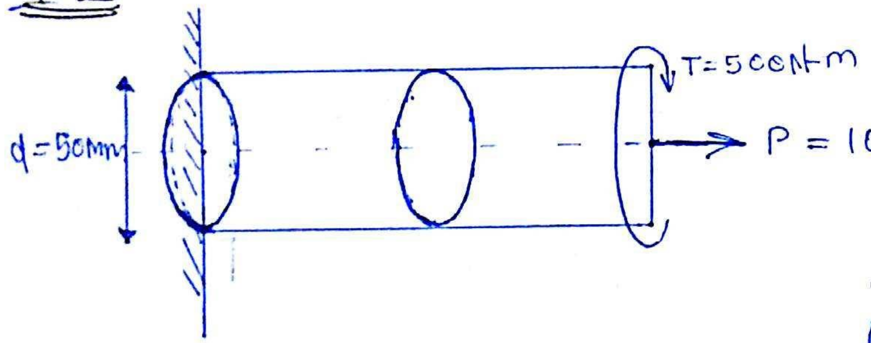
★★ CHAPTER - 07

* Principal stress & Principal strain *

Aim :- Aim of this chapter to derive expression for major principle stress, minor principle stress and max shear stress at a critical point in a component under biaxial state of stress (i.e combined stress)

- The major principle stress, minor principle plane & Max shear stress eq^n are use in the design of a machine component under combined stress (I.e. Theories failure equation)

Ques



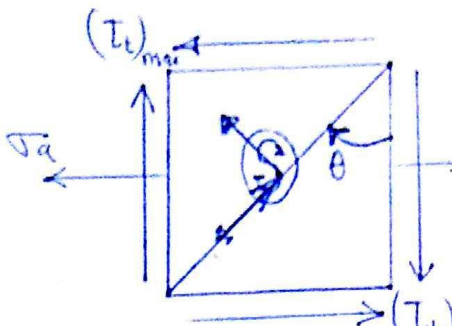
Determine
(a) max. σ_t & Max τ_s induced in bar
(b) max. axial stress & max. torsional shear stress in the bar
(or) max σ_t & Max. shear stress on the x-s/c of bar.

when A.L. = 0

$\Rightarrow$ Max. τ_s on the x-s/c = max. τ_s in the bar $= \dfrac{16T}{\pi d^3}$

when T.M. = 0

$\Rightarrow$ Max. σ_t on the x-s/c = max σ_t in the bar $= \dfrac{4P}{\pi d^2}$



$\sigma_a = \sigma_x = \dfrac{P}{A}$

$(\tau_t)_{max} = \tau_{xy} = \dfrac{16T}{\pi d^3}$

Fig: Bi-axial state of stress at any point on surface of bar (or) any point on the periphery of the x-sc

Expression for σ_n & τ_s developed on a oblique plan passing through a point under bi-axial state of stress

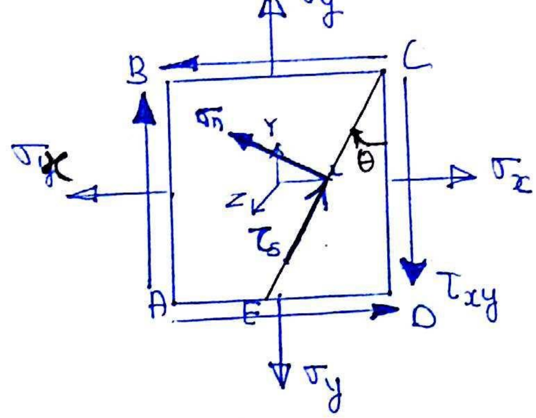
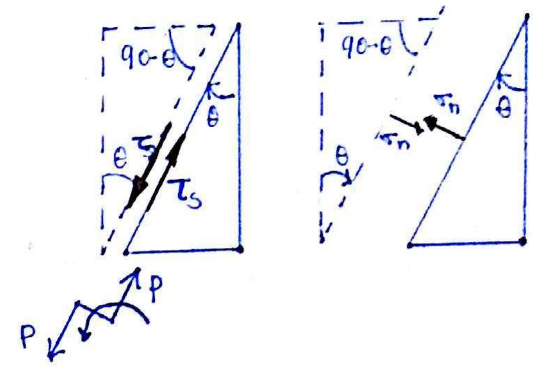


Fig :- Bi-axial state of stress at a critical point



Assume dir^n is ACW

$$(\sigma_n)_\theta = \frac{1}{2}[\sigma_x+\sigma_y] + \frac{1}{2}[\sigma_x-\sigma_y]\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{—(I)}$$

$$(\tau_s)_\theta = -\frac{1}{2}[\sigma_x-\sigma_y]\sin 2\theta + \tau_{xy}\cos 2\theta \quad \text{—(II)}$$

Above eq^n are used to determine normal stress and shear stress on a oblique plane passing through a point under bi-axial state of stress & uni-axial state of stress (i.e. $\sigma_x = \sigma$, $\sigma_y = \tau_{xy} = 0$) following sign convention used

1. tensile normal stress should be considered as positive & vice-versa

2. shear stress on x-face (τ_{xy}) should be considered positive when it causes a couple in cw dirn and vice-versa

3. shear stress on oblique plane (τ_s) should be considered (+ve) when it causes a couple in the ACW dirn

4. Inclination of o.p. (θ) should be considered as positive when it measured in cw dirn from x-face.

let $(\sigma_n)'$ & $(\tau_s)'$ are the normal & shear stresses on a oblique plane which is complimentory to the given oblique plane (CE)

$$(\sigma_n') = (\sigma_n)_{\theta = 90+\theta} = \frac{1}{2}[\sigma_x+\sigma_y] + \frac{1}{2}[\sigma_x-\sigma_y]\cos(180+2\theta) + \tau_{xy}\sin(180+2\theta)$$

$$(\sigma_n') = \frac{1}{2}[\sigma_x+\sigma_y] - \frac{1}{2}[\sigma_x-\sigma_y]\cos 2\theta - \tau_{xy}\sin 2\theta \quad \text{—(III)}$$

$$\boxed{(\sigma_n)_\theta + (\sigma_n)'_{90+\theta} = \sigma_x + \sigma_y = \text{Constant}}$$

$$\boxed{\begin{aligned}&\sigma_1+\sigma_2 = \sigma_x+\sigma_y\\ &\sigma_1+\sigma_2+\sigma_3 = \sigma_x+\sigma_y+\sigma_z\end{aligned}} \quad \text{(IV)}$$

$$(\tau_s)' = (\tau_s)_{\theta = 90+\theta} = -\frac{1}{2}[\sigma_x-\sigma_y](-\sin 2\theta) + \tau_{xy}(-\cos 2\theta)$$

$$(\tau_s)' = -\left[-\frac{1}{2}(\sigma_x-\sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta\right]$$

$$\boxed{(\tau_s)' = -[\tau_s] \Rightarrow \tau_s + \tau_s' = 0}$$ for all state of stress.

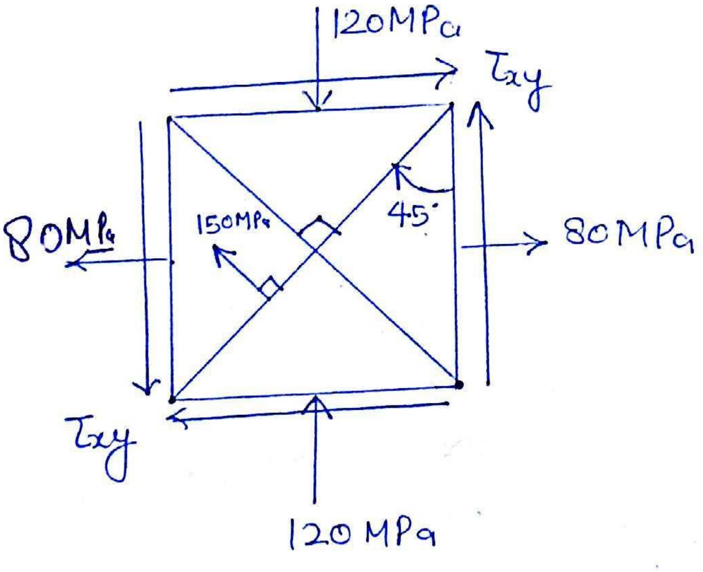
Imp

* Complimentory shear stress at a point are always equal (mag) and unlike in nature.

* Complimentory normal stress at a point are equal in mag. & unlike in nature when $\sigma_x + \sigma_y = 0$
  e.g. pure shear state of stress, N.A.

* When $\sigma_x + \sigma_y \neq 0$ then complimentory normal stress equal in mag or unequal in mag. or like in nature or unlike in nature.

Example



$(\sigma_n)_{\theta=135} = ?$

Sol^n

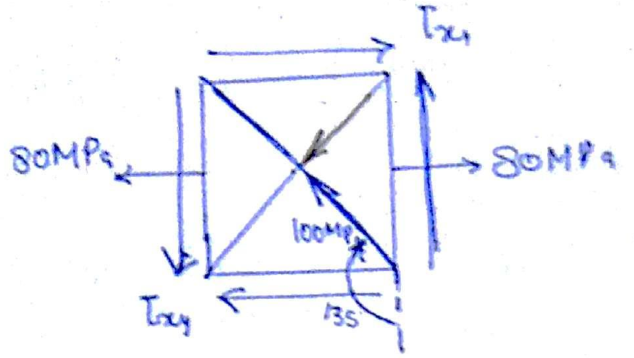
$$(\sigma)_{\theta=45} + (\sigma)_{\theta=135} = \sigma_x + \sigma_y$$

$$(150) + (\sigma_n)_{\theta=135} = 80 - 120$$

$$(\sigma_n)_{\theta=135} = -190\ \text{MPa}$$

$$= 190\ \text{MPa Comp.}$$

eg

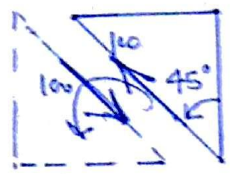


$(\tau_s)_{\theta=45} = ?$

$(\tau_s)_{\theta=135} = 100\ (MPa)\ (ACW)$

Complimentory shear stress are equant & unlike in natur

$$(\tau_s)_{135} + (\tau_s)_{45} = 0$$



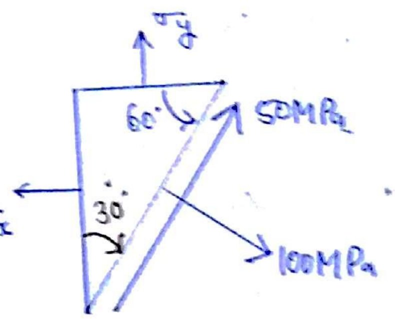
$$(\tau_s)_{45} = -100\ MPa$$

$$= 100\ MPa\ (CW)$$

Given 100 MPa (ACW)

Question

For biaxial state of stress as shown in fig determine value of σ_x & σ_y



$\sigma_x = ?$ $\sigma_y = ?$

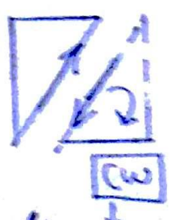
$\tau_{xy} = 0$

$\theta = 90-60 = 30°$ (x-face)

Solm

$\theta = 30°$; $\tau_{xy} = 0$ $\qquad \sigma_n = 100\,MPa$

$\tau_s = -50\,MPa$



-ve माना था eqn derive करते समय

$$(\sigma_n)_{\theta=30} = \frac{1}{2}[\sigma_x+\sigma_y] + \frac{1}{2}[\sigma_x-\sigma_y]\cos 60° + 0$$

$$100 = \frac{1}{2}[\sigma_x+\sigma_y] + \frac{1}{2}[\sigma_x-\sigma_y]\left(\frac{1}{2}\right)$$

$$3\sigma_x + \sigma_y = 400$$

$$(\tau_s)_{\theta=30} = -\frac{1}{2}\left[\sigma_x - \sigma_y\right]\sin 60^\circ + 0$$

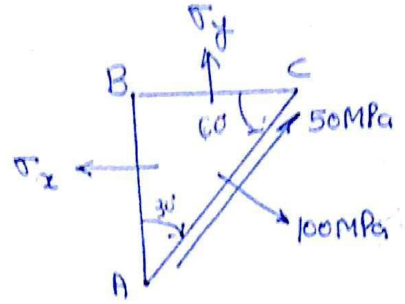
$$-50 = -\frac{1}{2}(\sigma_x - \sigma_y)\frac{\sqrt{3}}{2}$$

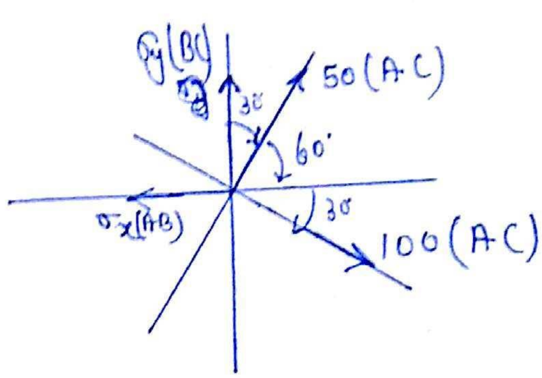
$$\sigma_x - \sigma_y = \frac{200}{\sqrt{3}} \qquad -(11)$$

$$\sigma_x = 128.8 \text{ MPa } (T)$$

$$\sigma_y = 13.39 \text{ MPa } (T)$$

$\boxed{OR}$





$$\sin 30^\circ = \frac{BC}{AC}$$

$$\cos 30^\circ = \frac{AB}{AC}$$

$$\sigma_x(AB) = 50(AC)\cos 60^\circ + 100(AC)\cos 30^\circ \qquad -(1)$$

$$(BC)\,\sigma_y + 50(AC)\sin 60^\circ = 100(AC)\sin 60^\circ \qquad -(2)$$

$$\sigma_x = 50\left(\frac{AC}{AB}\right)\cos 60^\circ + 100\left(\frac{AC}{AB}\right)\cos 30^\circ$$

$$\cancel{\sigma_x = 50\left(\frac{1}{\cos 30^\circ}\right)\cos 60^\circ + 100\left(\frac{1}{\cos 30}\right)\cos 30^\circ}$$

