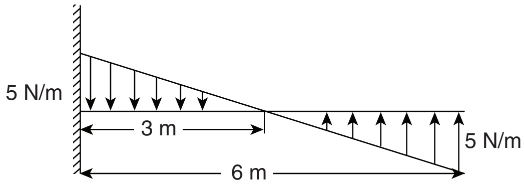
Solid Mechanics Test I

Number of Questions: 25 **Time: 60 min.**

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1.



A cantilever beam of 6 m span is subjected to a uniformly varying load as shown. The bending moment at the middle of the beam is

(A) 27.5 N–m (B) 15.0 N–m
(C) 22.0 N–m (D) 18.7 N–m

2.

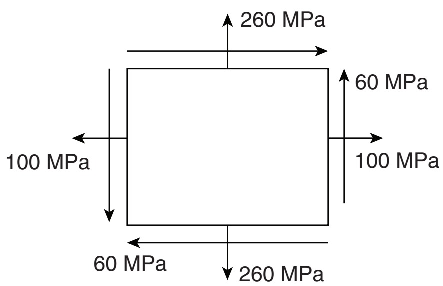
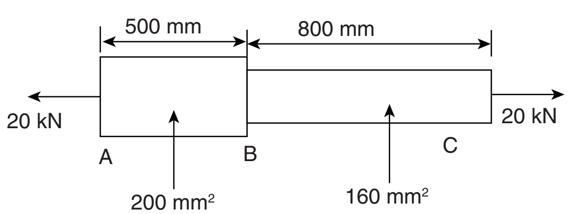


Figure shows state of stress at a point in a stressed body. Radius of Mohr's circle representing the state of stress is

(A) 60 (B) 80
(C) 120 (D) 100

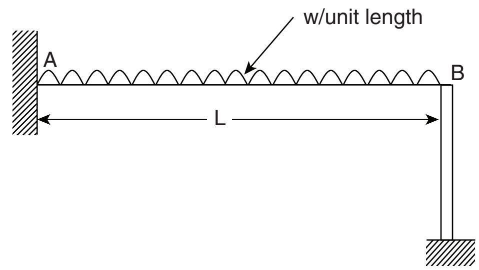
3.



A bar *ABC* with cross sectional area 200 mm^2 at portion *AB* and 150 mm^2 at portion *BC* is subjected to an axial pull of 20 kN. If $E = 2 \times 10^5$ N/mm^2, strain energy stored in the bar is

(A) 15.5 Nm (B) 18.6 Nm
(C) 8.7 Nm (D) 10.5 Nm

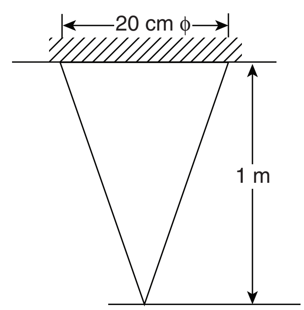
4.



The free end of the cantilever *AB* is supported by a prop. The cantilever is loaded by a uniformly distributed load as shown in the figure. Assuming that there is no deflection at the free end, force on the prop is

(A) $\frac{4}{5}wL$ (B) $\frac{3}{8}wL$
(C) $\frac{3}{4}wL$ (D) $\frac{3}{5}wL$

5. A solid conical bar of uniformly varying cross section is hung vertically as shown



If specific weight is 80000 N/m^3 and modulus of elasticity is $E = 2 \times 10^5$ N/mm^2, extension of its length due to self weight is

(A) 6.67×10^{-5} m (B) 1.33×10^{-4} m
(C) 1×10^{-4} m (D) 4.45×10^{-5}m

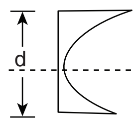
6. The bulk modulus is *K*, modulus of elasticity *E*, and passion ratio is $\frac{1}{m}$ then which of the following is true?

