CHAPTER

12 Mathematical Methods for Economics

"The master economist must possess a rare combination of gifts. He must be mathematician, historian, statesman, philosopher to some degree"

- J.M.Keynes

Learning Objectives

1 To understand why mathematics is required for economics,

2 To learn the knowledge of mathematical methods, as a facility for self-expression not only in descriptive economics, but also in quantitative economics.

12.1 Introduction

Economic analysis is a systematic approach to (a) determine the optimum use of scarce resources and (b)choose available alternatives and select the best alternative to achieve a particular objective. Mathematical methods are helpful for achieving the objectives of the economic analysis.

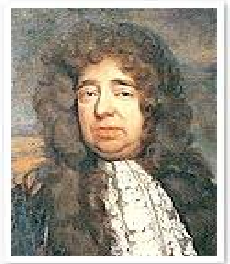
12.1.1 Why Study Mathematics?

The subject Economics deals with many quantitative variables and functions, in consumption, production, distribution and policy making. Hence, the mathematical methods would help economists to use the quantitative variables in a better way and to obtain accurate results.

The lengthy and descriptive economic contents can be clearly set in simple notation in mathematical models for clear and easy understanding. For example, **the number of pens demanded in a given time period in a Higher Secondary School is 200 when price is zero. This decreases by 10 for every ₹1 rise in the price of pen.** It is expressed mathematically as

$Q = 200 - 10P$, here Q is the quantity demanded and P is the price. Thus large information can be expressed and communicated with simple functions and equations.

12.1.2 Mathematics in Economics



Sir William Petty
1623-1687

Sir William Petty declared that he wanted to reduce political and economic matters in terms of number, weight and measure. He was the first one to use mathematics in economics. The first known writer to apply mathematical method to economic problems was Giovanni Ceva (1711), an Italian.

12.1.3 Uses of Mathematical Methods in Economics

1. Mathematical Methods help to present the economic problems in a more precise form.
2. Mathematical Methods help to explain economic concepts.
3. Mathematical Methods help to use a large number of variables in economic analyses.
4. Mathematical Methods help to quantify the impact or effect of any economic activity implemented by Government or anybody. There are of course many other uses.

Think and Do

- Who is the father of Economics? Did he use any of the mathematical tools in his contributions? If yes, list out.
- Find out the mathematical tools, which are used by you in your daily routine life.

12.2 Functions

12.2.1 Definition

A function is a mathematical relationship in which the values of a dependent variable are determined by the values of one or more independent variables.

Functions with a single independent variable are called Simple Univariate functions. There is a one to one correspondence. Functions,with more than one independent variable, are called Multivariate functions. The independent variable is often designated by X. The dependent variable is often designated by Y. For example, Y is function of X which means Y depends on X or the value of Y is determined by the value of X. Mathematically one can write $Y = f(X)$.

12.2.2 Linear Equation

A statement of relationship between two quantities is called an equation. In an equation, if the largest power of the independent variable is one, then it is called as Linear Equation. Such equations when graphed give straight lines. For example $Y = 100 - 10X$.

For a straight line, there are two variables namely X and Y. X is called independent variable and Y is called dependent variable.

When 'X' value increases by one unit, then the corresponding change in the 'Y' value is called as the slope of the line. Slope of the line is obtained by the formula,

m = slope (marginal change)

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}, \frac{Change\, in\, Y}{Change\, in\, X}.$$

Where (X_1, Y_1) and (X_2, Y_2) are two arbitrary points

Slope or Gradient of the line represents the ratio of the changes in vertical and horizontal lines.

The formula for constructing a straight line is

$$(Y - Y_1) = m(X - X_1)$$

If the two points are (0, 0) and (X, Y) then the formula is Y = mX

Example 12.1

Find the equation of a straight line which passes through two points (2, 2) and (4, –8) which are (X_1, Y_1) and (X_2, Y_2) respectively.

Note: For drawing a straight line, at least two points are required. Many straight lines can pass through a single point.

Solution

Here $X_1 = 2, \quad Y_1 = 2$

$X_2 = 4, \quad Y_2 = -8$

Formula for construction of straight line

$$\frac{Y - Y_1}{Y_2 - Y_1} = \frac{X - X_1}{X_2 - X_1}$$

Applying the values

$$\frac{Y-2}{-8-2} = \frac{X-2}{4-2}$$

$$\frac{Y-2}{-10} = \frac{X-2}{2}$$

$$2(Y-2) = -10(x-2)$$

$$2Y-4 = -10x + 20$$

$$2Y = -10x+24$$

$$Y = -5x + 12$$

–5 is slope, denoted by **m**

12 is **Y** intercept, or constant denoted by **c**

This is of the form Y = mX + c

Y = 12–5X when X = 0; Y = 12

When Y = 0; X = 12/5 = 2.4

(This line looks likes demand line in micro economics)

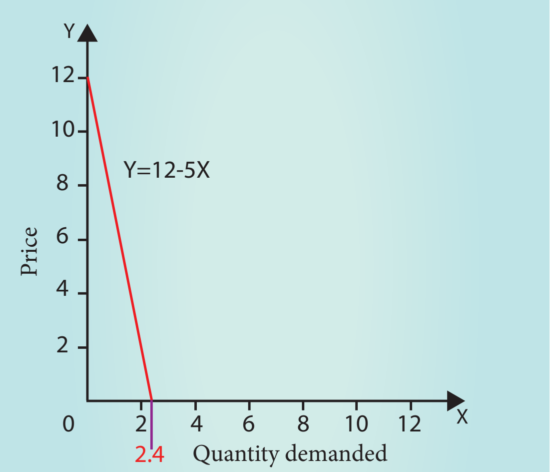


Diagram 12.1

12.2.3 Application in Economics

By applying the above method, the demand and supply functions are obtained.

Demand Function: $Q_d = f(P_x)$ where Q_d stands for Quantity demand of a commodity and P_x is the price of that commodity.

Supply Function: $Q_s = f(P_x)$ where 'Q_s' stands for Quantity supplied of a commodity and P_x is the price of that commodity.

In the example 12.1 the equation Y = –5X + 12 has been obtained. It is a linear function. Since slope is negative here, this function could be a demand function.

Demand Line

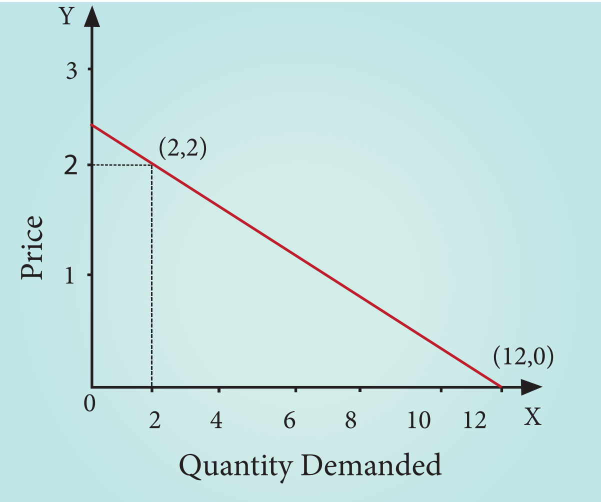


Diagram 12.2

Supply Line

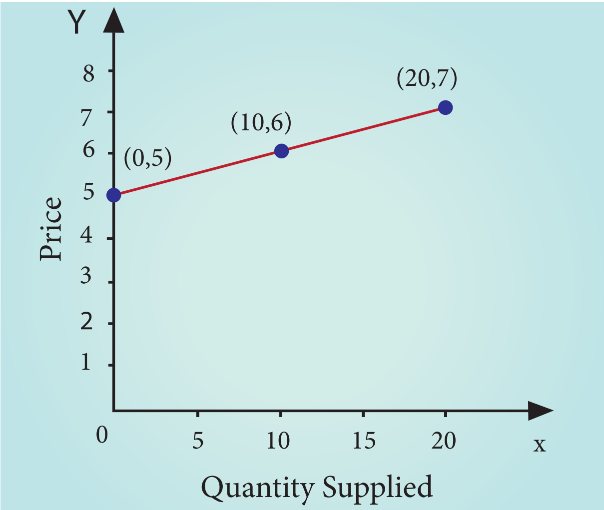


Diagram 12.3

Price-quantity relationship is negative in demand function. $Q_d = 12-5X$ or $Q_d = 12-5P$. If P = 2, $Q_d = 2$.

