CHAPTER

6.7

THE STATE-VARIABLE ANALYSIS

1. Consider the SFG shown in fig. P6.7.1

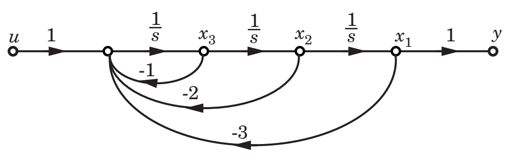

Fig. P6.7.1

For this system dynamic equation is

(A) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}$

(B) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}$

(C) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}$

(D) None of the above

Statement for Q.2–4:

Represent the given system in state-space equation $\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u}$. Choose the correction option for matrix **A**.

2.

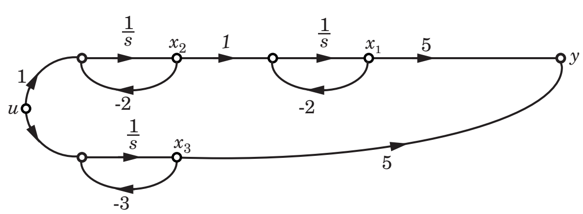

Fig. P6.7.2

(A) $\begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 1 & -2 \\ 0 & -2 & 0 \\ -3 & 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & -1 & 2 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

3.

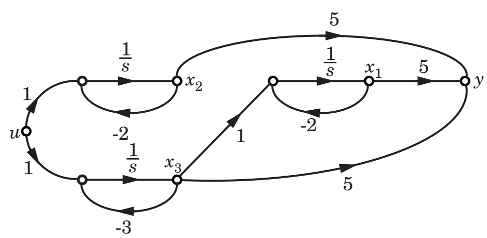

Fig. P6.7.3

(A) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & -2 & 0 \\ -3 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

4.

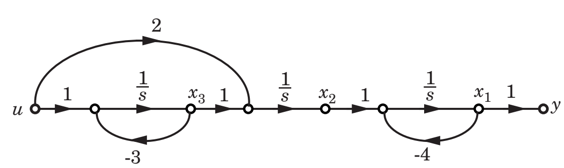

Fig. P6.7.4

(A) $\begin{bmatrix} 0 & 1 & -4 \\ 1 & 0 & 0 \\ -3 & 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -1 & 4 \\ -1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix}$ (D) $\begin{bmatrix} 4 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

5. The state equation of a LTI system is represented by

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\mathbf{u}$$

The Eigen values are

(A) $-1, +1$ (B) $-0.5 \pm j1.323$

(C) $-1, -1$ (D) None of the above

6. The state equation of a LTI system is

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}$$

The state-transition matrix $\Phi(t)$ is

(A) $\begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$ (B) $\begin{bmatrix} -e^{-3t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$

(C) $\begin{bmatrix} -e^{-3t} & 0 \\ 0 & -e^{-3t} \end{bmatrix}$ (D) $\begin{bmatrix} e^{-3t} & 0 \\ 0 & -e^{-3t} \end{bmatrix}$

7. The state equation of a LTI system is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}$$

The state transition matrix is

(A) $\begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$ (B) $\begin{bmatrix} \cos 2t & -\sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$

(C) $\begin{bmatrix} \sin 2t & \cos 2t \\ -\cos 2t & \sin 2t \end{bmatrix}$ (D) $\begin{bmatrix} \sin 2t & -\cos 2t \\ \cos 2t & \sin 2t \end{bmatrix}$

Statement for Q.8–9:

The state-space representation of a system is given by $\dot{\mathbf{x}}(t) = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{B}\cdot\mathbf{u}(t)$, where

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

If $\mathbf{x}(0)$ is the initial state vector, and the component of the input vector $\mathbf{u}(t)$ are all unit step function, then the state transition equation is given by $\dot{\mathbf{x}}(t) = \Phi(t)\mathbf{x}(0) + \theta(t)$, where $\Phi(t)$ is a state transition matrix and $\theta(t)$ is a vector matrix.

8. The $\Phi(t)$ is

(A) $\begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$ (B) $\begin{bmatrix} \cos 2t & -\sin 2t \\ \sin 2t & \cos 2t \end{bmatrix}$

(C) $\begin{bmatrix} \sin 2t & \cos 2t \\ -\cos 2t & \sin 2t \end{bmatrix}$ (D) $\begin{bmatrix} \sin 2t & -\cos 2t \\ \cos 2t & \sin 2t \end{bmatrix}$

9. The $\theta(t)$ is

(A) $\begin{bmatrix} 0.5(1-\sin 2t) \\ 0.5\cos 2t \end{bmatrix}$ (B) $\begin{bmatrix} \sin 2t \\ \cos 2t \end{bmatrix}$

(C) $\begin{bmatrix} 0.5(1-\cos 2t) \\ 0.5\sin 2t \end{bmatrix}$ (D) $\begin{bmatrix} \cos 2t \\ \sin 2t \end{bmatrix}$

10. From the following matrices, the state-transition matrices can be

(A) $\begin{bmatrix} -e^{-t} & 0 \\ 0 & 1-e^{-t} \end{bmatrix}$ (B) $\begin{bmatrix} 1-e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 1-e^{-t} & e^{-t} \end{bmatrix}$ (D) $\begin{bmatrix} 1-e^{-t} & e^{-t} \\ 0 & e^{-t} \end{bmatrix}$

Statement for Q.11–13:

A system is described by the dynamic equations $\dot{\mathbf{x}}(t) = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{B}\cdot\mathbf{u}(t)$, $y(t) = \mathbf{C}\cdot\mathbf{x}(t)$ where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{C} = [1 \ 0 \ 0]$$

11. The Eigen values of $\mathbf{A}$ are

(A) $0.325, -1.662 \pm j0.562$

(B) $2.325, 0.338 \pm j0.562$

(C) $-2.325, -0.338 \pm j0.562$

(D) $-0.325, 1.662 \pm j0.562$

12. The transfer-function relation between $X(s)$ and $U(s)$ is

(A) $\frac{1}{s^3+3s^2+2s-1}\begin{bmatrix} 1 \\ -s \\ s^2 \end{bmatrix}$ (B) $\frac{1}{s^3+3s^2+2s+1}\begin{bmatrix} 1 \\ s \\ s^2 \end{bmatrix}$

(C) $\frac{1}{s^3+3s^2+2s+1}\begin{bmatrix} 1 \\ -s \\ s^2 \end{bmatrix}$ (D) None of the above

13. The output transfer function $Y(s)/U(s)$ is

(A) $s(s^3+3s^2+2s+1)^{-1}$ (B) $s(s^3+3s^2+2s-1)^{-1}$

(C) $(s^3+3s^2+2s+1)^{-1}$ (D) None of the above

14. A system is described by the dynamic equation $\dot{\mathbf{x}}(t) = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{B}\cdot\mathbf{u}(t),\ y(t) = \mathbf{C}\cdot\mathbf{x}(t)$ where

$$\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \mathbf{C} = [1 \quad 1]$$

The output transfer function $Y(s)/U(s)$ is

(A) $\frac{(s+1)}{(s+2)^2}$

(B) $\frac{s+1}{s+2}$

(C) $\frac{(s+2)}{(s+1)}$

(D) None of the above

15. The state-space representation of a system is given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\mathbf{u}(t),\ y(t) = [1 \quad 1]\mathbf{x}(t)$$

The transfer function of this system is

(A) $(s^2 + 3s + 2)^{-1}$

(B) $(s+2)^{-1}$

(C) $s(s^2 + 3s + 2)^{-1}$

(D) $(s+1)^{-1}$

16. The state-space representation for a system is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}\mathbf{u},\ y = [1 \quad 0 \quad 0]\mathbf{x}$$

The transfer function $Y(s)/U(s)$ is

(A) $\frac{10(2s^2 + 3s + 1)}{s^3 + 3s^2 + 2s + 1}$

(B) $\frac{10(2s^2 + 3s + 1)}{s^3 + 2s^2 + 3s + 1}$

(C) $\frac{10(2s^2 + 3s + 2)}{s^3 + 3s^2 + 2s + 1}$

(D) $\frac{10(2s^2 + 3s + 2)}{s^3 + 2s^2 + 3s + 1}$

Statement for Q.17–18:

Determine the state-space representation for the transfer function given in question. Choose the state variable as follows

$$x_1 = c = y,\ x_2 = \frac{dc}{dt} = \dot{c},\ x_3 = \frac{d^2c}{dt^2} = \ddot{c},\ x_4 = \frac{d^3c}{dt^2} = \dddot{c}$$

17. $\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$

(A) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}\mathbf{r}$

(B) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 24 & 26 & 9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}\mathbf{r}$

(C) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 9 & 26 & 24 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}\mathbf{r}$

(D) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -9 & -26 & -24 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}\mathbf{r}$

18. $\frac{C(s)}{R(s)} = \frac{100}{s^4 + 20s^3 + 10s^2 + 7s + 100}$

(A) $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 100 & 7 & 10 & 20 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \end{bmatrix}\mathbf{r},$

$y = [1\ 0\ 0\ 0]\mathbf{x}$

(B) $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -100 & -7 & -10 & -20 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \end{bmatrix}\mathbf{r}$

$y = [1\ 0\ 0\ 0]\mathbf{x}$

(C) $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 20 & 10 & 7 & 100 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\mathbf{r}$

$y = [100\ 0\ 0\ 0]\mathbf{x}$

(D) $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -20 & -10 & -7 & -100 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\mathbf{r}$

