. A five digit number is chosen at random. The probability that all the digit are distinct and digits at odd places are odd and digits at even place are even, is $\frac{7}{17}$

Q4. If X follows a binomial distribution with parameters $n = 6$ and P. If $4(P(X = 4)) = P(X = 2)$, then P = $\frac{1}{3}$

Q5. Probability of throwing 16 in one throw with three dice is $\frac{1}{36}$

ASSERTION AND REASONING

Directions: (Q1-Q5) : In the following questions , A statement of assertion (a) is followed by a statement of Reason (R) .Mark the correct choice as

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q1. **Assertion (a) :** If P (a) = 0.4, P (b) = 0.48 and $P(A \cap (b)) = 0.16$, then the value of P(A / B) is $\frac{1}{3}$
Reason (R): A and B are independent events then $P(A \cap B) = P(A).P(B)$

Q2. **Assertion (a) :** If each outcome of an experiment is associated with at least one of the events E_1, E_2, E_3, ...E_n, then collectively the events are said to be exhaustive
Reason (R): Mathematically we write $E_1 + E_2 + E_3$........ $E_n = S$

Q3. **Assertion (a) :** If A and B are two events, then P(A/ (b) $= \frac{P(A \cap B)}{P(B)}$
Reason (R): For mutually exclusive events $P(A/B) \neq 0$.

Q4. **Assertion (a) : De Morgan's laws:** If A & B are two subsets of a universal set U, then
(a) $(A \cup B)^C \neq A^C \cap B^C$
Reason (R): When two or more than two events occur simultaneously, the event is said to be a compound event.

Q5. **Assertion (a) :** The probability distribution for a binomial variate 'X' is given by :
$P(X = r) = {}^nC_r\, p^r q^{n-r}$ where P(X = r) is the probability of r successes
Reason (R): If an event A can occur with one of the n mutually exclusive and exhaustive events B_1, B_2, ..., B_n and the probabilities $P(A/B_1)$, $P(A/B_2)$ .... $P(A/B_n)$ are known, then

$$P(B_i / (a)) = \frac{P(B_i).P(A/B_i)}{\sum_{i=1}^{n} P(B_i).P(A/B_i)}$$

SOLUTIONS

MCQ

S1. (b) $P(A/B)=\frac{P(A\cap B)}{P(B)}$

$=P(A\cap B)=P(B)P(A/B)$

$=\frac{5}{13}\times\frac{2}{5}$

$=\frac{2}{13}$

S2. (d) Given,

$P_1=\frac{1}{6}+y, P_6=\frac{1}{6}-y,$

$P_3=P_4=P_5=P_2=\frac{1}{6}$

$P(\text{ sum }=7)=2P(6,1)+2P(5,2)+2P(4,3)$

$\Rightarrow 2\left(\frac{1}{6}+y\right)\left(\frac{1}{6}-y\right)+2\left(\frac{1}{6}\right)^2+2\left(\frac{1}{6}\right)^2=\frac{13}{96}$

$\Rightarrow 2\left(\frac{1}{36}-y^2+\frac{1}{36}+\frac{1}{36}\right)=\frac{13}{96}$

$\Rightarrow 2\left(\frac{1}{12}-y^2\right)=\frac{13}{96}\Rightarrow 2y^2=\frac{1}{6}-\frac{13}{96}=\frac{3}{96}=\frac{1}{32}$

$y^2=\frac{1}{64}\Rightarrow y=\frac{1}{8}$

$64y^2-1=64\left(\frac{1}{8}\right)^2-1=0$

S3. (a)

$\mathrm{P(A/B)}$

$=\frac{\mathrm{P(A\cap B)}}{\mathrm{P(B)}}$

$=\frac{4}{13}\div\frac{9}{13}$

$=\frac{4}{7}$

S4. (a)

$\mathrm{P}\left(\bar{E_1}\cap\bar{E_2}\right)=\mathrm{P}\left(\bar{E_1}\right)\times\mathrm{P}\left(\bar{E_2}\right)$

since, $P(E_1)=0.3$ and $P(E_2)=0.4$

$\Rightarrow \mathrm{P}\left(\bar{E_1}\right)=1-\mathrm{P}(\mathrm{E}_1)=0.7$ and $\mathrm{P}\left(\bar{E_2}\right)=1-\mathrm{P}(\mathrm{E}_2)=0.6$

Since, E_1 and E_2 are two independent events

$\Rightarrow \bar{E_1}$ and $\bar{E_2}$ are also independent events.

Therefore, $\mathrm{P}\left(\bar{E_1}\cap\bar{E_2}\right)=0.7\times0.6=0.42$

S5. (d) Let S : Standard quality We want to find P(A|S), i.e. probability that selected standard scooter is from plant A

$\mathrm{P(A\mid S)}=\frac{\mathrm{P(A)\cdot P(S|A)}}{\mathrm{P(A)\cdot P(S|A)+P(B)\cdot P(S|B)}}$

Where, P(A)= probability that scooter is from A=$\frac{80}{100}$

P(B)= probability that scooter is from B = $\frac{20}{100}$

P(S|A)= probability that standard scooter from A=$\frac{85}{100}$

P(S|B)= probability that standard scooter from B=$\frac{65}{100}$

$\mathrm{P(A\mid S)}=\frac{(80)(85)}{(80)(85)+(20)(65)}$

$=\frac{6800}{6800+1300}=\frac{68}{81}$

S6. (a)

Here, $\mathrm{P(A)}=\frac{3}{5},\mathrm{P(B)}=\frac{1}{5}$

For mutually exclusive events A and B,

$P(A\text{ or }B)=P(A)+P(B)$

$P(A\text{ or }B)=\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$

S7. (c) We know that $n(\mathrm{B}\cap\mathrm{A}')=n(\mathrm{B})-n(\mathrm{A}\cap\mathrm{B})$

$\Rightarrow\frac{n(\mathrm{B}\cap\mathrm{A}')}{n(\mathrm{S})}=\frac{n(\mathrm{B})}{n(\mathrm{S})}-\frac{n(\mathrm{A}\cap\mathrm{B})}{n(\mathrm{S})}$

$\therefore \mathrm{P}(\mathrm{B}\cap\mathrm{A}')=\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A}\cap\mathrm{B})$

$\therefore \mathrm{P}(\mathrm{B}\cap\mathrm{A}')=0.69-0.35=0.34$

S8. (d)

$P(A\mid B)=P(B\mid A)$

$\frac{P(A\cap B)}{P(B)}=\frac{P(B\cap A)}{P(A)}$

$\Rightarrow P(A)=P(B)$

Thus option (d)is correct.

