JEE Advanced

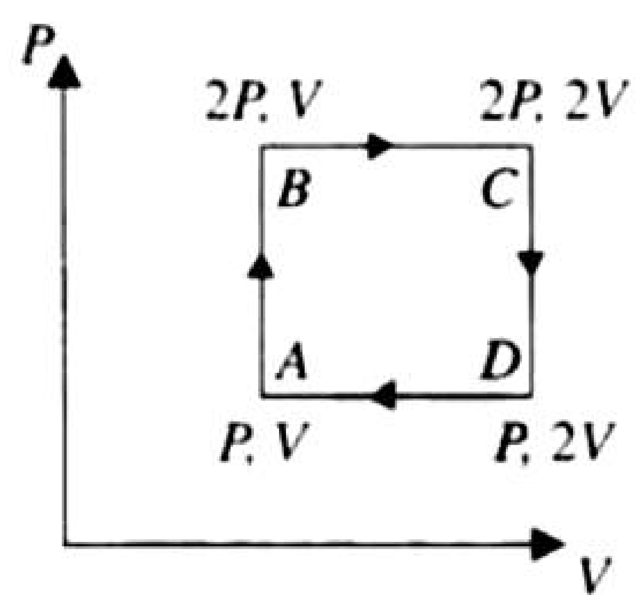
Single Correct Answer Type

1. A wall has two layers A and B, each made of different materials. Both the layers have the same thickness. The thermal conductivity of the material of A is twice that of B. Under thermal equilibrium, the temperature difference across the wall is 36°C. The temperature difference across the layer A is

a. 6°C **b.** 12°C **c.** 18°C **d.** 24°C

(IIT-JEE 1980)

2. An ideal monatomic gas is taken round the cycle $ABCDA$ as shown in the P–V diagram (see figure). The work done during the cycle is



a. PV **b.** $2PV$
c. 1/2 **d.** zero

(IIT-JEE 1983)

3. At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is

a. H_2 **b.** F_2 **c.** O_2 **d.** Cl_2

(IIT-JEE 1984)

4. 70 calories of heat is required to raise the temperature of 2 moles of an ideal diatomic gas at constant pressure from 30°C to 35°C. The amount of heat required (in calorie) to raise the temperature of the same gas through the same range (30°C to 35°C) at constant volume is

a. 30 **b.** 50 **c.** 70 **d.** 90

(IIT-JEE 1985)

5. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of the steam condensed in kilograms is

a. 0.130 **b.** 0.065 **c.** 0.260 **d.** 0.135

(IIT-JEE 1986)

6. If 1 mole of a monatomic gas ($\gamma = 5/3$) is mixed with 1 mole of a diatomic gas ($\gamma = 7/5$), the value of γ for the mixture is

a. 1.40 **b.** 1.50 **c.** 1.53 **d.** 3.07

(IIT-JEE 1988)

7. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in the steady state. The effective thermal conductivity of the system is

a. $K_1 + K_2$ **b.** $\dfrac{K_1 K_2}{(K_1 + K_2)}$

c. $\dfrac{(K_1 + 3K_2)}{4}$ **d.** $\dfrac{(3K_1 + K_2)}{4}$

(IIT-JEE 1988)

8. When an ideal diatomic gas is heated at constant pressure, the fraction of the heat energy supplied which increases the internal energy of the gas is

a. $\dfrac{2}{5}$ **b.** $\dfrac{3}{5}$ **c.** $\dfrac{3}{7}$ **d.** $\dfrac{5}{7}$

(IIT-JEE 1990)

9. Three closed vessels A, B and C, are at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of the O_2 molecules in vessel A is v_1, that of the N_2 molecules in vessel B is v_2, the average speed of the O_2 molecules in vessel C is

a. $\dfrac{(v_1 + v_2)}{2}$ **b.** v_1

c. $(v_1\ v_2)^{1/2}$ **d.** $\sqrt{\dfrac{3kT}{M}}$

(IIT-JEE 1992)

10. Three rods of identical cross-sectional area are made from the same metal and form the sides of an isosceles triangle ABC, right-angled at B. The points A and B are maintained at temperatures T and $(\sqrt{2})T$, respectively. In the steady state, the temperature of the point is T_c. Assuming that only heat conduction takes place, T_c/T is

a. $\dfrac{1}{2(\sqrt{2}-1)}$ **b.** $\dfrac{3}{\sqrt{2}+1}$

c. $\dfrac{1}{\sqrt{3}(\sqrt{2}-1)}$ **d.** $\dfrac{1}{\sqrt{2}+1}$ **(IIT-JEE 1995)**

11. Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that of S_2 is

a. $\dfrac{1}{3}$ **b.** $\dfrac{1}{\sqrt{3}}$

c. $\dfrac{\sqrt{3}}{1}$ **d.** $\left(\dfrac{1}{3}\right)^{\frac{1}{3}}$ **(IIT-JEE 1995)**

12. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root mean square velocity of the gas molecules is v, at 480 K it becomes

a. $4v$ **b.** $2v$ **c.** $v/2$ **d.** $v/4$

(IIT-JEE 1996)

13. The average translational energy and the rms speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 m/s, respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour)

a. 12.42×10^{-21} J, 968 m/s
b. 8.78×10^{-21} J, 684 m/s
c. 6.21×10^{-21} J, 968 m/s
d. 12.42×10^{-21} J, 684 m/s **(IIT-JEE 1997)**

14. The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the North Star has the maximum value at 350 nm. If these stars behave like black bodies, the ratio of the surface temperature of the sun and the North Star is

a. 1.46 **b.** 0.69 **c.** 1.21 **d.** 0.83

(IIT-JEE 1997)

15. The average translational kinetic energy of O_2 (relative molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 (relative molar mass 28) molecules in eV at that temperature is

a. 0.0015 **b.** 0.003 **c.** 0.048 **d.** 0.768

(IIT-JEE 1997)

16. A vessel contains 1 mole of O_2 gas (relative molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing 1 mole of He gas (relative molar mass 4) at a temperature $2T$ has a pressure of

a. $P/8$ **b.** P **c.** $2P$ **d.** $8P$

(IIT-JEE 1997)

17. A spherical black body with a radius of 12 cm radiates 450 W power at 50 K. If the radius were halved and the temperature doubled, the power radiated in watts would be

a. 225 **b.** 450 **c.** 900 **d.** 1800

(IIT-JEE 1997)

18. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per O_2 molecule to per N_2 molecule is

a. 1:1
b. 1:2
c. 2:1
d. depends on the moment of inertia of the two molecules

(IIT-JEE 1998)

19. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The changes in the pressure in A and B are found to be Δp and $1.5\ \Delta p$, respectively. Then

a. $4\ m_A = 9\ m_B$ **b.** $2\ m_A = 3\ m_B$
c. $3\ m_A = 2\ m_B$ **d.** $9\ m_A = 4\ m_B$

(IIT-JEE 1998)

20. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is

a. 30 K **b.** 18 K **c.** 50 K **d.** 42 K

(IIT-JEE 1998)

21. A black body is at a temperature of 2880 K. The energy of radiation emitted by this body with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm, is U_2 and between 1499 nm and 1500 nm is U_3. Wien's constant, $b = 2.88 \times 10^6$ nm-K. Then

a. $U_1 = 0$ **b.** $U_3 = 0$ **c.** $U_1 > U_2$ **d.** $U_2 > U_1$

(IIT-JEE 1998)

22. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

a. $4RT$ **b.** $15RT$ **c.** $9RT$ **d.** $11RT$

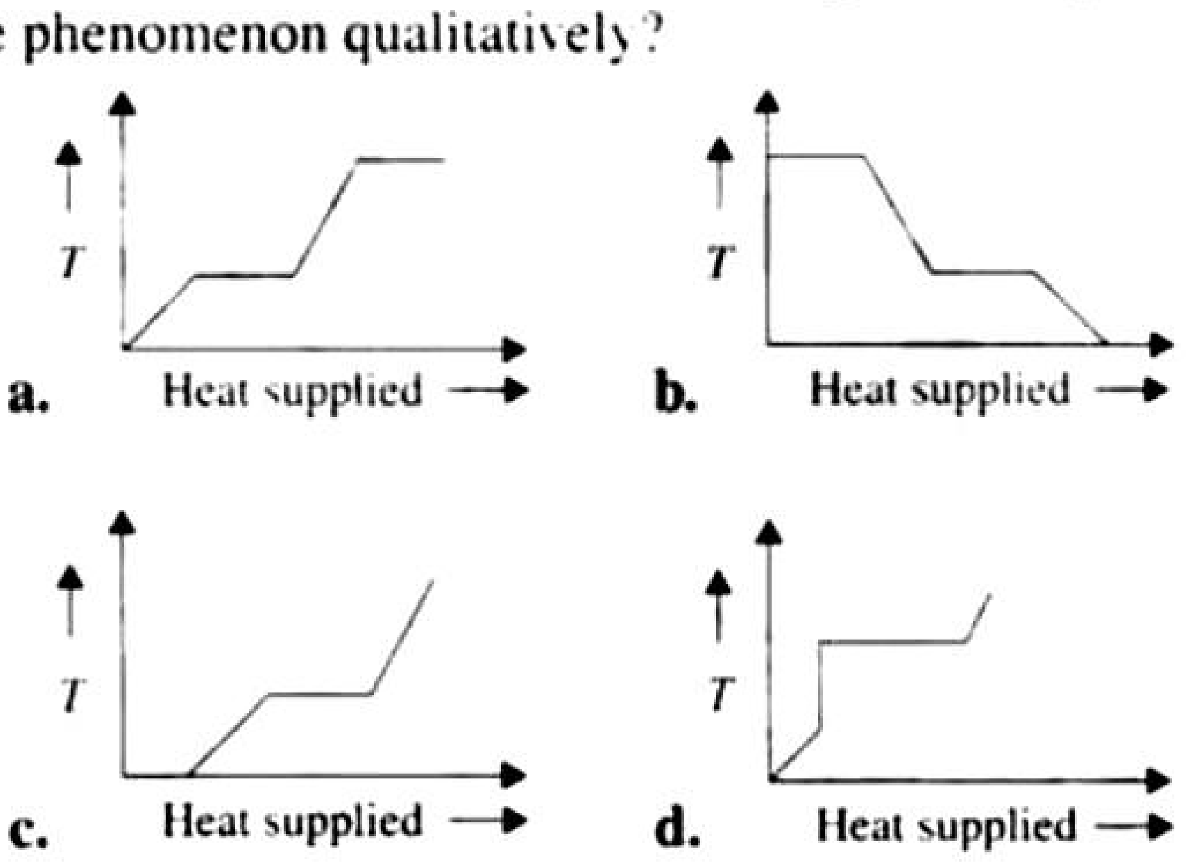
(IIT-JEE 1999)

23. A monatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 are the lengths of the gas column before and after expansion, respectively, then T_1/T_2 is given by

a. $\left(\frac{L_1}{L_2}\right)^{2/3}$ **b.** $\frac{L_1}{L_2}$

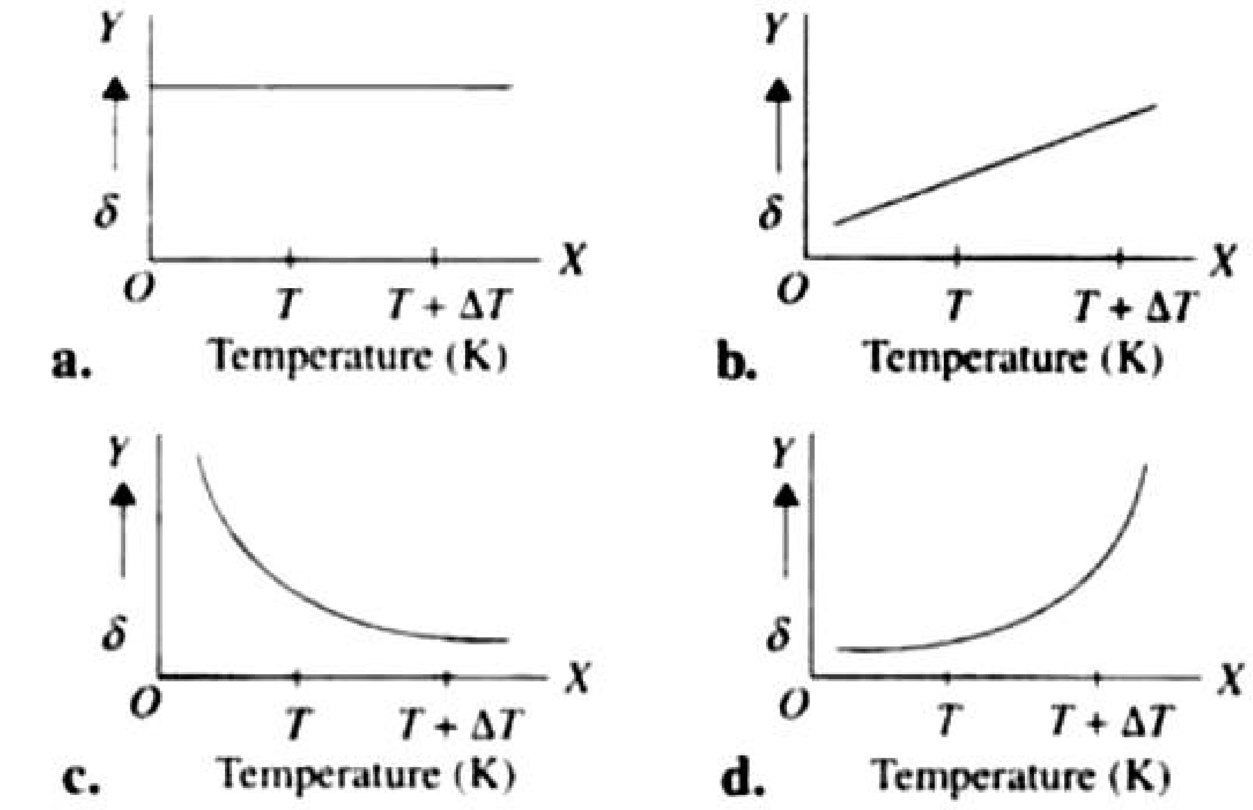
c. $\frac{L_2}{L_1}$ **d.** $\left(\frac{L_2}{L_1}\right)^{2/3}$ **(IIT-JEE 2000)**

24. A block of ice at $-10°C$ is slowly heated and converted to steam at $100°C$. Which of the following curves represents the phenomenon qualitatively?



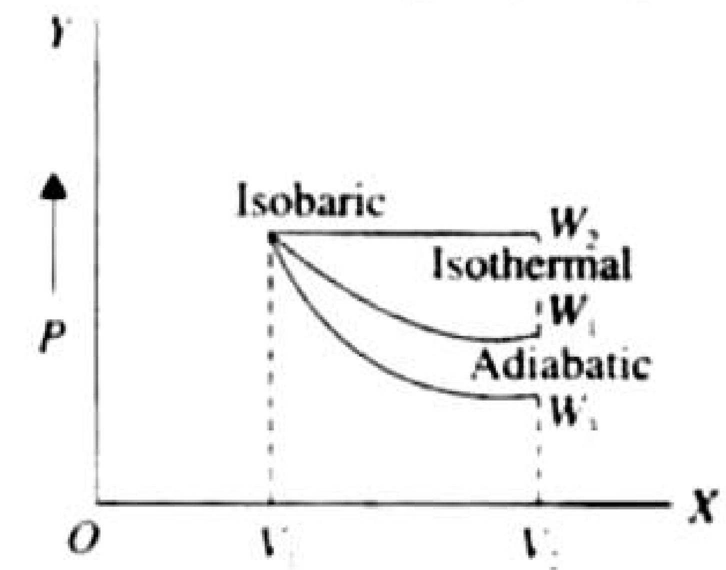
(IIT-JEE 2000)

25. An ideal gas is initially at temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta T / V \Delta T$ varies with temperature as



(IIT-JEE 2000)

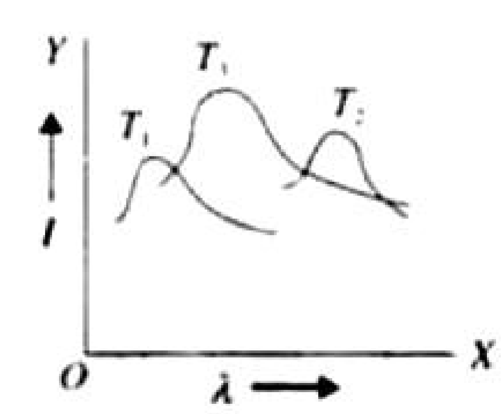
26. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then



a. $W_2 > W_1 > W_3$ **b.** $W_2 > W_3 > W_1$
c. $W_1 > W_2 > W_3$ **d.** $W_1 > W_3 > W_2$

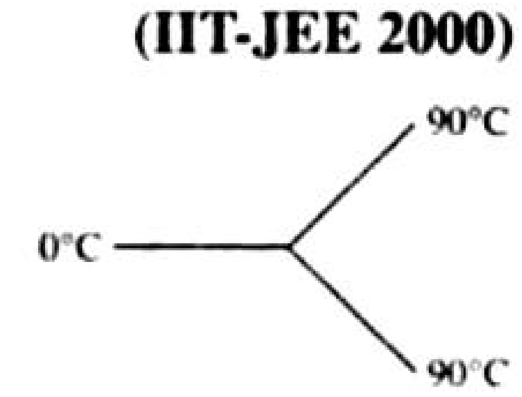
(IIT-JEE 2000)

27. The plots of intensity versus wavelength for three black bodies at temperatures T_1, T_2 and T_3, respectively are as shown. Their temperatures are such that



a. $T_1 > T_2 > T_3$
b. $T_1 > T_3 > T_2$
c. $T_2 > T_3 > T_1$
d. $T_3 > T_2 > T_1$ **(IIT-JEE 2000)**

28. Three rods made of same material and having the same cross section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C, respectively. The temperature of the junction of the three rods will be



a. 45°C **b.** 60°C **c.** 30°C **d.** 20°C

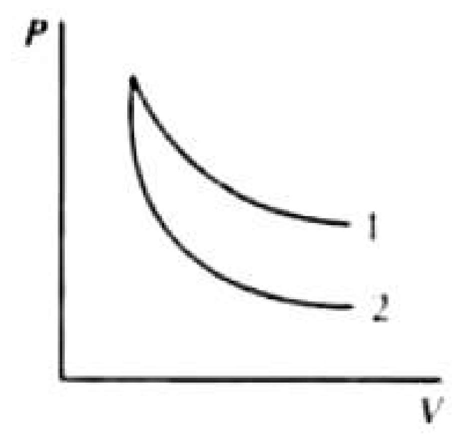
(IIT-JEE 2001)

29. In a given process on an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas

a. the temperature will decrease
b. the volume will increase
c. the pressure will remain constant
d. the temperature will increase

(IIT-JEE 2001)

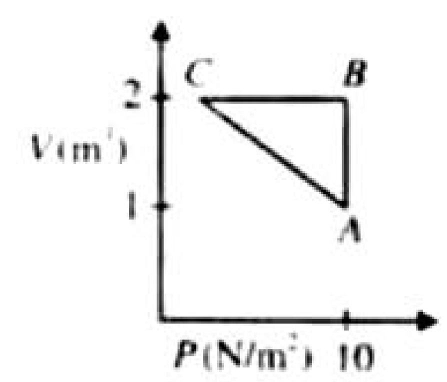
30. P–V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond, respectively, to



a. He and O_2
b. O_2 and He
c. He and Ar
d. O_2 and N_2

(IIT-JEE 2001)

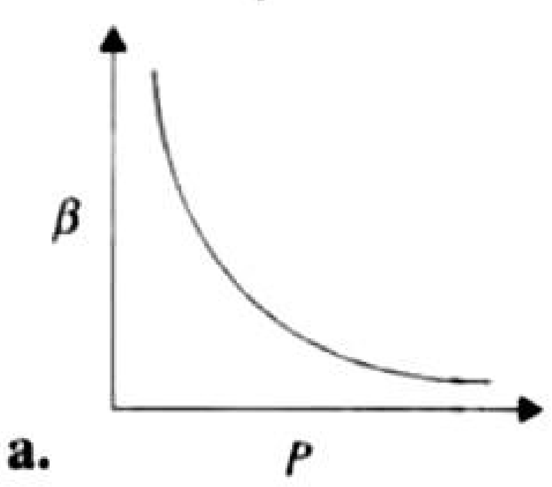
31. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \to A$ is



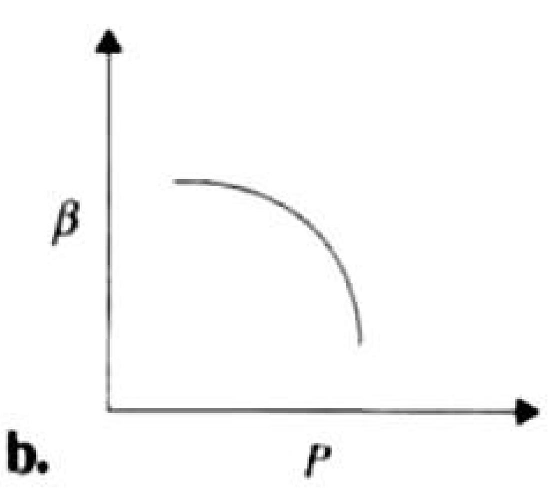
a. –5 J b. –10 J c. –15 J d. –20 J

(IIT-JEE 2002)

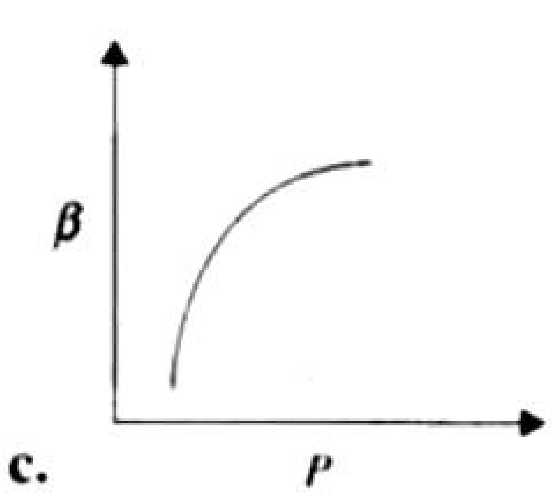
32. Which of the following graphs correctly represents the variation of $\beta = -\frac{(dV/dP)}{V}$ with P for an ideal gas at constant temperature?



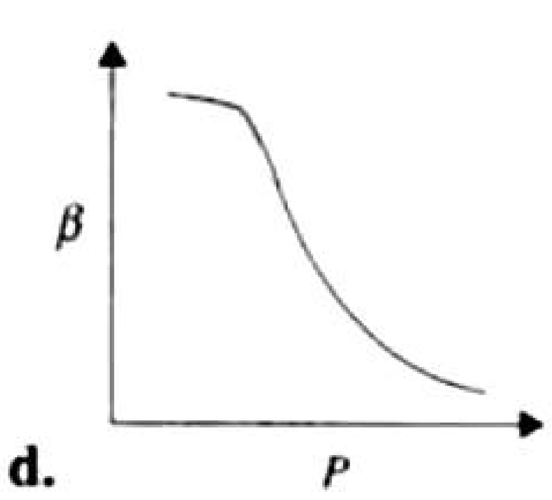
a.



b.



c.



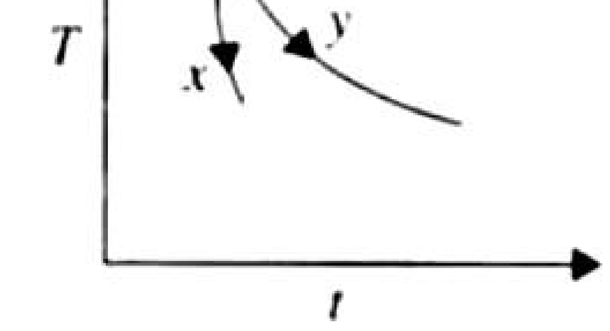
d.

(IIT-JEE 2002)

33. An ideal black body at room temperature is thrown into a furnace. It is observed that

a. initially it is the darkest body and later the brightest
b. it is the darkest body at all times
c. it cannot be distinguished at all times
d. initially it is the darkest body and later it cannot be distinguished **(IIT-JEE 2002)**

34. The graph, shown in the diagram, represents the variation of temperature (T) of two bodies, x and y, having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of two bodies.



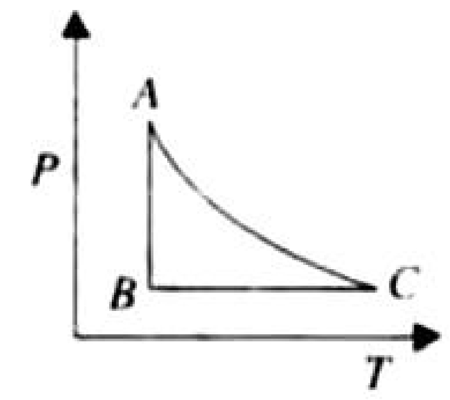
a. $E_x > E_y$ and $a_x < a_y$
b. $E_x < E_y$ and $a_x > a_y$
c. $E_x > E_y$ and $a_x > a_y$
d. $E_x < E_y$ and $a_x < a_y$ **(IIT-JEE 2003)**

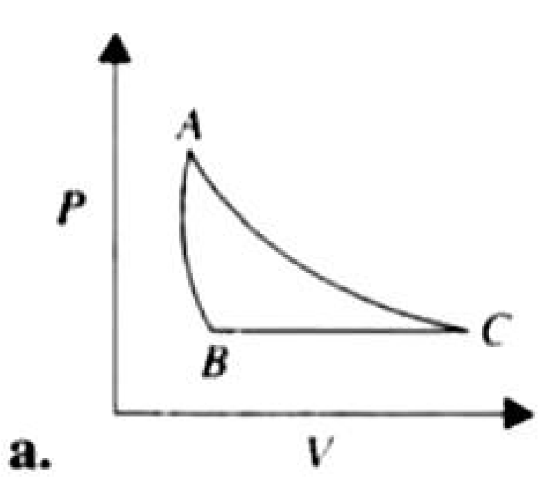
35. Two rods, one of aluminium and the other made of steel, having initial lengths l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_a and α_s, respectively. If the length of each rod increases by the same amount when their temperatures are raised by t°C, then find the ratio $\frac{l_1}{l_1 + l_2}$.

a. α_s/α_a b. α_a/α_s
c. $\alpha_s/(\alpha_a + \alpha_s)$ d. $\alpha_a/(\alpha_a + \alpha_s)$

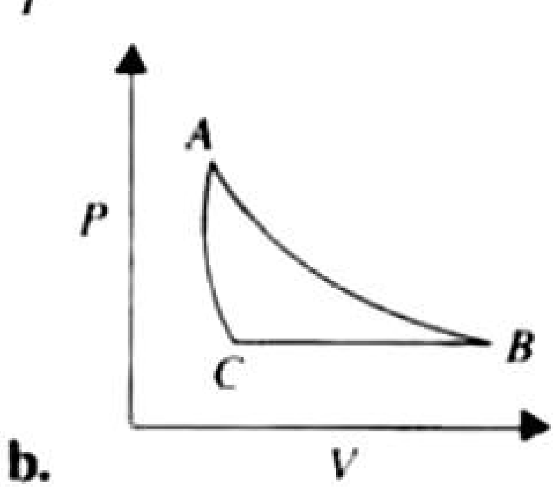
(IIT-JEE 2003)

36. The P–T diagram for an ideal gas is shown in the figure, where AC is an adiabatic process, find the corresponding P–V diagram.

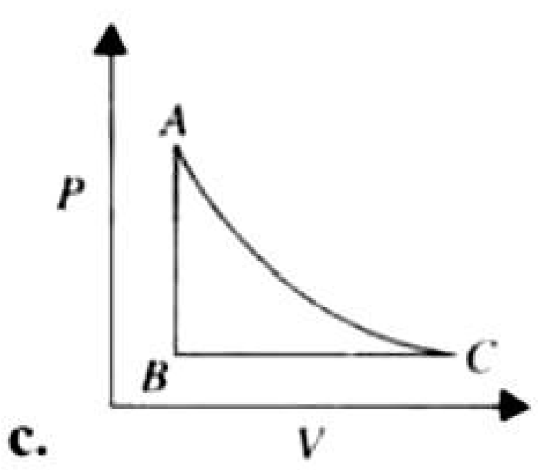




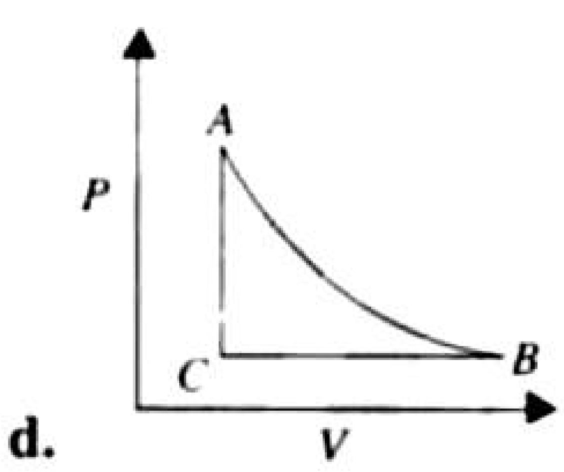
a.



b.



c.



d.

(IIT-JEE 2003)

37. 2 kg of ice at –20°C is mixed with 5 kg of water at 20°C in an insulating vessel having negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/kg/°C and 0.5 kcal/kg/°C respectively, while the latent heat of fusion of ice is 80 kcal/kg.

a. 7 kg b. 6 kg c. 4 kg d. 2 kg

(IIT-JEE 2003)

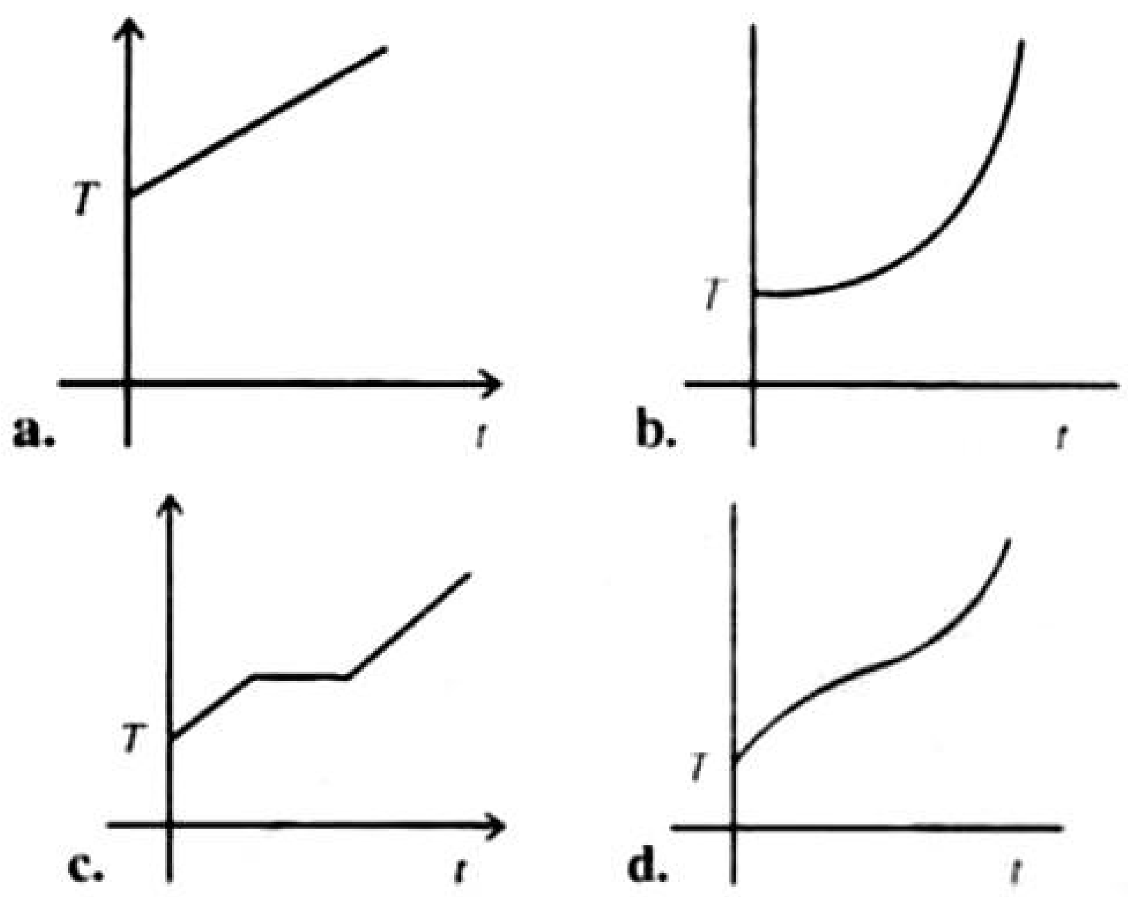
38. Three discs A, B and C having radii 2, 4 and 6 cm, respectively, are coated with carbon black. Wavelengths for maximum intensity for the three discs are 300, 400 and 500 nm, respectively. If Q_A, Q_B and Q_C are powers emitted by A, B and C, respectively, then

a. Q_A will be maximum
b. Q_B will be maximum

c. Q_C will be maximum

d. $Q_A = Q_B = Q_C$ **(IIT-JEE 2004)**

39. If liquefied oxygen at 1 atmospheric pressure is heated from 50 K to 300 K by supplying heat at constant rate, the graph of temperature vs. time will be



(IIT-JEE 2004)

40. Two identical rods are connected between two containers one of them is at 100°C and another is at 0°C. If rods are connected in parallel then the rate of melting of ice is q_1 g/s. If they are connected in series then the rate is q_2. The ratio q_2/q_1 is

a. 2 b. 4 c. 1/2 d. 1/4

(IIT-JEE 2004)

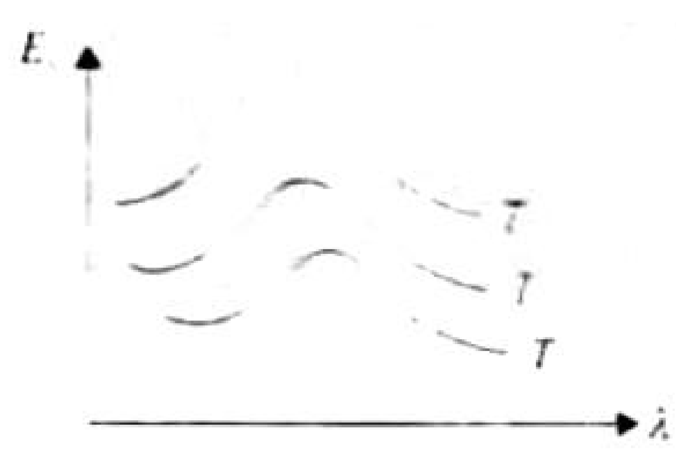
41. An ideal gas is initially at P_1, V_1 is expanded to P_2, V_2 and then compressed adiabatically to the same volume V_1 and pressure P_3. If W is the net work done by the gas in the complete process, which of the following is true?

a. $W > 0$: $P_3 > P_1$ b. $W < 0$: $P_3 > P_1$

c. $W > 0$: $P_3 < P_1$ d. $W < 0$: $P_3 < P_1$

(IIT-JEE 2004)

42. Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc is a function of its wavelength as shown in the figure. Which of the following option is the correct match?



a. Sun – T_3, tungsten filament – T_1, welding arc – T_2

b. Sun – T_2, tungsten filament – T_1, welding arc – T_3

c. Sun – T_3, tungsten filament – T_2, welding arc – T_1

d. Sun – T_1, tungsten filament – T_2, welding arc – T_3

(IIT-JEE 2005)

43. In which of the following processes, convection does not take place primarily?

a. sea and land breeze

b. boiling of water

c. heating air around a furnace

d. warming of glass of bulb due to filament

(IIT-JEE 2005)

44. A spherical body of area A and emissivity $e = 0.6$ is kept inside a perfectly black body. Total heat radiated by the body at temperature T is

a. $0.4\ \sigma AT^4$ b. $0.8\ \sigma AT^4$

c. $0.6\ \sigma AT^4$ d. $1.0\ \sigma AT^4$

(IIT-JEE 2005)

45. Calorie is defined as the amount of heat required to raise temperature of 1 g of water by 1°C and it is defined under which of the following conditions?

a. From 14.5°C to 15.5°C at 760 mm of Hg

b. From 98.5°C to 99.5°C at 760 mm of Hg

c. From 13.5°C to 14.5°C at 76 mm of Hg

d. From 3.5°C to 4.5°C at 76 mm of Hg

(IIT-JEE 2005)

46. Two litres of water in a container is heated with a coil of 1 kW at 27°C. The lid of the container is open and energy dissipates at the rate of 160 J/s. In how much time temperature will rise from 27°C to 77°C (given specific heat of water is 4.2 kJ/kg)?

a. 7 min b. 6 min 2 s

c. 8 min 20 s d. 14 min

(IIT-JEE 2005)

47. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

a. $\frac{1}{T}$ b. $\frac{2}{T}$ c. $\frac{3}{T}$ d. $\frac{4}{T}$

(IIT-JEE 2008)

48. A real gas behaves like an ideal gas if its

a. pressure and temperature are both high

b. pressure and temperature are both low

c. pressure is high and temperature is low

d. pressure is low and temperature is high

(IIT-JEE 2010)

49. 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T_1, the work done in the process is

a. $\frac{9}{8}RT_1$ b. $\frac{3}{2}RT_1$ c. $\frac{15}{8}RT_1$ d. $\frac{9}{2}RT_1$

(IIT-JEE 2011)

50. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e., second) plate under steady state condition is

a. $\left(\frac{65}{2}\right)^{1/4} T$ b. $\left(\frac{97}{4}\right)^{1/4} T$

c. $\left(\frac{97}{2}\right)^{1/4} T$ d. $(97)^{1/4} T$

(IIT-JEE 2012)

51. A mixture of 2 moles of helium gas (atomic mass = 4 amu), and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is:

a. 0.32 **b.** 0.45 **c.** 2.24 **d.** 3.16

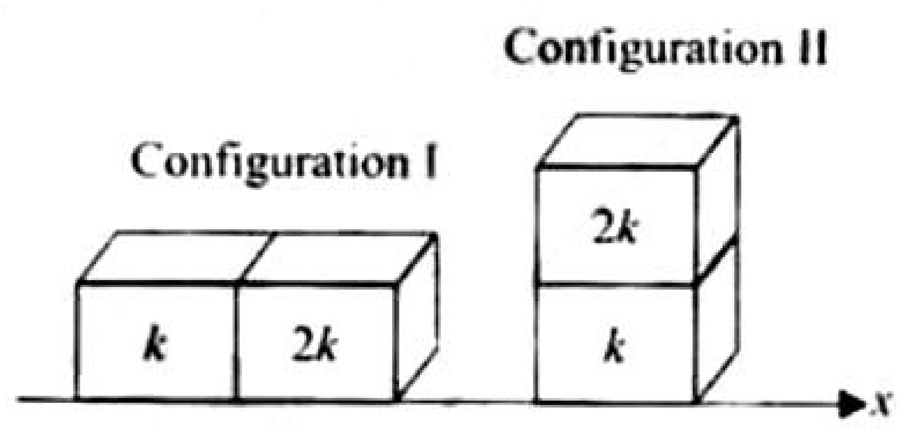
(IIT-JEE 2012)

52. Two moles of ideal gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take $R = 8.31$ J/mol.K)

a. 62 J **b.** 104 J **c.** 124 J **d.** 208 J

(IIT-JEE 2012)

53. Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in the figure. One of the blocks has thermal conductivity k and the other $2k$. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is



a. 2.0 s **b.** 3.0 s **c.** 4.5 s **d.** 6.0 s

(JEE Advanced 2013)

54. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is

a. 1 : 4 **b.** 1 : 2 **c.** 6 : 9 **d.** 8 : 9

(JEE Advanced 2013)

55. Parallel rays of light of intensity $I = 912$ Wm^{-2} are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}$ Wm^{-2} K^{-4} and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to

a. 330 K **b.** 660 K **c.** 990 K **d.** 1550 K

(JEE Advanced 2014)

Multiple Correct Answer Type

1. For an ideal gas:

a. The change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nCv(T_2 - T_1)$, where Cv is the molar specific heat at constant volume and n the number of moles of the gas.

b. The change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process.

c. The internal energy does not change in an isothermal process.

d. No heat is added or removed in an adiabatic process.