When P assumes 0, only 12 alone remains in the equation. This is called Intercept or Constant, if P = 0 and $Q_d = 12$.

In Marshallian analysis, money terms measured in Y-axis and physical units are measured in X-axis. Accordingly, price is measured in Y-axis and quantity demanded is measured in X-axis

Example: 12.2

Find the supply function of a commodity such that the quantity supplied is zero, when the price is ₹5 (or below) and the supply (quantity) increases continuously at the constant rate of 10 units for each one rupee rise when the price is above ₹5.

Solution:

To construct the linear supply function atleast two points are needed. First data point of supply function is obtained from the statement that the quantity supplied is zero, when the price is ₹5, that is (0, 5). The second and third data points of supply function can be obtained from the statement that supply increases 10 units for each one rupee rise in price, that is (10, 6) & (20, 7).

> When p = 5, supply is zero. When p = 6, supply is 10 and so on. When p is less than 5, say 4, supply is -10, which is possible in mathematics. But it is meaningless in Economics. Normally supply curve originates from zero, noting that when price is zero, supply is also zero.

The equation of the straight line joining two data points (10, 6) and (20, 7) is given as

The equation of the straight line is

$$\frac{Y-Y_1}{Y_2-Y_1} = \frac{X-X_1}{X_2-X_1}$$

substituting the values of (x_1, y_1) (x_2, y_2) by (10, 6) , (20, 7) respectively,

$$\frac{Y-6}{7-6} = \frac{X-10}{20-10}$$

$$\frac{Y-6}{1} = \frac{X-10}{10}$$

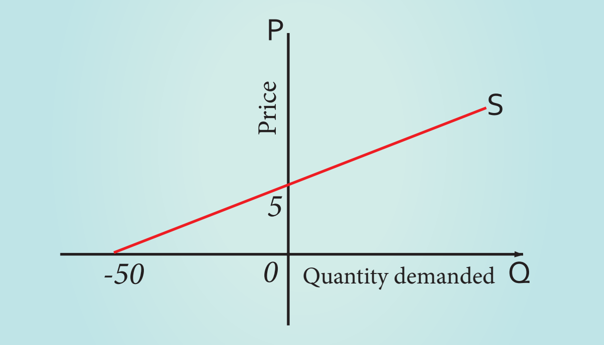


Diagram 12.4

Then $Y - 6 = \frac{X - 10}{10}$

$$10(Y - 6) = X - 10$$
$$10Y - 60 = X - 10$$
$$10Y - 60 + 10 = X$$
$$10Y - 50 = X$$
$$-X = -10Y + 50$$

Multiplying both sides by minus (–), we get

$$X = -50 + 10Y$$

Considering X as quantity supplied and Y as price (P)

Then $X = 10P - 50$ (or)

$$\boxed{X = -50 + 10\,P}$$

If Price = 0; Q = –50

If Q = 0; P = 5

Note: The coefficient of 'P' is – in demand function.

The coefficient of 'P' is + in supply function.

12.2.4 Equilibrium

The point of intersection of demand line and supply line is known as equilibrium. The point of equilibrium is obtained by using the method of solving a set of equations. One can obtain the values of two unknowns with two equations. At equilibrium point,

Demand = Supply

(These are hypothetical examples)

$$100 - 10P = 50 + 10\,P$$
$$100 - 50 = 20P$$
$$50 = 20P$$
$$\frac{50}{20} = P$$
$$P = 2.5$$

When P = 2.5, Demand = 100–10 (2.5)
= 75

When P = 2.5, Supply = 50 +10 (2.5)
= 75

Example: 12.3

Find the equilibrium price and quantity by using the following demand and supply functions Qd = 100–5P and $Q_s = 5P$ respectively.

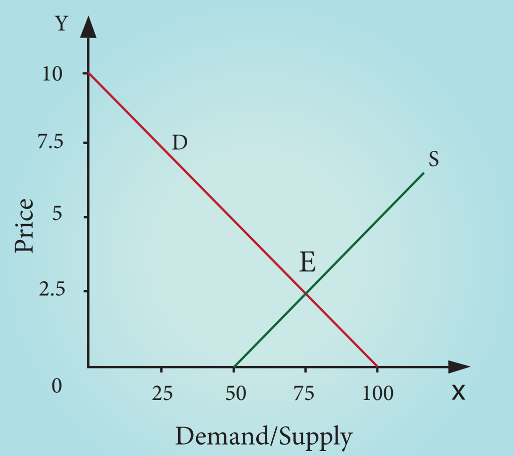


Diagram 12.5

Solution:

Equilibrium is attained when,

$$Q_s = Q_d$$
$$5\,P = 100 - 5\,P$$
$$10P = 100$$
$$P = 10$$

When P = 10

In supply function

$$Q_s = 5\,P = 5 \times 10 = 50$$

In demand function,

$Q_d = 100 - 5\,P = 100 - 5(10) = 50$

Hence at

$\boxed{P = 10,\ Q_d = 50,\ Qs = 50.}$

Quantity demanded is equal to supply at 50 units when price is ₹10

Example: 12.4

The market demand curve is given by D = 50 – 5P. Find the maximum price beyond which nobody will buy the commodity.

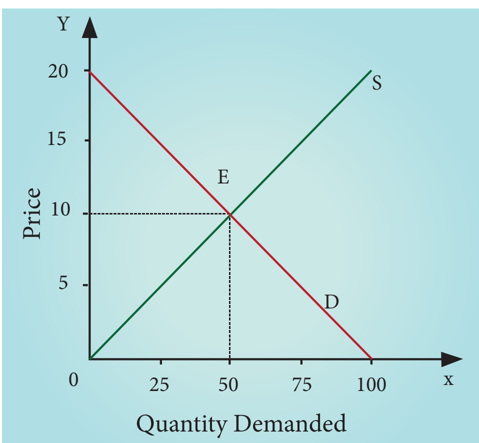


Diagram 12.6

Solution:

Given

$Q_d = 50 - 5P$

$5P = 50 - Q_d$

$5P = 50$ when Q_d is zero.

$$P = \frac{50}{5}$$

P = 10 When P = 10, Demand is 0

Hence P = 10, which is the maximum price beyond which nobody will demand the commodity.

Example: 12.5

The demand for milk is given by

Price (Y)	1	2	3
Demand (X)	100	50	0

Find the linear demand function and its slope.

Solution:

Equation of demand function joining two data points (100, 1) and (50, 2) are (x_1, y_1) and (x_2, y_2) respectively.