$y = [100\ 0\ 0\ 0]\mathbf{x}$

19. A state-space representation of a system is given by $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}\mathbf{x},\ y = [1 \quad -1]\mathbf{x}$, and $\mathbf{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

The time response of this system will be

(A) $\sin\sqrt{2}t$

(B) $\frac{3}{\sqrt{2}}\sin\sqrt{2}t$

(C) $-\frac{1}{\sqrt{2}}\sin\sqrt{2}t$

(D) $\sqrt{3}\sin\sqrt{2}t$

20. For the transfer function

$$\frac{Y(s)}{U(s)} = \frac{s+3}{(s+1)(s+2)}$$

The state model is given by $\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x} + \mathbf{B}\cdot\mathbf{u}$, $y = \mathbf{C}\cdot\mathbf{x}$. The $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are

29. $\dot{\mathbf{x}} = \begin{bmatrix} -5 & -4 & -2 \\ -3 & -10 & 0 \\ -1 & 1 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\mathbf{r},\ y = [-1\ \ 2\ \ 1]\mathbf{x}$

$e_{step}(\infty)$	$e_{ramp}(\infty)$
(A) 1.0976	0
(B) 1.0976	∞
(C) 0	1.0976
(D) ∞	1.0976

30. $\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -9 & 7 \\ -1 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\mathbf{r},\ y = [1\ \ 0\ \ 0]\mathbf{x}$

$e_{step}(\infty)$	$e_{ramp}(\infty)$
(A) 0	0.714
(B) ∞	0.714
(C) 0	4.86
(D) ∞	4.86

Statement for Q.31–33:

Consider the system shown in fig. P6.7.31-33

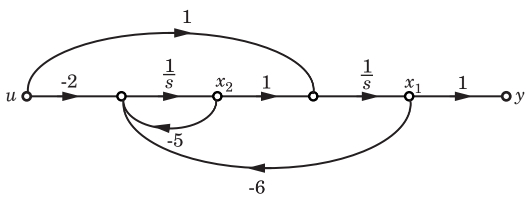

Fig. P6.7.31-33

31. The controllability matrix is

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$

32. The observability matrix is

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$

33. The system is

(A) Controllable and observable

(B) Controllable only

(C) Observable only

(D) None of the above

Statement for Q.34–36:

Consider the system shown in fig. P6.7.34-36.

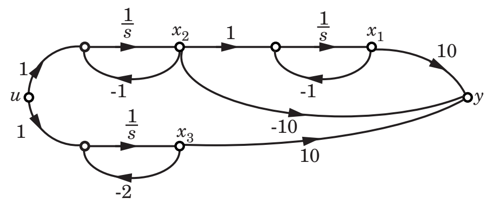

Fig. P6.7.34-36

34. The controllability matrix for this system is

(A) $\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & -20 \\ 10 & -20 & 40 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & 20 \\ 10 & -20 & -40 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & -1 \\ 1 & 6 & -1 \\ 1 & -4 & -4 \end{bmatrix}$

35. The observability matrix is

(A) $\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & -20 \\ 10 & -10 & 40 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & 20 \\ 10 & -10 & -40 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$

36. The system is

(A) Controllable and observable

(B) Controllable only

(C) Observable only

(D) None of the above

Statement for Q.37–38:

A state flow graph is shown in fig. P6.7.37-38

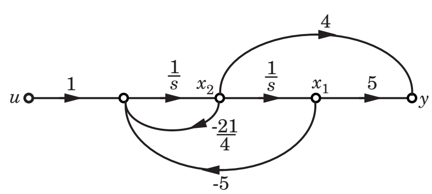

Fig. P6.7.37-38

37. The state and output equation for this system is

(A) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 5 & \frac{21}{4} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u},\ y = [5\ \ 4]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(B) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{21}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u},\ y = [5 \quad 4] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(C) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 5 & \frac{21}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{u},\ y = [4 \quad 5] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(D) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{21}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{u},\ y = [4 \quad 5] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

38. The system is

(A) Observable and controllable

(B) Controllable only

(C) Observable only

(D) None of the above

39. Consider the network shown in fig. P6.7.39. The state-space representation for this network is

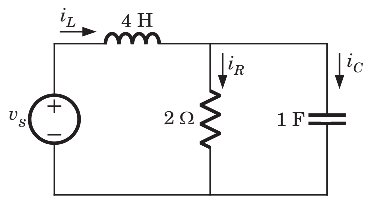

Fig. P6.7.39

(A) $\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -0.25 & 1 \\ -0.5 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 1 \\ 0.25 \end{bmatrix} v_s,\ i_R = [0.5 \quad 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix}$

(B) $\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -0.5 & 1 \\ -0.25 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.25 \\ 1 \end{bmatrix} v_s,\ i_R = [0.5 \quad 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix}$

(C) $\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} 1 & 0.25 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} v_s,\ i_R = [0.5 \quad 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix}$

(D) $\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -1 & 0.25 \\ 0 & -0.5 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} v_s,\ i_R = [0.5 \quad 0] \begin{bmatrix} v_C \\ i_L \end{bmatrix}$

40. For the network shown in fig. P6.7.40. The output is

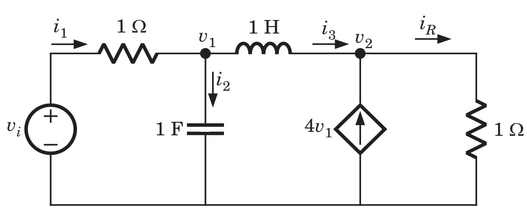

Fig. P6.7.40

$i_R(t)$. The state space representation is

(A) $\begin{bmatrix} \dot{v}_1 \\ \dot{i}_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_i,\ i_R = [4 \quad 1] \begin{bmatrix} v_1 \\ i_3 \end{bmatrix}$

(B) $\begin{bmatrix} \dot{v}_1 \\ \dot{i}_3 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ i_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_i,\ i_R = [4 \quad 1] \begin{bmatrix} v_1 \\ i_3 \end{bmatrix}$

(C) $\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} v_i,\ i_R = [1 \quad 4] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

(D) $\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -1 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} v_i,\ i_R = [1 \quad 4] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

Statement for Q.41–43:

Consider the network shown in fig. P6.7.41-43. This system may be represented in state space representation $\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u}$

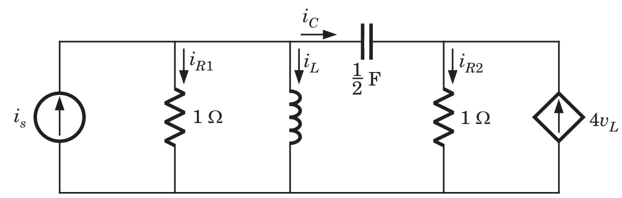

Fig. P6.7.41-43

41. The state variable may be

(A) i_{R1}, i_{R2}

(B) i_L, i_C

(C) v_C, i_L

(D) None of the above

42. If state variable are chosen as in previous question, then the matrix **A** is

(A) $\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 3 \\ 1 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$

43. The matrix **B** is

(A) $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

(C) $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$

Statement for Q.44–47:

Consider the network shown in fig. P6.7.44-47

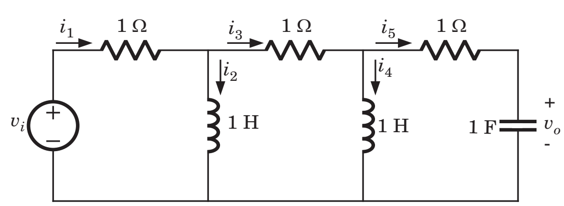

Fig. P6.7.44-47

44. The state variable may be

(A) i_2 , i_4 (B) i_2 , i_4 , v_o

(C) i_1 , i_3 (D) i_1 , i_3 , i_5

45. In state space representation matrix **A** is

(A) $\begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

46. The matrix **B** is

(A) $\begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$

47. If output is v_o , then matrix **C** is

(A) [–1 0 0] (B) [1 0 0]

(C) [0 0 –1] (D) [0 0 1]

****

SOLUTIONS

1. (B) From the SFG

$\dot{x}_3 = -3x_1 - 2x_2 - x_3 + u$

$x_2 = \frac{x_3}{s} \quad \Rightarrow \quad \dot{x}_2 = x_3$

$x_1 = \frac{x_2}{s} \quad \Rightarrow \quad \dot{x}_1 = x_2$

2. (A) $\dot{x}_1 = -2x_1 + x_3$, $\dot{x}_2 = -2x_2 + u$, $\dot{x}_3 = -3x_3 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}$$

3. (C) $\dot{x}_1 = -2x_1 + x_3$, $\dot{x}_2 = -2x_2 + u$, $\dot{x}_3 = -3x_3 + u$

$y = 5x_1 + 5x_2 + 5x_3$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \mathbf{u}$$

4. (C) $\dot{x}_1 = -4x_1 + x_2$, $\dot{x}_2 = x_3 + 2u$, $\dot{x}_3 = -3x_3 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -4 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \mathbf{u}$$

5. (B) $\Delta s = |s\mathbf{I} - \mathbf{A}| = s^2 + s + 2 = 0 \Rightarrow s = -0.5 \pm j1.323$

6. (A) $(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s+3 & 0 \\ 0 & s+3 \end{bmatrix}$, $|s\mathbf{I} - \mathbf{A}| = \frac{1}{(s+3)^2}$

$$(s\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} \frac{1}{s+3} & 0 \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$\Phi(t) = \mathrm{L}^{-1}\{(s\mathbf{I} - \mathbf{A})\} == \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-3t} \end{bmatrix}$$