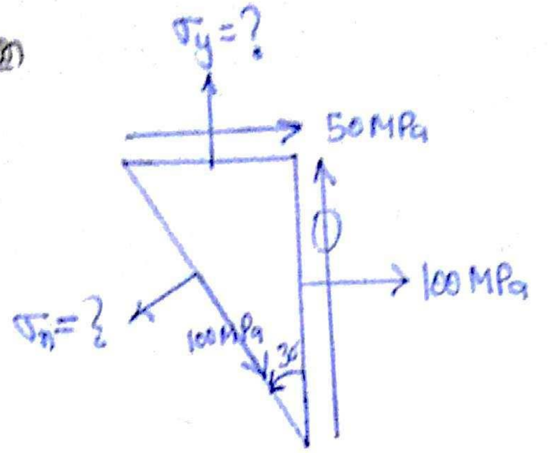
$$\cancel{\sigma_x = \frac{50}{\sqrt{3}/2}\times\left(\frac{1}{2}\right) + \frac{100}{(1/2)}\times\frac{\sqrt{3}}{2} = \frac{50\times 2}{2\sqrt{3}} + \frac{2\times 100\times\sqrt{3}}{2}}$$

$$\sigma_x = 50\left[\frac{1}{\cos 30^\circ}\right]\cos 60^\circ + 100\left[\frac{1}{\cos 30^\circ}\right]\cos 30$$

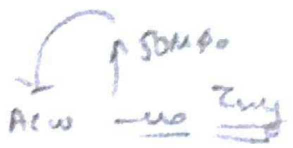
$$\sigma_x = 50\,\frac{1}{\sqrt{3}/2}\left(\frac{1}{2}\right) + 100 = 128.8 \text{ MPa } (T)$$

$$\sigma_y = 100\left(\frac{1}{\sin 30^\circ}\right)\sin 60^\circ - 50\left(\frac{1}{\sin 30}\right)\sin 60^\circ = 13.39 \text{ MPa } (T)$$

Question



And $\sigma_y = ?$ & $\sigma_n = ?$



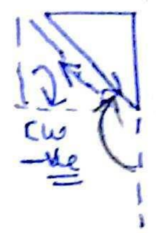
Sol^n

$\sigma_x = 100$ MPa

$\sigma_y = ?$

$\tau_{xy} = -50$ MPa (consider vertical dir^n ($\updownarrow$) it cause couple in ACW dir^n so −ve)

$\tau_s = -100$ MPa



$\theta = -30^\circ$

or $\theta = 150^\circ$

$$-100 = -\frac{1}{2}[\sigma_x - \sigma_y]\sin 2\theta + \tau_x \cos 2\theta$$

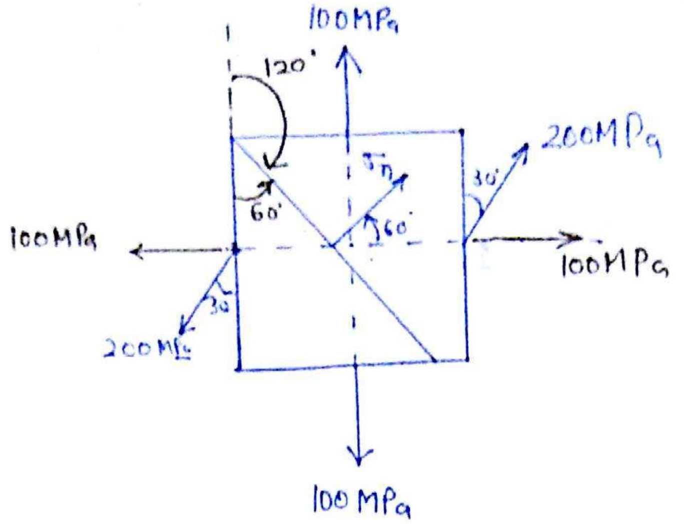
$$-100 = -\frac{1}{2}(100 - \sigma_y)(\sin(-60^\circ)) + (-50)\cos(-60^\circ)$$

$$\sigma_y = 273.2 \text{ MPa } (\uparrow)$$

$$(\sigma_n)_{\theta=-30} = \frac{1}{2}(100 + 273.2) + \frac{1}{2}[100 - 273.2]\cos(-60^\circ) + (-50)\sin(-60^\circ)$$

$$(\sigma_n) = 186.6 \text{ MPa } (\uparrow)$$

Question



Find normal and shear stress on oblique plane.

$\sigma_n = ?$

$\tau_s = ?$

Sol^n $-\sigma_y = 100\,MPa$ $\qquad \theta = -60°$

$-\sigma_x = 200 \sin 30° = 100\,MPa$

$-\tau_{xy} = -200 \cos 30° = -200 \times \frac{\sqrt{3}}{2} = -100\sqrt{3}\,MPa$

$\sigma_x = 100\,MPa$

$\sigma_y = 100\,MPa$ $\qquad \tau_{xy} = -100\sqrt{3}\,MPa$

$\theta = -60°$ or $120°$

$$\sigma_n = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{1}{2}[100+100] + \frac{1}{2}[100-100]\cos(120°) + (-100\sqrt{3})\sin(120°)$$

$$\sigma_n = 100 + 100\sqrt{3}\sin(120°)$$

$$\sigma_n = 100 + 150 = 250\,MPa\,(T)$$

$\sigma_n = 250\,(T$
$\tau_s = 86.602\,(ACW)$

$$\tau_s = -\frac{1}{2}[\sigma_x - \sigma_y]\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_s = -\frac{1}{2}[100-100)\sin(-120°) - 100\sqrt{3}\cos(-120)$$

$$\tau_s = +100\sqrt{3}\cos 120$$

$$\tau_s = 100\frac{\sqrt{3}}{2} = 50\sqrt{3} = 86.602\,MPa$$

Principal Planes & Principal stresses :-

(or) planes of zero τ_s

(or) planes of pure complimentary σ_n

(or) — " — max. & min. σ_n (when $\sigma_{1,2}$ are like in nature)

(or) — " — max. σ_t & Max σ_c (when $\sigma_{1,2}$ are Unlike in nature)

location of principal plance :-

$$(\tau_s)_\theta = \text{——} = 0 \quad \text{(or)} \quad \frac{d}{d\theta}(\sigma_n)_\theta = 0$$

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}} \Rightarrow 2\theta = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

$$2\theta = \text{——}, \text{——}$$

$$\theta, \theta' = \text{—}\text{"}\text{—}$$

$$\boxed{\theta' = \theta + 90}$$

I Method

Principal ~~planes~~ stress :-

$$(\sigma_n)_\theta = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$(\sigma_n)_\theta = \text{——} \text{ MPa} \quad ; \quad (\sigma_n)_{\theta'} = \text{——}$$

$$(\sigma_n)_{\theta'} = \sigma_x + \sigma_y - (\sigma_n)_\theta \Rightarrow (\sigma_n)_{\theta'} = \text{——MPa}$$

σ_1 = major principal stress ~~at~~ at a given point.

= larger of $[(\sigma_n)_\theta, (\sigma_n)_{\theta'}]$

eg Assume $(\sigma_n)_\theta = 50$ MPa, $(\sigma_n)_{\theta'} = -100$ MPa

then $\sigma_1 = -100$ MPa (or) 100 MPa = max Comp. stress at the given point.

= Major principal plane

θ_1 = location of major principal plane

= — " — max. comp. stress plane at a given point

For previouse eq. $\theta_1 = \theta'$

σ_2 = Minor principal stress at a Given point.

= Smaller of $[\underline{(\sigma_n)_\theta}, \underline{(\sigma_n)_{\theta'}}]$

For previouse eg.. σ_2 = 50 MPa

= Max. tensile stress at the given point.

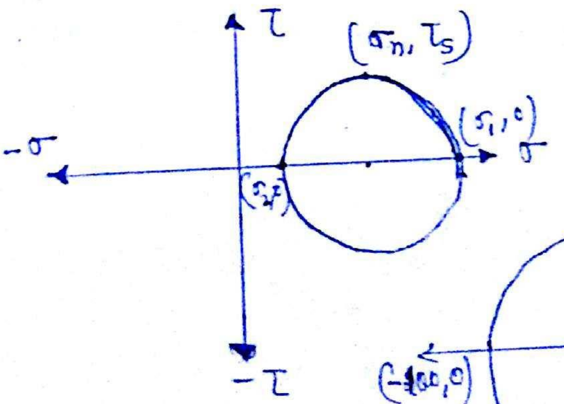
θ_2 = location of minor principal plane at the given point

= — " — max. tensile stress plane at a given point

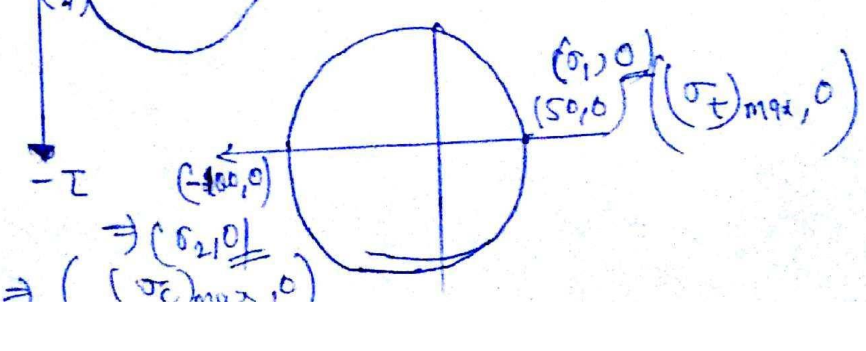
$\theta_2 = \theta$ (For previous eg.)

eg. $(\sigma_n)_\theta$ = 150 MPa $(\sigma_n)_{\theta'}$ = 100 MPa

σ_1 = 150 MPa (Major) σ_2 = 100 MPa (Minor)



$(\sigma_1, 0) = (150, 0)$
$(\sigma_2, 0) = (100, 0)$



II Method for $\sigma_{1,2}$:—

$$\boxed{\sigma_{1,2} = \frac{1}{2}\left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]}$$

$\sigma_{1,2} =$ ———, ——— MPa

$$\boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

$2\theta =$ ———, ———

θ & $\theta' =$ ———

- This method should be use to determine principal stess values only. (it will not gives location)
- To determine principal plane value and corresponding principal stess locations it is always batter to use first method only.

From above eqn—

$$\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$$

$$\sigma_1 - \sigma_2 = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \text{Diameter of Mohr Circle.}$$

Radius of Mohr' circle $= \dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$

Max. shear stress, In plane τ_{max} & Absolute τ_{max} :-

location of τ_{max} planes :-

$$\frac{d}{d\theta}(\tau_s)_\theta = 0$$

$$\tan 2\theta = \frac{\sigma_y - \sigma_x}{2\tau_{xy}} = \frac{-1}{\left(2\tau_{xy}/\sigma_x - \sigma_y\right)}$$

2θ = —, —

$\theta_{3,4}$ = —, —

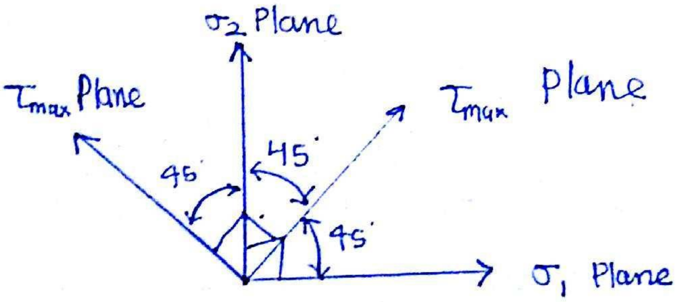
$\theta_4 = \theta_3 + 90^\circ$

$\Rightarrow$ θ_1

$\theta_2 = \theta_1 \pm 90^\circ$ $\theta > 90^\circ$ '−', $\theta < 90$ '+'

$\theta_3 = \theta_1 + 45^\circ$

$\theta_4 = \theta_3 + 90^\circ$



let σ_n^* = normal stress on τ_{max} Plane

In-plane τ_{max} = shear stress on τ_{max} plane

I Method

$$(\sigma_n)^* = (\sigma_n)_{\theta=\theta_3 \text{ or } \theta_4} = \frac{1}{2}[\sigma_x+\sigma_y] + \frac{1}{2}[\sigma_x-\sigma_y]\cos 2\theta_3 + \tau_{xy}\sin 2\theta_3$$

$$(\sigma_n)^* = \_\_\_\_ \text{ MPa}$$

IInd Method

$$\boxed{(\sigma_n)^* = \frac{\sigma_1+\sigma_2}{2}} = \text{Avg. of major \& minor principal stress.}$$

I Method for IN-Plane τ_{max} :-

$$\text{In-plane } \tau_{max} = (\tau_s)_{\theta=\theta_3 \text{ or } \theta_4} = -\frac{1}{2}[\sigma_x-\sigma_y]\sin 2\theta_3 + \tau_{xy}\cos 2\theta_3$$

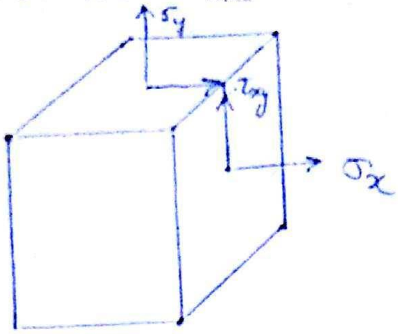
$$\text{In plane } \tau_{max} = \pm \_\_\_\_ \text{MPa}$$

II Method for IN-Plane τ_{max} :-

$$\text{In-plane } \boxed{\tau_{max} = \pm\left(\frac{\sigma_1-\sigma_2}{2}\right)} = \pm\left(\text{half of the diff of } \sigma_1 \,\&\, \sigma_2\right)$$

$$= \pm(\text{Radius of Mohr circle})$$

Absolute τ_{max} :-



$$\text{Abs.}\,\tau_{max} = \text{larger of}\left[\left|\frac{\sigma_1-\sigma_2}{2}\right|, \left|\frac{\sigma_2-\sigma_3}{2}\right|, \left|\frac{\sigma_3-\sigma_1}{2}\right|\right]$$

In plane

τ_{max} = radius of mohr circle

Bi-axial state of stress ($\sigma_3 = 0$)

$$* \ \text{Abs.}\,\tau_{max} = \text{larger of}\left[\left|\frac{\sigma_1-\sigma_2}{2}\right|, \left|\frac{\sigma_1}{2}\right|, \left|\frac{\sigma_2}{2}\right|\right]$$

when $\tau_{1,2}$ are <u>like</u> in nature, Abs $\tau_{max} = \left|\frac{\sigma_1}{2}\right|$

when $\tau_{1,2}$ are <u>Unlike</u> in nature, Abs $\tau_{max} = \left|\frac{\sigma_2}{2}\right|$

In plane τ_{max} & Abs τ_{max} are equal in mag. under following state of stress condition.