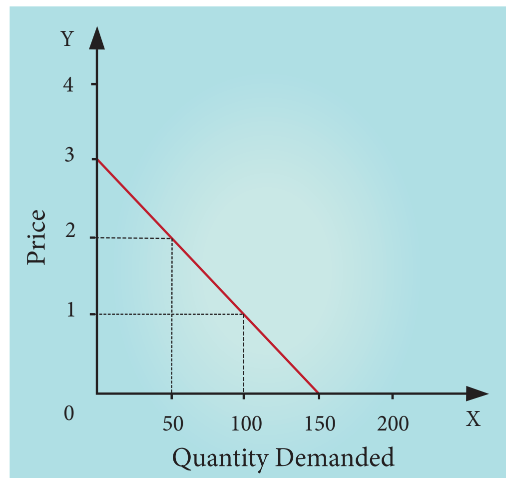


Diagram 12.7

$$\frac{Y - Y_1}{Y_2 - Y_1} = \frac{X - X_1}{X_2 - X_1}$$

$$\frac{Y - 1}{2 - 1} = \frac{X - 100}{50 - 100}$$

$$\frac{Y - 1}{1} = \frac{X - 100}{-50}$$

$$-50\,(Y - 1) = 1\,(X - 100)$$

$$-50Y + 50 = X - 100$$

$$-50Y + 50 + 100 = X$$

$$-50Y + 150 = X$$

$$X = 150 - 50Y$$

Hence the demand function is

$\boxed{Q_d = 150 - 50P \text{ and Slope } m = -50}$

Think and Do for Water Management in your area

Try to find the demand function for water in your street and the daily total demand for water in litre for all purposes.

12.3 Matrices

12.3.1 Matrices

'Matrix' is a singular while 'matrices' is a plural form. Matrix is a rectangular array of numbers systematically arranged in rows and columns within brackets. In a matrix, if the number of rows and columns are equal, it is called a square matrix.

12.3.2 Determinants

For every square matrix, there exists a determinant. This determinant is an arrangement of same elements of the corresponding matrix into rows and columns by enclosing vertical lines.

For example,

$\begin{pmatrix} 1 & 3 & 5 \\ 6 & 2 & 4 \\ 7 & 8 & 9 \end{pmatrix}$ is a square matrix of order 3 x 3,then

$\begin{vmatrix} 1 & 3 & 5 \\ 6 & 2 & 4 \\ 7 & 8 & 9 \end{vmatrix}$ is a determinant.

$\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$ is a square matrix of order 2 x 2, then

$\begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix}$ is a determinant.

In general, if $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is a matrix then,

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is a determinant of the matrix A denoted by $|A|$.

The value of the determinant is expressed as a single number.

Calculation of the value of determinant for a 2 x 2 matrix is shown below

$$\text{If } |A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \; then |A| = a_1 b_2 - a_2 b_1$$

Calculation of determinant value for a 3 x 3 matrix is shown below

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

Example: 12.6

Find the value of the determinant for the matrix A= $\begin{pmatrix} 3 & 4 \\ 10 & -2 \end{pmatrix}$

Solution:

Given matrix A = $\begin{pmatrix} 3 & 4 \\ 10 & -2 \end{pmatrix}$ then, the Determinant

$$|A| = \begin{vmatrix} 3 & 4 \\ 10 & -2 \end{vmatrix} = 3(-2) - 10(4)$$

= − 6 − 40 = − 46 is the value of the determinant.

Example: 12.7

Find the value of the determinant of the matrix

$$A = \begin{pmatrix} 3 & 4 & 7 \\ 2 & 1 & 3 \\ 7 & 2 & 1 \end{pmatrix}$$

Solution:

Determinant of the given matrix is,

$$|A| = \begin{vmatrix} 3 & 4 & 7 \\ 2 & 1 & 3 \\ 7 & 2 & 1 \end{vmatrix} = 3\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 7 & 1 \end{vmatrix} + 7\begin{vmatrix} 2 & 1 \\ 7 & 2 \end{vmatrix}$$

$$= 3\ (1 - 6)\ -4\ (2 - 21) + 7\ (4 - 7)$$

$$= 3\ (-5)\ -4\ (-19) + 7\ (-3)$$

$$= -15 + 76 - 21$$

$$\boxed{|A| = 40}$$

The value of determinant is 40.

12.3.3 Cramer's Rule

G. CRAMER
(1704-1752)

Cramer's rule provides the solution of a system of linear equations with '**n**' variables and '**n**' equations. It helps to arrive at a unique solution of a system of linear equations with as many equations as unknowns.

If the given equations are

$$a_{11}x + a_{12}\,y + a_{13}\,z = b_1$$

$$a_{21}x + a_{22}\,y + a_{23}\,z = b_2$$

$$a_{31}x + a_{32}\,y + a_{33}\,z = b_3$$

then

$$x = \frac{\Delta x}{\Delta},\quad y = \frac{\Delta y}{\Delta},\quad z = \frac{\Delta z}{\Delta}$$

where, $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix},$$

$$\Delta z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

Key Note

If the determinant $\Delta = 0$, then solution does not exist.

Example: 12.8

Find the value of x and y in the equations by using Cramer's rule. x + 3y = 1 and 3x – 2y = 14

Solution:

Given equations are

$$x + 3y = 1$$

$$3x - 2y = 14$$

Then the equations in the matrix form :

$$\begin{pmatrix} 1 & 3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 14 \end{pmatrix}$$

Calculating Δ, $\Delta = \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix}$

$$= -2 - 9$$

$$= -11$$

$\Delta \neq 0$, Hence solution exists.

$$\Delta_x = \begin{vmatrix} 1 & 3 \\ 14 & -2 \end{vmatrix} = -2 - 42 = -44$$

$$\Delta_y = \begin{vmatrix} 1 & 1 \\ 3 & 14 \end{vmatrix} = 14 - 3 = 11$$

Hence $x = \frac{\Delta x}{\Delta} = \frac{-44}{-11} = 4$, $y = \frac{\Delta y}{\Delta} = \frac{11}{-11} = -1$

$\therefore x = 4$ and $y = -1$

Answer checking:

Substituting in equation the values of x and y,

4 + 3(-1) = 1,

3(4) – 2(-1)= 14

Example: 12.9

Find the solution of the system of equations.

$$5x_1 + 3x_2 = 30$$

$$6x_1 - 2x_2 = 8$$

Solution:

The coefficient and the constant terms are given below for the equations

$$\Delta = \begin{vmatrix} 5 & 3 \\ 6 & -2 \end{vmatrix} = -10 - 18 = -28$$

$$\Delta x_1 = \begin{vmatrix} 30 & 3 \\ 8 & -2 \end{vmatrix} = -60 - 24 = -84$$

$$\Delta x_2 = \begin{vmatrix} 5 & 30 \\ 6 & 8 \end{vmatrix} = +40 - 180 = -140$$

$$\therefore x_1 = \frac{\Delta x_1}{\Delta} = \frac{-84}{-28} = 3$$

$$x_2 = \frac{\Delta x_2}{\Delta} = \frac{-140}{-28} = 5$$

$$\therefore x_1 = 3, \quad x_2 = 5$$

Example: 12.10

Find the solution of the system of equation

$$7x_1 - x_2 - x_3 = 0$$

$$10x_1 - 2x_2 + x_3 = 8$$

$$6x_1 + 3x_2 - 2x_3 = 7$$

Solution:

The matrix form of the given equation is written as

$$\begin{bmatrix} 7 & -1 & -1 \\ 10 & -2 & 1 \\ 6 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 7 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -1 & -1 \\ 10 & -2 & +1 \\ 6 & 3 & -2 \end{vmatrix}$$

$$= 7(4-3) - (-1)(-20-6) + (-1)(30+12)$$

$$= 7(1) + 1(-26) - 1(42)$$

$$= 7 - 26 - 42 = -61$$

$$\Delta x_1 = = \begin{vmatrix} 0 & -1 & -1 \\ 8 & -2 & 1 \\ 7 & 3 & -2 \end{vmatrix}$$

$$= 0(4-3) - (-1)(-16-7) + (-1)(24+14)$$

$$= 0 + 1(-23) - 1(38)$$

$$= -23 - 38 = -61$$

$$\Delta x_2 = = \begin{vmatrix} 7 & 0 & -1 \\ 10 & 8 & 1 \\ 6 & 7 & -2 \end{vmatrix}$$

$$= 7(-16-7) - 0(-20-6) + (-1)(70-48)$$

$$= 7(-23) + 0 - 1(22)$$

$$= -161 - 22 = -183$$

$$\Delta x_3 = = \begin{vmatrix} 7 & -1 & 0 \\ 10 & -2 & 8 \\ 6 & 3 & 7 \end{vmatrix}$$

$$= 7(-14-24) - (-1)(70-48) + 0(30+12)$$

$$= 7(-38) + 1(22) + 0$$

$$= -266 + 22 = -244$$

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{-61}{-61} = 1$$

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$$x_2 = \frac{\Delta x_2}{\Delta} = \frac{-183}{-61} = 3$$

$$x_3 = \frac{\Delta x_3}{\Delta} = \frac{-244}{-61} = 4$$

12.3.4 Application in Economics

Example 12.11

Mr.Anbu, purchased 2 pens, 3 pencils and 1 note book. Mr.Barakath , purchased 4 pens, 3 pencils and 2 notebooks. Mr.Charles purchased 2 pens, 5 pencils and 3 notebooks. They spent ₹32, ₹52 and ₹60 respectively. Find the price of a pen, a pencil and a note book.

Solution:

Let x be the price of a pen, y be the price of a pencil and z be the price of a notebook,

In equations:

2x + 3y + 1z = 32,

4x + 3y + 2z = 52,

2x + 5y +3z = 60

In matrix form

$$\begin{bmatrix} 2 & 3 & 1 \\ 4 & 3 & 2 \\ 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 32 \\ 52 \\ 60 \end{bmatrix}$$

$$\begin{aligned} \Delta &= 2(9-10)-3(12-4)+1(20-6) \\ &= 2(-1)-3(8)+1(14) \\ &= -2-24+14=-12 \end{aligned}$$

To find Δ_x

$$\Delta_x = \begin{vmatrix} 32 & 3 & 1 \\ 52 & 3 & 2 \\ 60 & 5 & 3 \end{vmatrix}$$

$$\begin{aligned} \Delta_x &= 32(9-10)-3(156-120)+1(260-180) \\ &= 32(-1)-3(36)+1(80) \\ &= -32-108+80=-60 \end{aligned}$$

To find Δ_y

$$\Delta_y = \begin{vmatrix} 2 & 32 & 1 \\ 4 & 52 & 2 \\ 2 & 60 & 3 \end{vmatrix}$$

$$\begin{aligned} \Delta y &= 2(156-120)-32(12-4)+1(240-104) \\ &= 2(36)-32(8)+1(136) \\ &= 72-256+136=-48 \end{aligned}$$

To findΔ_z

$$\Delta_z = \begin{vmatrix} 2 & 3 & 32 \\ 4 & 3 & 52 \\ 2 & 5 & 60 \end{vmatrix}$$

$$\begin{aligned} \Delta z &= 2(180-260)-3(240-104)+32(20-6) \\ &= 2(-80)-3(136)+32\,(14) \\ &= -160-408+448=-120 \end{aligned}$$

$$x = \frac{-60}{-12} = 5 \quad (\text{Price of a pen})$$

$$y = \frac{-48}{-12} = 4 \quad (\text{Price of a pencil})$$

$$z = \frac{-120}{-12} = 10\,(\text{Price of a notebook})$$

Answer checking

2(5)+3(4)+1(10)=32

4(5)+3(4)+2(10)=52

2(5)+5(4)+3(10)=60

Think and Do

Fathima, purchased 6 pens and 5 Pencils spending ₹49, Rani purchased 3 Pens and 4 pencils spending ₹32. What is the price of a pen and pencil?

$$\begin{bmatrix} 6 & 5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 49 \\ 32 \end{bmatrix}$$

Solution : Price of a pen = ₹4

Price of a pencil = ₹5

12.4 DIFFERENTIAL CALCULUS

12.4.1 Meaning

The fundamental operation of calculus is differentiation. Derivative is used to express the rate of change in any function. Derivative means a change in the dependent variable with respect to small change (closer to zero) in independent variable.

Let the function be,

y = f(x)

Differentiating y with respect to x is,

$$\frac{d(y)}{dx} = \frac{df(x)}{dx}$$

12.4.2 Some Standard Forms of Differentiation

(Constant, addition and subtraction only)

1. $\frac{d(c)}{dx} = 0$ where C is a constant.

   (Read differentiation of 'C' with respect to 'x' is)

2. $\frac{d(x^n)}{dx} = nx^{n-1}$

3. $\frac{d(x)}{dx} = 1x^{1-1} = 1x^0 = 1$

4. $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

5. $\frac{d(u-v)}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

Key Note

If the power of any non-zero real number is zero, it's value is 1 $x^0 = 1$ when $x \neq 0$

Example: 12.12

If Y = 4, then find $\frac{dy}{dx}$

Solution:

Y = 4, here 4 is a constant.Differentiation of constant function is zero.

So, $\frac{dy}{dx} = \frac{d(4)}{dx} = 0$

Example: 12.13

Find the slope of the function y = $6x^3$ for any value of x.

Solution:

Given y = $6x^3$

Slope = $\frac{dy}{dx}$

$\frac{dy}{dx} = 6(3)x^{3-1} = 18x^2$ for any value of x.

Example: 12.14

What is the slope of the function y = $5x^4$ when x = 10?

Solution:

Given function y = $5x^4$

Slope = $\frac{dy}{dx}$

$\frac{dy}{dx} = 5(4)x^{4-1}$

$= 20x^3$

When x = 10, then slope = $20\ (10)^3$

= 20,000,

Therefore Slope is 20,000.

Example: 12.15

Differentiate the function $Y = 3x^2 + 16x^3$ with respect to x.

Solution:

$$Y = 3x^2 + 16x^3$$

Differentiating,

$$\frac{dy}{dx} = 3(2)x^{2-1} + 16(3)x^{3-1}$$

$$= 6x^1 + 48x^2$$

$$\frac{dy}{dx} = 6x + 48x^2$$

Example: 12.16

If $Y = 2x^3 - 6x$, then find $\frac{dy}{dx}$

Solution:

$Y = 2x^3 - 6x$

Differentiate 'y' with respect to x,

$$\frac{dy}{dx} = 2(3)x^{3-1} - 6(1)x^{1-1}$$

$$= 6x^2 - 6x^0$$

$$\frac{dy}{dx} = 6x^2 - 6$$

12.4.3 Application of Differential Calculus

The relation between two or more variables can be expressed by means of a function. Continuous functions alone are differentiable. For instance, the differential calculus is applicable for finding the following:

(1) The rate of change in demand with respect to price (in micro economics)

(2) The rate of change in income with respect to the investment. (in macroeconomics) .

12.4.4 Marginal concepts

Marginal concept is concerned with variations of Y (on the margin of X), that is, it is the variation corresponding in Y to a very small variation in X. (X is the independent variable and Y is the dependent variable)

12.4.5 Marginal Product

Marginal product of a factor of production refers to addition to total product due to the use of an additional unit of a factor.

$$MP = d(TP)/dQ = \Delta TP/\Delta Q$$

12.4.6 Marginal Cost

Marginal cost is an addition to the total cost caused by producing one more unit of output. In symbols:

$$MC = \frac{d(TC)}{dQ} \text{ or } MC = \frac{\Delta(TC)}{\Delta Q}$$

Where, ΔTC represents a change in total cost and ΔQ represents a small change in output or quantity. (in economics one worker, one output etc are assumed to be very small units)

Example: 12.17

Given the total cost function, $TC = 15 + 3Q^2 + 7Q^3$, drive the marginal cost function.