S9. (d) Using Bernoulli's Trial P(Success =x)= ${}^nC_x\cdot p^x\cdot q^{(n-x)}, x=0,1,2,\ldots n$ and $q=(1-p), n=8$

The probability of success, i.e. the bulb is defective =$p=\frac{6}{100}$

$q=1-\frac{6}{100}=\frac{94}{100}$

probability of that there is not more than one defective piece = P(0 defective items) + P(1 defective item) =

${}^8C_0\cdot\left(\frac{6}{100}\right)^0\left(\frac{94}{100}\right)^8+{}^8C_1\cdot\left(\frac{6}{100}\right)^1\left(\frac{94}{100}\right)^7$

$\Rightarrow\left(\left(\frac{47}{50}\right)^7\times\left(\frac{71}{50}\right)\right)$

S10. (d) The probability that any one of them has an accident is 0.1. The probability any car reaches safely is 0.9. The probability that all the cars reach the finishing line without any accident is = (0.9) (0.9) (0.9) = 0.729

S11. (a) The probability that the bird will be shot, is 1/3

Using Bernoulli's Trial we have, $P(\text{Success }=x)={}^nC_x\cdot p^x\cdot q^{(n-x)}$

$x=0,1,2,\ldots n$ and $q=(1-p), n=3$

$p=\frac{1}{3}\quad q=2/3$

$\Rightarrow\quad {}^3C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2+{}^3C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1+{}^3C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0$

$\Rightarrow\frac{19}{27}$

S12. (b) Given failures are twice as likely as success $\Rightarrow p=2q$ and we know that $p+q=1$

$\Rightarrow p=2/3, q=1/3$

The probability of at least 5 failures is

$={}^6\mathrm{C}_5\mathrm{P}^5\mathrm{q}+{}^6\mathrm{C}_6\mathrm{P}^6$

$=6\times\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)+1\times\left(\frac{2}{3}\right)^6=\frac{256}{729}$

S13. (a) $P(A \cup B)' = \frac{1}{6} \Rightarrow P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}$

$P(\bar{A}) = \dfrac{1}{4} \Rightarrow P(A) = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$

$P(B) = \frac{1}{3}$

$\because P(A) \neq P(B)$ so they are not equally likely.

Also $P(A) \times P(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

$= P(A \cap B)$

$\because P(A \cap B) = P(A) \cdot$

$P(B)$ so $A \& B$ are independent.

S14. (a) $\mathrm{P}(\text{ at least one of them solves correctly }) = 1 -$ P (none of them solves correctly)

$= 1 - \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}\right) = \frac{235}{256}$

S15. (a) Given: A box containing 3 black and 4 white marbles. Each trail is independent of the other trial.

Hence the sample space is given by $\mathrm{S} = \{1\mathrm{B}, 2\mathrm{B}, 3\mathrm{B}, 1\mathrm{W}, 2\mathrm{W}, 3\mathrm{W}, 4\mathrm{W}\}$

To find: the probability that both the marbles are drawn are black.

Let, success : marble drawn is black. i.e., $\frac{3}{7}$

Now, the Probability of success in the first trial is $\mathrm{P}_1(\text{ success }) = \frac{3}{7}$

Probability of success in the second trial without replacement of the first draw is given by $P_2(\text{ success }) = \frac{2}{6}$

Hence, the probability that both the marbles are drawn are black, with each trial being independent is given by

$P_1 \times P_2 = \frac{3}{7} \times \frac{2}{6} = \frac{1}{7}$

S16. (a) Given: A box containing 30 bulbs of which 5 are defective. Each trail is independent of the other trial.

To find: the probability that both the bulbs are chosen are defective.

Let, success :bulb chosen is defective .i.e $\dfrac{5}{30}$

Now , the Probability of success in the first trial is

$P_1(\text{ success }) = \frac{5}{30}$

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{ success }) = \frac{4}{29}$

Hence, the probability that both the bulbs are chosen are defective, with each trial being independent is Given by

$P_1 \times P_2 = \frac{5}{30} \times \frac{4}{29} = \frac{2}{87}$

S17. (b) Given : Here probability of A and B that can solve the same problem is given, i.e., P(A)=$\frac{2}{3}$ and $P(B) = \frac{3}{5} \Rightarrow$ $P(\bar{A}) = \frac{1}{3}$ and $\mathrm{P}(\bar{B}) = \frac{2}{5}$

Also, A and B are independent. Not A and not B are independent.

"At least one of " A" and " B" will solve the problem"

Now , P(at least one of them will solve the problem)

= 1-P(both are unable to solve)

$= 1 - \mathrm{P}(\bar{A} \cap \bar{B})$

$= 1 - \mathrm{P}(\bar{A}) \times \mathrm{P}(\bar{B})$

$= 1 - \left(\frac{1}{3} \times \frac{2}{5}\right)$

$= \frac{13}{15}$

Therefore, at least one of A and B will solve the problem is $\frac{13}{15}$

S18. (c) Given: let A,B and C be three students whose chances of solving a problem is given i.e.,

$P(A) = \frac{1}{4}, P(B) = \frac{1}{5}$ and $P(C) = \frac{1}{6}$

$\Rightarrow \mathrm{P}(\bar{A}) = \frac{3}{4}, \mathrm{P}(\bar{B}) = \frac{4}{5}$ and $\mathrm{P}(\bar{C}) = \frac{5}{6}$

To Find: The probability that the problem is solved.

