UNIT 13

PLAYING WITH NUMBERS

(A) Main Concepts and Results

- Numbers can be written in general form. For example, a two digit number *ab* is written as $ab = 10a + b$; a three digit number *abc* is written as $abc = 100a + 10b + c$.
- The general form of numbers are helpful in solving various problems related to numbers.
- Rationale for the divisibility of numbers by 11, 10, 5, 2, 9 or 3 can be explained by writing the numbers in general form.
- Many number puzzles involving different letters for different digits are solved using rules of number operations.

(B) Solved Examples

In examples 1 to 4, out of four options only one is correct. Write the correct answer.

Example 1 : Generalised form of a three-digit number *xyz* is

(a) $x + y + z$ (b) $100x + 10y + z$

(c) $100z + 10y + x$ (d) $100y + 10x + z$

Solution : The correct answer is (b).

Example 2 : The usual form of $100a + b + 10c$ is

(a) *abc* (b) *cab* (c) *bac* (d) *acb*

Solution : The correct answer is (d).

Example 3 : If $5 \times A = CA$ then the values of A and C are

(a) A = 5, C = 1 (b) A = 4, C = 2

(c) A = 5, C = 2 (d) A = 2, C = 5

Solution : The correct answer is (*c*).

Example 4 : If 5 A + 25 is equal to B 2, then the value of A + B is

(a) 15 (b) 10 (c) 8 (d) 7

Solution : The correct answer is (*a*).

In examples 5 to 7, fill in the blanks to make the statements true.

Example 5 : The number $ab - ba$ where a and b are digits and $a > b$ is divisible by ________.

Solution : 9.

Example 6 : When written in usual form $100a + 10c + 9$ is equal to ________.

Solution : ac 9

Example 7 : If AB × B = 9B, then A = _________, B = _________.

Solution : 9, 1

In examples 8 to 10, state whether the statements are true (T) or false (F).

Example 8 : If *abc*, *cab*, *bca* are three digit numbers formed by the digits *a*, *b*, and c then the sum of these numbers is always divisible by 37.

Solution : True.

Example 9 : Let *ab* be a two-digit number, then $ab + ba$ is divisible by 9.

Solution : False.

Example 10 : If a number is divisible by 2 and 4, then it will be divisible by 8.

Solution : False.

Example 11 : A three-digit number 42x is divisible by 9. Find the value of x.

Solution : Since 42x is divisible by 9, the sum of its digits, i.e. $4 + 2 + x$ must be divisible by 9.

i.e. $6 + x$ is divisible by 9

i.e. $6 + x = 9$ or 18, _____.

Since x is a digit, therefore $6 + x = 9$ or, $x = 3$.

Example 12 : Find the value of A and B if

$$\begin{array}{r} 41\,A \\ +\,B\;\;4 \\ \hline 5\,1\,2 \\ \hline \end{array}$$

Solution : From ones column A + 4 gives a number whose ones digit is 2. So, A = 8. The value of B can be obtained by solving 2 + B is a number whose ones digit is 1. So, B = 9.

$$\begin{array}{r} 418 \\ +\;94 \\ \hline 512 \\ \hline \end{array}$$

Example 13 : Suppose that the division $x \div 5$ leaves a remainder 4 and the division $x \div 2$ leaves a remainder 1. Find the ones digit of x.

Solution : Since $x \div 5$ leaves a remainder 4, so ones digit of x can be 4 or 9. Also, since $x \div 2$ leaves a remainder 1, so ones digit must be 9 only.

Application on Problem Solving Strategy

Example 14 :

If 756*x* is divisible by 11, where *x* is a digit find the value of *x*.

Understand and Explore the problem

- What is given in the question?

  A four digit number 756x is divisible by 11.
- Which property is required to solve the problem?

  Divisibility of a number by 11.

Plan a Strategy

- Find the sum of the digits of given number 756x at odd places.
- Find the sum of the digits of 756x at even places.
- Find the difference of step 1 and step 2.

