4. Sequences and series

(→) Means step continued on new line.

For example:

ii) $i\frac{(4+3i)}{(1-i)} = \frac{4i+3i^2}{1-i} \times \frac{1+i}{1+i}$

ii) $i\frac{(4+3i)}{(1-i)} =$

→ $\frac{4i+3i^2}{1-i} \times \frac{1+i}{1+i}$

---x---

Exercise: 4.1

1. Verify whether the following sequences are G.P. If so, write tn.

i) 2,6, 18,54, ...

Solution:

Here $t_1 = 2,$

$t_2 = 6,$

$t_3 = 18,$

$t_4 = 54, \ldots..$

$\frac{t2}{t1} = \frac{6}{2} = 3$

$\frac{t3}{t2} = \frac{18}{6} = 3$

$\frac{t4}{t3} = \frac{54}{18} = 3$

$\text{i.e.} \frac{t2}{t1} = \frac{t3}{t2} = \frac{t4}{t3}$

$\rightarrow = \ \ldots \ 3 = r$

Which is constant.

Given sequence is a G.P

$t_n = a.r^{n-1}$

$= 2(3)^{n-1}$

$a = t1 = 2$

and $r = 3$

$t_n = 2(3)^{n-1}$

ii) 1, -5, 25, -125, ……

Solution:

Here, $t_1 = 1,$

$t_2 = 5,$

$t_3 = 25,$

$t_4 = -125, \ldots$

$\frac{t2}{t1} = \frac{-5}{1} = -5$

$\frac{t3}{t2} = \frac{25}{-5} = -5$

$\frac{t4}{t3} = \frac{-125}{25} = -5$

$\text{i.e.} \frac{t2}{t1} = \frac{t3}{t2}$

$= \frac{t4}{t3} = -5 = \text{r}$

Which is constant.

Given sequence is a G.P

$\text{t}_\text{n} = \text{a}.\text{r}^{\text{n}-1}$

$= 1(-5)^{\text{n}-1}$

$\text{t}_\text{n} = (-5)^{\text{n}-1}$

iii) $\sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, \ldots.$

Solution:

Here, $\text{t1} = \sqrt{5},$

$\text{t2} = \frac{1}{\sqrt{5}},$

$\text{t3} = \frac{1}{5\sqrt{5}},$

$\text{t4} = \frac{1}{25\sqrt{5}}, \ldots.$

$\frac{t2}{t1} = \frac{1/\sqrt{5}}{\sqrt{5}} = \frac{1}{5}$

$\frac{t3}{t2} = \frac{1/5\sqrt{5}}{1/\sqrt{5}} = \frac{1}{5}$

$\frac{t4}{t3} = \frac{1/25\sqrt{5}}{1/5\sqrt{5}} = \frac{1}{5}$

i. e. $\frac{t2}{t1} = \frac{t3}{t2} = \frac{t4}{t3}$

$= \frac{1}{5} = r$

Which is constant.

Given sequence is a G.P

$t_n = a.r^{n-1}$

$= \sqrt{5}\left(\frac{1}{5}\right)$ ....(a = t1

$\rightarrow = \sqrt{5}$ and $r = \frac{1}{5}$)

$= 5^{\frac{1}{2}}.\frac{1}{5^{n-1}}$

$= \frac{1}{5^{n-\frac{3}{2}}} = 5^{\frac{3}{2}-n}$

$t_n = 5\frac{3}{2} - n$

iv) 3, 4, 5, 6, ...

Solution:

Here, t1 =3,

t2 =4,

t3 =5,

t4 =6

$\frac{t2}{t1} = \frac{4}{3}$

$\frac{t3}{t2} = \frac{5}{4}$

i. e. $\frac{t2}{t1} \neq \frac{t3}{t2}$

Given sequence is not in G.P

v)7, 14, 21, 28, ...

Solution:

Here, t1 =7,

t2 =14,

t3 =21,

t4 =28, ...

$\frac{t2}{t1} = \frac{14}{7}$

$\frac{t3}{t2} = \frac{21}{14}$

$\text{i.e.} \frac{t2}{t1} \neq \frac{t3}{t2}$

Given sequences is not in G.P

2) For the G.P

i) If $\mathbf{r = \frac{1}{3},}$
$\mathbf{a = 9}$, Find t7

Solution:

$\text{Here, } r = \frac{1}{3},$

$a = 9, \text{Find t7}$

We have,

$t_n = a.r^{n-1}$

$T7 = 9 \text{ x } (\frac{1}{3})^6$

$= 9 \text{ x } \frac{1}{729} = \frac{1}{81}$

$t7 = \frac{1}{81}$

ii) If $\mathbf{a = \frac{7}{243}, r = \frac{1}{3}}$ **, find t3**

Solution:

Here, $a = \frac{7}{243}$,

$r = \frac{1}{3}$,

find t3 $= ?$

We have, $t_n = a.r^{n-1}$

$t3 = \left(\frac{7}{243}\right) \text{x} \left(\frac{1}{3}\right)2$

$= \frac{7}{243} \text{ x } \frac{1}{9}$

$t3 = \frac{7}{2187}$

iii) If a = 7,

and r = −3,

Find $\mathbf{t_6}$.

Solution: Here,

a=7, r=-3, t6 = ?

$t6 = (7)(-3)^{6-1}$

$= (7)(-3)^5$

$= 7 \times (-243)$

t6 = -1701

Thus, t6 = -1701

iv) If $a = \frac{2}{3}$,

$t6 = 162$, find r

Solution:

Here, $a = \frac{2}{3}$,

$t6 = 162$,

We have

$t_n = a.r^{n-1}$

$t6 = a.r^5$

$162 = \frac{2}{3} \times r^5$

$r^5 = \frac{162 \times 3}{2}$

=243

$= 3^5$

r = 5

3) Which term of the G.P 5,25,

$125, 625, \ldots$ Is 5^{10}?

Solution:

Here, $a = 5,$

$$r = \frac{25}{5} = 5$$

and $t_n = 5^{10}$

We have

$t_n = a.r^{n-1}$

$5^{10} = 5 \times 5^{n-1}$

$5^{10} = 5^n$

$N = 10$

Hence,tenth term of the

G.P is 5^{10}

4) For what values of x,the term

$$\frac{4}{3}, x\ \frac{4}{27}$$

are in G.P.?

Solution:

Here, $t1 = \frac{4}{3},$

$t2 = x$

$, t3 = \frac{4}{27}$

As these terms are in G.P

$$\frac{t2}{t1} = \frac{t3}{t2}$$

i.e. $t_2{}^2 = t1 \times t3$

$x^2 = \frac{4}{3} \text{ x } \frac{4}{27}$

$= \frac{16}{81}$

$x = \pm\frac{4}{9}$

5) if for a sequences,

$t_n = \frac{5^{n-3}}{2^{n-3}},$

show that the sequence is a G.P Find its first term and the common ratio.