Here, P(the problem is solved)=1-P (the problem is not solved)

$= 1 - \mathrm{P}(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - [\mathrm{P}(\bar{A}) \times \mathrm{P}(\bar{B}) \times \mathrm{P}(\bar{C})]$

$= 1 - \left[\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}\right]$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

Therefore, The probability that the problem is solved is "$\frac{1}{2}$.

S19. (c) Given: Let A,B, C and D be first second third and fourth shots whose probability of hitting the plane is given i.e, $\mathrm{P(A)} = 0.4, \mathrm{P(B)} = 0.3, \mathrm{P(C)} = 0.2$ and $\mathrm{P(D)} = 0.1$ respectively

$\Rightarrow \mathrm{P}(\bar{A}) = 0.6$ and $\mathrm{P}(\bar{B}) = 0.7$ and $\mathrm{P}(\bar{C}) = 0.8$ and $\mathrm{P}(\bar{D}) = 0.9$

To Find: The probability that at least one shot hits the plane.

Here, P(at least one shot hits the plane)=1-P (none of the shots hit the plane)

$= 1 - \mathrm{P}(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$= 1 - [\mathrm{P}(\bar{A}) \times \mathrm{P}(\bar{B}) \times \mathrm{P}(\bar{C}) \times \mathrm{P}(\bar{D})]$

$= 1 - [0.6 \times 0.7 \times 0.8 \times 0.9]$

$= 1 - 0.3024$

$= 0.6976$

Therefore, The probability that at least one shot hits the plane is 0.6976.

S20. (c) Let R: Red ball

W: White ball

A: Bag A

B : Bag B

Assuming, selecting bags is of equal probability *i.e.* $\frac{1}{2}$

We want to find " P(A|W)", i.e. the selected white ball is from bag " A

$$P(A \mid W)=\frac{P(A) \cdot P(W \mid A)}{P(A) \cdot P(W \mid A)+P(B) \cdot P(W \mid B)}$$

$$=\frac{\left(\frac{1}{2}\right)\left(\frac{1}{7}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{7}\right)+\left(\frac{1}{2}\right)\left(\frac{4}{7}\right)}$$

$$=\frac{1}{5}$$

Conclusion: Therefore, the probability of selected white ball is from bag A is $\frac{1}{5}$

S21. (d) Let X: Car produced from plant X

Y: Car produced from plant Y

S: Car rated as standard quality we want to find P(X|S), i.e., selected standard quality car is from plant X

$$\mathrm{P}(\mathrm{X} \mid \mathrm{S})=\frac{\mathrm{P}(\mathrm{X}) \cdot \mathrm{P}(\mathrm{S} \mid \mathrm{X})}{\mathrm{P}(\mathrm{X}) \cdot \mathrm{P}(\mathrm{S} \mid \mathrm{X})+\mathrm{P}(\mathrm{Y}) \cdot \mathrm{P}(\mathrm{S} \mid \mathrm{Y})}$$

$$=\frac{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right)}{\left(\frac{70}{100}\right)\left(\frac{80}{100}\right)+\left(\frac{30}{100}\right)\left(\frac{90}{100}\right)}$$

$$=\frac{56}{83}$$

Conclusion: Therefore, the probability of selected standard quality car is from plant X is $\frac{56}{83}$

S22. (a) Let M: Motorcycle

S: Scooter

A: Accident vehicle

We want to find P(M|A), i.e., probability of accident vehicle was a motorcycle

$$\mathrm{P}(\mathrm{M} \mid \mathrm{A})=\frac{\mathrm{P}(\mathrm{M}) \cdot \mathrm{P}(\mathrm{A} \mid \mathrm{M})}{\mathrm{P}(\mathrm{M}) \cdot \mathrm{P}(\mathrm{A} \mid \mathrm{M})+\mathrm{P}(\mathrm{S}) \cdot \mathrm{P}(\mathrm{A} \mid \mathrm{S})}$$

$$=\frac{\left(\frac{3000}{5000}\right)(0.02)}{\left(\frac{3000}{5000}\right)(0.02)+\left(\frac{2000}{5000}\right)(0.01)}$$

$$=\frac{6}{8}$$

$$=\frac{3}{4}$$

Conclusion: Therefore, the probability of accident vehicle was motorcycle is $\frac{3}{4}$

S23. (d) The probability of safe arrival of the ship is 1/5

Using Bernoulli's Trial we have,

$P(\text{ Success }=x)={ }^n C_x \cdot p^x \cdot q^{(n-x)}$

x=0, 1, 2, ………n and q = (1-p), n =5 p = 1/5, q = 4/5

Probability of safe arrival of at least 3 ships is = P (3) +P (4) +P (5)

$\Rightarrow \quad { }^5 C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2+{ }^5 C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^1+{ }^5 C_5\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^0$

$\Rightarrow \quad \frac{181}{3125}$

S24. (a) The probability of occurrence of an event E in one trial is 0.4

Using Bernoulli's Trial we have,

$P(\text{ Success }=x)={ }^n C_x \cdot p^x \cdot q^{(n-x)}$

$x=0,1,2$ and $q=(1-p), n=3$

p = 0.4 , q = 0.6

The probability that E occurs at least once is, P(1) +P(2)+P(3)

$\Rightarrow \quad { }^3 C_1\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^2+{ }^3 C_2\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^1+{ }^3 C_3\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^0$

$\Rightarrow \quad \frac{98}{125}=0.784$

25. © Using Bernoulli's Trial P(Success =x)= ${ }^n C_x \cdot p^x \cdot q^{(n-x)}$

x=0, 1, 2, ………n and q = (1-p)

As the coin is tossed 5 times the total number of outcomes will be 2^5.