Solve

- Given $y = 2x$
- Sum of digits at odd places = $x + 5$
- Sum of digits at even places = 6 + 7 = 13
- Difference = (x + 5) – 13

  = $x - 8$

  Now $(x - 8)$ should be equal to 0 or a multiple of 11 (i.e. 11, 22, 33, …, etc.)

  $x - 8 = 0$

  $x = 8$ or $x - 8 = 11$

  $x = 11 + 8 = 19$
- Since x is a digit so it can take values from 0 – 9

  Hence $x = 8$

  Required number is 7568.

Revise

- 7568

  Sum of digits at odd places = 5 + 8 = 13

  Sum of digits at even places = 6 + 7 = 13

  Difference = 13 – 13

  = 0

  So Value of x is correct.

Think and Discuss

1. What would be the value of y, if $277y$ is divisible by 11?

(C) Exercise

In each of the questions 1 to 17, out of the four options, only one is correct. Write the correct answer.

1. Generalised form of a four-digit number *abdc* is

   (a) $1000\ a + 100\ b + 10\ c + d$

   (b) $1000\ a + 100\ c + 10\ b + d$

   (c) $1000\ a + 100\ b + 10\ d + c$

   (d) $a \times b \times c \times d$

2. Generalised form of a two-digit number xy is

(a) $x + y$ (b) $10x + y$ (c) $10x - y$ (d) $10y + x$

3. The usual form of $1000a + 10b + c$ is

(a) *abc* (b) *abco* (c) *aobc* (d) *aboc*

4. Let *abc* be a three-digit number. Then *abc* – *cba* is not divisible by

(a) 9 (b) 11 (c) 18 (d) 33

5. The sum of all the numbers formed by the digits x, y and z of the number *xyz* is divisible by

(a) 11 (b) 33 (c) 37 (d) 74

6. A four-digit number *aabb* is divisible by 55. Then possible value(s) of b is/are

(a) 0 and 2 (b) 2 and 5 (c) 0 and 5 (d) 7

7. Let *abc* be a three digit number. Then *abc* + *bca* + *cab* is not divisible by

(a) $a + b + c$ (b) 3 (c) 37 (d) 9

8. A four-digit number 4*ab*5 is divisible by 55. Then the value of $b - a$ is

(a) 0 (b) 1 (c) 4 (d) 5

9. If abc is a three digit number, then the number $abc - a - b - c$ is divisible by

(a) 9 (b) 90 (c) 10 (d) 11

10. A six-digit number is formed by repeating a three-digit number. For example 256256, 678678, etc. Any number of this form is divisible by

(a) 7 only (b) 11 only (c) 13 only (d) 1001

11. If the sum of digits of a number is divisible by three, then the number is always divisible by

(a) 2 (b) 3 (c) 6 (d) 9

12. If $x + y + z = 6$ and z is an odd digit, then the three-digit number *xyz* is

(a) an odd multiple of 3 (b) odd multiple of 6

(c) even multiple of 3 (d) even multiple of 9

13. If 5 A + B 3 = 65, then the value of A and B is

(a) A = 2, B = 3 (b) A = 3, B = 2

(c) A = 2, B = 1 (d) A = 1, B = 2

14. If A 3 + 8 B = 150, then the value of A + B is

(a) 13 (b) 12 (c) 17 (d) 15

15. If 5 A × A = 399, then the value of A is

(a) 3 (b) 6 (c) 7 (d) 9

16. If 6 A × B = A 8 B, then the value of A – B is

(a) –2 (b) 2 (c) –3 (d) 3

17. Which of the following numbers is divisible by 99

(a) 913462 (b) 114345 (c) 135792 (d) 3572406

In questions 18 to 33, fill in the blanks to make the statements true.

18. 3134673 is divisible by 3 and ______.

19. 20x3 is a multiple of 3 if the digit x is ______ or ______ or ______.

20. 3x5 is divisible by 9 if the digit x is __________.

21. The sum of a two–digit number and the number obtained by reversing the digits is always divisible by __________.

22. The difference of a two–digit number and the number obtained by reversing its digits is always divisible by ___________.