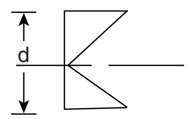
(A) $E = 3K\left(1+\frac{2}{m}\right)$ (B) $E = 3K\left(1-\frac{1}{m}\right)$
(C) $E = 3K\left(1-\frac{2}{m}\right)$ (D) $E = 3K\left(1+\frac{1}{m}\right)$

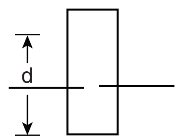
7. A solid circular shaft is subjected to bending & twist. The ratio of maximum shear to maximum bending stress at any point would be ($M = T$)

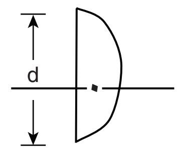
(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 2 : 3

8. The shear stress distribution diagram of a beam of rectangular cross section, subjected, to transverse loading will be

(A) 

(B) 

(C) 

(D) 

Where '*d*' is the depth of the beam

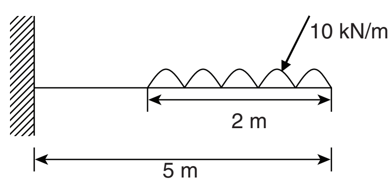
9. Proof resilience is the maximum energy stored at
(A) limit of proportionality
(B) elastic limit
(C) plastic limit
(D) None of these

10. Which of the following will give the value of deflection at any point?

(A) $EI\dfrac{dy}{dx} = M$ (B) $E\,I\,Y = M$

(C) $E\,I\,Y = \int M$ (D) $E\,I\,Y = \int\int M$

11.



The displacement of the free end of the cantilever beam shown in figure is
[Take $E = 2 \times 10^5$ N/mm^2,
$I = 180 \times 10^6$ mm^4]
(A) 16.39 mm (B) 14.93 mm
(C) 12.72 mm (D) 10.68 mm

12. During an experiment on a steel column using Rankine's formula, the following results were available

Slenderness ratio	65	160
Average stress at failure	200 N/mm^2	70 N/mm^2

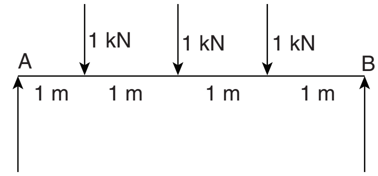
Rankine's constant for the material of the column is
(A) 1.865×10^{-4} (B) 2.194×10^{-4}
(C) 1.623×10^{-4} (D) 1.373×10^{-4}

Common Data for Questions 13 and 14:
At a cross section in a shaft of diameter 100 mm it is subjected to a bending moment of 2.5 kNm, and a twisting moment of 5 kNm.

13. Maximum principal stresses induced in the section in N/mm^2 are
(A) 37.5, 12.64 (B) 41.2, −15.74
(C) 52.8, −17.92 (D) 49.3, −16.78

14. The direct stress in N/mm^2 that produces same strain on that produced by the principal stresses is
(Poisson's ratio is 0.3)
(A) 36.78 (B) 52.76
(C) 45.92 (D) 39.62

Statement for Linked Answer Questions 15 and 16:



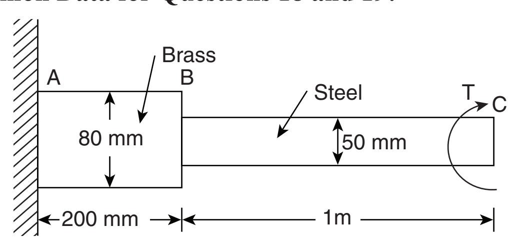
A simply supported beam *AB* is loaded as shown in the figure. The beam has a rectangular cross section of 100 mm width and 240 mm depth.

15. At a section 1.5 m from *A* maximum shearing stress is
(A) 0.0625 N/mm^2
(B) 0.0848 N/mm^2
(C) 0.0313 N/mm^2
(D) 0.0565 N/mm^2

16. Principal stresses at a point in neutral axis of the above section in N/mm^2 is
(A) +0.0313, 0
(B) +0.0313, −0.0313
(C) +0.0625, 0.0313
(D) +0.0625, −0.0625

17. A 2 m long wooden column bottom fixed and top end free, has a square cross section and has to take a load of 100 kN. Modulus of elasticity is 12 GPa. Size of the column, using Euler's formula and a factor of safety 3, is
(A) 148.5 mm (B) 135.8 mm
(C) 162.3 mm (D) 156.7 mm

Common Data for Questions 18 and 19:

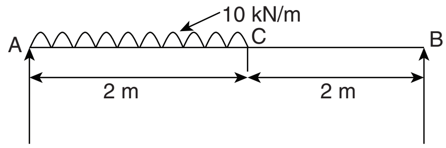


For the stepped shaft *ABC*, fixed at *A*, portion *AB* is made of brass and portion *BC* is made of steel, Allowable shear stress for brass is 80 N/mm^2 and for steel is 100 N/mm^2. Modulus of rigidity for brass is 40 kN/mm^2 and for steel is 80 kN/mm^2