7. (A) $(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -2 \\ 2 & s \end{bmatrix}$, $|s\mathbf{I} - \mathbf{A}| = s^2 + 4$

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{s^2+4}\begin{bmatrix} s & 2 \\ -2 & s \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2+4} & \frac{2}{s^2+4} \\ \frac{-2}{s^2+4} & \frac{s}{s^2+4} \end{bmatrix}$$

$$\Phi(t) = \mathrm{L}^{-1}\{(s\mathbf{I} - \mathbf{A})\} = \begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$$

8. (A) $(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -2 \\ 2 & s \end{bmatrix}$, $\Delta s = |s\mathbf{I} - \mathbf{A}| = s^2 + 4$

$$(s\mathbf{I}-\mathbf{A})^{-1} = \frac{1}{s^2+4}\begin{bmatrix} s & 2 \\ -2 & s \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2+4} & \frac{2}{s^2+4} \\ \frac{-2}{s^2+4} & \frac{s}{s^2+4} \end{bmatrix}$$

$$\Phi(t) = L^{-1}\{(s\mathbf{I}-\mathbf{A})\} = \begin{bmatrix} \cos 2t & \sin 2t \\ -\sin 2t & \cos 2t \end{bmatrix}$$

9. (C) $\theta(t) = L^{-1}\{(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}R(s)\}$

$$= L^{-1}\left\{\frac{1}{s^2+4}\begin{bmatrix} s & 2 \\ -2 & s \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\frac{1}{s}\right\} = L^{-1}\left\{\frac{1}{s(s^2+4)}\begin{bmatrix} 2 \\ s \end{bmatrix}\right\}$$

$$= L^{-1}\left\{\begin{bmatrix} \frac{2}{s(s^2+4)} \\ \frac{1}{(s^2+4)} \end{bmatrix}\right\} = \begin{bmatrix} 0.5(1-\cos 2t) \\ 0.5\sin 2t \end{bmatrix}$$

10. (C) (A) is not a state-transition matrix, since $\Phi(0) \neq \mathbf{I}$

(B) is not a state-transition matrix since $\Phi(0) \neq \mathbf{I}$

(C) is a state-transition matrix since $\Phi(0) = \mathbf{I}$ and $[\Phi(t)]^{-1} = \Phi(-t)$

11. (C) $(s\mathbf{I}-\mathbf{A}) = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{bmatrix}$

$|s\mathbf{I}-\mathbf{A}| = s^3 + 3s^2 + 2s + 1,$

$\Rightarrow \quad s = -2.325, \; -0.338 \pm j0.562$

12. (B) $\dfrac{X(s)}{U(s)} = (s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$

$$= \frac{1}{s^3+3s^2+2s+1}\begin{bmatrix} s^3+3s+2 & s+3 & 1 \\ -1 & s(s+3) & s \\ -s & -2s-1 & s^2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^3+3s^2+2s+1}\begin{bmatrix} 1 \\ s \\ s^2 \end{bmatrix}$$

13. (C) $\dfrac{Y(s)}{U(s)} = \dfrac{CX(s)}{U(s)}$

$$= [1 \;\; 0 \;\; 0]\begin{bmatrix} \frac{1}{s^3+3s^2+2s+1} \\ \frac{s}{s^3+3s^2+2s+1} \\ \frac{s^2}{s^3+3s^2+2s+1} \end{bmatrix} = \frac{1}{s^3+3s^2+2s+1}$$

14. (D) $\dfrac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}$

$$\mathbf{B} = [1 \;\; 1]\frac{1}{\Delta s}\begin{bmatrix} s+1 & 1 \\ 0 & s+1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{s+2}{(s+1)^2}$$

15. (D) $T(s) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}(s\mathbf{I}-\mathbf{A})^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$(s\mathbf{I}-\mathbf{A})^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$T(s) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{s+1}$$

16. (C) $\dot{\mathbf{x}} = \mathbf{A}\cdot\mathbf{x} + \mathbf{B}\cdot\mathbf{u}, \; y = \mathbf{C}\cdot\mathbf{x} + \mathbf{D}\mathbf{u}$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}, \mathbf{C} = [1 \;\; 0 \;\; 0], \mathbf{D} = 0$$

$$T(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

$$(s\mathbf{I}-\mathbf{A})^{-1} = \frac{1}{s^3+3s^2+2s+1}\begin{bmatrix} s^3+3s+2 & s+3 & 1 \\ -1 & s(s+3) & s \\ -s & -2s-1 & s^2 \end{bmatrix}$$

Substituting the all values,

$$T(s) = \frac{10(2s^2+3s+2)}{s^3+3s^2+2s+1}$$

17. (A) $\dfrac{C(s)}{R(s)} = \dfrac{b_0}{s^3+a_2s^2+a_1s+a_0} = \dfrac{24}{(s^3+9s^2+26s+24)}$

$(s^3+a_2s^2+a_1s+a_0)C(s) = b_0R(s)$

Taking the inverse Laplace transform assuming zero initial conditions

$\dddot{c} + a_2\ddot{c} + a_1\dot{c} + a_0c = b_0r$

$x_1 = c = y, \;\; x_2 = \dot{c}\;, \;\; x_3 = \ddot{c}$

$\dot{x}_1 = \dot{c} = x_2\;, \;\; \dot{x}_2 = \ddot{c} = x_3$

$\dot{x}_3 = \dddot{c} = b_0r - a_2\ddot{c} - a_1\dot{c} - a_0c$

$= -a_0x_1 - a_1x_2 - a_2x_3 + b_0r,$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_0 \end{bmatrix}\mathbf{r}$$

$a_0 = 24\,, \;\; a_1 = 26, \;\; a_2 = 9, \;\; b_0 = 24$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}\mathbf{r}$$

$$y = [1 \;\; 0 \;\; 0]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

18. (B) Fourth order hence four state variable

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix}\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_0 \end{bmatrix}\mathbf{r},\ y = [1\ \ 0\ \ 0\ \ 0]\mathbf{x}$$

$a_0 = 100,\ \ a_1 = 7,\ \ a_2 = 10,\ \ a_3 = 20,\ \ b_0 = 100$

19. (B) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix},\ (s\mathbf{I} - \mathbf{A})^{-1} = \dfrac{1}{s^2 + 2}\begin{bmatrix} s & 1 \\ -2 & s \end{bmatrix}$

$$\Phi(t) = \mathrm{L}^{-1}\{(s\mathbf{I} - \mathbf{A})\} = \begin{bmatrix} \cos\sqrt{2}t & \dfrac{\sin\sqrt{2}t}{\sqrt{2}} \\ -\sqrt{2}\sin\sqrt{2}t & \cos\sqrt{2}t \end{bmatrix}$$

$$\mathrm{x}(t) = \Phi(t)\mathrm{x}(0) = \begin{bmatrix} \cos\sqrt{2}t + \dfrac{\sin\sqrt{2}t}{\sqrt{2}} \\ -\sqrt{2}\sin\sqrt{2}t + \cos\sqrt{2}t \end{bmatrix}$$

$$y = x_1 - x_2 = \frac{3}{\sqrt{2}}\sin\sqrt{2}t$$

20. (C) Find the transfer function of option

For (A) $,\dfrac{Y(s)}{U(s)} = \dfrac{1}{s-2},$

For (B) $,\dfrac{Y(s)}{U(s)} = \dfrac{1}{s-2}$

For (C), $\dfrac{Y(s)}{U(s)} = [0\ \ 1]\dfrac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 & 0 \\ 2 & s+1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= [0\ \ 1]\dfrac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 \\ s+3 \end{bmatrix} = \dfrac{s+3}{(s+1)(s+3)}$

So (C) is correct option.

21. (C) $\mathbf{A} = \begin{bmatrix} -2 & -1 \\ -3 & -5 \end{bmatrix},$

$|s\mathbf{I} - \mathbf{A}| = s^2 + 7s + 7 \quad \Rightarrow \quad s = -5.79,\ -1.21$

22. (B) $(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -2 & -3 \\ 0 & s-6 & -5 \\ -1 & -4 & s-2 \end{bmatrix}$

$|s\mathbf{I} - \mathbf{A}| = s^3 - 8s^2 - 11s + 8 \quad \Rightarrow \quad s = 9.11,\ 0.53,\ -1.64$

23. (D) $X(s) = (s\mathbf{I} - \mathbf{A})^{-1}(\mathbf{x}(0) + \mathbf{B}\cdot\mathbf{u})$

$= \begin{bmatrix} s-1 & -2 \\ 3 & s+1 \end{bmatrix}^{-1}\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\dfrac{3}{s^2+9}\right)$

$\dfrac{1}{(s^2+5)(s^2+9)}\begin{bmatrix} 2s^3 + 4s^2 + 21s + 45 \\ s^3 - 7s^2 + 12s - 7 \end{bmatrix}$

$Y(s) = [1\ \ 2]X(s) = \dfrac{4s^3 - 10s^2 + 45s - 105}{(s^2+5)(s^2+9)}$

24. (B) $X(s) = (s\mathbf{I} - \mathbf{A})^{-1}(\mathbf{x}(0) + \mathbf{B}\cdot\mathbf{u})$

$= \dfrac{1}{(s+1)(s+2)}\begin{bmatrix} s+1 & 0 \\ -1 & s+2 \end{bmatrix}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\dfrac{1}{s}\right)$

$= \begin{bmatrix} \dfrac{(s+1)}{s(s+2)} \\ \dfrac{1}{s(s+1)(s+2)} \end{bmatrix}$