23. The difference of three-digit number and the number obtained by putting the digits in reverse order is always divisible by 9 and ___________.

24. If $\begin{array}{rcc} & 2 & B \\ + & A & B \\ \hline & 8 & A \end{array}$ then A = ______ and B = ______.

25. If $\begin{array}{rcc} & A & B \\ \times & & B \\ \hline & 9 & 6 \end{array}$ then A = ______ and B = ______.

26. If $\begin{array}{rcc} & B & 1 \\ \times & & B \\ \hline & 4 & 9B \end{array}$ then B = _______.

27. 1 x 35 is divisible by 9 if x = _______.

28. A four-digit number *abcd* is divisible by 11, if $d + b$ = _______ or _____

29. A number is divisible by 11 if the differences between the sum of digits at its odd places and that of digits at the even places is either 0 or divisible by ______.

30. If a 3-digit number *abc* is divisible by 11, then ______ is either 0 or multiple of 11.

31. If A × 3 = 1A, then A = ______.

32. If B × B = AB, then either A = 2, B = 5 or A = ______, B = ______.

33. If the digit 1 is placed after a 2-digit number whose tens is t and ones digit is u, the new number is ______.

State whether the statements given in questions 34 to 44 are true (T) or false (F):

34. A two-digit number *ab* is always divisible by 2 if b is an even number.

35. A three-digit number *abc* is divisible by 5 if c is an even number.

36. A four-digit number *abcd* is divisible by 4 if *ab* is divisible by 4.

37. A three-digit number *abc* is divisible by 6 if c is an even number and $a + b + c$ is a multiple of 3.

38. Number of the form 3N + 2 will leave remainder 2 when divided by 3.

39. Number 7N + 1 will leave remainder 1 when divided by 7.

40. If a number a is divisible by b, then it must be divisible by each factor of b.

41. If AB × 4 = 192, then A + B = 7.

42. If AB + 7C = 102, where B ≠ 0, C ≠ 0, then A + B + C = 14.

43. If 213x27 is divisible by 9, then the value of x is 0.

44. If N ÷ 5 leaves remainder 3 and N ÷ 2 leaves remainder 0, then N ÷ 10 leaves remainder 4.

Solve the following :

45. Find the least value that must be given to number a so that the number 91876a2 is divisible by 8.

46. If $\begin{array}{r} 1\ \ P \\ \times\ \ \ P \\ \hline Q\ \ 6 \end{array}$ where Q – P = 3, then find the values of P and Q.

47. If 1AB + CCA = 697 and there is no carry–over in addition, find the value of A + B + C.

48. A five-digit number AABAA is divisible by 33. Write all the numbers of this form.

49. Find the value of the letters in each of the following questions.

$$\begin{array}{r} A\ \ A \\ +A\ \ A \\ \hline XA\ \ Z \\ \hline \end{array}$$

50. $\begin{array}{r} 8\ 5 \\ +4\ A \\ \hline B\ C\ 3 \\ \hline \end{array}$

51. $\begin{array}{r} B\ 6 \\ +8\ A \\ \hline C\ A\ 2 \\ \hline \end{array}$

52. $\begin{array}{r} 1\ B\ A \\ +\ A\ B\ A \\ \hline 8\ A\ 2 \\ \hline \end{array}$

53. $\begin{array}{r} C\ B\ A \\ +C\ B\ A \\ \hline 1\ A\ 3\ 0 \\ \hline \end{array}$

54. $\begin{array}{r} B\ A\ A \\ +B\ A\ A \\ \hline 3\ A\ 8 \\ \hline \end{array}$

55. $\begin{array}{r} A\ 0\ 1\ B \\ +1\ 0\ A\ B \\ \hline B\ 1\ 0\ 8 \\ \hline \end{array}$

56. $\begin{array}{r} A\ B \\ \times\ \ 6 \\ \hline C\ 6\ 8 \\ \hline \end{array}$