Solution:

$t_n = \frac{5^{n-3}}{2^{n-3}}$

$t_n + 1 = \frac{5^{n+1-3}}{2^{n+1-3}}$

$= \frac{5^{n-2}}{2^{n-2}}$

Now, $\frac{tn+1}{tn}$

$\rightarrow = \frac{5^{n-2}}{2^{n-2}} \div \frac{5^{n-3}}{2^{n-3}}$

$= \frac{5^{n-2}}{2^{n-2}} \text{x } \frac{2^{n-3}}{5^{n-3}}$

$= \frac{5^{n-2-n+3}}{2^{n-2-n+3}}$

$= \frac{5}{2}$ which is constant, for all n € N

The given sequences is a G.P. with

$r = \frac{5}{2}$ and $a = t1$

$= \frac{5^{1-3}}{2^{1-3}}$

$= \frac{5^{-2}}{3^{-2}}$

$= \frac{4}{25}$

6) Find three numbers in G.P such that their sum is 21 and sum of their squares is 189.

Solution:

Let the three numbers in G.P be.
$\frac{a}{r}$, a, ar.

From the first condition, their sum is 21.

$\frac{a}{r}, + a + ar = 21$

$a\left(\frac{1}{r} + 1 + r\right) = 21 \,..(1)$

From the second condition, sum of their squares is 189

$\frac{a^2}{r^2} + a^2 + a^2 r^2 = 189$

$a^2\left(\frac{1}{r^2} + 1 + r^2\right)$

$= 189\ (2)$ On squaring equation (1), we get

$a^2\left(\frac{1}{r^2} + 1 + r^2 + \frac{2}{r} + 2 + 2r\right)$

$= 441$

$\text{i.e. } a^2\left(\frac{1}{r^2} + 1 + r^2\right) +$

$\rightarrow 2a\left[a\left(\frac{1}{r} + 1 + r\right)\right]$

$= 441$

From equation 1 and 2

189 + 2a x 21 = 441

42a = 441 – 189

42a = 252

$$a = \frac{252}{42}$$

a =6
From 1,

$$6\left(\frac{1}{r} + 1 + r\right) = 21$$

$$\text{i.e. } 2\left(\frac{1 + r + r^2}{r}\right) = 7$$

$$2 + 2r + 2r^2 = 7r$$

$$2r^2 - 5r + 2 = 0$$

$$2r^2 - 4r - 1r + 2 = 0$$

2r(r-2) -1 (r-2) = 0

(r-2) (2r-1) = 0

r-2 = 0 or 2r -1 = 0

$$r = 2 \text{ or } r = \frac{1}{2}$$

For a =6 and r =2 the three numbers are 3, 6, 12

For $a = 6$ and $r = \frac{1}{2}$,

the three number are 12,6,3

7) Find four numbers in G.P such that sum of the middle two numbers is 10/3 and their product is 1

Solution:

Let the four number in G.P be

$\frac{a}{r^3}, \frac{a}{r}, ar, a^3$

From the first condition:

Sum of middle two numbers

is $\frac{10}{3}$

$\frac{a}{r} + ar = \frac{10}{3}$ ….(1)

From the second condition their products is 1

$\frac{a}{r^3} \times \frac{a}{r} \times ar \times ar^3 = 1$

$a^4 = 1$

$= a = 1$ ………… $a > 0$

From equation (1)

$\frac{1}{r} + r = \frac{10}{3}$

$\frac{1+r^2}{r} = \frac{10}{3}$

$3 + 3r^2 = 10r$

$3r^2 - 10r + 3 = 0$

$3r^2 - 9r - 1r + 3 = 0$

3r (r-3) -1 (r-3) = 0

(r-3) (3r-1) = 0

r-3 = 0 or 3r – 1 = 0

$r = 3$ or $r = \frac{1}{3}$

For a =1 and r =3 the four numbers are

$\frac{1}{27}, \frac{1}{3}, \quad 3, 27.$

For $a = 1$ and $r = \frac{1}{3}$,

then four numbers are

$27, 3, \quad \frac{1}{3}, \frac{1}{27}$

8) Find five numbers in G.P such that their products is 1024 and fifth term is square of the third term

Solution:

Let the five numbers in G.P be,

$\frac{a}{r^2}, \frac{a}{r}, \quad a, ar, ar^2$

From the first condition their products is 1024

$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar$

$\rightarrow \times ar^2 = 1024$

$a^5 = 4^5 = a = 4$

From the second condition,

Fifth term is square of the third term.

$ar^2 = a^2$

$r^2 = a$

$r^2 = 4$

$r = \pm 2$

For a =4 and r =2

The five numbers are 1, 2, 3, 4, 8, 16.

For a =4, and r =-2,

The five numbers are 1, -2, 4, -8, 16.

9) The fifth term of a G.P is x , eighth term of the G.P is y, and eleventh term of the G.P is z. Verify whether $y^2 = x . z$.

Solution:

Given t5 =x , t8 =y and t11 =z

$ar^4 = x \; ax^7 = y$

→ and $ar^{10} = z$

Now , $x.z = (ar^4)(ar^{10})$

$= a^2r^{14}$

$= (ar^7)^2$

$= y^2$

Hence, $y^2 = x.z$

10) If p, q, r, s are in G.P show that p + q, q+r, r+s are also in G.P

Solution:

Given p. q. r, s and are in G.P

$\frac{q}{p} = \frac{r}{q} = \frac{s}{r} = k(\text{say})$

$q = pk$
$r = qk = pk^2$

$s = rk = pk^3$

$t1 = p + q$

$= p + pk$

$= p(1+k)$

$t2 = q + r$

$\rightarrow = pk + pk^2$

$\rightarrow = pk(1 + k)$

$t3 = r + s$

$\rightarrow = pk^2 + pk^3$

$\rightarrow = pk^2 (1 + k)$

$\frac{t2}{t1} = \frac{pk(1+k)}{p(1+k)} = k$

$\frac{t3}{t2} = \frac{pk(1+k)}{pk(1+k)} = k$

$\frac{t2}{t1} = \frac{t3}{t2}$

$\frac{q+r}{p+q} = \frac{r+s}{q+r}$

P + q, q +r, r + s are in G.P

Exercise 4.2

1) For the following G.P . s, Find S_n.

i) 3, 6, 12, 24.

Solution:

Here, a =3 r =2

$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$ r > 1

$S_n = 3\left(\frac{2^n - 1}{2 - 1}\right)$

$S_n = 3(2n - 1)$

ii) $p, q, \frac{q^2}{p}, \frac{q^3}{p^2}, \ldots\ldots$

Solution:

Here, a = p,

$r = \frac{q}{p}$

Case (i): $r = \frac{q}{p} > 1$

$\rightarrow = i.e.\ q > p$

Then, $S_n = a\left(\frac{r^n - 1}{r - 1}\right)$

$= \frac{p\left(\frac{q}{p}\right)n-1}{(\frac{q}{p}-1)}$

$S_n = \frac{p^2}{q-p}\left[\left(\frac{q}{p}\right)^n - 1\right]$

Case ii) $r = \frac{q}{p} < 1$

$= i.e. q > p$

Then $S_n = \frac{p^2}{p-q}[1 - (\frac{q}{p})n]$

2) For a G.P if

i) $a = 2, r = -\frac{2}{3}$

We have,

$S_n = a\left(\frac{1 - r^n}{1 - r}\right)$

$r < 1$

$S_6 = 2 \text{ x } \left[\frac{1 - (\frac{2}{3})^6}{1 + \frac{2}{3}}\right]$

$= 2 \text{ x } \left[\frac{1-\frac{64}{729}}{\frac{5}{3}}\right]$

$= 2 \text{ x } \frac{3}{5} \times \frac{729-64}{729}$

$= \frac{2}{5} \text{ x } \frac{665}{243}$

$= \frac{2 \times 133}{243}$

$= S_6 = \frac{266}{243}$

ii) $S_5 = 1023$,

$r = 4$, Find a

Solution:

Here, $S_5 = 1023$,

$r = 4$ $a = ?$

$a\left(\frac{r^n - 1}{r - 1}\right) = S_n \; r > 1$

$a\left(\frac{r^5 - 1}{r - 1}\right) = 1023$

$a\left(\frac{4^5 - 1}{4 - 1}\right) = 1023$

$a\left(\frac{1024 - 1}{3}\right) = 1023$

$a\left(\frac{1023}{3}\right) = 1023$

$a = 3$

3) For a G.P if

i) $a = 2$

$r = 3$

$S_n = 242$ $n = ?$

Solution:

Here

$a = 2,$

$r = 3$

$S_n = 242$

$n = ?$

$a\left(\frac{r^n - 1}{r - 1}\right) = S_n$ r > $12\left(\frac{3^n - 1}{3-1}\right) = 242$

$3^n - 1 = 242$

$3^n = 245 = 3^5$

$N = 5$

ii) Sum of first 3 term is 125 and sum of next 3 terms is 27 find the values of r

Solution:

Here, $S_3 = 125$

i.e. $t1 + t2 + t3 = 125$

and $t4 + t5 + t6 = 27$

$S_6 = 125 + 27 = 152$

$\frac{S_6}{S_3} = \frac{152}{125}$

$125 \text{ x } S_6 = 152 \text{ x } S_3$

$125 \times a\left(\frac{r^6 - 1}{r - 1}\right)$

$\rightarrow = 152 \times a\left(\frac{r^3-1}{r-1}\right)$

$125\times\ (r^3-1)(r^3+\ 1)$

$\rightarrow =\ 152\times\ (r^3-1)$

$125(r^3+\ 1) = 152$

$125r^3 + 125\ = 152$

$125r^3 + 125 = 152$

$125r^3 = 152 - 125\ = 27$

$R^3 = \frac{27}{125} = \left(\frac{3}{5}\right)^3$

$r\ = \frac{3}{5}$

4) For a G.P

i) If t3 = 20,

t6 = 160 Find S_7

Solution:

Here, t3 =20

t6 =160 $S_7 = ?$

$\frac{t6}{t3} = \frac{160}{20}$

$$\frac{a.r^5}{a.r^2} = 8$$

$r^3 = 2^3$

$r = 2$

Also, t3 = 20

$a.r^2 = 20$

a x 4 = 20

a =5

Now,

$$S_7 = a\left(\frac{r^7 - 1}{r-1}\right) r > 1$$

$$S_7 = 5\left(\frac{2^7 - 1}{2-1}\right)$$

= 5 × (128-1)

= 5 ×127

$= 5 \text{ x } (128 - 1) = 5 \text{ x } 127$

$S_7 = 635$

ii) if t4 = 16, t9 = 512,

find S_{10}?

Solution:

Here, t4 = 16,

t9 = 512 S_{10}?

$\frac{t9}{t4} = \frac{512}{16}$

$\frac{a.r^8}{a.r^3} = 32$

$r^5 = 2^5$

r =2

Also, t4 = 16

$a.r^3 = 16$
$a \text{ x } (2)^3 = 16$

8a = 16

A = 2

$\text{Now}, S_{10} = a\left(\frac{r^{10}-1}{r-1}\right)$
$r > 1$

$S_{10} = 2\left(\frac{2^{10}-1}{2-1}\right)$

$= 2 \text{ x } (1024 - 1)$

= 2 x 1023 = 2046

$S_{10} = 2043$

5) Find the sum to n terms:

i) 3 + 33 + 333 + 3333 + ….

Solution:

$S_n = 3 + 33 +$

333 + ...upto n term

= 3 [1+ 11+ 111 + ... Upto n term

$= \frac{3}{9}[9 + 99 +$

→ 999 + ... Upto n term

$= \frac{1}{3}[(10-1) + (10^2-1)$

→ $+ (10^3 - 1) + \ldots$

→ $+ (10^n - 1)]$

$= \frac{1}{3}[(10 + 10^2 +$

→ $10^3 + \ldots + 10^n)$

→ $-(1 + 1 + 1 \ldots$ upto n term)

Clearly 1st bracketed term are in G.P with a = r = 10

$S_n = \frac{1}{3}\left[10 \times \left(\frac{10n-1}{10-1}\right) - n\right]$

$S_n = \frac{1}{3}\left[\frac{10}{9} x (10n - 1) - n\right]$

$S_n =$

→ $\frac{1}{27}[10(10n-1) - 9n]$

ii)8+ 88+ 888+ 8888+ ...

Solution: let

$S_n = 8 + 88 + 888$

$\rightarrow + \ldots \text{Upto n terms}$

= 8 [1+ 11+ 111+ ... upto n term]

$= \frac{8}{9}[\,9 + 99 + 999$

$\rightarrow + \cdots \text{ Upto n term }]$

$= \frac{8}{9}[\,(10 - 1) + (10^2$

$\rightarrow -1) + (10^3 - 1)$

$\rightarrow + \ldots . + (10^n - 1)]$

$= \frac{8}{9}[\,(10 + 10^2 + 10^3$

$\rightarrow + \cdots + 10^n\,) -$

$\rightarrow (1 + 1 + 1 \ldots \text{upto n term})]$

Clearly first bracketed terms are in a G.P with a = r = 10

$S_n = \frac{8}{9}\left[10 \times \left(\frac{10^n - 1}{10 - 1}\right) - n\right]$

$S_n = \frac{8}{9}\left[\frac{10}{9} \times (10^n - 1) - n\right]$

$S_n = \frac{8}{81}[10\,(10^n - 1) - 9n]$

6) Find the sum to n term:

i) 0.4 + 0.44+ 0.444+ ....

Solution: Let

$S_n = 0.4 + 0.44 +$

$\rightarrow 0.444 + \cdots.$ Upto n term

= 4 [0.1+ 0.11+ 0.111+.... Upto n term]

$= \frac{4}{9} [\ 0.9 + 0.99 + 0.999$

$\rightarrow + \cdots$ Upto n term]

$= \frac{4}{9}[(1 - 0.1) + (1 - 0.11) +$

$\rightarrow (1 - 0.111) + \cdots$ upto n term]

$= \frac{4}{9}[\ (1 + 1 + \ 1\ + \cdots$

Upto n term) – (0.1+0.01+ 0.001+⋯ upto n term)]

Clearly the term in the second bracket are in a G.P with a = 0.1 and r= 0.1

$S_n = \frac{4}{9}\left[n - 0.1\left(\frac{1 - (0.1)^n}{1 - 0.1}\right)\right]$

$S_n =$

$\rightarrow \frac{4}{9}\left[n - \frac{1}{9}\ (1 - (0.1)^n)\right]$

$S_n = \frac{4}{81}\left[9n - \ (1 - \frac{1}{10^n})\right]$

ii) 0.7 + 0.77 + 0.777 + ...

