

## 19. Differential Equations with Variable Separable

### Exercise 19A

#### 1. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

#### Answer

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

Rearranging the terms, we get:

$$\Rightarrow \frac{dy}{1 + y^2} = (1 + x^2)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y^2} = \int (1 + x^2)dx + c$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + c \dots \left( \int \frac{dy}{1 + y^2} = \tan^{-1} y, \int x^n = \frac{x^{n+1}}{n+1} \right)$$

$$\text{Ans: } \tan^{-1} y = x + \frac{x^3}{3} + c$$

#### 2 Ans.: Question

Find the general solution of each of the following differential equations:

$$x^4 \frac{dy}{dx} = -y^4$$

#### Answer

$$x^4 \frac{dy}{dx} = -y^4$$

$$\Rightarrow \frac{dy}{-y^4} = \frac{dx}{x^4}$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{-y^4} = \int \frac{dx}{x^4} + c'$$

$$\Rightarrow \frac{-y^{-4+1}}{-4+1} = \frac{x^{-4+1}}{-4+1} + c'$$

$$\Rightarrow \frac{1}{3y^3} = -\frac{1}{3x^3} + c'$$

$$\Rightarrow \frac{1}{y^3} + \frac{1}{x^3} = 3c'$$

$$\Rightarrow \frac{1}{x^3} + \frac{1}{y^3} = c \dots (3c' = c)$$

#### 3. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = 1 + x + y + xy$$

**Answer**

$$\frac{dy}{dx} = 1 + x + y + xy = 1 + y + x(1 + y)$$

$$\Rightarrow \frac{dy}{dx} = (1 + y)(1 + x)$$

Rearranging the terms we get:

$$\Rightarrow \frac{dy}{1 + y} = (1 + x)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y} = \int (1 + x)dx + c$$

$$\Rightarrow \log|1 + y| = x + \frac{x^2}{2} + c \dots (\int \frac{dy}{1 + y} = \log|1 + y|)$$

$$\text{Ans: } \log|1 + y| = x + \frac{x^2}{2} + c$$

#### 4. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = 1 - x + y - xy$$

**Answer**

$$\Rightarrow \frac{dy}{dx} = 1 - x + y - xy = 1 + y - x(1 + y)$$

$$\Rightarrow \frac{dy}{dx} = (1 + y)(1 - x)$$

Rearranging the terms we get:

$$\Rightarrow \frac{dy}{1 + y} = (1 - x)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y} = \int (1 - x)dx + c$$

$$\Rightarrow \log|1 + y| = x - \frac{x^2}{2} + c \dots (\int \frac{dy}{1 + y} = \log|1 + y|)$$

$$\text{Ans: } \log|1 + y| = x - \frac{x^2}{2} + c$$

#### 5. Question

Find the general solution of each of the following differential equations:

$$(x - 1)\frac{dy}{dx} = 2x^3y$$

**Answer**

$$(x - 1)\frac{dy}{dx} = 2x^3y$$

Separating the variables we get:

$$\Rightarrow \frac{dy}{y} = 2x^3 \frac{dx}{(x-1)}$$

$$\Rightarrow \frac{dy}{y} = \frac{2((x-1)(x^2 + x + 1) + 1)}{(x-1)} dx$$

$$\Rightarrow \frac{dy}{y} = 2 \left( x^2 + x + 1 + \frac{1}{x-1} \right) dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{y} = \int 2 \left( x^2 + x + 1 + \frac{1}{x-1} \right) dx + c$$

$$\Rightarrow \log|y| = \frac{2x^3}{3} + \frac{2x^2}{2} + 2x + 2\log|x-1| + c$$

$$\Rightarrow \log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$$

$$\text{Ans: } \log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$$

## 6. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = e^{x+y}$$

**Answer**

$$\frac{dy}{dx} = e^x e^y$$

Rearranging the terms we get:

$$\Rightarrow \frac{dy}{e^y} = e^x dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{e^y} = \int e^x dx + c$$

$$\Rightarrow \frac{e^{-y}}{-1} = e^x + c$$

$$\Rightarrow e^x + e^{-y} = c$$

$$\text{Ans: } e^x + e^{-y} = c$$

## 7. Question

Find the general solution of each of the following differential equations:

$$(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0$$

**Answer**

$$(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0$$

$$\Rightarrow dy = \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

Integrating both the sides we get,

$$\Rightarrow \int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx + c$$

$$\Rightarrow y = \log|e^x + e^{-x}| + c \dots \left(\frac{d}{dx}(e^x + e^{-x}) = e^x - e^{-x}\right)$$

$$\text{Ans: } y = \log|e^x + e^{-x}| + c$$

### 8. Question

Find the general solution of each of the following differential equations:

#### Answer

$$\text{Given: } \frac{dy}{dx} = e^x e^{-y} + x^2 e^{-y}$$

$$\Rightarrow \frac{dy}{dx} = e^{-y}(e^x + x^2)$$

$$\Rightarrow \frac{dy}{e^{-y}} = (e^x + x^2) dx$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dy}{e^{-y}} = \int (e^x + x^2) dx + c$$

$$\Rightarrow e^y = e^x + \frac{x^3}{3} + c$$

$$\text{Ans: } e^y = e^x + \frac{x^3}{3} + c$$

### 9. Question

Find the general solution of each of the following differential equations:

$$e^{2x-3y} dx + e^{2y-3x} dy = 0$$

#### Answer

$$e^{2x} e^{-3y} dx + e^{2y} e^{-3x} dy = 0$$

Rearranging the terms we get:

$$\Rightarrow \frac{e^{2x} dx}{e^{-3x}} = - \frac{e^{2y} dy}{e^{-3y}}$$

$$\Rightarrow e^{2x+3x} dx = - e^{2y+3y} dy$$

$$\Rightarrow e^{5x} dx = - e^{5y} dy$$

Integrating both the sides we get:

$$\Rightarrow \int e^{5x} dx = - \int e^{5y} dy + c'$$

$$\Rightarrow \frac{e^{5x}}{5} = - \frac{e^{5y}}{5} + c'$$

$$\Rightarrow e^{5x} + e^{5y} = 5c' = c$$

$$\text{Ans: } e^{5x} + e^{5y} = c$$

### 10. Question

Find the general solution of each of the following differential equations:

$$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$$

### Answer

Rearranging all the terms we get:

$$\frac{e^x dx}{1 - e^x} = -\frac{\sec^2 y \, dy}{\tan y}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{e^x dx}{1 - e^x} = -\int \frac{\sec^2 y \, dy}{\tan y} + c$$

$$\Rightarrow \frac{\log|1 - e^x|}{-1} = -\log|\tan y| + \log c$$

$$\Rightarrow \log|1 - e^x| = \log|\tan y| - \log c$$

$$\Rightarrow \log|1 - e^x| + \log c = \log|\tan y|$$

$$\Rightarrow \tan y = c(1 - e^x)$$

$$\text{Ans: } \tan y = c(1 - e^x)$$

### 11. Question

Find the general solution of each of the following differential equations:

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

### Answer

Rearranging the terms we get:

$$\frac{\sec^2 x \, dx}{\tan x} = -\frac{\sec^2 y \, dy}{\tan y}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{\sec^2 x \, dx}{\tan x} = -\int \frac{\sec^2 y \, dy}{\tan y} + c$$

$$\Rightarrow \log|\tan x| = -\log|\tan y| + \log c$$

$$\Rightarrow \log|\tan x| + \log|\tan y| = \log c$$

$$\Rightarrow \tan x \cdot \tan y = c$$

$$\text{Ans: } \tan x \cdot \tan y = c$$

### 12. Question

Find the general solution of each of the following differential equations:

$$\cos x(1 + \cos y)dx - \sin y(1 + \sin x)dy = 0$$

### Answer

Rearranging the terms we get:

$$\frac{\cos x \, dx}{(1 + \sin x)} = \frac{\sin y \, dy}{(1 + \cos y)}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{\cos x \, dx}{(1 + \sin x)} = \int \frac{\sin y \, dy}{(1 + \cos y)} + c$$

$$\Rightarrow \log|1 + \sin x| = -\log|1 + \cos y| + \log c$$

$$\Rightarrow \log|1 + \sin x| + \log|1 + \cos y| = \log c$$

$$\Rightarrow (1 + \sin x)(1 + \cos y) = c$$

$$\text{Ans: } (1 + \sin x)(1 + \cos y) = c$$

### 13. Question

For each of the following differential equations, find a particular solution satisfying the given condition :

$$\cos\left(\frac{dy}{dx}\right) = a, \text{ where } a \in \mathbb{R} \text{ and } y = 2 \text{ when } x = 0.$$

### Answer

$$\cos\left(\frac{dy}{dx}\right) = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

$$\Rightarrow dy = \cos^{-1} a \, dx$$

Integrating both the sides we get:

$$\Rightarrow \int dy = \int \cos^{-1} a \, dx + c$$

$$\Rightarrow y = x \cos^{-1} a + c$$

$$\text{when } x = 0, y = 2$$

$$\therefore 2 = 0 + c$$

$$\therefore c = 2$$

$$\therefore y = x \cos^{-1} a + 2$$

$$\Rightarrow \frac{y-2}{x} = \cos^{-1} a$$

$$\Rightarrow \cos\left(\frac{y-2}{x}\right) = a$$

$$\text{Ans: } \cos\left(\frac{y-2}{x}\right) = a$$

### 14. Question

For each of the following differential equations, find a particular solution satisfying the given condition :

$$\frac{dy}{dx} = -4xy^2, \text{ it being given that } y = 1 \text{ when } x = 0.$$

### Answer

Rearranging the terms we get:

$$\frac{dy}{y^2} = -4x \, dx$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dy}{y^2} = -\int 4x \, dx + c$$

$$\Rightarrow \frac{y^{-1}}{-1} = -\frac{4x^2}{2} + c$$

$$\Rightarrow y^{-1} = 2x^2 + c$$

$$y = 1 \text{ when } x = 0$$

$$\Rightarrow (1)^{-1} = 2(0)^2 + c$$

$$\Rightarrow c = 1$$

$$\Rightarrow \frac{1}{y} = 2x^2 + 1$$

$$\Rightarrow \frac{1}{2x^2 + 1} = y$$

$$\text{Ans: } y = \frac{1}{2x^2 + 1}$$

### 15. Question

For each of the following differential equations, find a particular solution satisfying the given condition :

$x \, dy = (2x^2 + 1) \, dx$  ( $x \neq 0$ ), given that  $y = 1$  when  $x = 1$ .

### Answer

Rearranging the terms we get:

$$dy = \frac{2x^2 + 1}{x} dx$$

$$\Rightarrow dy = 2x \, dx + \frac{1}{x} \, dx$$

Integrating both the sides we get:

$$\Rightarrow \int dy = \int 2x \, dx + \int \frac{1}{x} \, dx + c$$

$$\Rightarrow y = x^2 + \log|x| + c$$

$$y = 1 \text{ when } x = 1$$

$$\therefore 1 = 1^2 + \log 1 + c$$

$$\therefore 1 - 1 = 0 + c \dots (\log 1 = 0)$$

$$\Rightarrow c = 0$$

$$\therefore y = x^2 + \log|x|$$

$$\text{Ans: } y = x^2 + \log|x|$$

### 16. Question

For each of the following differential equations, find a particular solution satisfying the given condition :

$$\frac{dy}{dx} = y \tan x, \text{ it being given that } y = 1 \text{ when } x = 0.$$

### Answer

Rearranging the terms we get:

$$\frac{dy}{y} = \tan x \, dx$$

$$\Rightarrow \int \frac{dy}{y} = \int \tan x \, dx + c$$

$$\Rightarrow \log|y| = \log|\sec x| + \log c$$

$$\Rightarrow \log|y| - \log|\sec x| = \log c$$

$$\Rightarrow \log|y| + \log|\cos x| = \log c$$

$$\Rightarrow y \cos x = c$$

$$y = 1 \text{ when } x = 0$$

$$\therefore 1 \times \cos 0 = c$$

$$\therefore c = 1$$

$$\Rightarrow y \cos x = 1$$

$$\Rightarrow y = 1/\cos x$$

$$\Rightarrow y = \sec x$$

$$\text{Ans: } y = \sec x$$

## Exercise 19B

### 1. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{x-1}{y+2}$$

#### Answer

$$(y+2)dy = (x-1)dx$$

Integrating on both sides,

$$\int (y+2)dy = \int (x-1)dx$$

$$\frac{y^2}{2} + 2y = \frac{x^2}{2} - x + C$$

$$y^2 + 4y - x^2 + 2x = C$$

### 2. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{x}{(x^2+1)}$$

#### Answer

$$dy = \frac{x}{x^2+1} dx$$

Multiply and divide 2 in numerator and denominator of RHS,

$$y = \frac{1}{2} \cdot \left( \frac{2x}{x^2+1} dx \right)$$

Integrating on both sides

$$y = \frac{1}{2} \cdot \log(x^2+1) + C$$

### 3. Question

Find the general solution of each of the following differential equations:



$$\frac{dy}{dx} = (1+x)(1+y^2)$$

**Answer**

$$\frac{1}{1+y^2} dy = (1+x) dx$$

Integrating on both sides

$$\int \frac{1}{1+y^2} dy = \int (1+x) dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + C$$

#### 4. Question

Find the general solution of each of the following differential equations:

$$(1+x^2) \frac{dy}{dx} = xy$$

**Answer**

$$\frac{1}{y} \cdot dy = \frac{x}{x^2+1} dx$$

Multiply and divide 2 in numerator and denominator of RHS,

$$\frac{1}{y} \cdot dy = \frac{1}{2} \cdot \left( \frac{2x}{x^2+1} dx \right)$$

Integrating on both sides

$$\log y = \frac{1}{2} \cdot \log(1+x^2) + \log C$$

$$\log y = \log \sqrt{1+x^2} + \log C$$

$$\Rightarrow y = \sqrt{1+x^2} \cdot C_1$$

#### 5. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + y = 1 \quad (y \neq 1)$$

**Answer**

$$\frac{dy}{dx} = 1 - y$$

$$\frac{1}{1-y} dy = dx$$

Integrating on both sides

$$\int \frac{1}{1-y} dy = \int dx$$

$$\Rightarrow \log|1-y| = x + C$$

#### 6. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

**Answer**

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\frac{1}{\sqrt{1-y^2}} dy = -\frac{1}{\sqrt{1-x^2}} dx$$

Integrating on both sides

$$\int \frac{1}{\sqrt{1-y^2}} dy = \int -\frac{1}{\sqrt{1-x^2}} dx$$

$$\sin^{-1} y = \sin^{-1} x + C$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = C$$

