Quadrilaterals

FUNDAMENTAL

Polygons: A simple closed figure made up of line segments only, is known as a polygon.

(i) Minimum no of sides in a polygon is three, which gives a triangle (a).

(ii) More than three sides of polygon are as follows:

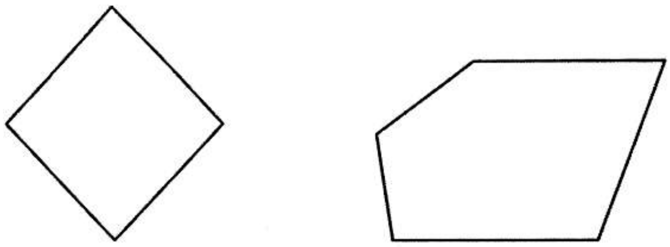
(a) 4-sided figure = quadrilateral

(b) 5 -sided figure = pentagon

(c) 6-sided figure = hexagon and so on.

Type of polygons

(1) Convex polygon: Polygon in which each angle is less than 180°



e.g.: Figure – 1

(2) Concave Polygon : Polygon in which at least one angle is more than 180°

e.g.:

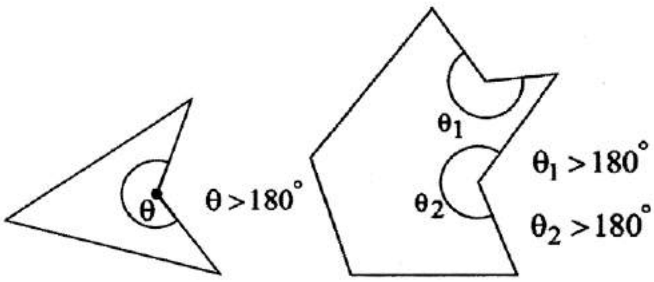


Figure - 2

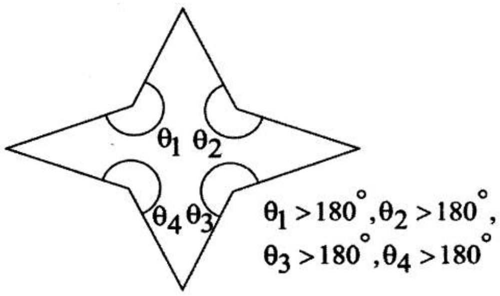


Figure – 3

According to another basis, polygons are classified as;

(i) Regular Polygons: Polygons in which all sides **and** all angles are equal, are called Regular Polygons. Specific names are given to each of them depending upon no of sides.

(a) In Triangles:

Equilateral $\Delta 1e$

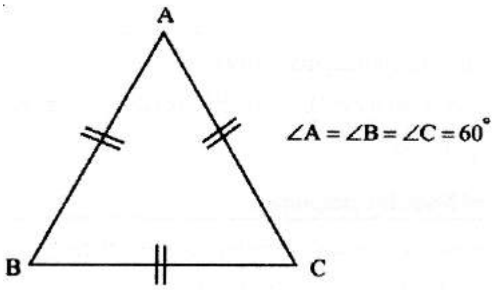


Figure-4

(b) In quadrilaterals: Square

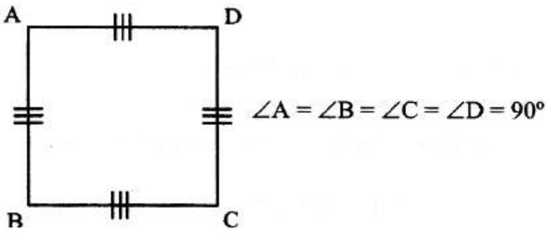


Figure-5

(c) In Pentagons: Regular pentagon

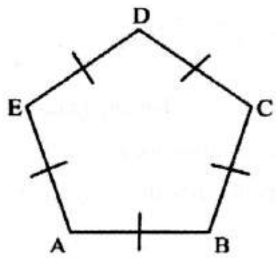


Figure-6

$\angle A = \angle B = \angle C = \angle D = \angle E = 108^\circ$

(d) In Hexagons: Regular hexagon

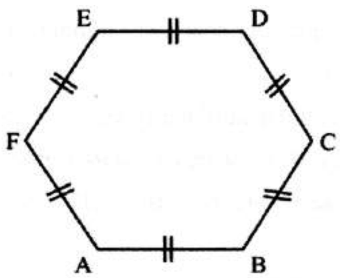


Figure-7

$\angle A = \angle B = \angle C = \angle D = \angle E = \angle F = 120^\circ$ & so on.

(ii) Irregular polygons: Polygons which do not nave equal sides or equal angles, are called Irregular Polygons. Thus, RHOMBUS, which has equal sides but unequal angles, is irregular polygon.