Solution:

$$TC = 15 + 3Q^2 + 7Q^3$$

$$MC = \frac{d(15)}{dQ} + \frac{d(3Q^2)}{dQ} + \frac{d(7Q^3)}{dQ}$$

$$= 0 + 3(2)Q^{2-1} + 7(3)Q^{3-1}$$

$$MC = 6Q + 21Q^2$$

12.4.7 Marginal Revenue

Marginal Revenue is the revenue earned by selling an additional unit of the product. In other words, Marginal Revenue is an addition made to the total revenue by selling one more unit of the good.

$$MR = \frac{d(TR)}{dQ} \text{ or } = \frac{\Delta TR}{\Delta Q}$$

Where ΔTR stands for change in the total revenue, and ΔQ stands for change in output.

Example: 12.18

Given TR = 50Q – $4Q^2$,find marginal revenue when Q = 3.

Solution:

$TR = 50Q - 4Q^2$

$MR = d(TR)/dQ$

$MR = 50(1)Q^{1-1} - 4(2)Q^{2-1}$

$= 50(1)Q^0 - 8\,Q^1$

$= 50(1) - 8Q \quad (\therefore Q^0 = 1, Q^1 = Q)$

$MR = 50 - 8Q$

When Q = 3

$MR = 50 - 8(3) = 26$

Example: 12.19

A producer has the total cost function $TC(Q) = Q^3 - 18Q^2 + 91Q + 10$ where costs are given in rupees. Find the marginal cost (MC) and the average variable cost (AVC), when Q =3.

Solution:

Given $TC(Q) = Q^3 - 18Q^2 + 91Q + 10$, To find MC differentiate the function with respect to Q.

$$MC(Q) = \frac{d(TC)}{dQ} = 3Q^{3-1} - 18(2)Q^{2-1} + 91(1)Q^{1-1} + 0$$

$= 3Q^2 - 36Q^1 + 91Q^0 + 0$

$MC(Q) = 3Q^2 - 36Q + 91 \quad (\therefore Q^0 = 1)$

When Q = 3

$MC(Q) = 3(3^2) - 36(3) + 91$

$= 3(9) - 108 + 91$

$= 27 - 108 + 91$

$= 118 - 108$

$= 10$

To find AVC

Given $TC(Q) = Q^3 - 18Q^2 + 91Q + 10$

We know $TVC(Q) = Q^3 - 18Q^2 + 91Q$ ($\therefore$constant value is fixed cost)

$AVC(Q) = TVC(Q)/Q$

$$\boxed{AVC(Q) = Q^2 - 18Q + 91}$$

When Q = 3

$AVC(Q) = 3^2 - 18(3) + 9$

$= 9 - 54 + 91$

$= 100 - 54 = 46$

$$\therefore AVC(Q) = \frac{Q^3 - 18Q^2 + 91Q}{Q}$$

$= Q^2 - 18Q + 91$

Note : Fixed cost= 10

Average fixed cost = $\frac{10}{Q}$.

$$\text{Average cost} = \frac{Q^3 - 18Q^2 + 91Q + 10}{Q}$$

$$= Q^2 - 18Q + 91 + \frac{10}{Q}$$

So Average cost = AVC + AFC

AC = AVC + AFC

$$AC = \frac{TC}{Q}$$

Example: 12.20

A manufacturer estimates that, when units of a commodity are produced each month the total costs will be TC(Q) = 128 + 60Q + $8Q^2$ Find the marginal cost, average cost, fixed cost, variable cost, average fixed cost and average variable cost.

Solution:

Given that $TC(Q) = 128 + 60Q + 8Q^2$

We know TC = Fixed cost + variable cost

$$MC(Q) = \frac{d(TC)}{dQ}$$

$$= 0 + 60(1)Q^{1-1} + 8(2)Q^{2-1}$$

$$= 0 + 60Q^0 + 16Q^1 \text{(Since, } Q^0 = 1)$$

$$\boxed{MC = 60 + 16Q}$$

$$Average\ Cost = \frac{TC}{Q}$$

$$= \frac{128 + 60Q + 8Q^2}{Q}$$

$$AC = \frac{128}{Q} + 60 + 8Q$$

Constant value is known as fixed cost

Fixed cost = 128

FC = 128

$$\text{Average Fixed cost} = \frac{128}{Q}$$

$$AFC = \frac{128}{Q}$$

Average Variable cost =60 + 8Q (total variable cost divided by Q)

$$\therefore AVC = 60 + 8Q$$

12.4.8 Elasticity of Demand

Elasticity of Demand is the ratio of the proportionate change in quantity demanded to the proportionate change in price. In mathematical terms,

$$e_d = \left(\frac{P}{x}\right)\left(\frac{dx}{dp}\right)$$

In demand function Q = a–bP

$$e_d = (dQ/dP)(P/Q)$$

Example 12.21

If the demand function is $x = \frac{100}{P}$, find e_d with respect to price at the point where P = 2

Note

By taking supply function, the elasticity of supply can be calculated

Solution:

Given

$$x = \frac{100}{P} = 100P^{-1}$$

$$\therefore \frac{dx}{dp} = 100(-1)P^{-1-1}$$

$$= 100(-1)P^{-2}$$

$$= -100(P^{-2})$$

$$= \frac{-100}{P^2}$$

At P=2,

$$\frac{dx}{dp} = \frac{-100}{4} = -25 \text{ and } x = \frac{100}{2} = 50$$

Substituting the values in formula

$$e_d = \frac{Pdx}{xdp} = \left(\frac{2}{50}\right)\left(\frac{-100}{4}\right) = \frac{-200}{200} = -1$$

$$e_d = -1$$

12.5 Integral Calculus

12.5.1 Integration

Differential calculus measures the rate of change of functions. In Economics it is also necessary to reverse the process of differentiation and find the function F(x) whose rate of change has been given. This is called integration. The function F(x) is termed an integral or anti- derivative of the function f(x).

The integral of a function f(x) is expressed mathematically as

$$\int f(x)dx = F(x)+C$$

Here the left hand side of the equation is read "the integral of f(x) with respect to x" The symbol $\int$ is an integral sign, f(x) is integrand, C is the constant of integration, and F(x)+c is an indefinite integral. It is so called because, as a function of x, which is here unspecified, it can assume many values.

12.5.2 Meaning

If the differential coefficient of $F(x)$ with respect to x is f(x), then an integral of f(x) with respect to x is F(x). It is a reverse process of differentiation. In symbols:

$$\text{If } \frac{d[F(x)]}{d_x} = f(x), \text{ then } \int f(x)dx = F(x)+C$$

Following points need to be remembered:

a. $\int$ is used to denote the process of integration. In fact, this symbol is an elongated 'S' denoting sum.

b. The differential symbol 'dx' is written by the side of the function to be integrated.

c. $\int f(x)dx = F(x)+C$, C is the integral constant

$\int f(x)dx$ means, integration of f(x) with respect to x.

12.5.3 Basic Rule of Integration

(i) Power Rule $\int x^n \, dx = \frac{x^{(n+1)}}{n+1} + C$

(ii) $\int k.dx = kx + c$, where k is a constant

(iii) $\int a.x^n \, dx = a\int x^n \, dx$

Example 12.22

$$\int 4x^3 dx = 4\int x^3 dx$$
$$= 4\frac{x^{3+1}}{3+1} + c$$
$$= 4\frac{x^4}{4} + c$$
$$= x^4 + c$$

Example12.23

$$\int (x^2 + x - 1)dx = \int x^2 dx + \int x dx - \int dx$$
$$= \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} - x + c$$
$$= \frac{x^3}{3} + \frac{x^2}{2} - x + c$$

Example 12.24

$$\int 5dx = 5x + c$$

Example12.25

$$\int 4x dx = 4\frac{x^{1+1}}{1+1} + c$$
$$= 4\frac{x^2}{2} + c$$
$$= 2x^2 + c$$

12.5.4 Application of Integration

Example 12.26

Let the marginal cost function of a firm be 100–10x+0.1x^2 where x is the output. Obtain the total cost function of the firm under the assumption that its fixed cost is ₹500.