## 7. Question

Find the general solution of each of the following differential equations:

$$x \frac{dy}{dx} + y = y^2$$

**Answer**

$$\Rightarrow x \cdot \frac{dy}{dx} + y = y^2$$

$$x \cdot \frac{dy}{dx} = y^2 - y$$

$$\frac{1}{y^2 - y} dy = \frac{1}{x} dx$$

$$\frac{1}{y(y-1)} dy = \frac{1}{x} dx$$

Integrating on both the sides,

$$\int \frac{1}{y(y-1)} dy = \int \frac{1}{x} dx$$

LHS:

$$\text{Let } \frac{1}{y(y-1)} dy = \frac{A}{y} + \frac{B}{(y-1)}$$

$$\frac{1}{y(y-1)} dy = \frac{A(y-1)}{y} + \frac{By}{(y-1)}$$

$$1 = A(y-1) + By$$

$$1 = Ay + By - A$$

Comparing coefficients in both the sides,

$$A = -1, B = 1$$

$$\frac{1}{y(y-1)} dy = -\frac{1}{y} + \frac{1}{(y-1)}$$

$$\int \frac{1}{y(y-1)} dy = \int \left[ -\frac{1}{y} + \frac{1}{(y-1)} \right] dy$$

$$\int -\frac{1}{y} dy + \int \frac{1}{(y-1)} dy$$

$$-\log y + \log(y-1)$$

$$\Rightarrow \log\left(\frac{y-1}{y}\right)$$

RHS:

$$\int \frac{1}{x} dx$$

$$\int \frac{1}{x} dx = \log x + \log C$$

Therefore the solution of the given differential equation is

$$\log\left(\frac{y-1}{y}\right) = \log x + \log C$$

$$\frac{y-1}{y} = x.C$$

$$y-1 = yxC$$

$$\Rightarrow y = 1 + xyC$$

## 8. Question

Find the general solution of each of the following differential equations:

$$x^2 (y + 1) dx + y^2 (x - 1) dy = 0$$

**Answer**

$$x^2(y + 1)dx + y^2(x - 1)dy = 0$$

$$x^2(y + 1)dx = -y^2(x - 1)dy$$

$$x^2(y + 1)dx = y^2(1 - x)dy$$

$$\frac{x^2}{(1-x)} dx = \frac{y^2}{y+1} dy$$

Add and subtract 1 in numerators of both LHS and RHS,

$$\frac{x^2 - 1 + 1}{(1-x)} dx = \frac{y^2 - 1 + 1}{y+1} dy$$

$$\frac{(x^2 - 1) + 1}{(1-x)} dx = \frac{(y^2 - 1) + 1}{y+1} dy$$

By the identity,  $(a^2 - b^2) = (a + b).(a - b)$

$$\frac{(x + 1)(x - 1) + 1}{(1-x)} dx = \frac{(y + 1)(y - 1) + 1}{(y + 1)} dy$$

Splitting the terms,

$$-(x + 1)dx + \frac{1}{(1-x)}dx = (y-1)dy + \frac{1}{(y+1)}dy$$

Integrating,

$$\int -(x + 1)dx + \int \frac{1}{(x-1)}dx = \int (y-1)dy + \int \frac{1}{(y+1)}dy$$

$$-\left(\frac{x^2}{2} + x\right) + \log|x-1| = \left(\frac{y^2}{2} - y\right) + \log|1 + y| + C$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + x - y + \log|x-1| + \log|1 + y| = C$$

### 9. Question

Find the general solution of each of the following differential equations:

$$y(1-x^2)\frac{dy}{dx} = x(1+y^2)$$

**Answer**

$$\frac{y}{1+y^2}dy = \frac{x}{1-x^2}dx$$

Multiply 2 in both LHS and RHS,

$$\frac{2y}{1+y^2}dy = \frac{2x}{1-x^2}dx$$

Integrating on both the sides,

$$\int \frac{2y}{1+y^2}dy = \int \frac{2x}{1-x^2}dx$$

$$\log(1+y^2) = -\log(1-x^2) + \log C$$

$$\log(1+y^2) + \log(1-x^2) = \log C$$

$$= (1+y^2).(1-x^2) = C$$

### 10. Question

Find the general solution of each of the following differential equations:

$$y \log y \, dx - x \, dy = 0$$

**Answer**

$$y \cdot \log y \, dx = x \, dy$$

$$\frac{1}{x}dx = \frac{1}{y \cdot \log y}dy$$

Integrating on both the sides,

$$\int \frac{1}{x}dx = \int \frac{1}{y \cdot \log y}dy$$

LHS:

$$\int \frac{1}{x}dx = \log x$$

RHS:

$$\int \frac{1}{y \cdot \log y} dy$$

Let  $\log y = t$

$$\text{So, } \frac{1}{y} dy = dt$$

$$\int \frac{1}{y \cdot \log y} dy = \int \frac{1}{t} dt$$

$$= \log t$$

$$= \log(\log y)$$

Therefore the solution of the given differential equation is

$$\log x = \log(\log y) + \log C$$

$$x = \log y \cdot C$$

### 11. Question

Find the general solution of each of the following differential equations:

$$x(x^2 - y^2) dy + y(y^2 + x^2) dx = 0$$

**Answer**

$$x \cdot x^2(1 - y^2)dy + y \cdot y^2(1 + x^2)dx = 0$$

$$x^3(1 - y^2)dy + y^3(1 + x^2)dx = 0$$

$$\frac{1 + x^2}{x^3} dx + \frac{1 - y^2}{y^3} dy = 0$$

$$\frac{1}{x^3} dx + \frac{1}{x} dx + \frac{1}{y^3} dy - \frac{1}{y} dy = 0$$

Integrating ,

$$\int \frac{1}{x^3} dx + \int \frac{1}{x} dx + \int \frac{1}{y^3} dy - \int \frac{1}{y} dy = C$$

$$\frac{x^{-3+1}}{-3+1} + \log x - \log y + \frac{y^{-3+1}}{-3+1} = C$$

$$-\frac{1}{2x^2} + -\frac{1}{2y^2} + \log x - \log y = C$$

$$-\frac{1}{2x^2} + -\frac{1}{2y^2} + \log\left(\frac{x}{y}\right) = C$$

### 12. Question

Find the general solution of each of the following differential equations:

$$(1 - x^2) dy + xy(1 - y) dx = 0$$

**Answer**

$$(1 - x^2)dy = -xy(1 - y)dx$$

$$(1 - x^2)dy = xy(y - 1)dx$$

$$\frac{1}{y(y - 1)} dy = \frac{x}{1 - x^2} dx$$

Integrating on both the sides,

$$\int \frac{1}{y(y-1)} dy = \int \frac{x}{1-x^2} dx$$

LHS:

$$\text{Let } \frac{1}{y(y-1)} dy = \frac{A}{y} + \frac{B}{(y-1)}$$

$$\frac{1}{y(y-1)} dy = \frac{A(y-1)}{y} + \frac{By}{(y-1)}$$

$$1 = A(y-1) + By$$

$$\Rightarrow 1 = Ay + By - A$$

Comparing coefficients in both the sides,

$$A = -1, B = 1$$

$$\frac{1}{y(y-1)} dy = -\frac{1}{y} + \frac{1}{(y-1)}$$

$$\int \frac{1}{y(y-1)} dy = \int \left[ -\frac{1}{y} + \frac{1}{(y-1)} \right] dy$$