① Uni-axial state of stress Condition (i.e. In plane τ_{max} = Abs τ_{max} = $\frac{\sigma_1}{2}$)

② Bi axial state of stress when principal stress are unlike in nature (i.e. Inplane τ_{max} = Abs τ_{max} = $\frac{\sigma_1-\sigma_2}{2}$

Imp

(3) Absolute τ_{max} represents maximum shear stress develope at a given point hence for all design calculation absolute τ_{max} has to be considered.

(4) In plane τ_{max} represent max. shear stress in the given plane only.

Imp

Ques In a biaxial state of stress problem major & minor principal stresses are 200 MPa & 100 MPa, respectively. Det (a) Value of Max τ_s. (b) Value of max τ_s at the given point

(a) = 50 MPa

(b) = 100 MPa

(a) 50 MPa (b) 100 MPa (c) 150 MPa (D) 200 MPa

$$\frac{\sigma_1 - \sigma_2}{2} = 50 \text{ MPa}$$

In-plane τ_{max}

$$\frac{\sigma_1}{2} = 100 \text{ MPa}$$

Abs. τ_{max}

Eg:- $\sigma_1 = 200 \text{ MPa}$; $\sigma_2 = -100 \text{ MPa}$

$$\text{In plane } \tau_{max} = \frac{200 - (-100)}{2} = 150 \text{ MPa}$$

then In-plane τ_{max} = Abs τ_{max} = 150 MPa

Planes of Pure shear stress

(or) Planes of zero normal stress :-

* These planes will exists when $\sigma_{1,2}$ are Unlike in nature only.

* These plane are located by equating σ_n eqn to zero

* These plane are non complimentory planes except when $\sigma_1 = \sigma_2$

* These plane become max. τ_s plane when $\sigma_1 = \sigma_2$

* $\tau_s^* =$ shear stress on planes of pure shear $\leq \tau_{max}$

I Method :-

$$\sigma_n = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 2\theta + \tau_{xy}\sin 2\theta = 0$$

$$2\theta = \text{—}, \text{—}$$

$$\theta_{5,6} = \text{—}, \text{—}$$

$$(\tau_s)^* = (\tau_s)_{\theta = \theta_5 \& \theta_6} = -\frac{1}{2}[\sigma_x - \sigma_y]\sin 2\theta_5 + \tau_{xy}\cos 2\theta_5$$

$$= \pm \text{ —— MPa}$$

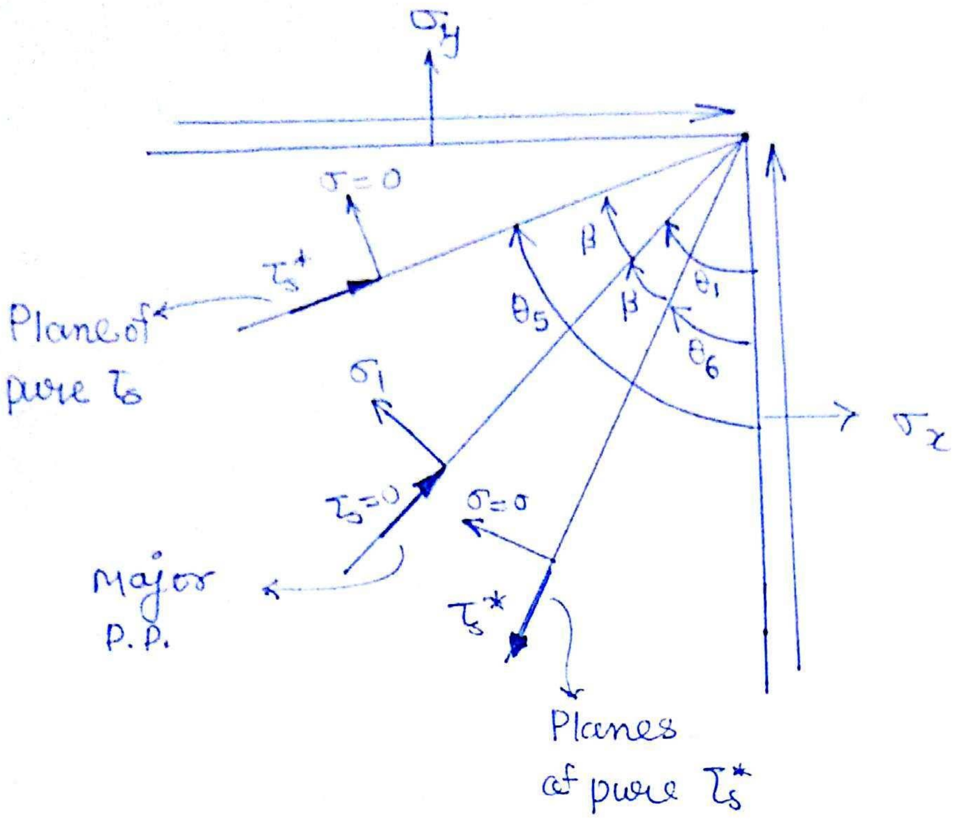
II Method :-

$$\tau_s^* = \sqrt{-\sigma_1 \sigma_2}$$

$$\tan\beta = \sqrt{\frac{-\sigma_1}{\sigma_2}}$$

β = Angle betn planes of pure shear & σ_1 plane

$$\theta_{5,6} = \theta_1 \pm \beta$$



$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \_\_\_\_, \_\_\_\_ \text{ MPa}$$

$$\sigma_n^* = \frac{\sigma_1 + \sigma_2}{2}$$

In-plane $\tau_{max} = \pm\left(\frac{\sigma_1 - \sigma_2}{2}\right)$

Abs. $\tau_{max} = \left|\frac{\sigma_1}{2}\right|$ or $\left|\frac{\sigma_1 - \sigma_2}{2}\right|$

$\tau_s^* = \sqrt{-\sigma_1\sigma_2}$ $\quad$ σ_1 & σ_2 are <u>Unlike</u>

<u>location</u>:-

① $\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$

$\theta, \theta' = \underline{30'}, \underline{120'}$

② $(\sigma_n)_\theta = \underline{100 MPa}$

$(\sigma_n)_{\theta'} = (\sigma_x + \sigma_y) - (\sigma_n)_\theta = \underline{50 MPa}$

$\sigma_1 = \underline{100}$; $\theta_1 = \underline{30'}$

$\sigma_2 = \underline{50}$; $\theta_2 = \underline{120'}$

③ $\theta_3 = \theta_1 + 45^\circ = 75'$

$\theta_4 = \theta_3 + 90^\circ = 165'$

④ $\theta_5 = \theta_1 + \beta$; $\theta_6 = \theta_1 - \beta$

$\because \beta = \tan^{-1}\sqrt{\frac{-\sigma_1}{\sigma_2}}$

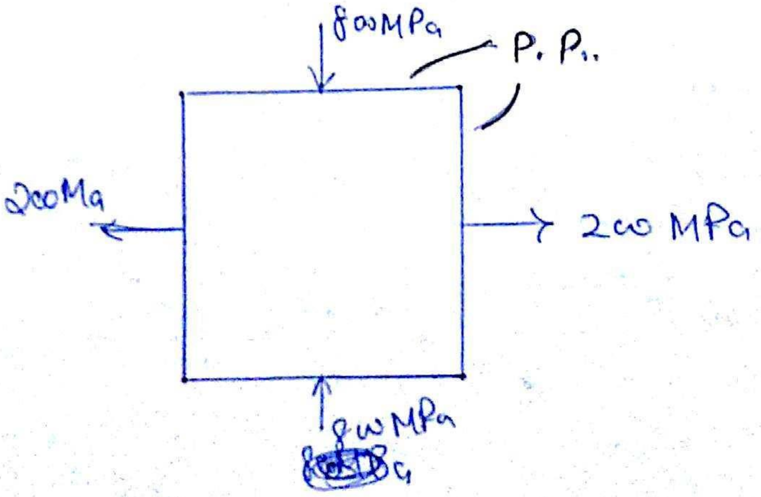
Eg:- For the Bi-axial state of stress at a point as shown in fig det.

① principal planes & corresponding principal stress

② τ_{max} planes and resultant stress on τ_{max} plane.

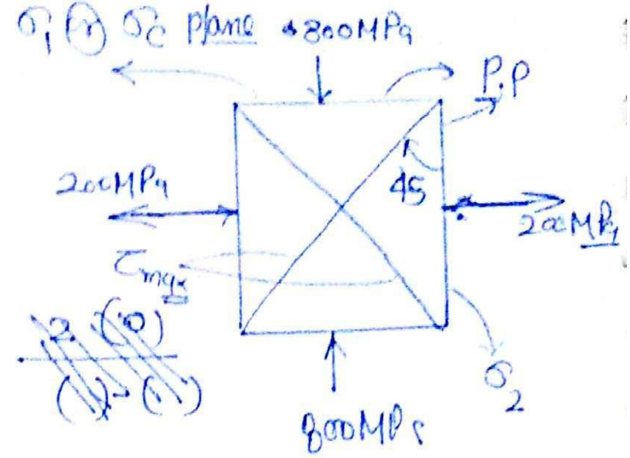
③ location of planes of pure shear. and shear stress on planes of pure shear

④ max. tensile, max. comp. & max. shear stress at the given point.



(a) $\sigma_x = 200 \text{ MPa}$

$\sigma_y = -80 \text{ MPa}$



$$\cancel{\tan 2\theta} = \cancel{\frac{2\tau_{xy}}{\sigma_x - \sigma_y}} = \cancel{\frac{2(0)}{(\ )-(\ )}}$$

$$\cancel{\sigma_1 (\text{or}) \sigma_2} = \cancel{\sigma_x (\text{or}) \sigma_y}$$

$\sigma_1 = -800 \text{ MPa} \qquad \theta_1 = 90^\circ$ (Major)

$\sigma_2 = 200 \text{ MPa} \qquad \theta_2 = 0^\circ$ (Minor)

(b)

Planes of τ_{max}

$\theta_{3,4} = 45^\circ$ & 135°

$$\sigma_n^* = \frac{\sigma_1 + \sigma_2}{2} = \frac{-800+200}{2} = -300 \text{ MPa}$$

$$\text{In plane } \tau_{max} = \pm\left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

$$= \pm\left(\frac{-800-200}{2}\right)$$

$$\tau_{max} = \pm 500 \text{ MPa}$$

$$(\sigma_r)_{\text{on } \tau_{max} \text{ plane}} = \sqrt{\sigma_n^{*2} + (\text{In plane } \tau_{max})^2}$$

$$= \sqrt{(300)^2 + (500)^2} = 583.09 \text{ MPa}$$

(c) Plane of $(\sigma_n)_\theta = 0$

$$\tau_s^* = \sqrt{-\sigma_1 \sigma_2} = \sqrt{(-800)(200)} = \pm 400 \text{ MPa}$$

$$\beta = \tan^{-1}\sqrt{\frac{-\sigma_1}{\sigma_2}} = \tan^{-1}\sqrt{4} = 63.4^\circ$$

$\theta_5 = \theta_1 + \beta = 153.4'$

$\theta_6 = \theta_1 - \beta = 26.6°$

(d) $(\sigma_t)_{max} = 200\ MPa$

$(\sigma_c)_{max} = 800\ MPa$

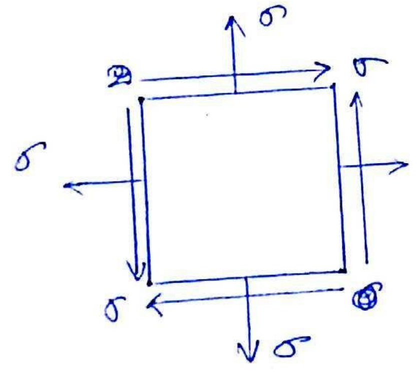
Abs τ_{max} = Inplane τ_{max} = 500 MPa

Eq #

For the bi-axial state of stress as shown below

det. (a) Principle stress & corrosponding plane location.

(b) $(\sigma_c)_{max}, (\sigma_t)_{max}, (\tau_s)_{max}$ at given point



(a)

$\sigma_x = \sigma$

$\sigma_y = \sigma$

$\tau_{xy} = -\sigma$

(i) $\tan 2\theta = \dfrac{2(-\sigma)}{\sigma - \sigma} = \infty$

$2\theta = 90, 270'$

$\theta = 45°, 135°$

(ii) $(\sigma_n)_{\theta=45} = \frac{1}{2}(\sigma+\sigma) + \frac{1}{2}(\sigma-\sigma)\cos 90' + (-\sigma)\sin 90°$

$(\sigma_n)_{\theta=45} = 0$

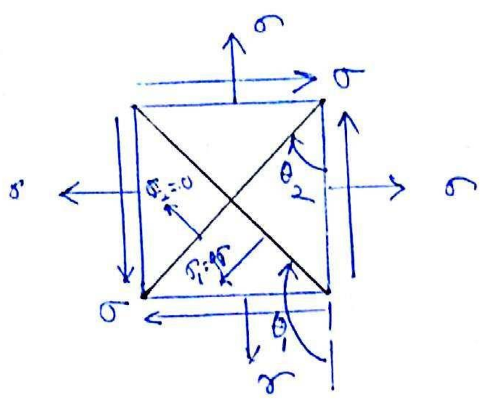
$$\cancel{(\sigma_n)_{\theta=135} = \frac{1}{2}(\sigma+\sigma) + \frac{1}{2}(\ldots)}$$

$$(\sigma_n)_{\theta=135} = \sigma_x + \sigma_y - (\sigma_n)_{\theta=45^\circ}$$

$$(\sigma_n)_{\theta=135} = \sigma + \sigma - 0 = 2\sigma$$

<u>So</u> $\sigma_1 = 2\sigma \;;\; \theta_1 = 135^\circ$

$\sigma_2 = 0 \;;\; \theta_2 = 45^\circ$



(iii) $\theta_{3,4} = 0^\circ \;\&\; 90^\circ$

$\sigma_n^+ = \sigma$

In plane $\tau_{max} = \pm\sigma$

(iv) $(\sigma_t)_{max} = 2\sigma \Rightarrow (135)$

$(\sigma_c)_{max} = 0$

Abs. τ_{max} = In plane $\tau_{max} = \sigma$

$$\frac{(\sigma_t)_{max}}{\tau_{max}} = \frac{2\sigma}{\sigma} = 2$$

Que. Tri- axial state of stress at apoint is represent by ~~strain~~ stress tensor as shown below det. principal stress at that point max. shear stress at that point. Ans

$$[\sigma]_{3D} = \begin{vmatrix} 60 & 30 & 0 \\ 30 & 40 & 0 \\ 0 & 0 & 20 \end{vmatrix} \text{ MPa}$$