Properties of Regular polygons:-

If n = no. of sides of a regular polygon, then,

(i) Exterior angle, $'\theta' = \left(\frac{360^\circ}{n}\right)$

(ii) Interior angle, = $\left(180^\circ - \theta\right)$

(iii) No. of diagonals of polygons of 'n' sides $= \frac{n(n-2)}{2}$

❖ In convex regular polygon,

(i) Sum of exterior $\angle^{1e}s = 360^\circ$

(ii) Sum of interior $\angle^{1e}s = \left(\frac{n-2}{2}\right) * 360^\circ$

Now, we shall concentrate our studies on quadrilaterals.

Quadrilaterals: On the basis of our knowledge of polygons, quadrilaterals are simple closed figures made up of four line segments. Another way to define quadrilateral is as follows:

Quadrilateral

Let, A, B, C, D be four points in a plane. Let 3 or more points be not collinear. The figure formed by four line segments joining points A, B, C and D is called the quadrilateral ABCD.

For Example, look at the quadrilateral ABCD,

(i) the four points A, B, C, D are called its **vertices,**

(ii) The four line segments AB, BC, CD, and DA are called its **sides,**

(iii) $\angle DAB, \angle ABC, \angle BCD,$ and $\angle CDA$ are known as its **angles**

(iv) Line segments AC and BD are known as its diagonals.

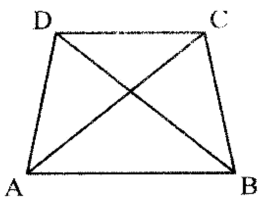


Figure-8

Adjacent sides of a quadrilateral

Sides of a quadrilateral having a common end point are called its adjacent sides. In Figure-8, (AB, BC), (BC, CD), (CD., DA) and (DA, AB) are four pairs of adjacent- sides of quad. ABCD.

Opposite sides of a quadrilateral

Two sides of a quadrilateral are known as its opposite sides if they do have a common end point. In the given figure 8, (AB, DC) and (AD, BC) are two pairs of opposite sides of quad. ABCD.

Adjacent Angles of a Quadrilateral

Two angles of a quadrilateral having common arm are called its adjacent angles. In figure 8, (ZA, ZB), (ZB, ZC), (ZC, ZD) and (ZD, ZA) are four pairs of adjacent angles of quad. ABCD.

Opposite Angles of a Quadrilateral

Non-adjacent angles are known as opposite angles. In figure-8, $(\angle A, \angle C)$, and $(\angle B, \angle D)$ are two pairs of opposite angles.

Angle Sum property of a Quadrilateral Idea / Thinking behind proof:

Draw a quadrilateral ABCD, draw any diagonal which will divide ABCD into two

Δ les ABC and ACD. Use angle sum property in both these Δ les.

To prove that: The sum of the angles of a quadrilateral is $360°$

Proof: Let ABCD be a quadrilateral. Join AC.

Construction: Join diagonal AC. Let $\angle CAD = \angle 1$ & $\angle CAB = \angle 2$

Now, $\angle 1 + \angle 2 = \angle A$ ...(i)

Also, $\angle 3 + \angle 4 = \angle C$ ...(ii)

$\therefore$ the sum of the angles of a triangle is $= 180^{\circ}$.

$\therefore$ In ΔABC,

$\angle 2 + \angle 4 + \angle B = 180^{\circ}$. ....(iii)

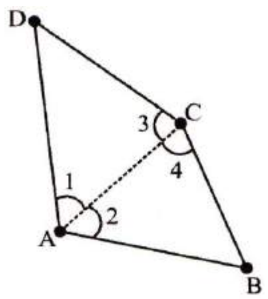


Figure – 9

In Δ ACD,

$\angle 1 + \angle 3 + \angle D = 180^{\circ}$. .....(iv)

Adding (iii) & (iv) we get, $\angle 2 + \angle 4 + \angle B + \angle 1 + \angle 3 + \angle D = 360^{\circ}$

$\Rightarrow (\angle 1 + \angle 2) + \angle B + (\angle 3 + \angle 4) + \angle D = 360^{\circ}$

$\therefore \angle A + \angle \mathbf{B} + \angle C + \angle \mathbf{D} = \mathbf{360}^{\circ}$

Hence, the sum of the angles of a quadrilateral is 360°.