$Y(s) = [0\ \ 1]X(s) = \dfrac{1}{s(s+1)(s+2)}$

$\Rightarrow \quad y(t) = \dfrac{1}{2} - e^{-t} + \dfrac{1}{2}e^{-2t}$

25. (B) $X(s) = (s\mathbf{I} - \mathbf{A})^{-1}(\mathbf{x}(0) + \mathbf{B}\cdot\mathbf{u})$

$= \begin{bmatrix} s+2 & -1 & 0 \\ 0 & s & -1 \\ 0 & 0 & s+1 \end{bmatrix}^{-1}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\dfrac{1}{s}\right) = \begin{bmatrix} \dfrac{1}{s(s+2)} \\ \dfrac{1}{s^2(s+2)} \\ \dfrac{1}{s^2(s+1)(s+2)} \end{bmatrix}$

$Y(s) = [1\ \ 0\ \ 0], \qquad X(s) = \dfrac{1}{s(s+2)}$

$y(t) = \dfrac{1}{2} - \dfrac{1}{2}e^{-2t}$

26. (D) For a unit step input $e_{step}(\infty) = 1 + \mathbf{CA}^{-1}\mathbf{B}$

$\mathbf{A} = \begin{bmatrix} -5 & 1 & 0 \\ 0 & -2 & 1 \\ 20 & -10 & 1 \end{bmatrix}, \qquad \mathbf{A}^{-1} = \begin{bmatrix} -0.4 & 0.05 & -0.05 \\ -1 & -0.25 & -0.25 \\ -2 & 1.5 & -0.5 \end{bmatrix}$

$e_{step}(\infty) = 1 + [-1\ \ 1\ \ 0]\begin{bmatrix} -0.4 & 0.05 & -0.05 \\ -1 & 0.25 & -0.25 \\ -2 & 1.5 & -0.5 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$= 1 - 0.2 = 0.8$

27. (A) $e_{step}(\infty) = 1 + \mathbf{CA}^{-1}\mathbf{B},\ \mathbf{A}^{-1} = \begin{bmatrix} -2 & -\dfrac{1}{3} \\ 1 & 0 \end{bmatrix}$

$e_{step}(\infty) = 1 + [1\ \ 1]\begin{bmatrix} -2 & -\dfrac{1}{3} \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

28. (C) $e_{ramp}(\infty) = \lim\limits_{t\to\infty}[(1 + \mathbf{CA}^{-1}\mathbf{B})t + \mathbf{C}(\mathbf{A}^{-1})^2\mathbf{B}]$

$1 + \mathbf{CA}^{-1}\mathbf{B} = \dfrac{2}{3}, \qquad e_{ramp}(\infty) = \lim\limits_{t\to\infty}\left[\dfrac{2}{3}t + \mathbf{C}(\mathbf{A}^{-1})^2\mathbf{B}\right] = \infty$

29. (B) $e_{step}(\infty) = 1 + \mathbf{CA}^{-1}\mathbf{B}$

$\mathbf{A} = \begin{bmatrix} -5 & -4 & -2 \\ -3 & -10 & 0 \\ -1 & 1 & -5 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},\ \mathbf{C} = [-1\ \ 2\ \ 1]$

$$\mathbf{A}^{-1} = \begin{bmatrix} -0.305 & 0.134 & 0.122 \\ 0.091 & -0.140 & -0.037 \\ -0.079 & -0.055 & -0.232 \end{bmatrix}$$

$$e_{step}(\infty) = 1 + 0.0976 = 1.0976$$

$$e_{ramp}(\infty) = \lim_{t\to\infty} [(1 + \mathbf{CA}^{-1}\mathbf{B})t + \mathbf{C}(\mathbf{A}^{-1})^2\mathbf{B}] = \infty$$

30. (B) $\mathbf{A}^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 1.286 & 0.143 & -0.714 \end{bmatrix}$

$$e_{step}(\infty) = 1 + [1\ \ 0\ \ 0]\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 1.286 & 0.143 & -0.714 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$(\mathbf{A}^{-1})^2 = \begin{bmatrix} -1.286 & -0.143 & 0.714 \\ 0 & 0 & -1 \\ -0.776 & -0.102 & -0.776 \end{bmatrix}$$

$\mathbf{C}(\mathbf{A}^{-1})^2\mathbf{B} = 0.714$, $e_{ramp}(\infty) = 0.714$

31. (B) $\dot{x}_1 = x_2 + u$, $\dot{x}_2 = -5x_2 - 6x_1 - 2u$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}u, \qquad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\mathbf{C}_M = [\mathbf{B}\ \ \mathbf{AB}] = \left[\begin{bmatrix} 1 \\ -2 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right] = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$$

32. (A) $y = x_1$, $y = [1\ \ 0]\mathbf{x}$,

$\mathbf{C} = [1\ \ 0]$, $\mathbf{CA} = [1\ \ 0]$, $\mathbf{O}_M = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

33. (C) det $\mathbf{C}_M = 0$. Hence system is not controllable. det $\mathbf{O}_M = 1$. Hence system is observable.

34. (B) $\dot{x}_1 = -x_1 + x_2$, $\dot{x}_2 = -x_2 + u$, $\dot{x}_3 = -2x_3 + u$

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}u, \ \mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$\mathbf{A}^2\mathbf{B} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}$$

$$\mathbf{C}_m = [\mathbf{B}\ \ \mathbf{AB}\ \ \mathbf{A}^2\mathbf{B}] = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$$

35. (A) $y = 10x_1 - 10x_2 + 10x_3$, $y = [10\ \ -10\ \ 10]\underline{\mathbf{x}}$

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}, \ \mathbf{C} = [10\ \ -10\ \ 10],$$

$$\mathbf{CA} = [10\ \ -10\ \ 10]\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = [10\ \ 0\ \ -20]$$

$$\mathbf{CA}^2 = [10\ \ 0\ \ -20]\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = [10\ \ -10\ \ 40]$$

$$\mathbf{O}_M = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix} = \begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & -20 \\ 10 & -10 & 40 \end{bmatrix}$$

36. (A) $\det \mathbf{C}_m = \det\begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix} = -1,$

Since the determinant is not zero, the 3×3 matrix is nonsingular and system is controllable

$$\det \mathbf{O}_M = \det\begin{bmatrix} 10 & -10 & 10 \\ -10 & 0 & -10 \\ 10 & -20 & 40 \end{bmatrix} = -3000$$

The rank of $\mathbf{O}_M$ is 3. Hence system is observable.

37. (B) $\dot{x}_2 = -5x_1 - \frac{21}{4}x_2 + u$, $\dot{x}_1 = x_2$, $y = 5x_1 + 4x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -\frac{21}{4} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\mathbf{u}, \ y = [5\ \ 4]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

38. (B) $\mathbf{O}_M = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -20 & 1 \end{bmatrix}$

det $\mathbf{O}_M = 0$. Thus system is not observable

$$\mathbf{C}_M = [\mathbf{B}\ \ \mathbf{AB}] = \begin{bmatrix} 0 & 1 \\ 1 & -\frac{21}{4} \end{bmatrix}$$

det $\mathbf{C}_M = -1$. Thus system is controllable.

39. (B) $\frac{dv_c}{dt} = i_c$, $\frac{di_L}{dt} = \frac{v_L}{4} = 0.25v_L$

v_C and i_L are state variable.

$i_L = i_C + i_R$, $i_C = i_L - i_R = i_L - \frac{v_C}{2}$, $v_L = v_s - v_C$

Hence equations are $\frac{dv_L}{dt} = i_L - \frac{v_C}{2} = -0.5v_C + i_L$

$$\frac{di_L}{dt} = 0.25(v_s - v_C) = -0.25v_C + 0.25v_s$$

$$\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -0.5 & 1 \\ -0.25 & 0 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0.25 \\ 1 \end{bmatrix}v_s,$$

$$i_R = \frac{v_C}{2} = 0.5 v_C, \qquad i_R = [0.5 \quad 0]\begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

40. (B) $\frac{dv_1}{dt} = i_2$, $\frac{di_3}{dt} = v_L$

Hence v_1 and i_3 are state variable.

$i_2 = i_1 - i_3 = (v_i - v_1) - i_3$, $i_2 = - v_1 - i_3 + v_i$

$v_L = v_1 - v_2 = v_1 - i_R$, $= v_1 - (i_3 + 4v_1) = -3v_1 - i_3$

$$\frac{dv_1}{dt} = - v_1 - i_3 + v_i \; , \; \frac{di_3}{dt} = - 3v_1 - i_3, \qquad y = i_R = 4v_1 + i_3$$

$$\begin{bmatrix} \dot{v}_1 \\ \dot{i}_3 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} v_1 \\ i_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_i, \; i_R = [4 \quad 1]\begin{bmatrix} v_1 \\ i_3 \end{bmatrix}$$

41. (C) Energy storage elements are capacitor and inductor. v_C and i_L are available in differential form and linearly independent. Hence v_C and i_L are suitable for state-variable.