Solⁿ

$\tau_{xy} = 30$ MPa

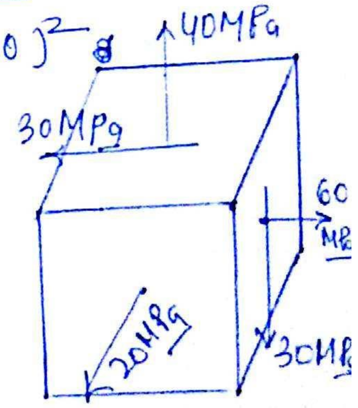
$\sigma_x = 60$ MPa; $\sigma_y = 40$ MPa, $\sigma_z = 20$ MPa

$$\tan 2\theta = \frac{2 \times 30}{20} \Rightarrow 2\theta = 71.56, 251.56$$

$$\tan 2\theta = 3$$

$$\theta = 35.78, \; 125.78 \text{ (Not reqd)}$$

$$\sigma_1, \sigma_2 = \frac{1}{2}(60+40) \pm \sqrt{(60-40)^2 + 4(30)^2}$$



$$\sigma_1, \sigma_2 = 81.62 \text{ MPa}, \; 18.38 \text{ MPa}$$

$\sigma_1 = 81.62$ MPa

$\sigma_2 = 20$ MPa

$\sigma_3 = 18.38$ MPa

$$\text{Abs } \tau_{max} = \left|\frac{\sigma_1 - \sigma_3}{2}\right| = \frac{81.62 - 18.38}{2}$$

$$\tau_{max} = 31.62 \text{ MPa}$$

Eg Deter σ_x, σ_y & τ_{xy} if Normal stress on oblique plane incline at an angle of $0°, 60°, 120°$ are 100 MPa, 150 MPa, 75 MPa

Sol^n

$$(\sigma_n)_\theta = \frac{1}{2}[\sigma_x + \sigma_y] + \frac{1}{2}[\sigma_x - \sigma_y]\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$100 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 0° + \tau_{xy}\sin 0°$$

$$100 = \frac{1}{2}\sigma_x + \cancel{\frac{\sigma_y}{2}} + \frac{\sigma_x}{2} - \cancel{\frac{\sigma_y}{2}}$$

$$\sigma_x = 100 \text{ MPa} \quad -(1)$$

σ_n'

$$150 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 120 + \tau_{xy}\sin 120°$$

$$150 = \frac{100}{2} + \frac{\sigma_y}{2} + \left(\frac{100}{2} - \frac{\sigma_y}{2}\right)\left(-\frac{1}{2}\right) + \tau_{xy}\left(\frac{\sqrt{3}}{2}\right)$$

$$\cancel{150 = \frac{3}{2}\sigma_y + \tau_{xy}}$$

$$150 = 50 + \frac{\sigma_y}{2} - 25 + \frac{\sigma_y}{4} + \tau_{xy}\frac{\sqrt{3}}{2}$$

$$150 = 25 + \frac{3}{4}\sigma_y + \tau_{xy}\frac{\sqrt{3}}{2} \quad -(2)$$

$$75 = \frac{1}{2}(100 + \sigma_y) + \frac{1}{2}(100 - \sigma_y)\left(-\frac{1}{2}\right) - \tau_{xy}\frac{\sqrt{3}}{2} \quad -(3)$$

from eq (2) & (3)

$$\sigma_y = 116.67 \text{ MPa}; \quad \tau_{xy} = 43.3 \text{ MPa}$$

$$\sigma_x = 100 \text{ MPa}$$

Eg Determine the following when a point is under pure shear state of stress

(a) Principle plane & Corresponding principal stress

(b) τ_{max} planes & Corresponding stress

(c) Max tensile, Max. Comp. & Max. shear stress at given point

$$\sigma_x = \sigma_y = 0 \; ; \quad \tau_{xy} = \tau$$

(a) P.P. location

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \infty$$

$$2\theta = 90^\circ ;\ 270^\circ$$

$$\theta, \theta' = 45^\circ ,\ 135^\circ$$

shaft under pure tension

(b)

$$(\sigma_n)_{\theta=45} = \frac{1}{2}(0+0) + \frac{1}{2}(0-0)\cos(90) + \tau \sin 90^\circ$$

$$(\sigma_n)_{\theta=45} = \tau$$

$$(\sigma_n)_{\theta=135} = \sigma_x + \sigma_y - (\sigma_n)_{\theta=45} = -\tau$$

$$\sigma_1 = \tau \quad , \quad \theta_1 = 90^\circ$$

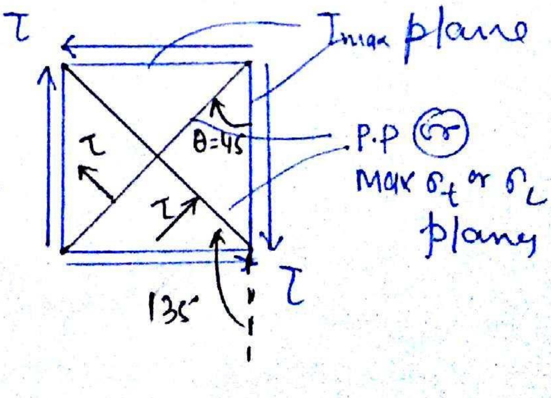
$$\sigma_2 = -\tau \quad , \quad \theta_2 = 135$$

(c) $\sigma_1 / \sigma_2 = -1$

(d) $(\sigma_t)_{max} = (\sigma_c)_{max} =$ Inplane τ_{max} = Abs. τ_{max}

$$= \tau = \frac{16T}{\pi d^3}$$

(e) $\theta_{3,4} = \theta_{5,6} = 0^\circ \; \& \; 90^\circ$



Note:-

Mild Steel

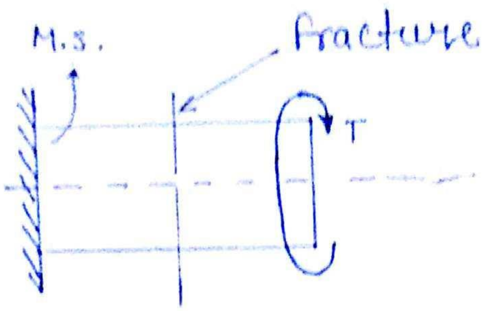
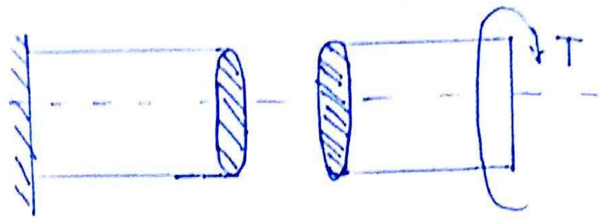


Fig. smooth transverse fracture of M.S. shaft under pure torsion



Cast Iron

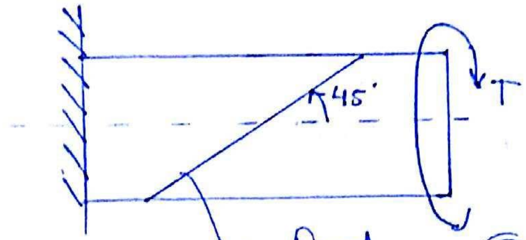
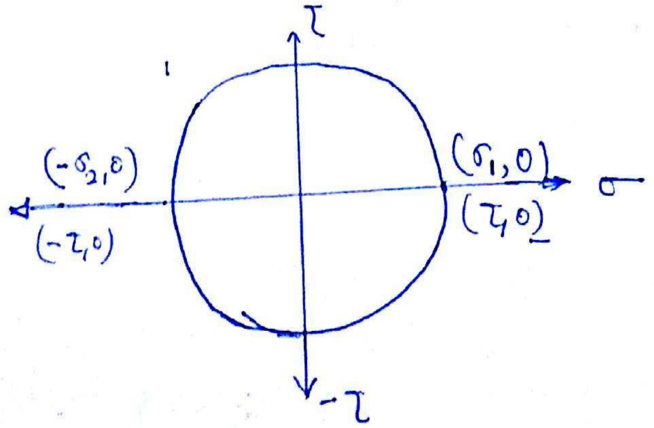


Fig :- Granular helicoidal fracture of CI shaft under pure torsion.



Mohr Circle
Centre (0,0)
Radius = τ

workbook
Q. 13
P.g. 42

$\sigma_a = -\sigma$

$\sigma_b = \sigma$

$\tau_s = \sqrt{3}\sigma$

Sol^n

$\sigma_a = -\sigma$

either (+) or (−) $\rightarrow$ $\sigma_b = \pm\sigma$

$\tau_s = \sqrt{3}\sigma$

$\tau_{xy} = \sqrt{3}\sigma$

$\sigma_x = \sigma_a \pm \sigma_b$

$\sigma_x = -\sigma - \sigma$

$\sigma_x = -2\sigma$

$\sigma_y = 0$

objective

$$\sigma_1, \sigma_2 = \frac{1}{2}\left\{(\sigma_x + \sigma_x) \pm \sqrt{4\tau_{xy}^2 + (\sigma_x - \sigma_y)^2}\right\}$$

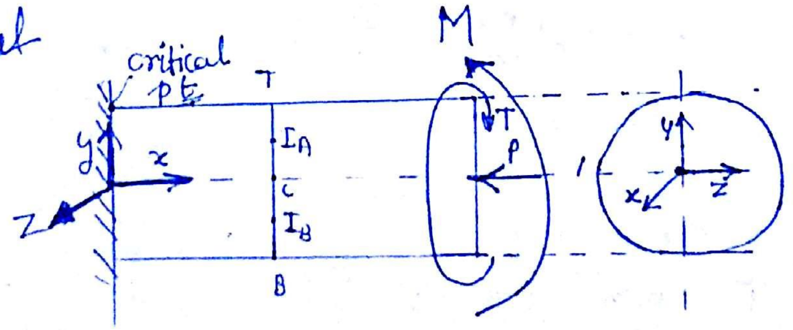
$$\sigma_{1,2} = \frac{1}{2}\left\{(-2\sigma) \pm \sqrt{4 \times 3\sigma^2 + 4\sigma^2}\right\}$$

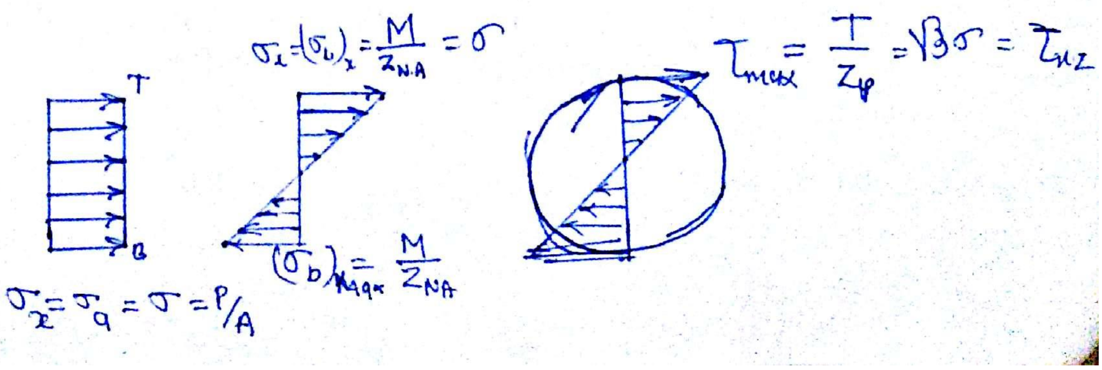
$$\sigma_{1/2} = -\sigma \pm 2\sigma$$

$\sigma_{1} = -3\sigma$ or 3σ Comp.

$\sigma_2 = \sigma$ tensile

Conventional





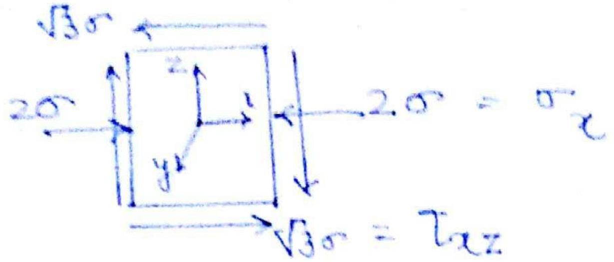


Fig Bi-axial state of stress at a ~~point~~ point on the top fibre of X-S/C

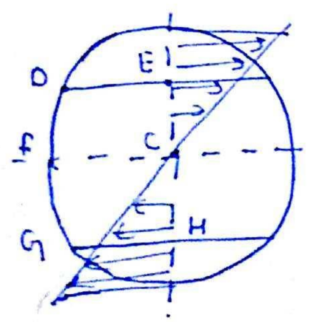
$$(\sigma_{1,2})_T = \frac{1}{2}[\sigma_x+\sigma_y] \pm \sqrt{(\sigma_x-\sigma_y) + 4(\tau_{xy})^2}$$

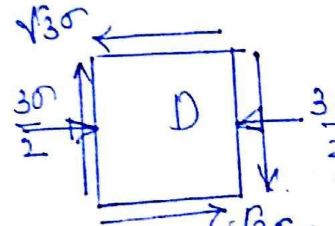
$$= \frac{1}{2}\left[(-2\sigma+0) \pm \sqrt{(-2\sigma)^2 + 4(\sqrt{3}\sigma)^2}\right]$$