$$\int -\frac{1}{y} dy + \int \frac{1}{(y-1)} dy$$

$$-\log y + \log(y-1)$$

$$= \log\left(\frac{y-1}{y}\right)$$

RHS:

$$\int \frac{x}{1-x^2} dx$$

Multiply and divide 2

$$\frac{1}{2} \cdot \int \frac{2x}{1-x^2} dx$$

$$-\frac{1}{2} \cdot \log(1-x^2) + \log C$$

$$-\log \sqrt{1-x^2} + \log C$$

Therefore the solution of the given differential equation is

$$\log\left(\frac{y-1}{y}\right) = -\log \sqrt{1-x^2} + \log C$$

$$-\log\left(\frac{y-1}{y}\right) = \log \sqrt{1-x^2} + \log C$$

$$\log\left(\frac{y}{y-1}\right) = \log \sqrt{1-x^2} + \log C$$

$$\frac{y}{y-1} = \sqrt{1-x^2} \cdot C$$

$$= y = (y-1) \cdot \sqrt{1-x^2} \cdot C$$

### 13. Question

Find the general solution of each of the following differential equations:

$$(1 - x^2)(1 - y) dx = xy(1 + y) dy$$

**Answer**

$$\frac{1 - x^2}{x} dx = \frac{y(1 + y)}{(1 - y)} dy$$

$$\left[ \frac{1}{x} - x \right] dx = \left[ \frac{y + y^2}{1 - y} \right] dy$$

$$\left[ \frac{1}{x} - x \right] dx = \left[ \frac{y}{1 - y} + \frac{y^2}{1 - y} \right] dy$$

Integrating on both the sides,

$$\int \left[ \frac{1}{x} - x \right] dx = \int \left[ \frac{y}{1 - y} + \frac{y^2}{1 - y} \right] dy$$

LHS:

$$\int \left[ \frac{1}{x} - x \right] dx = \log x - \frac{x^2}{2}$$

RHS:

$$\int \frac{y}{1 - y} dy = \int \frac{y - 1 + 1}{1 - y} dy$$

$$\int \frac{y - 1}{1 - y} dy + \int \frac{1}{1 - y} dy$$

$$\int -1 dy + \int \frac{1}{1 - y} dy$$

$$-y + \log|1 - y|$$

$$\int \frac{y^2}{1 - y} dy$$

Add and subtract 1 in numerators of both LHS and RHS,

$$\frac{y^2 - 1 + 1}{(1 - y)} dy$$

$$\frac{(y^2 - 1) + 1}{(1 - y)} dy$$

By the identity,  $(a^2 - b^2) = (a + b)(a - b)$

$$\frac{(y + 1)(y - 1) + 1}{(1 - y)} dy$$

Splitting the terms,

$$-(y + 1)dy + \frac{1}{(1 - y)} dy$$

Integrating,

$$\int -(y + 1)dy - \int \frac{1}{(y - 1)} dy$$

$$-\left(\frac{y^2}{2} + y\right) + \log|y-1|$$

Therefore the solution of the given differential equation is

$$\begin{aligned}\log x - \frac{x^2}{2} &= -y + \log|1-y| - \left(\frac{y^2}{2} + y\right) + \log|y-1| \\ &= \log|x.(1-y)^2| = \frac{x^2}{2} - \frac{y^2}{2} - 2y + C\end{aligned}$$

#### 14. Question

Find the general solution of each of the following differential equations:

$$(y + xy) dx + (x - xy^2) dy = 0$$

**Answer**

$$y(1+x)dx + x(1-y^2)dy = 0$$

$$\frac{1+x}{x}dx + \frac{1-y^2}{y}dy = 0$$

$$\frac{1}{x}dx + 1.dx + \frac{1}{y}dy - ydy = 0$$

Integrating ,

$$\int \frac{1}{x}dx + \int 1.dx + \int \frac{1}{y}dy - \int ydy = C$$

$$\log|x| + x + \log|y| - \frac{y^2}{2} = C$$

$$= \log|xy| + x - \frac{y^2}{2} = C$$

#### 15. Question

Find the general solution of each of the following differential equations:

$$(x^2 - yx^2) dy + (y^2 + xy^2) dx = 0$$

**Answer**

$$x^2(1-y)dy + y^2(1+x)dx = 0$$

$$\frac{1+x}{x^2}dx + \frac{1-y}{y^2}dy = 0$$

$$\frac{1}{x^2}dx + \frac{1}{x}dx + \frac{1}{y^2}dy - \frac{1}{y}dy = 0$$

Integrating,

$$\int \frac{1}{x^2}dx + \int \frac{1}{x}dx + \int \frac{1}{y^2}dy - \int \frac{1}{y}dy = C$$

$$-\frac{1}{x} + \log|x| - \frac{1}{y} - \log|y| = C$$

$$\log\left|\frac{x}{y}\right| = \frac{1}{x} + \frac{1}{y} + C$$

#### 16. Question



Find the general solution of each of the following differential equations:

$$(x^2y - x^2)dx + (xy^2 - y^2)dy = 0$$

**Answer**

$$x^2(y-1)dx + y^2(x-1)dy = 0$$

$$\frac{x^2}{x-1}dx + \frac{y^2}{y-1}dy = 0$$

Add and subtract 1 in numerators ,

$$\frac{x^2-1+1}{(x-1)}dx + \frac{y^2-1+1}{(y-1)}dy$$

$$\frac{(x^2-1)+1}{(x-1)}dx + \frac{(y^2-1)+1}{(y-1)}dy$$

By the identity,  $(a^2 - b^2) = (a + b).(a - b)$

$$\frac{(x+1)(x-1)+1}{(x-1)}dx + \frac{(y+1)(y-1)+1}{(y-1)}dy$$

Splitting the terms,

$$(x+1)dx + \frac{1}{(x-1)}dx + (y+1)dy + \frac{1}{(y-1)}dy$$

Integrating,

$$\int (x+1)dx + \int \frac{1}{(x-1)}dx + \int (y+1)dy + \int \frac{1}{(y-1)}dy = c$$

$$\frac{x^2}{2} + x + \log|x-1| + \frac{y^2}{2} + y + \log|y-1|$$

$$\frac{1}{2} \cdot (x^2 + y^2) + (x + y) + \log|(x-1)(y-1)|$$

### 17. Question

Find the general solution of each of the following differential equations:

$$x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$$

**Answer**

$$\frac{x}{\sqrt{1+x^2}}dx + \frac{y}{\sqrt{1+y^2}}dy = 0$$

Integrating,

$$\int \frac{x}{\sqrt{1+x^2}}dx + \int \frac{y}{\sqrt{1+y^2}}dy$$

$$= c \text{ formula: } \left\{ \frac{d}{dx}(\sqrt{1+x^2}) = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}} \right\}$$

$$\sqrt{1+x^2} + \sqrt{1+y^2} = c$$

### 18. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

**Answer**

$$\frac{dy}{dx} = e^x \cdot e^y + x^2 \cdot e^y$$

$$\frac{dy}{dx} = e^y (e^x + x^2)$$

$$\frac{1}{e^y} dy = (e^x + x^2) dx$$

Integrating on both the sides,

$$\int \frac{1}{e^y} dy = \int (e^x + x^2) dx$$

$$-e^{-y} = e^x + \frac{x^3}{3} + C$$

$$e^x + e^{-y} + \frac{x^3}{3} = C$$

### 19. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{3e^{2x} + 3d^{4x}}{e^x + e^{-x}}$$

**Answer**

Considering 'd' as exponential 'e'

