6. Factorisation of Algebraic expressions

- Factorisation of quadratic polynomials of the form $ax^2 + bx + c$ can be done using Factor theorem and splitting the middle term.

Example 1:

Factorize $x^2 – 7x + 10$ using the factor theorem.

Solution:

Let $p(x) = x^2 - 7x + 10$
The constant term is 10 and its factors are ±1, ±2, ±5and ±10.
Let us check the value of the polynomial for each of these factors of 10.
$p(1) = 1^2 - 7 \cdot 1 + 10 = 1 - 7 + 10 = 4 \neq 0$
Hence, $x - 1$ is not a factor of $p(x)$.
$p(2) = 2^2 - 7 \cdot 2 + 10 = 4 - 14 + 10 = 0$
Hence, $x - 2$ is a factor of $p(x)$.
$p(5) = 5^2 - 7 \cdot 5 + 10 = 25 - 35 + 10 = 0$
Hence, $x - 5$ is a factor of $p(x)$.
We know that a quadratic polynomial can have a maximum of two factors. We have obtained the two factors of the given polynomial, which are $x – 2$ and $x – 5$.
Thus, we can write the given polynomial as:
$p(x) = x^2 - 7x + 10 = (x - 2)(x - 5)$

Example 2:

Factorize $2x^2 – 11x + 15$ by splitting the middle term.

Solution:

The given polynomial is $2x^2 - 11x + 15$.
Here, $a\,c= 2 \times 15 = 30$. The middle term is − 11. Therefore, we have to split − 11 into two numbers such that their product is 30 and their sum is –11. These numbers are − 5 and − 6 [As (–5) + (–6) = –11 and (–5) × (–6) = 30].
Thus, we have:

$$\begin{aligned} 2x^2 - 11x + 15 &= 2x^2 - 5x - 6x + 15 \\ &= x(2x - 5) - 3(2x - 5) \\ &= (2x - 5)(x - 3) \end{aligned}$$

Note: A quadratic polynomial can have a maximum of two factors.

- Identities for sum and difference of two cubes are:
  - $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
  - $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

For example, $x^6 - 729y^6$ can be factorized as:

$x^6 - 729y^6$

$= (x^3)^2 - (27y^3)^2$

$= (x^3 + 27y^3)(x^3 - 27y^3)$ [Using $a^2 - b^2 = (a + b)(a - b)$]

$= [(x)^3 + (3y)^3][(x)^3 - (3y)^3]$

$= (x + 3y)(x^2 + 9y^2 - 3xy)(x - 3y)(x^2 + 9y^2 + 3xy)$