Solution

$$MC = 100 - 10x + 0.1\,x^2$$

$$TC = \int \left(100 - 10x + 0.1x^2\right) dx$$

$$= 100x - 10\frac{x^2}{2} + 0.1\frac{x^3}{3} + c$$

$$= 100x - 5x^2 + \frac{x^3}{30} + c$$

Fixed cost is given as ₹500

$$\therefore TC = 100x - 5x^2 + \frac{x^3}{30} + 500$$

$$= \frac{x^3}{30} - 5x^2 + 100x + 500$$

Example 12.27

The marginal cost function for producing x units is y = 23 + 16x – 3x^2 and the total cost for producing zero unit is ₹40. Obtain the total cost function and the average cost function.

Solution:

Given the marginal cost function y = 23 + 16x – 3x^2; c = 40

₹40 is the fixed cost.

We know that

Total Cost function = ∫ (Marginal cost function) dx+c

$$TC = \int y\,dx + c$$

$$= \int (23 + 16x - 3x^2)dx + c,$$ where c is a constant

$$= \int 23dx + \int 16x dx - \int 3x^2\,dx + c$$

$$= 23x + 16\left[\frac{x^2}{2}\right] - 3\left[\frac{x^3}{3}\right] + c$$

$$TC = 23x + 8x^2 - x^3 + c$$

c = 40 given

$$\therefore TC = 23x + 8x^2 - x^3 + 40$$

$$\text{Average cost function} = \frac{TC}{x}$$

$$= 23 + 8x - x^2 + \frac{40}{x}$$

12.5.5 Consumer's Surplus

This theory was developed by the Alfred Marshall. The demand function P(x) reveals the relationship between the quantities that the people would buy at given price. It can be expressed as

$$P = f(x)$$

Consumer surplus is the difference between the price one is willing to pay and the price that is actually paid.

It is represented in the following diagram.

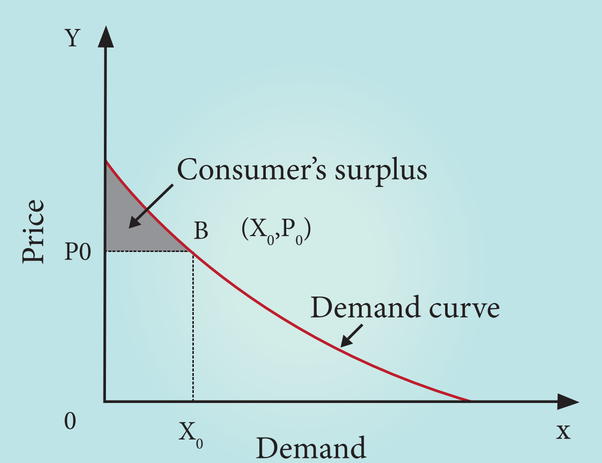


Diagram 12.8

Mathematically, the consumer's surplus (CS) can be defined as

CS = (Area under the demand curve from x = 0 to x = x_0) – (Area of the rectangle OX_0BP_0)

$$CS = [\int_0^{x0} p(x)\,dx] - x_0 p_0$$

Example:12.28

If the demand function is $P = 35 - 2x - x^2$ and the demand x_0 is 3, what will be the consumer's surplus?

Solution

Given demand function,

$$P = 35 - 2x - x^2$$

for x = 3

$$P = 35 - 2(3) - 3^2$$

$$= 35 - 6 - 9$$

$$P = 20$$

Therefore,

CS = (Area of the curve below the demand curve from 0 to 3) – Area of the rectangle (20 x 3 = 60)

$$CS = \int_0^3 (35-2x-x^2)dx - (20\times3)$$

$$= \left[35x - 2\frac{x^2}{2} - \frac{x^3}{3}\right]_0^3 - 60$$

$$= 35(3) - 2\left(\frac{3^2}{2}\right) - \frac{3^3}{3} - 60$$

$$= 105 - 9 - 9 - 60$$

= 27 Units.

12.5.6 Producer's surplus

$$PS = P_o x_o - \int_o^{x_o} g(x)\,dx$$

Example 12.29

Given the demand function $P_d = 25 - Q^2$ and the supply function $P_s = 2Q + 1$. Assuming pure competition, find (a) consumers surplus and (b) producers surplus. (P_d = Demand price; P_s = Supply price)

Solution:

For market equilibrium, $P_d = P_s$

$$25 - Q^2 = 2Q+1$$

$$0 = -25 + Q^2 + 2Q + 1$$

$$0 = -24 + Q^2 + 2Q$$

$$Q^2 + 2Q - 24 = 0$$

$$Q^2 + 6Q - 4Q - 24 = 0$$

$$Q(Q+6) - 4(Q+6) = 0$$

$$(Q+6)(Q-4) = 0$$

So, Q = 4 or Q = –6. Since Q cannot be equal to –6,

$$Q = 4$$

When Q=4, $P_d = 25 - 4^2 = 9$;

$$P_s = 2(4) + 1 = 9$$

Consumers' surplus = $\int_0^4 (25 - Q^2)dQ$ – (9 X 4)

$$= \left[25Q - \frac{Q^3}{3}\right]_0^4 - 36$$

$$= [(25)(4) - \frac{1}{3}(4)^3] - (0) - 36$$

$$= [100 - \frac{64}{3}] - (0) - 36 = 42.67$$

Producers' surplus PS

$$(PS) = (9\times4) - \int_0^4 (2Q+1)dQ$$

$$= 36 - \left[(Q^2+Q)\right]_0^4$$

$$= 36 - (16+4) = 16$$

Think and Do

- Find your change in mark by additional hour of study in any of your subject
- Find your consumption of petrol for an additional unit of kilometer travelled
- Ask your parents about their spending with respect to every additional unit of wage or salary or income

12.6 Information and Communication Technology (ICT)

Information and Communication Technology (ICT) is the infrastructure that enables computing faster and accurate. The following table gives an idea of range of technologies that fall under the category of ICT.

S.No	Information	Technologies
1	Creation	Personal Computers, Digital Camera, Scanner, Smart Phone
2	Processing	Calculator, PC, Smart Phone
3	Storage	CD, DVD, Pen Drive, Microchip, Cloud
4	Display	PC, TV, Projector, Smart Phone
5	Transmission	Internet, Teleconference, Video conferencing, Mobile Technology, Radio
6	Exchange	E mail, Cell phone

The evaluation of ICT has five phases. They are evolution in

(a) Computer
(b) PC
(c) Microprocessor
(d) Internet and
(e) Wireless links

In Economics, the uses of mathematical and statistical tools need the support of ICT for data compiling, editing, manipulating and presenting the results. In general, SPSS and Excel packages are often used by researchers in economics. Such Software is designed to do certain user tasks.Word processor, spread sheet and web browser are some of the examples which are frequently used while undertaking analysis in the study of economics.

12.6.1 MS Word

MS word is a word processor, which helps to create, edit, print and save documents for future retrieval and reference.

The features of word processor are

a) Document can be created, copied, edited and formatted.

b) Words and sentences can be inserted, changed or deleted.

c) Formatting can be applied.

d) Margins and page size can be adjusted.

f) Spell check can be availed.

g) Multiple documents – files can be merged.

