PROBABILITY

INTRODUCTION

- **Experiment :-** An operation which can produce some well-defined outcomes call an experiment.

- **Random Experiment :-** An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

  Examples -
  (i) Rolling a dice.
  (ii) Tossing a coin.
  (iii) picking a object.

- **Sample Space :-** set of all possible out comes of an experiment is called as sample space.
  (Set)

  Examples -
  (i) In tossing a coin, $S_1 = \{H, T\}$
  (ii) In tossing two coin, $S_2 = \{HH, HT, TH, TT\}$
  (iii) In rolling a dice, $S_3 = \{1, 2, 3, 4, 5, 6\}$
  (iv) rolling two dice, $S_4 = \{(1,1)\ (1,2)\ (1,3)\ (1,4),\ (1,5)\ (1,6)\ \ldots\ (6,6)\}$
  (v) life time of a care, $S_5 = [0, \infty)$.

- **Event :-** Any subset of a sample space is an event.
  (Set)

  Examples -
  (i) Tossing a coin that can be event, $E_1 = \{H\}$, $E_6 = \{T\}$ $\boxed{E_1 \subseteq S}$
  (ii) $E_3 = \{2, 4, 6\}$, ~~even out~~ (set of even outcomes)
  (iii) $E_2 = \{HH, HT, TH\}$, (at least 1 head).
  (iv) $E_4 = \{(1,2)\ (2,1)\}$, (sum is ~~of~~ 3).
  (v) $E_5 = [3, 6]$, (car life time is atleast 3 and atmost 6 - year).

→ sample space and Events both are set, so we can apply all the rules of set. (union, intersection, complement)

Ex-

(1) $E_1 \cup E_6 = \{H, T\}$

(2) $E_1 \cap E_2 = \emptyset$ (Mutual Exclusion = no common - Events)

(3) $\overline{E_1} = S - E_1$ ($\overline{E_1} = \{T\}$)

* → For each event 'E' in the sample Space 'S', we assign a number $P(E)$ such that:

(I) $0 \leq P(E) \leq 1$

(II) $P(S) = 1$

(III) For any sequence of events E_1, E_2 --- that are mutually exclusive,

$$P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n).$$

$\boxed{P(E) = \frac{n(E)}{n(S)}}$

Ex- tossing 1 coin-

$S = \{H, T\}$

$E_1 = \{H\}$, $E_2 = \{T\}$

$P(E_1) = \frac{1}{2}$, $P(E_2) = 1/2$

Ex- tossing 2 coin-

$S = \{HH, HT, TH, TT\}$
$\quad\;\; 1/4 \quad 1/4 \quad 1/4 \quad 1/4$

$E_1 = \{HH\}$, $P(E_1) = 1/4$

$E_2 = \{TT\}$, $P(E_2) = 1/4$

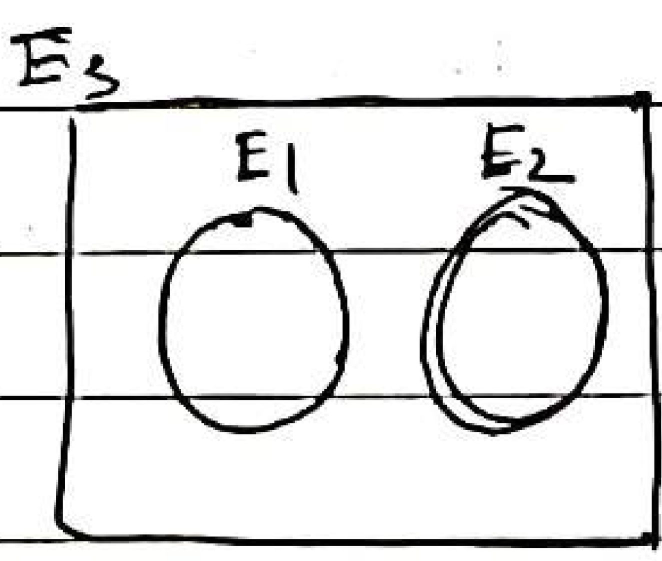
$E_3 = E_1 \cup E_2$

$P(E_3) = P(E_1 \cup E_2)$

* $\boxed{P(E_3) = P(E_1) + P(E_2)}$

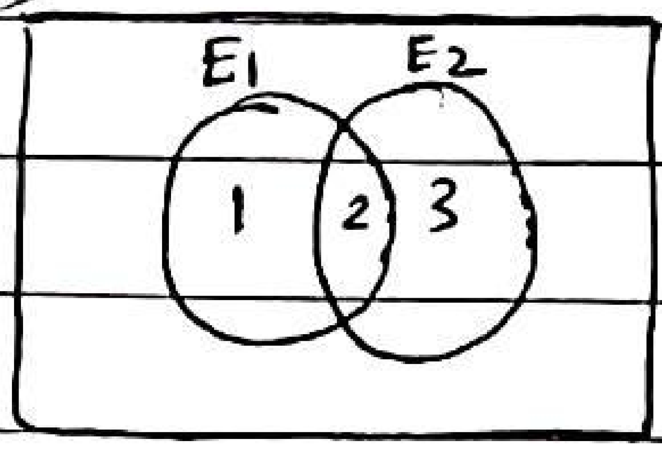
$= 1/4 + 1/4$

$= 1/2$



Individual probability (ME)

E_s



not individual probability
(not-ME)

$P \quad E_1 \cup E_2 = (1,2) \cup (2,3)$

** $\boxed{P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)}$

Ex- tossing two coin-

→ $S = \{HH, HT, TH, TT\}$

$E_1 = \{HH, HT\}$ - first coin going to head.

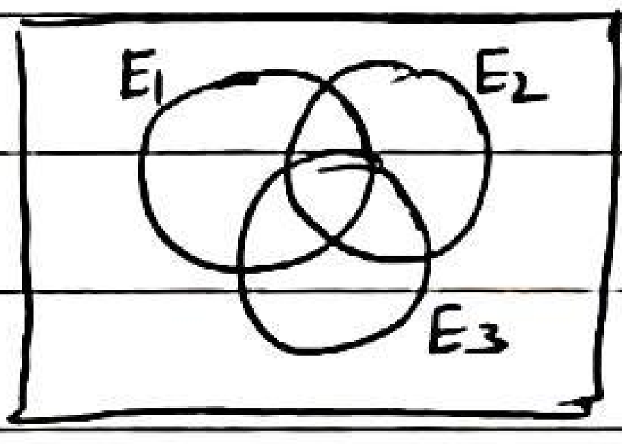
$E_2 = \{HH, TH\}$ - second coin going to be head.

$E_3 = \{HH, HT, TH\}$ at least one head.
$= E_1 \cup E_2$

$P(E_3) = \boxed{P(E_1 \cup E_2) \Rightarrow P(E_1) + P(E_2) - P(E_1 \cap E_2)}$

$\Rightarrow \frac{2}{4} + \frac{2}{4} - \frac{1}{4}$

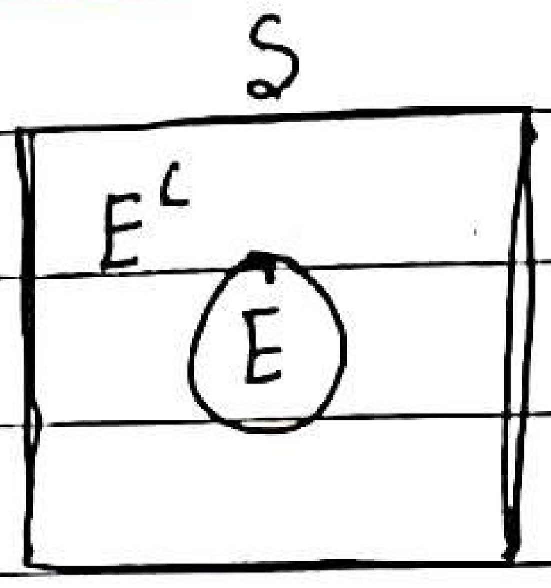
$\Rightarrow \frac{1}{2} + \frac{1}{2} - \frac{1}{4} \Rightarrow \frac{3}{4}.$



(not-Mutual exclusive)

** $\boxed{P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3)}$

→ When ever trying to take the union you have to see whether they are mutual exclusive or not.



(ME, so add them)

$$S = E \cup E^c$$

$$P(S) = P(E \cup E^c)$$

$$P(S) = P(E) + P(E^c)$$

$$\Rightarrow \quad 1 = P(E) + P(E^c)$$

$$* \quad \boxed{P(E^c) = 1 - P(E)}$$