(IIT-JEE 1989)

2. An ideal gas is taken from the state A (pressure P, volume V) to the state B (pressure $P/2$, volume $2V$) along a straight line path in the P–V diagram. Select the correct statement(s) from the following:

a. The work done by the gas in process A to B exceeds the work that would be done by it if the system were taken from A to B along the isotherm.

b. In the T–V diagram, the path AB becomes a part of a parabola.

c. In the P–T diagram, the path AB becomes a part of a hyperbola.

d. In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases.

(IIT-JEE 1993)

3. Two bodies A and B have thermal emissivities of 0.01 and 0.81, respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power of the same rate. Wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A, by 1.00 μm. If the temperature of A is 5802 K

a. the temperature of B is 1934 K

b. $\lambda_B = 1.5$ μm

c. the temperature of B is 11604 K

d. the temperature of B is 2901 K **(IIT-JEE 1994)**

4. From the following statements concerning ideal gas at any given temperature T, select the correct one(s).

a. The coefficient of volume expansion at constant pressure is the same for all ideal gases.

b. The average translational kinetic energy per molecule of oxygen gas is $3kT$, k being the Boltzmann constant.

c. The mean free path of molecules increases with decrease in pressure.

d. In a gaseous mixture, the average translational kinetic energy of the molecules of each component is different.

(IIT-JEE 1995)

5. During the melting of a slab of ice at 273 K at atmosphere pressure,

a. positive work is done by the ice–water system on the atmosphere

b. positive work is done on the ice–water system by the atmosphere

c. the internal energy of the ice–water system increases

d. the internal energy of the ice–water system decreases

(IIT-JEE 1998)

6. Let $\bar{v}$, v_{rms} and v_p, respectively, denote the mean speed, root mean square speed and most probable speed of the molecules in an ideal monatomic gas at absolute temperature T. The mass of a molecule is m. Then

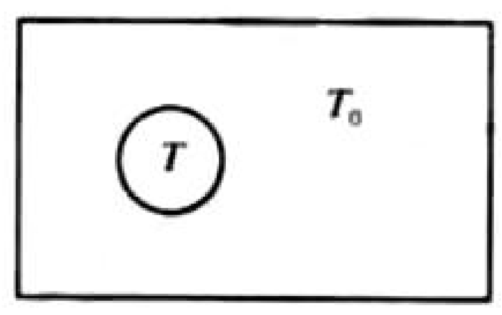
a. no molecule can have a speed greater than $\sqrt{2}v_{rms}$
b. no molecule can have speed less than $v_p/\sqrt{2}$
c. $v_p < \bar{v} < v_{rms}$
d. the average kinetic energy of a molecule is $3/4mv_p^2$

(IIT-JEE 1998)

7. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is
a. proportional to ΔT
b. inversely proportional to ΔT
c. proportional to $|\alpha_B - \alpha_C|$
d. inversely proportional to $|\alpha_B - \alpha_C|$

(IIT-JEE 1999)

8. A black body of temperature T is inside the chamber of T_0 temperature initially. Sun rays are allowed to fall from a hole in the top of the chamber. If the temperatures of black body (T) and chamber (T_0) remain constant, then

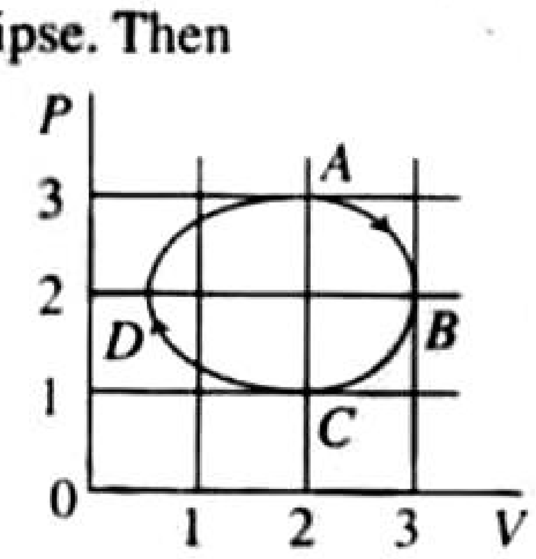


a. Black body will absorb radiation.
b. Black body will absorb less radiation.
c. Black body will emit more energy.
d. Black body will emit energy equal to energy absorbed by it.

(IIT-JEE 2006)

9. Cv and Cp denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then
a. $Cp - Cv$ is larger for a diatomic ideal gas than for a monatomic ideal gas.
b. $Cp + Cv$ is larger for a diatomic ideal gas than for a monatomic ideal gas.
c. Cp/Cv is larger for a diatomic ideal gas than for a monatomic ideal gas.
d. $Cp \cdot Cv$ is larger for a diatomic ideal gas than for a monatomic ideal gas. **(IIT-JEE 2009)**

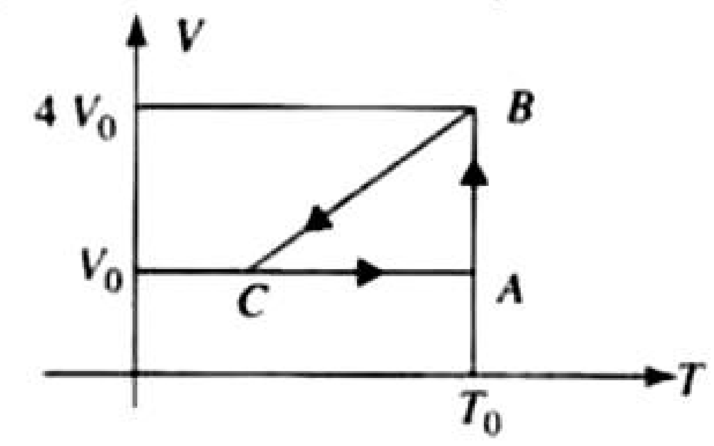
10. Figure shows the P–V plot of an ideal gas taken through a cycle $ABCDA$. The part ABC is a semi-circle and CDA is half of an ellipse. Then



a. the process during the path $A \rightarrow B$ is isothermal
b. heat flows out of the gas during the path $B \rightarrow C \rightarrow D$
c. work done during the path $A \rightarrow B \rightarrow C$ is zero
d. positive work is done by the gas in the cycle $ABCDA$

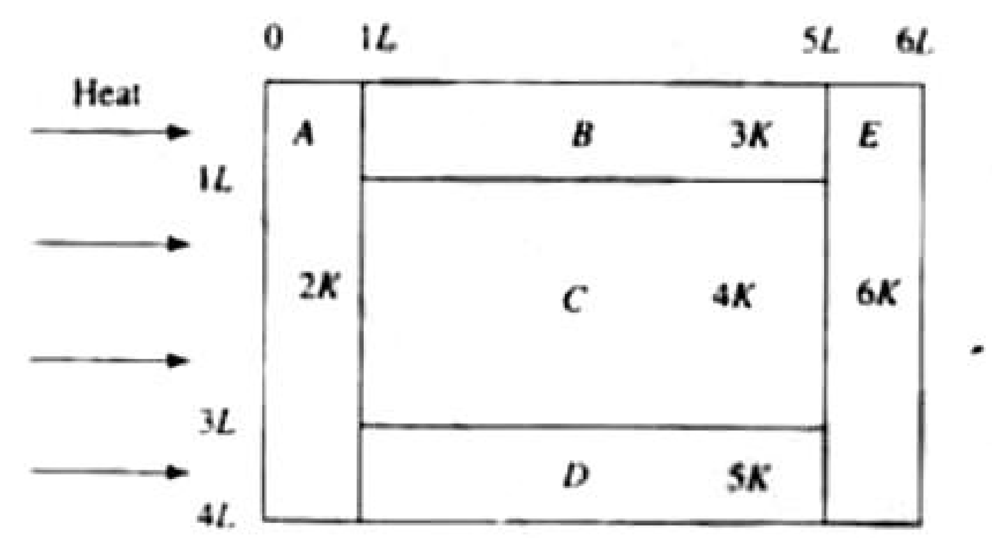
(IIT-JEE 2009)

11. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following



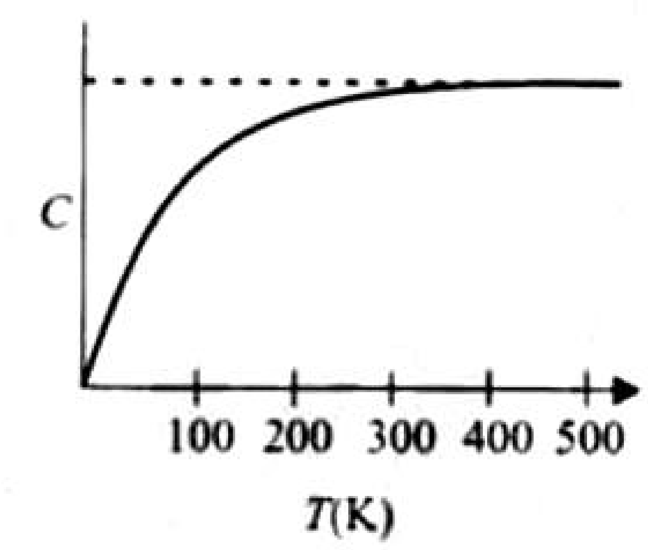
a. Internal energies at A and B are the same
b. Work done by the gas in process AB is $P_0V_0 \ln 4$
c. Pressure at C is $P_0/4$
d. Temperature at C is $T_0/4$ **(IIT-JEE 2010)**

12. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width.



a. Heat flow through slabs A and E is same
b. Heat flow through slab E is maximum
c. Temperature difference across slab E is smallest.
d. Heat flow through C = heat flow through B + heat flow through D. **(IIT-JEE 2011)**

13. The figure shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation.



a. the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.
b. heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400 to 500 K.
c. there is no change in the rate of heat absorption in range 400–500 K.
d. the rate of heat absorption increases in the range 200–300 K. **(JEE Advanced 2013)**

14. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is(are):

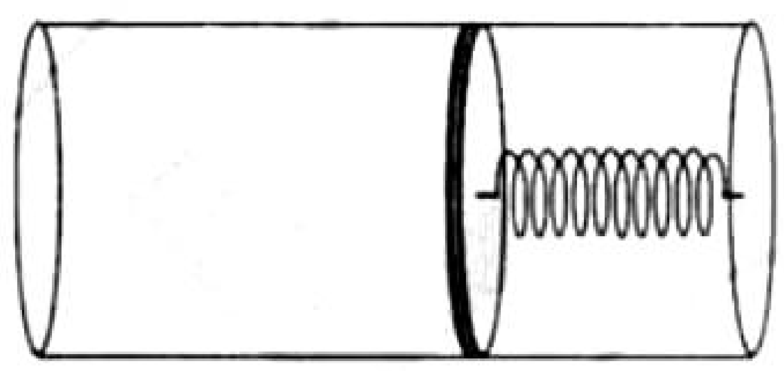
a. The average energy per mole of the gas mixture is 2RT

b. The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$

c. The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{2}$

d. The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$ **(JEE Advanced 2015)**

15. An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are):



a. If $V_2 = 2V_1$ and $T_2 = 3T$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

b. If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

c. If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

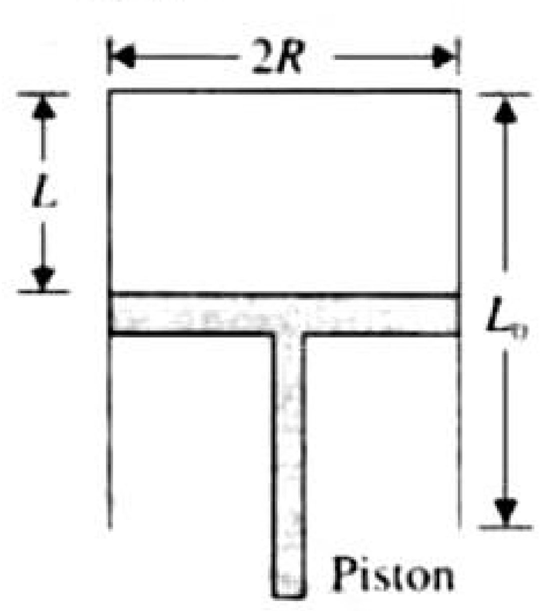
d. If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$ **(JEE Advanced 2015)**

Linked Comprehension Type

For Problems 1–3

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface as shown in the figure. The atmospheric pressure is p_0.

(IIT-JEE 2007)



1. The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be

a. p_0 b. $\frac{p_0}{2}$

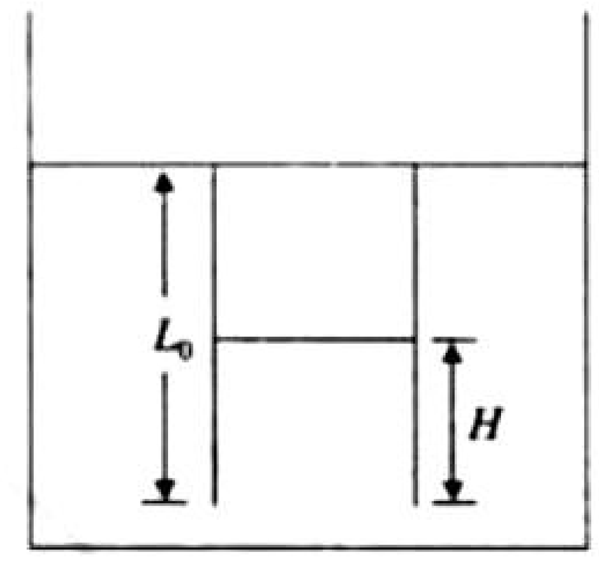
c. $\frac{p_0}{2}+\frac{Mg}{\pi R^2}$ d. $\frac{p_0}{2}-\frac{Mg}{\pi R^2}$

2. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

a. $\left(\frac{2p_0\pi R^2}{\pi R^2 p_0 + Mg}\right)(2L)$ b. $\left(\frac{p_0\pi R^2 - Mg}{\pi R^2 p_0}\right)(2L)$

c. $\left(\frac{p_0\pi R^2 + Mg}{\pi R^2 p_0}\right)(2L)$ d. $\left(\frac{p_0\pi R^2}{\pi R^2 p_0 - Mg}\right)(2L)$

3. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



a. $\rho g(L_0 - H)^2 + p_0(L_0 - H) + L_0 p_0 = 0$

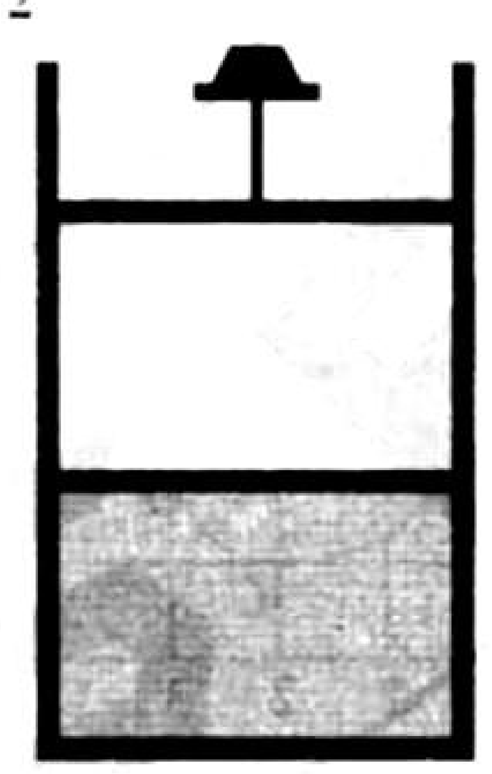
b. $\rho g(L_0 - H)^2 - p_0(L_0 - H) - L_0 p_0 = 0$

c. $\rho g(L_0 - H)^2 + p_0(L_0 - H) - L_0 p_0 = 0$

d. $\rho g(L_0 - H)^2 - p_0(L_0 - H) + L_0 p_0 = 0$

For Problems 4 and 5

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R, C_P = \frac{5}{2}R$ and those for an ideal diatomic gas are $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$.



(JEE Advanced 2014)

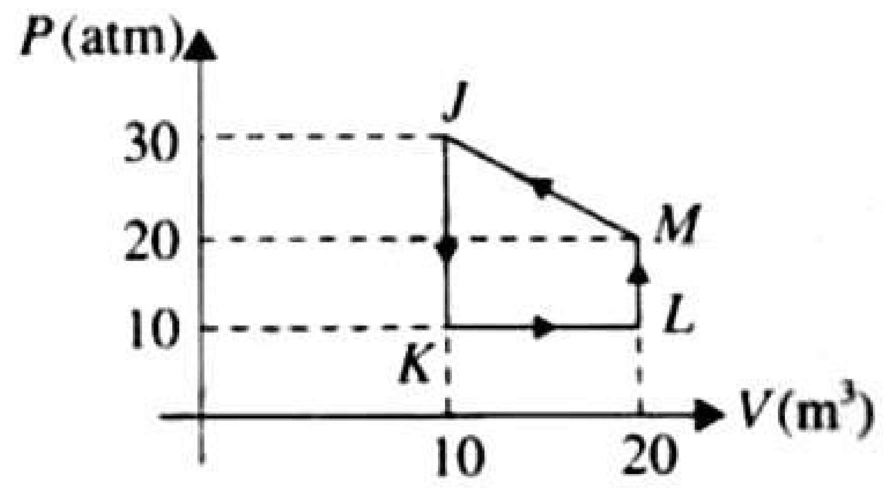
4. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

a. 550 K b. 525 K c. 513 K d. 490 K

5. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be
a. 250 R **b.** 200 R **c.** 100 R **d.** −100 R

Matching Column Type

1. Heat given to process is positive. Match the following options of Column I with the corresponding options in Column II:



Column I	Column II
(i) JK	(a) $\Delta W > 0$
(ii) KL	(b) $\Delta Q < 0$
(iii) LM	(c) $\Delta W < 0$
(iv) MJ	(d) $\Delta Q > 0$

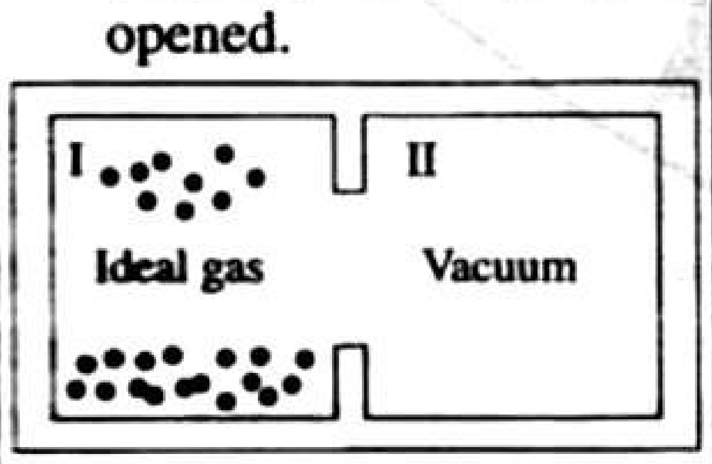
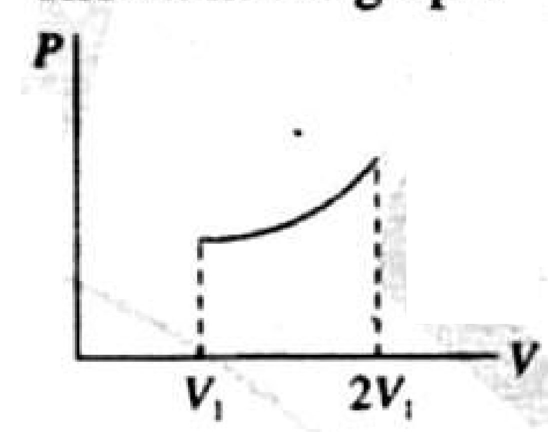
(IIT-JEE 2006)

2. Column I gives some devices and Column II gives some processes on which the functioning of these devices depends. Match the devices in Column I with the processes in Column II.

Column I	Column II
(i) Bimetallic strip	(a) Radiation from a hot body
(ii) Steam engine	(b) Energy conversion
(iii) Incandescent lamp	(c) Melting
(iv) Electric fuse	(d) Thermal expansion of solids

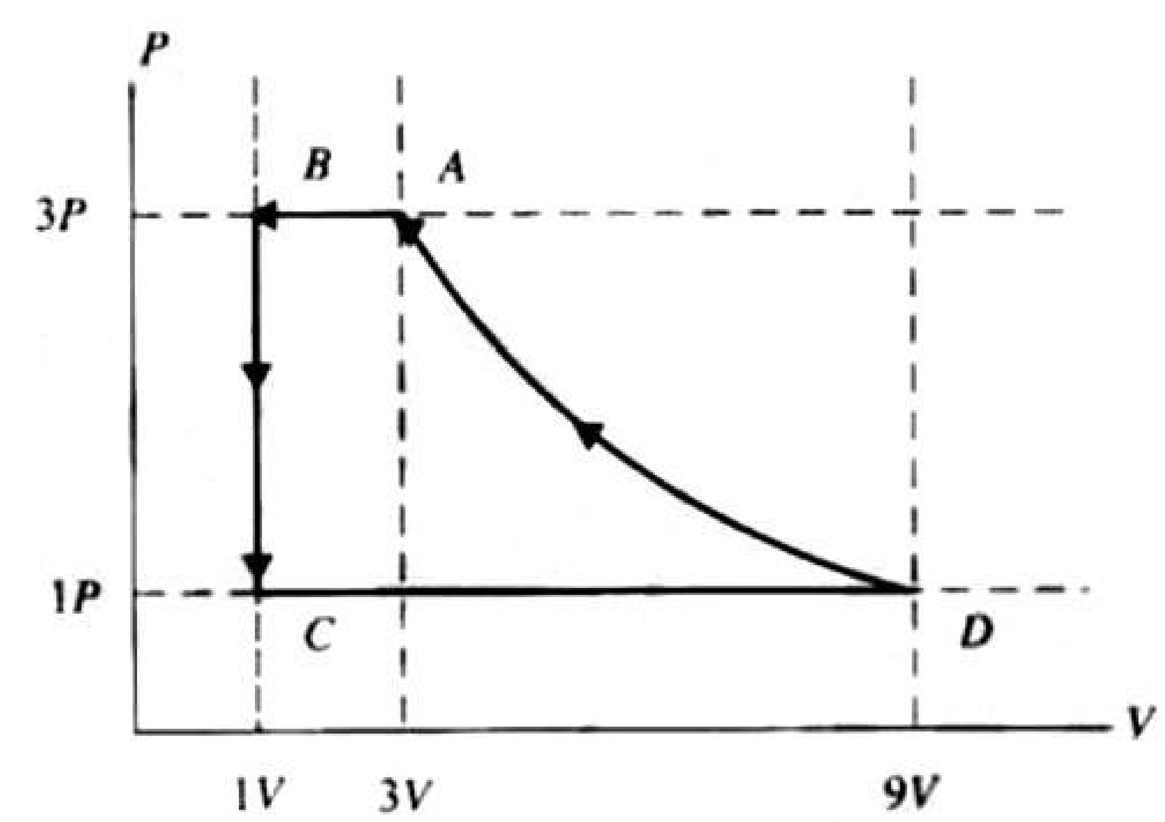
(IIT-JEE 2007)

3. Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process.

Column I	Column II
(i) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and Chamber II has vacuum. The valve is opened. 	(a) The temperature of the gas decreases
(ii) An ideal monatomic gas expands to twice its original volume such that its pressure $P \propto 1/V^2$, where V is the volume of the gas.	(b) The temperature of the gas increases or remains constant
(iii) An ideal monatomic gas expands to twice its original volume such that its pressure $P \propto 1/V^{4/3}$, where V is its volume.	(c) The gas loses heat
(iv) An ideal monatomic gas expands such that its pressure P and volume V follow the behaviour shown in the graph. 	(d) The gas gains heat

(IIT-JEE 2008)

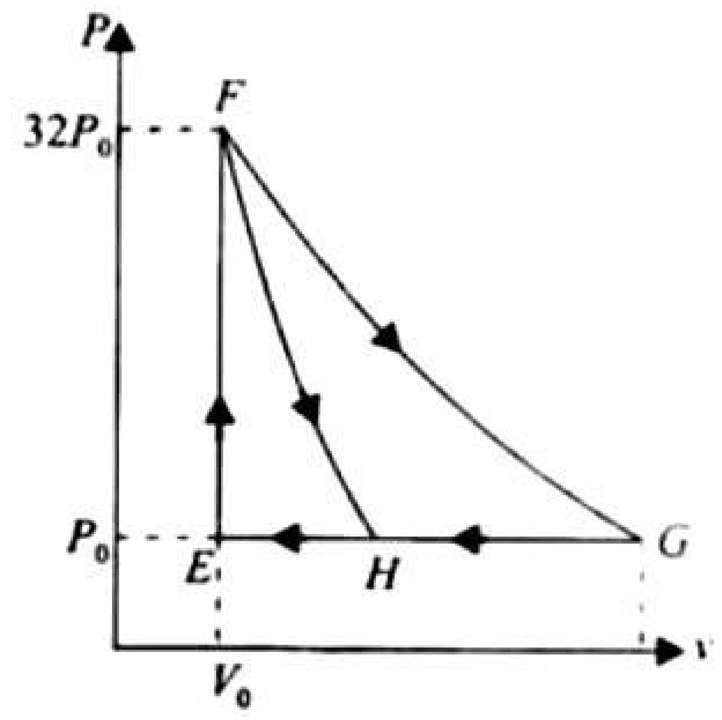
4. One mole of a monatomic ideal gas is taken through a cycle $ABCDA$ as shown in the P-V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I.



Column I	Column II
(i) Process $A \to B$	(a) Internal energy decreases
(ii) Process $B \to C$	(b) Internal energy increases
(iii) Process $C \to D$	(c) Heat is lost
(iv) Process $D \to A$	(d) Heat is gained
	(e) Work is done on the gas

(IIT-JEE 2011)

5. One mole of mono-atomic ideal gas is taken along two cyclic processes $E \to F \to G \to E$ and $E \to F \to H \to E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



Match the paths in Column I with the magnitudes of the work done in Column II and select the correct answer using the codes given below the lists.

Column I	Column II
(i) $G \to E$	1. $160\, P_0 V_0 \ln 2$
(ii) $G \to H$	2. $36\, P_0 V_0$
(iii) $F \to H$	3. $24\, P_0 V_0$
(iv) $F \to G$	4. $31\, P_0 V_0$

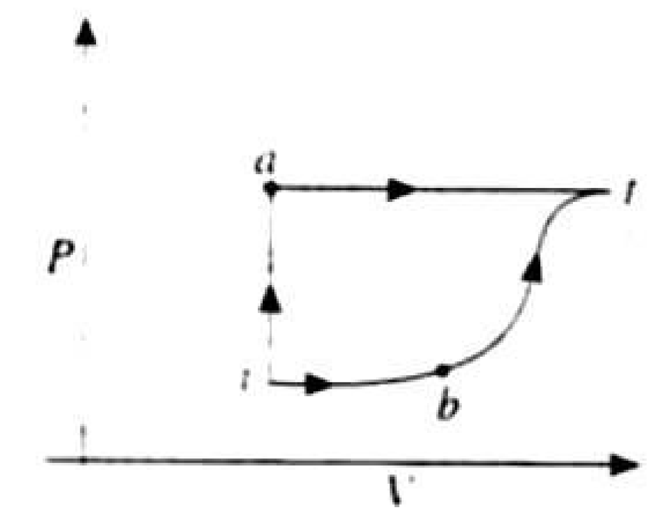
Codes:

	i.	ii.	iii.	iv.
a.	4	3	2	1
b.	4	3	1	2
c.	3	1	2	4
d.	1	3	2	4

(JEE Advanced 2013)

Integer Answer Type

1. A metal rod AB of length $10x$ has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ. (Neglect any heat loss to the surrounding.) **(IIT-JEE 2009)**
2. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B? **(IIT-JEE 2010)**
3. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is **(IIT-JEE 2010)**
4. Steel wire of length 'L' at 40°C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40°C to 30°C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5}/°C. Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that $L >>$ diameter of the wire. Then the value of 'm' in kg is nearly.
5. A thermodynamic system is taken from an initial state i with internal energy U = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio Q_{bf} / Q_{ib} is **(JEE Advanced 2014)**



6. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is ______. **(JEE Advanced 2015)**

Assertion-Reasoning Type

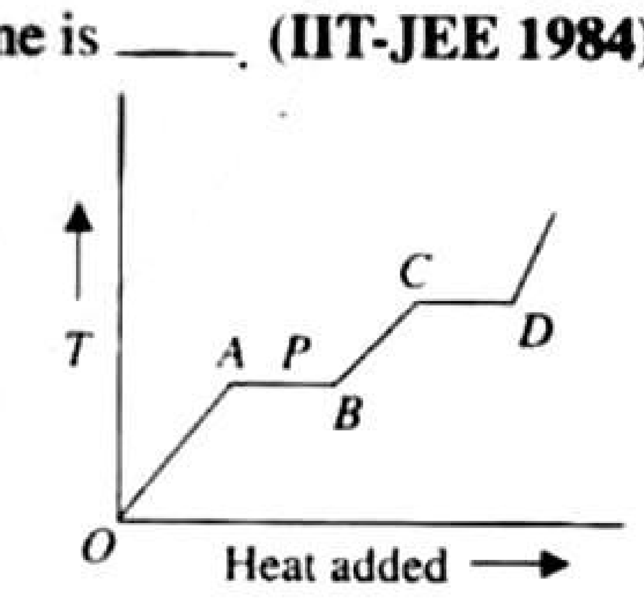
In the following question, assertion (A) is given by corresponding statement of reason (R) of the statements, mark the correct answer.

a. If both assertion and reason are true and the reason is the correct explanation of the assertion.
b. If both assertion and reason are true but reason is not the correct explanation of assertion.
c. If assertion is true but the reason is false.
d. If assertion is false but the reason is true.

1. **Assertion:** The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.
Reason: The molecules of a gas collide with each other and the velocities of the molecules change due to the collision. **(IIT-JEE 2007)**

Fill in the Blanks Type

1. One mole of a monatomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is ____. **(IIT-JEE 1984)**
2. The variation of temperature of a material as heat is given to it at a constant rate is shown in the figure. The material is in solid state at point O. The state of the material at point P is ____. **(IIT-JEE 1985)**



3. During an experiment, an ideal gas is found to obey an additional law VP^2 = constant. The gas is initially at a temperature T and volume V. When it expands to a volume $2V$, the temperature becomes——°C. **(IIT-JEE 1987)**

4. 300 grams of water at 25°C is added to 100 g of ice at 0°C. The final temperature of the mixture is ____. **(IIT-JEE 1989)**

5. The earth receives on its surface radiation from the sun at the rate of 1400 Wm^{-2}. The distance of the centre of the sun from the surface of the earth is 1.5×10^{11}, and the radius of the sun is 7×10^8 m. Treating the sun as a black body, it follows from the above data that its surface temperature is ____ K. **(IIT-JEE 1989)**

6. A solid copper sphere (density ρ and specific heat c) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0 K. The time required for the temperature of the sphere to drop to 100 K is ____. **(IIT-JEE 1991)**

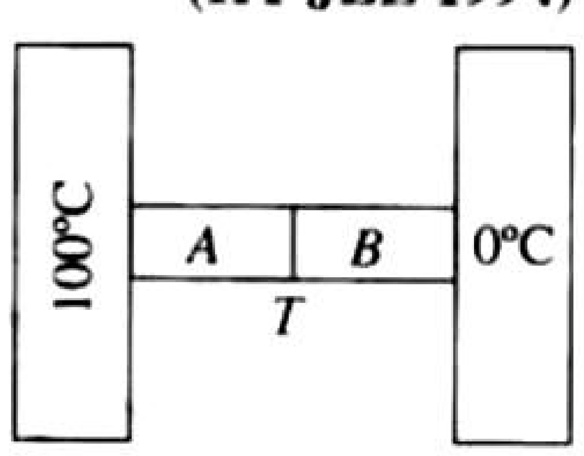
7. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. If the temperature difference between the outer and inner surface of the shell is not to exceed T, the thickness of the shell should not be less than ____. **(IIT-JEE 1991)**

8. A substance of mass M kilograms requires a power input of P watts to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in time t seconds. The latent heat of fusion of the substance is ____. **(IIT-JEE 1992)**

9. A container of volume 1 m^3 is divided into two equal parts by a partition. One part has an ideal gas at 300 K and the other part is vacuum. The whole system is thermally isolated from the surrounding. When the partition is removed, the gas expands to occupy the whole volume. Its temperature now will be ____. **(IIT-JEE 1993)**

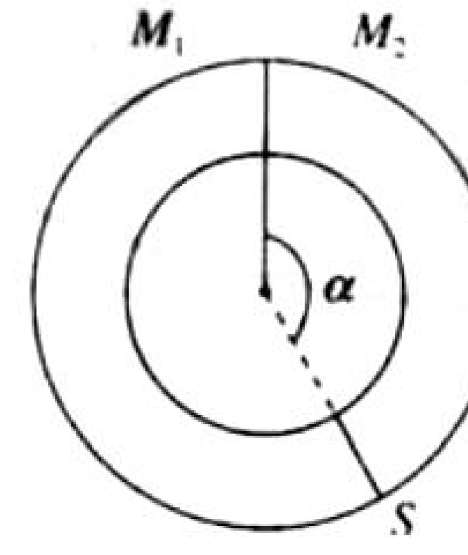
10. An ideal gas with pressure P, volume V and temperature T is expanded isothermally to a volume $2V$ and a final pressure P_i. If the same gas is expanded adiabatically to a volume $2V$, the final pressure is P_a. The ratio of the specific heats of the gas is 1.67. The ratio p_a/p_i is ____. **(IIT-JEE 1994)**

11. Two metal cubes A and B of same size are arranged as shown in the figure. The extreme ends of the combination are maintained at the indicated temperature.