How to open a word Document?

One can open MSWord from various options.

Click start → All program → MS word or Double click the MS word icon from the desktop.

Uses of Menu

Home menu	→	It is used to change the fonts, font size, change the text color and apply text style bold, italic, underline etc.
Insert	→	It is used to insert page numbers, charts, tables, shapes, word art forms, equations, symbols and pictures.
Page Layout	→	It is used to change the margin size, split the text into more columns, background colour of a page.
Reference	→	Insert table of authors, endnote, footnote
Review	→	Spell check, Grammar, Translate.
View	→	Print layout, full screen reading, document view

12.6.2 Microsoft Office Excel

It is used in data analysis by using formula. A spread sheet is a large sheet of paper which contains rows and columns. The intersection of rows and columns is termed as 'cell'. MS Excel 2007 version supports up to 1 million rows and 16 thousand columns per work sheet.

Start

You can start excel from various options.

- Click Start → Program → Micro Soft Excel.
- Double Click the MS Excel Icon from the Desk top.

Work Sheet

A worksheet is a table like document containing rows and columns with data and formula. There are four kinds of calculation operators. They are arithmetic, comparison, text concatenation (link together) and reference. MS Excel helps to do data analysis and data presentation in the form of graphs, diagrams, area chart, line chart etc.

12.6.3 Microsoft Power Point

It is a software used to perform computer based presentation.

Steps involved in making presentation:

(i) Click Start Menu

(ii) Click Program

(iii) Select Microsoft Power point – Click.

(iv) New Power Point file will open, and then type the title and subtitle if wanted.

(v) A new slide can be inserted by 'click' on icon 'new slide' or using short key 'Ctrl + M'

(vi) We can type the content, insert the table, pictures, movies, sounds, etc., with the content.

(vii) Tab 'Design' helps to design the slides (can select common design for all slides or separate slide for each slide)

(viii) Click icon slide show, one can run slide show either starting from the first slide or starting from the current slide.

The power point presentation (PPT) facilitates the key points to be kept in memory and understand the particular topic. Recently, the smart class room teaching uses the PPT to deliver the information in an effective way to enhance the quality of teaching.

Think and Do

- Make a Document with MS word on "Incredible India".
- Prepare an Excel Sheet for your daily pocket expenses for each category/item in last month
- Prepare and present a "Power Point" for "Day out with your parents"

CONCLUSION

This chapter provide the knowledge of necessity of mathematics in economics by explaining the application of linear algebra, calculus and Information Communication and Technology. Specifically the knowledge of functions, matrices , differential calculus, Integral calculus ,MS word, MS Excel and Power Point Presentation are depicted with suitable applications. The activities are also added for students to learn it reality about the use of mathematical methods in economics.

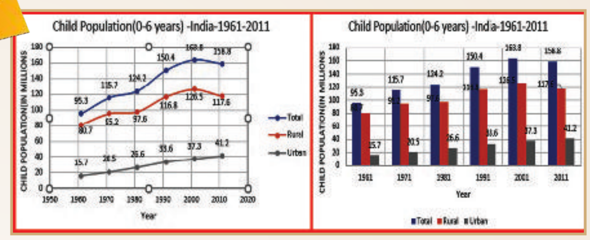
FORMULAE

1. m= y_2-y_1/x_2-x_1 for Slope
2. (y-y_1) = m (x-x_1) for straight Line
3. $|A| = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$ for 3x3 matrices
4. Differentiation of constant is zero
5. Differentiation of x^n is $nx^{(n-1)}$
6. e_d = Marginal function / Average function
7. $e_d = \frac{-P}{x}\frac{dx}{dp}$
8. Integration of x^n is $\frac{x^{n+1}}{n+1} + C$
9. $CS = \left[\int_0^{x_0} f(x)dx\right] - x_o p_o$
10. PS = x_0p_0 − integration of supply function within limit $= x_o p_o - \int_0^{x_0} g(x)\,dx$

ICT CORNER

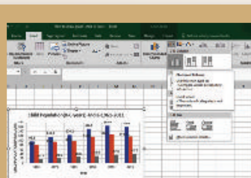
Drawing Graphs for the Data Collected

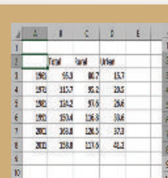
Graphs using EXCEL for the given data is given

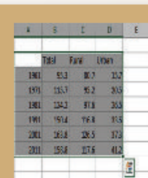


Steps:

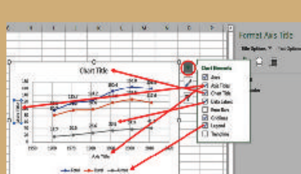
- Collection of data of Child population (0-6 years) from 1961 to 2011 in Rural and Urban areas in India. Let us draw the graph for the data.
- Open Microsoft Excel workbook, Type the X-axis data in the First column and then type respective data in consecutive columns.
- Now select all the typed data, After selecting the data Click “Insert” to get Charts. select scatter type to get scroll down menu.
- Select “Scatter with Smooth Lines and Markers” you will get the required graph as shown here.
- By selecting 3 icons on the right side to edit “chart elements” Particularly Check on the boxes Axis Titles and Chart Title.
- Type x-axis and y-Axis, followed by Chart Title. Click “Legend” to change the position
- Now right click on the graph (a) to copy the graph and Then paste in a word page (or)Select move chart to move In other excel page, Menu willappear to place it in new sheet.
- Now If you want to change the graph type as bar chart or any other type,click on the graph to select and then click on any type of graph given in the top menu


Step1


Step2


Step3


Step4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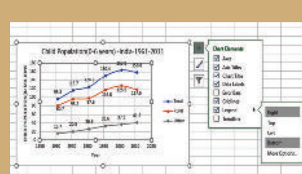
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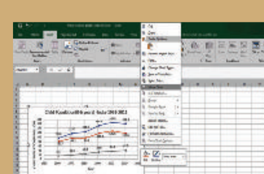
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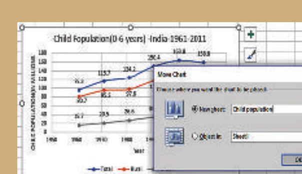

Step8

Pictures are indicatives only*

URL:

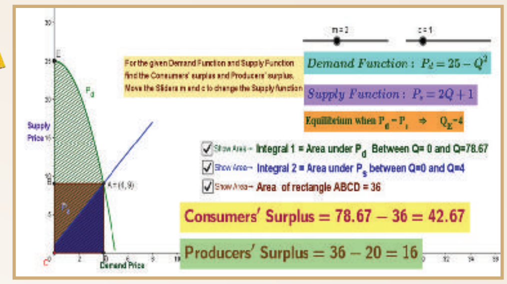
https://youtu.be/Xn7Sd5Uu42A

(or) scan the QR Code

ICT CORNER

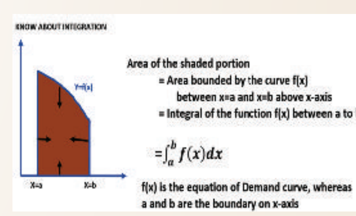
Consumer's and Producer's Surplus

Lets use Integration to find Consumer's and Producer's Surplus

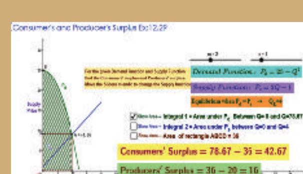


Steps:

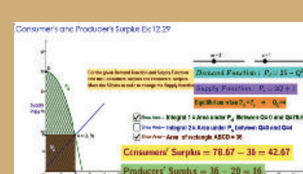
- Open the Browser type the URL given (or) Scan the QR Code.
- GeoGebra Work book called "XI STD ECONOMICS" will appear, Open the worksheet named "Consumer's and Producer's Surplus Ex:12.29"
- Without integration we cannot find the Area under the curve. For Higher studies atleast you should know what is Integration and why it is needed.
- In the worksheet Green colour is the Demand Curve and Blue colour is the Supply curve. They intersect at Point A (4,9). In which x axis value 4 is the demand price. If you integrate the Demand curve between 0 and 4 we get the area as shown. Click "Show Area Integral1" integrating the demand price between 0 and 4.
- If you click on "Show Area of Rectangle" you can see the area of the rectangle which is obtained by Multiplying the length 4 and Breadth 9 (Point A(4,9))
- If you subtract: the area under the curve PD -Area of the rectangle you get the Consumer's Surplus.
- Click on "Show Area Integral 2" you see Blue colour area which is obtained by Integrating Supply Price line between 0 and 4. Subtract: Area of the rectangle – Area under the line PS you get The Producer's Surplus. You can change PS line by moving the sliders 'm' and 'c'. you can see the changes in Consumer's Surplus and Producer's Surplus.