And we know that the favorable outcomes of getting the odd tail number of times ,successes will be, getting a tail

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

${ }^5 C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4+{ }^5 C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2+{ }^5 C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$

$\Rightarrow \frac{16}{32}=\frac{1}{2}$

S26. (a) A die is tossed twice, The probability of getting a 4, 5 or 6 in the first trial is 3/6 = P(a)

The probability of getting a 1, 2, 3 or 4 in the second trial is 4/6 =P(B)

As the events are independent, the probability of these two events together will be, P(A).P(b) = 1/3.

S27. (b) Using Bernoulli's Trial $P(\text{ Success }=x)={ }^n C_x \cdot p^x \cdot q^{(n-x)}$

x=0, 1, 2, ………n and q = (1-p)

As the coin is thrown 6 times the total number of outcomes will be 2^6.

And we know that the favorable outcomes of getting at least 3 successes will be, getting a head

The probability of success is $\frac{1}{2}$ and of failure is also $\frac{1}{2}$

& ${ }^6 C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3+{ }^6 C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2+{ }^6 C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1+{ }^6 C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0$

& $\Rightarrow \frac{21}{32}$

S28. (c) Let us assume that X be the number of spade cards

Using the Bernoulli trial, X has a binomial distribution

P (X = x) = $n_{C_x p^x q^{n-x}}$

Thus, the number of cards drawn, n = 5

Probability of getting spade card, p = 13/52 = ¼

Thus the value of the q can be found using

q = 1 − p = 1 − (1/4) = ¾

Now substitute the p and q values in the formula,

Hence, P (X = x) = ${}^5C_x\,(3/4)^{5-x}\,(1/4)^x$

Probability of Getting all the spade cards:

P (all the five cards should be spade)
$= {}^5C_5\,(1/4)^5(3/4)^0$
$= (1/4)^5$
$= 1/1024$

S29. (a) $P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{4}{6} = \frac{2}{3}$

S30. (a) Let us consider two independent events A and B, then $P(A \cap B) = P(A).\,P(B)$
when an unbiased die is thrown twice
S = {(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}
Let us describe two events as
A: odd number on the first throw
B: odd number on the second throw

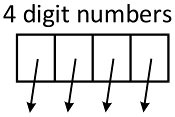
To find P(A)
A = {(1, 1), (1, 2), (1, 3), ..., (1, 6)
(3, 1), (3, 2), (3, 3), ..., (3, 6)
(5, 1), (5, 2), (5, 3), ..., (5, 6)}
Thus, P (a) = 18/36 = ½

To find P(B)
B = {(1, 1), (2, 1), (3, 1), ..., (6, 1)
(1, 3), (2, 3), (3, 3), ..., (6, 3)
(1, 5), (2, 5), (3, 5), ..., (6, 5)}
Thus, P (b) = 18/36 = ½
$A \cap B$ = odd number on the first & second throw
= { (1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)}
So, $P(A \cap B) = 9/36 = 1/4$
Now, P(A). P(b) = (1/2) × (1/2) = ¼
As $P(A \cap B) = P(A).\,P(B)$,
Hence, the two events A and B are independent events.

SUBJECTIVE QUESTIONS

S1. Sample space S = {1, 2, 3, 4, 5, 6} ; event A = {2, 3, 5}
∴ n((a) = 3 and n(S) = 6
∴ P((a) $= \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$.

S2. Total 4 digit numbers formed



4 × 4 × 3 × 2 = 96

Each of these 96 numbers are equally likely & mutually exclusive of each other.

Now, A number is divisible by 3, If sum of digits is divisible by 3
we can either use 0, 2, 3, 4 or 0, 1, 2, 3
so total favorable cases = (3 × 3 × 2) × 2 = 36
probability $= \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{36}{96} = \frac{3}{8}$

S3. Let A be the event 'the ball drawn is white' and B be the event 'the ball drawn is green'. P(The ball drawn is white or green) = $P(A \cup B)$ = P(a) + P(b) – $P(A \cap B) = \frac{8}{11}$

S4. $P(\bar{A} \cap B)$ = P((b) – $P(A \cap$ (b)
Also $P(A/B) = \frac{P(A\cap B)}{P(B)} \Rightarrow P(A \cap B) = 0.1$
From given data, $P(\bar{A} \cap B) = 0.5 - 0.1 = 0.4$

S5. In a single throw of a pair of dice probability of getting a doublet is $\frac{1}{6}$ considering it to be a success,
$p = \frac{1}{6}$
$\therefore q = 1 - \frac{1}{6}$ = number of success r = 2
$\therefore P(r = 2) = {}^5C_2\, p^2 q^3 = 10 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^3 = \frac{625}{3888}$

NUMERICAL TYPE QUESTIONS

S1. $\left(\frac{1}{6}\right)$ Total number of possible cases = 36
Favorable cases of getting a total of at least 10 ={(4,6),(5,5),(5,6),(6,4),(6,5),(6,6)} Total number of favorable cases = 6
$$P(\text{Total of at least } 10) = \frac{6}{36} = \frac{1}{6}$$

S2. $\left(\frac{1}{5}\right)$ $n =$ Total no.of ways $= {}^{120}C^1 = 120$.
m= Favourable no. of ways is the number of terms in the arithmetical series 5,10,15,20,25,30,....,120.
$$\therefore 120 = 4 + (m-1)5 \text{ or } m = 24$$
Hence required probability is $= \frac{m}{n} = \frac{24}{120} = \frac{1}{5}$.