The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m°C and 200 W/m°C, respectively. After steady state is reached, the temperature t of the interface will be ____. **(IIT-JEE 1996)**

12. A ring-shaped tube contains two ideal gases with equal masses and relative molar masses $M_1 = 32$ and $M_2 = 28$. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle α as shown in the figure is ____.



(IIT-JEE 1997)

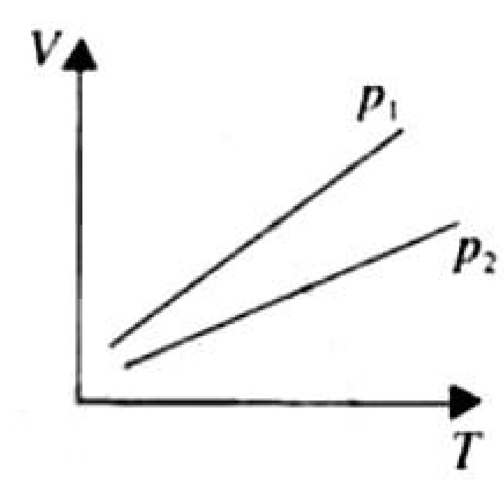
13. A gas thermometer is used as a standard thermometer for measurement of temperature. When the gas container of the thermometer is immersed in water at its triple point 273.16 K, the pressure in the gas thermometer reads 3.0×10^4 N/m^2. When the gas container of the same thermometer is immersed in another system, the gas pressure reads 3.5×10^4 N/m^2. The temperature of this system is therefore ____°C. **(IIT-JEE 1997)**

14. Earth receives 1400 W/m^2 of solar power. If all the solar energy falling on a lens of area 0.2 m^2 is focused on to a block of ice of mass 280 g, the time taken to melt the ice will be ____ minutes (latent heat of fusion of ice = 3.3×10^5 J/kg). **(IIT-JEE 1997)**

True/False Type

1. The root mean square speeds of the molecules of different ideal gases maintained at the same temperature are the same. **(IIT-JEE 1981)**

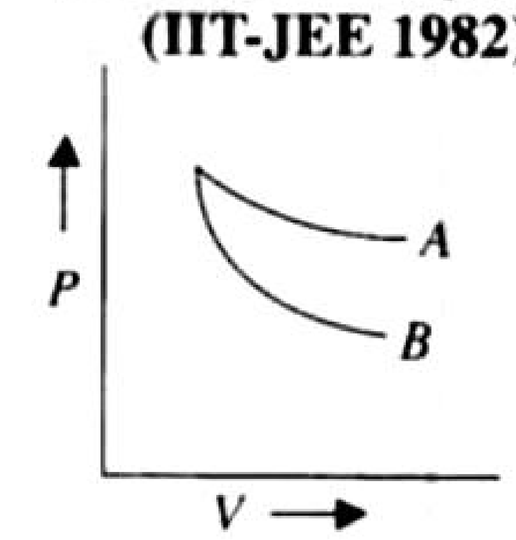
2. The volume V versus temperature T graphs for a certain amount of a perfect gas at two pressures p_1 and p_2 are as shown in the figure. It follows from the graphs that p_1 is greater than p_2.



(IIT-JEE 1982)

3. Two different gases at the same temperature have equal root mean square velocities. **(IIT-JEE 1982)**

4. The curves A and B in the figure show P–V graphs for an isothermal and an adiabatic process of an ideal gas. The isothermal process is represented by the curve A.



(IIT-JEE 1985)

5. The root mean square (rms) speed of oxygen molecules (O_2) at a certain temperature T (degree absolute) is V. If the temperature is doubled and oxygen gas dissociates into atomic oxygen, the rms speed remains unchanged. **(IIT-JEE 1985)**

6. At a given temperature, the specific heat of a gas at constant pressure is always greater than its specific heat at constant volume. **(IIT-JEE 1987)**

7. Two spheres of the same material have radii 1 m and temperatures 4000 K and 2000 K, respectively. The energy radiated per second by the first sphere is greater than that by the second. **(IIT-JEE 1988)**

Subjective Type

1. A composite rod is made by joining a copper rod, end to end, with a second rod of different material but of the same area of cross section. At 25°C, the composite rod is 1 m long and the copper rod is 30 cm long. At 125°C the length of the composite rod increases by 1.91 mm. When the composite rod is prevented from expanding by holding

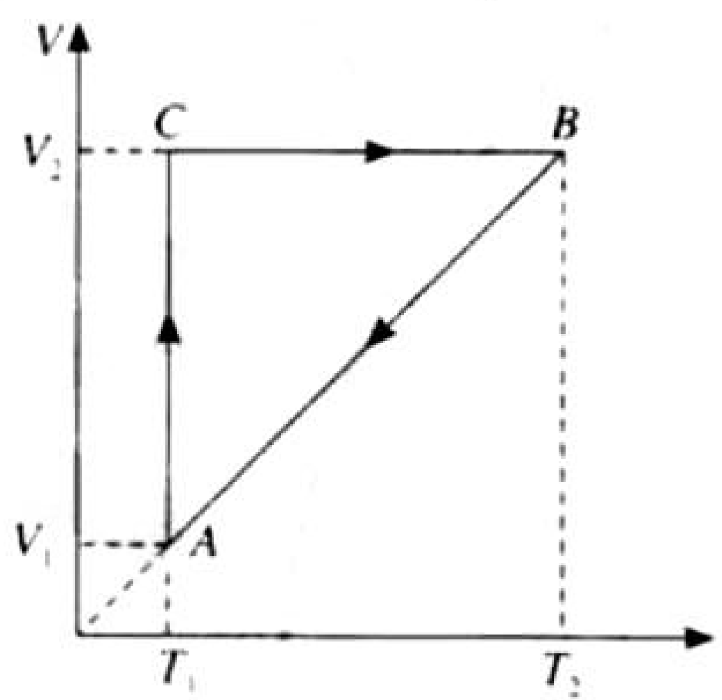
it between two rigid walls, it is found that the constituent rods have remained unchanged in length in spite of rise of temperature. Find Young's modulus and the coefficient of linear expansion of the second rod (Y of copper = 1.3×10^{10} N/m^2 and α of copper = 17×10^{-6}/K).

(IIT-JEE 1979)

2. A body of mass 2 kg is being dragged with a uniform velocity of 2 m/sec on a rough horizontal plane. The coefficient of friction between the body and the surface is 0.20, J = 4.2 J/cal and g = 9.8 m/sec^2. Calculate the amount of heat generated in 5 sec. **(IIT-JEE 1980)**

3. A lead bullet just melts when stopped by an obstacle. Assuming that 25 per cent of the heat is absorbed by the obstacle, find the velocity of the bullet if its initial temperature is 27°C.
(Melting point of lead = 327°C, specific heat of lead = 0.03 calories /gm/°C, latent heat of fusion of lead = 6 calories /gm, J = 4.2 joules / calorie). **(IIT-JEE 1981)**

4. A cyclic process $ABCA$ shown in the V-T diagram (fig) is performed with a constant mass of an ideal gas. Show the same process on a P-V diagram (1981)



5. An ideal gas is enclosed in a vertical cylindrical container and supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. Atmospheric pressure is P_0, and when the piston is in equilibrium, the volume of the gas is V_0. The piston is now displaced slightly from equilibrium position. Assuming that the system is completely isolated from its surroundings, show that the piston executes simple harmonic motion and find the frequency of oscillation. **(IIT-JEE 1981)**

6. Calculate the work done when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 6 litres respectively. The final volume of the gas is 2 litres. Molar specific heat of the gas at constant volume is $3R/2$. **(IIT-JEE 1982)**

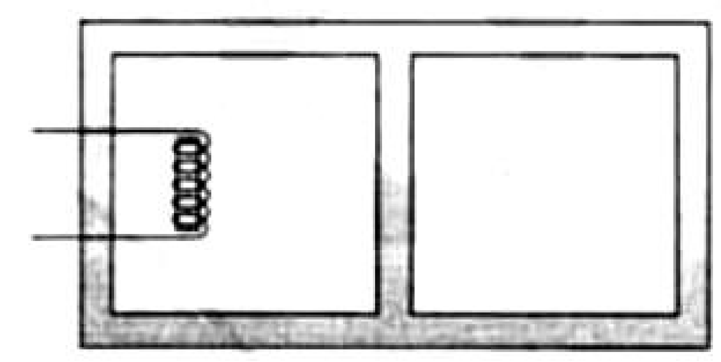
7. A solid sphere of copper of radius R and a hollow sphere of the same material of inner radius r and outer radius A are heated to the same temperature and allowed to cool in the same environment. Which of them starts cooling faster? **(IIT-JEE 1982)**

8. One gram mole of oxygen at 27° and one atmospheric pressure is enclosed in a vessel.

i. Assuming the molecules to be moving with v_{rms}, find the number of collision per second which the molecules make with one square-metre area of the vessel wall.

ii. The vessel is next thermally insulated and moved with a constant speed v_0. It is then suddenly stopped. process results in a rise of temperature of the gas by 1°C. Calculate the speed v_0. **(IIT-JEE 1983)**

9. The rectangular box shown in the figure has a partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has one mole of a mono-atomic ideal gas ($\gamma = 5/3$) at a pressure P_0, volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater. The walls of the box and the partition are thermally insulated. Heat loss through the lead wires of the heater is negligible. The gas in the left chamber expands pushing the partition until the final pressure in both chambers becomes $243\,P_0/32$. Determine (i) the final temperature of the gas in each chamber and (ii) the work done by the gas in the right chamber



(IIT-JEE 1984)

10. Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in a water bath maintained at 62°C. What is the new value of the pressure inside the bulbs? The volume of the connecting tube is negligible. **(IIT-JEE 1985)**

11. An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of +20°C inside it, when the outside temperature is −10°C. The walls have three different layers materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of the electric-heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 watt/m/°C respectively. **(IIT-JEE 1986)**

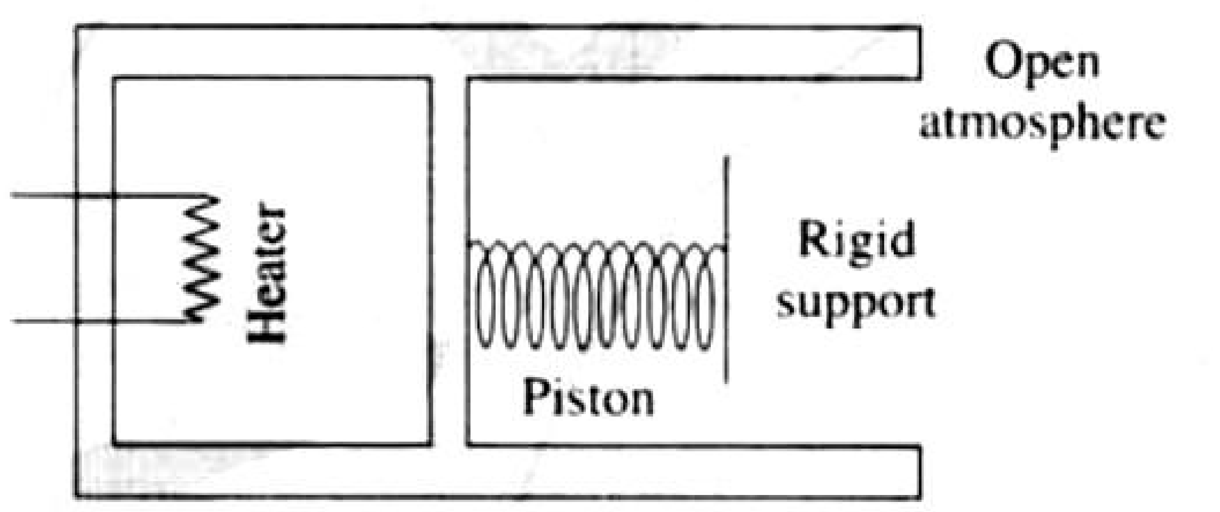
12. An ideal gas has a specific heat at constant pressure $C_P = \dfrac{5R}{2}$. The gas is kept in a closed vessel of volume 0.0083 m^3, at a temperature of 300 K and a pressure of 1.6×10^6 N/m^2. An amount of 2.49×10^4 joules of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas. **(IIT-JEE 1987)**

13. Two moles of helium gas ($\phi = 5/3$) is initially at temperature 27°C and occupies a volume of 20 L. The gas is first expanded at constant pressure until the volume is doubled. Then, it undergoes an adiabatic change until the temperature returns to its initial value.

a. Sketch the process on a P–V diagram.
b. What are the final volume and pressure of the gas?
c. What is the work done by the gas?

(IIT-JEE 1988)

14. An ideal monatomic gas is confined in a cylinder by a spring-loaded piston of cross section 8×10^{-3} m^2. Initially, the gas is at 300 K and occupies a volume of 2.4×10^{-3} m^3 and the spring is in its relaxed (unstretched, uncompressed) state (see figure). The gas is heated by a small electric heater until the piston moves out slowly by 0.1 m. Calculate the final temperature of the gas and the heat supplied (in joule) by the heater. The force constant of the spring is 8000 N/m, atmospheric pressure 1×10^5 N/m^2. The cylinder and the piston are thermally insulated. The piston is massless and there is no friction between the piston and the cylinder. Neglect heat loss through the lead wires of the heart. The heat capacity of the heater coil is negligible. Assume the spring to be massless.



(IIT-JEE 1989)

15. An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to $\frac{T}{2}$.

i. How many degrees of freedom do the gas molecules have?
ii. Obtain the work done by the gas during the expansion as a function of the initial pressure P and volume V.

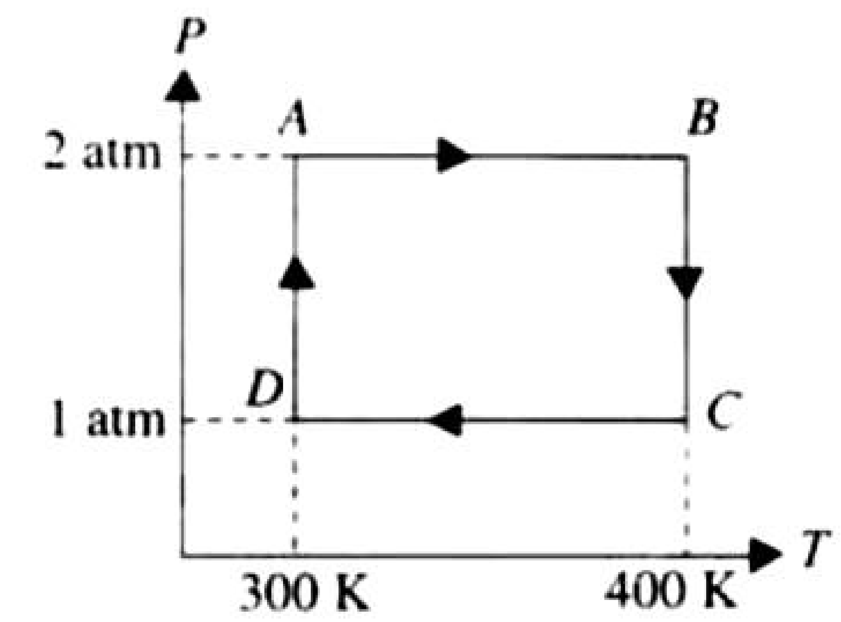
(IIT-JEE 1990)

16. Three moles of an ideal gas ($C_P = 7/2R$) at pressure P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally, the gas is compressed at constant volume to its original pressure P_A.

a. Sketch P–V and P–T diagrams for the complete process.
b. Calculate the new work done by the gas and net heat supplied to the gas during the complete process.

(IIT-JEE 1991)

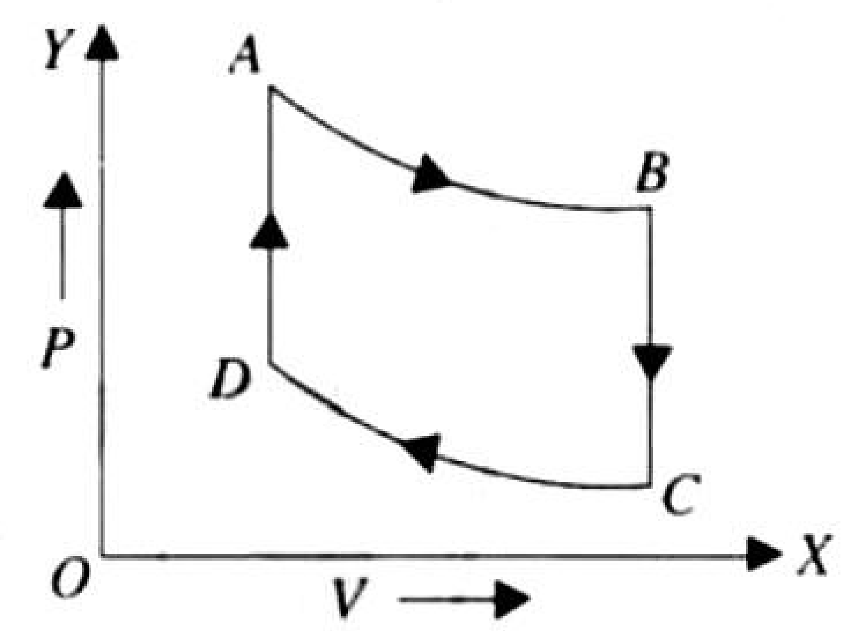
17. Two moles of helium gas undergoes a cyclic process as shown in the figure. Assuming the gas to be ideal, calculate the following quantities in this process.



a. The net change in the heat energy.
b. The net work done.
c. The net change in internal energy. **(IIT-JEE 1992)**

18. A cylindrical block of length 0.4 m and area of cross section 0.04 m^2 is placed coaxially on a thin metal disc of mass 0.4 Kg and the same cross section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 watt/m-k and the specific heat of the material of the disc is 600 J/kg·K, how long will it take for the temperature of the disc to increase to 350 K? Assume, for purpose of calculation, the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder. **(IIT-JEE 1992)**

19. One mole of monatomic ideal gas is taken through the cycle as shown in the figure.



$A \rightarrow B$ adiabatic, expansion
$B \rightarrow C$ cooling at constant volume
$C \rightarrow D$ adiabatic compression
$D \rightarrow A$ heating at constant volume

The pressure and temperature at A, B, etc., are denoted by P_A, P_B, . . . and, T_A, T_B . . . T, respectively. Given that $T_A = 1000$ K, $P_B = (2/3)P_A$ and $P_C = (1/3)P_A$.
Calculate the following quantities:

a. The work done by the gas in process $A \rightarrow B$.
b. The work done by the gas in process $B \rightarrow C$.
c. The temperature T_D.

(Given that $(2/3)^{2/5} = 0.85$) **(IIT-JEE 1993)**

20. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = -2980$ J and $Q_4 = 3645$ J respectively.
The corresponding quantities of work involved are $W_1 = 2200$ J, $W_2 = -825$ J, $W_3 = -1100$ J and W_4 respectively.

1. Find the value of W_4
2. What is the efficiency of the cycle **(IIT-JEE 1994)**

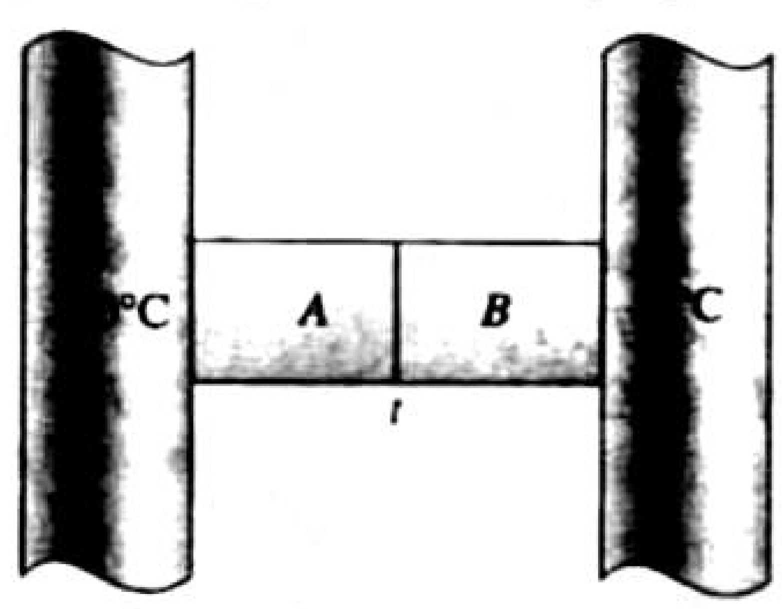
21. A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases, at a temperature of 27°C and pressure of 1×10^5 Nm^{-2}. The total mass of the mixture is 28 g. If the molar masses of neon and argon are 20 and 40 g mol^{-1} respectively, find the masses of the individual gases in the container assuming them to the ideal (Universal gas constant $R = 8.314$ J/mol K). **(IIT-JEE 1994)**

22. A gaseous mixture enclosed in a vessel of volume V consists of one mole of a gas A with $\gamma (= C_p/C_v) = 5/3$ and another gas B with $\gamma = \frac{7}{5}$ at a certain temperature T. The relative molar masses of the gases A and B are 4 and 32, respectively. The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $PV^{19/13}$ = constant, in adiabatic processes.

a. Find the number of moles of the gas B in the gaseous mixture.
b. Compute the speed of sound in the gaseous mixture at $T = 300$ K.
c. If T is raised by 1K from 300 K, find the % change in the speed of sound in the gaseous mixture.
d. The mixture is compressed adiabatically to 1/5 of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities. **(IIT-JEE 1995)**

23. At 27°C two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabatically to a volume $2V$. Calculate (i) the final temperature of the gas, (ii) change in its internal energy, and (iii) the work done by the gas during this process. **(IIT-JEE 1996)**

24. The temperature of 100 g of water is to be raised from 24°C to 90°C by adding steam to it. Calculate the mass of the steam required for this purpose.

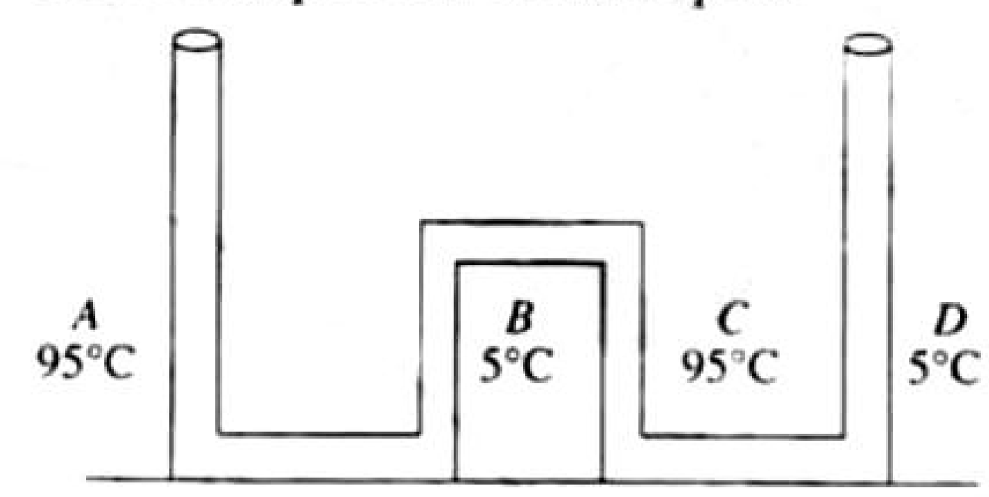


(IIT-JEE 1996)

25. A double-pane window used for insulating a room thermally from outside consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by a 0.05 m thick stagnant air space. In the steady state, the room glass inter-face and the glass-outdoor interface are at constant temperatures of 27°C and 0°C respectively. Calculate the rate of heat flow through the window pane. Also find the temperatures of other interfaces. Given thermal conductivities of glass and air as 0.8 and 0.08 $Wm^{-1}K^{-1}$ respectively. **(IIT-JEE 1997)**

26. One mole of a diatomic ideas gas ($\gamma = 1.4$) is taken through a cyclic process starting from point A. The process $A \to B$ is an adiabatic compression, $B \to C$ is isobaric expansion, $C \to D$ is a adiabatic expansion, and $D \to A$ is isochoric. The volume ratios are $V_A/V_B = 16$ and $V_C/V_B = 2$ and temperature at A is $T_A = 300$ K. Calculate the temperature of the gas at the points B and D and find the efficiency of the cycle. **(IIT-JEE 1997)**

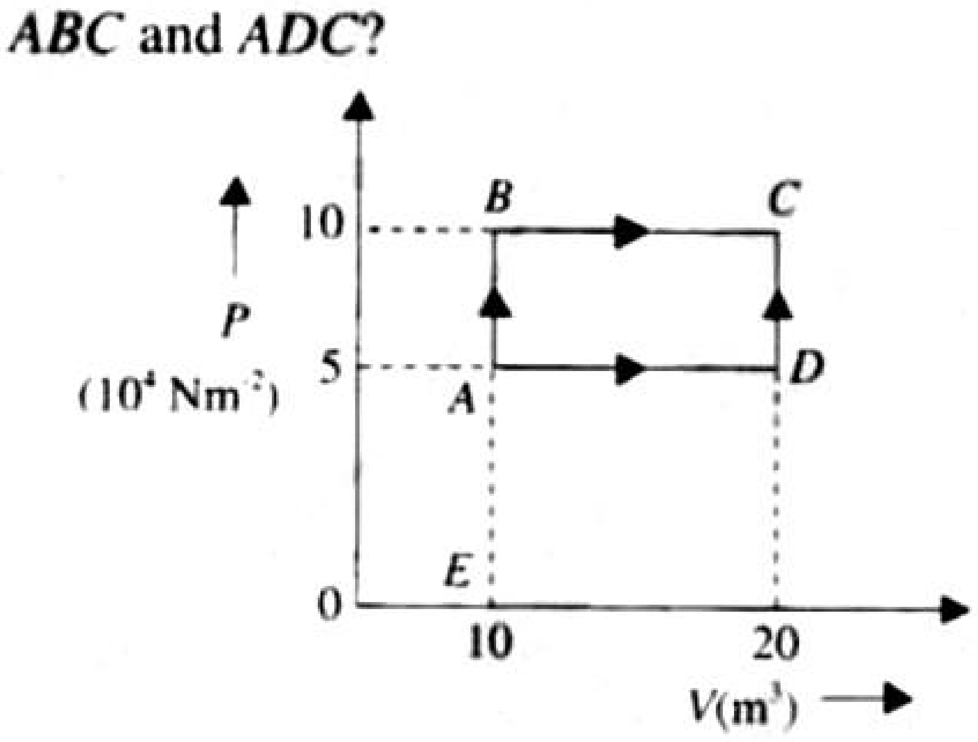
27. The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A and D are open to the atmosphere. A and C are maintained at a temperature of 95°C while the column B and D are maintained at 5°C. The height of the liquid in A and D measured from the base the are 52.8 cm and 51 cm respectively. Determine the coefficient of thermal expansion of the liquid.



(IIT-JEE 1997)

28. A sample of 2 kg of monatomic helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC as shown in the figure. Given molecular mass of helium = 4.

a. What is the temperature of helium in each of the states A, B, C and D?
b. Is there any way of telling afterwards which sample of helium went through the process ABC and which went through the process ADC? Write yes or no.
c. How much is the heat involved in each of the processes ABC and ADC?



(IIT-JEE 1997)

29. A solid body X of heat capacity C is kept in an atmosphere whose temperature $T_A = 300$ K. at time $t = 0$ the temperature of X is $T_0 = 400$ K. It cools according to Newton's laws of cooling. At time t_1, its temperature 350 K.

At this time (t_1). The body connected to a large box Y at atmospheric temperature through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time $t = 3t_1$.

(IIT-JEE 1998)

30. Two moles of an ideal monatomic gas, initially at pressure p_1 and volume V_1, undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2.

a. Sketch the complete process on a p–V diagram.

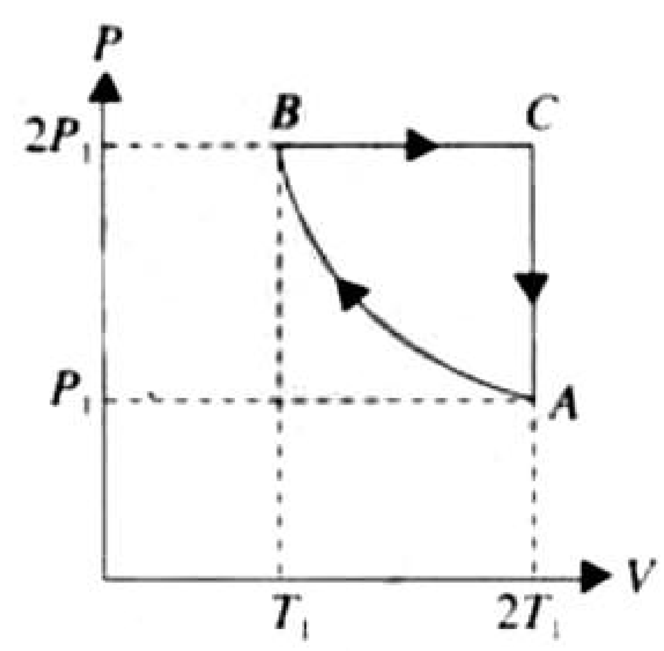
b. Find the total work done by the gas, the total change in its internal energy and the final temperature of the gas. [Give your answer in terms of p_1, V_1, V_2, Q and R] **(IIT-JEE 1999)**

31. Two moles of an ideal monatomic gas is taken through a cycle $ABCA$ as shown in the P–T diagram. During the process AB, pressure and temperature of the gas vary such that PT = constant. If T_1 = 300 K, calculate

a. the work done of the gas in the process AB, and

b. heat absorbed or released by the gas in each of the process.

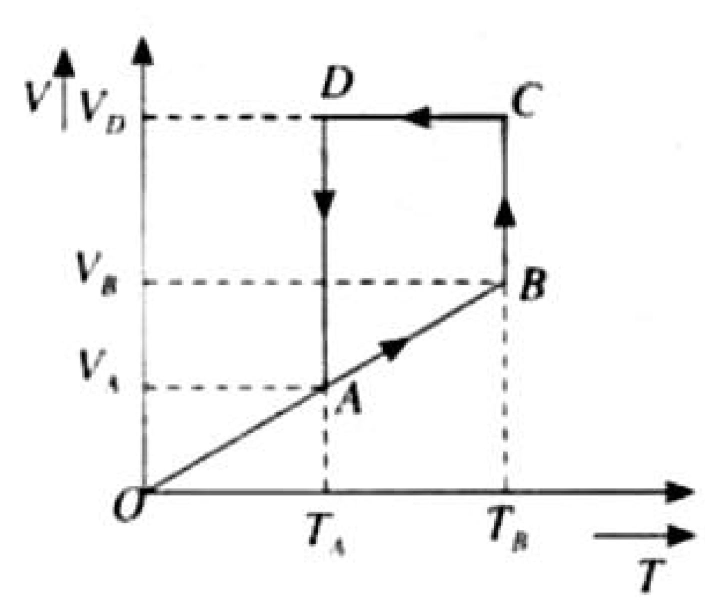
Give answers in terms of the gas constant R.



32. An ice cube of mass 0.1 kg of 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according to the empirical relation $S = A + BT$, where A = 100 cal/kg-K and $B = 2 \times 10^{-2}$ cal/kg-K^2. If the final temperature of the container is 27°C, determine the mass of the container. (Latent heat of fusion of water = 8×10^4 cal/kg, Specific heat of water = 10^3 cal/kg–K). **(IIT-JEE 2001)**

33. A 5 m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically form a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring radiation losses. (For the steel wire: Young's modulus = 2.1×10^{11} Pa; density = 7860 kg/m^3; specific heat = 420 J/kg-k). **(IIT-JEE 2001)**

34. A monoatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in figure. The volume ratios are $\frac{V_B}{V_A} = 2$ and $\frac{V_D}{V_A} = 4$. If the temperature T_A at A is 27°C.



Calculate,

a. the temperature of the gas at point B.

b. heat absorbed or released by the gas in each process.

c. the total work done by the gas during the complete cycle.

Express your answer in terms of gas constant R. **(IIT-JEE 2001)**

35. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 N/m^2. During an observation time of 1 second, an atom travelling with the root-mean-square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. Take $R = \frac{25}{3}$ J/mol-K and $k = 1.38 \times 10^{-23}$ J/K.