18. Maximum value of torque that can be applied at the end of the shaft is
(A) 8042 Nm (B) 6053 Nm
(C) 2454 Nm (D) 3064 Nm

19. Total rotation at the free end in degrees is
(A) 4.18° (B) 3.04°
(C) 3.62° (D) 2.51°

Common Data for Questions 20 and 21:

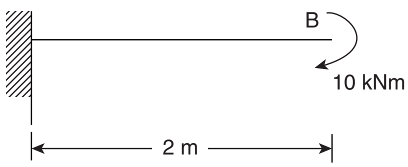


The simply supported beam loaded as shown above has a flexural rigidity 833.33 kN – m^2

20. Slope at end *A* is
(A) 12 rad (B) 15 rad
(C) 10 rad (D) 17 rad

21. Maximum deflection occurs between *A* and *C*. Distance from *A* is
(A) 2.00 m (B) 1.00 m
(C) 1.84 m (D) 1.95 m

22.

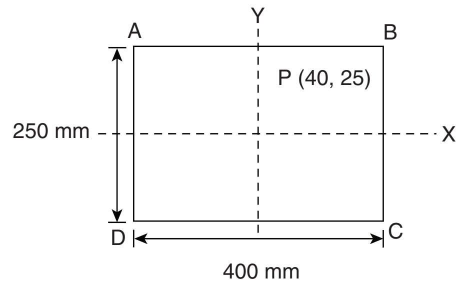


A cantilever *AB* of length 2 m and 100 mm breadth and 200 mm depth is fixed at end *A*. It is subjected to a moment of 10 kNm at the free end *B*. Flexural rigidity is 13,340 kN–m^2. Magnitude of maximum deflection is
(A) 1.8 mm (B) 2.2 mm
(C) 1.5 mm (D) 2.8 mm

23. A cylindrical tank of 750 mm internal diameter and 4 m length is made of 18 mm thick sheet. If it is subjected to an internal fluid pressure of 2 N/mm^2, maximum intensity of shear stress induced is
(Take $E = 2 \times 10^5$ N/mm^2 and $\mu = 0.3$)
(A) 11.78 N/mm^2 (B) 9.57 N/mm^2
(C) 8.62 N/mm^2 (D) 10.42 N/mm^2

24. Change in volume of the tank in cm^3 is
(A) 699 (B) 1119
(C) 1520 (D) 386

25.



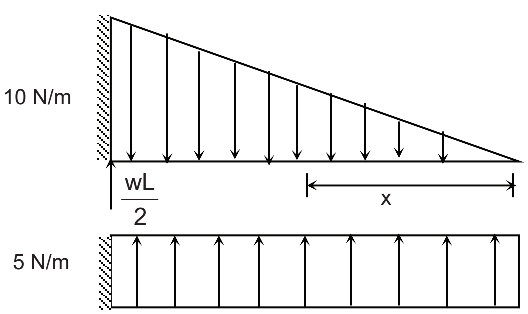
On a short masonry column of cross section as shown above, a concentrated load of 500 kN is applied at point *P*, + 40 mm from *y* axis and + 25 mm from *x* – axis. Moment of inertia about *x*– axis = 520.833 × 10^6 and moment of inertia about *y* – axis = 1333 × 10^6
Stress developed at point *D* is
(A) 2.10 N/mm^2 (B) 1.15 N/mm^2
(C) 1.00 N/mm^2 (D) 2.67 N/mm^2

Answer Keys

1. B	**2.** D	**3.** D	**4.** B	**5.** A	**6.** C	**7.** B	**8.** D	**9.** B	**10.** D
11. A	**12.** D	**13.** B	**14.** C	**15.** C	**16.** B	**17.** A	**18.** C	**19.** B	**20.** B
21. C	**22.** C	**23.** D	**24.** A	**25.** C					

Hints and Explanations

1. The load acting can be split in to two as shown in figure
(i) A uniformly varying load of 10 N/m at fixed end and zero at free end.
(ii) A uniformly distributed negative load of 5 N/m acting through out the beam.