S3. $\left(\frac{1}{6}\right)$ $\Rightarrow$ To prove $= P(\bar{A})/P(A) = 5:1$
For Round Table arrangement
If there are n persons,
Possible arrangement $\Rightarrow (n-1)!$
For thirteen persons $\rightarrow (13-1)! = 12!$
For particular two person consider them one.

Now, for twelve persons $\rightarrow(12-1)!=11!$

Expected

$\Rightarrow 11!2!(2 \rightarrow \text{Arranging them together})$

Total outcomes $=12!$

Probability person sit together $=\frac{11!2!}{12!}=\frac{1}{6}$

S4. $\left(\frac{5}{12}\right)$

Given $P(A \cup B)=\frac{3}{4}$

$P(A \cap B)=\frac{1}{4}$

$P(\overline{A})=\frac{2}{3}$

$\therefore P(B)=\frac{2}{3}$

By using P(B) = $P(A \cup B)+P(A \cap B)-P(A)$

$\therefore P(\overline{A} \cap B)=\frac{2}{3}-\frac{1}{4}=\frac{5}{12}$

S5. $\left(\frac{25}{56}\right)$ The probability of solving the question by these three students are $P(A)=\frac{1}{3}, P(B)=\frac{2}{7}, P(C)=\frac{3}{8}$.

Then probability of question solved by only one student

$$P(A\overline{B}\overline{C} \text{ or } \overline{A}B\overline{C} \text{ or } \overline{A}\overline{B}C)$$

$=P(A)P(\overline{B})P(\overline{C})+P(\overline{A})P(B)P(\overline{C})+P(\overline{A})P(\overline{B})P(C)$

$=\frac{1}{3}\times\frac{5}{7}\times\frac{5}{8}+\frac{2}{3}\times\frac{2}{7}\times\frac{5}{8}+\frac{2}{3}\times\frac{5}{7}\times\frac{3}{8}$

$=\frac{25+20+30}{168}$

$=\frac{25}{56}$

TRUE AND FALSE

S1. (False) p(probability of a success)

Probability of at least one failure = 1-(Probability of no failure)

For 5 events, probability of no failure $=p^5$

Hence, $1-p^5 \geq \frac{31}{32}$

$\Rightarrow p^5 \leq \left(\frac{1}{2}\right)^5$

$\Rightarrow p \leq \frac{1}{2}$

But $p \geq 0$

So, P lies in the interval $\left[0,\frac{1}{2}\right]$.

Sol 2.(True)

$P(\overline{A \cup B})=\frac{1}{6}, P(A \cap B)=\frac{1}{4}, P(\bar{A})=\frac{1}{4}$

$\therefore P(A \cup B)=1-P(\overline{A \cup B})=1-\frac{1}{6}=\frac{5}{6}$ and P(a) = $1-P(\bar{A})=1-\frac{1}{4}=\frac{3}{4}$

$P(A \cup B) = P(a) + P(b) - P(A \cap B)$

$\Rightarrow \frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$ P(b) $=\frac{1}{3}$

$\Rightarrow$ A and B are not equally likely.

Also, $P(A \cap B) = P(A).P(b)$ $=\frac{1}{4}$

So, events are independent.

S3. (True) $P(E_1)=\frac{1}{6}, P(E_2)=\frac{1}{6}, P(E_3)=\frac{1}{2}$

Also

$P(E_1 \cap E_2)=\frac{1}{36}, P(E_2 \cap E_3)=\frac{1}{12}, P(E_1 \cap E_3)=\frac{1}{12}$

$And P(E_1 \cap E_2 \cap E_3)=0 \neq P(E_1)\cdot P(E_2)\cdot P(E_3)$

Hence, E_1, E_2, E_3 are independent.

S4. (False) $P(A)+P(B)-2P(A \cap B)=\frac{1}{4}$

$P(B)+P(C)-2P(B \cap C)=\frac{1}{4}$

$P(A)+P(C)-2P(A \cap C)=\frac{1}{4}$

$\Rightarrow \sum P(A)-\sum P(A \cap B)=\frac{3}{8}$

$\Rightarrow P(A \cup B \cup C)=\sum P(A)--\sum P(A \cap B)+P(A \cap B \cap C)$

$=\frac{3}{8}+\frac{1}{16}=\frac{7}{16}$

S5. (True) The probability of getting head and tail in one toss is $\frac{1}{2}$

$P(\text{at least one H})=1-P(\text{no head in four rows})$

$=1-P(\text{four tails})$

$=1-\left(\frac{1}{2}\right)^4=1-\frac{1}{16}=\frac{15}{16}$

ASSERTION AND REASONING

S1. (a) **Assertion (a)** : Let A and B be two events such that P(a) $=\frac{1}{5}$, while P(A or B) = $\frac{1}{2}$.Let P (b) = P , then for P = $\frac{3}{8}$

,A and B independent.

Reason (R): For independent events, $P(A \cap B)=P(A)P(B)$

$P(A \cup B)=P(A)+P(B)-P(A \cap B)$

$=P(A)+P(B)-P(A)P(B)$

$= \frac{1}{5} + p - \frac{p}{5}$

$\Rightarrow \frac{1}{2} = \frac{1}{5} + \frac{4}{5}p$

$\Rightarrow p = \frac{3}{8}$

Assertion (a) and Reason (R) both are correct and Reason (R) is the correct explanation of assertion (A)

S2. (b) Assertion(a) is correct

$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(B)} = \frac{P(A)}{P(\bar{B})}$

[Since , given A and B are two mutually exclusive events]

$P\left(\frac{A}{\bar{B}}\right) = \frac{\left(1-\frac{5}{6}\right)}{\left(1-\frac{1}{3}\right)} = \frac{1}{4}$

Reason (R) is also correct

For independent events, $P(A|\bar{B}) = P(A) = 0.2$

Both A and R are true but R is NOT the correct explanation of A

S3. (c) Assertion (a) is correct.