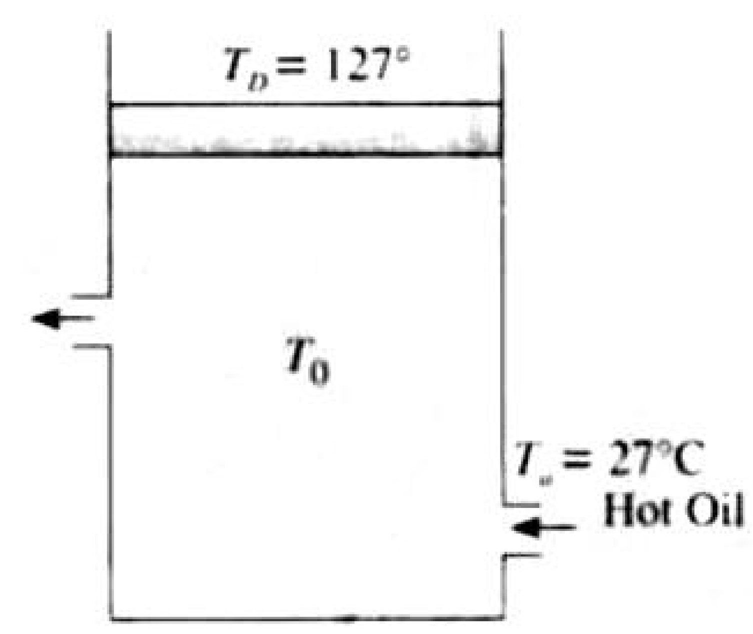
a. Evaluate the temperature of the gas.

b. Evaluate the average kinetic energy per atom.

c. Evaluate the total mass of helium gas in the box. **(IIT-JEE 2002)**

36. An insulated container containing monatomic gas of molar mass M is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature. **(IIT-JEE 2003)**

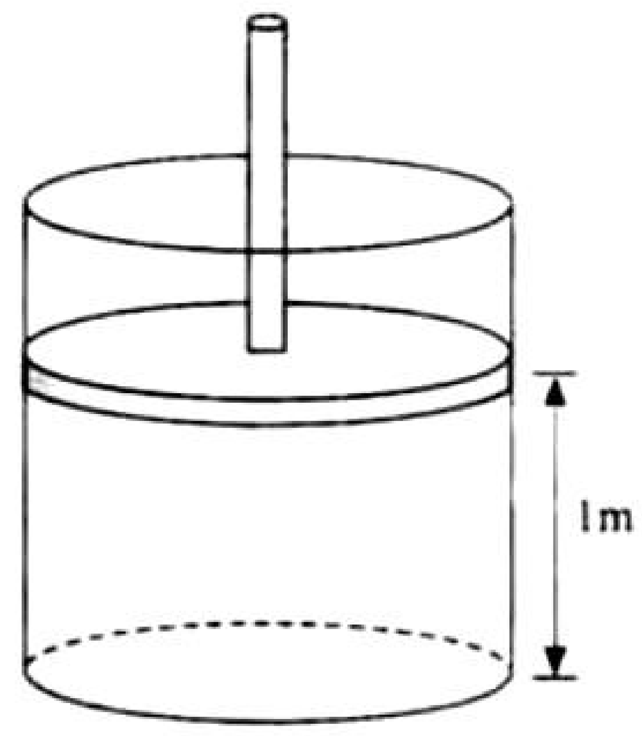
37. Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity K = 0.149 J/(m–°C-sec), thickness t = 5 mm, emissivity = 0.6. Temperature of the top of the lid is maintained at T_t = 127°C. If the ambient temperature T_a = 27°C. Calculate **(IIT-JEE 2003)**



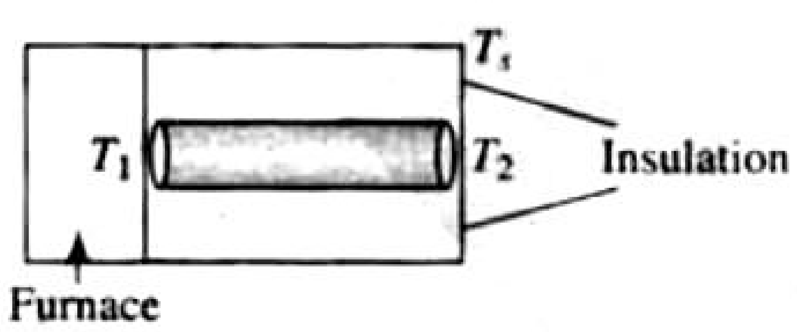
a. rate of heat loss per unit area due to radiation from the lid.

b. temperature of the oil. $\left(\text{Given } \sigma = \frac{17}{3} \times 10^{-8}\right)$

38. A diatomic gas is enclosed in a vessel fitted with massless movable piston. Area of cross section of vessel is 1m^2. Initial height of the piston is 1 m (see the figure). The initial temperature of the gas is 300 K. The temperature of the gas is increased to 400 K, keeping pressure constant, calculate the new height of the piston. The piston is brought to its initial position with now heat exchange. Calculate the final temperature of the gas. You can leave answer infraction. **(IIT-JEE 2004)**



39. A cylindrical rod of length l, thermal conductivity K and area of cross section A has one end in the furnace at temperature T_1 and the other end in surrounding at temperature T_2. Surface of the rod exposed to the surrounding has emissivity e. Also $T_2 = T_S + \Delta T$ and $T_S >> \Delta T$. If $f_1 - T_S \propto \Delta T$, find the proportionality constant.



(IIT-JEE 2004)

40. A cubical block of co-efficient of linear expansion α_s is submerged partially inside a liquid of co-efficient of volume expansion γ_l. On increasing the temperature of the system by ΔT, the height of the cube inside the liquid remains unchanged. Find the relation between α_S and γ_l **(IIT-JEE 2004)**

41. A metal of mass 1 kg at constant atmospheric pressure and at initial temperature 20°C is given a heat of 20000 J. Find the following:

a. change in temperature

b. work done

c. change in internal energy

(**Given:** Specific heat = 400 J/kg/°C, coefficient of cubical expansion, $\gamma = 9 \times 10^{-5}$/°C, density $\rho = 9000$ kg/m^3, atmospheric pressure = 105 N/m^2) **(IIT-JEE 2005)**

42. In an insulated vessel, 0.05 kg steam at 373 K and 0.45 kg of ice at 253 K are mixed. Find the final temperature of the mixture (in Kelvin).

Given, L_{fusion} = 80 cal/g = 336 J/g

$L_{vapourization}$ = 540 cal/g = 2268 J/g

S_{ice} = 2100 J/kg-K= 0.5 cal/g-K

And S_{water} = 4200 J/kg-K = 1 cal/g-K **(IIT-JEE 2006)**

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. b.	**2.** a.	**3.** a.	**4.** b.	**5.** a.
6. b.	**7.** c.	**8.** d.	**9.** b.	**10.** b.
11. d.	**12.** b.	**13.** d.	**14.** b.	**15.** c.
16. c.	**17.** d.	**18.** a.	**19.** c.	**20.** d.
21. d.	**22.** d.	**23.** d.	**24.** a.	**25.** c.
26. a.	**27.** b.	**28.** b.	**29.** a.	**30.** b.
31. a.	**32.** a.	**33.** a.	**34.** c.	**35.** c.
36. None	**37.** b.	**38.** b.	**39.** c.	**40.** d.
41. b.	**42.** a.	**43.** d.	**44.** c.	**45.** a.
46. c.	**47.** c.	**48.** d.	**49.** a.	**50.** c.
51. d.	**52.** d.	**53.** a.	**54.** d.	**55.** a.

Multiple Correct Answers Type

1. a, b, c, d.	**2.** a, b, d.	**3.** a, b.	**4.** a, c.
5. b, c.	**6.** c, d.	**7.** b, d.	**8.** a, d.
9. b, d.	**10.** b, d.	**11.** a, b.	**12.** a, c, d.
13. a, b, c, d.	**14.** a., b., d.	**15.** a., b., c.	

Linked Comprehension Type

1. a. **2.** d. **3.** c. **4.** d. **5.** d

Matching Column Type

1. (i) – (b); (ii) – (a), (d); (iii) – (d); (iv) – (b), (c)

2. (i) – (d); (ii) – (b); (iii) – (a), (b); (iv) – (b), (c)

3. (i) – (b); (ii) – (a), (c); (iii) – (a), (d); (iv) – (b), (d)

4. (i) – (a), (c), (e); (ii) – (a), (c); (iii) – (b), (d); (iv) – (c), (e)

5. (a) (i) – (4); (ii) – (3); (iii) – (2); (iv) – (1)

Integer Answer Type

1. (9) **2.** (9) **3.** (4) **4.** (3) **5.** (2)
6. (2)

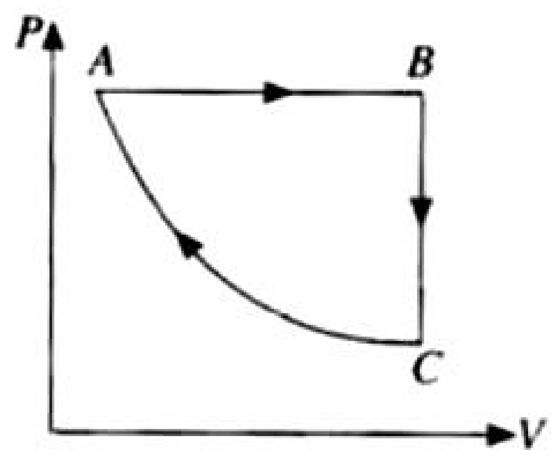
Assertion–Reasoning Type

1. b.

True/False Type

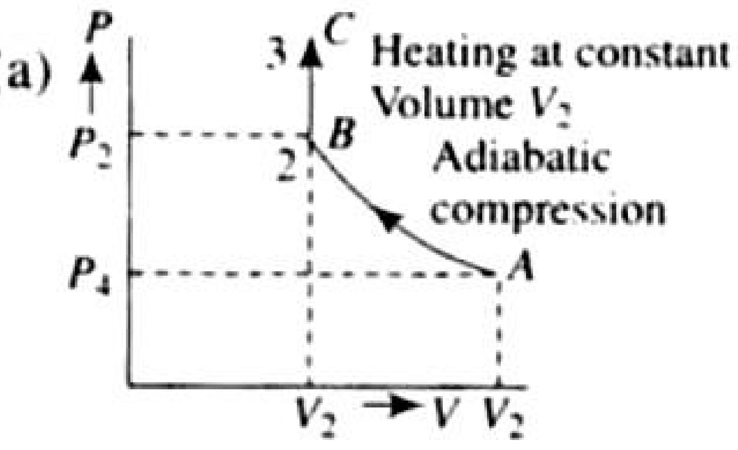
1. False **2.** False **3.** False **4.** True **5.** False
6. True **7.** True

Subjective Type

1. 1.205×10^{11} N/m^2, 2×10^{-5} per °C
2. 9.33 cal 3. 409.87 m/s
4. 

5. $\dfrac{A}{2\pi}\sqrt{\dfrac{p_0\gamma}{V_0 M}}$
6. –973.1 J 7. Hollow sphere
8. (i) 1.97×10^{27} (ii) 36 m/s
9. (i) $T_1 = 12.9T_0$, $T_2 = 2.25T_0$ (ii) $-15.58T_0$ joule
10. 83.75 cm Hg 11. 9000 W 12. 375 K, 3.6×10^6 Nm^{-2}
13. (b) 113 L, 0.44×10^5 N/m^2 (c) 12459 J
14. 800 K, 720 J 15. (i) $f = 5$ (ii) 1.25 PV
16. (b) $0.58\ RT_A$
17. (a) 1152 J (b) 1152 J (c) zero 18. 166.38 sec
19. (a) 1869.75 J (b) 5297.6 J (b) 500 K
20. (i) 765 J (ii) 10.82%
21. Mass of Neon = 4 g ∴ Mass of Argon = 24 g
22. (a) 2 mol (b) 400.03 ms^{-1}

    (c) $\left(\dfrac{1}{6}\right)$% (d) -8.27×10^{-5} V (Pa^{-1})
23. (i) 189 K (ii) 2767 J (iii) –2767 J
24. 12 g 25. 41.54 J/s, 26.48 °C, 0.52 °C
26. $T_B = 909$ K, $T_D = 791$ K, 61.4 % 27. 6.67×10^{-5} °C^{-1}
28. (a) T_A = 120.34 K, T_B = 240.68 K, T_C = 481.36 K, T_D = 240.68 K

    (b) No (c) $Q_{ABC} = 3.25 \times 10^6$ J, $Q_{ADC} = 2.75 \times 10^6$ J
29. $T = [300 + 12.5e^{\frac{-2KAt_1}{CL}}]$ Kelvin
30. (a) 

    (b) (i) $W_{\text{Total}} = \dfrac{3}{2}P_1V_1\left[1 - \left(\dfrac{V_1}{V_2}\right)^{2/3}\right]$

    (ii) $\Delta U_{\text{Total}} = \dfrac{3}{2}P_1V_1\left[1 - \left(\dfrac{V_1}{V_2}\right)^{2/3}\right]$

    (iii) $T_{\text{Final}} = \dfrac{P_1V_1^{5/3}V_2^{-2/3}}{2R} + \dfrac{Q}{3R}$
31. (a) 1200 R (b) $Q_{AB} = -2100$ R, $Q_{BC} = 1500$ R, $Q_{CA} = 831.6$ R
32. 0.495 kg 33. 0.00457 °C
34. (a) 600 K

    (b) (i) 1500 R (ii) 831.8 R (iii) –900 R (iv) –831.8 R

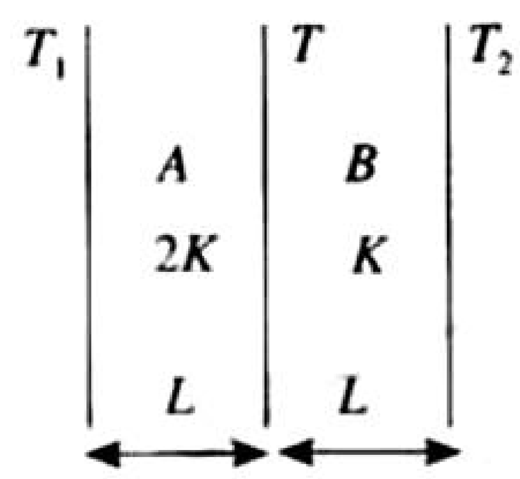
    (c) 600 R
35. (a) 160 K (b) 3.312×10^{-12} J (c) 0.3012 gm
36. $\Delta T = \dfrac{mV_0^2}{3R}$ 37. (a) 595 Watt/m^2 (b) 419.83 K
38. $\dfrac{4}{3}$m, $400\left(\dfrac{4}{3}\right)^{2/5}$ K
39. The proportionality constant = $\left(1 + \dfrac{4\sigma\Delta l T_s^3}{K}\right)$
40. $\gamma = 2$ 41. (a) 50°C (b) 0.05 J (c) 19999.95 J
42. 273 K

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. b. $H = \frac{2kA(T_1 - T)}{L} = \frac{kA(T - T_2)}{L}$



$\Rightarrow \quad 2T_1 - 3T = -T_2$

Adding T_1 on both sides:

$3T_1 - 3T = T_1 - T_2$

$\Rightarrow \quad T_1 - T = \frac{T_1 - T_2}{3}$

$= \frac{36}{3} = 12°C$

2. a. Work done during the cycle = area enclosed in the curve

$= (2P - P)(2V - V) = PV$

3. a. $v_{rms} = \sqrt{\frac{3RT}{M}}$

Room temperature, $T \approx 300$ K

$$1930 = \sqrt{\frac{3 \times 8.31 \times 10^3 \times 300}{M}}$$

$M = 2$ g

or the gas is H_2.

4. b. $Q_1 = nC_P\Delta T$

$Q_2 = nC_V\Delta T$

$$\frac{Q_2}{Q_1} = \frac{C_V}{C_P} = \frac{1}{\gamma}$$

$$Q_2 = \frac{Q_1}{\gamma} = \frac{70}{1.4} = 50 \text{ cal}$$

5. a. Heat required,

$Q = (1.1 + 0.02) \times 10^3 \times 1 \times (80 - 15) = 72800$ cal

Let m gram of steam is condensed, then heat loss

$= m \times 540 + m \times 1 \times 20$

$= 560$ m

Heat loss = Heat gain

$\Rightarrow \quad 560 \text{ m} = 72800$

$\Rightarrow \quad m = 130 \text{ g} = 0.130 \text{ kg}$

6. b. $\gamma_1 = 5/3$ means gas is monatomic or $C_{V_1} = \frac{3}{2}R$

$\gamma_2 = 7/5$ means gas is diatomic or $C_{V_2} = 5/2\,R$

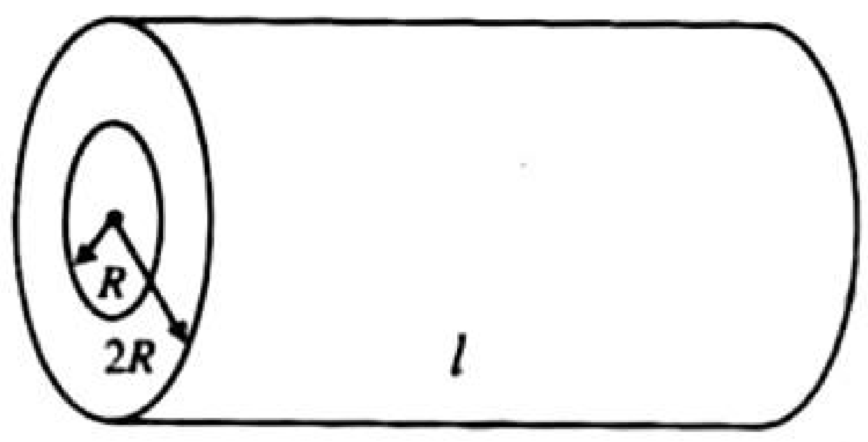
C_V (of the mixture)

$$= \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}\right)R}{1+1} = 2R$$

C_P (of the mixture) $= C_V + R = 3R$

$$\gamma_{\text{mixture}} = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$$

7. c. Let R_1 and R_2 be the thermal resistances of inner and outer portions. Since temperature difference at both ends is same, the resistances are in parallel. Hence,



$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K(4\pi R^2)}{l} = \frac{K_1(\pi R^2)}{l} + \frac{K_2(3\pi R^2)}{l} \Rightarrow K = \frac{3K_2 + K_1}{4}$$

8. d. The desired fraction is

$$f = \frac{\Delta U}{\Delta Q} = \frac{nC_V\Delta T}{nC_P\Delta T} = \frac{C_V}{C_P} = \frac{1}{\gamma}$$

$$f = \frac{5}{7} \qquad \left(\text{as } \gamma = \frac{7}{5}\right)$$

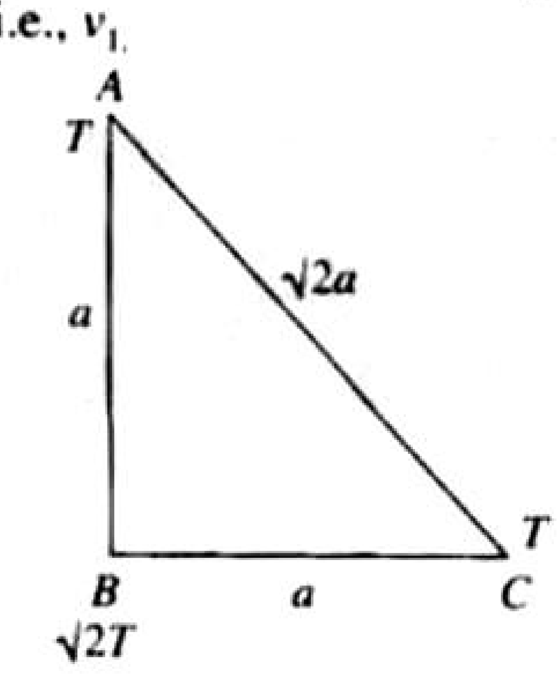
9. b. The average speed of molecules of an ideal gas is given by

$$\langle v \rangle = \sqrt{\frac{8RT}{\pi M}}$$

i.e., $\langle v \rangle \propto \sqrt{T}$ for same gas.

Since temperature of A and C are same, average speed of O_2 molecules will be equal in A and C, i.e., v_1.

10. b. $$\left(\frac{\Delta Q}{\Delta t}\right)_{BC} = \left(\frac{\Delta Q}{\Delta t}\right)_{CA}$$



$$\Rightarrow \frac{kA(\sqrt{2}T - T_C)}{a} = \frac{kA(T_C - T)}{\sqrt{2}a}$$

Solve to get

$$\frac{T_C}{T} = \frac{3}{\sqrt{2}+1}$$

11. d. According to Stefan's law

$$\Delta Q = e\sigma AT^4\Delta t$$

also $\Delta Q = mc\Delta T \Rightarrow \Delta Q = mc\Delta T = e\sigma AT^4\Delta t$

$$\Rightarrow \frac{\Delta T}{\Delta t} = \frac{e\sigma AT^4}{mc} = \frac{e\sigma T^4}{mc}\left[\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}\right]$$

$$= k\left(\frac{1}{m}\right)^{1/3}$$

$$\therefore \frac{\Delta T_1/\Delta t_1}{\Delta T_2/\Delta t_2} = \left(\frac{m_2}{m_1}\right)^{1/3} = \left(\frac{1}{3}\right)^{1/3}$$

12. b. $$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$v_{\text{rms}} \propto \sqrt{T}$$

When temperature is increased from 120 K to 480 K (i.e., four times), the root mean square speed will become $\sqrt{4}$ or 2 times, i.e., $2v$.

13. d. The formula for average kinetic energy is

$$\overline{(\text{KE})} = \frac{3}{2}KT$$

$$\therefore \frac{\overline{(\text{KE})}_{600\text{K}}}{\overline{(\text{KE})}_{300\text{K}}} = \frac{600}{300}$$

$$\Rightarrow \overline{(\text{KE})}_{600\text{K}} = 2 \times 6.21 \times 10^{-21}\,\text{J} = 12.42 \times 10^{-21}\,\text{J}$$

Also the formula for rms velocity is

$$C_{\text{rms}} = \sqrt{\frac{3KT}{m}}$$

$$\therefore \frac{(C_{\text{rms}})_{600\text{K}}}{(C_{\text{rms}})_{300\text{K}}} = \sqrt{\frac{600}{300}}$$

$$\Rightarrow (C_{\text{rms}})_{600\text{K}} = \sqrt{2} \times 484 = 684\ \text{m/s}$$

14. b. According to Wein's displacement law, $\lambda_m T$ = constant where λ_m is the wavelength for which intensity of radiation emitted is maximum

$$(\lambda_m)_S T_S = (\lambda_m)_{NS} \times T_{NS}$$

S = sun

NS = north star

$$\Rightarrow \frac{T_S}{T_{NS}} = \frac{(\lambda_m)_{NS}}{(\lambda_m)_S} = \frac{350}{510} = 0.69$$

15. c. Average translational kinetic energy of an ideal gas molecule is $3/2KT$ which depends on temperature only. Therefore if the temperature is the same, translational kinetic energy of O_2 and N_2 both will be equal.

16. c. $PV = nRT$ or $P = \frac{nRT}{V}$ or $P \propto T$

If V and n are the same. Therefore, if T is doubled, pressure also becomes two times, i.e., $2P$.

17. d. The energy radiated per second by a black body is given by Stefan's law,

$$\frac{E}{t} = \sigma T^4 \times A$$

where A is the surface area of the black body

$$\frac{E}{t} = \sigma T^4 \times 4\pi r^2$$

Since black body is a sphere, $A = 4\pi r^2$

Case (i)

$$\frac{E}{t} = 450, \quad T = 500\ \text{K}, \quad r = 0.12\ \text{m},$$

$$\therefore \quad 450 = 4\pi\sigma(500)^4(0.12)^2 \qquad \text{(i)}$$

Case (ii)

$$\frac{E}{t} = ?, \quad T = 1000\ \text{K}, \quad r = 0.06\ \text{m} \qquad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{E/t}{450} = \frac{(1000)^4(0.06)^2}{(500)^4(0.12)^2} = \frac{2^4}{2^2} = 4$$

$$\Rightarrow \frac{E}{t} = 450 \times 4 = 1800 \text{ W}$$

18. a. Average kinetic energy per molecule per degree of freedom = $1/2kT$. Since both the gases are diatomic and are at same temperature (300 K), both will have the same number of rotational degree of freedom, i.e., two. Therefore, both the gases will have the same average rotational kinetic energy per molecule

$$= 2 \times \frac{1}{2}kT = kT$$

Thus, the ratio will be 1:1.

19. c. Process is isothermal. Therefore, T = constant. Volume is increasing; therefore, pressure will decrease $\left(p \propto \frac{1}{V}\right)$.

In chamber $A \rightarrow$

$$\Delta p = (p_A)_i - (p_A)_f = \frac{n_ART}{V} - \frac{n_ART}{2V}$$

$$= \frac{n_ART}{2V} \quad \text{(i)}$$

In chamber $B \rightarrow$

$$1.5\Delta p = (p_B)_i - (p_B)_f = \frac{n_BRT}{V} - \frac{n_BRT}{2V}$$

$$= \frac{n_BRT}{2V} \quad \text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{n_A}{n_B} = \frac{1}{1.5} = \frac{2}{3}$$

$$\frac{m_A/M}{m_B/M} = \frac{2}{3}$$

$$\frac{m_A}{m_B} = \frac{2}{3}$$

$$3m_A = 2m_B$$

20. d. A is free to move; therefore, heat will be supplied at constant pressure.

$$dQ_A = nC_PdT_A \quad \text{(i)}$$

B is held fixed, therefore, heat will be supplied at constant volume.

$$dQ_B = nC_VdT_B \quad \text{(ii)}$$

But $dQ_A = dQ_B$

$$nC_PdT_A = nC_VdT_B$$

$$dT_B = \left(\frac{C_P}{C_V}\right)dT_A$$

$$= \gamma(dT_A) \quad [\gamma = 1.4 \text{ (diatomic)}]$$
$$(dT_A = 30 \text{ K})$$

$$= (1.4)(30 \text{ K})$$

$$dT_B = 42 \text{ K}$$

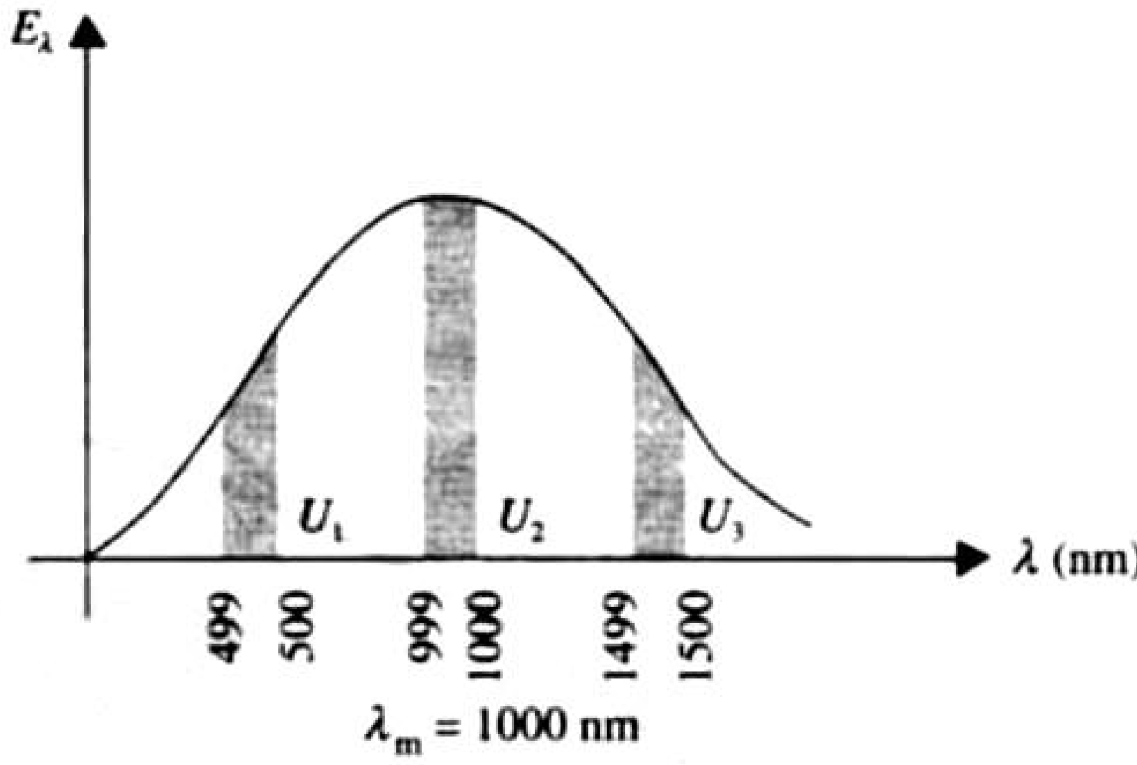
21. d. Wien's displacement law is

$$\lambda_m T = b \quad (b = \text{Wien's constant})$$

$$\lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6 \text{ nm-K}}{2880 \text{ K}}$$

$$\lambda = 1000 \text{ nm}$$

Energy distribution with wavelength will be as follows:



From the graph it is clear that

$$U_2 > U_1 \quad \text{(in fact } U_2 \text{ is maximum)}$$

22. d. The internal energy of n moles of a gas is $u = \frac{1}{2}nFRT$

where F = number of degrees of freedom.

The internal energy of 2 moles of oxygen at temperature T is

$$u_1 = \frac{1}{2} \times 2 \times 5RT = 5RT \quad (F = 5 \text{ for oxygen molecule})$$

Total internal energy of 4 moles of argon at temperature T is

$$= u_2 = \frac{1}{2} \times 4 \times 3RT = 6RT$$

Total internal energy = $u_1 + u_2 = 11RT$

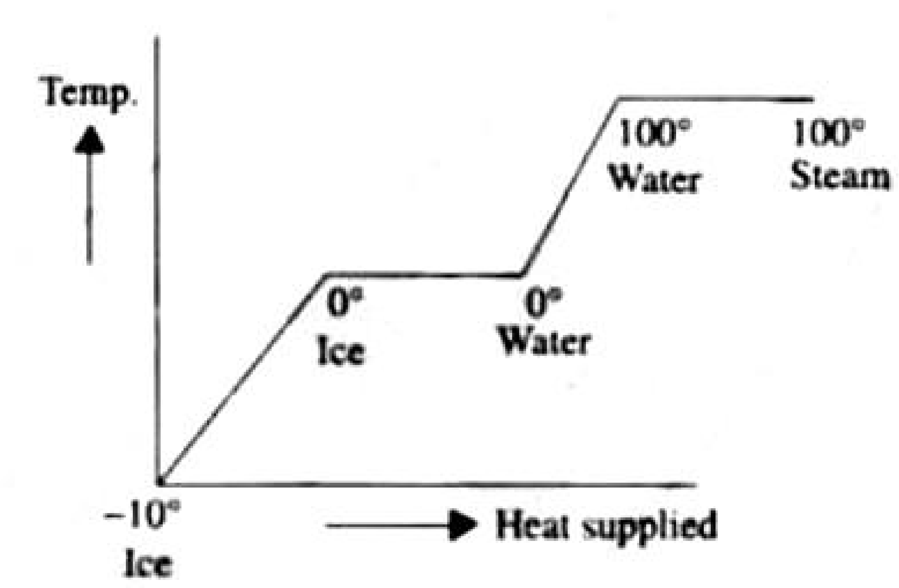
23. d. Here $TV^{\gamma-1}$ = constant

As $\gamma = 5/3$, hence $TV^{2/3}$ = constant

Now $T_1L_1^{2/3} = T_2L_2^{2/3}$ $\quad (\because V \propto L)$

Hence $\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$

24. a.



25. c. We know that V/T = constant

$$\frac{V + \Delta V}{T + \Delta T} = \frac{V}{T} \quad \text{or} \quad VT + T\Delta V = VT + V\Delta T$$

$$\text{or} \quad T\Delta V = V\Delta T \quad \text{or} \quad \frac{\Delta V}{V\Delta T} = \frac{1}{T}$$

This is represented by graph (c).

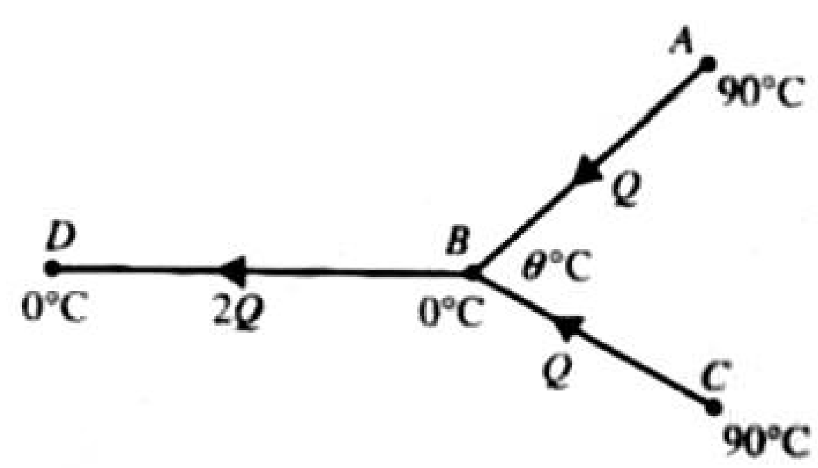
26. a. Work done is equal to area under the curve on PV diagram. (**a**) is the correct option.

27. b. According to Wien's law, λT = constant.
From the graph

$$\lambda_1 < \lambda_3 < \lambda_2$$

$$\therefore \quad T_1 > T_3 > T_2$$

28. b. Let θ°C be the temperature at B. Let Q be the heat flowing per second from A to B on account of temperature difference by conductivity.



$$\therefore \quad Q = \frac{KA(90-\theta)}{l} \quad \text{(i)}$$

where k = thermal conductivity of the rod, A = Area of cross section of the rod, l = length of the rod. By symmetry, the same will be the case for heat flow from C to B.
$\therefore$ The heat flowing per second from B to D will be

$$2Q = \frac{KA(\theta - 0)}{l} \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i)

$$2 = \frac{\theta}{90-\theta} \Rightarrow \theta = 60°\text{C}$$

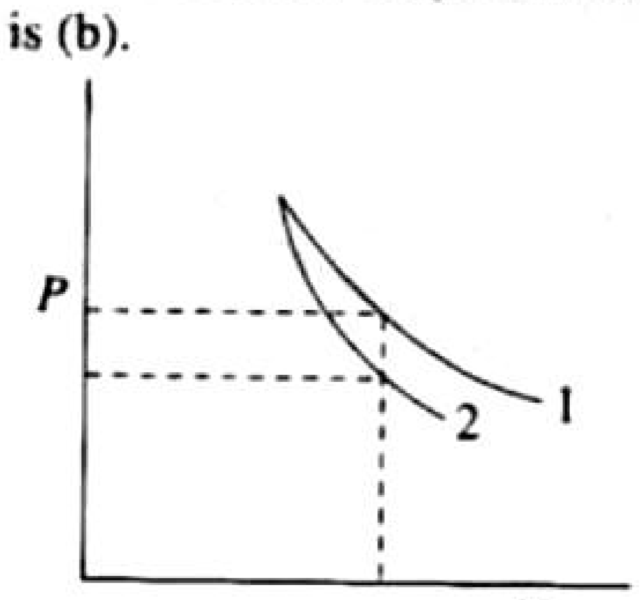
29. a. From the first law of thermodynamics
$dQ = dU + dW$
Here $dW = 0$ (given)
$\therefore \quad dQ = dU$
Now since $dQ < 0$ (given)
$\therefore \quad dQ$ is negative $\Rightarrow dU = -$ve
$\Rightarrow \quad dU$ decreases $\Rightarrow$ Temperature decreases.
$\therefore$ The correct option is (a).

30. b. For adiabatic process, PV^γ = constant

For monatomic gas, $\gamma = \dfrac{C_p}{C_v} = 1.67$

For diatomic gas, $\quad \gamma = 1.4$

Since $\gamma_{\text{diatomic}} < \gamma_{\text{monatomic}}$, so with increase in volume, decrease in pressure will be more for monatomic gas.
$\Rightarrow$ Graph 1 is for diatomic and Graph 2 is for monatomic.
Correct option is (b).



31. a. For cyclic process,

$$Q_{\text{cyclic}} = W_{AB} + W_{BC} + W_{CA}$$

$$= 10\text{ J} + 0 + W_{CA} = 5\text{ J} \quad \Rightarrow W_{CA} = -5\text{ J}$$

32. a. PV = constant

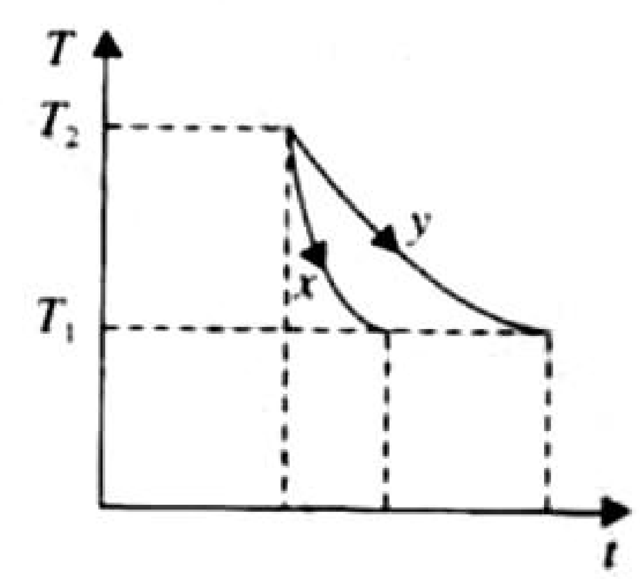
Differentiating, $\dfrac{PdV}{dP} = -V$

$$\beta = -\left(\frac{1}{V}\right)\left(\frac{dV}{dP}\right) = \left(\frac{1}{P}\right) \quad \Rightarrow \beta \times P = 1$$

Therefore, the graph between β and P will be a rectangular hyperbola. (a) is the correct option.

33. a. When the temperature of black body becomes equal to the temperature of furnace, it will radiate maximum energy, so it will be brightest. Initially it will absorb all radiations, so it will be darkest.

34. c. The graph shows that for the same temperature difference $(T_2 - T_1)$, less time is taken for x. This means the emissivity is more for x. According to Kirchhoff's law, a good emitter is a good absorber as well.