Special Types of Quadrilaterals

Parallelogram: A quadrilateral is called a parallelogram if both pairs of its opposite sides are parallel. In figure-10, ABCD is a parallelogram $(|| grn)$

$AB || DC and AD || BC.$

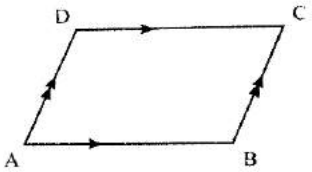


Figure – 10

Rhombus: A parallelogram $(|| gm)$ having all sides equal is called a rhombus. ABCD is a rhombus in figure-11

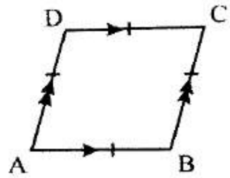


Figure – 11

By property of $|| gm, AB || DC; AD || BC$

By property of rhombus, AB = BC = CD = DA

Rectangle: A parallelogram in which each angle is a right angle is a rectangle.

Figure-12, ABCD is a quadrilateral in which

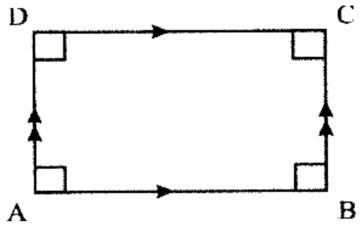


Figure – 12

$AB || DC, AD || BC$ and $\angle A = \angle B = \angle C = \angle D = 90^{\circ}$.

Note: However, in ABCD, if we do not write $AB || DC \& AD || BC$, and we simply write $\angle A = \angle B = \angle C = \angle D = 90^{\circ}$, then also it will be a rectangle (because by property of interior $\angle les, \angle A + \angle B = 180^{\circ} \Rightarrow AB$ is transversal on $AD \& BC \Rightarrow AD || BC$)

Square: It is a parallelogram in which all me sides are equal and each angle measures 90. In the figure-13,

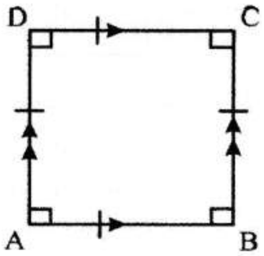


Figure – 13

$AB \parallel DC, AD \parallel BC, AB = BC = CD$ and $\angle A = \angle B = \angle C = \angle D = 90^\circ$.

Although, it looks like a rhombus (because we have drawn it like that), ABCD is a square. In fact, square is a special case of rhombus in which $\angle A = B = C = D = 90^\circ$

Trapezium: is a quadrilateral having exactly one pair of parallel sides in figure-14, $AB \parallel DC$

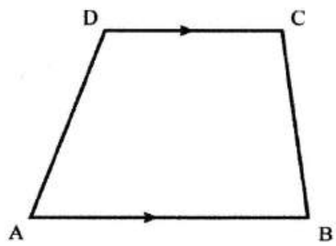


Figure – 14

Isosceles Trapezium: It is trapezium in which non parallel sides are equal. In figure-15,

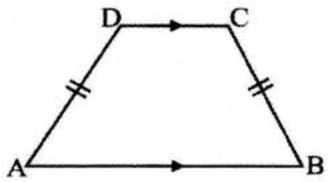


Figure – 15

$AD = BC \Rightarrow ABCD$ is an isosceles trapezium.

Thus, ABCD will be an isosceles trapezium if $AB \parallel DC \, and \, AD = BC$.

Kite: It is a quadrilateral in which two pairs of adjacent sides are equal but opposite sides are unequal.

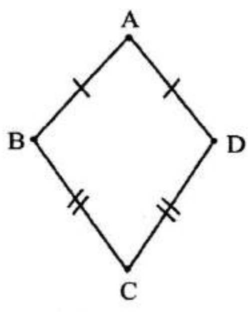


Figure – 16

See figure-16, which is self-explanatory, ABCD is a kite.

Key Properties of Quadrilateral with Proofs:

Property 1

In a parallelogram, ($\|$ gm),

(a) Opposite sides are equal (b) Opposite $\angle les$ are equal (iii) diagonals bisect each other

The converse of the above result can also be used:

(i) A quadrilateral is a $\| gm$ if opposite sides are equal

(ii) A quadrilateral is a $\| gm$ if opposite $\angle les$ are equal

(iii) A quadrilateral is a $\| gm$ if diagonals bisect each other.

PROOF: Let ABCD be a parallelogram

Construction: Draw diagonal AC.

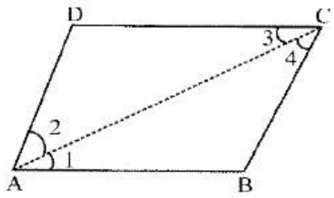


Figure – 17

In A ABC and A CDA, We get,

$\angle 1 = \angle 3$ (alternate angles)

$\angle 2 = \angle 4$ (alternate angles)

And $AC = CA$ (common side)

$\therefore$ by ASA congruence,

$\therefore \Delta ABC \cong \Delta CDA$

$\Rightarrow AB = CD, BC = DA$ and $\angle B = \angle D.$ (see the nomenclature, we are not writing AB = DC or $BC = AD) \Rightarrow$ (a) proved

Similarly, by constructing the diagonal BD, we get

$\Delta ABD \cong \Delta CDB.$ $\therefore \angle A = \angle C$

Thus, (b) proved,

In order to prove (c) consider parallelogram ABCD and draw diagonals AC and BD, intersecting each other at O.