Bending moment due to uniformly varying load at $x = 3$ m.
Bending moment due to the uniformly distributed load
$$= +\frac{wx^2}{2} = \frac{5\times(3)^2}{2} = 22.5 \text{ Nm}$$
Total bending moment = 22.5 – 7.5 = 15 Nm
Choice (B)

2. Radius of Mohr's circle is $= \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$
$$= \sqrt{\left(\frac{260-100}{2}\right)^2 + 60^2}$$

$= \sqrt{80^2 + 60^2}$

$= \sqrt{6400 + 3600}$

$= \sqrt{1000} = 100.$ Choice (D)

3. Stress in portion AB $\sigma_1 = \frac{20000}{200} = 100$ N/mm^2

Stress in portion BC $\sigma_2 = \frac{20000}{100} = 200$ N/mm^2

Strain energy stored $= \sum\left(\frac{\sigma^2}{2E} \times \text{volume}\right)$

$$= \frac{(100)^2}{2\times 2\times 10^5}\times(200\times 500) + \frac{200^2}{2\times 2\times 10^5}\times(100\times 800)$$

= 10,500 N – mm = 10.5 Nm. Choice (D)

4. Deflection of beam at the free end if no prop is there $= \frac{wL^4}{8EI}$

Upward deflection of the end due to the force $F = \frac{FL^3}{3EI}$

When there is no resultant deflection

$$\frac{FL^3}{3EI} = \frac{wL^4}{8EI}$$

$\therefore$ $F = \frac{3}{8}wL$. Choice (B)

5. Extension of vertical conical bar due to self weight

$$= \frac{wl^2}{6E} = \frac{80,000\times 1^2}{6\times 2\times 10^5\times 10^6}\text{ m}$$

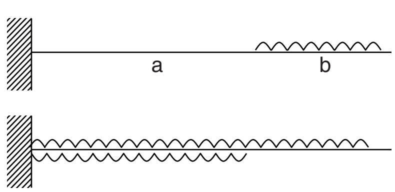
$= 6.67 \times 10^{-5}$ m Choice (A)

7. $\tau = \frac{16T}{\pi d^3}$

$\sigma = \frac{32M}{\pi d^3}$

$\frac{\tau}{\sigma} = \frac{16T}{\pi d^3}\times\frac{\pi d^3}{32M} = 1 : 2$ Choice (B)

11.



The given uniformly distributed loading of the cantilever can be treated as a combination as shown above. i.e., a full loading + a negative loading through length a

$\therefore$ Deflection at free end

$$= \frac{w\ell^4}{8EI} - \left[\frac{wa^4}{8EI} + \frac{wa^3.b}{6EI}\right]$$

$$= \frac{10\times 5^4}{8EI} - \left[\frac{10\times 3^4}{8EI} + \frac{10\times 3^3\times 2}{6EI}\right]$$

$$= \frac{1}{EI}\left[781.25 - (101.25 + 90)\right] = \frac{590}{EI}$$

E = 2 × 10^5 N/mm^2 = 2 × 10^8 kN/m^2

I = 180 × 10^6 mm^4 = 180 × 10^{-6} m^4

$\therefore$ Deflection $= \frac{590}{2\times 10^8\times 180\times 10^{-6}}$

= 0.01639 m = 16.39 mm. Choice (A)

12. Rankine's formula is

$$P_{cr} = \frac{\sigma_c A}{1 + a\left(\ell/k\right)^2} \text{ Or } \frac{P_{cr}}{A} = \frac{\sigma_c}{1 + a\left(\ell/k\right)^2}$$

When ℓ/k = slenderness ratio = 65

$\frac{P_{cr}}{A} = 200$ N/mm^2

$\therefore$ $200 = \frac{\sigma_c}{1 + a(65)^2}$ ——————————(1)

When $\ell/k = 160$

$\frac{P_{cr}}{A} = 70$

$\therefore$ $70 = \frac{\sigma_c}{1 + a(160)^2}$ —————————(2)

(1) ÷ (2) gives,

$$\frac{200}{70} = \frac{1 + a(160)^2}{1 + a(65)^2}$$

200 + 845,000 a = 70 + 1792,000 a

947000 a = 130

$a = 1.373 \times 10^{-4}$. Choice (D)

13. M = 2.5 kNm = 2.5 × 10^6 Nmm

T = 5 kNm = 5 × 10^6 Nmm

Maximum principal stress

$$p_1 = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right]$$

$$= \frac{16}{\pi 100^3}\left[2.5 + \sqrt{2.5^2 + 5^2}\right]\times 10^6 = 41.2 \text{ N/mm}^2$$

$$p_2 = \frac{16}{\pi d^3}\left[M - \sqrt{M^2 + T^2}\right]\times 10^6$$

$$= \frac{16}{\pi 100^3}\left[2.5 - \sqrt{2.5^2 + 5^2}\right]\times 10^6$$

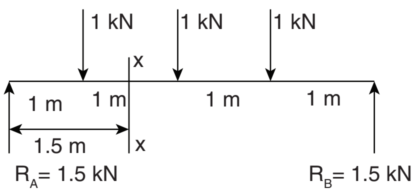
= – 15.74 N/mm^2. Choice (B)