35. c. The length of each rod increases by the same amount

$$\therefore \quad \Delta l_a = \Delta l_s \quad \Rightarrow l_1\alpha_a t = l_2\alpha_s t$$

$$\Rightarrow \frac{l_2}{l_1} = \frac{\alpha_a}{\alpha_s} \quad \Rightarrow \frac{l_2}{l_1} + 1 = \frac{\alpha_a}{\alpha_s} + 1$$

$$\Rightarrow \frac{l_2 + l_1}{l_1} = \frac{\alpha_a + \alpha_s}{\alpha_s} \quad \Rightarrow \frac{l_1}{l_1 + l_2} = \frac{\alpha_s}{\alpha_a + \alpha_s}$$

36. None. Process AC cannot be adiabatic, because in adiabatic expansion, temperature decreases.

37. b. Heat required to convert 5 kg of water at 20°C to 5 kg of water at 0°C = $mC_w\Delta T = 5 \times 1 \times 20 = 100$ kcal.
Heat released by 2 kg ice at –20°C to convert 2 kg of ice at 0°C = $mC_{ice}\Delta T = 2 \times 0.5 \times 20 = 20$ kcal.
How much ice at 0°C will convert into water at 0°C for giving another 80 kcal of heat $Q = mL$
$\Rightarrow 80 = m \times 80 \Rightarrow m = 1$ kg
Therefore, the amount of water at 0°C = 5 kg + 1 kg = 6 kg.
Thus, at equilibrium we have
(6 kg water at 0°C + 1 kg ice at 0°C)

38. b. We know that $\lambda_m T = c \quad \lambda_A < \lambda_B < \lambda_C$

So, $\quad T_A > T_B > T_C$

$$\left\{\because T_A = \frac{C}{3\times10^{-7}},\ T_B = \frac{C}{4\times10^{-7}},\ T_C = \frac{C}{5\times10^{-7}}\right\}$$

$Q = e\sigma AT^4$ (e = 1 black body)

$\therefore \quad Q = \sigma AT^4$

$\therefore \quad Q_A = \sigma\pi(2\times10^{-2})^2 \times \dfrac{C^4}{27\times10^{-28}}$

and $\quad Q_B = \sigma\pi(4\times10^{-2})^2 \times \dfrac{C^2}{64\times10^{-28}}$

$Q_C = \sigma\pi(6\times10^{-2})^2 \times \dfrac{C^2}{625\times10^{-28}}$

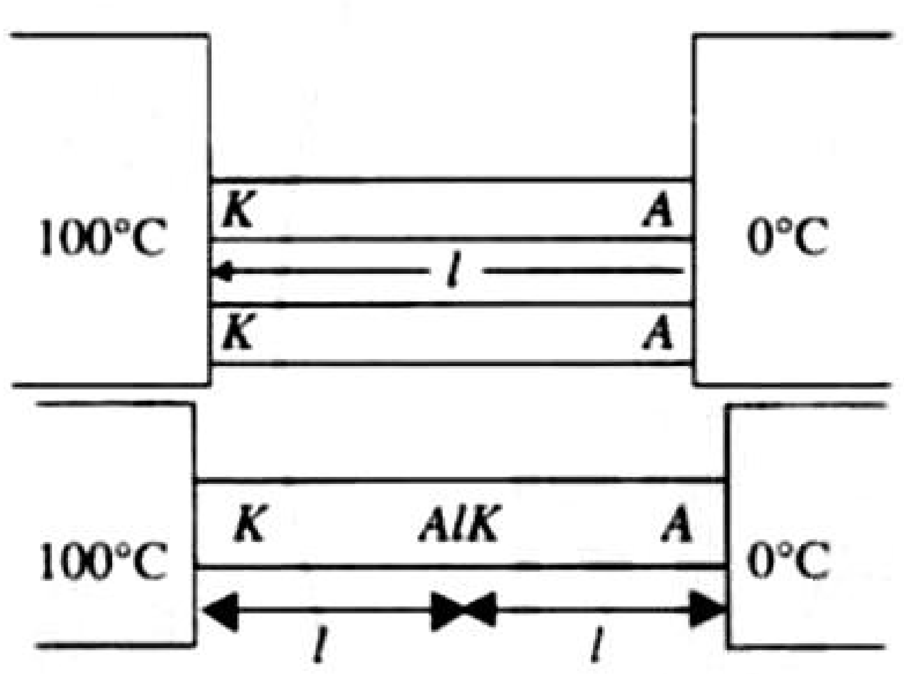
From comparison, Q_B is maximum.

39. c. Temperature of liquid oxygen will first increase till boiling point.
Then phase change will take place from liquid to gas during which temperature remains same. After this temperature of gaseous oxygen will further increase. All this is correctly shown by graph (c).

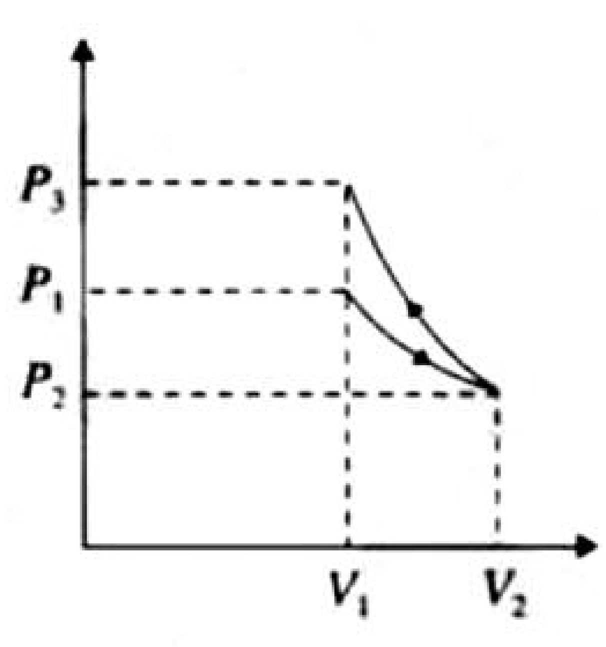
40. d. $q_1 = \dfrac{K2A(100)}{l}$

$q_2 = \dfrac{A(100)}{\frac{l}{k}+\frac{l}{k}} = \dfrac{KA(100)}{2l}$

$\therefore \quad \dfrac{q_2}{q_1} = \dfrac{KA(100)}{2l} \times \dfrac{l}{K2A(100)} = \dfrac{1}{4}$



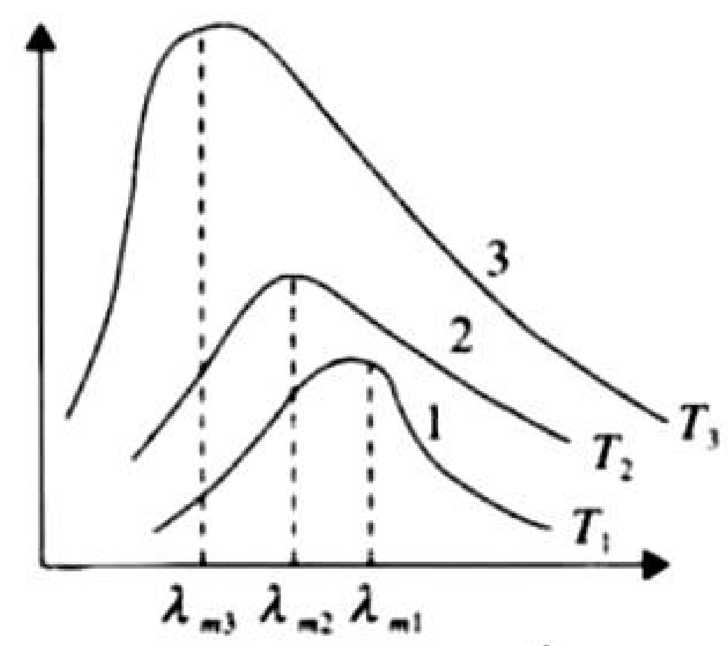
41. b. In the first process W is +ve as ΔV is +ve, in the second process W is –ve as ΔV is –ve and area under the curve of second process is more. Therefore, the net work < 0 and also $P_3 > P_1$.



42. a. According to Wien's displacement law
$\lambda_m \times T$ = constant

Here $\lambda_{m3} < \lambda_{m2} < \lambda_{m1}$

$\Rightarrow T_3 > T_2 > T_1$



The temperature of the sun is higher than that of the welding arc which in turn is greater than the tungsten filament. (a) is the correct option.

43. d. Heating of glass bulb through filament is through radiation.

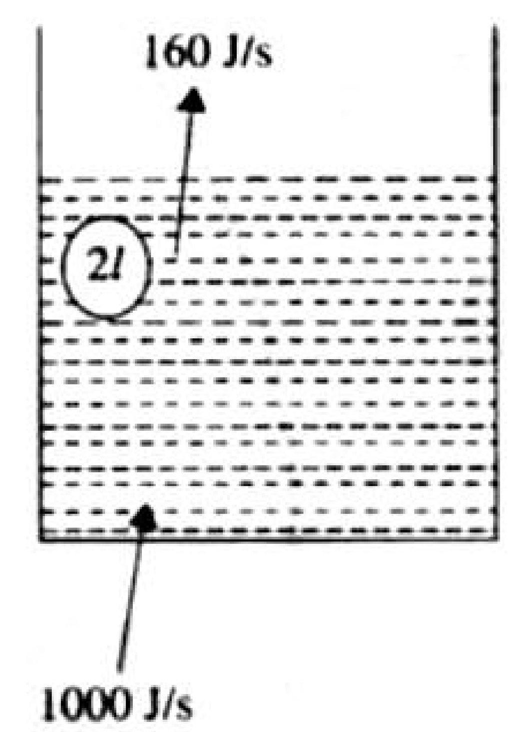
44. c. Heat radiated at temperature T:
$Q = e\sigma AT^4 = 0.6\sigma AT^4$

45. a. One calorie is the heat required to raise the temperature of 1 g of water from 14.5°C to 15.5°C at 760 mm of Hg.

46. c. As shown in the figure, the net heat absorbed by the water to raise its temperature = (1000 – 160) = 840 J/s. Now, the heat required to raise the temperature of water from 27°C to 77°C is $Q = mc\Delta T = 2 \times 4200 \times 50$ J. Therefore the time required

$$t = \frac{Q}{840} = \frac{2\times4200\times50}{840} = 500 \text{ s}$$

= 8 min 20 s



47. c. pT^2 = constant

$\left(\dfrac{nRT}{V}\right)T^2$ = constant

T^3V^{-1} = constant

Differentiating the equation, we get

$\dfrac{3T^2}{V}dT - \dfrac{T^3}{V^2}dV = 0$

$3dT = \dfrac{T}{V}dV$ (i)

From the equation, $dV = V\gamma dT$
γ = coefficient of volume expansion of gas = dV/VdT.

From Eq. (i), $\gamma = \dfrac{dV}{VdT} = \dfrac{3}{T}$

48. d. Theory based

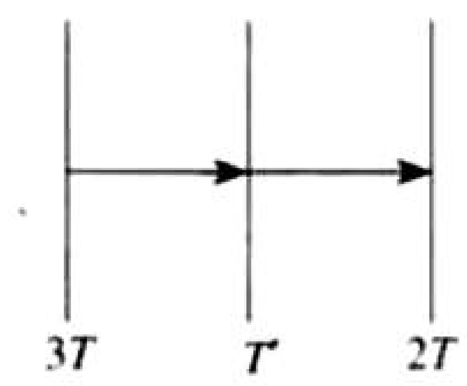
49. a. Number of moles of He = 5.6/22.4 = 1/4

Now $\quad T(5.6)^{\gamma-1} = T_2(0.7)^{\gamma-1}$

$T_1 = T_2\left(\dfrac{1}{8}\right)^{2/3} \Rightarrow 4T_1 = T_2$

$$\text{Work done} = -\frac{nR[T_2 - T_1]}{\gamma - 1} = \frac{\frac{1}{4}R[3T_1]}{\frac{2}{3}} = -\frac{9}{8}RT_1$$

50. c.



In steady state energy absorbed by middle plate is equal to energy released by middle plate.

$$\sigma A(3T)^4 - \sigma A(T')^4 = \sigma A(T')^4 - \sigma A(2T)^4$$

$$(3T)^4 - (T')^4 = (T')^4 - (2T)^4$$

$$(2T')^4 = (16 + 81)T^4$$

$$T' = \left(\frac{97}{2}\right)^{1/4} T$$

51. d. $$\frac{v_{rms_{He}}}{v_{rms_{Ar}}} = \frac{\sqrt{\frac{3RT}{m_{He}}}}{\sqrt{\frac{3RT}{m_{Ar}}}} = \sqrt{\frac{m_{Ar}}{m_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$$

52. d. $\Delta Q = nC_P\Delta T$

$$= 2\left(\frac{f}{2}R + R\right)\Delta T$$

$$= 2\left[\frac{3}{2}R + R\right] \times 5$$

$$= 2 \times \frac{5}{2} \times 8.31 \times 5 = 208 \text{ J}$$

53. a. $$R_1 = \frac{L}{kA} + \frac{L}{2kA} = \frac{3L}{2kA}$$

$$\frac{1}{R_2} = \frac{1}{\left(\frac{L}{kA}\right)} + \frac{1}{\left(\frac{L}{2kA}\right)} = \frac{3kA}{L}$$

$$R_2 = \frac{1}{3kA}$$

$$\Delta Q_1 = \Delta Q_2$$

$$\frac{\Delta T_1}{R_1}t_1 = \frac{\Delta T_2}{R_2}t_2$$

$$\Rightarrow \quad t_2 = \frac{R_2}{R_1}t_1 = 2\text{s}$$

54. d. $PV = nRT = \frac{m}{M}RT$

$$\Rightarrow PM = \rho RT$$

$$\frac{\rho_1}{\rho_2} = \frac{P_1M_1}{P_2M_2} = \left(\frac{P_1}{P_2}\right) \times \left(\frac{M_1}{M_2}\right) = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$

Here ρ_1 and ρ_2 are the densities of gases in the vessel containing the mixture.

55. a. Rate of radiation energy last by the sphere = rate of radiation energy incident

$$= \sigma(4\pi r^2)[T^4 - (300)^4] = 912 \times \pi r^2$$

$$\sigma \times T^4 = \sigma(300)^4 + \frac{912}{4}$$

$$T^4 = (300)^4 + \frac{912}{(4 \times 5.7 \times 10^{-8})}$$

$$= (300)^4 + \frac{912}{22.8} \times 10^8$$

$$= (300)^4 + (40 \times 10^8) = (81 + 40) \times 10^8$$

$$= 121 \times 10^8 \quad \therefore T = 330 \text{ K}$$

Multiple Correct Answer Type

1. a, b, c, d.

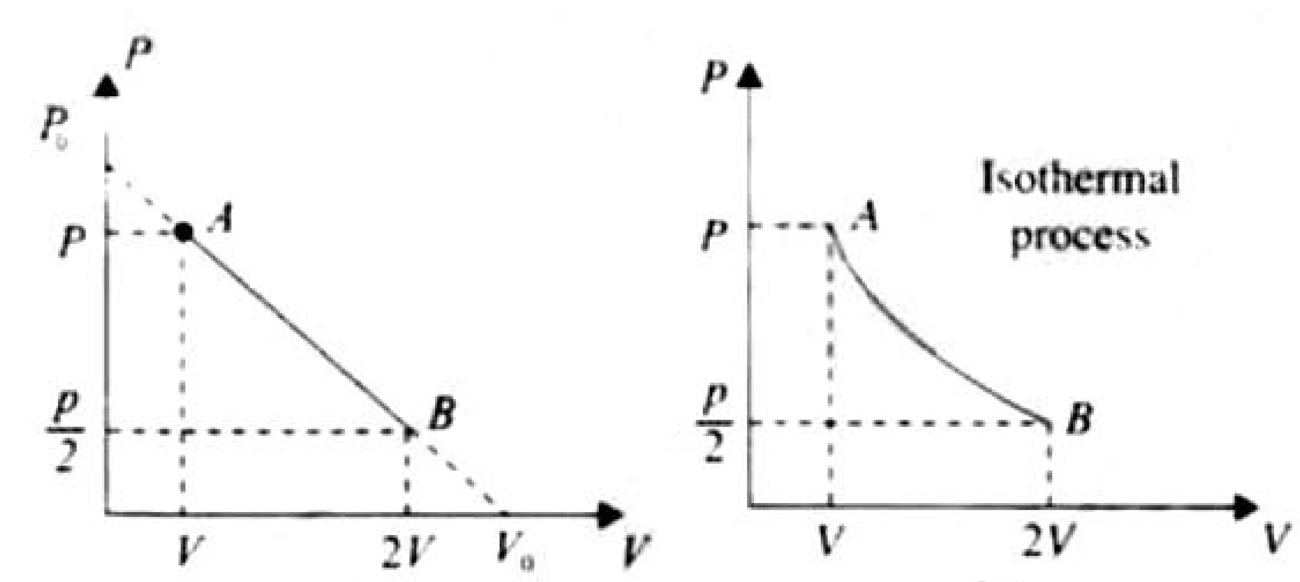
$\Delta U = nC_v\Delta T$ → for any process

In adiabatic process: $\Delta Q = 0$, so $\Delta U = -\Delta w \Rightarrow |\Delta U| = |\Delta w|$

In isothermal process, $\Delta T = 0$, so $\Delta U = 0$

2. a, b, d.

Work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along the isotherm. This is because the work done is the area under the P–V indicator diagram. As shown, the area under the graph in the first diagram will be more than that in the second diagram. When we extrapolate the graph shown in figure(a), let P_0 be the intercept on the P-axis and V_0 be the intercept on the V-axis. The equation of the line AB can be written as



(a) (b)

$$P = -\frac{P_0}{V_0}V + P_0 \quad [\because y = mx + c] \tag{i}$$

To find a relationship between P and T, we use

$$PV = RT \quad \Rightarrow V = \frac{RT}{P} \tag{ii}$$

From Eqs. (i) and (ii),

$$P = -\frac{P_0}{V_0} \times \frac{RT}{P} + P_0$$

$$\Rightarrow P^2V_0 - PP_0V_0 = -P_0RT \tag{iii}$$

Relation between P and T is the equation of a parabola.

Also $PV = RT$

$$\therefore \quad P = \frac{RT}{V} \qquad \text{(iii)}$$

From Eqs. (i) and (ii),

$$\frac{RT}{V} = -\frac{P_0}{V_0}V + P_0$$

$$\Rightarrow RT = -\frac{P_0}{V_0}V^2 + P_0 V \qquad \text{(iv)}$$

The above equation is of a parabola (between T and V)

$$T = -\frac{P_0}{V_0 R}V^2 + \frac{P_0}{R}V$$

Differentiating the above equation w.r.t. V we get

$$\frac{dT}{dV} = -\frac{P_0}{V_0 R} \times 2V + \frac{P_0}{R}$$

when $\frac{dT}{dV} = 0$,

then $\frac{P_0}{V_0 R} \times 2V = \frac{P_0}{R} \quad \Rightarrow V = \frac{V_0}{2}$

Also $\frac{d^2T}{d^2V} = \frac{-2P_0}{V_0 R} = -\text{ve}$

$\Rightarrow V = V_0/2$ is the value of maxima of temperature

Also $P_A V_A = P_B V_B \quad \Rightarrow T_A = T_B$ (From Boyle's law)

$\Rightarrow$ In going from A to B, the temperature of the gas first increases to a maximum (at $V = V_0/2$) and the decreases and reaches back to the same value.

3. a, b.

Energy emitted per second by body $A = \varepsilon_A \sigma T_A{}^4 A$ where A is the surface area.

Energy emitted per second by body $B = \varepsilon_B \sigma T_B{}^4 A$

Given that power radiated is equal

$$\varepsilon_A \sigma T_A{}^4 A = \varepsilon_B \sigma T_B{}^4 A, \quad \varepsilon_A T_A{}^4 = \varepsilon_B T_B{}^4$$

$$\Rightarrow T_B = \left(\frac{\varepsilon_A}{\varepsilon_B}\right)^{1/4} T_A = 1934 \text{ K}$$

According to Wien's displacement law $(\lambda_m) \propto \frac{1}{T}$

Since temperature of A is more, therefore $(\lambda_m)_A$ is less

$$\therefore \quad (\lambda_m)_B - (\lambda_m)_A = 1 \times 10^{-6}\text{m} \quad \textbf{(given)} \qquad \text{(i)}$$

Also according to Wien's displacement law

$$(\lambda_m)_A T_A = (\lambda_m)_B T_B$$

$$\Rightarrow \frac{(\lambda_m)_A}{(\lambda_m)_B} = \frac{T_B}{T_A} = \frac{1934}{5802} = \frac{1}{3} \qquad \text{(ii)}$$

On solving Eqs. (i) and (ii),

we get $\lambda_B = 1.5 \times 10^{-6}\text{m}$.

4. a, c.

For 1 mole of an ideal gas

$$pV = RT \qquad \text{(i)}$$

at constant pressure:

$$PdV = RdT \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{dV}{V} = \frac{dT}{T}$$

The coefficient of volume expansion at constant pressure is given by

$$\frac{dV}{VdT} = \frac{1}{T}$$

same for all gases at same temperature.

The average translational kinetic energy per molecule is $(3/2)kT$ and not $3kT$. With decrease in pressure, volume of the gas increases so its mean free path increases. [Option (c)]

The average translational kinetic energy of the molecules is independent of their nature, so each component of the gaseous mixture will have the same value of average translational kinetic energy.

5. b, c.

There is a decrease in volume during melting of an ice slab at 273 K. Therefore, negative work is done by ice–water system on the atmosphere or positive work is done on the ice–water system by the atmosphere. Hence option (**b**) is correct. Second, heat is absorbed during melting (i.e., dQ is positive) and as we have seen, work done by ice–water system is negative (dW is negative.) Therefore, from the first law of thermodynamics, $dU = dQ - dW$, with change in internal energy of ice–water system, dU will be positive or internal energy will increase.

6. c, d.

$$v_{rms} = \sqrt{\frac{3RT}{M}}, \ \bar{v} = \sqrt{\frac{8}{\pi} \cdot \frac{RT}{M}} \approx \sqrt{\frac{2.5RT}{M}}$$

and $v_p = \sqrt{\frac{2RT}{M}}$

From these expressions we can see that

$$v_p < \bar{v} < v_{rms}$$

Second, $v_{rms} = \sqrt{\frac{3}{2}} v_p$

and average kinetic energy of a gas molecule

$$= \frac{1}{2} m v_{rms}^2$$

$$= \frac{1}{2} m \left(\sqrt{\frac{3}{2}} v_p\right)^2 = \frac{3}{4} m v_p^2$$

7. b, d.

The expression of radius of curvature R is

$$R = \frac{d}{(\alpha_1 - \alpha_2)\Delta t}$$

Thus, $R \propto \frac{1}{\Delta t}$ and $R \propto \frac{1}{|\alpha_B - \alpha_C|}$

8. a, d.

Since the sun rays fall on the black body, it will absorb radiations and since its temperature is constant, it will emit radiations. The temperature will remain same only when energy emitted is equal to energy absorbed.

9. b, d.

For monatomic gas,

$$C_v = \frac{3}{2}R, \quad C_p = \frac{5}{2}R$$

For diatomic gas,

$$C_v = \frac{5}{2}R, \quad C_p = \frac{7}{5}R$$

10. b, d.

$$\Delta Q = \Delta U + W$$

For process $B \to C \to D$

ΔU is negative and W is also negative, so ΔQ is also negative, hence heat flows out during this process.

A to B, work is +ve. B to C work is –ve. But +ve work is more, so net work is not zero in process $A \to B \to C$.

11. a, b.

Process AB is isothermal. Temperature at A and B is the same, so internal energy at A and B is the same.

$$\Delta W_{AB} = nRT_0 \ln(V_B/V_A)$$

For point A: $P_0V_0 = nRT_0$

So $\Delta W_{AB} = P_0V_0 \ln(4V_0/V_0) = P_0V_0 \ln(4)$

Nothing can be said about pressure and temperature at C.

12. a., c., d.

a. At steady state, heat flow through A and E are same. So option (a) is correct and option (b) is incorrect.

c. $\Delta T = H \times R$, where H is heat current.

'H' is same for A and E but R is smallest for E. So temperature difference across slab E is smallest. So option (c) is correct

d. $H_B = \frac{\Delta T}{R_B}$, $H_C = \frac{\Delta T}{R_C}$ and $H_D = \frac{\Delta T}{R_D}$.

From this option if, $H_C = H_B + H_D$

$$\Rightarrow \frac{1}{R_C} = \frac{1}{R_B} + \frac{1}{R_D}$$

$$\Rightarrow \frac{4K(2Lb)}{4L} = \frac{3K(Lb)}{4L} + \frac{5K(Lb)}{4L}$$

which is true. Here b is the width of slab.

Hence option (d) is also correct.

13. a., b., c., d.

Option (a) is correct because the graph between 0–100 K appears to be a straight line upto a reasonable approximation.

Option (b) is correct because area under the curve in the temperature range 0–100 K is less than in range 400–500 K.

Option (c) is correct because the graph of C versus T is constant in the temperature range 400–500 K.

Option (d) is correct because in the temperature range 200–300 K specific heat capacity increases with temperature.

14. a, b, d.

For the mixture equivalent degrees of freedom is calculated using the relation $F_{eq} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$

$$f_{eq} = \frac{5+3}{2} = 4$$

$$U = \frac{f}{2}nRT$$

For mixture, $U = \frac{4}{2}nRT$

$\therefore$ Energy per mole: $\frac{U}{n} = 2RT$

For mixture, $\gamma = 1 + \frac{2}{f} = 1 + \frac{1}{2} = \frac{3}{2}$

$$M_{mix} = \frac{1 \times 2 + 1 \times 4}{2} = 3$$

$$\gamma_{He} = 1 + \frac{2}{3} = \frac{5}{3}$$

Ratio of speed of sound = $\sqrt{\frac{\gamma_{mix}}{M_{mix}} \frac{M_{He}}{\gamma_{He}}} = \sqrt{\frac{6}{5}}$

$$V_{rms} = \sqrt{\frac{3RT}{M}} \text{ ratio} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

15. a, b, c.

We will assume that the gas on the side of container having spring is connected to atmosphere. Hence, its pressure remains constant at P_1.

In final position of Piston, force balance gives:

$$Kx + P_1A = P_2A$$

Where A = cross sectional area of piston.

So, $Kx = (P_2 - P_1)A$ (i)

Also, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

If $V_2 = 2V_1, T_2 = 3T_1$, then $P_2 = \frac{3}{2}P_1$

Also, $V_2 - V_1 = Ax$ (ii)

(i) and (ii) give: $Kx = (P_2 - P_1)\frac{(V_2 - V_1)}{x}$

$\therefore$ $\frac{1}{2}Kx^2 = \frac{1}{2}(P_2 - P_1)(V_2 - V_1)$ (iii)

$$= \frac{1}{2}\left(\frac{P_1}{2}\right)(V_1) = \frac{P_1V_1}{4}$$

Hence, (a) is correct.

$$\Delta U = \frac{f}{2}(P_2V_2 - P_1V_1)$$

$$= \frac{3}{2}\left[\frac{3}{2} \times 2P_1V_1 - P_1V_1\right] = 3P_1V_1$$

Hence, (b) is correct as well.

Net work done on piston by all forces is zero. So,

$$W_{gas} = \frac{1}{2}Kx^2 - P_1(V_2 - V_1) = 0$$

$\therefore$ $W_{gas} = \frac{P_1V_1}{3} + 2P_1V_1$

$$W_{gas} = \frac{7}{3}P_1V_1$$

So, (c) also turns out to be correct.

$$Q = \Delta U + W$$

$$\Delta U = \frac{3}{2}(P_2V_2 - P_1V_1) = \frac{3}{2}\left(\frac{4}{3}P_1 \cdot 3V_1 - P_1V_1\right)$$

$$\Rightarrow \quad \Delta U = \frac{9}{2}P_1V_1$$

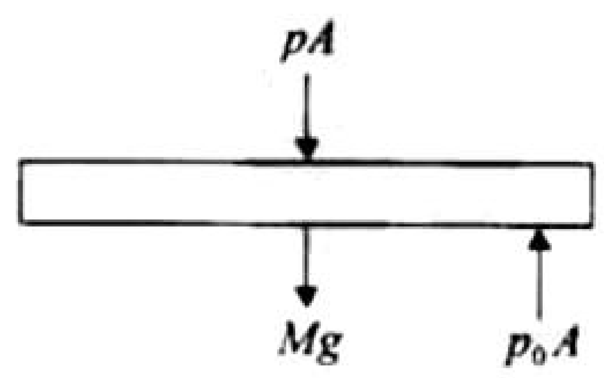
$$\therefore \quad Q = \frac{9}{2}P_1V_1 + \frac{7}{3}P_1V_1 \quad \Rightarrow Q = \frac{41}{6}P_1V_1$$

Hence (d) is not correct.

Linked Comprehension Type

1. a. Since it is open from top, pressure will be p_0. Therefore, option (a) is correct.

2. d. Let p be the pressure in equilibrium, then $pA = p_0A - Mg$



$$p = p_0 - \frac{Mg}{A} = p_0 - \frac{Mg}{\pi R^2}$$

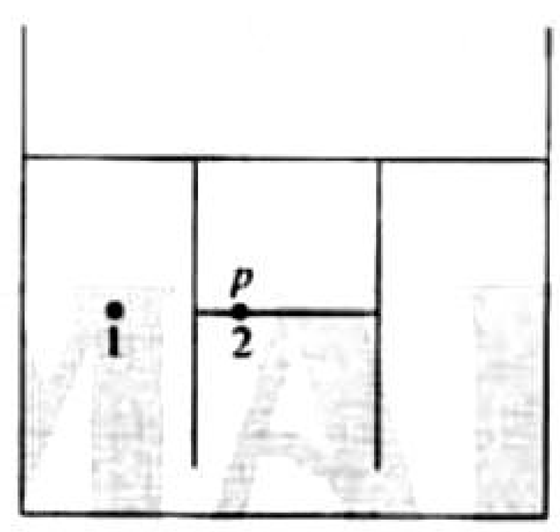
Applying $p_1V_1 = p_2V_2$

$$L' = \frac{2p_0L}{p} = \left(\frac{p_0}{p_0 - \dfrac{Mg}{\pi R^2}}\right)(2L)$$

$$= \left(\frac{p_0\pi R^2}{\pi R^2 p_0 - Mg}\right)(2L)$$

Therefore, option (d) is correct.

3. c. $p_1 = p_2$



$$p_0 + \rho g(L_0 - H) = p \quad \text{(i)}$$

Now, applying $p_1V_1 = p_2V_2$ for the air inside the cylinder, we have

$$p_0(L_0) = p(L_0 - H)$$

$$p = \frac{p_0L_0}{L_0 - H}$$

Substituting in Eq. (i), we have

$$p_0 + \rho g(L_0 - H) = \frac{p_0L_0}{L_0 - H}$$

$$\Rightarrow \quad \rho g(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0 = 0$$

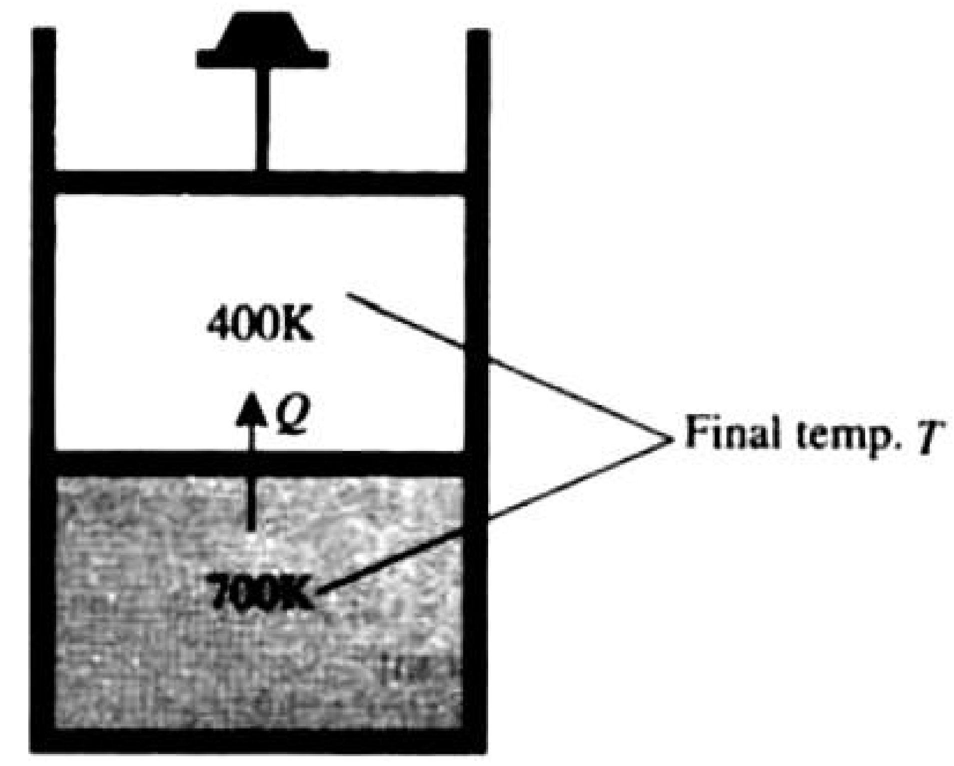
Therefore, option (c) is correct.

4. d. Let final temperature of both compartments is T. Heat given by lower compartment

$$Q = nC_v\Delta T = 2 \times \frac{3}{2}R \times (700 - T) \quad \text{(i)}$$

Heat obtained by upper compartment

$$Q = nC_p\Delta T = 2 \times \frac{7}{2}R \times (T - 400) \quad \text{(ii)}$$



equating (i) and (ii)

$3(700 - T) = 7(T - 400)$

$2100 - 3T = 7T - 2800$

$4900 = 10T \Rightarrow T = 490$ K

5. d. Let final temperature is T. Heat given by lower compartment

$$Q = nC_p\Delta T = 2 \times \frac{5}{2}R \times (700 - T) \quad \text{(i)}$$

Heat obtained by upper compartment

$$Q = nC_p\Delta T = 2 \times \frac{7}{2}R \times (T - 400) \quad \text{(ii)}$$

By equating (i) and (ii)

$5(700 - T) - 7(T - 400)$

$3000 - 5T = 7T - 2800$

$6300 = 12T$

$T = 525$ K

$\therefore$ Work done by lower gas $W_L = P\Delta V = nR\Delta T = -350\,R$

Work done by upper gas $W_U = P\Delta V = nR\Delta T = +250\,R$

Net work done $W_L + W_U = -350 + 250 = -100\,R$

Matching Column Type

1. (i) – (b); (ii) – (a), (d); (iii) – (d); (iv) – (b), (c).

In process $J \rightarrow K$: V is constant whereas p is decreasing. Therefore, T should also decrease.

$W = 0$, $\Delta U =$ negative and $Q < 0$

In process $K \rightarrow L$: p is constant while V is increasing. Therefore, temperature should also increase.

$W > 0$, $\Delta U > 0$ and $Q > 0$

In process $L \rightarrow M$: This is inverse of process $J \rightarrow K$.

$W = 0$, $\Delta U > 0$ and $Q > 0$

In process $M \rightarrow J$: V is decreasing.

Therefore, $W < 0$.

$(pV)_J < (pV)_M$

$T_J < T_M$

$\Delta U < 0$

Therefore, $Q < 0$.

2. (i) – (d); (ii) – (b); (iii) – (a), (b); – (iv) – (b), (c).

Bimetallic strip is based on thermal expansion of materials.

In steam engine, internal energy of fuel (say coal) is converted into mechanical work.

In incandescent lamp and fuse also energy is converted from electrical to heat.

Fuse is based on melting of fuse wire if suddenly current increases.

3. (i) – (b); (ii) – (a), (c); (iii) – (a), (d); (iv) – (b), (d).

(i) In case of free expansion under adiabatic condition, change in internal energy $\Delta U = 0$.
Therefore, internal energy and temperature will remain constant.

(ii) $p \propto \frac{1}{V^2}$

$\therefore \quad pV^2 = \text{constant} \quad \text{(i)}$

$$\left(\frac{nRT}{V}\right) \cdot V^2 = \text{constant}$$

$$T \propto \frac{1}{V} \quad \text{(ii)}$$

If volume is doubled, temperature will decrease as per Eq. (ii).
Further, molar heat capacity in process pV^x = constant.

$$C = C_V + \frac{R}{1-x}$$

From Eq. (i), $x = 2$

$$C = \frac{3}{2}R + \frac{R}{1-2} = +\frac{R}{2}$$

Since molar heat capacity is positive, according to $Q = nC\Delta T$, Q will be negative if ΔT is negative, or gas loses heat if temperature is decreasing.