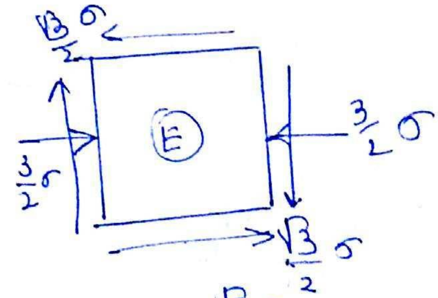
$$= -\sigma \pm 2\sigma$$

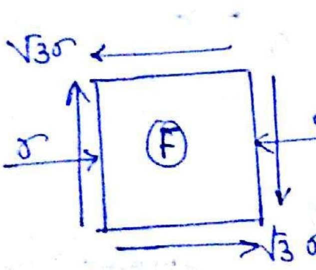
$$(\sigma_{1,2})_T = \sigma, -3\sigma$$

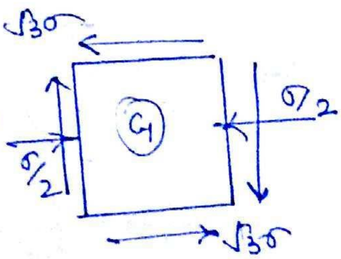
$$(\sigma_1)_{top} = -3\sigma \qquad (\sigma_2)_{top} = \sigma$$

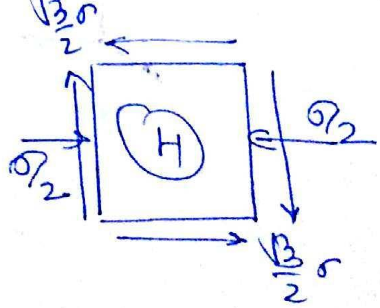












Principal strains & Max. Shear strain :-

I/P Data → state of strain at a point

$$[\epsilon]_{2D} = \begin{bmatrix} \epsilon_x & \frac{\gamma_{xy}}{2} \\ \frac{\gamma_{xy}}{2} & \epsilon_y \end{bmatrix} \qquad \begin{matrix} \sigma \rightarrow \epsilon \\ \tau \rightarrow \gamma/2 \end{matrix}$$

let ϵ_n & γ_s are the normal strain & shear strain on an O.P. passing through a point under bi-axial state of strain.

$$\boxed{(\epsilon_n)_\theta = \frac{1}{2}[\epsilon_x+\epsilon_y] + \frac{1}{2}[\epsilon_x-\epsilon_y]\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta} \quad -(I)$$

$$\frac{(\gamma_s)_\theta}{2} = -\frac{1}{2}[\epsilon_x-\epsilon_y]\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta \quad -(II)$$

$$\boxed{(\gamma_s)_\theta = -[\epsilon_x-\epsilon_y]\sin 2\theta + \gamma_{xy}\cos 2\theta} \quad -(II)$$

$$\boxed{(\epsilon_n)_\theta + (\epsilon_n)_{90+\theta} = \epsilon_x+\epsilon_y}$$

$$\boxed{(\gamma_s)_\theta + (\gamma_s)_{90+\theta} = \text{zero}}$$

location of P.P. →

$$(\gamma_s)_\theta = \_\_\_ = 0 \quad \text{or} \quad \frac{d}{d\theta}(\epsilon_n)_\theta = 0$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\epsilon_x-\epsilon_y} \qquad \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

II Method for $\epsilon_{1,2}$:-

$$(\epsilon_{1,2}) = \frac{1}{2}\left[(\epsilon_x + \epsilon_y) \pm \sqrt{(\epsilon_x - \epsilon_y)^2 + 4\left(\frac{\gamma_{xy}}{2}\right)^2}\,\right]$$

$$\epsilon_1 + \epsilon_2 = \epsilon_x + \epsilon_y$$

$$\epsilon_1 - \epsilon_2 = \sqrt{(\epsilon_x - \epsilon_y)^2 + 4\left(\frac{\gamma_{xy}}{2}\right)^2} = \text{Dia. Mohr circle for strain}$$

II Method for Abs γ_{max}

$$\text{Abs } \frac{\gamma_{max}}{2} = \max.\left[\left|\frac{\epsilon_1 - \epsilon_2}{2}\right|, \left|\frac{\epsilon_2 - \epsilon_3}{2}\right|, \left|\frac{\epsilon_3 - \epsilon_1}{2}\right|\right]$$

$$\text{Abs. Max. } \gamma_{max} = \text{larger of}\left[\underbrace{|\epsilon_1 - \epsilon_2|, |\epsilon_2 - \epsilon_3|, |\epsilon_3 - \epsilon_1|}\right]$$

In plane γ_{max} = dia of mohr circle. for strain

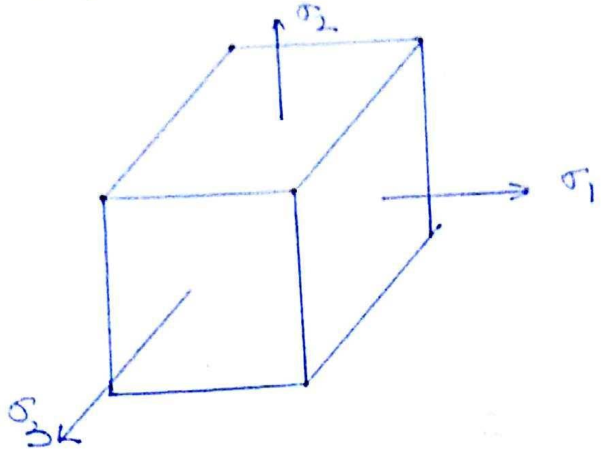
For bi-axial, $\epsilon_3 = 0$

When $\epsilon_{1,2}$ are like in nature, Abs $\gamma_{max} = |\epsilon_1|$

when $\epsilon_{1,2}$ are Unlike in nature, Abs $\gamma_{max} = |\epsilon_1 - \epsilon_2|$

Relationship between principal stress & Principal strain :-

Case-I Expression for $\epsilon_{1,2,3}$ in term of $\sigma_{1,2,3}$:-



$$\left.\begin{aligned} \epsilon_1 &= \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2+\sigma_3)\right] \\ \epsilon_2 &= \frac{1}{E}\left[\sigma_2 - \mu(\sigma_1+\sigma_3)\right] \\ \epsilon_2 &= \frac{1}{E}\left[\sigma_3 - \mu(\sigma_1+\sigma_2)\right] \end{aligned}\right\} \quad -①$$

$$\epsilon_v = \epsilon_1 + \epsilon_2 + \epsilon_3 = \frac{(1-2\mu)}{E}\left[\sigma_1+\sigma_2+\sigma_3\right]$$

Under plane stress problem $(\sigma_3 = 0)$

$$\left.\begin{aligned} \epsilon_1 &= \frac{1}{E}\left[\sigma_1 - \mu\sigma_2\right] \quad -(a) \\ \epsilon_2 &= \frac{1}{E}\left[\sigma_2 - \mu\sigma_1\right] \quad -(b) \\ \epsilon_3 &= \frac{-\mu}{E}\left[\sigma_1+\sigma_2\right] \quad -(c) \end{aligned}\right\} \quad ②$$

eqn ① & ⑪ are used to determine principal strains at a point when state of stress or principal stresses at that point are know.

Case (II) Expression for σ_1, σ_2 in term of $\epsilon_{1,2}$

From eq^n (a), $\sigma_1 = E\,\epsilon_1 + \mu\,\sigma_2$ — (1)

From eq^n (b), $\sigma_2 = E\,\epsilon_2 + \mu\,\sigma_1$ — (2)

by sub. eq^n (2) in eq^n (1)

$$\sigma_1 = E\,\epsilon_1 + \mu(E\,\epsilon_2 + \mu\,\sigma_1)$$

$$\boxed{\sigma_1 = \left(\frac{E}{1-\mu^2}\right)\left[\epsilon_1 + \mu\,\epsilon_2\right]} \quad - (3)$$

Similarly by sub eq^n (1) in eq^n (2)

$$\boxed{\sigma_2 = \frac{E}{1-\mu^2}\left(\epsilon_2 + \mu\,\epsilon_1\right)} \quad - (4)$$

eq^n (3) & (4) are used to determine principal stresses at a point when state of strain (or) principal strain at that point are know.

Quest

$$\sigma_{2D} = \begin{bmatrix} 100 & 40 \\ 40 & 50 \end{bmatrix} \text{MPa}$$

det
(a) $\epsilon_{1,2}$
(b) Abs. τ_{max}
(c) Abs γ_{max}

$E = 200$ GPa
$\mu = 0.3$

Ans_ $\sigma_x = 100$ MPa

$\sigma_y = 50$ MPa

$\tau_{xy} = 40$ MPa

$$\sigma_{1,2} = \frac{1}{2}\left((100+50) \pm \sqrt{(50)^2 + 4\times(40)^2}\right)$$

$$\sigma_{1,2} = \frac{1}{2}\left[150 \pm \sqrt{2500 + 6400}\right]$$

$$\sigma_{1,2} = \frac{1}{2}\left[150 \pm 94.33\right]$$

$$\sigma_1 = 122.16 \qquad \sigma_2 = 27.83 \text{ MPa}$$

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_2) = \frac{1}{200\times10^3}(122.16 - 0.3\times27.83)$$

$$\epsilon_1 = 5.69\times10^{-4}$$

$$\epsilon_2 = \frac{1}{200\times10^3}(27.83 - 0.3\times122.16)$$

$$\epsilon_2 = -4.409\times10^{-5} = -0.4409\times10^{-4}$$

$$\tau_{max} = \frac{\sigma_1}{2} = \frac{122.16}{2} = \underline{56.43 \text{ MPa}}$$

$$\gamma_{max} = |\epsilon_1 - \epsilon_2| = 5.69\times10^{-4} + 0.4409\times10^{-4}$$

$$\gamma_{max} = 6.1309\times10^{-4}$$

Que $[\epsilon]_{2D} = \begin{bmatrix} 100 & 250 \\ 250 & 600 \end{bmatrix} \times 10^{-6}$

$E = 200$ GPa

$G = 80$ GPa

$\mu = 0.25$

det : (a) $\sigma_{1,2}$ (b) Abs γ_{max} (c) Abs τ_{max}

$$\epsilon_{1,2} = \frac{1}{2}\left[(\epsilon_x + \epsilon_y) \pm \sqrt{(\epsilon_x - \epsilon_y)^2 + 4\left(\frac{\gamma_{xy}}{2}\right)^2}\right] \times 10^{-6}$$

$$= \frac{1}{2}\left[(100 + 600) \pm \sqrt{(100-600)^2 + 4\left(\frac{250}{\cancel{2}}\right)^2}\right] = \cancel{\frac{(700 \pm 559)}{2}}$$

$\epsilon_1 = \cancel{629.56\times10^{-6}}$ 703.55×10^{-6} $\epsilon_2 = -3.55 \times 10^{-6}$

$$\sigma_1 = \frac{E}{1-\mu^2}[\epsilon_1 + \mu\epsilon_2] = \frac{\cancel{200\times10^3}}{\cancel{0.9375}} \frac{200\times10^3}{0.9375}(703 - 0.9375 \times 3.55) \times 10^{-6}$$

$$= 149.38$$

$$\sigma_2 = \frac{E}{1-\mu^2}[\epsilon_2 + \mu\epsilon_1) = \frac{200 \times 10^3}{0.9375}(-3.55 + 0.25 \times 703.55) \times 10^{-6}$$

$\sigma_1 = 149.38$ MPa ; $\sigma_2 = 36.76$ MPa

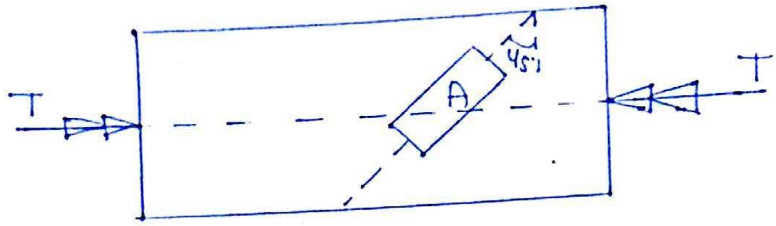
$$\text{Abs } \tau_{max} = \frac{149.38}{2} = 74.69 \text{ MPa}$$

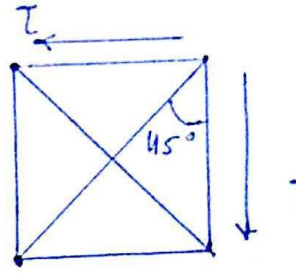
$$\text{Abs } \gamma_{max} = |\epsilon_1 - \epsilon_2|$$

$$\gamma_{max} = 707.1 \times 10^{-6}$$

Ques Determine the power transmission capacity of circular shaft rotating 500 rpm, Given d = 40 mm (diameter) strain gauge ~~reating~~ reading which mounted on the surface of shaft at an angle of 45° is equal = 250×10^{-6} $E = 200\,GPa$, $\mu = 0.3$

Sol





$$(\epsilon_n)_{\theta=45^\circ} = \epsilon_1 = 250 \times 10^{-6}$$

$$(\epsilon_n)_{\theta=135} = \epsilon_2 = -250 \times 10^{-6}$$

$$\sigma_1 = \frac{E}{1-\mu^2}\left[\epsilon_1 + \mu\epsilon_2\right] = \frac{200\times10^3}{1-(0.3)^2}\left[250 - 0.3\times250\right]\times10^{-6}$$

$$\sigma_1 = 38.46\ MPa \qquad \sigma_2 = -38.46\ MPa$$

$$\text{Abs } \tau_{max} = \sigma_1 = 38.46\ MPa$$

$$T_{max} = \frac{\pi}{16} d^3 \tau_{max} = \frac{\pi}{16}(40)^3 \times 38.46 = 483.30\ N\text{-}m$$

$$P = \frac{2\pi T N}{60} = \frac{2\times\pi\times500\times483.30}{60}$$

$$P = 25.306\ kW$$

$$\text{Abs. } \gamma_{max} = \epsilon_1 - \epsilon_2 = 500\times10^{-6}$$

$$\text{Abs } \tau_{max} = G\,\gamma_{max} = 38.46\ MPa$$

Strain Rosettes:-

It is define as the arrangment of strain ~~gausage~~ gauges in three arbitory dirn.

strain gauges are used to measure normal strain along the direction

Based on the arrangment of strain gauges Strain rosettes are classifield into

① rectangular strain rosettes ($\alpha = 45°$; $\theta = 0°, 45°, 90°$)

② Delta strain rosettes ($\alpha = 60°$, $\theta = 0°; 60°, 120°$)

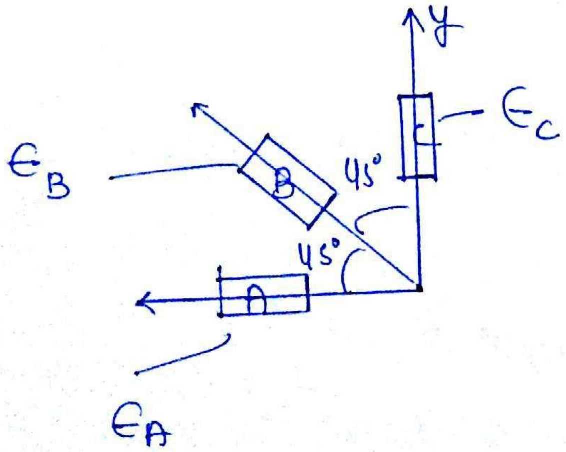
③ Star strain rosettes ($\alpha = 120°$, $\theta = 0°; 120°, 240°$)

where α = angle between strain gauges

θ = Inclination of oblique plane from refrence plane.