Solution: Let

$S_n = 0.7 + 0.77 + 0.777 +$

.... Upto n term

= 7[0.1 + 0.11+ 0.111+ ..... upto n term]

$= \frac{7}{9}[0.9 + 0.99 + 0.999 +$

.... Upto n term]

$= \frac{7}{9}[(1 - 0.1) + (1 - 0.01)$

→ $+(1 - 0.001) +$

.... Upto n term]

$= \frac{7}{9}[(1 + 1 + 1 +$

...... upto n term) – (0.1+ 0.11+0.001... upto n term)]

Clearly the term in the second bracket are in a G.P with a =0.1 r= 0.1

$S_n = \frac{7}{9}[n - 0.1\ (\frac{1-(0.1)n}{1-0.1})]$

$S_n = \frac{7}{9}[n - \frac{1}{9}\ (1 - (0.1)^n)]$

$S_n = \frac{7}{81}[9n - (1 - \frac{1}{10n})]$

7) Find the sum to n terms of the sequences.

i) 0.5, 0.05, 0.005, ...

Solution:

Let $S_n = 0.5 + 0.05 + 0.005$

→ $+$ ... upto n terms

$= \frac{5}{10} + \frac{5}{10^2} + \frac{5}{10^3}$

→ $+ \ldots + \frac{5}{10^n}$

$= 5\left[\frac{1}{10} + \left(\frac{1}{10}\right)2 + \right.$

$\rightarrow \left.\left(\frac{1}{10}\right)3 + \cdots + (\frac{1}{10})n\right]$

Clearly the term in bracket are in G.P with

$\rightarrow a = r = \frac{1}{10}$

$S_n = 5 \times \frac{1}{10}\left[\frac{1-\left(\frac{1}{10}\right)n}{1-\frac{1}{10}}\right]$

$= \frac{5}{10} \times \frac{10}{9}\left[1-\left(\frac{1}{10}\right)n\right]$

$S_n = \frac{5}{9}\left[1-\left(\frac{1}{10}\right)n\right]$

ii) 0.2, 0.02, 0.002, ...

Solution:

let $S_n = 0.2 + 0.02$

$+ 0.002 +$ ....upto n terms

$= \left[2\frac{1}{10} + \left(\frac{1}{10}\right)2 + \left(\frac{1}{10}\right)3\right.$

$\rightarrow \left. + \cdots + \left(\frac{1}{10}\right)n\right]$

Clearly the term in bracket are in G.P

with $a = r = \frac{1}{10}$

$S_n = 2 \times \frac{1}{10}\left[\frac{1-\left(\frac{1}{10}\right)n}{1-\frac{1}{10}}\right] =$

$\rightarrow \frac{2}{10} \text{ x } \frac{10}{9}\left[1-\left(\frac{1}{10}\right)n\right]$

$S_n = \frac{2}{9}\left[1-\left(\frac{1}{10}\right)n\right]$

8) For a sequences,if

$\mathbf{S_n = (3^n - 1)}$
find the $\boldsymbol{n^{th}}$ term,

hence show that the sequences.

Solution:

We have,

$S_n = 2(3^n - 1)$

$S_n = 2(3^{n-1} - 1)$

$\text{Now}, t_n = S_n - S_{n-1}$

$= 2(3^n - 1) - 2(3^{n-1} - 1)$

$= 2[3^n - 1 - 3^{n-1} + 1]$

$= 2[3^n - 3 - 3^{n-1}]$

$= 2 \text{ x } 3^{n-1} [3 - 1]$

$t_n = 4 \text{ x } 3^{n-1}$

$t_{n+1} = 4 \text{ x } 3^n$

$\frac{t_{n+1}}{t_n} = \frac{4 \times 3^n}{4 \times 3^{n-1}}$

= 3 = Constant for all n € N

Hence, Given sequences is a G.P with a = 4 r =3

9) If s, p, r are the sum , product and sum of the reciprocals of n terms of a G.P

respectively then verify that

$$\left[\frac{S}{R}\right]^{N} = p^2$$

Solution:

Let the n terms of the G.P be

$a, ar, ar^2, \ldots \ldots, r^{n-1}$

$S_n = a + ar + ar^2$

$\rightarrow + \ldots + ar^{n-1}$

$= \frac{a(r^n - 1)}{(r-1)} \ldots \ldots . (1)$

Now,

$P = a \times ar$

$\rightarrow \times ar^2 \times ar^3$

$\rightarrow \times \ldots . . \times ar^{n-1}$

$= (a \times a \times a \ldots n \text{ times}) (r$

$\rightarrow \times r^2 \times r^3 \times \ldots . \times r^{n-1})$

$= (a^n)(r)^{1+2+3+\cdots..+(n-1)}$

$= (a^n).(r)\frac{n-1}{2} \ldots (2)$

$\text{and } R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} +$

$\rightarrow \ldots \ldots + \frac{1}{ar^{n-1}}$

This is G.P. where the first term

is $\frac{1}{a}$ and

→ the common ratio is $\frac{1}{r}$

$$R = \frac{\frac{1}{a}\left[\frac{1}{r}\right]n-1}{\frac{1}{r}}$$

$$\rightarrow = \frac{\frac{1}{a}\left(\frac{1-r^2}{r^n}\right)}{\frac{1-r}{r}}$$

$$= \frac{1-r^n}{a.r^n} \times \frac{r}{1-r}$$

$$= \frac{1-r^n}{a.r^{n-1}(1-r)}$$

$$\frac{1}{R} = \frac{a.r^{n-1}(1-r)}{1-r^n} \quad \ldots\ldots..(3)$$

Now, LHS. $= P^2$

$$= \left[(a^n).(r)\frac{(n-1)n}{2}\right]2$$

$$= a^{2n}.r^{(n-1)n} \quad \ldots.(4)$$

$$\text{And } \frac{s}{r} = \frac{a.(r^{n-1})}{r-1}$$

$$\rightarrow \times \frac{a.r^{n-1}(1-r)}{(1-r^n)}$$

From 1 and 3

$$= \frac{a.(r^n-1)}{r-1} \times$$

$\rightarrow \dfrac{a.r^{n-1}(r-1)}{(r^n-1)}$

$= a^2 . r^{(n-1)n} \dots (5)$

From 4 and 5

LHS = RHS

$P^2 = \left[\frac{S}{R}\right] N$

10) If S_n, S_{2n}, S_{3n}

are the sum of n,2n,3n Term f a G.P respectively then verify that

$\mathbf{S_n (S_{3n} - S_{2n})}$

$\rightarrow = \mathbf{(S_{2n} - S_n)^2}$.