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$$

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + \frac{1}{e^x}}$$

$$\frac{dy}{dx} = \frac{(3e^{2x} + 3e^{4x}) \cdot e^x}{e^{2x} + 1}$$

$$\frac{dy}{dx} = \frac{3 \cdot e^{2x}(1 + e^{2x}) \cdot e^x}{e^{2x} + 1}$$

$$\frac{dy}{dx} = 3 \cdot e^{3x}$$

$$dy = 3 \cdot e^{3x} dx$$

Integrating on both the sides,

$$\int dy = \int 3 \cdot e^{3x} dx$$

$$y = 3 \cdot \frac{e^{3x}}{3} + C$$

$$y = e^{3x} + C$$

### 31. Question

Find the general solution of each of the following differential equations:

$$(\cos x) \frac{dy}{dx} + \cos 2x = \cos 3x$$

**Answer**

$$\text{Given: } \frac{dy}{dx} + \frac{\cos 2x}{\cos x} = \frac{\cos 3x}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(x+2x) - \cos 2x}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(\cos x \cos 2x - \sin x \sin 2x) - (\cos^2 x - 1)}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \cos 2x - \frac{2 \sin x \cos x \sin x}{\cos x} - 2 \cos x + \sec x$$

$$\Rightarrow \frac{dy}{dx} = \cos 2x - 2 \sin^2 x - 2 \cos x + \sec x$$

$$\Rightarrow y = \int (\cos 2x - 2 \sin^2 x - 2 \cos x + \sec x) dx$$

$$\Rightarrow y = \int \cos 2x dx - \int 2 \sin^2 x dx - \int 2 \cos x dx + \int \sec x dx$$

$$\Rightarrow y = \int \cos 2x dx - \int (1 - \cos 2x) dx - \int 2 \cos x dx + \int \sec x dx$$

$$\Rightarrow y = \frac{\sin 2x}{2} - 2 \sin x - x + \log |\sec x + \tan x| + c$$

## 20. Question

Find the general solution of each of the following differential equations:

$$3e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$$

**Answer**

$$\Rightarrow 3. e^x \cdot \tan y dx = (e^x - 1) \sec^2 y dy$$

$$3. \frac{e^x}{e^x - 1} dx = \frac{\sec^2 y}{\tan y} dy$$

$$3. \left[ \frac{1}{\frac{e^x - 1}{e^x}} \right] dx = \frac{\sec^2 y}{\tan y} dy$$

$$3. \left[ \frac{1}{1 - e^{-x}} \right] dx = \frac{\sec^2 y}{\tan y} dy$$

Integrating on both the sides,

$$\int 3. \left[ \frac{1}{1 - e^{-x}} \right] dx = \int \frac{\sec^2 y}{\tan y} dy$$

$$3. \log |1 - e^{-x}| = \log |\tan y| + \log C \text{ formula: } \left\{ \frac{d}{dy} \tan y = \frac{1}{\tan y} \cdot \sec^2 y \right\}$$

$$\log(1 - e^{-x})^3 = \log |\tan y| + \log C$$

$$\tan y = (1 - e^{-x})^3 \cdot C$$

## 32. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \frac{(1 + \cos 2y)}{(1 - \cos 2x)} = 0$$

**Answer**

$$\text{Given: } \frac{dy}{dx} + \frac{1 + \cos 2y}{1 - \cos 2x} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2\cos^2 y}{2\sin^2 x}$$

$$\Rightarrow \sec^2 y \frac{dy}{dx} = -\operatorname{cosec}^2 x$$

$$\Rightarrow \int \sec^2 y dy = -\int \operatorname{cosec}^2 x dx$$

$$\Rightarrow \tan y = \cot x + c$$

## 21. Question

Find the general solution of each of the following differential equations:

$$e^y (1 + x^2) dy - \frac{x}{y} dx = 0$$

**Answer**

$$e^y (1 + x^2) dy = \frac{x}{y} dx$$

$$e^y \cdot y dy = \frac{x}{1 + x^2} dx$$

Integrating on both the sides,

$$\int e^y \cdot y dy = \int \frac{x}{1 + x^2} dx$$

LHS:

$$\int e^y \cdot y dy$$

By ILATE rule,

$$\int e^y \cdot y dy = y \cdot \int e^y dy - \int \left[ \frac{d}{dy} (y) \cdot \int e^y dy \right] dy$$

$$y \cdot e^y - \int e^y dy$$

$$y \cdot e^y - e^y$$

$$e^y (y - 1)$$

RHS:

$$\int \frac{x}{1 + x^2} dx$$

Multiply and divide by 2

$$\frac{1}{2} \int \frac{2x}{1 + x^2} dx$$

$$\frac{1}{2} \cdot \log |1 + x^2|$$

$$\log \sqrt{1+x^2}$$

Therefore the solution of the given differential equation is

$$\Rightarrow e^y(y-1) = \log \sqrt{1+x^2} + C$$

### 33. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \frac{\cos x \sin y}{\cos y} = 0$$

### Answer

$$\text{Given: } \frac{dy}{dx} = -\frac{\cos x \sin y}{\cos y}$$

$$\Rightarrow \frac{dy}{dx} = -\cos x \tan y$$

$$\Rightarrow \int \cot y dy = -\int \cos x dx$$

$$\Rightarrow \log|\sin y| = -\sin x + c$$

### 34. Question

Find the general solution of each of the following differential equations:

$$\cos x(1 + \cos y)dx - \sin y(1 + \sin x)dy = 0$$

### Answer

$$\text{Given: } \cos x(1+\cos y)dx - \sin y(1+\sin x)dy=0$$

Dividing the whole equation by  $(1+\sin x)(1+\cos y)$ , we get,

$$\Rightarrow \frac{\int \cos x dx}{1+\sin x} = \frac{\int \sin y dy}{1+\cos y}$$

$$\Rightarrow \log|1+\sin x| + \log|1+\cos y| = \log c$$

$$\Rightarrow (1+\sin x)(1+\cos y) = c$$

### 22. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = e^{x+y} + e^{x-y}$$

### Answer

$$\frac{dy}{dx} = e^x \cdot e^y + e^x \cdot e^{-y}$$

$$\frac{dy}{dx} = e^x(e^y + e^{-y})$$

$$\frac{1}{e^y + e^{-y}} dy = e^x dx$$

$$\frac{1}{e^y + \frac{1}{e^y}} dy = e^x dx$$

$$\frac{e^y}{(e^y)^2 + 1} dy = e^x dx$$

Integrating on both the sides,

$$\int \frac{e^y}{(e^y)^2 + 1} dy = \int e^x dx \text{ formula: } \left\{ \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2} \right\}$$

$$\Rightarrow \tan^{-1} e^{-y} = e^x + C$$

### 35. Question

Find the general solution of each of the following differential equations:

$$\sin^3 x \, dx - \sin y \, dy = 0$$

#### Answer

$$\text{Using } \sin^3 x = \frac{3 \sin x - \sin 3x}{4}$$

We have,

$$\Rightarrow \frac{3 \sin x - \sin 3x}{4} dx - \sin y dy = 0$$

$$\Rightarrow \frac{3}{4} \sin x dx - \frac{\sin 3x}{4} dx - \sin y dy = 0$$

$$\Rightarrow \int \frac{3}{4} \sin x dx - \int \frac{\sin 3x}{4} dx - \int \sin y dy = 0$$