42. (B) $\frac{1}{2}\frac{dv_C}{dt} = i_C \;\Rightarrow\; \frac{dv_C}{dt} = 2i_C$

$\frac{1}{2}\frac{di_L}{dt} = v_L \;\Rightarrow\; \frac{di_L}{dt} = 2v_L$

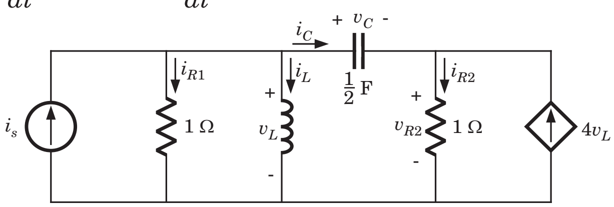

Fig. S6.7.42

$v_L = v_C + v_{R2} = v_C + i_{R2}$, $i_C + 4v_L = i_{R2}$

$v_L = v_C + i_C + 4v_L$, $-3v_L = v_C + i_C$ ...(i)

$i_C = i_s - i_{R1} - i_L$, $i_C = i_s - \frac{v_L}{1} - i_L$ ...(ii)

Solving equation (i) and (ii)

$-3(i_s - i_L - i_C) = v_C + i_C$, $2i_C = v_C - 3i_L + 3i_s$

$-3v_L = v_C + i_s - v_L - i_L$, $2v_L = - v_C + i_L - i_s$

$$\frac{dv_C}{dt} = v_C - 3i_L + 3i_s \; , \; \frac{di_L}{dt} = - v_C + i_L - i_s$$

$$\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix} i_s$$

43. (A) $\mathbf{B} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

44. (B) There are three energy storage elements, hence 3 variable. i_2 , i_4 and v_o are available in differentiated form hence these are state variable.

45. (A) $\frac{di_2}{dt} = v_2$, $\frac{di_4}{dt} = v_4$, $\frac{dv_o}{dt} = i_5$

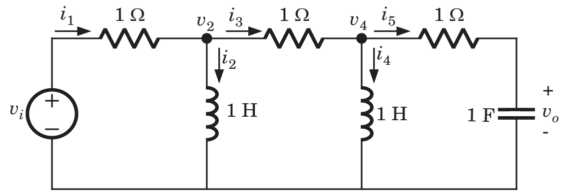

Fig. S6.7.45

Now obtain v_2 , v_4 and i_5 in terms of the state variable

$-v_i + i_1 + i_3 + i_5 + v_o = 0$

But $i_3 = i_1 - i_2$ and $i_5 = i_3 - i_4$

$-v_i + i_1 + (i_1 - i_2) + (i_3 - i_4) + v_o = 0$

$$i_1 = \frac{2}{3} i_2 + \frac{1}{3} i_4 - \frac{1}{3} v_o + \frac{1}{3} v_i$$

$$v_2 = v_i - i_1 = -\frac{2}{3} i_2 - \frac{1}{3} i_4 + \frac{1}{3} v_o + \frac{2}{3} v_i$$

$$i_3 = i_1 - i_2 = -\frac{1}{3} i_2 + \frac{1}{3} i_4 - \frac{1}{3} v_o + \frac{1}{3} v_i$$

$$i_5 = i_3 - i_4 = -\frac{1}{3} i_2 - \frac{2}{3} i_4 - \frac{1}{3} v_o + \frac{1}{3} v_i$$

$$v_4 = i_5 + v_o = -\frac{1}{3} i_2 - \frac{2}{3} i_4 + \frac{2}{3} v_o + \frac{1}{3} v_i$$

$$\begin{bmatrix} \dot{i}_2 \\ \dot{i}_4 \\ \dot{v}_o \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix}\begin{bmatrix} i_2 \\ i_4 \\ v_o \end{bmatrix} + \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} v_i$$

$$\mathbf{A} = \begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{2}{3} & -\frac{1}{3} \end{bmatrix},$$

46. (B) $\mathbf{B} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$

47. (D) v_o is state variable

$$y = v_o \; , \qquad y = [0 \quad 0 \quad 1] = \begin{bmatrix} i_2 \\ i_4 \\ v_o \end{bmatrix}$$

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10. If machine is not properly adjusted, the product resistance change to the case where $a_x = 1050\,\Omega$. Now the rejected fraction is

(A) 5046% (B) 10.57%
(C) 2.18% (D) 6.43%

11. Cannon shell impact position, as measured along the line of fire from the target point is described by a gaussian random variable X. It is found that 15.15% of shell falls 11.2 m or farther from the target in a direction toward the cannon, while 5.05% fall farther from the 95.6 m beyond the target. The value of a_x and σ_x for X is (Given that $F(1.03) = 0.8485$ and $F(1.64) = 0.9495$)

(A) $T + 40$ m and 50 m (B) $T + 40$ m and 30 m
(C) $T + 10$ m and 50 m (D) $T + 30$ m and 40 m

12. A gaussian random voltage X for which $a_X = 0$ and $\sigma_X = 4.2$ V appears across a $100\,\Omega$ resistor with a power rating of 0.25 W. The probability, that the voltage will cause an instantaneous power that exceeds the resistor's rating, is

(A) $2Q\left(\frac{5}{4.2}\right)$ (B) $Q\left(\frac{5}{4.2}\right)$
(C) $1 + Q\left(\frac{5}{4.2}\right)$ (D) $1 - Q\left(\frac{5}{4.2}\right)$

Statement for Question 13 -14 :

Assume that the time of arrival of bird at Bharatpur sanctuary on a migratory route, as measured in days from the first year (January 1 is the first day), is approximated as a gaussian random variable X with $a_X = 200$ and $\sigma_x = 20$ days. Given that : $F(0.5) = 0.6915$, $F(1.0) = 0.8413.$, $F(1.5) = 0.8531$, $F(1.55) = 0.9394$ and $F(2.0) = 0.9773$.

13. What is the probability that birds arrive after 160 days but on or before the 210th day ?

(A) 0.6687 (B) 0.8413
(C) 0.8531 (D) 0.9773

14. What is the probability that bird will arrive after 231st day ?

(A) 0.0432 (B) 0.1123
(C) 0.0606 (D) 0.0732

Statement for Question 15-16:

The life time of a system expressed in weeks is a Rayleigh random variable X for which

$$f_X(x) = \begin{cases} \frac{x}{200} e^{-\frac{x^2}{400}} & 0 \le x \\ 0 & x < 0 \end{cases}$$

15. The probability that the system will not last a full week is

(A) 0.01% (B) 0.25%
(C) 0.40% (D) 0.60%

16. The probability that the system lifetime will exceed in year is

(A) 0.01% (B) 0.05%
(C) 0.12% (D) 0.22%

17. The cauchy random variable has the following probability density function

$$f_X(x) = \frac{b/\pi}{b^2 + (x - a)^2}$$

For real numbers $0 < b$ and $-\infty < a < \infty$. The distribution function of X is

(A) $\frac{1}{\pi}\tan^{-1}\left(\frac{x-a}{b}\right)$

(B) $\frac{1}{\pi}\cot^{-1}\left(\frac{x-a}{b}\right)$

(C) $\frac{1}{2} + \frac{1}{\pi}\tan^{-1}\left(\frac{x-a}{b}\right)$

(D) $\frac{1}{2} + \frac{1}{\pi}\cot^{-1}\left(\frac{x-a}{b}\right)$

Statement for Question 18 - 19

The number of cars arriving at ICICI bank drive-in window during 10-min period is Poisson random variable X with $b = 2$.

18. The probability that more than 3 cars will arrive during any 10 min period is

(A) 0.249 (B) 0.143
(C) 0.346 (D) 0.543

19. The probability that no car will arrive is

(A) 0.516 (B) 0.459
(C) 0.246 (D) 0.135

20. The power reflected from an aircraft of complicated shape that is received by a radar can be described by an exponential random variable W. The density of W is

$$f_W(w) = \begin{cases} \frac{1}{W_0} e^{-w/W_0} & w > 0 \\ 0 & w < 0 \end{cases}$$

where W_0 is the average amount of received power. The probability that the received power is larger than the power received on the average is

(A) e^{-2} (B) e^{-1}

(C) $1 - e^{-1}$ (D) $1 - e^{-2}$

Statement for Question 21-23:

Delhi averages three murder per week and their occurrences follow a poission distribution.

21. The probability that there will be five or more murder in a given week is

(A) 0.1847 (B) 0.2461

(C) 0.3927 (D) 0.4167

22. On the average, how many weeks a year can Delhi expect to have no murders ?

(A) 1.4 (B) 1.9

(C) 2.6 (D) 3.4

23. How many weeds per year (average) can the Delhi expect the number of murders per week to equal or exceed the average number per week ?

(A) 15 (B) 20

(C) 25 (D) 30

24. A discrete random variable X has possible values $x_i = i^2$, $i = 1, 2, 3, 4$ which occur with probabilities 0.4, 0.25, 0.15, 0.1,. The mean value $\overline{X} = E[X]$ of X is

(A) 6.85 (B) 4.35

(C) 3.96 (D) 1.42

25. The random variable X is defined by the density

$$f_X(x) = \frac{1}{2} u(x) e^{-\frac{x}{2}}$$

The expected value of $g(X) = X^3$ is

(A) 48 (B) 192

(C) 36 (D) 72

26. The mean of random variable X is

(A) 1/4 (B) 1/6

(C) 1/3 (D) 1/5

27. The variance of random variable X is

(A) 1/10 (B) 3/80

(C) 5/16 (D) 3/16

28. A Random variable X is uniformly distributed on the interval (–5, 15). Another random variable $Y = e^{-\frac{X}{5}}$ is formed. The value of $E[Y]$ is

(A) 2 (B) 0.667

(C) 1.387 (D) 2.967

29. A random variable X has $\overline{X} = -3$, $\overline{x^2} = 11$ and $\sigma_X^2 = 2$ For a new random variable $Y = 2x - 3$, the $\overline{Y}$, $\overline{Y}^2$ and σ_Y^2 are

(A) 0, 81, 8 (B) -6, 8, 89

(C) -9, 89, 8 (D) None of the above

Statement for Question 31-32 :

A joint sample space for two random variable X and Y has four elements (1, 1), (2, 2), (3, 3) and (4, 4). Probabilities of these elements are 0.1, 0.35, 0.05 and 0.5 respectively.