Solution:

Let a be the first term and r be the common ratio of the G.P
Then the G.P is
$a, ar, ar^2, \dots \dots Ar^n, \dots \dots$

Now sum of first n term of the G.p

$S_n = \frac{a(r^n-1)}{r-1}$

Sum of the first 2n terms of the G.P

$S_{2n} = \frac{a(r^{2n}-1)}{r-1}$

Sum of the first 3n term of the G.P

$S_{3n} = \frac{a(r^{3n}-1)}{r-1}$

$S_{3n} - S_{2n}$

$$= \frac{a(r^{3n}-1)}{r-1} - \frac{a(r^{2n}-1)}{r-1}$$

$$= \frac{a}{r-1}(r^{3n} - 1 - r^{2n} + 1)$$

$$= \frac{a}{r-1}(r^{3n} - r^{2n})$$

$$= \frac{a}{r-1} \times r^{2n}(r^n - 1)$$

Now,

LHS. = $S_n (S_{3n} - S_{2n})$

$$= \frac{a(r^n-1)}{r-1} x \frac{a(r^{2n}-1)}{r-1}$$

$$= \frac{a^2 . r^{2n}(r^n - 1)2}{(r-1)2}$$

$$= \frac{a^2}{(r-1)2} . r^{2n}(r^n - 1)^2$$

………… (1)

Also, $S_{2n} - S_n$ =

$$\rightarrow \frac{a(r^{2n}-1)}{r-1} - \frac{a(r^n-1)}{r-1}$$

$$= \frac{a(r^{2n}-1) - a(r^n-1)}{r-1}$$

$$= \frac{a}{r-1}(r^{2n} - 1 - r^n - 1)$$

$$= \frac{a}{r-1}(r^{2n} - r^n)$$

$$S_{2n} - S_n = \frac{a}{r-1} r^n (r^n - 1)$$

$$\text{RHS} = (S_{2n} - S_n)^2$$

$$= \left[\frac{a}{r-1} . r^{2n}(r^n - 1)\right]^2$$

$$= \frac{a^2}{(r-1)} . r^{2n} (r^n - 1)^2$$

$\ldots \ldots . . (2)$

From 1 and 2 LHS = RHS

$S_n(S_{3n} - S_{2n})$

$\rightarrow = (S_{2n} - S_n)^2$

Exercise. 4.3

1. Determine whether the sum to infinity of the following G.P exist if exist find them.

i) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \ldots$

Solution:

Here,

$$a = t1 = \frac{1}{2}$$

and

$$r = \frac{t2}{t1} = \frac{1/4}{1/2} = \frac{1}{2}$$

Clearly, $| r | < 1$, the sum to infinity of the G.P exists.

Sum to infinity

$$= \frac{a}{1-r} = \frac{\frac{1}{2}}{1 - \frac{1}{2}}$$

$$= \frac{1/2}{1/2} = 1$$

Thus the sum to infinity of the G.P is 1.

ii) $\mathbf{2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \dots \dots}$

Solution:

Here , a $= t1 = 2$

and

$$r = \frac{t2}{t1} = \frac{\frac{4}{3}}{2}$$

$$= \frac{2}{3}$$

Cleary, l r l < 1

The sum to infinity of the G.P exists.

Sum to infinity,

$$= \frac{a}{1-r}$$

$$= \frac{2}{1-\frac{2}{3}}$$

$$= \frac{2}{\frac{3-2}{3}}$$

$$= 2 \times 3 = 6$$

Thus, the sum to infinity of the G.P is 6

iii) $\mathbf{-3, 1, \frac{-1}{3},}$

$\rightarrow \mathbf{\frac{1}{9}}, \dots \dots \dots \dots$

Solution:

Here

$a = -3 = t1$ and

$$r = \frac{t2}{t1} = \frac{1}{-3}$$

$$= -\frac{1}{3}$$

Cleary,l r l < 1

The sum to infinity of the G.P exist

Sum to infinity

$$= \frac{a}{1-r}$$

$$= \frac{-3}{1-\left(-\frac{1}{3}\right)}$$

$$= \frac{-3}{1+\frac{1}{3}}$$

$$= \frac{-3}{\frac{4}{3}}$$

$$= \frac{-9}{4}$$

Thus the sum to infinity of the

G.P is $\frac{-9}{4}$

iv) $\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5},$

Solution:

Here, $a = t1, = \frac{1}{5}$

and

$$r = \frac{t2}{t1}$$

$$= \frac{-\frac{2}{5}}{\frac{1}{5}} = -2$$

l r l = 2

\l r l < 1

The sum to infinity of the G.P does not exist

2) Express the following recurring decimals a rational number.

i) 0.32

Solution: 0.32 = 0.32 + 0.0032 + 0.000032 + ………

Here, the term form a G.P.

With   a = 0.32

$$\text{And } r = \frac{0.0032}{0.32} 0.01$$

$$= \frac{1}{100}$$

$$\text{l r l} = \frac{1}{100} < 1$$

The sum to the infinity of the G.P exist and

$$= \frac{a}{1-r}$$

$$= \frac{0.32}{1-0.01}$$

$$= \frac{0.32}{0.99}\frac{32}{99}$$

Thus 0.32 $= \frac{32}{99}$

ii) 3.5

Solution: 3.5 = 3 + 0.5+ 0.05+ 0.005 + ……

Here,the term after the first term form a

G.P with a =0.5 and

$r = \frac{0.05}{0.5} = 0.1$

$= \frac{1}{10}$

$| r | = \frac{1}{10} < 1$

The sum to the infinity of the G.P exists and

$= \frac{a}{1-r}$

$= \frac{0.5}{1-0.1}$

$= \frac{0.5}{0.9}$

$= \frac{5}{9}$

$3.5 = 3 + \frac{5}{9}$

$= \frac{27+5}{9}$

$= \frac{32}{9}$

Thus, $3.5 = \frac{32}{9}$

iii) 4.18

Solution: $4.18 = 4 + 0.18 + 0.0018 + 0.000018 + \ldots\ldots$

Here, the term after the first term form a G.P

With a =0.18

$$\text{And } r = \frac{0.0018}{0.18}$$

$$= 0.01 = \frac{1}{100}$$

$$|r| = \frac{1}{100} < 1$$

The sum to the infinity of the G.P exist and

$$= \frac{a}{1-r} = \frac{0.18}{1-0.01}$$

$$= \frac{0.18}{0.99} = \frac{2}{11}$$

$$4.18 = 4 + \frac{2}{11}$$

$$= \frac{44+2}{11} + \frac{46}{11}$$

$$\text{Thus, } 4.18 = \frac{46}{11}$$

iv) 0.345

Solution: $0.345 = 0.3 + 0.045 + 0.00045 + 0.0000045 + \ldots\ldots.$

Here, the term after the first term form a G.P

With a =0.045

$$R = \frac{0.00045}{0.045} = 0.01$$

$= \frac{1}{100}$

$|r| = \frac{1}{100} < 1$

The sum to the infinity of the G.P exists and

$$= \frac{a}{1-r} = \frac{0.045}{1-0.01}$$

$$= \frac{0.045}{0.99} + \frac{4.5}{90}$$

$$\underline{\quad} 0.345 = \frac{3}{10} + \frac{4.5}{99}$$

$$= \frac{3}{10} + \frac{45}{990}$$

$$= \frac{267+45}{990}$$

$$= \frac{342}{990} = \frac{19}{55}$$

$$\underline{\quad} \text{Thus}, 0.345 = \frac{19}{55}$$

v) 3.456

Solution: 3.456 = 3.4 + 0.056 + 0.00056 + 0.0000056 + ……..

Here, the term after the first term form a G.P.