$$\Rightarrow \frac{3}{4} (-\cos x) + \frac{1}{12} \cos 3x + \cos y = k$$

$$\Rightarrow 12 \cos y + \cos 3x - 9 \cos x = c$$

### 23. Question

Find the general solution of each of the following differential equations:

$$(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$$

#### Answer

$$\frac{\cos x}{\sin x} dx + \frac{e^y}{e^y + 1} dy = 0$$

$$\cot x \, dx + \frac{e^y}{e^y + 1} dy = 0$$

Integrating,

$$\int \cot x \, dx + \int \frac{e^y}{e^y + 1} dy = C$$

$$\log |\sin x| + \log |e^y + 1| = \log C$$

$$\log |\sin x \cdot (e^y + 1)| = \log C$$

$$\Rightarrow \sin x \cdot (e^y + 1) = C$$

### 36. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \sin(x + y) = \sin(x - y)$$

#### Answer

$$\frac{dy}{dx} + \sin(x + y) = \sin(x - y)$$

$$\Rightarrow \frac{dy}{dx} = \sin(x-y) - \sin(x+y)$$

$$\Rightarrow \frac{dy}{dx} = -2\sin y \cos x \quad (\text{Using } \sin(A+B) - \sin(A-B) = 2\sin B \cos A)$$

$$\Rightarrow -\operatorname{cosec} y dy = \cos x dx$$

$$\Rightarrow -\int \operatorname{cosec} y dy = \int \cos x dx$$

$$\Rightarrow -\log|\operatorname{cosec} y - \cot y| = \sin x + c$$

$$\Rightarrow \sin x + \log|\operatorname{cosec} y - \cot y| + c = 0$$

#### 24. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \frac{xy + y}{xy + x} = 0$$

#### Answer

$$\frac{dy}{dx} + \frac{y(1+x)}{x(1+y)} = 0$$

$$\frac{1+y}{y} dy + \frac{1+x}{x} dx = 0$$

$$\frac{1}{y} dy + 1 \cdot dy + \frac{1}{x} dx + 1 \cdot dx = 0$$

Integrating ,

$$\int \frac{1}{y} dy + \int 1 \cdot dy + \int \frac{1}{x} dx + \int 1 \cdot dx = C$$

$$\log|y| + y + \log|x| + x = C$$

$$\Rightarrow \log|xy| + x + y = C$$

#### 37. Question

Find the general solution of each of the following differential equations:

$$\frac{1}{x} \cos^2 y dy + \frac{1}{y} \cos^2 x dx = 0$$

#### Answer

$$\text{Given: } \frac{1}{x} \cos^2 y dy + \frac{1}{y} \cos^2 x dx = 0$$

$$\Rightarrow y \cos^2 y dy + x \cos^2 x dx = 0$$

$$\Rightarrow \frac{y}{2} (1 + \cos^2 y) dy + \frac{x}{2} (1 + \cos^2 x) dx = 0 \quad (\text{Using, } 2\cos^2 a = 1 + \cos 2a)$$

$$\Rightarrow y dy + y \cos^2 y dy + x dx + x \cos^2 x dx = 0$$

$$\Rightarrow \frac{y^2}{2} + \frac{y}{2} \sin^2 y - \int \frac{\sin^2 y}{2} dy$$

$$\Rightarrow \frac{y^2}{2} + \frac{y}{2} \sin^2 y + \frac{\cos^2 y}{4} + \frac{x^2}{2} + \frac{x}{2} \sin^2 x + \frac{\sin^2 x}{4} = c$$

#### 25. Question

Find the general solution of each of the following differential equations:

$$\sqrt{1-x^4} dy = x dx$$

**Answer**

$$dy = \frac{x}{\sqrt{1-x^4}} dx$$

Multiply and divide by 2,

$$dy = \frac{1}{2} \cdot \frac{2x}{\sqrt{1-x^4}} dx$$

$$dy = \frac{1}{2} \cdot \frac{2x}{\sqrt{1-(x^2)^2}} dx$$

Integrating on both the sides,

$$\int dy = \frac{1}{2} \cdot \int \frac{2x}{\sqrt{1-x^4}} dx \text{ formula: } \left\{ \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \right\}$$

$$\Rightarrow y = \frac{1}{2} \cdot \sin^{-1} x^2 + c$$

### 38. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \sin^3 x \cos^2 x + xe^x$$

**Answer**

$$\text{Here we have, } y = \int (\sin^3 x \cos^2 x + xe^x) dx$$

$$\Rightarrow \int \cos^2 x (1 - \cos^2 x) \sin x dx + \int xe^x dx$$

Taking  $\cos x$  as  $t$  we have,

$$\Rightarrow \cos x = t,$$

$$\Rightarrow -\sin x dx = dt,$$

So we have,

$$\Rightarrow y = \int \cos^2 x \sin x dx - \int \cos^4 x \sin x dx + \int xe^x dx$$

$$\Rightarrow y = -\int t^2 dt - \int t^4 (-dt) + \int xe^x dx$$

$$\Rightarrow y = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + xe^x - e^x + c$$

### 26. Question

Find the general solution of each of the following differential equations:

$$\operatorname{cosec} x \log y \frac{dy}{dx} + x^2 y = 0$$

**Answer**

$$\frac{\log y}{y} dy + \frac{x^2}{\csc x} dx = 0$$

$$\frac{\log y}{y} dy + x^2 \cdot \sin x dx = 0$$

Integrating ,



$$\int \frac{\log y}{y} dy + \int x^2 \cdot \sin x dx = C$$

Consider the integral  $\int \frac{\log y}{y} dy$

Let  $\log y = t$

$$\text{So, } \frac{1}{y} dy = dt$$

$$\int \frac{\log y}{y} dy = \int t \cdot dt$$

$$\frac{t^2}{2}$$

$$\frac{(\log y)^2}{2}$$

Consider the integral  $\int x^2 \cdot \sin x dx$

By ILATE rule,

$$\int x^2 \cdot \sin x dx = x^2 \int \sin x dx - \int \left[ \frac{d}{dx}(x^2) \int \sin x dx \right] dx$$

$$-x^2 \cdot \cos x - \int [2x \cdot \int \sin x dx] dx$$

$$-x^2 \cos x + 2 \int [x \cdot \cos x] dx$$

Again by ILATE rule,

$$-x^2 \cos x + 2 \left[ x \cdot \int \cos x dx - \int \left\{ \frac{d}{dx} x \cdot \int \cos x dx \right\} dx \right]$$

$$-x^2 \cos x + 2 \left[ x \sin x - \int \sin x dx \right]$$

$$-x^2 \cos x + 2 [x \sin x + \cos x]$$

$$-x^2 \cos x + 2x \sin x + 2 \cos x$$

$$\cos x (2 - x^2) + 2x \sin x$$

Therefore the solution of the given differential equation is,

$$\frac{(\log y)^2}{2} + \cos x (2 - x^2) + 2x \sin x = C$$

## 27. Question

Find the general solution of each of the following differential equations:

$$y dx + (1 + x^2) \tan^{-1} x dy = 0$$

## Answer

$$\frac{1}{\tan^{-1} x \cdot (1 + x^2)} dx + \frac{1}{y} dy = 0$$

Integrating,

$$\int \frac{1}{\tan^{-1} x \cdot (1 + x^2)} dx + \int \frac{1}{y} dy = C$$

Consider the integral  $\int \frac{1}{\tan^{-1} x \cdot (1 + x^2)} dx$

Let  $\tan^{-1} x = t$

So,  $\frac{1}{1+x^2} dx = dt$

$$\int \frac{1}{\tan^{-1} x \cdot (1 + x^2)} dx = \int \frac{1}{t} dt$$

$\log t$

$\log(\tan^{-1} x)$

Consider the integral  $\int \frac{1}{y} dy$

$\log y$

Therefore the solution of the differential equation is

$$\log(\tan^{-1} x) + \log y = \log C$$

$$\tan^{-1} x \cdot y = C$$

### 39. Question

Find the particular solution of the differential equation  $\frac{dy}{dx} = 1 + x + y + xy$ , given that  $y = 0$  when  $x = 1$ .