30. The probability of the event$\{X \le 2.5, Y \le 6\}$ is

(A) 0.45 (B) 0.50

(C) 0.55 (D) 0.60

31. The probability of the event $\{X \le 3\}$ is

(A) 0.45 (B) 0.50

(C) 0.55 (D) 0.60

Statement for Question 32-34 :

Random variable X and Y have the joint distribution

$$F_{X,Y}(x, y) = \begin{cases} \frac{5}{4}\left(\frac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right) u(y), & 0 \le x \le 4 \\ 0 & x < 0 \text{ or } y < 0 \\ 1 + \frac{1}{4} e^{-5y^2} - \frac{5}{4} e^{-y^2}, & 4 \le x \text{ and any } y \ge 0 \end{cases}$$

32. The marginal distribution function $F_X(x)$ is

(A) $\begin{cases} 0, & x, > 0 \\ \dfrac{5x}{4(x+1)}, & -4 < x \le 4 \\ 1, & x \le -4 \end{cases}$

(B) $\begin{cases} 0, & x < 0 \\ \dfrac{5x}{4(x+1)}, & 0 \le x < 4 \\ 1, & x \ge 4 \end{cases}$

(C) $\begin{cases} 1, & x > 0 \\ \dfrac{5x}{4(x+1)}, & -4 < x \le 0 \\ 0, & x \le -4 \end{cases}$

(D) $\begin{cases} 1, & x < 0 \\ \dfrac{5x}{4(x+1)}, & 0 \le x < 4 \\ 0, & x \ge 4 \end{cases}$

33. The marginal distribution function $F_Y(y)$ is

(A) $\begin{cases} -\dfrac{5}{4}e^{-y^2}, & y > 0 \\ 1 + \dfrac{4}{4}e^{-5y^2}, & y \le 0 \end{cases}$

(B) $\begin{cases} 0, & y > 0 \\ 1 + \dfrac{1}{4}e^{-5y^2} - \dfrac{5}{4}e^{-y^2}, & y \le 0 \end{cases}$

(C) $\begin{cases} \dfrac{-5}{4}e^{-y^2}, & y < 0 \\ 1 + \dfrac{1}{4}e^{-5y^2} - \dfrac{5}{4}e^{-y^2}, & y \ge 0 \end{cases}$

(D) $\begin{cases} 0, & y < 0 \\ 1 + \dfrac{1}{4}e^{-5y^2} - \dfrac{5}{4}e^{-y^2}, & y \ge 0 \end{cases}$

34. The probability $P\{3 < X \le 5,\ 1 < y \le 2\}$ is

(A) 0.001 (B) 0.002

(C) 0.003 (D) 0.004

Statement for Question 35-39 :

Two random variable X and Y have a joint density

$$F_{X,Y}(x, y) = \frac{10}{4}[u(x) - u(x-4)]u(y)y^3 e^{-(x+1)y^2}$$

35. The marginal density $f_X(x)$ is

(A) $5\dfrac{u(x) - u(x-4)}{(x+1)^2}$ (B) $5\dfrac{u(x) - u(x-4)}{(x+1)}$

(C) $\dfrac{5}{4}\dfrac{u(x) - u(x-4)}{(x+1)^2}$ (D) $\dfrac{5}{4}\dfrac{u(x) - u(x-4)}{(x+1)}$

36. The marginal density $f_Y(y)$ is

(A) $\frac{5}{4}y^2[e^{-y^2} - e^{-5y^2}]u(y)$

(B) $\frac{5}{2}y^2[e^{-y^2} - e^{-5y^2}]u(y)$

(C) $\frac{5}{4}y[e^{-y^2} - e^{-5y^2}]u(y)$

(D) $\frac{5}{2}y[e^{-y^2} - e^{-5y^2}]u(y)$

37. The marginal distribution function $F_X(x)$ is

(A) $\dfrac{5}{4}\dfrac{1}{(x+1)^2}[4(x) - 4(x-4)]$

(B) $\dfrac{5}{2}\dfrac{1}{(x+1)^2}[4(x) - 4(x-4)]$

(C) $\dfrac{5}{4}\dfrac{1}{(x+1)}[4(x) - 4(x-4)]$

(D) None of the above

38. The marginal distribution function $F_Y(y)$ is

(A) $[1 - e^{-y^2}]u(y)$ (B) $\frac{5}{2}[1 - e^{-y^2}]u(y)$

(C) $\frac{5}{4}[1 - e^{-y^2}]u(y)$ (D) None of the above

39. The joint distribution function is

(A) $\begin{cases} \dfrac{5}{4}\left[\dfrac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right], & 0 \le x \le 4 \text{ and } y > 0 \\ 1 + \dfrac{1}{4}[e^{-5y^2} - 5e^{-y^2}], & x > 4 \text{ and } y > 0 \end{cases}$

(B) $\begin{cases} \dfrac{5}{8}\left[\dfrac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right], & 0 \le x \le 4 \text{ and } y > 0 \\ 1 + \dfrac{1}{2}[e^{-5y^2} - 5e^{-y^2}], & x > 4 \text{ and } y > 0 \end{cases}$

(C) $\begin{cases} \dfrac{5}{8}\left[\dfrac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right], & 0 \le x \le 4 \text{ and } y > 0 \\ 1 + \dfrac{1}{4}[e^{-5y^2} - 5e^{-y^2}], & x > 4 \text{ and } y > 0 \end{cases}$

(D) $\begin{cases} \dfrac{5}{4}\left[\dfrac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right], & 0 \le x \le 4 \text{ and } y > 0 \\ 1 + \dfrac{1}{2}[e^{-5y^2} - 5e^{-y^2}], & x > 4 \text{ and } y > 0 \end{cases}$

40. The function

$$F_{X,Y}(x, y) = \frac{a}{2}\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{2}\right)\right]\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{y}{3}\right)\right]$$

is a valid joint distribution function for random variables X and Y if the constant a is

(A) $\dfrac{1}{\pi^2}$ (B) $\dfrac{2}{\pi^2}$

(C) $\dfrac{4}{\pi^2}$ (D) $\dfrac{8}{\pi^2}$

41. Random variable X and Y have the joint distribution function

$$F_{X,Y}(x,y)=\begin{cases}0, & x<0 \text{ or } y<0\\ \frac{27}{26}x\left(1-\frac{y^2}{27}\right), & 0\le x<1 \text{ and } 1\le y\\ \frac{27}{26}y\left(1-\frac{y^2}{27}\right), & 1\le x \text{ and } 0\le y<1\\ \frac{27}{26}xy\left(1-\frac{x^2y^2}{27}\right), & 0\le x<1 \text{ and } 0\le y<1\\ 1, & 1\le x \text{ and } 1\le y\end{cases}$$

The probability of the event $\{0<X\le 0.5,\ 0<Y\le 0.25\}$ is

(A) 0.13 (B) 0.24

(C) 0.69 (D) 1

Statement for question 42-43 :

The joint probability density function of random variable X and Y is given by

$$p_{XY}(x,y)=xye^{-\frac{(x^2+y^2)}{2}}u(x)u(y)$$

42. The $p_X(x)$ is

(A) $2xe^{-x^2}u(x)$ (B) $xe^{-\frac{x^2}{2}}u(x)$

(C) $xe^{-x^2}u(x)$ (D) $2xe^{\frac{x^2}{2}}u(x)$

43. The $p_{Y/X}(y/x)$ is

(A) $\frac{1}{2}ye^{-y^2}u(y)$ (B) $ye^{-y^2}u(y)$

(C) $ye^{-\frac{y^2}{2}}u(y)$ (D) $\frac{1}{2}ye^{-\frac{y^2}{2}}u(y)$

44. The probability density function of a random variable X is given as $f_X(x)$. A random variable Y is defined as $y=ax+b$ where $a<0$. The PDF of random variable Y is

(A) $bf_X\left(\frac{y-b}{a}\right)$ (B) $af_X\left(\frac{y-b}{a}\right)$

(C) $\frac{1}{a}f_X\left(\frac{y-b}{a}\right)$ (D) $\frac{1}{b}f_X\left(\frac{y-b}{a}\right)$

45. The function

$$f_{X,Y}(x,y)=\begin{cases}be^{-(x+y)} & 0<x<a \text{ and } 0<y<\infty\\ 0 & \text{else where}\end{cases}$$

is a valid joint density function if b is

(A) $\frac{a^2}{1-e^{-a}}$ (B) $\frac{a}{1-e^{-a}}$

(C) $\frac{1}{1-e^{-a}}$ (D) None of the above

Statement for Question 46-47 :

Random variable X and Y have the joint density

$$f_{X,Y}(x,y)=\frac{1}{2}u(x)u(y)e^{-\frac{x}{4}-\frac{y}{3}}$$

46. The probability of the event $\{2<X\le 4,\ -1<Y\le 5\}$ is

(A) 0.1936 (B) 6.2964

(C) 0 (D) None of the above

47. The probability of the event $\{0<X<\infty,\ <y\le -2\}$ is

(A) 0.2349 (B) 0.3168

(C) 0.4946 (D) None of the above

48. Let X and Y be two statistically independent random variables uniformly distributed in the ranges (–1, 1) and (-2, 1) respectively. Let $Z=X+Y$. Then the probability that $(Z\le -2)$ is