With a =0.056

$$\text{And } r = \frac{0.00056}{0.056}$$

$$= 0.01 = \frac{1}{100}$$

$$|r| = \frac{1}{100} < 1$$

The sum to the infinity to the G.P exists and

$$= \frac{a}{1-r} = \frac{0.056}{1-0.01}$$

$$= \frac{0.056}{0.99} = \frac{5.6}{99}$$

$$\overline{\quad}3.456 = 3.4 + \frac{5.6}{99}$$

$$= \frac{34}{10} + \frac{56}{990}$$

$$= \frac{3366+56}{990}$$

$$= \frac{3422}{990}$$

$$= \frac{1711}{495}$$

$$\text{Thus}, 3.456 = \frac{1711}{495}$$

3) If the common ratio a G.P. is

is $\frac{2}{3}$ and sum of

its terms to infinity is 12.Find the first terms.

Solution:

Here, $r = \frac{2}{3}$

and sum to infinity $= 12$

$$\frac{a}{1-r} = 12$$

$$A = 12(1-r)$$

$$= 12\left(1-\frac{2}{3}\right)$$

$= 12(\frac{3-2}{3})$

The first term = 4.

4) If the first term of the G.P is 16 and its sum to infinity is

$\frac{\mathbf{176}}{\mathbf{5}}$

find the common ratio.

Solution:

Here, a =16 and sum to

infinity is $\frac{176}{5}$

i. e. $\frac{a}{1-r} = \frac{176}{5}$

5a = 176(1-r)

5 x 16 = 176(1-r)

$\frac{80}{176} = 1 - \mathrm{r}$

$\frac{5}{11} = 1 - \mathrm{r}$

$\mathrm{r} = 1 - \frac{5}{11}$

$\mathrm{r} = \frac{6}{11}$

5) The sum of the terms of an infinity G.P is 5 and the sum of the squares of those terms is 15 find the G.P

Solution:

Here, the sum of an infinity G.P is 5

$\frac{a}{1-r} = 5$

Also, the sum of the squares of these terms is 15.

$\frac{a^2}{1-r^2} = 15$

$\frac{a}{1-r} \times \frac{a}{1+r} = 15$

$5 \times \frac{a}{1+r} = 15$

$\frac{a}{1+r} = 3$ …………..(2)

From 1 a =5(1-r) = 5-5r

From 2 a =3(1+r) = 3+3r

5- 5r = 3+3r

-8r = -2

$r = \frac{1}{4}$

From 1 $a = 5 - \frac{5}{4}$

$= \frac{20-5}{4}$

$= \frac{15}{4}$

Required terms of G.P is given by
$a, ar, ar^2, ar^3, \ldots.$

$\text{i. e.} \frac{15}{4}, \frac{15}{16}, \frac{15}{64}, \ldots\ldots$

Exercise 4.4

1. verify whether the following sequences are H.P.

i) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \dots \dots$

Solution: Given sequences

is $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \dots \dots$

The reciprocals of the number are 3, 5, 7, 9, ...

These numbers are in A.P with a =3 and d =5 – 3 = 7-5 = 9-7 = 2

The sequences

$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \dots \dots$

Is a H.P.

ii) $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \dots \dots$

Solution:

Given sequences is

$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \dots \dots$

The reciprocals of the number are 3, 6, 9, 12,....

These numbers are in A.P with a =3 and d =6-3 = 9-6 =12-9 =3

The sequences

$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \dots \dots$

Is a H.P.

iii) $\frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \frac{1}{15}, \dots \dots$

Solution:

Given sequences is

$\frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \frac{1}{15}, \dots \dots$

The reciprocals of the number are

7, 9, 11, 13, 15, ...

These numbers are in A.P with a=7 and d=9 -7   11-9 = 13-11= 15-13=2

The sequences

$\frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \frac{1}{15},$

$\dots \dots \dots \dots$ is a H. P

2) Find the nth terms and hence find the 8th terms of the following H.P

i) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11},$

solution:

Given sequences is in H.P

2, 5, 8, 11, ...are in A.P with a =2 and d =3

$t_n = a + (n-1)d = 2 + (n-1)3$

= 2+3n -3   = 3n -1

n th terms of

$$\text{H. P} = \frac{1}{3n-1}$$

8 th terms of

$$\text{H. P} = \frac{1}{3 \times 8 - 1}$$

$$= \frac{1}{24-1}$$

$= \frac{1}{23}$

The 8th terms of the

$\rightarrow$ H. P is $\frac{1}{23}$.

ii) $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \ldots$

Solution: Given sequences is in H.P

4, 6, 8, 10,.... are in A.P with a =4 and d =2.

$t_n = a + (n-1)d$

$\rightarrow = 4 + (n-1)2$

= 4+ 2n -2 = 2n+ 2

N th terms of H.P.

$\rightarrow = \frac{1}{2n+2}$

8th terms of H. P

$= \frac{1}{2(8)+2}$

$= \frac{1}{16+2}$

$= \frac{1}{18}$

The 8th terms of the

$\rightarrow$ H. P is $\frac{1}{18}$.

iii) $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \ldots$

Solution:

Given sequences is in H.P

5, 10, 15, 20,... are in A.P with a =5 and d =5

$t_n = a + (n-1)d$

$\rightarrow = 5 + (n-1)5$

$= 5 + 5n - 5 \quad = 5n$

Nth terms of

$\rightarrow$ H.P $= \frac{1}{5n}$

8th terms of

$\rightarrow$ H.P $= \frac{1}{5 \times 8}$

$= \frac{1}{40}$

The 8th terms of the

$\rightarrow$ H.P is $\frac{1}{40}$.

3) Find A.M of two positive numbers whose G.M and H.M are 4 and $\frac{16}{5}$ respectively.

Solution:

Here, G = 4 and

$H = \frac{16}{5}$

We have $G^2 = AH$

$$16 = A \times \frac{16}{5}$$

$A = 5$

4) Find H.M of two positive numbers

Whose A. M and

Solution:

Here, $A = \frac{15}{2}$

And $G = 6$

We have $G^2 = AH$

$$36 = \frac{15}{2} X H$$

$$H = \frac{2 \times 36}{5}$$

$$H = \frac{24}{5}$$

5) Find G.M of two positive numbers whose A.M and H.M are 75 and 48.

Solution:

Here, A= 75 and H =48

We have $G^2 = AH$

$G^2 = 75 X 48$

$G^2 = 3600$

$G = 60$

6) Insert two numbers

$\rightarrow$ **between $\frac{1}{7}$ and $\frac{1}{13}$**

so that the resulting sequences is a H.P

Solution:

Let the required number

$\rightarrow$ be $\frac{1}{H_1}$ and $\frac{1}{H_2}$.

$\frac{1}{7}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{13}$ are in H.P

7, H1, H2, 13, and t4 =13

Here, a =7 and t4 =13

a + 3d = 13

7 + 3d = 13

3d =13 -7 = 6

d = 2

H1 = t2 = a+b = 7 + 2 =9

H2 = t3 = a + 2b = 7 + 4 = 11

Hence required numbers are

7) Insert two numbers between 1 and -27 so that the resulting sequences is a G.P

Solution:

let required numbers be G1 and G2.

1, G1, G2, -27 are in G.P

Here, a= 1 and t4 = -27

$a.r^3 = -27$

$r^3 = (-3)^3$

r = -3

G1 = T2 = a.r = -3

$G2 = T3 = a.r = (-3)^2 = 9$

Hence, the required number are -3 and 9.