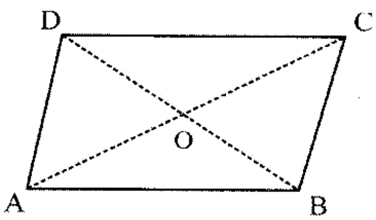


Figure – 18

In ΔOAB and ΔOCD,

$\angle OAB = \angle OCD$ (alternate angles)

$\angle AOB = \angle COD$ (vertically opposite angles)

$AB = CD$ (opposite sides of a parallelogram) By ASA congruence,

$\therefore \Delta OAB \cong OCD$

$\therefore OA = OC$ and $OB = OD \Rightarrow O$ is the midpoint of AC as well as BD.

Thus, diagonals of a parallelogram bisect each other.

Homework: Prove the three converse results, namely, (i), (ii), & (iii)

Remarks: As discussed earlier, a rectangle, a square and a rhombus are special types of parallelograms. So all the properties of a parallelogram apply to all of them.

Property 2

The diagonals of a rhombus are orthogonal and bisect each other,

PROOF: In figure - 19, ABCD is a rhombus whose diagonals intersect at the point O.

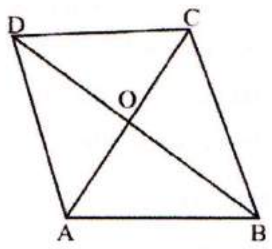


Figure – 19

We know that the diagonals of a parallelogram bisect each other.

Every rhombus is a parallelogram.

$\therefore$ By property of $|| gm$, diagonals bisect each other.

$\therefore$ $OA = OC$ and $OB = OD$

Now, the other part of proof is to prove that they are orthogonal, i.e. they intersect at right angles.

From $\Delta^{1e} AOB$ and $\Delta^{1e} AOD$, we have:

$AO = AO$ (common side)

$AB = AD$ (Sides of a rhombus)

$OB = OD$ (already proved)

$\therefore \Delta AOB \cong \Delta AOD$ (by SSS congruence)

$\Rightarrow \angle AOB = \angle AOD.$

But, $\angle AOB + \angle AOD = 2$ linear pair of $\angle^{1e} s$

$\therefore \angle AOB = \angle AOD =$ right angle $= 90°$

Hence, the diagonals of a rhombus are also orthogonal.

Property 3

The diagonals of a rectangle are equal and bisect each other.

PROOF: Let ABCD be a rectangle (figure - 20) whose diagonals AC and BD intersect at the point O.

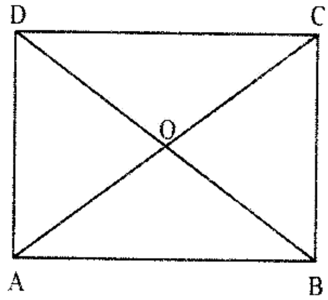


Figure – 20

Consider $\Delta^{les} ABC \,\&\, ABD$ on same base AB

Form $\Delta ABC \text{ and } \Delta BAD$, We have

$\angle ABC = \angle BAD$ (each equal to 90°)

$AB = BA$ (common side)

BC = AD (opposite sides of a rectangle).

$\therefore$ By SAS congruence,

$\therefore ABC \cong \Delta BAD \quad \Rightarrow AC = BD.$

Hence, the diagonals of a rectangle are equal. As regards bisection, this has already been proved for $\|$ *gm*.

Hence, the diagonals of a rectangle are equal and bisect each other.

Property 4

The diagonals of a square are equal, orthogonal (at 90°) and bisect each other.

PROOF: A square is both (a) rectangle and (b) a rhombus.

(a) By property of rectangle, diagonals of square are equal & bisect each other.

(b) By property of rhombus, diagonals of square are orthogonal & bisect each other.

Combing (a) & (b) diagonal of square are (i) orthogonal (ii) equal & (iii) bisect.

Note 1. If the diagonals of a quadrilateral are equal, then it is not necessarily a rectangle.

Note 2. If the diagonals of a quadrilateral intersect at right angles, then it is not necessarily a rhombus.

Homework:

Suitably think of and construct quadrilaterals according to note 1 and note 2 and show it to your teacher.