### Answer

Given:

$$\frac{dy}{dx} = (1 + x)(1 + y)$$

$$\Rightarrow \frac{dy}{1+y} = (1 + x) dx$$

$$\Rightarrow \log|y + 1| = \left(x + \frac{x^2}{2} + c\right)$$

$\Rightarrow$  now, for  $y = 0$  and  $x = 1$ ,

We have,

$$\Rightarrow 0 = 1 + \frac{1}{2} + c$$

$$\Rightarrow c = -\frac{3}{2}$$

$$\Rightarrow \log|y + 1| = \frac{x^2}{2} + x - \frac{3}{2}$$

### 28. Question

Find the general solution of each of the following differential equations:

$$\frac{1}{x} \cdot \frac{dy}{dx} = \tan^{-1} x$$

### Answer

$$dy = x \cdot \tan^{-1} x dx$$

Integrating on both the sides,

$$\int dy = \int x \cdot \tan^{-1} x \, dx$$

$$y = \tan^{-1} x \int x \, dx - \int \left[ \frac{d}{dx} (\tan^{-1} x) \cdot \int x \, dx \right] dx \text{ (by ILATE rule)}$$

$$y = \tan^{-1} x \cdot \frac{x^2}{2} - \int \left[ \frac{1}{1+x^2} \right] \cdot \frac{x^2}{2} dx$$

$$y = \tan^{-1} x \cdot \frac{x^2}{2} - \frac{1}{2} \cdot \int \frac{x^2}{x^2+1} dx$$

$$y = \tan^{-1} x \cdot \frac{x^2}{2} - \frac{1}{2} \int \left[ \frac{x^2-1+1}{x^2+1} \right] \text{ (adding and subtracting 1)}$$

$$y = \tan^{-1} x \cdot \frac{x^2}{2} - \frac{1}{2} \int \left[ 1 - \frac{1}{x^2+1} \right] dx$$

$$y = \tan^{-1} x \cdot \frac{x^2}{2} - \frac{1}{2} [x - \tan^{-1} x] + C$$

$$y = \tan^{-1} x \cdot \frac{x^2}{2} - \frac{1}{2} x + \frac{\tan^{-1} x}{2} + C$$

$$y = \frac{1}{2} \cdot \tan^{-1} x (x^2 + 1) - \frac{1}{2} x + C$$

#### 40. Question

Find the particular solution of the differential equation  $x(1+y^2) \, dx - y(1+x^2) \, dy = 0$ , given that  $y = 1$  when  $x = 0$ .

#### Answer

$$\frac{2x \, dx}{1+x^2} - \frac{2y \, dy}{1+y^2} = 0$$

$$\Rightarrow \frac{\log(1+x^2)}{1+y^2} = 0$$

$$\Rightarrow (1+x^2) = c(1+y^2)$$

$$\Rightarrow y = 1, x = 0$$

$$\Rightarrow 1 = c(2)$$

$$\Rightarrow c = \frac{1}{2}$$

$$\Rightarrow 2(1+x^2) = 1+y^2$$

$$\Rightarrow 2+2x^2-1=y^2$$

$$\Rightarrow 2x^2+1=y^2$$

$$\Rightarrow y = \sqrt{2x^2+1}$$

#### 29. Question

Find the general solution of each of the following differential equations:

$$e^x \sqrt{1-y^2} \, dx + \frac{y}{x} \, dy = 0$$

#### Answer

$$e^x \cdot x \, dx + \frac{y}{\sqrt{1-y^2}} dy = 0$$

Integrating,

$$\int e^x \cdot x \, dx + \int \frac{y}{\sqrt{1-y^2}} dy = C$$

Consider the integral  $\int e^x \cdot x \, dx$

By ILATE rule,

$$\int e^x \cdot x \, dx = x \cdot \int e^x \, dx - \int \left[ \frac{d}{dx}(x) \cdot \int e^x \, dx \right] dx$$

$$x \cdot e^x - \int e^x \, dx$$

$$x \cdot e^x - e^x$$

$$e^x(x-1)$$

Consider the integral  $\int \frac{y}{\sqrt{1-y^2}} dy$

$$\text{Its value is } -\sqrt{1-y^2} \text{ as } \frac{d}{dx}(\sqrt{1-y^2}) = \frac{-2y}{2\sqrt{1-y^2}} = \frac{-y}{\sqrt{1-y^2}}$$

Therefore the solution of the given differential equation is

$$e^x(x-1) - \sqrt{1-y^2} = C$$

#### 41. Question

Find the particular solution of the differential equation  $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ , given that  $y = 0$  when  $x = 0$ .

#### Answer

$$\log\left(\frac{dy}{dx}\right) = 3x + 4y$$

$$\Rightarrow y = 0$$

$$\Rightarrow x = 0$$

$$\Rightarrow \frac{dy}{dx} = e^{3x} e^{4y}$$

$$\Rightarrow e^{-4y} dy = e^{3x} dx$$

$$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$$

$$\Rightarrow \text{For } y = 0, x = 0, \text{ we have}$$

$$\Rightarrow -\frac{1}{4} = \frac{1}{3} + C$$

$$\Rightarrow C = -\frac{7}{12}$$

$$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$$

Hence, the particular solution is:

$$\Rightarrow 4e^{3x} + 3e^{-4x} = 7$$

#### 30. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

**Answer**

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$dy = \frac{1 - \cos x}{1 + \cos x} dx$$

$$\cos x \text{ can be written as } \cos x = \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$$

$$dy = \frac{1 - \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}}{1 + \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}} dx$$

$$dy = \frac{\left[ \frac{1 + \tan^2\left(\frac{x}{2}\right) - (1 - \tan^2\left(\frac{x}{2}\right))}{1 + \tan^2\left(\frac{x}{2}\right)} \right]}{\frac{1 + \tan^2\left(\frac{x}{2}\right) + (1 - \tan^2\left(\frac{x}{2}\right))}{1 + \tan^2\left(\frac{x}{2}\right)}} dx$$

$$dy = \frac{1 + \tan^2\left(\frac{x}{2}\right) - 1 + \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right) + 1 - \tan^2\left(\frac{x}{2}\right)} dx$$

$$dy = \frac{2 \tan^2\left(\frac{x}{2}\right)}{2} dx$$

$$dy = \tan^2\left(\frac{x}{2}\right) dx$$

Integrating on both the sides,

$$\int dy = \int \tan^2\left(\frac{x}{2}\right) dx$$

$$y = \int \left[ \sec^2\left(\frac{x}{2}\right) - 1 \right] dx \text{ formula: } \{\sec^2 x - \tan^2 x = 1\}$$

$$y = 2 \cdot \tan\left(\frac{x}{2}\right) - x + C \text{ formula: } \left\{ \frac{d}{dx} \tan\left(\frac{x}{2}\right) = \sec^2\left(\frac{x}{2}\right) \cdot \frac{1}{2} \right\}$$

#### 42. Question

Solve the differential equation  $(x^2 - yx^2)dy + (y^2 + x^2y^2) dx = 0$ , given that  $y = 1$  when  $x = 1$ .