A is proper subset of B and B is proper sunset of A $\Rightarrow$ $A = B$

Hence , P(a) = P(B)

But (R) is wrong .

A is proper subset of B $\Rightarrow$ $\bar{B}$ is proper subset of $\bar{A}$

Therefore , P$(\bar{A}) \geq P(\bar{B})$

S4. (d) Assertion (a) is wrong.

If the probability of an event is 0 , then it is called as an impossible event.

But reason (R) is correct.

From Basic Theorem of probability, P(B – A) = P(b) – P(a) , this is true only if the condition given in the question is true.

S5. (a) Both A and R are true and R is the correct explanation of A

HOMEWORK

MCQ

S1. (c) Let B stand for boys and G for girls.

Given, $P(B)=\frac{2}{3}$

Hence,

$P(G)=1-\frac{2}{3}=\frac{1}{3}$

Thus, the required probability will be

$=\frac{0.28\times 2}{3}+\frac{0.25\times 1}{3}$

$=\frac{1}{3}[0.56+0.25]$

$=\frac{0.81}{3}$

$=0.27$

S2. (d)

The probability that both are apples $=\left(\frac{5}{12}\right)\times\left(\frac{4}{12}\right)=\frac{20}{144}$

The probability that both are oranges $=\left(\frac{7}{12}\right)\times\left(\frac{8}{12}\right)=\frac{56}{144}$

Hence, the probability that the fruits are both apples or both oranges $=\left(\frac{20}{144}\right)+\left(\frac{56}{144}\right)$

$=\frac{(20+56)}{144}$

$=\frac{76}{144}$

S3. (b) $P\left(\frac{A}{B}\right)=$ Probability of getting king card in 1st draw if there is a king card in 2nd draw

Required Probability

$=P\left(\frac{A}{B}\right)$

$=\frac{P(A\cap B)}{P(B)}$

$=\frac{\frac{4}{52}\times\frac{3}{51}}{\frac{48}{52}\times\frac{4}{51}+\frac{4}{52}\times\frac{3}{51}}$

$=\frac{12}{192+12}$

$=\frac{12}{204}$

$=\frac{1}{17}$

Thus, the required probability is $=\frac{1}{17}$.

S4. (a) The probability of getting a head on the rigged coin is 1.

The probability of getting a head on the fair coin is 0.5.

Now out of $n+1$ coins there is one rigged coin and n fair coins.

Hence, if one coin is drawn it can be either a rigged coin or a fair coin.

Therefore the required probability is

$\frac{1}{n+1}\cdot 1+\frac{n}{n+1}\cdot\frac{1}{2}$

$=\frac{2}{2n+2}+\frac{n}{2n+2}$

$=\frac{n+2}{2(n+1)}=\frac{7}{12}$

$\frac{n+2}{n+1}=\frac{7}{6}$

$6n+12=7n+7$

$n=12-7=5$

$\therefore n=5.$

S5. (a) $P=\frac{2}{6}=\frac{1}{3}$

$q=1-p=1-\left(\frac{1}{3}\right)=\frac{2}{3}$

$n=3$

By using binomial distribution,

Mean $=\text{np}=\mu=3\left(\frac{1}{3}\right)=1$

Variance $=\text{npq}=\sigma^2=3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)=\frac{2}{3}$

Therefore, $\mu=1, \sigma^2=\frac{2}{3}$

S6. (a) For a binomial experiment consisting of n trials, the probability of exactly k successes is

$P(k\,\text{successes})={}^{n}C_{k}p^{k}(1-p)^{n-k}$

Where the probability of success on each trial is p.

According to the question,

$n=3;\ P(2\,\text{successes})=12\times P(3\,\text{successes})$

To find:

The probability of a success in each trail, p

$P(2\text{ successes})=12\times P(3\text{ successes})$

$\Rightarrow {}^{3}C_{2}p^{2}(1-p)^{3-2}=12\times {}^{3}C_{3}p^{3}(1-p)^{3-3}$

$\Rightarrow 3\times p^{2}(1-p)^{1}=12\times 1\times p^{3}(1-p)^{0}$

$\Rightarrow 3p^{2}-3p^{3}=12p^{3}$

Divide throughout by $3p^2$, we get

$1-p=4p$

$\Rightarrow 5p=1$

$\Rightarrow p=\frac{1}{5}$

S7. (c) Let the probability of a vessel wrecking be q and of safe arrival be p so that $q=\frac{1}{10}$ and $p=1-\frac{1}{10}=\frac{9}{10}$.

In the binomial exp ion,

$(q+p)^5=q^5+{}^5C_1q^4p+{}^5C_2q^3p^2+{}^5C_3q^2p^3+{}^5C_4qp^4+p^5$

$\therefore$ The probability of at least 4 vessels arriving safely is the sum of last two terms.

Hence the required probability $={}^5C_4QP^4+P^5$

$=5\frac{1}{10}\left(\frac{9}{10}\right)^4+\left(\frac{9}{10}\right)^5=\frac{45927}{50000}$

S8. (d) $P(X=r)={}^nC_r\cdot p^r\cdot q^{n-r}$

$={}^nC_r\cdot\left(\frac{1}{2}\right)^r\cdot\left(\frac{1}{2}\right)^{n-r}$

$={}^nC_r\cdot\left(\frac{1}{2}\right)^n$

$Now, P(X=4), P(X=5)\ and\ P(X=6)\ are\ in\ A.P.$

$\Rightarrow 2P(X=5)=P(X=4)+P(X=6)$

$\Rightarrow 2\cdot{}^nC_5\cdot\left(\frac{1}{2}\right)^n={}^nC_4\cdot\left(\frac{1}{2}\right)^n+{}^nC_6\cdot\left(\frac{1}{2}\right)^n$