(iii) $p \propto \frac{1}{V^{4/3}}$

$$pV^{4/3} = \text{constant}$$

$$\left(\frac{nRT}{V}\right)V^{4/3} = \text{constant}$$

$$T \propto \frac{1}{V^{1/3}}$$

Further, with increase in volume, temperature will decrease.
Here, $T \propto \frac{1}{V^{1/3}}$

$$C = \frac{3}{2}R + \frac{R}{1-\frac{4}{3}} = -15\,R$$

As molar heat capacity is negative, Q will be positive if ΔT is negative. The gas gains heat with decrease in temperature.

(iv) $T \propto pV$
In expansion from V_1 to $2V_2$, product of pV is increasing. Therefore, temperature will increase or ΔU = positive. Further, in expansion, work done is also positive.
Hence, $Q = W + \Delta U$ = positive, gas gains heat.

4. (i) – (a), (c), (e); (ii) – (a), (c); (iii) – (b), (d); (iv) – (c), (e)

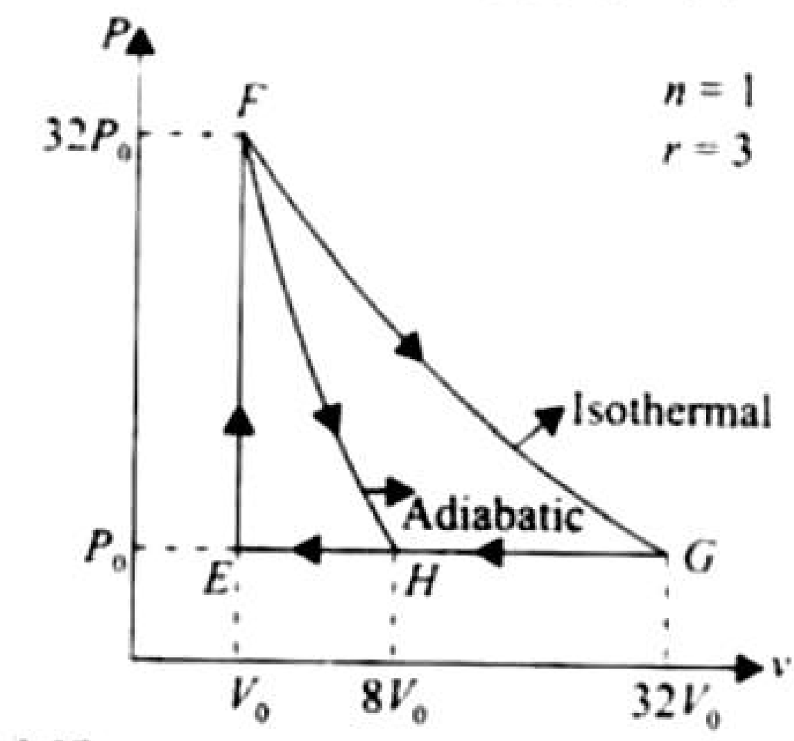
(i) $A \to B: V\downarrow, P$ constant $\to T\downarrow, U\downarrow$ and ΔW is –ve.

(ii) $B \to C$: V is same, $P\downarrow T\downarrow$
No work is done.

(iii) $C \to D$: $V\uparrow \Rightarrow T\uparrow, \Delta U \Rightarrow$ +ve
ΔW = +ve

(iv) $D \to A$: V decreases so $\Delta W \Rightarrow$ –ve
Final temperature is same so $\Delta U = 0$
and then $\Delta Q \Rightarrow$ –ve.

5. (a) (i) – (4); (ii) – (3); (iii) – (2); (iv) – (1).



Apply $PV^{1+2/3}$ = constant for F to H.

$$(32P_0)V_0^{5/3} = P_0V_H^{5/3} \Rightarrow V_H = 8V_0$$

For path FG, PV = constant

$$\Rightarrow (32P_0)V_0 = P_0V_G \Rightarrow V_G = 32V_0$$

Work done in $GE = 31\,P_0V_0$

Work done in $GH = 24\,P_0V_0$

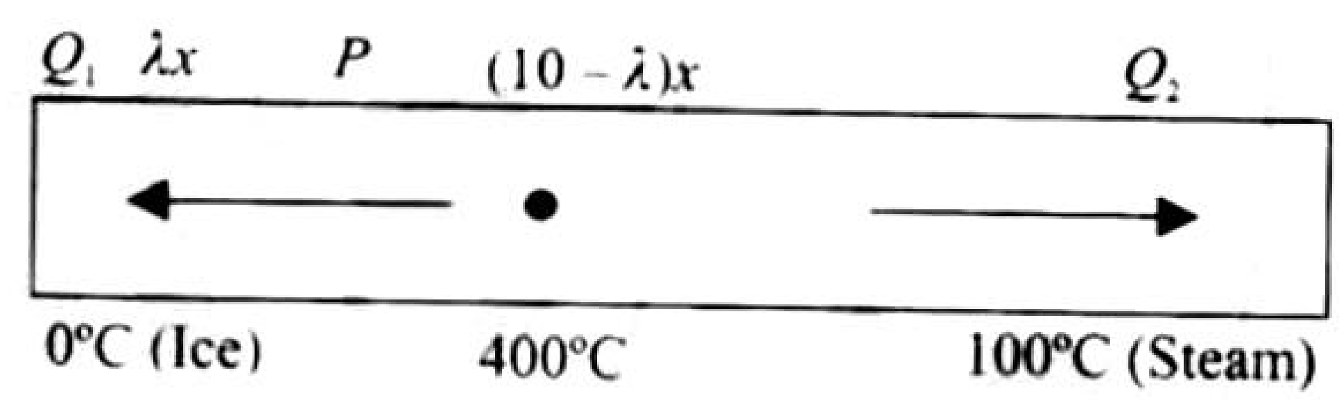
Work done in $FH = \frac{P_HV_H - P_FV_F}{(-2/f)} = 36P_0V_0$

Work done in $FG = RT\ln\left(\frac{V_G}{V_F}\right) = 160P_0V_0\ln 2$

Integer Answer Type

1. (9)

$$\frac{dm_{\text{ice}}}{dt} = \frac{dm_{\text{vapour}}}{dt}$$



$$Q_1 = \frac{KA400}{t} = m \times 80$$

$$Q_1 = \frac{KA(400-100)t}{(10-\lambda)x}t = m \times 540$$

Dividing both, $\lambda = 9$

2. (9)

$\lambda_m T$ = constant, $\lambda_A T_A = \lambda_B T_B$

Now rate of total energy radiated $\propto AT^4$

3. (4)

$$TV^{\lambda-1} = \text{constant}$$

$$TV^{7/5-1} = aT\left(\frac{V}{32}\right)^{7/5-1} \Rightarrow a = 4$$

4. (3)

$$\frac{F}{A} = Y\frac{\Delta L}{L} \Rightarrow \frac{mg}{A} = Y(\alpha\Delta\theta)$$

$$m = \frac{Ay\alpha(\Delta\theta)}{g} = \frac{\pi r^2 y\alpha(\Delta\theta)}{g}$$

$$= \frac{\pi(10^{-3})^2 \times 10^{11} \times 10^{-5} \times 10}{10} = \pi \approx 3\ \text{kg}$$

5. (2)

$\Delta Q_{iaf} = \Delta U_{iaf} + W_{iaf} \Rightarrow 500 = \Delta U_{iaf} + 200$ J

$\Delta U_{iaf} = 300$ J $\therefore \Delta U_{ibf} = 300$ J

$W_{ibf} = W_{ib} + W_{bf} = 50 + 100 = 150$ J

$Q_{ibf} = W_{ibf} + \Delta U_{ibf} = 150 + 300 = 450$

$\therefore \quad \Delta Q_{ibf} = 450\ \text{J} = Q_{ib} + Q_{bf}$

$U_i = 100$ J $U_b = 200$ J $\Delta U_{ib} = 100$ J $W_{ib} = 50$ J

$Q_{ib} = \Delta U_{ib} + W_{ib} \therefore Q_{ib} = 150$ J

$\therefore \quad 450 = Q_{ib} + Q_{bf} \Rightarrow 450 = 150\ \text{J} + Q_{bf}$

$$Q_{bf} = 300\ \text{J} \Rightarrow \frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$$

6. (2) Let A be the surface area of smaller body then,

$$P_A = \sigma \cdot (16 \times 10^4) A T_A^4 \Rightarrow P_B = \sigma A T_B^4$$

$$\frac{16 \times 10^4 T_A^4}{T_B^4} = 10^4 \Rightarrow \left(\frac{T_A}{T_B}\right)^4 = \frac{1}{16}$$

$$\Rightarrow \quad \frac{T_A}{T_B} = \frac{1}{2}$$

By Wein's law, $\lambda T =$ const $\therefore$ ratio $\frac{\lambda_A}{\lambda_B} = 2$

Assertion-Reasoning Type

1. **b.** Total translational kinetic energy

$$= \frac{3}{2} nRT = \frac{3}{2} pV = 1.5\ pV$$

Fill in the Blank Type

1. $$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{(1)\left(\frac{3}{2}R\right) + (1)\left(\frac{5}{2}R\right)}{1+1} = 2R$$

2. AB represents a process when physical state changes from solid to liquid and the temperature remains unchanged. Since P is a point between A and B, the material is partly solid and partly liquid.

3. $PV = RT$ (ideal gas equation)

$$\Rightarrow P = \frac{RT}{V} \quad \text{(i)}$$

Given that

$$VP^2 = \text{constant} \quad \text{(ii)}$$

From Eqs. (i) and (ii),

$$V \times \frac{R^2 T^2}{V^2} = \text{constant}$$

$$\therefore \quad \frac{T^2}{V} = \text{constant}$$

$$\therefore \quad \frac{T_1^2}{V_1} = \frac{T_2^2}{V_2}$$

$$\Rightarrow T_2 = T_1 \sqrt{\frac{V_2}{V_1}} = T\sqrt{\frac{2V}{V}} = \sqrt{2}T$$

4. The heat required for 100 g of ice at 0°C to change its temperature to 0°C

$$= mL = 100 \times 80 \times 4.2 = 33600\ \text{J} \quad \text{(i)}$$

The heat released by 300 g of water at 25°C to change its temperature to 0°C

$$= mc\Delta = 300 \times 4.2 \times 25 = 31{,}500\ \text{J} \quad \text{(ii)}$$

Since the energy in Eq. (ii) is less than that of Eq. (i), the final temperature will be 0°C.

5. The energy received per second per unit area from sun at a distance of 1.5×10^{11} m is 1400 J/sm². The total energy released by sun per second

$$= 1400 \times 4\pi \times (1.5 \times 10^{11})^2$$

Therefore, the total energy released per second per unit surface area of the sun

$$= \frac{1400 \times 4\pi \times (1.5 \times 10^{11})^2}{4\pi \times (7 \times 10^8)^2}$$

This energy E is also equal to $E = \sigma T^4$

$$\Rightarrow \quad T = \left[\frac{1400 \times 4\pi \times (1.5 \times 10^{11})^2}{4\pi \times (7 \times 10^8)^2 \times 5.67 \times 10^{-8}}\right]^{1/4} = 5803\ \text{K}$$

6. The energy emitted per second when the temperature of the copper sphere is T and the surrounding temperature T_0

$$= \sigma(T^4 - T_0^4) \times A$$

where A = surface area

$$= \sigma T^4 A \quad \text{(i)}$$

Here $T_0 = 0$ K

We know that

$$dQ = mcdt$$

$$\therefore \quad \frac{dQ}{dt} = -mc\frac{dT}{dt} \quad \text{(ii)}$$

Here the –ve sign shows that the temperature is decreasing with time.

Energy emitted per second from Eqs. (i) and (ii)

$$\sigma T^4 A = -mc\frac{dT}{dt}$$

$$\Rightarrow \quad dt = -\frac{mcdT}{\sigma T^4 A} = -\frac{\rho \times \frac{4}{3}\pi r^3 c dT}{\sigma T^4 \times 4\pi r^2}$$

$$\left[\therefore m = \rho \times \frac{4}{3}\pi r^3\right]$$

$$\Rightarrow \quad dt = -\frac{\rho rc}{3\sigma}\frac{dT}{T^4}$$

Integrating both sides,

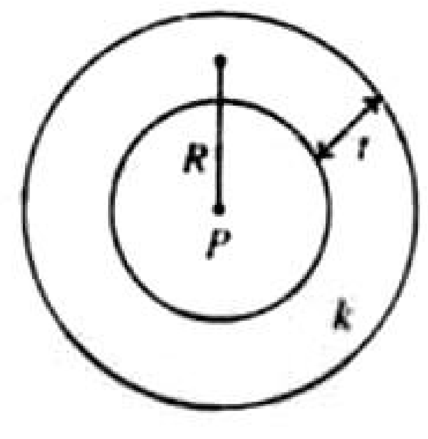
$$\int_0^t dt = -\frac{\rho rc}{3\sigma}\int_{200}^{100}\frac{dT}{T^4} = -\frac{\rho rc}{3\sigma}\left[-\frac{1}{3T^3}\right]_{200}^{100}$$

$$t = \frac{\rho rc}{9\sigma}\left[\frac{1}{(100)^3} - \frac{1}{(200)^3}\right]$$

$$t = \frac{7\rho rc}{(72\times 10^6)\sigma}$$

7. When the spherical shell is thin, $t \ll R$

In this case, the rate of flow of heat from the sphere to the surroundings

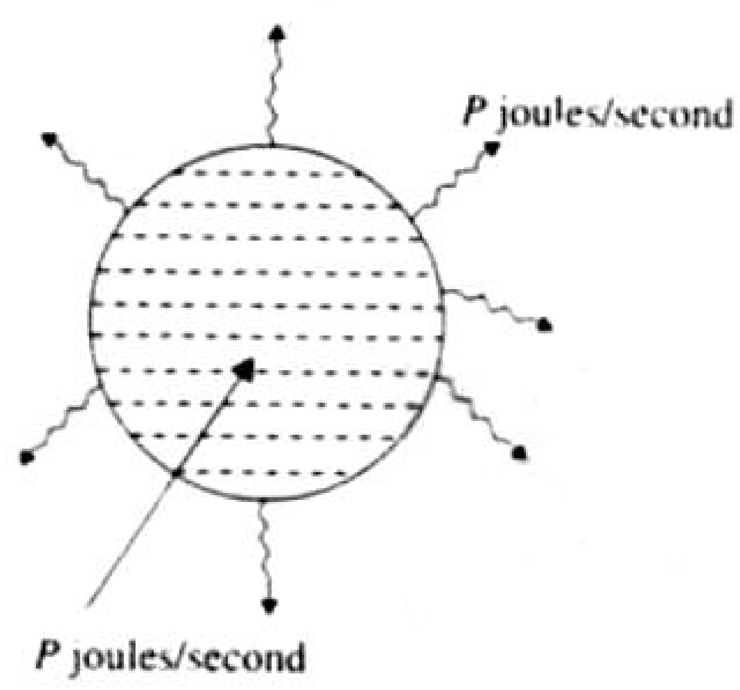


$$P = \frac{K(4\pi R^2)T}{t}$$

where T is the temperature difference and t is the thickness of steel.

$$t = \frac{4\pi R^2 KT}{P}$$

8. Since P joules per second of heat is supplied to keep the substance in molten state, it means that the substance in the molten state at its meeting point releases P joules of heat in one second.



When the power is turned off, the heat input becomes zero. But heat output continues. It takes t seconds for the substance to solidify (given). Therefore total heat released in t seconds $= P \times t$

This is equal to $ML_{\text{fusion}} \Rightarrow ML_{\text{fusion}} = P \times t$

$$L_{\text{fusion}} = \frac{P\times t}{M}$$

9. This is a case of free expansion. Here $\Delta Q = 0$ and $\Delta W = 0$, because gas is doing work on vacuum. So $\Delta U = 0$. Hence no change in temperature, it will remain 300 K.

10. For isothermal expansion,

$$P\times V = P_i 2V \Rightarrow P_1 = \frac{P}{2}$$

For adiabatic expansion,

$$PV^\gamma = P_a \times (2V)^\gamma \Rightarrow P_a = \frac{P}{2^\gamma} = \frac{P}{2^{1.67}}$$

$$\therefore \quad \frac{P_a}{P_i} = \frac{P}{2^{1.67}}\times\frac{2}{P} = \frac{1}{2^{0.67}}$$

11. The heat temperature through A per second

$Q_1 = K_1A(100 - t)/l$

The heat transferred through B per second

$Q_2 = K_2A(t - 0)/l$

At steady state. $\quad K_1A(100 - t)/l = K_2A(t - 0)/l$

$\Rightarrow \quad 300(100 - t) = 200(t - 0)$

$\Rightarrow \quad 300 - 3t = 2t \Rightarrow t = 60°C$

12. Pressure on both sides will be equal

$p_1 = p_2$

i.e., $\frac{n_1RT}{V_1} = \frac{n_2RT}{V_2}$ $\quad \left(n = \frac{m}{M}\right)$

$$\frac{m}{M_1V_1} = \frac{m}{M_2V_2} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{M_1}{M_2} = \frac{32}{28} = \frac{8}{7}$$

but $\frac{V_2}{V_1} = \frac{\alpha}{360 - \alpha}$

Solve to get $\alpha = \left(\frac{360°}{8+7}\right)\times 8 = 192°$

13. In case of gas thermometer, the volume of the gas container remains constant. Therefore

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{3\times 10^4}{273.16} = \frac{3.5\times 10^4}{T_2}$$

$$\Rightarrow T_2 = 318.6\text{ K}$$

14. Solar power received by earth = 1400 W/m^2

Solar power received by 0.2 m^2 area = (1400 W/m^2) (0.2 m^2) = 280 W

Mass of ice = 280 g = 0.280 kg

Heat required to melt ice

= (0.280) (3.3 × 10^5)

= 9.24 × 10^4 J

If t is the time taken for the ice to melt, we will have

$$(280)\, t = 9.24\times 10^4 \text{ J} \left[\because P = \frac{E}{t}\right]$$

$$t = \frac{9.24\times 10^4}{280}\text{ s} = 330\text{ s} = 5.5\text{ min}$$

True/False Type

1. **False.** At the same temperature $\bar{c} \propto \frac{1}{\sqrt{M}}$

i.e., dependent on molar mass and hence $\bar{c}$ will be different for different ideal gases.

2. **False.** $\frac{PV}{T}$ = constant

For a particular temperature T_i

We have $\quad p \propto \frac{1}{V}$

Since, $\quad V_i > V_2, \quad p_2 < p_i$

$\Rightarrow p_1 > p_2$

3. **False.** We know that $c_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

For a particular temperature, $c_{\text{rms}} \propto \sqrt{\frac{1}{M}}$

i.e., c_{rms} will have different values for different gases.

4. **True.** The slope of p-V curve is more for adiabatic process than for isothermal process. From the graph it is clear that slope for B is greater than the slope for A.

5. **False.** $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$v_{rms} \propto \sqrt{\frac{T}{M}} \qquad \text{(i)}$$

When oxygen gas dissociates into atomic oxygen, its atomic mass M will become half. Temperature is doubled. So, from Eq. (i) v_{rms} will become two times.

6. **True.** $C_p > C_v$
This is because at constant pressure when heat is supplied to the gas for increasing temperature, some heat is used up in doing work for increasing volume.

7. **True.** $E \propto T^4$

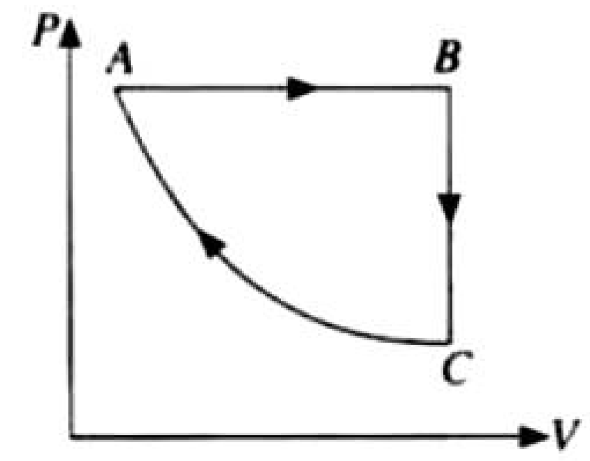
Subjective Type

1. Since total expansion = expansion of the constituent rods
$1.91 \times 10^{-3} = 0.3 \times 17 \times 10^{-6} \times (125 - 25) + 0.7 \times \alpha \times (125 - 25)$
or $1.91 \times 10^{-3} = 5.1 \times 10^{-4} + 70\alpha$
or $70\alpha = (1.91 - 0.51) \times 10^{-3} = 1.4 \times 10^{-3}$
or $\alpha = 2 \times 10^{-5}$/K
When prevented from expanding thermal expansion is accompanied by elastic contraction. Since the lengths remain unchanged, thermal expansion is equal to elastic contraction.
$\therefore$ Strain $= \frac{l\alpha \Delta t}{l} = \alpha \Delta t = 17 \times 10^{-6} \times 100 = 17 \times 10^{-4}$
and stress $= Y \times$ strain $= 1.3 \times 10^{10} \times 17 \times 10^{-4} = 22.1 \times 10^6$
For the second rod
Strain $= \alpha\Delta t = 2 \times 10^{-5} \times 100 = 2 \times 10^{-3}$
Stress $= Y \times$ strain $= Y \times 2 \times 10^{-3}$
But the same stress is effective throughout the composite rod.
$\therefore$ $Y \times 2 \times 10^{-3} = 22.1 \times 10^6$ or $Y = 11.1 \times 10^9$ N/m^2

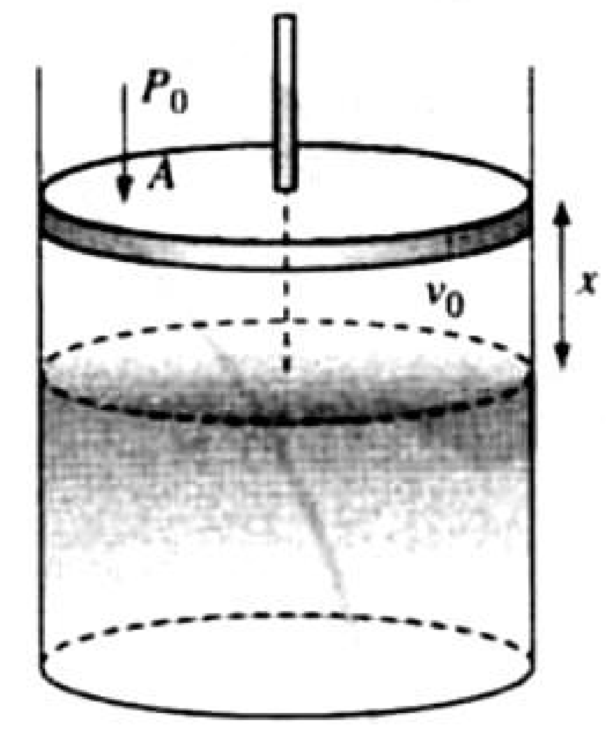
2. Heat generated = work done by the frictional force = f.(displacement)
$W_f = \mu mg \times (v \times t)$
$= 0.2 \times 2 \times 9.8 \times 2 \times 5 = 39.2$ J
Hence amount of heat generated
$$H = \frac{W_f}{4.2} = \frac{39.2}{4.2} \text{cal} = 9.33 \text{ cal}$$

3. The lead bullet just melts when stopped by an obstacle.
Also given that 25% of the heat is absorbed by the obstacle. Therefore 75% heat is utilized in melting of lead. Initial temp. = 27°C
Melting point = 300°C
(0.75) K.E. = Heat utilized in increasing the temperature. From 27°C to 327°C
$$(0.75) \times \frac{1}{2} M v^2 = Mc\Delta T + ML$$
$$(0.75) \times \frac{1}{2} v^2 = (0.03 \times 300 + 6) \times (4.2 \times 1000)$$
[4.2 × 1000 to convert into S.I. system]
which gives $v = 409.87$ m/s

4. In V-T diagram:
The line $A \to B$ indicates
$V \propto T \Rightarrow$ Pressure is constant, i.e., isobaric process.
$B \to C$ Volume is constant. Since the temperature is decreasing, the pressure should also decrease.
$C \to A$ The temperature is constant but volume decreases. The process is isothermal. Hence, p-v diagram should be



5. The initial pressure (equilibrium pressure) of the gas
$$P = P_0 + \frac{Mg}{A} \qquad \text{(i)}$$
When the position of the piston is slightly displaced by a distance x, the change in volume of gas
$dV = -Ax$ (ii)
For adiabatic process PV^γ = constant (iii)
Differentiating equation (iii)
We get $P(\gamma V^{(\gamma-1)} dV) + V^\gamma \cdot dP = 0$
$$dP = -\left(\frac{\gamma P}{V}\right) dV$$
Hence net restoring force on piston (upward direction)



$$\Rightarrow \quad F = dP \cdot A = \left(P_0 + \frac{Mg}{A}\right)\frac{\gamma A^2 x}{V_0}$$
$$\Rightarrow \quad Ma = \left(P_0 + \frac{Mg}{A}\right)\frac{\gamma A^2 x}{V_0}$$
$$\Rightarrow \quad \vec{a} = -\left(P_0 + \frac{Mg}{A}\right)\frac{\gamma A^2}{V_0 M} \cdot \vec{x}$$
Comparing it with $\vec{a} = -\omega^2 \vec{x}$ we get
$$\omega^2 = \left(P_0 + \frac{Mg}{A}\right)\frac{\gamma A^2}{V_0 M}$$
$$\therefore \quad \omega = \sqrt{\left(p_0 + \frac{Mg}{A}\right)\frac{\gamma A^2}{V_0 M}}$$
If $\frac{Mg}{A}$ is small as compared to p_0 then
$$\omega = \sqrt{\frac{p_0 \gamma A^2}{V_0 M}} = 2\pi f \quad \therefore f = \frac{A}{2\pi}\sqrt{\frac{p_0 \gamma}{V_0 M}}$$

6. In adiabatic process the work done is given by
$$W = \frac{1}{\gamma - 1}[P_1 V_1 - P_2 V_2]$$
Here, $P_1 = 10^5$ N/m^2, $V_1 = 6l = 6 \times 10^{-3}$ m^3
$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma, V_2 = 2l = 2 \times 10^{-3} \text{ m}^3$$

Give that $C_\upsilon = \frac{3}{2}R$

According to Mayer's relationship $C_p - C_\upsilon = R$

$$\therefore \quad C_p = \frac{5}{2}R \quad \therefore \gamma = \frac{C_p}{C_\upsilon} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = 1.67$$

$$\therefore \quad P_2 = 10^5\left[\frac{6}{2}\right]^{1.67} = 10^5 \times (3)^{1.67} = 6.26 \times 10^5 \text{ N/m}^2$$

$$\therefore \quad W = \frac{1}{(1.67-1)}[10^5 \times 6 \times 10^{-3} - 6.26 \times 10^5 \times 2 \times 10^{-3}]$$
$$= -973.1 \text{ J}$$

Work done is negative because the gas is compressed.

7. Here the temperature and surface area of both the spheres are same, therefore the Energy emitted per second by both spheres is the same.

We know that $Q = mc\Delta T$

Since Q is same and c is same (both copper)

$$\therefore \quad m \propto \frac{1}{\Delta T}$$

Mass of hollow sphere is less

∴ Temperature change will be more.

∴ Hollow sphere will cool faster.

8. We know that $P = \frac{F}{A}$

$$\therefore \quad F = P \times A = 10^5 \times 1 = 10^5 \text{ N}$$

But $\quad F = \frac{\Delta p}{\Delta t}$

$$\therefore \quad \Delta P = F \times \Delta t = F \times 1 = 10^5 \text{ [From (i)]} \qquad \text{(iii)}$$

Where n is the number of collisions per second per square metre area

From (ii) and (iii)

$$n \times 2mv = 10^5 \Rightarrow n = \frac{10^5}{2mv}$$

Root mean square velocity

$$v = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.314 \times 300}{32/1000}} = 483.4 \text{ m/s}$$

According to mole concept 6.023×10^{23} molecules will have mass 32 g

$$\therefore \quad \text{I molecule will have mass } \frac{32}{6.023 \times 10^{23}} \text{ g}$$

$$\therefore \quad n = \frac{10^5 \times 6.023 \times 10^{23}}{2 \times 32 \times 483.4} = 1.97 \times 10^{27}$$

ii. The kinetic energy of motion of molecules will be converted in to heat energy.

K.E. of 1 gm mole of oxygen $= \frac{1}{2}Mv_0^2 \qquad$ (i)

Where v_0 is the velocity with which the vessel was moving.

The heat gained by 1 gm mole of molecules at constant volume

$$= nCv\Delta T = 1 \times C_v \times 1 = C_v \qquad \text{(ii)}$$

From (i) and (ii)

$$\frac{1}{2}Mv_0^2 = Cv \qquad \text{(iii)}$$

Now $C_p - C_V = R$

$$\Rightarrow \quad \frac{C_p}{C_V} - \frac{C_V}{C_V} = \frac{R}{C_V} \Rightarrow \gamma - 1 = \frac{R}{C_V}$$

$$\therefore \quad C_V = \frac{R}{\gamma - 1} \qquad \text{(iv)}$$

From (iii) and (iv)

$$\frac{1}{2}Mv_0^2 = \frac{R}{\gamma - 1}$$

$$\therefore \quad v_0 = \sqrt{\frac{2R}{M(\gamma-1)}} = \sqrt{\frac{2 \times 8.314}{\frac{32}{100} \times (1.41 - 1)}}$$

$\gamma = 1.41$ for O_2 (diatomic gas)

$$\Rightarrow \quad v_0 = 35.6 \text{ m/s}$$

9. For the left chamber the number of moles will be same hence, applying $\frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2}$

We get $\frac{P_0V_0}{T_0} = \frac{P_0 \times 243}{32 \times T_1} \times V_1$

$$\Rightarrow \quad T_1 = \frac{243}{32} \times \frac{V_1T_0}{V_0} \qquad \text{(i)}$$

The right chamber is adiabatic chamber applying

$P_1V_1^\gamma = P_2V_2^\gamma$ (adiabatic compression)

We get $P_0V_0^\gamma = P_0 \times \frac{243}{32} \times V_2^\gamma$

$$\Rightarrow \quad \left(\frac{V_2}{V_0}\right)^\gamma = \frac{32}{243} \Rightarrow \frac{V_2}{V_0} = \left(\frac{32}{243}\right)^{3/5} \Rightarrow V_2 = \frac{8}{27}V_0$$

But $V_1 + V_2 = 2V_0$

$$\therefore \quad V_1 = 2V_0 - V_2 = 2V_0 - \frac{8}{27}V_0 = \frac{46}{27}V_0 \qquad \text{(ii)}$$

From (i) and (ii)

$$T_1 = \frac{243}{32} \times \frac{46 \times V_0}{V_0 \times 27} \times T_0$$

or $\quad T_1 = \frac{207}{16}T_0 = 12.9T_0$ (approx.)

To find the temperature in the second chamber (right), we apply $\left(\frac{T_1}{T_2}\right)^\gamma = \left(\frac{P_2}{P_1}\right)^{1-\gamma}$

$$\Rightarrow \quad \left(\frac{T_0}{T_2}\right)^{5/3} = \left(\frac{243P_0}{32P_0}\right)^{1-5/3}$$

$$\Rightarrow \quad T_2 = \frac{9}{4}T_0 \; T_2 = 2.25T_0$$

Work done in right chamber (adiabatic process)

$$W = \frac{1}{(\gamma - 1)}[P_0V_0 - P_2V_2]$$
$$= \frac{3}{2}\left[P_0V_0 - \frac{243}{32}P_0 \times \frac{8}{27}V_0\right]$$
$$= -\frac{3}{2}\left(\frac{9}{4} - 1\right)P_0V_0 = -\frac{15}{8} \times RT_0$$
$$= -15.58T_0 \text{ joule}$$

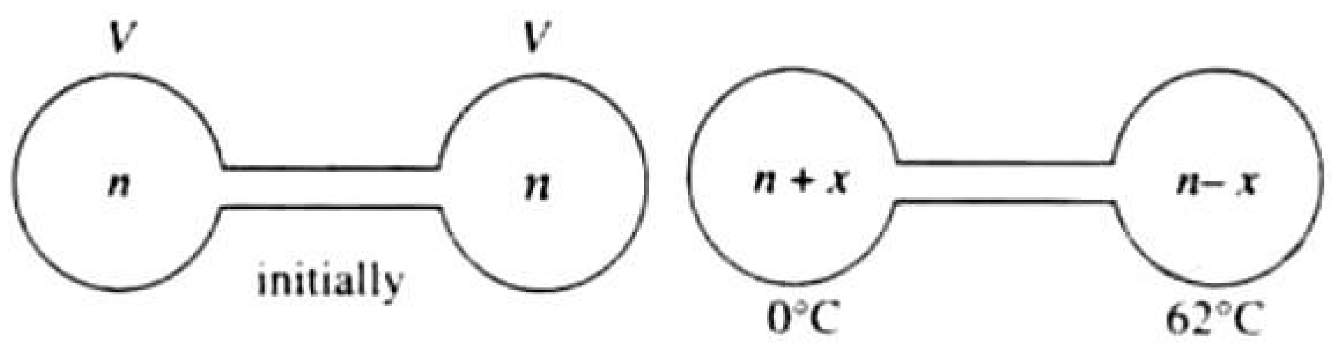
10. Let x moles of the gas shift from high temperature bulb to low temperature bulb.

For left bulb using $PV = nRT$

$$76 \times V = nR \times 273 \text{ Initially} \quad \text{(i)}$$

and $P' \times V = (n + x)R \times 273$ finally (ii)

Dividing (i) by (ii), we get $\frac{P'}{76} = \frac{n+x}{n}$ (iii)



For right bulb

$$76 \times V = nR \times 273 \text{ Initially} \quad \text{(iv)}$$

$$P' \times V = (n - x)R \times 335 \text{ Finally} \quad \text{(v)}$$

On dividing $\frac{P'}{76} = \frac{n-x}{n} \times \frac{335}{273}$ (vi)

From (iii) and (vi)

$$\frac{n+x}{n} = \frac{n-x}{n} \times \frac{335}{273} \Rightarrow n = \frac{608}{62}x \quad \text{(vii)}$$

Substituting the value of (vii) in (iiii) we get

$$\frac{P'}{76} = 1 + \frac{62}{608}$$

$$\Rightarrow \quad P' = \frac{670}{608} \times 76 = 83.75 \text{ cm Hg}$$

11. Thermal resistance of composite wall

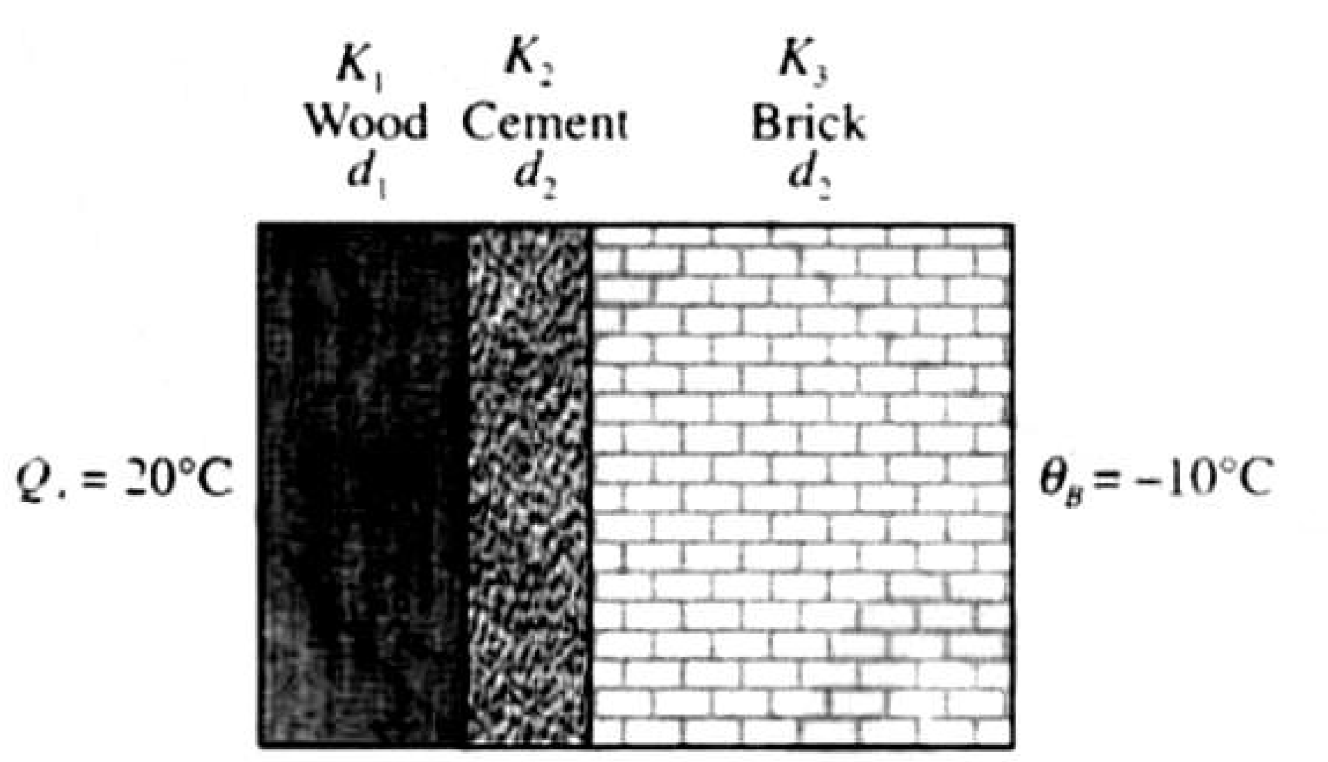
$$R = R_{wood} + R_{cement} + R_{brick}$$

$$= \frac{d_1}{K_1A} + \frac{d_2}{K_2A} + \frac{d_3}{K_3A} = \frac{1}{A}\left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right)$$

Heat flow per second $H = \frac{\text{temperature difference}}{\text{thermal resistor}}$

$$= \frac{(Q_A - Q_B)}{R}$$

$$Q_A = 20°C \; \theta_B = -10°C$$



$$H = \frac{(\theta_A - \theta_B)A}{\frac{d_1}{k_1} + \frac{d_2}{k_2} + \frac{d_3}{k_3}}$$

$$= \frac{[20 - (-10)] \times 137}{\left[\frac{0.25}{0.125} + \frac{1}{1.5} + \frac{25}{1}\right] \times 10^{-2}} = 9000 \text{ W}$$

This rate of heat has to be supplied by the heater to keep the temperature constant.