Expression for ϵ_x, ϵ_y & γ_{xy} in term of rectangular strain rosette reading

$$\epsilon_A = (\epsilon_n)_{\theta=0°} ; \quad \epsilon_B = (\epsilon_n)_{\theta=45°} ; \quad \epsilon_C = (\epsilon_n)_{\theta=90°} =$$



$$(\epsilon_n)_\theta = \frac{1}{2}[\epsilon_x + \epsilon_y] + \frac{1}{2}[\epsilon_x - \epsilon_y]\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$(\epsilon_n)_{\theta=0^\circ} = \frac{1}{2}[\epsilon_x + \epsilon_y] + \frac{1}{2}[\epsilon_x - \epsilon_y](1) + \frac{\gamma_{xy}}{2}(0)$$

$$\epsilon_x = \epsilon_A$$

$$(\epsilon_n)_{\theta=90^\circ} = \frac{1}{2}[\epsilon_x + \epsilon_y] + \frac{1}{2}[\epsilon_x - \epsilon_y](-1) + \frac{\gamma_{xy}}{2}(0) = \epsilon_c$$

$$\epsilon_y = \epsilon_c$$

$$(\epsilon_n)_{\theta=45^\circ} = \frac{1}{2}[\epsilon_x + \epsilon_y] + \frac{1}{2}[\epsilon_x - \epsilon_y](0) + \frac{\gamma_{xy}}{2}(1) = \epsilon_B$$

$$\gamma_{xy} = 2\epsilon_B - \epsilon_x - \epsilon_y$$

$$[\epsilon]_{2D} = \begin{bmatrix} \epsilon_A & \frac{2\epsilon_B - \epsilon_A - \epsilon_c}{2} \\ \frac{2\epsilon_B - \epsilon_A - \epsilon_c}{2} & \epsilon_c \end{bmatrix}$$

Quest

$\epsilon_{\theta=0} = 1000 \times 10^{-6}$

$\epsilon_{\theta=45} = 400 \times 10^{-6}$

$\epsilon_{\theta=90^\circ} = 800 \times 10^{-6}$

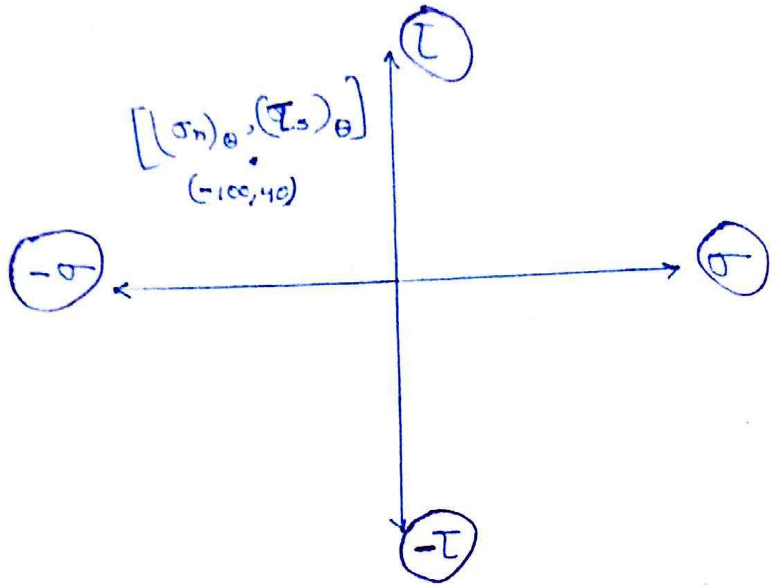
(a) $\epsilon_{1,2}$ (b) $\sigma_{1,2}$

(c) $\theta_{1,2}$ (d) Abs τ_{max}

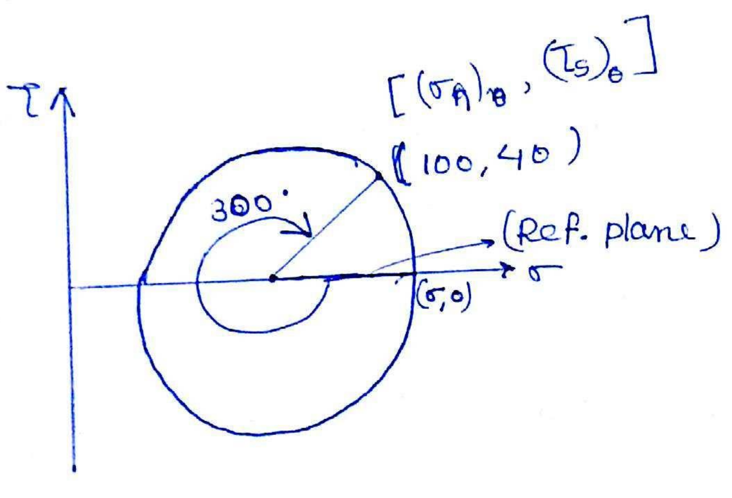
Abs γ_{max}.

MOHR'S CIRCLE :-
(Graphical method)

Sign convention used in mohr's circle for stress



Eg



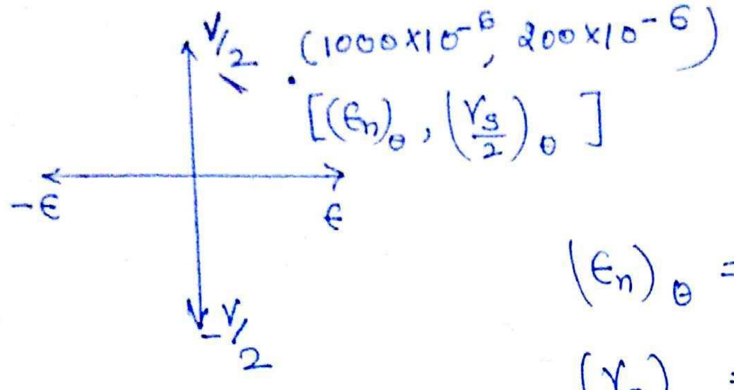
$(\sigma_n)_{\theta=150^\circ} = 100\,MPa$ (T)

$(\tau_s)_{\theta=150^\circ} = 40\,MPa$ (CW)

$$\begin{bmatrix}\text{X- Coordinate of} \\ \text{any point on Mohrs} \\ \text{Circle for stress}\end{bmatrix} = \begin{bmatrix}\text{Normal stress on} \\ \text{the corresponding} \\ \text{oblique plane}\end{bmatrix}$$

$$\begin{bmatrix}\text{Y- Coordinate of} \\ \text{any point on Mohr's} \\ \text{circle for stress}\end{bmatrix} = \begin{bmatrix}\text{Shear stress on the} \\ \text{Corresponding} \\ \text{oblique plane}\end{bmatrix}$$

* Sing convention used in Mohr's Circle for strain :-



$(\epsilon_n)_\theta = 1000 \times 10^{-6}$ (T)

$(\gamma_s)_\theta = 400 \times 10^{-6}$ (CW)

[Normal strain on any o.b.] = [x- Coordinate of corresponding point on the Mohr's circle for strain]

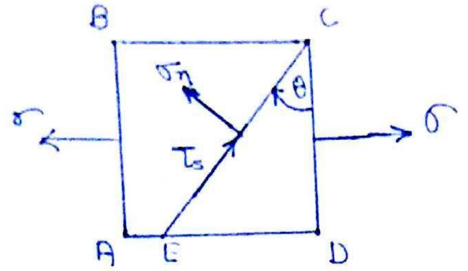
[Shear strain on any o.b.] = 2 [Y-Coordinate of corresponding on the Mohr's circle for strain]

Steps used to draw a Mohr's Circle :-

1. Draw the graphical representation of bi-axial state of stress of a critical point in a component
2. Draw the x-y axes.
3. Represent 'σ' on x-axis & 'τ' on Y-axis
4. Fix a common scale for normal and shear stress (σ & τ)
5. locate a point A corresponding to state on x-face.
6. locate a point B corresponding to state of stress on Y-face
7. Join the points A & B and bisects the line AB

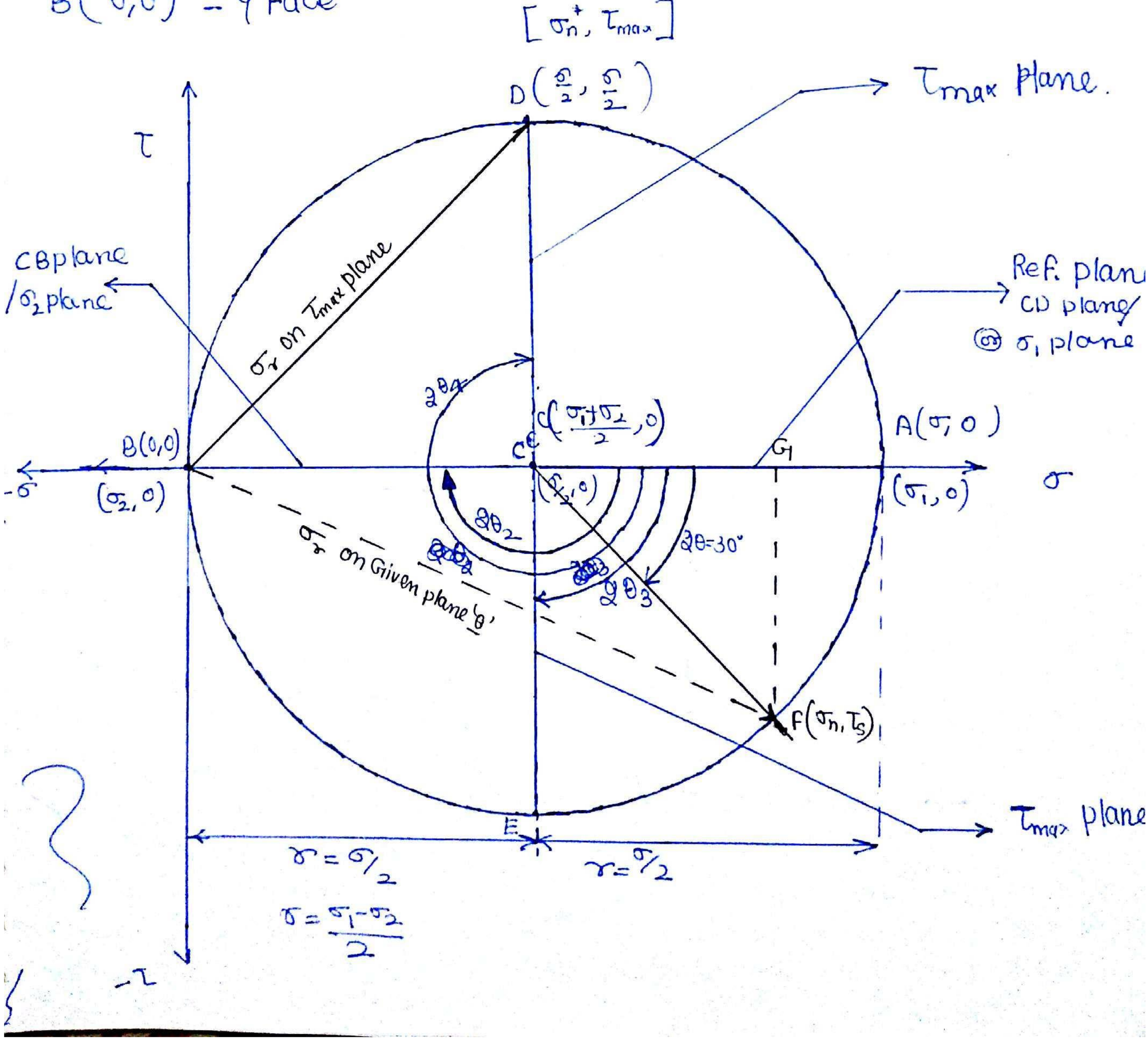
(8) By considering bisection point 'C' as centre of circle and radius is equal to CA & CB draw a circle.