$\Rightarrow 2\cdot{}^nC_5={}^nC_4+{}^nC_6$

$\Rightarrow 2\cdot\frac{n!}{(n-5)!\cdot 5!}=\frac{n!}{(n-4)!\cdot 4!}+\frac{n!}{(n-6)!\cdot 6!}$

$\Rightarrow \frac{2}{5(n-5)}=\frac{1}{(n-4)(n-5)}+\frac{1}{6\times 5}$

$\Rightarrow n^2-21n+98=0$

$\Rightarrow (n-7)(n-14)=0$

$\Rightarrow \boxed{n=7\ or\ 14}$

S9. (c) The probability of success in one strike is $p=\frac{1}{2}$

$\Rightarrow$ Probability of failure $=q=\frac{1}{2}$

Now, probability of r successes

$P(x=r)={}^nC_r\cdot\left(\frac{1}{2}\right)^r\cdot\left(\frac{1}{2}\right)^{n-r}={}^nC_r\left(\frac{1}{2}\right)^n$

According to the given condition, $P(x\ge 2)\ge 0.99$

$\Rightarrow 1-P(x<2)\ge 0.99$

$\Rightarrow 1-P(x=0)-P(x=1)\ge 0.99$

$\Rightarrow 1-0.99\ge\frac{1+n}{2^n}$

$\Rightarrow 2^n\ge 100+100^n$

For $n=10, 2^n<100+100n$

For $n=11,12,13,$

$2^n>100+100n.$

Hence, the wer is 11.

S10. (b) Third 6 is obtained on the eighth throw, so 6 should appear two times in first 7 throws.

So, the required probability is

$={}^7C_2\times\left(\frac{1}{6}\right)^2\times\left(\frac{5}{6}\right)^5\times\frac{1}{6}$

$={}^7C_2\left(\frac{5^5}{6^8}\right)$

$=\frac{{}^7C_2\times 5^5}{6^8}$

SUBJECTIVE QUESTIONS

S1. Since, $0\le P(a)\le 1, 0\le P(b)\quad\le 1, 0\ P(V)\le 1$ and $0\le P(a)\quad+P(b)\quad+P(c)\quad\le 1$

$\therefore\quad 0\le\frac{3x+1}{3}\le 1\Rightarrow -\frac{1}{3}\le x\le\frac{2}{3}\qquad\ldots(i)$

$0\le\frac{1-x}{4}\le 1\Rightarrow -3\le x\le 1\qquad\ldots(ii)$

$0\le\frac{1-2x}{2}\le 1\Rightarrow -\frac{1}{2}\le x\le\frac{1}{2}$

$\ldots(iii)$

And $\quad 0\le\frac{3x+1}{3}+\frac{1-x}{4}+\frac{1-2x}{2}\le 1$

$\Rightarrow\quad 0\le 13-3x\le\ 12$

$\Rightarrow\quad\frac{1}{3}\le x\frac{13}{3}$

From Eqs. (i), (ii), (iii) and (iv), we get

$\Rightarrow\quad\frac{1}{3}\le x\frac{13}{3}$

S2. Mean $=np=4$

Variance $=npq=2$

$\Rightarrow q=\frac{1}{2}\Rightarrow p=\frac{1}{2}$

$n=\frac{4}{p}=8$

$P(X=1)={}^8C_1p^1q^{8-1}=8\cdot\frac{1}{2}\cdot\frac{1}{2^7}=\frac{1}{32}$

S3. Given, probabilities of speaking truth are

$P(A)=\frac{4}{5}$ And $P(B)=\frac{3}{4}$

And their corresponding probabilities of not speaking truth are

$P(\bar{A})=\frac{1}{5}$

And $P(\bar{B})=\frac{1}{4}$

The probability that they contradict each other

$=P(A)\times P(\bar{B})+P(\bar{A})\times P(B)$

$=\frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}=\frac{1}{5}+\frac{3}{20}=\frac{7}{20}$

S4. Given, E = {X is a prime number} = {2, 3, 5, 7}

P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(= 7)

= 0.23 + 0.12 + 0.20 + 0.07 = 0.62

and F = {X < 4} = {1, 2, 3}

$\Rightarrow$ P(F) = P(X = 1) + P(X = 2) + P(X = 3)

= 0.15 + 0.23 + 0.12 = 0.5

and $E \cap F$ = {X is prime number as well as < 4}

= {2, 3}

$P(E \cap F) = P(X = 2) + P(X = 3)$

= 0.23 + 0.12 = 0.35

$\therefore$ Required probability,

$P(E \cap F) = P(E) + P(F) - P(E \cap F)$

= 0.62 + 0.5 – 0.35 = 0.77

S5. Given that, $P(A \cap B) = \frac{1}{4}$, $P(\bar{A}) = \frac{1}{4}$

and $P(\overline{A \cup B}) = \frac{1}{6}$

$\Rightarrow 1 - P(AÈB) = \frac{1}{6}$ $[\because P(a) + P(\bar{A}) = 1]$

$\Rightarrow$ $1 - P(a) - P(b) + P(A \cap B) = \frac{1}{6}$

$[\because P(A \cup B) = P(a) + P(b) - P(A \cap B)]$

$\Rightarrow$ $P(\bar{A}) - P(B) + \frac{1}{4} = \frac{1}{6}$

$\Rightarrow$ $P(b) = \frac{1}{4} + \frac{1}{4} - \frac{1}{6}$

$\Rightarrow$ $P(b) = \frac{1}{3}$ and $P(a) = \frac{3}{4}$

Now, $P(A \cap B) = \frac{1}{4} = \frac{3}{4} \times \frac{1}{3} = P(a) \quad P(B)$

Hence, the events A and B are independent events but not equally likely.

NUMERICAL TYPE QUESTIONS

S1. $(\frac{1}{9})$ All three persons has three options to apply a house.