(A) zero (B) 1/6

(C) 1/3 (D) 1/12

49. The probability density function of two statistically independent random variable X and Y are

$$f_X(x)=5u(x)e^{-5x}$$

$$f_Y(y)=24(y)e^{-2y}$$

The density of the sum $W=X+Y$ is

(A) $\frac{10}{6}[e^{-2\omega}-e^{5\omega}]u(w)$

((B) $\frac{10}{8}[e^{-2\omega}-e^{5\omega}]u(w)$

(C) $\frac{10}{13}[e^{-2\omega}-e^{-5\omega}]u(w)$

(D) $\frac{10}{2}[e^{-2\omega}-e^{-5\omega}]u(w)$

50. The density function of two random variable X and Y is

$$f_{X,Y}(x,y)=\begin{cases}\frac{1}{24} & 0<x<6 \text{ and } 0<y<4\\ 0 & \text{else where}\end{cases}$$

The expected value of the function $g(x,\ y)=(XY)$ is

(A) 64 (B) 96

(C) 32 (D) 48

51. The density function of two random variable X and Y is

$$f_{X,Y}(x,y) = \frac{e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}}{2\pi\sigma^2}$$

with σ^2 a constant. The mean value of the function $g(X,Y) = X^2 + Y^2$ is

(A) σ^2

(B) σ

(C) $2\sigma^2$

(D) 2σ

Statement for Question 52-54 :

The statistically independent random variable X and Y have mean values $\overline{X} = E[X] = 2$ and $\overline{Y} = E[Y] = Y$. They have second moments $\overline{X^2} = E[X^2] = 8$ and $Y^2 = E[Y^2] = 25$. Consider a random variable $W = 3X - Y$.

52. The mean value $E[W]$ is

(A) 2

(B) 4

(C) 8

(D) 25

53. The second moment of W is

(A) 145

(B) 49

(C) 97

(D) 0

54. The variance of the random variable is

(A) 4

(B) 45

(C) 49

(D) 54

55. Two random variable X and Y have the density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{xy}{9}, & 0 < x < 2 \text{ and } 0 < y < 3 \\ 0 & \text{elsewhere} \end{cases}$$

The X and Y are

(A) Correlated but statistically independent

(B) uncorrelated but statistically independent

(C) Correlated but statistically dependent

(D) Uncorrelated but statistically dependent

56. The value of σ_X^2, σ_Y^2, R_{XY} and ρ are respectively

(A) $\frac{11}{4}\ \frac{27}{2}, \frac{1}{2}.\left(2 + \frac{1}{\sqrt{3}}\right)$, and $-3\sqrt{\frac{33}{2}}$

(B) $\frac{11}{4}\ \frac{11}{2}, \frac{1}{2}\left(2 + \frac{1}{\sqrt{3}}\right)$, and $-3\sqrt{\frac{33}{2}}$

(C) $\frac{9}{4}\ \frac{11}{2}, \frac{1}{2}\left(2 - \frac{1}{\sqrt{3}}\right)$, and $-\frac{1}{3}\sqrt{\frac{2}{33}}$

(D) $\frac{9}{4}, \frac{11}{2}, \frac{1}{2}\left(2 - \frac{1}{\sqrt{3}}\right)$, and $-\frac{1}{3}\sqrt{\frac{2}{33}}$

57. The mean value of the random variable

$W = (X + 3Y)^2 + 2X + 3$ is

(A) $98 + \sqrt{3}$

(B) $98 - \sqrt{3}$

(C) $49 - \sqrt{3}$

(D) $49 + \sqrt{3}$

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SOLUTION

1. (A) $P\{X > 1\} = \int_1^{\infty} p_X(x)dx$

$= \int_1^{\infty} \frac{|x|}{2} e^{-|x|} dx = \frac{1}{2}\int_1^{\infty} x e^{-x} dx = 0.368$

2. (C) $P\{-1 < X \le 2\} = \int_{-1}^{0} -\frac{1}{2} xe^{x} dx + \int_0^2 \frac{1}{2} xe^{-x} dx$

$= 1 - \frac{1}{e} - \frac{3}{2e^2} = 0.429$

3. (A) Test 1: $f_X(x) \ge 0$ is true

Test 2: area must be 1 i.e. $\int_0^b \frac{\ell^{3x}}{4} dx = \frac{1}{4}\left[\frac{\ell^{3b}}{3} - \frac{1}{3}\right] = 1$

Thus $b = \frac{1}{3}\ln 13$

4. (C) $\int_{-\infty}^{\infty} \rho(v) = 1 \Rightarrow \frac{1}{2} 4k = 1 \Rightarrow k = \frac{1}{2}$

Thus $\rho(v) = \frac{kv}{4} = \frac{v}{8}$

Mean Square Value $= \int_{-\infty}^{\infty} x^2 \rho(x)dx = \int_0^4 x^2 \frac{x}{8} dx = 8$

5. (B) For $x = 0$, $F_X(x) = \int_0^x \frac{1}{2} e^{-\frac{x}{2}} dx = \left[1 - e^{-\frac{x}{2}}\right] u(x)$

$P(A) = F_X(3) - F_X(1) = e^{-\frac{1}{2}} - e^{-\frac{3}{2}} = 0.3834$

6. (D) $P(B) = F_X(2.5) = 1 - e^{-\frac{2.5}{2}} = 0.7135$

7. (D) $C = A \cap B = \{1 < X < 2.5\}$

$P(C) = F_X(2.5) - F_X(1) = e^{-\frac{1}{2}} - e^{-\frac{2.5}{2}} = 0.3200$

8. (A) $P(|X| > 2) = P\{2 < x\} + P\{x < -2\}$

$= 1 - P\{x \le 2\} + P\{x < -2\} = 1 - F(2) + F(-2)$

We know that for gaussian function $F(-x) = 1 - F(x)$

Thus $P(|X| > 2) = 1 - F(2) + 1 - F(2)$

$= 2 - 2F(2) = 2 - 2(0.9772) = 0.0456$

9. (C) Rejected resistor corresponds to $\{x < 900\,\Omega\}$ and $\{x > 1100\,\Omega\}$. Fraction rejected corresponds to probability of rejection.

P {resistor rejected} $= P\{X < 900\} + P\{X < 1100\}$

$= F_X(900) + [1 - F_X(1100)]$

$= F\left(\frac{900 - a_x}{\sigma_x}\right) + 1 - F\left(\frac{1100 - a_x}{\sigma_x}\right)$

$= F\left(\frac{900 - 1000}{40}\right) + 1 - F\left(\frac{11000 - 1000}{40}\right)$

$= F(-2.5) + 1 - F(2.5) = 1 - F(2.5) + 1 - F(2.5) = 2 - 2F(2.5)$

$= 2 - 2(0.9938) = 0.012$ or $1.2\ \%$

10. (B) P(resistor rejected) $= F\left(\frac{900 - 1050}{40}\right) + 1$

$-F\left(\frac{1100 - 1050}{40}\right) = F(-3.75) + 1 - F(1.25)$

$= 1 - F(3.75) + 1 - F(1.25)$

$= 2 - 0.9999 - 0.8944 = 0.1057$ or $20.57\ \%$

11. (D) $P\{x > T + 95.6\} = 0.0505$

$= 1 - F\frac{(T + 95.6 - a_x)}{\sigma_x} = F\left(\frac{T + 95.6 - a_x}{\sigma_x}\right) = 0.9495$

This occurs when $\frac{T + 95.6 - a_x}{\sigma_x} = 1.64$ ...(i)

$P\{x \le T - 11.2\} = 0.1515$

$= F\frac{(T - 11.2 - a_x)}{\sigma_x} = 1 - F\frac{(T - 11.2 - a_x)}{\sigma_x} = 8485$

This occur when $-\frac{T - 11.2 - a_x}{\sigma_x} = 1.03$ ...(ii)

Solving (i) and (ii) we get $a_x = T + 30$ and $\sigma_x = 40$

12. (A) 0.25 exceeds when $\frac{|x|^2}{100} > 0.21$ or $|x| > 5v$

$P(0.25\,\text{W exceeded}) = P\{|x| > 5\}$

$= P\{x > 5\} + P\{x < -5\} = 1 - P(x \le 5) + P\{x < -5\}$

$= 1 - P\left(\frac{5 - 0}{4.2}\right) + P\left(\frac{-5 - 0}{4.2}\right) = 1 - F\left(\frac{5}{4.2}\right) + F\left(\frac{-5}{4.2}\right)$

$= 1 - F\left(\frac{5}{4.2}\right) + 1 - F\left(\frac{5}{4.2}\right) = 2\left(1 - F\left(\frac{5}{4.2}\right)\right) = 2Q\left(\frac{5}{4.2}\right)$

13. (A) $P\{160 < X \le 120\} = F_X(210) - F_X(160)$

$= F\left(\frac{210 - 200}{20}\right) - F\left(\frac{160 - 200}{20}\right)$

$= F(0.5) - F(-2) = F(0.5) + F(2) - 1$

$= 0.6915 + 0.9773 - 1 = 0.6687$

14. (C) $P\{X > 231\} = 1 - P\{X \le 231\} = 1 - F_X(231)$

$= 1 - F\left(\frac{231 - 200}{20}\right) = 1 - F(1.55) = 1 - 0.9394 = 0.0606$

15. (B) We use the Rayleigh distribution with $a = 0$ and $b = 400$

For probability $P\{X \le 1\} = F_X(1) = 1 - e^{-\frac{1}{400}}$

$= 0.0025$ or 0.25 %

16. (C) $P\{X \ge 52\} = 1 - F_X(52)$

$= 1 - \left[1 - e^{-\frac{52^2}{400}}\right] = 1 - e^{-\frac{52^2}{400}} = 0.00116$ or 0.12 %