8) Find two numbers whose $\mathbf{A.M}$ exceedtheir

$\mathbf{G.M}$ by $\mathbf{\frac{1}{2}}$ and their

→ $\mathbf{H.M}$ by $\mathbf{\frac{25}{26}}$.

Solution:

Let a and b be the required numbers.

Then $A = \frac{a+b}{2} G$

$\rightarrow = \sqrt{ab} \quad H = \frac{2ab}{a+b}$

From the given condition,

$A - G = \frac{1}{2} \ldots (1)$

$A - H = \frac{25}{26} \ldots 2)$

Multiplying equation (2) by A

$A^2 - AH = \frac{25}{26} A$

$A^2 - G^2 = \frac{25}{26} A$

$$(A-G)(A+G)=\frac{25}{26}A$$

$\frac{1}{2}(A-G)=\frac{25}{26}A$ ... From 1)

$$A+G=\frac{25}{13}A$$

$$G=\frac{12}{13}A\ ...\ (3)$$

From (1)

$$A-\frac{12}{13}A=\frac{1}{2}$$

$A=\frac{13}{2}$ and

$$G=\frac{12}{13}\times\frac{13}{2}$$

= 6 .... from (3)

$$\text{Now}, A=\frac{a+b}{2}=\frac{13}{2}$$

$$\text{and } G=\sqrt{ab}=6$$

a + b = 13 and ab = 36

a(13-a) = 36

$$13a-a^2=36$$

$$a^2-13a+36=0$$

(a-9) (a-4) =0

a =9 or a =4

b = 4 or b =9

Hence the required number are 4 and 9.

9) Find two number whose A.M exceeds G.M by 7 and their H.M by $\frac{63}{5}$.

Solution:

Let a and b be the required numbers.

Then $A = \frac{a+b}{2}$,

$G = \sqrt{ab}$,

$H = \frac{2ab}{a+b}$

From the given condition:

$A - G = 7$ ……. (1)

$A - H = \frac{63}{5}$ ... (2)

Multiplying equation (2) by A

$A^2 - AH = \frac{63}{5}A$

$A^2 - G^2 = \frac{63}{5}A$

$(A - G)(A + G) = \frac{63}{5}A$

$7(A + G) = \frac{63}{5}A$

… … … From (1)

$5(A+G) = 9A$

$5A + 5G = 9A$

$5G = 4A$

$G = \frac{4}{5}A$ ... (3)

From (1)

$$A - \frac{4}{5}A = 7$$

5A – 4A = 35

A = 35

And

$$G = \frac{4}{5} \times 35 = 28$$

….. From (3)

$$\text{Now, } A = \frac{a+b}{2} = 35$$

$$\text{and } G = \sqrt{ab} = 28$$

a + b = 70 and ab= 784

a(70-a) = 784

$70a - a^2 = 784$

$a^2 - 70a + 784 = 0$

(a-56) (a-14) = 0

a = 56 or a =14

b =14 or b =56

Hence the required numbers are 14 and 56.

Exercise: 4.5

1) Find the sum

$$\sum_{r=1}^{n}(r+1)(2r-)$$

Solution:

$= \sum_{r=1}^{n}(r+1)(2r-)$

$= \sum_{r=1}^{n}(2r^2+2r-r-1)$

$= \sum_{r=1}^{n}(2r^2+r-1)$

$= 2\sum_{r=1}^{n}r^2 +$

$\rightarrow \sum_{r=1}^{n}r - \sum_{r=1}^{n}1$

$= 2 \text{ x } \left[\frac{n(n+1)(2n+1)}{6}\right]$

$\rightarrow + \frac{n(n+1)}{2} - \text{n}$

$= \text{n}[= \text{n}[\frac{(n+1)(2n+1)}{6}$

$\rightarrow + \frac{(n+1)}{2} - 1\,]$

$= \frac{n}{6}\,[2(2\text{n}^2 + 2\text{n} + \text{n} +$

$\rightarrow 1) + 3\text{n} + 3 - 6]$

$= \frac{n}{6}[2(2\text{n}^2 + 3\text{n} + 1)$

$\rightarrow + 3\text{n} - 3]$

$= \frac{n}{6}(4n^2 + 6n +$

$\rightarrow 2 + 3n - 3)$

$= \frac{n}{6}(4n^2 + 9n - 1)$

2) Find $\sum_{r=1}^{n}(3r^2 - 2r + 1)$

Solution:

$$= \sum_{r=1}^{n}(3r^2 - 2r + 1)$$

$= 3\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$

$= 3 \times \frac{n(n+1)(2n+1)}{2}$

$\rightarrow - \frac{2(n+1)n}{2} + n$

$= n\,[\frac{(n+1)\,(2n+1)}{2}$

$\rightarrow - \frac{2n+2}{2} + 1]$

$= n\,[\frac{2n^2+2n+n+1-2n-2+2}{2}]$

$= \frac{n}{2}(2n^2 + n + 1)$

3) Find $\sum_{r=1}^{n}(1 + 2 + 3 + \cdots . + r)$

Solution:

$= \sum_{r=1}^{n}(1+2+3+\cdots.+r)$

$= \sum_{r=1}^{n}[\frac{\frac{r(r+1)}{2}}{r}]$

$= \sum_{r=1}^{n}(\frac{r+1}{2})$

$= \frac{1}{2}\ [\sum_{r=1}^{n} \sum_{r=1}^{n} 1]$

$= \frac{1}{2}\ [\frac{n(n+1)}{2} + n]$

$= \frac{n}{4}(\mathrm{n}+1+2)$

$= \frac{n(n+3)}{4}$

4) Find $\sum_{r=1}^{n}(\frac{1^3+2^3+\cdots+r^3}{r(r+1)})$

Solution:

$$\sum_{r=1}^{n}(\frac{1^3+2^3+\cdots+\ r^3}{r(r+1)})$$

$= \sum_{r=1}^{n}[\frac{\frac{r^2(r+1)2}{4}}{r(r+1)}]$

$= \frac{1}{4}\sum_{r=1}^{n}(r^2+r)$

$= \frac{1}{4}\ [\sum_{r=1}^{n} r^2 + \ \sum_{r=i}^{n} r]$

$= \frac{1}{4}[\frac{n(n+1)(2n+1)}{6}$

$\rightarrow + \frac{n(n+1)}{2}]$

$= \frac{n(n+1)}{2\times4}\ [\frac{2n+1}{3} + 1]$

$= \frac{n(n+1)}{8} \times \frac{(2n+1+3)}{3}$

$= \frac{n(n+1)}{8} \times \frac{2n(n+2)}{3}$

$= \frac{n(n+1)(n+2)}{12}$

5) Find the sum 5 x 7 + 9 x 11 x 13 x 15 + …. Upto n term

Solution:

Here, the term of 1st part are 5, 9, 13….

And the term of 2nd parts are 7, 11, 15….