12. We have ideal gas equation that $PV = nRT$

$\therefore$ Number of moles

$$n = \frac{PV}{RT} = \frac{1.6 \times 10^6 \times 0.0083}{8.3 \times 300} = \frac{16}{3} = 5.33 \text{ moles}$$

According to Mayer's formulae

$$C_p - C_v = R \Rightarrow C_v = C_p - R = \frac{5R}{2} - R = \frac{3R}{2}$$

When 2.49×10^4 J of heat energy is supplied at constant volume, then we can use the following relationship to find change in temperature

$$Q = nC_v\Delta T$$

$$\therefore \quad \Delta T = \frac{Q}{nC_v} = \frac{2.49 \times 10^4}{5.33 \times \frac{3}{2} \times 8.3} = 375 \text{ K}$$

Therefore the final temperature = 300 + 375 = 675 K

Applying Gay Lussac's Law to find pressure.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow P_2 = \frac{P_1T_2}{T_1} = \frac{1.6 \times 10^6 \times 675}{300}$$

$$\Rightarrow \quad P_2 = 3.6 \times 10^6 \text{ Nm}^{-2}$$

13. For a perfect gas

$$PV = nRT$$

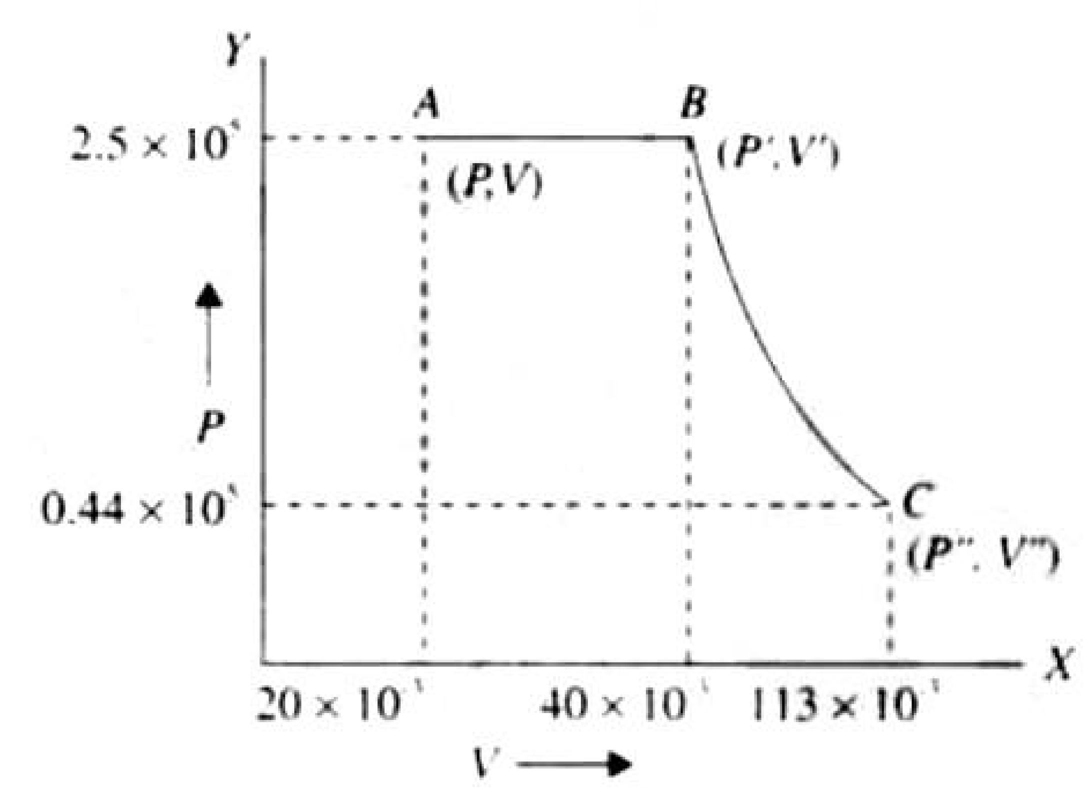
Given that $V = 20 \text{ L} = 20 \times 10^{-3} \text{ m}^3$

$T = 27°C = 300$ K and number of molecules $n = 2$

Thus, initial pressure is given as

$$P = \left(\frac{nRT}{V}\right) = \frac{(2 \times 8.3 \times 300)}{(20 \times 10^{-3})} = 2.5 \times 10^5 \text{ N/m}^2$$

a. Figure 2.77 shows the indicator diagram of the complete process.



b. At point B,

Pressure $P' = P = 2.5 \times 10^5$ N/m^2, and

$V' = 2V = 40 \times 10^{-3}$ m^3

As pressure is constant in the process AB, making its volume doubled, its temperature will also be doubled.

Thus, temperature at point B is $T' = 600$ K.

The gas now undergoes adiabatic expansions to cool down at $T'' = T = 300$ K

We know for an adiabatic process $TV^{\gamma-1}$ = constant

$$T'(V')^{\gamma-1} = T''(V'')^{\gamma-1}$$

$$\left(\frac{V''}{V'}\right) = \left(\frac{T'}{T''}\right)^{1/(\gamma-1)} = \left(\frac{600}{300}\right)^{\frac{1}{2.5-1}} = (2)^{3/2} = 2\sqrt{2}$$

Thus, final volume is

$V'' = (2\sqrt{2})V'$

$2 \times 1.414 \times 40 \times 10^{-3} = 113.14 \times 10^{-3}\ m^3$

Similarly, final pressure is given by process equation as $P'V'^{\gamma} = P''V''^{\gamma}$

or $P'' = P'\left(\frac{V'}{V''}\right)^{\gamma}$

$= 2.5 \times 10^5 \times \left(\frac{40 \times 10^{-3}}{113.14 \times 10^{-3}}\right)^{5/3}$

$= 4.42 \times 10^4$ Pa

c. Work done under isobaric process AB is

or $W_1 = P\Delta V$

or $W_1 = 2.5 \times 10^5 \times (40 - 20) \times 10^{-3}$

or $= 4980$ J

Work done during adiabatic process BC is given as

or $W_2 = \left(\frac{nR}{\gamma - 1}\right)[T_1 - T_2]$

or $= \frac{(2 \times 8.3)}{[1 - (5/3)]}[300 - 600] = 7470\,J$

Total work done = $W_1 + W_2 = 4980 + 7470 = 12450$ J

14. Initially, the pressure of the gas in the cylinder is atmospheric pressure as spring is in relaxed state. Therefore,

P_1 = atmospheric pressure = 1.0×10^5 N/m^2

V_1 = initial volume = 2.4×10^3 m^3

T_1 = initial temperature = 300 K

When the heat is supplied by the heater, the piston is compressed by 0.1 m. The reaction force of compression of spring is equal to kx which acts on the piston or on the gas as

$F = kx = 8000 \times 0.1 \times 800$ N

Pressure exerted on the piston by the spring is

$\Delta P = \frac{F}{A} = \frac{800}{8 \times 10^{-3}} = 1 \times 10^5 N/m^2$

The total pressure P_2 of the gas inside cylinder is

$P_2 = P_{atm} + \Delta P = 1 \times 10^5 + 1 \times 10^5 = 2 \times 10^5$ N/m^2

Since the piston has moved outwards, there has been an increase of ΔV in the volume of the gas, i.e.,

$\Delta V = A \times x = (8 \times 10^{-3}) \times (0.1)$

$= 8 \times 10^{-4}$ m^3

The final volume of the gas

$V_2 = V_1 + \Delta V = 2.4 \times 10^{-3} + 8 \times 10^{-4} = 3.2 \times 10^{-3}$ m^3

Let T_2 be the final temperature of gas. Then

$(P_1V_1/T_1) = (P_2V_2/T_2) \Rightarrow T_2 = (P_2V_2/P_1V_1)T_1$

$= 300 \times (2 \times 10^5 \times 3.2 \times 10^{-3})/(10^5 \times 2.4 \times 10^{-3})$

$= 800$K

Let the heat supplied by the heater be Q. This is used in two parts, i.e., a part is used in doing external work W due to expansion of the gas and the other part is used in increasing h internal energy of the gas. Hence,

$Q = W + \Delta U$

Now $W = \int_{V}^{V_2} P dV = \int_0^x \left(P_{atm} + \frac{kx}{A}\right) A dx$

$\left[\text{as pressure is } (P_{atm} + kx/A \text{ and } dV = Adx)\right]$

or $W = P_{atm} Ax + \left(\frac{kx^2}{2}\right)$

$= \left[10^5 \times (8 \times 10^{-3})(0.1) + \frac{8000 \times (0.1)^2}{2}\right] = 120\,J$

Further, $\Delta U = nC_v\Delta T$

Number of moles of gas can be obtained from initial conditions and gas law as

$n = \left(\frac{PV}{RT}\right) = \left(\frac{1 \times 10^5 \times 2.4 \times 10}{8.314 \times 300}\right) = 0.096$ mol

Thus, change in internal energy of gas is given as

$\Delta U = n\left(\frac{3}{2}R\right)\Delta T$ $\left(\text{as for monatomic gas } C_v = \frac{3}{2}R\right)$

or $U = 0.096 \times \left(\frac{3}{2}\right) \times 8.314 \times 500 = 598.6$ J.

Heat supplied by the heater = (120 + 598.6) = 718.6 J.

15. i. We have

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow TV^{\gamma-1} = \frac{T}{2}(5.66V)^{\gamma-1}$

Taking log on both sides log 2 = (γ − 1) log 5.66

On solving we get $\gamma = 1.4$

But $\gamma = 1 + \frac{2}{f} \Rightarrow 1.4 = 1 + \frac{2}{f} \Rightarrow f = \frac{2}{0.4} = 5.$

ii. For adiabatic process using relation

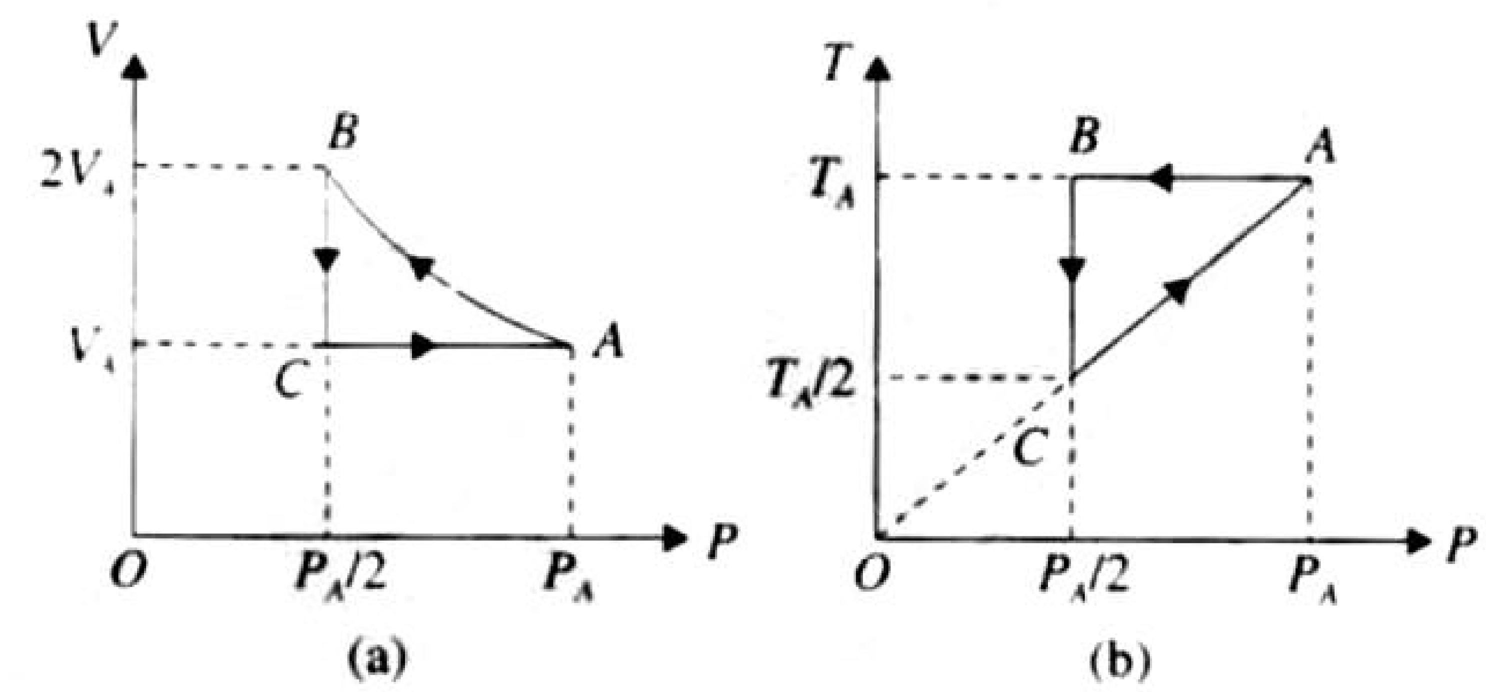
$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow P_2 = \frac{P}{(5.66)^{1.4}} = \frac{P}{11.32}$

Work done for adiabatic process

$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{PV - \left(\frac{P}{11.32}\right)(5.66V)}{1.4 - 1}$

$\Rightarrow$ $W = 1.25\,PV$

16. a. Diagrams of P–V and P–T are shown in figures (a) and (b)



The initial state of isothermal expansion is represented by A where pressure is P_A and volume V_A. Let the final state be represented by B where volume V_B is twice of V_A. Let the pressure be P_B. Then

$P_AV_A = P_BV_B = P_B(2V_A)$

or $P_B = P_A(V_A/2V_A) = P_A/2$

When the molecule is compressed to initial volume, the process is represented by BC. Finally, the gas is compressed at constant volume to its original pressure. The process is shown by curve CA.

Similarly, P–T diagram can be drawn.

b. Work done in the process AB is given by

$$W_1 = nRT \ln(V_B / V_A)$$
$$= 3 \times 8.314 \times T_A \times \ln 2$$
$$= 3 \times 8.314 \times T_A \times 0.693 = 17.29\ T_A$$

Work done in the process BC is given by

$$W_2 = P\Delta V = P_B \times (V_C - V_B)$$
$$= \left(\frac{P_A}{2}\right) \times (V_A - 2V_A)$$
$$= -P_A V_A/2 = -nRT_A/2$$
$$= -3RT_A/2 = -3 \times 8.314\ T_A/2 = -12.471\ T_A$$

Work done during process C_A is given by

$$W_3 = P\Delta V = 0 \qquad (\text{as } \Delta V = 0)$$

Net work $W = W_1 + W_2 + W_3$

$$= 17.26T_A - 12.45\ T_A = 4.81\ T_A$$

As initial and final states of the gas are the same

$$\Delta U = U_A - U_A = 0$$

From the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W = 0 + \Delta W$$
$$Q = W \text{ joules}$$

17. As we know in a cyclic process, the change in heat energy or heat supplied to the gas is equal to the net work done by the gas.

Here, AB is isobaric process. Hence, work done during this process from A to B is

$$W_{AB} = P(V_2 - V_1) = nR(T_2 - T_1)$$

or $\quad W_{AB} = 2 \times 8.314 \times (400 - 300) = 1662.8$ J

Work done during isothermal process from B to C is

$$W_{BC} = nRT_C \ln (V_2/V_1) = nRT_C \ln (P_1/P_2)$$
$$= 2 \times 8.314 \times 400 \times \ln (2) = 2 \times 8.314 \times 400 \times 0.693$$
$$= 4610.2 \text{ J}$$

Work done during isobaric process from C to D

$$W_{CD} = nR(T_D - T_C) = 2 \times 8.314 \times (300 - 400)$$
$$= -1662.8 \text{ J}$$

Work done during isothermal process from D to A

$$W_{DA} = nRT_D \ln (P_D/P_A)$$
$$= nRT_D \ln (2)$$
$$= -2 \times 8.314 \times 300 \times 0.693$$
$$= -3457.7 \text{ J}$$

Net work done

$$= W_{AB} + W_{BC} + W_{CD} + W_{DA}$$
$$= 1662.8 + 4610.2 - 1662.5 - 3457.7 = 1152.5 \text{ J}$$

Now from the first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

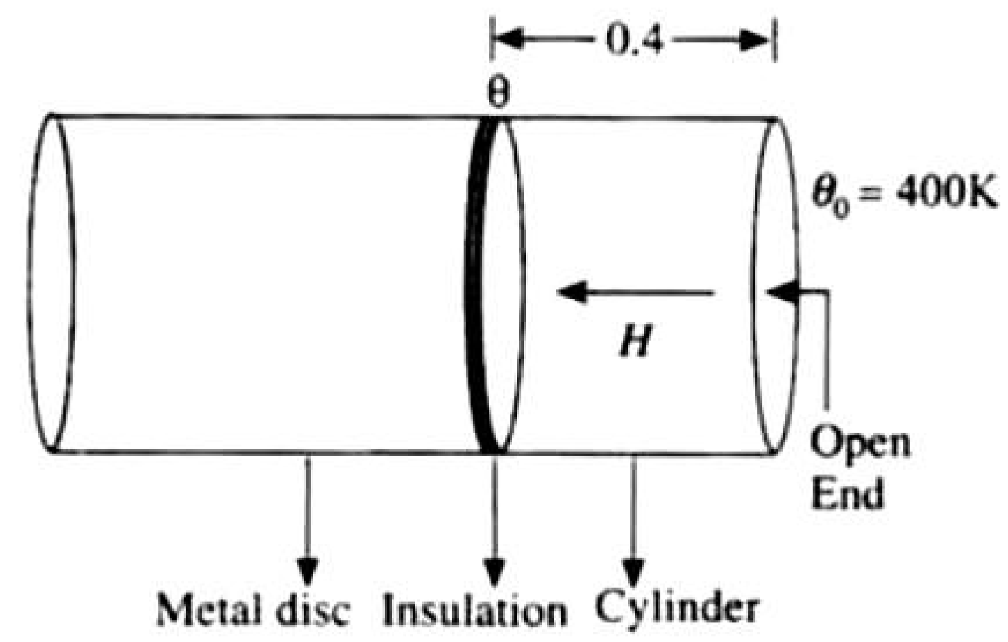
Here $\Delta U = 0$, thus we have

$$\Delta Q = \Delta W = 1152.5 \text{ J}$$

So the heat given to the system is 1152.5 J

As the gas returns to its original state, there is no change in internal energy.

18. The temperature at the open end of cylinder is 400 K and the temperature at the metal disc-cylinder interface is 300 K.



As heat passes through the cylinder and reaches the metal disc, the temperature of metal disc rises. Let at any instant of time, the temperature of the metal disc-cylinder interface is θ. At this instant the rate of heat crossing the cylinder.

$$H = \frac{dQ}{dt} = \frac{KA(400 - \theta)}{l} \qquad \text{(i)}$$

where K = thermal conductivity

A = area of cross section of cylinder

l = length of cylinder

The same amount of heat is received by the metal disc.

Therefore $\quad \dfrac{dQ}{dt} = mc\dfrac{d\theta}{dt} \qquad$ (ii)

m = mass of disc c = specified heat of metal disc

From (i) and (ii) $\quad mc\dfrac{d\theta}{dt} = \dfrac{KA(400 - \theta)}{l}$

$$\left(\frac{d\theta}{400 - \theta}\right) \times \frac{mcl}{KA} = dt$$

On integrating At $t = 0$, $\theta = 300$ K

And at $t = t$, $\theta = 350$ K

$$\int_0^t dt = \frac{mcl}{KA}\int_{300}^{350} \frac{d\theta}{(400 - \theta)}$$

$$\Rightarrow \quad t = \frac{-mcl}{KA}[\log(400 - \theta)]_{300}^{350}$$
$$= \frac{-0.4 \times 600 \times 0.4 \times 2.303}{10 \times 0.04} \log_{10} \frac{400 - 350}{400 - 300}$$
$$= 166.38 \text{ sec.}$$

19. a. The work done in adiabatic process AB is given by

$$W_1 \frac{R(T_A - T_B)}{\gamma - 1} = \frac{R(T_A - T_B)}{(5/3) - 1}$$

(as for monatomic gas $\gamma = 5/3$)

or $\quad = \dfrac{3}{2}R(T_A - T_B) \qquad$ (i)

For adiabatic change, we have

$$P_A^{\gamma-1}T_A^{\gamma} = P_B^{\gamma-1}T_B^{-\gamma}$$

or $\quad \left(\dfrac{P_A}{P_B}\right) = \left(\dfrac{T_A}{T_B}\right) \quad$ or $\quad \left(\dfrac{I_A}{I_B}\right) = \left(\dfrac{P_A}{P_B}\right)^{2/5}$

or $\quad \left(\dfrac{T_A}{T_B}\right) = \left(\dfrac{3}{2}\right)^{2/3} \qquad$ (as $P_B = (2/3)P_A$)

or $\quad T_B = T_A\left(\dfrac{2}{3}\right)^{2/3} \qquad$ (ii)

$$= 1000 \times 0.85 = 850 \text{ K}$$

From Eq. (i)

$$W_1 = \left(\frac{3}{2}\right) \times 8.31 \times (1000 - 850) = 1869.83 \text{ J}$$

b. Heat lost by the gas in process BC is given by

$$C_V(T_B - T_C) = \left(\frac{R}{\gamma - 1}\right)(T_B - T_C) = \left(\frac{3}{2}\right)R(T_B - T_C) \quad \text{(iii)}$$

Process BC is under constant volume; hence

$$\left(\frac{P_B}{T_B}\right) = \left(\frac{P_C}{T_C}\right) \quad \text{or} \quad T_C = \left(\frac{P_C}{P_B}\right) \times T_B$$

or $$T_C = \frac{(P_A/3)}{2P_A/3} T_B = \left(\frac{T_B}{2}\right) = 425 \text{K} \quad \text{(iv)}$$

From Eq. (ii), we get

$$\text{Heat lost} = \left(\frac{3}{2}\right) \times 8.31 \times (850 - 425) = 5297.63 \text{ J}$$

c. For path AB

$$P_A^{\gamma-1} T_A^{-\gamma} = P_B^{\gamma-1} T_B^{-\gamma}$$

or $$\left(\frac{P_A}{P_B}\right)^{\gamma-1} = \left(\frac{T_A}{T_B}\right)^{\gamma} \quad \text{(v)}$$

For path BC

$$\left(\frac{P_B}{T_B}\right) = \left(\frac{P_C}{T_C}\right) \quad \text{or} \quad \left(\frac{P_B}{P_C}\right) = \left(\frac{T_B}{T_C}\right) \quad \text{(vi)}$$

For path CD

$$\left(\frac{P_D}{T_B}\right)^{\gamma-1} = \left(\frac{T_D}{T_C}\right)^{\gamma} \quad \text{(vii)}$$

For path AD

$$\left(\frac{P_A}{P_D}\right) = \left(\frac{T_A}{T_C}\right) \quad \text{(viii)}$$

Dividing Eq. (v) by Eq. (vii), we get

$$\left(\frac{P_A}{P_B} \times \frac{P_C}{P_D}\right)^{\gamma-1} = \left(\frac{T_A}{T_B} \times \frac{T_C}{T_D}\right)^{\gamma} \quad \text{(ix)}$$

Dividing Eq. (viii) by Eq. (iv), we get

$$\left(\frac{P_A}{P_B} \times \frac{P_C}{P_D}\right) = \left(\frac{T_A}{T_B} \times \frac{T_C}{T_D}\right) \quad \text{or} \quad \left(\frac{P_A}{P_B} \times \frac{P_C}{P_D}\right)^{\gamma-1} = \left(\frac{T_A}{T_B} \times \frac{T_C}{T_D}\right)^{\gamma-1} \quad \text{(x)}$$

$$\left(\frac{T_A}{T_B} \times \frac{T_C}{T_D}\right)^{\gamma} = \left(\frac{T_A}{T_B} \times \frac{T_C}{T_D}\right)^{\gamma-1}$$

$$T_A T_C = T_D T_B$$

$$1000 \times 425 = T_D \times 850$$

$$T_D = 500 \text{ K}$$

20. i. In cyclic process $\Delta U = 0$

According to first law of thermodynamics $Q = \Delta U + W$

$\Rightarrow$ Here $\Delta U = 0$; $Q = \Delta W$

$\Rightarrow$ $Q_1 + Q_2 + Q_3 + Q_4 = W_1 + W_2 + W_3 + W_4$

$\Rightarrow$ $5960 - 5585 - 2980 + 3645$

$= 2200 - 825 - 1100 + W_4 \Rightarrow W_4 = 765 \text{ J}$

ii. $$\eta = \frac{\text{work done}}{\text{Heat supplied}} = \frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4}$$

$$= \frac{1040}{9605} = 10.82\%$$

21. Let in the mixture there be x gram of Neon. Then the mass of Argon will be $28 - x$.

Number of moles of Neon $= \dfrac{x}{20}$

Number of moles of Aragon $= \dfrac{28 - x}{40}$

Partial pressure due to Neon: $P_1 = \dfrac{(x/20)RT}{V}$

Partial pressure due to Argon $P_2 = \dfrac{[(28 - x)/40]RT}{V}$

But according to Dalton's law of partial pressure, total pressure

$$P = P_1 + P_2$$

$$10^5 = \frac{xRT}{20 \text{ V}} + \frac{(28 - x)RT}{40 \text{ V}}$$

$$\Rightarrow \quad \frac{10^5 \times 40 \text{ V}}{RT} = 2x + 28 - x$$

$$\Rightarrow \quad \frac{10^5 \times 40 \times 0.02}{8.314 \times 300} = x + 28$$

$\Rightarrow \quad x = 4$ g

$\Rightarrow$ Mass of Neon = 4 g $\therefore$ Mass of Argon = 24 g

22. a. The ratio of specific heat of mixture of gases

According to the relationship

$$PV\left(\frac{19}{13}\right) = \text{Constant we get } \gamma_m = \frac{19}{13}$$

For a mixture of gases

$\dfrac{(n_1 + n_2)}{\gamma_m - 1} = \dfrac{n_A}{\gamma_A - 1} + \dfrac{n_B}{\gamma_B - 1}$ where γ_m = Ratio of specific heats of mixture.

$$\frac{(1 + n)}{\frac{19}{13} - 1} = \frac{1}{\left(\frac{5}{3} - 1\right)} + \frac{n_B}{\left(\frac{7}{5} - 1\right)} \Rightarrow n_B = 2 \text{ mol}$$

b. We know that velocity of sound in air is given by the relationship

$$\upsilon = \sqrt{\frac{\gamma P}{\rho}} \text{ where } \rho = \text{density} = \frac{m}{V}$$

Also, $PV = (n_A + n_B)RT \Rightarrow PV = \dfrac{(n_A + n_B)}{V} RT$

$$\therefore \quad \upsilon = \sqrt{\frac{\gamma(n_A + n_B)RT}{V \times \frac{m}{V}}} = \sqrt{\frac{\gamma(n_A + n_B)RT}{m}}$$

Mass of the gas, $m = n_A M_A + n_B M_B$

$= 1 \times 4 + 2 \times 32 = 68$ g/molmol $= 0.068$ kg/mol

$$\therefore \quad \upsilon = \sqrt{\frac{19(1 + 2) \times 8.314 \times 300}{13 \times 0.068}} = 400.03 \text{ ms}^{-1}$$

c. We know that the velocity of sound

$$\upsilon = \sqrt{\frac{\gamma P}{d}} = \sqrt{\frac{\gamma RT}{M}} \text{ and } \upsilon + \Delta\upsilon = \sqrt{\frac{\gamma R(T + \Delta T)}{M}}$$

$$\Rightarrow \quad \frac{\upsilon + \Delta\upsilon}{\upsilon} = \sqrt{\frac{T + \Delta T}{T}} = \left(\frac{1 + \Delta T}{T}\right)^{1/2}$$

$$\Rightarrow \quad 1+\frac{\Delta v}{v}=1+\frac{1}{2}\frac{\Delta T}{T} \text{ when } \Delta T << T \text{ then } \frac{\Delta T}{T} << 1$$

Percentage charge $\frac{\Delta v}{v}\times 100=\frac{1}{2}\times\frac{\Delta T}{T}\times 100$

$$\frac{\Delta v}{\Delta v}\times 100=\frac{1}{2}\frac{1}{300}\times 100=\left(\frac{1}{6}\right)\%$$

d. PV^γ = const. (i)

Differentiating (i)

$$V^\gamma(dP)-P(\gamma V^{\gamma-1}dV)=0$$

$$\Rightarrow \quad V^\gamma dP=\gamma PV^{\gamma-1}dV$$

$$\Rightarrow \quad \frac{dP}{dV}=\frac{\gamma PV^{\gamma-1}}{V^\gamma}=\gamma PV^{\gamma-1-x}=\frac{\gamma P}{V}$$

$$\Rightarrow \quad \frac{-dP}{dV/V}=-\gamma P$$

$\therefore$ Bulk modulus $B=\gamma P$

$\therefore$ Compressibility $K=\frac{1}{\text{Bulk Modulus}}=\frac{1}{\gamma P}$

$$K_1=\frac{1}{\gamma P_1} \text{ and } K_2=\frac{1}{\gamma P_2}$$

$$\therefore \quad \Delta K=K_2-K_1=\frac{1}{\gamma P_2}-\frac{1}{\gamma P_1}=\frac{1}{\gamma}\left(\frac{1}{P_2}-\frac{1}{P_1}\right)$$

$\therefore$ Since the process is adiabatic, $P_2V_2^\gamma=P_1V_1^\gamma$

$$\therefore \quad P_2=P_1\left(\frac{V_1}{V_2}\right)^\gamma=P_1\left(\frac{V_1}{V_1/5}\right)^\gamma=P_15^\gamma$$

$$\therefore \quad \Delta K=\frac{1}{\gamma}\left(\frac{1}{P_15^\gamma}-\frac{1}{P_1}\right)=\frac{1}{\gamma P_1}\left(\frac{1}{5^\gamma}-1\right)$$

$$\Rightarrow \quad \Delta K=\frac{1}{\frac{19}{13}\times 24.93\times\frac{T}{V}}\left(\frac{1}{5^{19/13}}-1\right)$$

$$=-8.27\times 10^{-5}\text{ V (Pa}^{-1}\text{)}$$

23. Given $T_1=27+273=300$ K, $\gamma=\frac{5}{3}$ (for monoatomic gas)

$V_1=V$ and $V_2=2V$

We need to calculated final temperature T_2 = ?

As the gas expands adiabatically

$$T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$$

$$\Rightarrow \quad T_2=T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}=300\left[\frac{V}{2V}\right]^{5/3-1}$$

$$=\frac{300}{2^{2/3}}=189\text{ K}$$

Work done in adiabatic process

$$W=\frac{nR(T_1-T_2)}{(\gamma-1)}=\frac{2\times 8.31(300-189)}{(5/3-1)}=+2767\text{ J}$$

Change in internal energy

According to the first law of thermodynamics,

$Q=\Delta U+\Delta W$ But $Q=0$ (the process is adiabatic)

$\Delta U=-W=-2767$ J

24. Let m be the mass of the steam required to raise the temperature of 100 g of water from 24°C to 90°C.

Heat lost by steam = Heat gained by water

$$m[L+s\Delta\theta_1]=100s\Delta\theta_2$$

or $$m=\frac{(100)(s)(\Delta\theta_2)}{L+s(\Delta\theta_1)}$$

Here, s = specific heat of water = 1 cal/g°C

L = latent heat of vaporization = 540 cal/g.

$$\Delta\theta_1=(100-90)=10°\text{C}$$

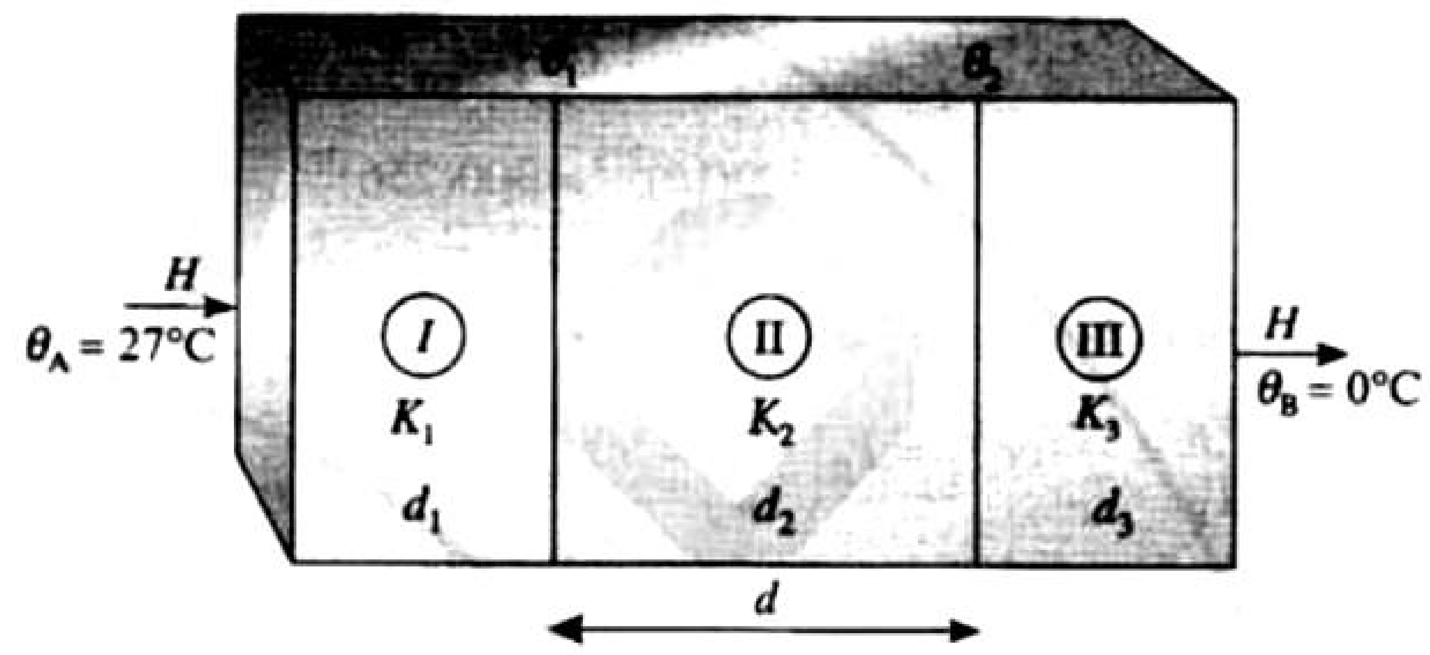
and $$\Delta\theta_2=(90-24)=66°\text{C}$$

Substituting the values, we have

$$m=\frac{(100)(1)(66)}{(540)+(1)(10)}=12\text{ g}$$

$\therefore \quad m=12$ g

25.