57. $\begin{array}{r} A\ B \\ \times\ A\ B \\ \hline 6\ A\ B \\ \hline \end{array}$

58. $\begin{array}{r} A\ A \\ \times\ A \\ \hline C\ A\ B \\ \hline \end{array}$

and B – A = 1

59. $\begin{array}{r} A\ B \\ -\ B\ 7 \\ \hline 4\ 5 \\ \hline \end{array}$

60. $\begin{array}{r} 8\ A\ B\ C \\ -A\ B\ C\ 5 \\ \hline D\ 4\ 8\ 8 \\ \hline \end{array}$

61. If 2A7 ÷ A = 33, then find the value of A.

62. 212 *x* 5 is a multiple of 3 and 11. Find the value of *x*.

63. Find the value of *k* where 31*k* 2 is divisible by 6.

64. 1*y*3*y*6 is divisible by 11. Find the value of *y*.

65. 756 *x* is a multiple of 11, find the value of *x*.

66. A three-digit number 2 *a* 3 is added to the number 326 to give a three-digit number 5*b*9 which is divisible by 9. Find the value of *b* –*a*.

67. Let E = 3, B = 7 and A = 4. Find the other digits in the sum

$$\begin{array}{r} \text{B A S E} \\ +\text{B A L L} \\ \hline \text{G A M E S} \\ \hline \end{array}$$

68. Let D = 3, L = 7 and A = 8. Find the other digits in the sum

$$\begin{array}{r} \text{M A D} \\ +\ \text{A S} \\ +\ \text{A} \\ \hline \text{B U L L} \\ \hline \end{array}$$

69. If from a two-digit number, we subtract the number formed by reversing its digits then the result so obtained is a perfect cube. How many such numbers are possible? Write all of them.

70. Work out the following multiplication.

$$\begin{array}{r} 12345679 \\ \times\ 9 \\ \hline \\ \hline \end{array}$$

Use the result to answer the following questions.

(a) What will be 12345679 × 45?

(b) What will be 12345679 × 63?

(c) By what number should 12345679 be multiplied to get 888888888?

(d) By what number should 12345679 be multiplied to get 999999999?

71. Find the value of the letters in each of the following:

(i)
$$\begin{array}{r} \text{PQ} \\ \times\ 6 \\ \hline \text{QQQ} \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} \text{2 L M} \\ +\ \text{L M 1} \\ \hline \text{M 1 8} \\ \hline \end{array}$$

72. If 148101B095 is divisible by 33, find the value of B.

73. If 123123A4 is divisible by 11, find the value of A.

74. If 56*x*32*y* is divisible by 18, find the least value of *y*.

(D) Application, Games and Puzzles

1. Polygonal Numbers

Study the patterns given below and extend it. We already know about square numbers.

*	* *	* * *
	* *	* * *
		* * *
1	4	9

Draw two more.

Here for the first square number, use 1^2; for the second square number, use 2^2. To find the third square number use 3^2 and so on. Write the nth square number.

Now let's move to triangular numbers.

*	*	*	*
	* *	* *	* *
		* * *	* * *
			* * * *
1	3	6	10

Find the next triangular number.

To find the nth triangular number we use the formula $\frac{n \times (n+1)}{2}$

Are you familiar with pentagonal numbers?