Step1



Step2



Step3

Step4

Pictures are indicatives only*

URL:

https://ggbm.at/ddY3wkjp

(or) scan the QR Code

MODEL QUESTIONS

Part-A Multiple Choice Questions

1. Mathematical Economics is the integration of
   - a. Mathematics and Economics
   - b. Economics and Statistics
   - c. Economics and Equations
   - d. Graphs and Economics

2. The construction of demand line or supply line is the result of using
   - a. Matrices
   - b. Calculus
   - c. Algebra
   - d. Analytical Geometry

3. The first person used the mathematics in Economics is
   - a. Sir William Petty
   - b. Giovanni Ceva
   - c. Adam Smith
   - d. Irving Fisher

4. Function with single independent variable is known as
   - a. Multivariate Function
   - b. Bivariate Function
   - c Univariate Function
   - d. Polynomial Function

5. A statement of equality between two quantities is called
   - a. Inequality
   - b. Equality
   - c. Equations
   - d. Functions

6. An incremental change in dependent variable with respect to change in independent variable is known as
   - a. Slope b. Intercept
   - c. Variant d. Constant

7. $(y - y_1) = m(x - x_1)$ gives the
   - a. Slope
   - b. Straight line
   - c. Constant
   - d. Curve

8. Suppose D = 50 – 5P. When D is zero then
   - a. P is 10 b. P is 20
   - c. P is 5 d. P is –10

9. Suppose D = 150 – 50P. Then, the slope is
   - a. –5 b. 50
   - c. 5 d. –50

10. Suppose determinant of a matrix $\Delta = 0$, then the solution
   - a. Exists
   - b. Does not exist
   - c. is infinity
   - d. is zero

11. State of rest is a point termed as

a. Equilibrium

b. Non-Equilibrium

c. Minimum Point

d. Maximum Point

12. Differentiation of constant term gives

a. one

b. zero

c. infinity

d. non-infinity

13. Differentiation of x^n is

a. $nx^{(n-1)}$

b. $n\,x^{(n+1)}$

c. zero

d. one

14. Fixed Cost is the -----------term in cost function represented in mathematical form.

a. Middle

b. Price

c. Quantity

d. Constant

15. The first differentiation of Total Revenue function gives

a. Average Revenue

b. Profit

c. Marginal Revenue

d. Zero

16. The elasticity of demand is the ratio of

a. Marginal demand function and Revenue function

b. Marginal demand function to Average demand function

c. Fixed and variable revenues

d. Marginal Demand function and Total demand function

17. If $x+y = 5$ and $x-y= 3$ then, Value of x

a. 4

b. 3

c. 16

d. 8

18. Integration is the reverse process of

a. Difference

b. Mixing

c. Amalgamation

d. Differentiation

19. Data processing is done by

a. PC alone

b. Calculator alone

c. Both PC and Calculator

d. Pen drive

20. The command **Ctrl** + **M** is applied for

a. Saving

b. Copying

c. getting new slide

d. deleting a slide

Part-A Answers

1	2	3	4	5	6	7	8	9	10
a	d	b	c	c	a	b	a	d	b
11	12	13	14	15	16	17	18	19	20
a	b	a	d	c	b	a	d	c	c

Part – B Answer the following questions in one or two sentences:

21. If $62 = 34 + 4x$ what is x? (Answer :x is 7)

22. Given the demand function $q = 150 - 3p$, derive a function for MR.

23. Find the average cost function where $TC = 60 + 10x + 15x^2$

24. The demand function is given by $x = 20 - 2p - p^2$ where p and x are the price and the quantity respectively. Find the elasticity of demand for $p = 2.5$.

25. Suppose the price p and quantity q of a commodity are related by the equation $q = 30 - 4p - p^2$ find (i) e_d at $p = 2$ (ii) MR

26. What is the formula for elasticity of supply if you know the supply function?

27. What are the Main menus of MS Word?

Part – C Answer the following questions in one paragraph:

28. Illustrate the uses of Mathematical Methodsm in Economics.

29. Solve for x quantity demanded *if* $16x - 4 = 68 + 7x$. (Ans: x is 8)

30. A firm has the revenue function $R = 600q - 0.03q^2$ and the cost function is $C = 150q + 60{,}000$, where q is the number of units produced. Find AR, AC, MR and MC. (Answersa: $AR = 600 - 0.03q$; $MR = 600 - 0.06\ q$; $AC = 150 + (60000/q)$)

31. Solve the following linear equations by using Cramer's rule.

$x_1 - x_2 + x_3 = 2$: $x_1 + x_2 - x_3 = 0$: $-x_1 - x_2 - x_3 = -6$

32. If a firm faces the total cost function $TC = 5 + x^2$ where x is output, what is TC when x is 10?

33. If $TC = 2.5q^3 - 13q^2 + 50q + 12$ derive the MC function and AC function.

34. What are the steps involved in executing a MS Excel Sheet?

Part – D Answer the following questions in about a page:

35. A Research scholar researching the market for fresh cow milk assumes that Qt= f(*P*t, *Y*,*A*,*N*, *P*c) where Qt is the quantity of milk demanded, *P*t is the price of fresh cow milk, *Y* is average household income, *A* is advertising expenditure on processed pocket milk, *N* is population and *P*c is the price of processed pocket milk .
   (a) What does Qt= f (*P*t, *Y*,*A*,*N*, *P*c) mean in words?
   (b) Identify the independent variables.
   (c) Make up a specific form for this function. (Use your knowledge of Economics to deduce whether the coefficients of the different independent variables should be positive or negative.)

36. Calculate the elasticity of demand for the demand schedule by using differential calculus method $P = 60 - 0.2Q$ where price is (i) zero, (ii) ₹20, (iii) ₹40.

37. The demand and supply functions are $p_d = 1600 - x^2$ and $p_s = 2x^2 + 400$ respectively. Find the consumer's surplus and producer's surplus at equilibrium point.

38. What are the ideas of information and communication technology used in economics?

ACTIVITY

1. The petrol consumption of your car is 16 Kilometers per litre. Let *x* be the distance you travel in Kilometers and *p* the price per litre of petrol in Rupees. Write expressions for demand for Petrol.

2. Make up your own demand function and then derive the corresponding MRfunction and find the output level which corresponds to zero marginal revenue.

3. Use an Excel spreadsheet to calculate values for Quantity of demand at various prices for the function Q = 100−10P then plot these values on a graph.

4. Open MS-Word and put the title as **PRESENT AND ABSENT OF STUDENTS** and insert the table and collect the data for all classes of your school and find the class of highest absentees in a month. Justify with reason for the absentees in a paragraph by using MS Word.

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