These parts are in A.P

nth terms of 1st part = 5 + (n-1)4 = 4n + 1

nth terms of 2nd parts = 7+ (n-1)4 = 4n + 3

$S_n = \sum_{r=1}^{n} tr$

$\rightarrow = \sum_{r=1}^{n}(4r+1)(4r+3)$

$= \sum_{r=1}^{n}(16r^2 + 16r + 3)$

$= 16\ \sum_{r=1}^{n} r^2$

$\rightarrow +16\ \sum_{r=1}^{n} r\ + \sum_{r=1}^{n} 3$

$= 16 \times \frac{n(n+1)(2n+1)}{6}$

→ $+ 16 \times \frac{n(n+1)}{2} + 3n$

$= n\left[\frac{8(n+1)(2n+1)}{3} +\right.$

→ $\left. 8(n+1) + 3\right]$

$= \frac{n}{3}$ [8(2n2 + 3n + 1)

→ + (8n + 11x3]

$= \frac{n}{3}[16n^2 + 48n + 41]$

6) Find the sum $2^2 + 4^2 +$

→ $6^2 + 8^2 + \cdots$. Upto n term.

Solution: Sum of $2^2 + 4^2$

→ $+ 6^2 + 8^2 + \cdots$ Upto n term.

$= \sum_{r=1}^{n}(2r)2$

$= 4 \sum_{r=1}^{n} r^2$

$= 4 \text{ x} \frac{n(n+1)(2n+1)}{6}$

$= \frac{2n(n+1)(2n+1)}{3}$

7) Find $(70^2 - 69^2) +$

→ $(68^2 - 67^2) + (66^2 -$

→ $\mathbf{65^2})\ + \cdots \ldots + (\mathbf{2^2} - \mathbf{1^2})$

Solution: $(70^2 - 69^2) +$

→ $(68^2 - 67^2) +\ (66^2 - 65^2)$

→ $+ \cdots \ldots + (2^2 - 1^2)$

$=\ (70^2 + 68^2 + 66^2 +$

→ $\cdots . +2^2) - (69^2 + 67^2$

→ $+65^2 + \cdots . +1^2)$

$=\ (2^2 + 4^2 + 6^2 + \cdots$

→ $+70^2) - (1^2 + 3^2 +$

→ $5^2 + \cdots . +69^2)$

$= \sum_{r=1}^{35}(2r)2 -$

→ $\sum_{r=1}^{35}(2r - 1)2$

$=\ \sum_{r=1}^{35}(2r)2 - \ (2\mathrm{r} - 1)^2]$

$=\ \sum_{r=1}^{35}(4r^2 - 4r^2 + 4r - 1)$

$= 4\ \sum_{r=1}^{35} r - \sum_{r=1}^{35} 1$

$=\ 4 \text{ x } \frac{35 \times 36}{2} - \ 36$

= 35 (72-1)

= 35 x 71

= 2485

8)Find the sum 1 x 3 x 5 + 3 x 5 x 7 + 5 x 7 x 9 + ... + (2n -1) (2n+1) (2n+3)

Solution:1 x 3 x 5 + 3 x 5 x 7 + 5 x 7 x 9 + ... + (2n -1) (2n+1) (2n+3)

Here, $\mathrm{t_n} = (2\mathrm{n} - 1)$

→ $(2\mathrm{n} + 1)\ (2\mathrm{n} + 3)$

$\mathrm{S_n} = \sum_{r=1}^{n} t_r =$

→ $\sum_{r=1}^{n}(2n - 1)$

→ $(2\mathrm{r} + 1)\ (2\mathrm{r} + 3)$

$= \sum_{r=1}^{n}(4r^2 - 1)_{\ (2r+3)}$

$= \sum_{r=1}^{n} 8r^3 - 12r^2 - 2\mathrm{r} - 3$

$= 8 \sum_{r=1}^{n} r^3$

→ $+ 12 \sum_{r=1}^{n} r - 2$

→ $\sum_{r=1}^{n} -3 \sum_{r=1}^{n}(1)$

$= 8 \times \frac{n^2(n+1)2}{4} +$

→ $12 \times \frac{n(n+1)(2n+1)}{6} -$

→ $2 \times \frac{n(n+1)}{2} - 3\mathrm{n}$

$= 2\mathrm{n}^2(\mathrm{n} + 1)^2 +$

→ $2\mathrm{n}(\mathrm{n} + 1)(2\mathrm{n} + 1)$

→ $- \mathrm{n}(\mathrm{n} + 1) - 3\mathrm{n}$

$= n(n+1)\,[2n(n+1) + 2\,(2n+1)\,-1]\,-3n$

$= n(n+1)\,[2n^2 + 2n$

$\rightarrow +4n^2 + 2 - 1] - 3n$

$= n(n+1)\,(6n^2 +$

$\rightarrow 2n + 1) - 3n$

$= n[(n+1)(6n^2$

$\rightarrow +2n + 1) - 3]$

$= n[6n^3 + 2n^2 + n +$

$\rightarrow 6n^2 + 2n + 1 - 3]$

$= n(6n^3 + 8n^2 + 3n - 2)$

9) If $\frac{1\times 2 + 2\times 3 + 3\times 4 + 4\times 5 \ldots .\, upto\ n\ term}{1+2+3+4+\cdots upto\ n\ term} = \frac{100}{3}$**, find n.**

Solution:

$$\frac{1\times2+2\times3+3\times4+4\times5 \ldots .upto\ n\ term}{1+2+3+4+\cdots upto\ n\ term} = \frac{100}{3}$$

$$\frac{\sum_{r=1}^{n} r(r+1)}{\sum_{r=1}^{n} r} = \frac{100}{3}$$

$$\frac{\sum_{r=1}^{n} r(r^2+1)}{\sum_{r=1}^{n} r} = \frac{100}{3}$$

$$\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r$$

$$\rightarrow = \frac{100}{3}\sum_{r=1}^{n} r$$

$$\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$\rightarrow = \frac{100}{3} \times \frac{n(n+1)}{2}$

$$\frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 1\right] =$$

$\rightarrow \frac{100}{3} \times \frac{n(n+1)}{2}$

$\frac{2n+1+3}{3} = \frac{100}{3}$

$2n + 4 = 100$

$2n = 96$

$n = 48$

10) If $S_1 S_2$ and S_3

are the sums of first n natural numbers their squares and their cubes respectively then show that

$$9{S_2}^2 = S_3\,(1 + 8S_1)$$

Solution:

S_1 is the sum of First n natural numbers.

$S_1 = \frac{n}{2}\,(n+1)..(1)$

$S_2 = \frac{n}{6}\,(n+1)$

$\rightarrow (2n+1)\ .....(2)$

S_3 is the sum

of cubes of first n natural numbers.

$S_3 = \frac{n^2}{4}\,(n+1)^2 \ldots (3)$

Now, L.H.S

$= 9(S_2)^2$

$= 9 \text{ x } \frac{n^2}{36} (n+1)^2$

$\rightarrow (2n+1)^2 \ldots$ From (2)

$= \frac{n^2}{36}(n+1)^2 [1+8$

$\rightarrow \times \frac{n}{2}(n+1)]$

from (1) and (3)

$= \frac{n^2}{4}(n+1)^2 [1$

$\rightarrow +4n(n+1)]$

$= \frac{n^2}{4}(n+1)^2 (4n^2$

$\rightarrow +4n+1)$

$= \frac{n^2}{4}(n+1)^2 (2n$

$\rightarrow +1)^2 \ldots\ldots\ldots\ldots (5)$

From $4, 5, 9(S_2)^2$

$\rightarrow = S_3 (1+8S_1).$