First three are given to you. Write the next one

*   *   *   *   *   *

1   *   *   *   *

*   *   *   *   *

5   *   *

*

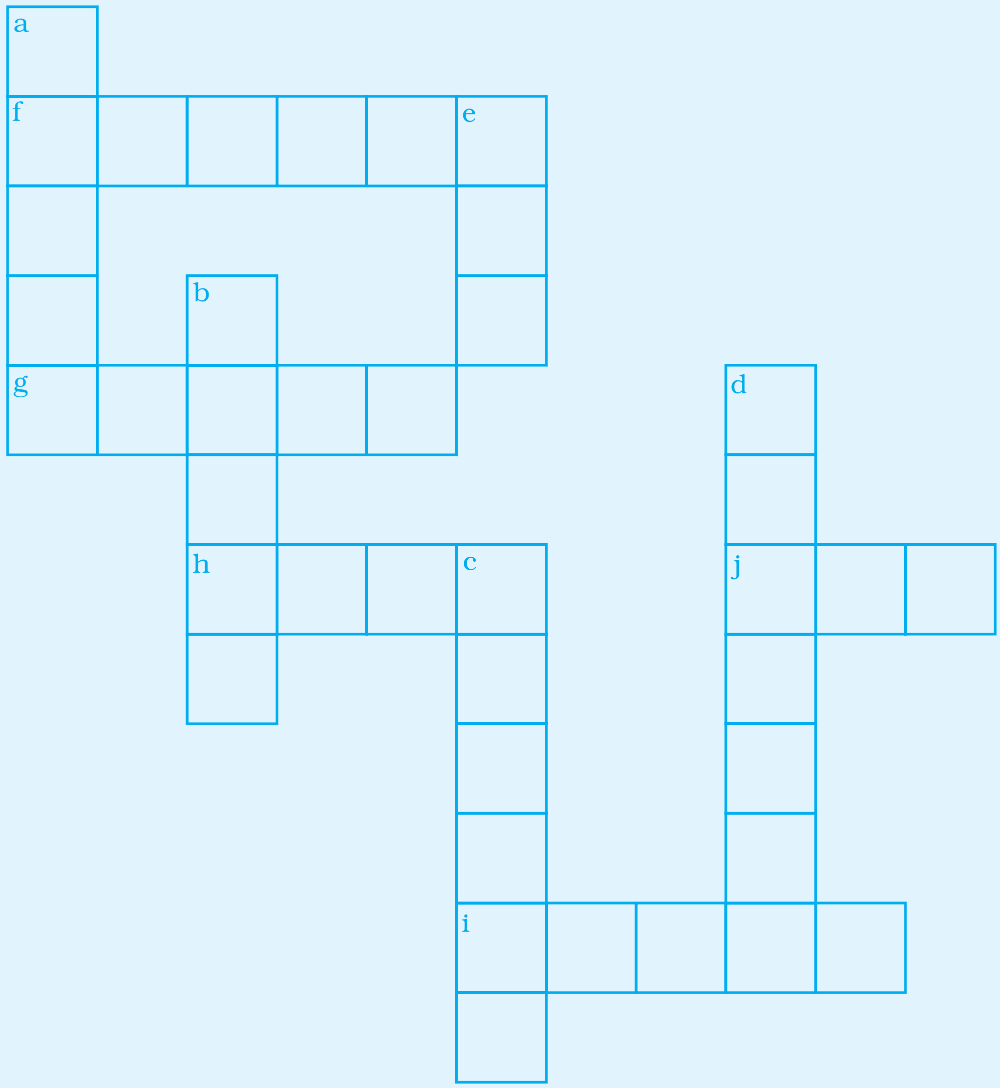
12

Draw the dot patterns for the next pentagonal number. Count the number of dots inside the entire shape and write the number under the shape.

2. Put tick mark in the appropriate boxes if the given numbers are divisible by any of 2, 3, 4, 5, 6, 8, 10, 11 numbers.

S.No.	Number	Divisible by									
		2	3	4	5	6	7	8	9	10	11
1.	40185										
2.	92286										
3.	56390										
4.	419562										
5.	10593248										

3. Cross Number Puzzle



Fill in the blank spaces in the cross number puzzle using following clues.

Down

(a) 59 ______ 63 ÷ 33

(b) 81 ______ 42 ÷ 6

(c) 7 ______ 6988 ÷ 11

(d) 37604 ______ 5 ÷ 15

(e) 56 ______ ÷ 10

Across

(f) 90 ______ 815 ÷ 15

(g) 3514 ______ ÷ 12

(h) 4 ______ 07 ÷ 7

(i) 8 ______ 558 ÷ 6

(j) 6 ______ 5 ÷ 55

Rough Work

Rough Work

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