14. Let p be the direct stress

$$\frac{p}{E} = \frac{p_1}{E} - \frac{\mu p_2}{E}$$

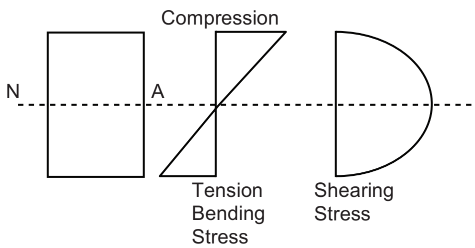
$P = 41.2 - 0.3\,(-15.74) = 45.92$ N/mm². Choice (C)

15.



Shear force at the section xx is $1.5 - 1.0 = 0.5$ kN

Average shearing stress $= \dfrac{0.5 \times 1000}{bd} = \dfrac{500}{100 \times 240}$



Shearing stress is maximum at neutral axis

Maximum shearing stress

$= 1.5 \times$ average shearing stress

$= 1.5 \times \dfrac{500}{24000} = 0.03125$ N/mm². Choice (C)

16. At Neutral axis bending stress (Normal stress) is zero

$\therefore \quad p_x = 0, p_y = 0$ and $q = 0.03125$ N/m²

Principal stress

$$= \frac{p_x + p_y}{2} + \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2} = 0 \pm \sqrt{0 + (0.03125)^2}$$

$= \pm\, 0.03125$ N/mm². Choice (B)

17. Critical load = working load × factor of safety

$= 100 \times 3 = 300$ kN

Applying Euler's formula

Critical load $= \dfrac{\pi^2 EI}{L^2}$

When L = Effective length = 2 × actual length

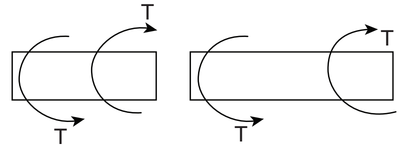
$= 2 \times 2 = 4$ m $= 4000$ mm

$$\therefore \quad 300 \times 10^3 = \frac{\pi^2 \times 12 \times 10^3 \times I}{(4000)^2}$$

$$I = 40{,}528{,}473 = \frac{a^4}{12}$$

$\therefore \quad a = 148.5$ mm. Choice (A)

18.



Both portions are subjected to same torque T

For brass portion maximum torque that can be applied

$$T_b = \frac{\pi}{16} d^3 \tau_b = \frac{\pi}{16}(80)^3 \times 80$$

$= 8042{,}477$ N–mm $= 8042.477$ Nm

For steel portion, $T_s = \dfrac{\pi}{16}(50)^3 \times 100$

$= 2454{,}369$ N–mm

$= 2454.369$ Nm

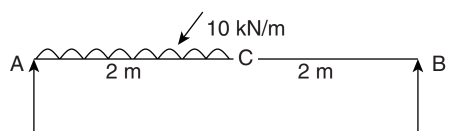
So maximum torque that can be applied at the end of the shaft is 2454 Nm = 2454 kN mm Choice (C)

19. Rotation of the free end $= \theta_B + \theta_s = \dfrac{TL_b}{G_b J_b} + \dfrac{TL_s}{G_s J_s}$

$$= 2454 \times \frac{32}{\pi}\left[\frac{200}{40 \times 80^4} + \frac{1000}{80 \times 50^4}\right]$$

$= 0.053$ rad $= 3.04°$. Choice (B)

20.