Case I :-



A $(\sigma, 0)$ - x face

B $(0, 0)$ - y face



Radial line	plane
CA	Ref. plane/σ_1 plane
CB	σ_2 plane
CD & CE	τ_{max} plane.
OD	Resultant on τ_{max} plane
OF	Resultant on oblique plane

$\sigma_1 = OA$; $\theta_1 = 0^\circ$

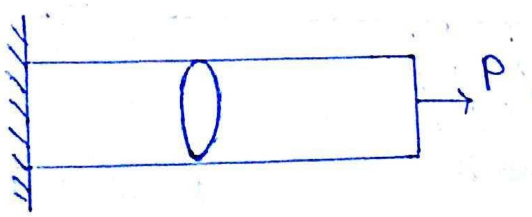
$\sigma_2 = OB = \text{zero}$; $\theta_2 = 90^\circ$

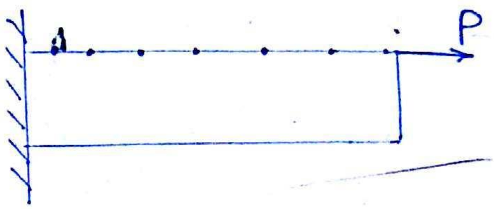
$\sigma_n^+ = OC = \frac{\sigma}{2}$

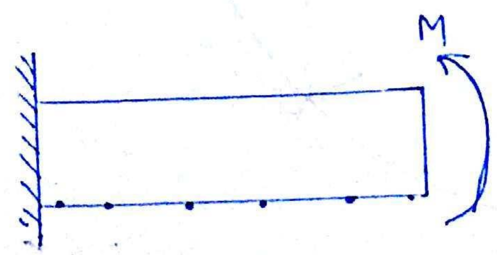
Inplane $\tau_{max} = CD = CE = \pm \frac{\sigma}{2}$

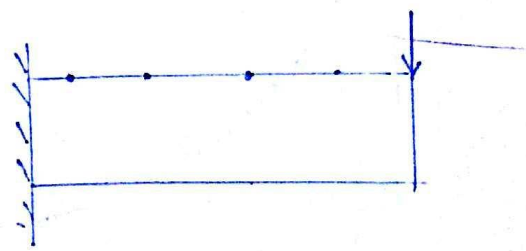
$\theta_3 = 45^\circ$; $\theta_4 = 135^\circ$

This mohr's circle valid for







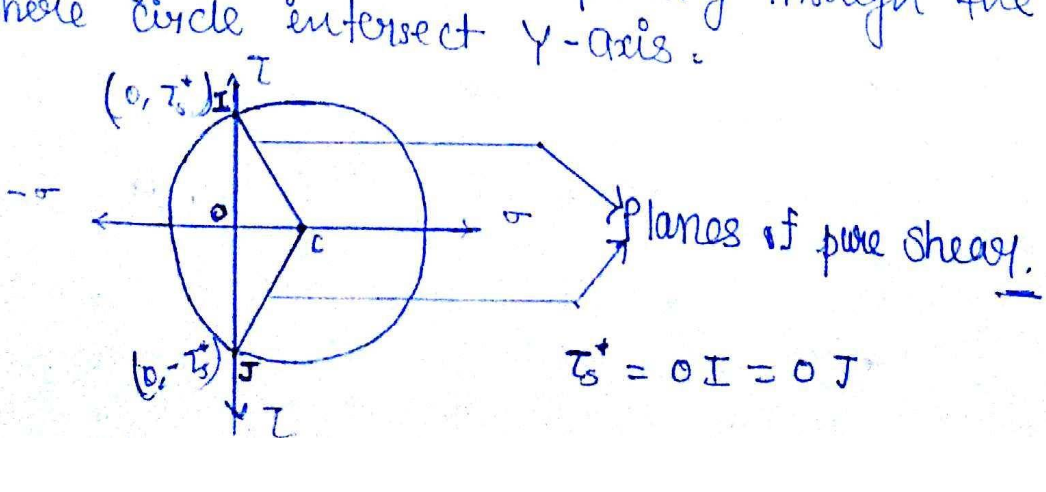


Following conclusion can be made

1. every plane passing through a point under bi-axial state of stress is represented by a radial line is a mohr circle.
2. coordinates of any point on the circle represent state of stress on the corresponding oblique plane.
3. every plane is represent the mohr circle by double its actual angle.
4. centre of mohr' circle always lies on x-axis
   [i.e. $c\left(\frac{\sigma_1+\sigma_2}{2}, 0\right)$]
5. Principal plane are representent on mohr circle by radial line lying on x-axis
6. principal stress are equal to x-coordinates of point of intersection of circle with x-axis
7. max shear stress plane are represented in mohr circle by radial lines which are ||el to y-axis
8. Normal stress on max. shear ~~ste~~ stress plane (σ_n^*) is equal to x-coordinate of center of mohr circle.
9. In plane τ_{max} is equal to radius of mohr's circle

$$\text{In-plane } \tau_{max} = \pm R = \pm\left(\frac{\sigma_1-\sigma_2}{2}\right) = \pm\frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$$

10. Plane of pure shear stress planes are represented on Mohr's circle by the radial line passing through the points where circle intersect y-axis.



$$\tau_s^* = OI = OJ$$

(11) Planes of pure shear are possible when mohr's intersect Y-axis i.e. when principal stress are unlike in nature.

$$\tau_s^{*} = OI = OJ = \sqrt{(CI)^2 - (OC)^2}$$

$$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 - \left(\frac{\sigma_1 + \sigma_2}{2}\right)^2}$$

$$\tau_s^{*} = \sqrt{-\sigma_1 \sigma_2}$$

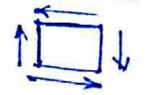
(12) Resultant stress on any oblique plane is equal to length of line which is joing the corresponding point on circle with origion.

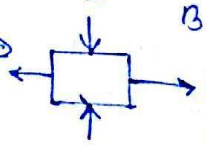
(13) when σ_1 & σ_2 are equal and unlike in nature following conslusion can be made

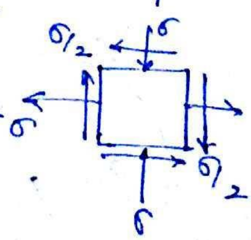
(i) centre of mohr's circle coincides with the origin

(ii) $\sigma_1 = \sigma_2$ = In plane τ_{max} = Abs τ_{max} = Resultant stress on any oblique plane = τ_s^{*} = Radius of mohr' circle

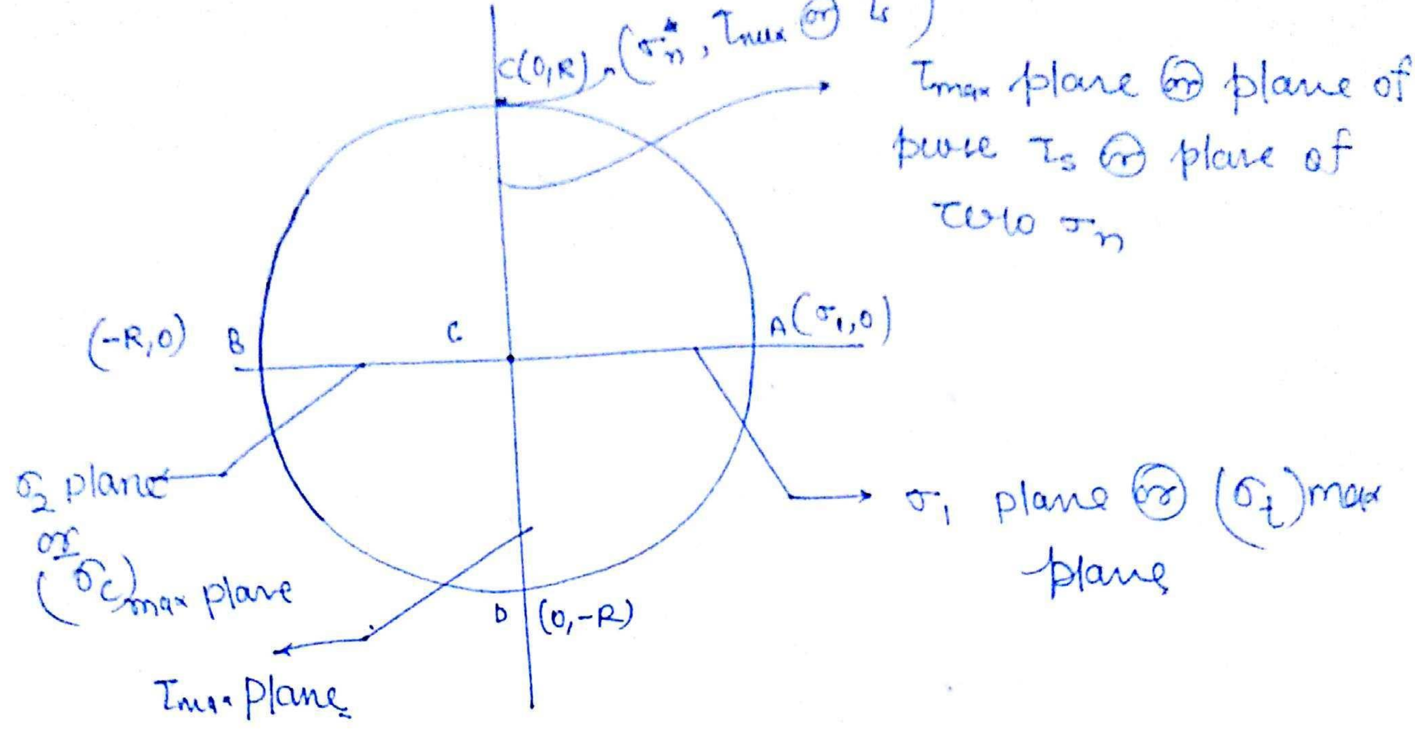
(iii) planes pure shear coincide with max. τ_s plane

(iv) eq: (a) $\sigma_x = \sigma_y = 0$; $\tau_{xy} = \tau \Rightarrow$  $A(0, \tau)$; $R = \tau$
$B(0, -\tau)$

(b) $\sigma_x = -\sigma_y = \sigma$; $\tau_{xy} = 0 \Rightarrow$  $A(\sigma, 0)$
$B(-\sigma, 0)$
$R = \underline{\sigma}$

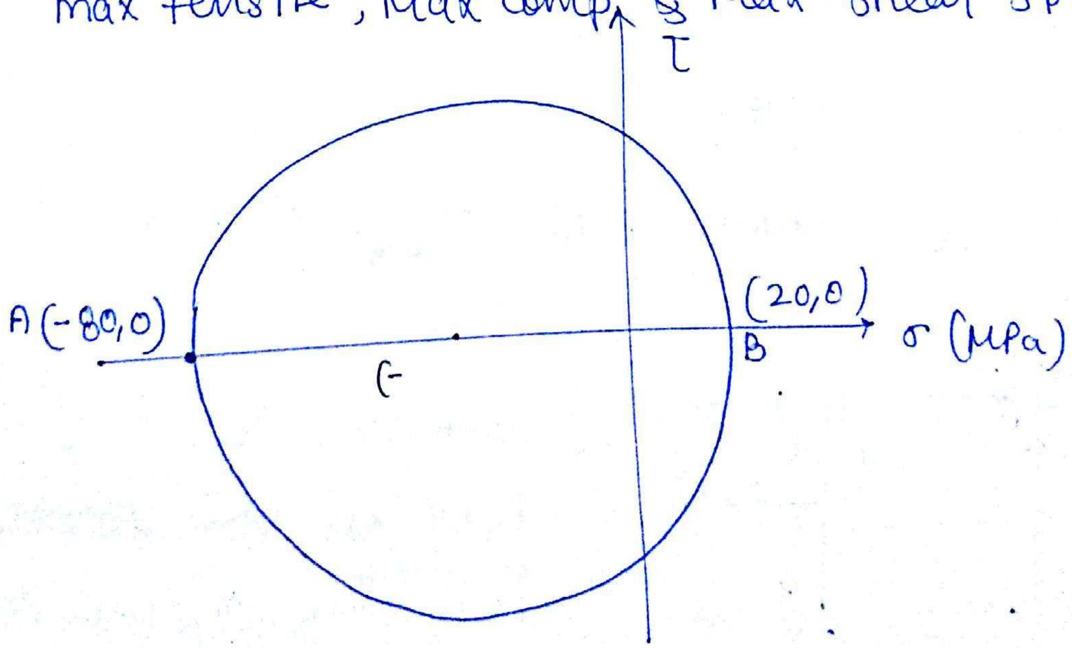
(c) $\sigma_x = -\sigma_y$; $\tau_{xy} = \tau$ $\Rightarrow$ 

$A(\sigma, \sigma/2)$
$B(\sigma, -\sigma/2)$

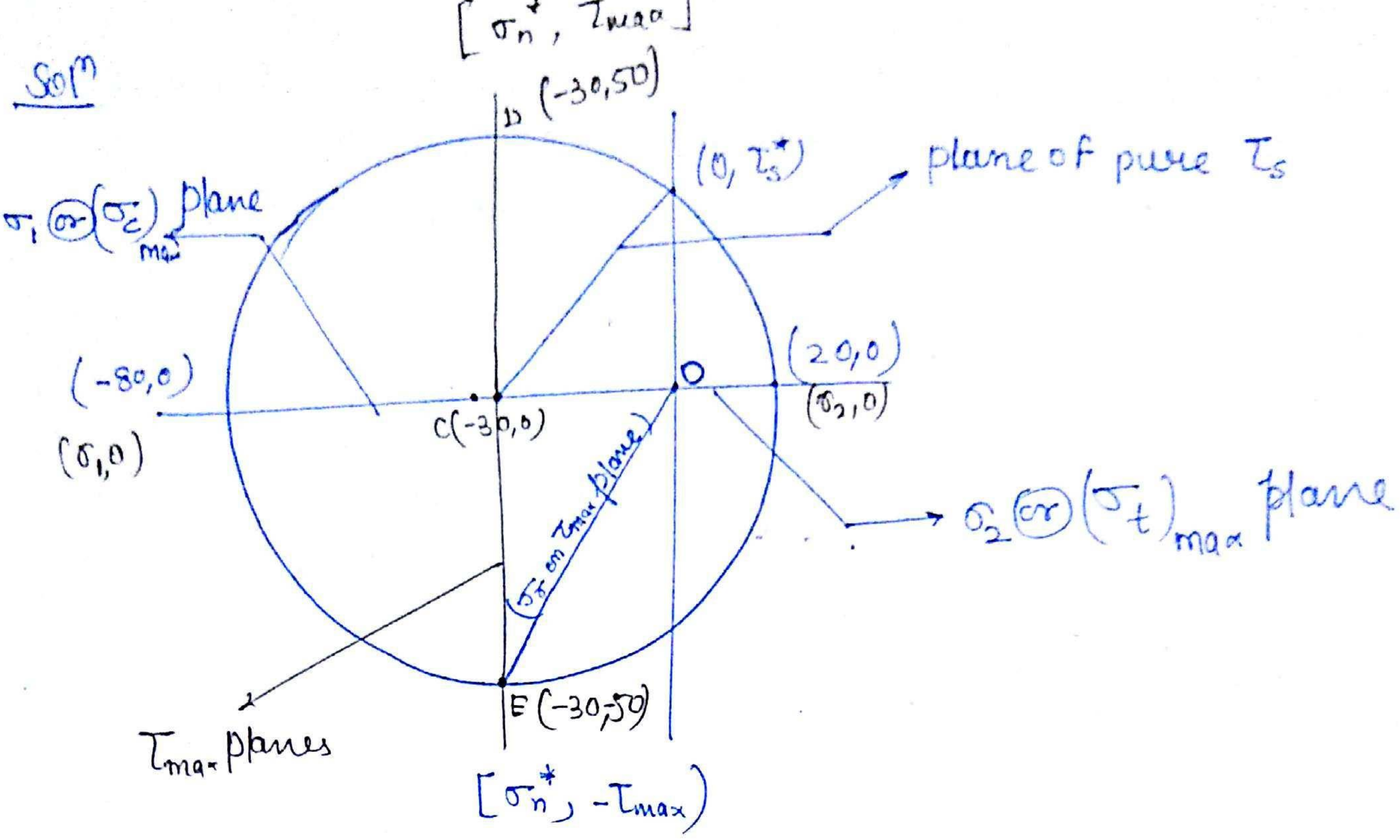


Eg For the mohr's circle as shown in the fig determine the following

1. Center Coordinates
2. Major & Minor principal stress
3. Resultant Stress on τ_{max} plane
4. Shear stress on planes of pure shear
5. max tensile, Max Comp. & Max shear stress