In steady-state the rate of heat flow through.

First material:

$$H=\frac{K_1}{d_1}A(\theta_A-\theta_1)\Rightarrow\theta_A-\theta_1=\frac{H}{A}\frac{d_1}{K_1} \quad \text{(i)}$$

Second material:

$$H=\frac{K_2}{d_2}A(\theta_1-\theta_2)\Rightarrow\theta_1-\theta_2=\frac{H}{A}\frac{d_2}{K_2} \quad \text{(ii)}$$

Third material:

$$H=\frac{K_3}{d_3}A(\theta_2-\theta_B)\Rightarrow\theta_2-\theta_B=\frac{H}{A}\frac{d_3}{K_3} \quad \text{(iii)}$$

$\Rightarrow$ Adding the above three equations we get

$$\theta_A-\theta_B=\frac{H}{A}\left(\frac{d_1}{K_1}+\frac{d_2}{K_2}+\frac{d_3}{K_3}\right)$$

$$\Rightarrow \quad H=\frac{(\theta_A-\theta_B)A}{\frac{d_1}{K_1}+\frac{d_2}{K_2}+\frac{d_3}{K_3}}$$

Substituting the values

$$H=\frac{(27-0)1}{\frac{0.01}{0.8}+\frac{0.05}{0.08}+\frac{0.01}{0.8}}=41.54\text{ J/s}$$

From (iv) $27-\theta_1=\frac{41.54}{1}\times\frac{0.01}{0.8}\Rightarrow\theta_1=26.48°\text{C}$

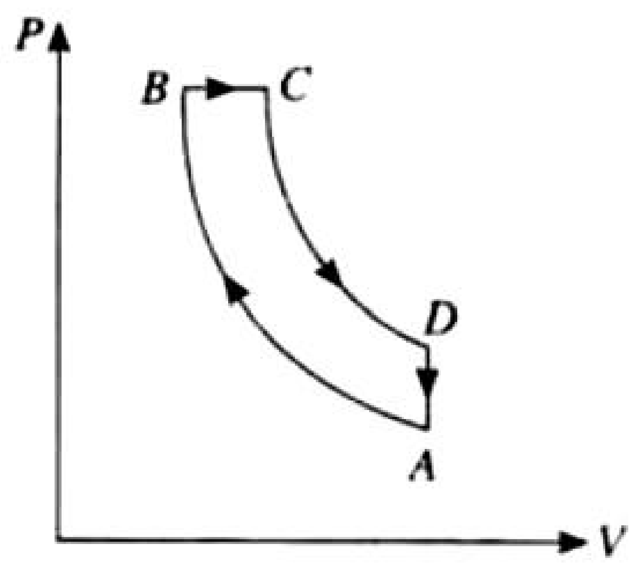
From (vi) $\theta_2-0=\frac{41.54}{1}\times\frac{0.01}{0.8}\Rightarrow\theta_2=0.52°\text{C}$

26. $A\to B$ adiabatic compression

$B\to C$, isobaric expansion

$C\to D$ adiabatic expansion

$D\to A$ isochoric process



Given $\frac{V_A}{V_B} = 16, \frac{V_C}{V_B} = 2$

$$T_A = 300\text{ K}, T_B = ?\ T_D = ?\ \eta = ?$$

For adiabatic compression process $A \rightarrow B$

$$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1} \text{ or } \frac{T_B}{T_A} = \left(\frac{V_A}{V_B}\right)^{\gamma-1} = (16)^{2/5}$$

$$\therefore \quad T_B = T_A(16)^{2/5} = 300(16)^{2/5} = 300(256)^{1/5}$$

$$= 300 \times 3.03 = 909\text{ K}$$

$\therefore$ For isobaric process $B \rightarrow C$

As $\frac{V_B}{T_B} = \frac{V_C}{T_C}$ or $T_C = T_B\left(\frac{V_C}{V_B}\right) = 909[2] = 1818\text{ K}$

Again for adiabatic expansion process $C \rightarrow D$

We have $\frac{V_A}{V_B} = 16$ and $\frac{V_C}{V_B} = 2$, hence $\frac{V_A}{V_C} = 8$

$$T_C V_C^{\gamma-1} = T_D V_D^{\gamma-1}$$

$$\therefore \quad T_D = T_C\left[\frac{V_C}{V_D}\right]^{\gamma-1} = 1818\left[\frac{1}{8}\right]^{2/5} = \frac{1818}{(64)^{1/5}} = 791\text{ K}$$

For Process $B \rightarrow C$: Heat absorbed $Q_1 = nC_p(T_c - T_B)$

$$= n\frac{\gamma R}{\gamma - 1}(T_C - T_B) = 1\frac{(7/5)R}{(2/5)}(1818 - 909)$$

$$= \frac{7R}{2} \times 909 \cong 3182R$$

For Process $D \rightarrow A$:

Heat released $Q_2 = nC_v(T_D - T_A)$

$$= n\left(\frac{R}{\gamma - 1}\right)(T_D - T_A)$$

$$= 1\frac{R}{(2/5)}(791 - 300) = \frac{5R}{2} \times 491$$

Now work done in adiabatic process

$$W_{AB} = \frac{nR}{\gamma - 1}(T_B - T_A)$$

$$= -\frac{R}{(2/5)}(909 - 300) = -\frac{5R}{2} \times 609$$

$$W_{BC} = -nR(T_C - T_B)$$

$$= 1 \times R(1818 - 909) = 909R$$

Also $W_{CD} = -\frac{nR}{\gamma - 1}(T_C - T_D) = +\frac{R}{(2/5)}(1818 - 791)$

$$= \frac{5R}{2} \times 1027$$

Total work done in process

$$W_{Net} = 909R + \frac{5R}{2}(1027 - 609)$$

$$= 909R + 1045R = 1954R$$

$$\therefore \text{ Efficiency} = \eta = \frac{W_{net}}{Q_{in}} \times 100$$

$$= \frac{1954R}{3182R} \times 100 = 61.4\%$$

27. Let the pressure at point O inside the horizontal tube be P_M

Since the liquid is at equilibrium at '1'

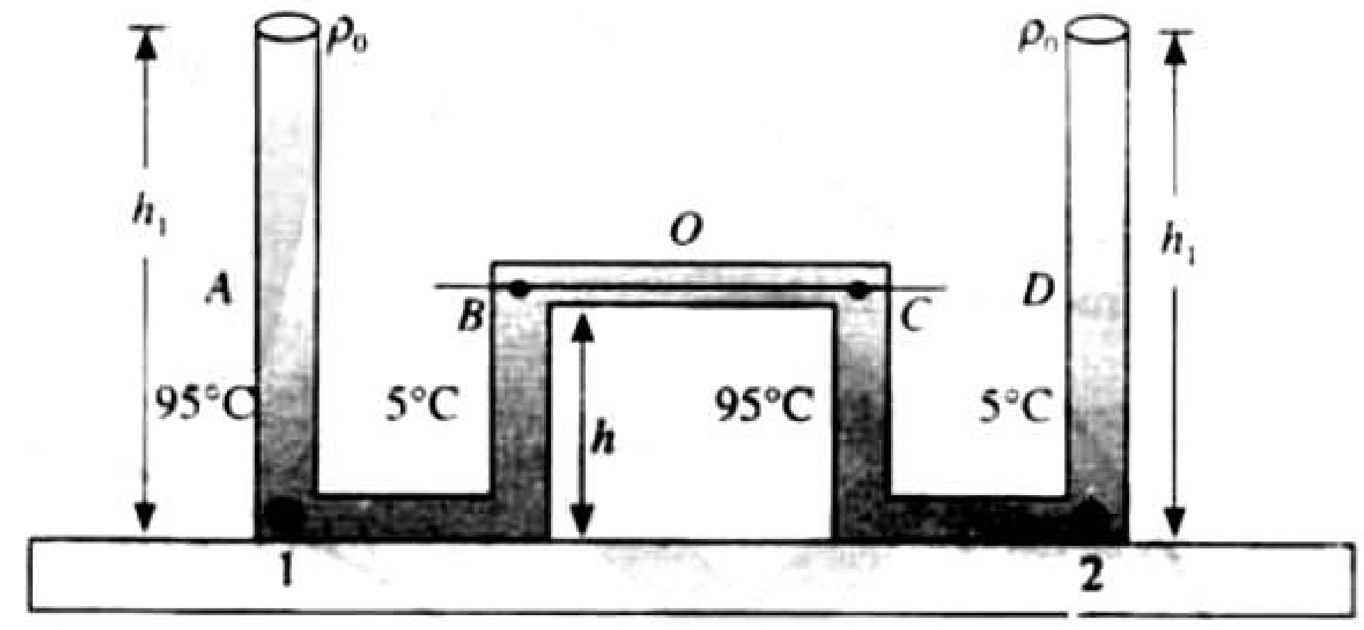
$$P_O + h_1\rho_{95^\circ}g = P_M + h\rho_{5^\circ}g$$

$$\Rightarrow \quad P_M = P_O + h_1\rho_{95^\circ}g - h\rho_{5^\circ}g \qquad \text{(i)}$$

Since the liquid is at equilibrium at '2'

$$P_O + h_2\rho_{5^\circ}g = P_M + h\rho_{95^\circ}g$$

$$\Rightarrow \quad P_M = P_O + h_2\rho_{5^\circ}g - h\rho_{95^\circ}g \qquad \text{(ii)}$$



From (i) and (ii)

$$P_O + h_1\rho_{95^\circ}g - h\rho_{5^\circ}g = P_O + h_2\rho_{5^\circ}g - h\rho_{95^\circ}g$$

$$\Rightarrow \quad \frac{\rho_{5^\circ}}{\rho_{95^\circ}} = 1.018 \qquad \text{(i)}$$

We know that elasticity at temperature ΔT

$$\rho_0 = \rho_t(1 + \gamma\Delta T)$$

$$\rho_t = \frac{\rho_0}{(1 + \gamma\Delta T)}$$

ρ_0 = density at 0°C

$$\therefore \quad \frac{\rho_{5^\circ}}{\rho_{95^\circ}} = \frac{1 + 95\gamma}{1 + 5\gamma} \qquad \text{(ii)}$$

From (i) and (ii) $\frac{1 + 95\gamma}{1 + 5\gamma} = 1.018$

$$\Rightarrow \quad 1 + 95\gamma = 1.018 + 5 \times 1.018\gamma \Rightarrow 89.91\gamma = 0.018$$

$$\Rightarrow \quad \gamma = 2.002 \times 10^{-4} \text{ But } \gamma = 3\alpha$$

$$\Rightarrow \quad \alpha = \frac{\gamma}{3} = \frac{2.002 \times 10^{-4}}{3} = 6.67 \times 10^{-5}\ {}^\circ\text{C}^{-1}$$

28. Given that mass of helium used in the process is $m = 2$ kg. number of moles can be given as

$$n = \left(\frac{m}{M}\right) = \frac{2}{(2 \times 10^{-3})} = 500$$

At different states, from the figure, the pressure and volume of gas are also givens as

$$P_A = P_D = 5 \times 10^4\text{ N/m}^2$$

$$P_B = P_C = 10^5\text{ N/m}^2$$

$$V_A = V_B = 10\text{ m}^3$$

$$V_C = V_D = 20\text{ m}^3$$

a. From gas law, we have

$$T_A = \frac{(P_A V_A)}{(nR)} = \frac{(5 \times 10^4 \times 10)}{(500 \times 8.314)} = 120.3\text{ K}$$

$$T_B = \frac{(P_BV_B)}{(nR)} = \frac{(10^5 \times 20)}{(500 \times 8.314)} = 481.11\,\text{K}$$

$$T_D = \frac{(P_DV_D)}{(nR)} = \frac{(5 \times 10^4 \times 20)}{(500 \times 8.314)} = 120.3\,\text{K}$$

b. Since the gas is taken from same initial state to same final state C, no matters whatever be the path, the answer is no.

c. In process ABC, the change in internal energy is

$$\Delta U_{ABC} = U_C - U_A = (f/2)nR(T_C - T_A)$$

$= (3/2) \times 500 \times 8.314\ (481.11 - 120.3)$

$= 2.25 \times 10^6$ J

Net work done in process ABC is

$W_{ABC} = W_{AB} + W_{BC}$

$= 0 +$ area below curve BC

$= 0 + 10^5 \times 10$

$= 10^6$ J

Thus, from the first law of thermodynamics, heat supplied in process ABC is

$Q = W + \Delta U$

or $Q = 10^6 + 2.25 \times 10^6$

$= 3.25 \times 10^6$ J

Similarly, in process ADC as being a state function, change in internal energy remains same as initial and final states are same. Thus,

$\Delta U_{ADC} = 2.25 \times 10^6$ J

Thus, work done by the gas in process ADC is

$W_{ADC} = W_{AD} + W_{DC}$

$=$ area below curve $AD + 0$

$= 5 \times 10^4 \times 10$

$= 0.5 \times 10^6$ J

Thus, from the first law of thermodynamics, heat supplied in the process ADC is given as

$Q = W + \Delta U$

or $= 0.5 \times 10^6 + 2.25 \times 10^6$

$= 2.75 \times 10^6$ J

29. For first situation ($t < t_1$)

According to Newton's law of cooling

$$\frac{dT}{dt} = -K'(T - T_A) \Rightarrow \frac{dT}{T - T_A} = -K'dt$$

On integrating we get $-\int_{400}^{350} \frac{dT}{T - T_A} = K\int_0^{t_1} dt$

$$-[\log_e(T - T_A)]_{400}^{350} = K'[t]_d^{t_1}$$

$$\Rightarrow \quad -\log_e \frac{350 - 300}{400 - 300} = K't_1$$

$$\Rightarrow \quad \log_e \frac{100}{50} = K't_1 \text{ or } K't_1 = \log_e 2 \qquad \text{(i)}$$

For first situation ($t > t_2$)

When the body X is connected to a large box Y. In this case cooling occurs by Newton's law of cooling as well as by conduction

$$\therefore \quad \frac{-dT}{dt} = K'(T - T_A) + \frac{KA(T - T_A)}{CL}$$

$$\Rightarrow \quad \frac{-dT}{dt} = \left[K' + \frac{KA}{CL}\right](T - T_A)$$

where k = coefficient of thermal conductivity of the rod.

$$\Rightarrow \quad \frac{-dT}{T - T_A} = \left[K' + \frac{KA}{CL}\right]dt$$

On integrating we get

$$-\int_{350}^{T} \frac{dT}{T - T_A} = \int_{t_1}^{3t_1}\left(K' + \frac{KA}{CL}\right)dt$$

$$\Rightarrow \quad -[\log_e(T - T_A)]_{350}^{T} = \left(K' + \frac{KA}{CL}\right)[t]_{t_1}^{3t_1}$$

$$\Rightarrow \quad -\log_e \frac{350 - 300}{T - 300} = \left(K' + \frac{KA}{CL}\right)2t = 2K'_{t_1}$$

$$\Rightarrow \quad \log_e \frac{50}{T - 300} = 2(\log_e 2) + \frac{2KA}{CL}t_1$$

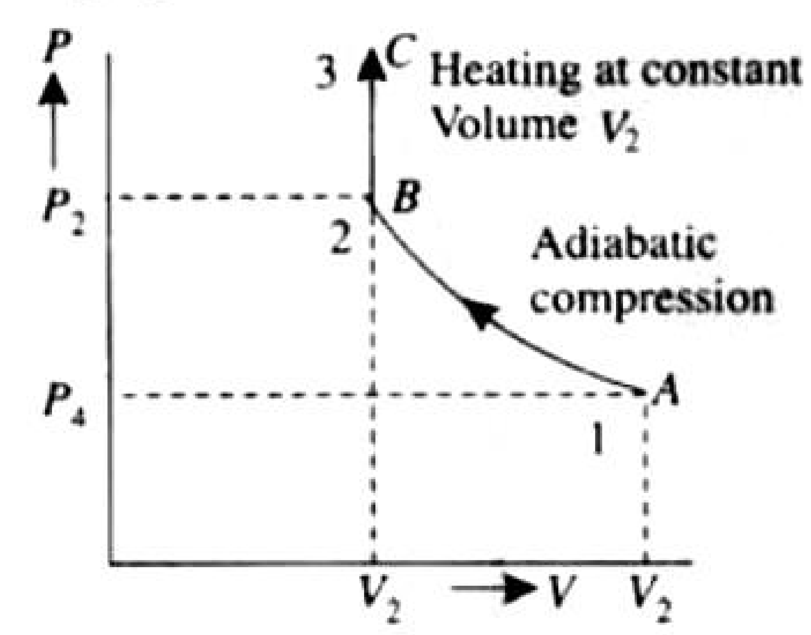
$$\frac{50}{T - 300} = e^{\left[\log_e 4 + \frac{2KA}{CL}t_1\right]} + \frac{2KA}{CL}t_1$$

$$\Rightarrow \quad T - 300 = 50e^{-[\log_e 4]} \times e^{\frac{-2KAt_1}{CL}}$$

$$\Rightarrow \quad T = [300 + 12.5e^{\frac{-2KAt_1}{CL}}]\ \textbf{Kelvin}$$

30. n = no. of moles = 2,

a. The complete process is shown on a P–V diagram in the adjoining figure.



b. i. Total work done

$$W = W_{AB} + W_{BC} = \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)} + 0$$

$[\because W_{BC} = P\Delta V = P \times 0 = 0]$

According to Poisson's law, $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

$$\therefore \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\therefore \quad W = \frac{1}{\gamma - 1}\left[P_1V_1 - P_1\left(\frac{V_1}{V_2}\right)^{\gamma} V_2\right]$$

$$= \frac{1}{\gamma - 1}\left[P_1V_1 - P_1 \cdot V_2 \cdot \frac{V_1}{V_2} \cdot \left(\frac{V_1}{V_2}\right)^{\gamma - 1}\right]$$

For monoatomic gas,

$$\gamma = 1 + \frac{2}{3} = \frac{5}{3}, W = \frac{3}{2}\left[P_1V_1 - P_1V_1\left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

$$= \frac{3}{2}P_1V_1\left[1 - \left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

ii. $\Delta U + \Delta U_{AB} + \Delta U_{BC}$

$$= Q - W = Q - \frac{3}{2}P_1V_1\left[1-\left(\frac{V_1}{V_2}\right)^{2/3}\right]$$

[according to the first law of thermodynamics]

$$\left[\begin{array}{l} B\to C\ Q = \Delta U_{BC} + 0 \\ A \to B\ 0 + \Delta U_{AB} + W \end{array}\right]$$

iii. For process $BC: \Delta U_{BC} = nC_v\Delta T = Q$ [$\because W_{BC} = 0$] For Monoatomic gas $C_v \frac{R}{\gamma - 1} = \frac{3}{2}R$.

$\therefore \quad \Delta U_{BC} = Q = 2 \times \frac{3R}{2}\cdot \Delta T$

Hence $\quad \Delta T = \frac{Q}{3R}$

According to Poission's Law:

For the process AB. $T_A V_B^{\gamma-1} = T_B V_B^{\gamma-1}$

or $\quad T_B = T_A\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{P_1V_1}{nR}\left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$\therefore \quad T_B = \frac{P_1}{2R}\cdot V_1^{\gamma}\cdot V_2^{1-\gamma} = \frac{P_1V_1^{5/3}\cdot V_2^{-2/3}}{2R}$.

Hence, $\quad T_C = T_B + \Delta T = \frac{P_1V_1^{5/3}V_2^{-2/3}}{2R} + \frac{Q}{3R}$

31. For the process **A–B**, it is given that

$PT =$ constant

Differentiating above equation partially, we have

$PdT + TdP = 0$ (i)

Equation of state for two moles of a gas

$PV = 2RT \quad$ or $\quad P = \frac{2RT}{V}$ (ii)

After differentiating Eq. (ii) partially, we get

$PdV + VdP = 2R\,dT$

From Eq. (ii) partially, we get

$PdV + VdP = 2R\,dT$ (iii)

From Eqs. (i) and (ii), we have

$\left(\frac{2RT}{V}\right)dT + T\,dP = 0$

or $\quad 2RTdT + VTdP = 0$

$VdP = -2\,RdT$ (iv)

Now from Eqs. (iii) and (iv), we have

$-2RdT + VdP = 2RdT$

or $\quad PdV = 4\,RdT$ (v)

a. The work done in the process AB

$$W_{AB}\int PdV = \int_{600}^{300} 4R\,dT$$

$$= 4R|T|_{600}^{300} = 4R(300 - 600)$$

$$= -1200\,R$$

b. i. As process $B \to C$ is isobaric, so

$$Q_{BC} = nC_P\Delta T = 2 \times \frac{5R}{2}\times(600 - 300)$$
$$= 1500\,R$$

ii. Process $C \to A$ is isothermal, so $\Delta U = 0$

$Q_{CA} = \Delta U + W_{CA} = W_{CA}$

$W_{CA} = nRT\ln(P_C/P_A)$

$= 2R \times 600\ln(2P_1/P_1) = 1200\,R\ln 2$

$Q_{CA} = 1200\,R\ln 2$

Again for process $A \to B$

$Q_{AB} = \Delta U + W_{AB}$

$= nC_v\Delta T + W_{AB}$

$= 2\times\left(\frac{3R}{2}\right)\times(300 - 600) - 1200R$

$= -900\,R - 1200\,R = -2100R$

32. Also according to the principle of calorimetry

Heat lost by container = Heat gained by ice.

Let dQ be the heat lost when the temperature decreases by dT at any instant when the temperature of the container is T.

$\therefore \quad dQ = mc\,dT$

Where m is the mass of the container and $C = A + BT$ is specific heat at that temperature

$\therefore \quad dQ = m(A + BT)dT$

On integrating we get

$$Q = \int_{500}^{300} m(A + BT)dT = m\left[AT + \frac{BT^2}{2}\right]_{500}^{300}$$

$= -21600$ m (heat lost)

Now consider heat gained by ice This heat is to be divided into two parts

i. For making 0° Ice to 0° water

ii. increasing the temperature of water from 0°C to 27°C

$Q_1 = mL$ and $Q_2 = mc\Delta T$

$= 0.1 \times 80{,}000 = 0.1 \times 10^3 \times 27$

8000Cal = 2700 Cal

$\therefore \quad Q_1 + Q_2 = 8000 + 2700 = 10{,}700$ Cal

From (i) and (ii)

21600 m = 10,700 $\Rightarrow m = 0.495$ kg

33. When the mass of 100 kg is attached, the string is under tension and hence in the deformed state. The potential energy (U) stored by string.

$U = \frac{1}{2}\times$ stress $\times$ strain $\times$ volume

$= \frac{1}{2}\times\frac{(\text{Stress})^2}{Y}\times\pi r^2 l$

$= \frac{1}{2}\times\frac{(\text{Mg}/\pi r^2)^2}{Y}\times\pi r^2 l = \frac{1}{2}\frac{\text{M}^2g^2l}{\pi r^2 Y}$ (i)

This potential energy is released in the form of heat, thereby raising the temperature of the wire

$Q = mc\Delta T$ (ii)

From (i) and (iii)

Since $U = Q$ Therefore

$\therefore \quad mc\Delta T = \frac{1}{2}\frac{M^2g^2l}{\pi r^2 Y}$

$$\therefore \quad \Delta T = \frac{1}{2}\frac{M^2g^2l}{\pi r^2 Ycm}$$

Here m = mass of string = density × volume of string = $\rho \times \pi r^2 l$

$$\therefore \quad \Delta T = \frac{1}{2}\frac{M^2g^2}{(\pi r^2)^2 Yc\rho}$$

$$= \frac{1}{2} \times \frac{(100 \times 10)^2}{(3.14 \times 2 \times 10^{-3})^2 \times 2.1 \times 10^{11} \times 420 \times 7860}$$

$$= 0.00457°C$$

34. a. Since AB is a straight line passing through O, hence in V–T graph

$$\therefore \frac{V}{T} = \text{Constant (Isobaric process)}$$

$$\therefore \quad \frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$T_B = \frac{V_B}{V_A} \times T_A = 2 \times 300 = 600 \text{ K} \left[\because \frac{V_B}{V_A} = 2\right]$$

b. i. A to B is a isobaric process

$$\therefore \quad Q = nC_p\Delta T = 2 \times \frac{5}{2} R \times 300 = 1500R$$

Heat is absorbed as Q is positive.

ii. B to C is an isothermal process.
Since the temperature is not changing
∴ Internal energy change = 0
∴ From first law of thermodynamics $Q = W$

$$\therefore \quad Q = 2.303 \times nRT \log_{10}\frac{V_f}{V_i}$$

$$= 2.303 \times 2 \times R \times 600 \times \log_{10} 2$$

$$= 2763.6 \times \log_{10} 2 \times R = 831.8R$$

Heat is absorbed.

iii. C to D is a isochoric process ∴ $dW = 0$

$$\therefore \quad Q = nC_p\Delta T = 2 \times \frac{3}{2} R \times (-300) = -900R$$

Since volume is decreasing this heat is released.

iv. D to A is an isothermal process

$$\therefore \quad Q = 2.303 \times nRT \log_{10}\frac{V_f}{V_i}$$

$$= 2.303 \times 2 \times R \times 300 \times \log\left(\frac{1}{4}\right) = -831.8R$$

Heat is released as Q is positive.

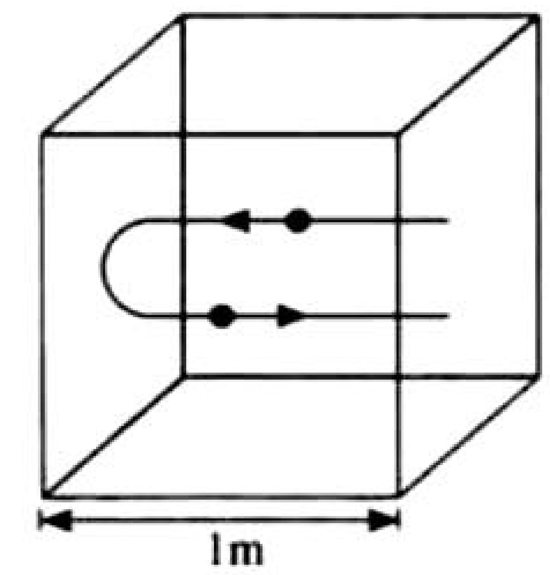
c. Total work done

$$= Q_{A \to B} + Q_{B \to C} + Q_{C \to D} + Q_{D \to A}$$

$$= (1500\,R + 831.8\,R) - (900\,R + 831.8\,R) = 600\,R$$

35. The observation time is 1 sec and during this time the atom makes 500 hits with the wall. The distance traveled by an atom of helium in $\frac{1}{500}$ sec (times between two successive collision) is 2 m. Therefore root mean square speed

$$v_{rms} = \frac{\text{distance}}{\text{time}} = \frac{2}{1/500} = 1000 \text{ m/s}$$



a. But $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow 1000 = \sqrt{\frac{3 \times 25/3 \times T}{4 \times 10^{-3}}}$

$$\Rightarrow \quad T = 160 \text{ K}$$

b. Average kinetic energy of an atom of a monoatomic gas $\frac{3}{2}kT$

$$\therefore E_{av} = \frac{3}{2}kT = 3.312 \times 10^{-12} \text{ Joules}$$

c. From gas equation $PV = \left(\frac{m}{M}\right)RT \Rightarrow m = 0.3012$ gm

36. When the container is stopped, the velocity of the container changes from V_0 to zero. Therefore the change in kinetic energy

$$K = \frac{1}{2}(nM)V_0^2 \qquad \text{(i)}$$

Here n = number of moles of gas present in the container
The kinetic energy at a given temperature for a monoatomic gas is

$$K = \frac{3}{2} \times nRT$$

$$\therefore \text{Change in kinetic energy} = \frac{3}{2}nR(\Delta T) \qquad \text{(ii)}$$

where ΔT = Change in temperature
From (i) and (ii)

$$\frac{3}{2}nR(\Delta T) = \frac{1}{2}(nm)V_0^2 \;\; \therefore \Delta T = \frac{mV_0^2}{3R}$$

37. From Stefan's–Boltzmann law

a. The rate of heat loss per unit area per second due to radiation is

$$E = \varepsilon\sigma(T^4 - T_0^4)$$

$$= 0.6 \times \frac{17}{3} \times 10^{-8}[(400)^4 - (300)^4]$$

$$= 595 \text{ watt/m}^2$$

b. Let T_{oil} be the temperature of the oil. The Rate of heat flow through conduction = Rate of heat flow through radiation

$$\frac{KA(T_{oil} - T)}{l} = 595 \times A$$

where A is the area of the top of lid

$$\Rightarrow \quad T_{oil} = \frac{595 \times l}{k} + T = \frac{595 \times 5 \times 10^{-3}}{0.149} + 400$$

$$= 419.83 \text{ K}$$

38. For isobaric process
we have $\frac{T_1}{V_1} = \frac{T_2}{V_2}$ also $V = A \times h$

$$\therefore \quad \frac{T_1}{Ah_1} = \frac{T_2}{Ah_2} \Rightarrow h_2 = \frac{T_2h_1}{T_1} = \frac{400}{300} \times 1 = \frac{4}{3}\text{m}$$

When the gas is compressed without heat exchange. The process is adiabatic

$$\therefore \quad T_1' = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1} = 400\left(\frac{4}{3}\right)^{2/5} \text{ K}$$

39. From the figure it is clear that emission takes places from the surface at temperature T_2 (circular cross section). Heat conduction and radiation through lateral surface is zero.

Heat conducted through rod is $Q = \dfrac{KA(T_1 - T_2)\Delta t}{l}$

Energy emitted by the surface of the rod in the same time Δt, id

$$E = \varepsilon\sigma A(T_2^4 - T_s^4)\Delta t$$

Since rod is at thermal equilibrium

$$\therefore \quad E = Q \text{ hence } \frac{KA(T_1 - T_2)\Delta t}{l} = \varepsilon\sigma A(T_2^4 - T_s^4)\Delta t$$

$$\Rightarrow \quad T_1 - T_2 = \frac{\varepsilon\sigma(T_2^4 - T_s^4)l}{K}$$

Using Binomial theorem $T_1 - T_2 = \dfrac{4\sigma\varepsilon l}{K}T_S^3\Delta T$

(Since $T_2 - T_s = \Delta T$ and $T_s >> \Delta T$)

$$\text{or} \quad T_1 - (T_S + \Delta T) = \frac{4\sigma\varepsilon l}{K}T_s^3\Delta T$$

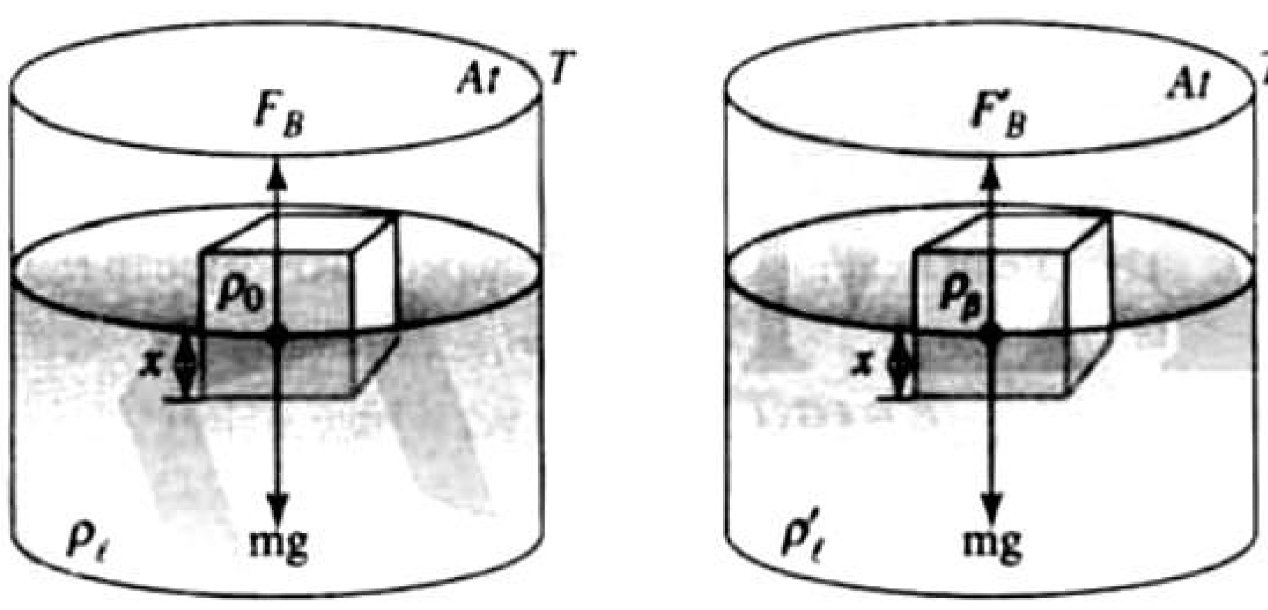
$$\text{or} \quad T_1 - T_S = \left(\frac{4\varepsilon\sigma l T_s^3 + 1}{K}\right)\Delta T$$

$\therefore$ The proportionality constant = $\left(1 + \dfrac{4\sigma\Delta l T_s^3}{K}\right)$

40. Initially, at temperature T, the block is floating in equilibrium have

$$F_B = \text{mg}$$

$$Ax\rho_l g = AL\rho_b g \Rightarrow x\rho_l = L\rho_b$$



At temperature $T + \Delta T$

Again the block is in equilibrium

$$F_B' = mg$$

$A'x\rho_l' g = AL\rho_b g$ [mg remains the same as above]

Now, $A' = A(1 + 2\alpha\Delta T)$

$\rho_l' = \rho_l(1 - \gamma\Delta T)$

$\therefore \quad A(1 + 2\alpha\Delta T)x\rho_l(1 - \gamma\Delta T)g = AL\rho_b g$

$\Rightarrow \quad x\rho_l(1 + 2\alpha\Delta T)(1 - \gamma\Delta T) = L\rho_b$

$\Rightarrow \quad x\rho_l(1 + 2\alpha\Delta T)(1 - \gamma\Delta T) = x\rho_l$

$\Rightarrow \quad 1 + 2\alpha\Delta T - \gamma\Delta T) = 1 \Rightarrow \gamma = 2\alpha$

41. a. From $\Delta Q = ms\Delta T$

$$\Delta T = \frac{\Delta Q}{ms} = \frac{20000}{1 \times 400} = 50^\circ\text{C}$$

b. $\Delta V = V\gamma\Delta T = \left(\dfrac{1}{9000}\right)(9 \times 10^{-5})(50) = 5 \times 10^{-7} \text{ m}^3$

$$W = p \cdot \Delta V = (10)^5(5 \times 10^{-7}) = 0.05 \text{ J}$$

c. $\Delta U = \Delta Q - W = (20000 - 0.05)\text{J}$

$= 19999.95$ J

42. 0.05 kg steam at 373 K $\xrightarrow{Q_1}$ 0.05 kg water at 373 K

0.05 kg water at 373 K $\xrightarrow{Q_2}$ 0.05 kg water at 273 K

0.45 kg ice at 253 K $\xrightarrow{Q_3}$ 0.45 kg ice at 273 K

0.45 kg ice at 273 K $\xrightarrow{Q_4}$ 0.45 kg water at 273 K

$Q_1 = (50)(540) = 27000$ cal = 27 kcal
$Q_2 = (50)(1)(100) = 5000$ cal = 5 kcal
$Q_3 = (450)(0.5)(20) = 4500$ cal = 4.5 kcal
$Q_4 = (450)(80) = 36000$ cal = 36 kcal

Now since $Q_1 + Q_2 > Q_3$ but $Q_1 + Q_2 < Q_4$ ice will come to 273 K from 253 K, but whole ice will not melt. Therefore, temperature of the mixture is 273 K.