$V_A + V_B = 2 \times 10 = 20$ kN

$V_A \times 4 = 2 \times 10 \times 3$

$V_A = 15$ kN

$V_B = 5$ kN

Using Macaulay's method

$$Mx = 5\,x - \frac{10(x-2)^2}{2} = \text{EI}\frac{d^2 y}{dx^2}$$

$$\text{EI}\frac{dy}{dx} = C_1 + \frac{5x^2}{2} - \frac{5(x-2)^3}{3}$$

$$\text{EI}y = C_2 + C_1 x + \frac{5}{2}\frac{x^3}{3} - \frac{5}{3}\frac{(x-2)^4}{4}$$

At $x = 0$, $y = 0$

$0 = C_2 + 0 + 0$

At $x = 4$, $y = 0$

$$0 = 0 + 4C_1 + \frac{5}{2} \times \frac{4^3}{3} - \frac{5}{3} \times \frac{2^4}{4}$$

$-4C_1 = 53.33 - 6.67 = 46.66$

$C_1 = -11.67$

$$\text{EI}\frac{dy}{dx} = -11.67 + \frac{5x^2}{2} - \frac{5(x-2)^3}{3}$$

At A, $x = 4$

$$\therefore \quad EI\left(\frac{dy}{dx}\right)_A = -11.67 + \frac{5\times4^2}{2} - \frac{5\times2^3}{3}$$

$$= -11.67 + 40 - \frac{40}{3} = 14.996 \text{ rad.}$$

Choice (B)

21. Maximum deflection occurs where slope is zero

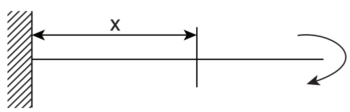
$$\therefore \quad -11.67 + \frac{5x^2}{2} - \frac{5}{3}(x-2)^3 = 0$$

$$2.5x^2 - \frac{5}{3}(x-2)^3 = 11.67$$

$\therefore$ $x = 2.16$ m (distance from C)

$\therefore$ Distance from $A = 4 - 2.16 = 1.84$ m Choice (C)

22.



At any section bending moment is 10 kNm
i.e., $M_x = 10$ kNm

$$EI\frac{d^2y}{dx^2} = 10 \text{ kNm}$$

$$EI\frac{dy}{dx} = 10x + C_1$$

At $x = 0$, $\frac{dy}{dx} = 0$

$\therefore$ $C_1 = 0$

$$EIy = \frac{10x^2}{2} + C_2$$

At $x = 0$, $y = 0$

$$\therefore \quad EIy = \frac{10x^2}{2}$$

$$EI\ y_{max} = \frac{10\times2^2}{2}$$

Flexural rigidity EI = 13, 340 kN – m^2

$$\therefore \quad y_{max} = \frac{20}{13340} \text{ m}$$

$= 1.5 \times 10^{-3}$ m
$= 1.5$ mm or

$$y_{max} = y_{max} = \frac{ML^2}{2EI} = \frac{10\times10^3\times2^2}{2\times13340}$$

$= 1.5$ mm Choice (C)

23. $d = 750$ mm
$t = 18$ mm

$$\frac{d}{t} = \frac{750}{18} = 41.67 > 15$$

$\therefore$ Thin cylinder formula can be applied

Hoop stress $\sigma_1 = \frac{pd}{2t} = \frac{2\times750}{2\times18} = 41.67$ N/mm^2

Longitudinal stress $\sigma_2 = \frac{pd}{4t} = \frac{41.67}{2} = 20.83$ N/mm^2

$q_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{20.83}{2} = 10.42$ N/mm^2. Choice (D)

24. Diametral strain $\frac{\delta d}{d} = e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$

$$= \frac{41.67}{2\times10^5} - 0.3\times\frac{20.83}{2\times10^5} = 17.71\times10^{-5}$$

Longitudinal strain $\frac{\delta L}{L} = e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$

$$= \frac{20.83 - 0.3\times41.67}{2\times10^5} = 4.165\times10^{-5}$$

Volumetric strain $\frac{\delta V}{V} = 2e_1 + e_2$

$= (2 \times 17.71 + 4.165) \times 10^{-5} = 39.58 \times 10^{-5}$

$\delta V = 39.58 \times 10^{-5} \times V$

$$= \frac{39.58}{10^5}\times\frac{\pi\times(0.75)^2}{4}\times4 \text{ m}^3$$

$= 69.94 \times 10^{-5}$ m^3 = 699.43 cm^3. Choice (A)

25. $\sigma = \frac{p}{A} + \frac{M_x}{I_x}y + \frac{M_y}{I_y}x$

$$= \frac{500\times10^3}{250\times400} + \frac{500\times10^3\times25}{520.833\times10^6}y + \frac{500\times10^3\times40}{1333\times10}x$$

$$f_D = 500\times10^3\left[\frac{1}{250\times400} - \frac{25\times125}{520.833\times10^6} - \frac{40\times200}{1333\times10^6}\right]$$

$= -1$ N/mm^2. Choice (C)