17. (C) $F_X = \int_{-\infty}^{x} f_X(u)du = \int_{-\infty}^{x} \frac{(b/\pi)du}{b^2 + (u-a)^2}$

Let $v = u - a$ and $dv = du$ to get

$F_X(x) = \frac{b}{\pi}\int_{-\infty}^{x-a} \frac{dv}{b^2 + v^2} = \frac{b}{\pi}\left[\frac{1}{b}\tan^{-1}\left(\frac{v}{b}\right)\right]_{-\infty}^{x-a}$

$= \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\left(\frac{x-a}{b}\right)$

18. (B) Here $f_X(x) = e^{-2}\sum_{k=0}^{\infty}\left(\frac{3^k}{k!}\right)\delta(x-k)$

$P\{x > 0\} = 1 - P\{x \le 3\}$

$= 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3)$

$= 1 - e^{-2}\left\{\frac{2^0}{0!} + \frac{2^1}{2!} + \frac{2^2}{2!} + \frac{2^3}{3!}\right\} = 1 - e^{-2}\left(\frac{19}{3}\right) = 0.1429$

19. (D) $P(x=0) = e^{-2}\left|\frac{2^0}{0!}\right| = 0.135$

20. (B) $P\{W > W_0\} = 1 - P\{W \le W_0\} = 1 - F_W(W_0)$

$= 1 - \left(1 - e^{-\frac{W_0}{W_0}}\right) = e^{-1}$

21. (A) P {5 or more} $= 1 - P(0) - P(1) - P(2) - P(4)$

$= 1 - e^{-3}\left[\frac{3^0}{0!} + \frac{3^1}{11} + \frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!}\right] = 1 - \frac{131}{8}e^{-3} = 0.1847$

22. (C) $P(0) = e^{-3} = 0.0498$

average number of week, per year with no murder $52e^{-3} = 2.5889$ week.

23. (D) P {3 or more} $= 1 - P(0) - P(1) - P(2)$

$= 1 - e^{-3}\left[1 + 3 + \frac{3^2}{2}\right] = 1 - \frac{17}{2}e^{-3} = 0.5768$

Average number of weeks per year that number of murder exceeds the average

$= 52\left(1 - \frac{17}{2}e^{-3}\right) = 29.994$ weeks

24. (B) $E[X] = \overline{X} = \sum_{i=1}^{4} x_i P(x_i)$

$= 1.0(0.4) + 4(0.25) + 9(0.15) + 16(0.1) = 4.35$

25. (A) $E[g(X)] = E[X^3] = \int_0^{\infty}\frac{1}{2}x^3 e^{-\frac{x}{2}} = \frac{1}{2}\left[\frac{6}{(1/2)^4}\right] = 48$

26. (A) Mean of $X = \int_{-\infty}^{x} x f_X(x)dx = \int_0^1 x3(1-x)^2 dx = \frac{1}{4}$

27. (B) Variance of X is $\sigma_x^2 = E[X^2] - \mu_x^2$

$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)dx = \int_0^1 x^2 3(1-x)^2 dx = \frac{1}{10}$

$\sigma_x^2 = \frac{1}{10} - \left(\frac{1}{4}\right)^2 = \frac{3}{80}$

Hence B is correct option

28. (B) Here $Y = g(X) = e^{-\frac{X}{3}}$

So $E[Y] = E[g(Y)] = \int_{-\infty}^{\infty} g(X)g_X(x)dx = \int_{-5}^{15} e^{-\frac{x}{5}}\frac{1}{15-(-5)}dx$

$= \frac{1}{20}\left[-5e^{-\frac{x}{5}}\right]_{-5}^{15} = \frac{1}{5}[e^1 - e^{-3}] = 0.667$

29. (C) $E[Y] = E[2X - 3] = 2\overline{X} - 3 = 2(-3) - 3 = -9$

$E[Y^2] = E[(2X-3)^2] = 4\overline{X^2} - 12\overline{X} - 9$

$= 4(11) - 12(-3) - 9 = 89$

$\sigma_Y^2 = \overline{Y^2} - \overline{Y}^2 = 89 - 9^2 = 8$

30. (A) $F_{XY}(x, y) = 0.1u(x-1)u(y-1) + 0.35u(x-2)u(y-2)$
$+ 0.05u(x-3)u(y-3) + 0.5u(x-y)u(y-4)$

$P\{X \le 2.5,\ Y \le 6.0\} = f_{XY}(2.5,\ 6.0) = 0.1 + 0.35 = 0.45$

31. (B) $P\{X \le 3.0\} = F_X(3.0) = F_{XY}(3.0, \infty)$

$= 0.1 + 0.35 + 0.05 = 0.5$

32. (B) $F_X(x) = F_{X,Y}(x,\ \infty)$

$\lim_{y\to\infty}\frac{5}{4}\left(\frac{x + e^{-(x+1)y^2}}{x+1} - e^{-y^2}\right)u(y) = \frac{5x}{4(x+1)}$

$\lim_{y\to\infty}\left(1 + \frac{1}{4}e^{-5y^2} - \frac{5}{4}e^{-y^2}\right) = 1$

33. (B) $F_Y(y) = F_{X,Y}(\infty,\ y)$

$$= \int_{-\infty}^{\infty} 5e^{-(w-y)}u(w-y)u(y)2e^{-2y}dy = 10\int_{0}^{w} e^{-5w+3y}dy, \quad w > 0$$

$$= \frac{10}{3}u(w)[e^{-2w} - e^{-5w}]$$

50. (A) $E[(XY)^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2y^2 f_{X,Y}(x,y)dx\,dy$

$$= \int_{y=0}^{4}\int_{x=0}^{6} \frac{x^2y^2}{24} dx\ dy = 64$$

51. (C) $E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x^2+y^2)\frac{e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}}{2\pi\sigma^2}dx\,dy$

$$= \int_{-\infty}^{\infty}\frac{x^2e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}dx\int_{-\infty}^{\infty}\frac{e^{-\frac{y^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}dy + \int_{-\infty}^{\infty}\frac{y^2e^{\frac{-y^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}dy\int_{-\infty}^{\infty}\frac{e^{\frac{-x^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}dx$$

Both double integral are of the same form. the second factors equal 1 because they are area of a gaussian density. The first factor equal σ^2 because they are second moment of gaussian density with zero mean and variance σ^2.

Thus $E[g(x,y)] = E[(x^2+y^2)] 2\sigma^2$

52. (A) $E[W] = E[3X - Y] = 3\overline{X} - \overline{Y} = 6 - 4 = 2$

53. (B) $E[W^2] = E[(3X-Y)^2] = E[9X^2 - 6XY + Y^2]$

$= 9\overline{X^2} - 6\overline{XY} + \overline{Y^2} = 9\overline{X^2} - 6\overline{X}\,\overline{Y} + \overline{Y^2}$

$= 9(8) - 6(2)(4) + 25 = 49$

54. (B) $\sigma_W^2 = E[(W-\overline{W})^2] = E[W^2 - 2W\overline{W} + \overline{W}^2]$

$= \overline{W^2} - \overline{W}^2 = 49 - 4 = 45$

55. (B) $R_{XY} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\ f_{X,Y}(x,y)\ dxdy = \int_0^3\int_0^2 \frac{x^2y^2}{9}dx\,dy = \frac{8}{3}$

$E[X] = \int_0^3\int_0^2 \frac{x^2y}{9}dx\,dy = \frac{4}{3}$

$E[Y] = \int_0^3\int_0^2 \frac{x^2y}{9}dx\,dy = 2$

Since $R_{XY} = \frac{8}{3} = E[X]E[Y] = 2\left(\frac{4}{3}\right) = \frac{8}{3}$, we have X and Y uncorrelated form

From marginal densities $f_X(x) = \int_0^3 \frac{xy}{9}dy = \frac{x}{2}, \quad 0 < x < 2$

$f_Y(y) = \int_0^2 \frac{xy}{9}dy = \frac{2y}{9}, \quad 0 < y < 3$

we have $f_X(x)\ f_Y(y) = \frac{xy}{9}$, $0 < x < 2$ and $0 < y < 3$

Thus $f_{X,Y}(x,y) = f_X(x)\ f_Y(y)$ and X and Y are statistically independent.

56. (C) $\sigma_X^2 = \overline{X^2} - \overline{X}^2 = \frac{5}{2} - \left(\frac{1}{2}\right)^2 = \frac{9}{4}$

$\sigma_Y^2 = \overline{Y^2} - \overline{Y}^2 = \frac{11}{2} - (2)^2 = \frac{11}{2}$

$R_{XY} = \overline{XY} = C_{XY} + \overline{X}\,\overline{Y} = -\frac{1}{2\sqrt{3}} + \frac{1}{2}(2) = \frac{1}{2}\left(2 - \frac{1}{\sqrt{3}}\right)$

$\rho = \frac{C_{XY}}{\sigma_X\sigma_Y} = \frac{-1/2\sqrt{3}}{(\sqrt{914})(\sqrt{11/2})} = \frac{-1}{3}\sqrt{\frac{2}{33}}$

57. (B) $\overline{W} = \overline{(X+3Y)^2 + 2X + 3}$

$= 3 + 2\overline{X} + \overline{X^2} + 6\overline{XY} + 9\overline{Y^2}$

$= 3 + 2\left(\frac{1}{2}\right) + \frac{5}{2} + 6\left(\frac{1}{2}\right)\left(2 - \frac{1}{\sqrt{3}}\right) + 9\left(\frac{19}{2}\right) = 98 - \sqrt{3}$