$\therefore$ Total number of cases = 3^3

Now, Favorable cases = 3 (as either all has applied for house 1 or 2 or 3)

$\therefore$ required probability = $\frac{3}{3^3} = \frac{1}{9}$

S2. $(\frac{1}{3})$ Given that, $P(A) = \frac{1}{4}$, $P\left(\frac{A}{B}\right) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{2}{3}$

We know that, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

...(i)

and $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$

...(ii)

$\therefore$ $P(B) = \frac{P\left(\frac{B}{A}\right) \cdot P(A)}{P\left(\frac{A}{B}\right)} = \frac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$

S3. **(1)** $\because$ A = {4, 5, 6} and B = {1, 2, 3, 4}

$\therefore$ $A \cap B = \{4\}$

[by addition theorem of probability]

$\therefore$ $P(A \cup B) = P(a) + P(b) - P(A \cap B)$

$\Rightarrow$ $P(A \cup B) = \frac{3}{6} + \frac{4}{6} - \frac{1}{6} = 1$

S4. $\left(\frac{1}{14}\right)$ = {00, 01, 02,....49}

Let A be the event that sum of the digits on the selected ticket is 8, then

A = {08, 17, 26, 35,44}

Let B be the event that the product of the digits is zero.

B = {00, 01, 02, 03,...,09, 10, 20, 30, 40}

$\therefore$ $A \cap B = \{8\}$

$\therefore$ Required probability

$= P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{50}}{\frac{14}{50}} = \frac{1}{14}$

S5. $(\frac{2}{7})$ Total number of cases = ${}^9C_3$ = 85

Number of favorable cases = ${}^3C_1 . {}^4C_1 . {}^2C_1$ = 24

$\therefore$ $P = \frac{24}{84} = \frac{2}{7}$

TRUE AND FALSE

S1. **(False)** Three balls can be selected in the following ways.

White: 3 2 1 0

Black: 0 1 2 3

$\therefore$ Total number of ways $= 4$,

clearly, there is only one favorable ways in which three balls are white.

Hence, required probability $= \frac{1}{4}$.

NOTE: It should be noted that the number of ways of selecting 3 white balls from 4 white balls is not equal to

${}^4C_3$, because all white balls are identical and all black balls are identical.

S2. **(True)** A box contains 10 mangoes out of which 4 are rotten.

Probability of taking one rotten mango $= \frac{4}{10} = \frac{2}{5}$

Probability of taking one good mango $= \frac{6}{10} = \frac{3}{5}$

If one of them is found to be good, the probability that the other is rotten is then first one is good and second is rotten or first one is rotten and second one is good

$= \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{6}{9} = \frac{48}{90} = \frac{8}{15}$

S3. (False) The odd places can be filled up in $5\times4\times3$ ways and the even places is 5×4 ways.

$\therefore$ the favorable number of ways $5\times4\times3\times5\times4 = 5^2.2^4.3$

The number of five digit number $= 9\times10^4$

Hence the probability the required probability $\frac{5^2.2^4.3}{9\times10^4} = \frac{1}{75}$

S4. (True) $4P(x=4) = P(x=2)$

$4\times\ {}^6C_4 \times (P^4) \times (1-P)^2 = {}^6C_4 \times (P^2) \times (1-P)^4$

$4\times P^2 = (1-P)^2$

$(3P-1)(P+1) = 0$

$P = \frac{1}{3}$

S5. (True) Three dices are thrown

Let values appeared on dices be x, y and z

For $x+y+z=16$ possible cases are

4 6 6, 6 6 4, 6 4 6,
5 5 6, 6 5 5, 5 6 5

Total cases are 6

Total number of cases $= 6\times6\times6$

Therefore probability $= \frac{6}{6\times6\times6} = \frac{1}{36}$

ASSERTION AND REASONING

S1. (a) Since A and B are independent events

$\therefore\ P(A\cap B) = P(A).P(B)$

Now, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= P(A) + P(B) - P(A).P(B)$

$0.6 = 0.4 + P - 0.4\text{P}$

$0.6\text{P} = 0.2$

$\therefore\ \text{P} = \frac{0.2}{0.6} = \frac{1}{3}$

Both A and R are true and R is the correct explanation of A

S2. (c) If each outcome of an experiment is associated with at least one of the events E_1, E_2, E_3, ...E_n, then collectively the events are said to be exhaustive. Mathematically we write $E_1 \cup E_2 \cup E_3$.........E_n = S. (Sample space)

Assertion is correct but Reason is False.

S3. (c) **Assertion (a)** : If A and B are two events, then $P(A/(b) = \frac{P(A\cap B)}{P(B)}$

A is correct

Reason (R): For mutually exclusive events P(A/B) $\neq$ 0.

R is not correct because for mutually exclusive events $P(A\cap B) = 0$, so P(A/B) = 0.

S4. (d) Assertion is wrong

De Morgan's laws: If A & B are two subsets of a universal set U, then

(a) $(A\cup B)^C = A^C \cap B^C$

Reason(R) is correct

When two or more than two events occur simultaneously, the event is said to be a compound event. Symbolically A $\cap$ B or AB represent the occurrence of both A & B simultaneously.

Note: "A $\cup$ B" or A + B represent the occurrence of either A or B.

S5. (a) Assertion is correct

The probability distribution for a binomial variate 'X' is given by :$P(X = r) = {}^nC_r\, p^r q^{n-r}$ where P(X = r) is the probability of r successes.

Reason is correct

Baye's theorem

If an event A can occur with one of the n mutually exclusive and exhaustive events B_1, B_2 , ..., B_n and the probabilities $P(A/B_1)$, $P(A/B_2)$ .... $P(A/B_n)$ are known, then

$$P(B_i / (a) = \frac{P(B_i)\,.\,P(A/B_i)}{\sum_{i=1}^{n} P(B_i)\,.\,P(A/B_i)}.$$