Solⁿ



(a) $C[-30,0]$ (b) $\sigma_1 = -80\text{ MPa}, \ \sigma_2 = 20\text{ MPa}$

(c) $\sigma_n^* = -30\text{ MPa}$ In plane $\tau_{max} = \pm R = \pm 50\text{ MPa}$

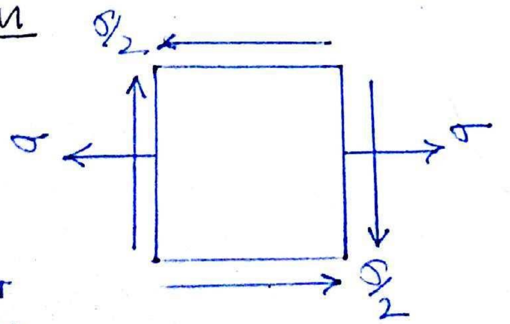
$$(\sigma_r) = \sqrt{(-30)^2 + (+50)^2} = 58.309\text{ MPa}$$

(d) $\tau_s^* = \pm 40\text{ MPa}$

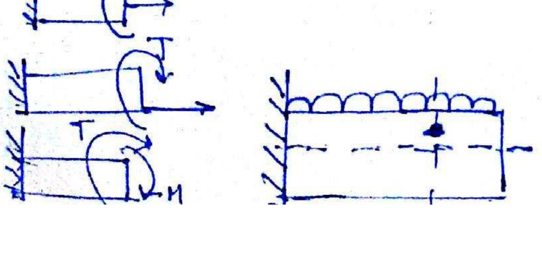
(e) $(\sigma_t)_{max} = 20\text{ MPa}$

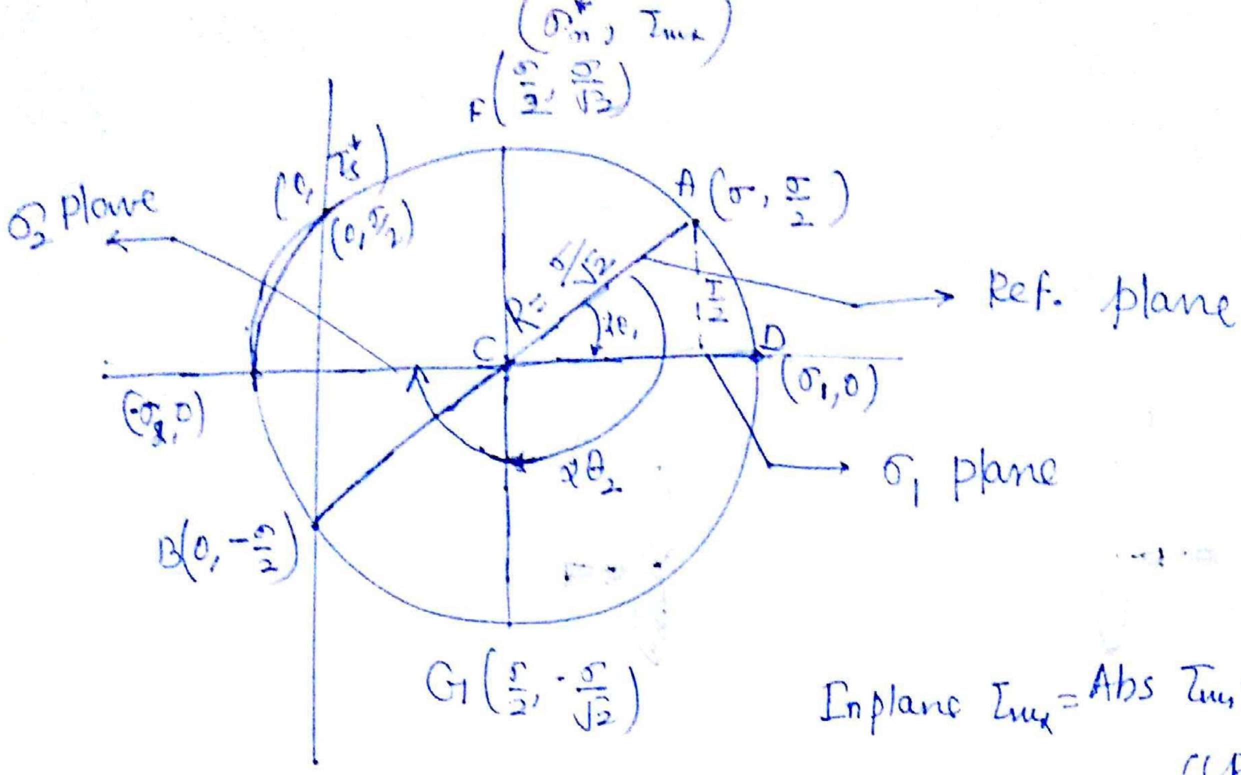
$(\sigma_c)_{max} = 80\text{ MPa}$; Abs $\tau_{max} = 50\text{ MPa}$

Question



Draw mohr's circle & find all ~~values~~ $\sigma_1, \sigma_2, \tau_{max}$, In plan τ_{in} and other





Inplane τ_{max} = Abs $\tau_{max} = \frac{\sigma}{\sqrt{2}}$
(CF & CG)

$$\sigma_1 = OD = OC + CD$$

$$\sigma_1 = \frac{\sigma}{2} + \frac{\sigma}{\sqrt{2}} = (\sigma_t)_{max} = OD$$

$$\sigma_2 = (-OE) = -(CE - OC)$$

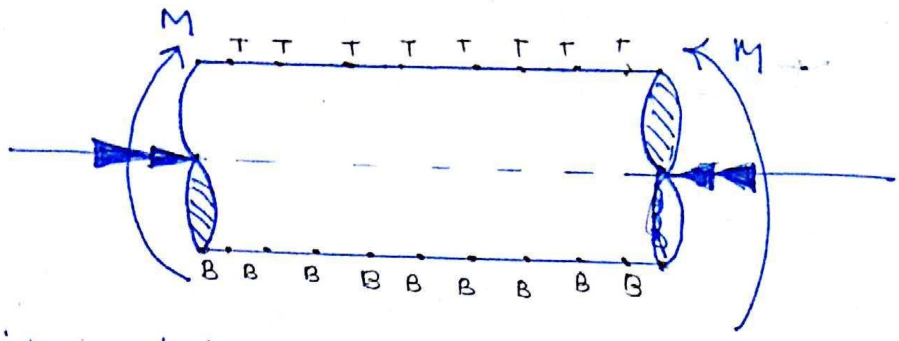
$$= -\left(\frac{\sigma}{\sqrt{2}} - \frac{\sigma}{2}\right) = OE$$

$\tau_s^* = \pm \sigma/2$
(OH & OB)

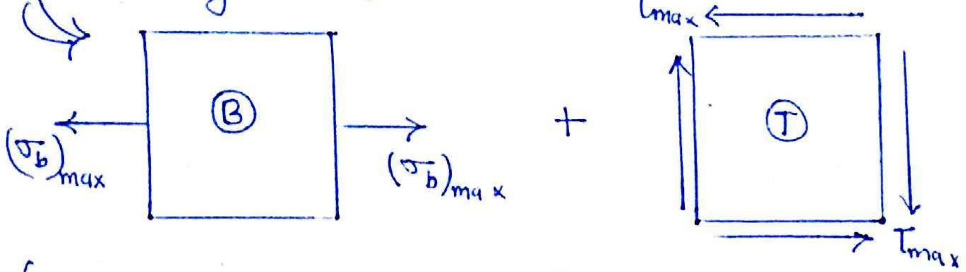
$$= \frac{\sigma}{2} - \frac{\sigma}{\sqrt{2}} = (\sigma_c)_{max}$$

Radial line	Plane
CA	Ref. plane
CD	σ_1 or $(\sigma_t)_{max}$ plane $2\theta_1$
CE	σ_2 or $(\sigma_c)_{max}$ plane $2\theta_2$
CG & CF	τ_{max} plane $2\theta_3$ & $2\theta_4$
CH & CB	Planes of pure shear $2\theta_5$ & $2\theta_6$

* Expression for σ_1, σ_2 & abs. τ_{max} when a solid circular shaft subjected to both bending moment & twisting moment simuntaneously.



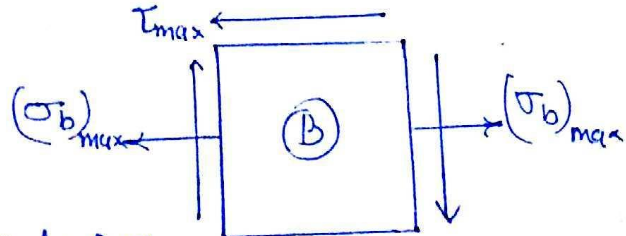
Considering bottom



(when B.m. acting Along)

(when T.M. acting alone)
(can take τ_{max} in any dirn while calculating σ_1 & σ_2)

when both are acting simunteneously



State of Stress at botton

$$(\sigma_b)_{max} = \frac{M}{Z_{N.A.}} = \frac{32M}{\pi d^3}$$

$$\tau_{max} = \frac{T}{Z_p} = \frac{16}{\pi d^3} T$$

$$(\sigma_{1,2})_B = \frac{1}{2}\left[\left(\frac{32M}{\pi d^3} + 0\right) \pm \sqrt{\left(\frac{32M}{\pi d^3} - 0\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}\right]$$

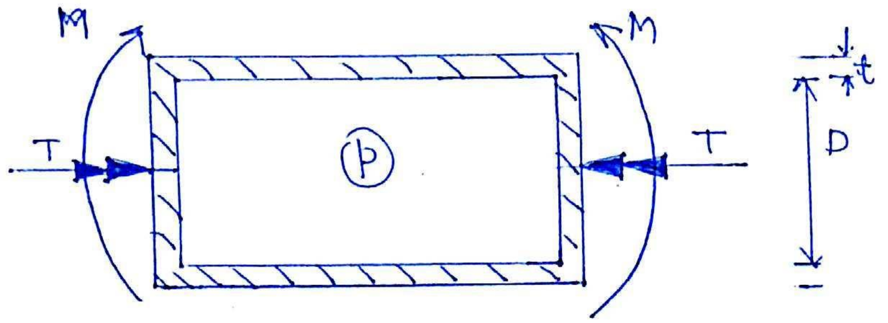
$$\sigma_{1,2} = \frac{1}{2} \times \frac{32}{\pi d^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

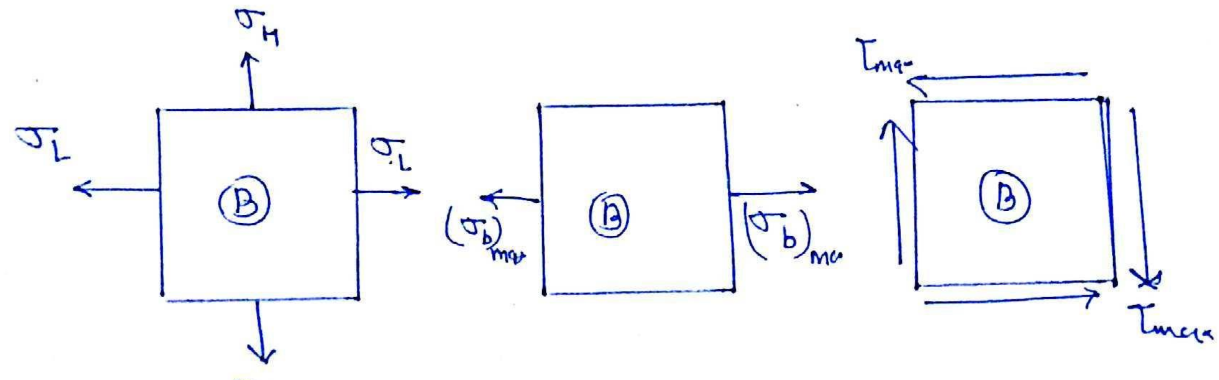
$$\boxed{(\sigma_1)_B = \frac{16}{\pi d^3}\left[M+\sqrt{M^2+T^2}\right] = (\sigma_t)_{max}}$$

$$\boxed{(\sigma_1)_B = \frac{16}{\pi d^3}\left[M-\sqrt{M^2+T^2}\right] = (\sigma_c)_{max}}$$

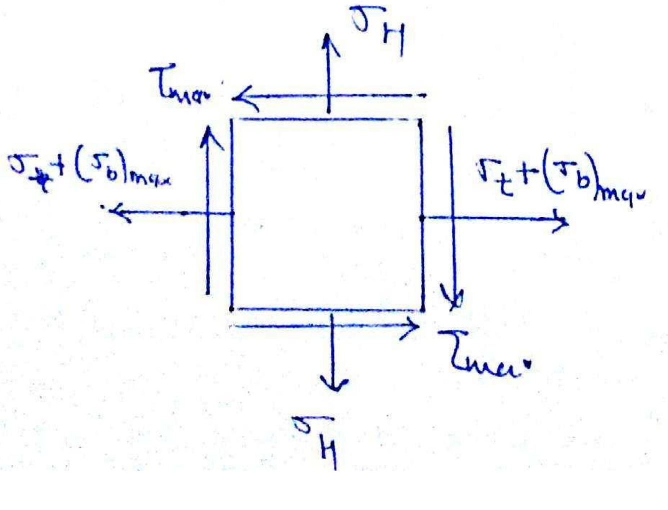
$$\boxed{\text{Abs } \tau_{max} = \text{In plane } \tau_{max} = \frac{\sigma_1-\sigma_2}{2} = \frac{16}{\pi d^3}\sqrt{M^2+T^2}}$$

Thin pressure vassel subjected to B.M. & T.M.





due to 'p' due to B.M due to T.M.



$$\sigma_x = \frac{PD}{4t} + \frac{4M}{\pi D^2 t}$$

$$\sigma_y = \frac{PD}{2t}$$

$$\tau_{xy} = \frac{2T}{\pi D^2 t}$$

Solve for $p = 20 MPa$

$D = 500 MM$

$t = 10 mm$

$M = 400 N\text{-}m$

$T = 300 N\text{-}m$