**Answer**

$$x^2(1 - y)dy + y^2(1 + x^2)dx = 0$$

$$\Rightarrow \frac{(1-y)}{y^2} dy + \frac{(1+x^2)}{x^2} dx = 0$$

$$\Rightarrow \int \frac{(1-y)}{y^2} dy + \int \frac{(1+x^2)}{x^2} dx = 0$$

$$\Rightarrow -\frac{1}{y} - \log y - \frac{1}{x} + x = c$$

For  $y=1, x=1$ , we have,

$$\Rightarrow -1 - 0 - 1 + 1 = c$$

$$\Rightarrow c = -1$$

Hence, the required solution is:

$$\Rightarrow \frac{1}{y} + \log y + \frac{1}{x} - x = 1$$

#### 43. Question

Find the particular solution of the differential equation  $e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$ , given that  $y = 1$  when  $x = 0$ .

#### Answer

Given:  $e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$  Separating the variables we get,

$$\Rightarrow x e^x dx + \frac{y}{\sqrt{1-y^2}} dy = 0$$

$$\Rightarrow \int x e^x dx + \int \frac{y}{\sqrt{1-y^2}} dy = 0 \text{ Substituting } \sqrt{1-y^2} = t, 1-y^2 = t^2, -2y dy = 2t dt, \text{ we have,}$$

$$\Rightarrow x e^x - e^x - \frac{1}{2} \log |\sqrt{1-y^2}| = c$$

For  $y=1$  and  $x=0$ , we have,

$$\Rightarrow 0 - 1 - 0 = c$$

$$\Rightarrow c = -1$$

$\Rightarrow$  Hence, the particular solution will be:-

$$\Rightarrow x e^x - e^x - \frac{1}{2} \log |\sqrt{1-y^2}| + 1 = 0$$

#### 44. Question

Find the particular solution of the differential equation  $\frac{dy}{dx} = \frac{x(2 \log x + 1)}{(\sin y + y \cos y)}$ , given that  $y = \frac{\pi}{2}$  when  $x = 1$ .

#### Answer

$$\text{Given: } \frac{dy}{dx} = \frac{x(2 \log x + 1)}{(\sin y + y \cos y)}$$

$$\Rightarrow \int \sin y dy + \int y \cos y dy = \int 2x \log x dx + \int x dx$$

Let  $\int y \cos y dy = I$  Then,

$$\int y \cos y dy = \left( \int \cos y dy \right) y - \int \left( \left( \int y \cos y dy \right) \cdot \frac{d}{dx} y \right) dy$$

$$\text{And } \int x \log x = \left( \int x dx \right) \log x - \int \left( \left( \int x dx \right) \frac{d}{dx} \log x \right) dx$$

We have,

$$\Rightarrow -\cos y + y \sin y + \cos y = x^2 \log x - \frac{x^2}{2} + \frac{x^2}{2} + c$$

For  $y = \frac{\pi}{2}, x = 1$  we have,

$$0 + \frac{\pi}{2} + 0 = 0 + c$$

$$c = \frac{\pi}{2}$$

$$\Rightarrow y \sin y = x^2 \log x + \frac{\pi}{2}$$

#### 45. Question

Solve the differential equation  $\frac{dy}{dx} = y \sin 2x$ , given that  $y(0) = 1$ .

#### Answer

We have,

$$\frac{dy}{dx} = y \sin 2x$$

$$\Rightarrow \frac{dy}{y} = \sin 2x dx$$

$$\Rightarrow \log y = -\frac{\cos 2x}{2} + c$$

For  $y=1$ ,  $x=0$ , we have,

$$c = \frac{1}{2}$$

$$\Rightarrow \log y = \frac{1}{2}(1 - \cos 2x)$$

$$\Rightarrow \log y = \sin^2 x$$

Thus,

The particular solution is:

$$y = e^{\sin^2 x}$$

#### 46. Question

Solve the differential equation  $(x+1)\frac{dy}{dx} = 2xy$ , given that  $y(2) = 3$ .

#### Answer

$$\text{Given: } (x+1)\frac{dy}{dx} = 2xy$$

$$\Rightarrow \frac{dy}{y} = 2 \frac{x}{x+1} dx$$

$$\Rightarrow \log y = \int 2 - \frac{2}{x+1} dx$$

$$\Rightarrow \log y = 2x - 2 \log(x+1) + c$$

For  $x=2$  and  $y=3$ , we have,

$$c = 3 \log 3 - 4$$

Hence, the particular solution is,

$$\Rightarrow y(x+1)^2 = 27e^{2x-4}$$

#### 47. Question

Solve  $\frac{dy}{dx} = x(2 \log x + 1)$ , given that  $y = 0$  when  $x = 2$ .

**Answer**

we have,  $\frac{dy}{dx} = 2x \log x + x$ , Integrating we get,

$$y = \int (2x \log x + x) dx,$$

$$y = \int 2x \log x dx + x dx$$

$$y = \left( \int 2x dx \right) \log x - \int \left[ \left( \int 2x dx \right) \left( \frac{d}{dx} \log x \right) \right] dx + \frac{x^2}{2} + c$$

given that  $y=0$  when  $x=2$

$$\Rightarrow y = x^2 \log x - \frac{x^2}{2} + \frac{x^2}{2} + c$$

now putting  $x=2$  and  $y=0$ ,

$$\Rightarrow 0 = 4 \log 2 + c$$

$$\Rightarrow c = -4 \log 2$$

Thus, the solution is:

$$y = x^2 \log x - 4 \log 2$$

**48. Question**

Solve  $(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$ , given that  $y = 1$  when  $x = 0$ .

**Answer**

we have,  $(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$ ,

$$\left\{ \right. \text{Given that: } y=1 \text{ when } x=0,$$

$$\Rightarrow (x^2 + 1)(x + 1) \frac{dy}{dx} = 2x^2 + x$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + x + x^2}{(x^2 + 1)(x + 1)} dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{x(x+1) + x^2 + 1 - 1}{(x^2 + 1)(x + 1)} dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{x^2 + 1} dx + \frac{x^2 + 1 - 1}{(x^2 + 1)(x + 1)} dx$$

$$\Rightarrow dy = \frac{x dx}{x^2 + 1} + \frac{dx}{x + 1} - \frac{dx}{(x^2 + 1)(x + 1)}$$

$$\Rightarrow \int dy = \int \frac{x dx}{x^2 + 1} + \int \frac{dx}{x + 1} - \int \frac{\frac{-1}{2} \frac{x+1}{x^2+1}}{x^2+1} dx + \int \frac{\frac{1}{2}}{x+1} dx$$

$$\Rightarrow y = \frac{1}{2} \log|x^2 + 1| + \log|x + 1| + \frac{1}{4} \log|x^2 + 1| - \frac{1}{2} \tan^{-1} x + \frac{1}{2} \log|x + 1| + c$$

$$\Rightarrow y = \frac{3}{4} \log|x^2 + 1| + \frac{1}{2} \log|x + 1| - \frac{1}{2} \tan^{-1} x + c$$

For  $y=1$ , when  $x=0$ , we have,

$$1 = 0 + 0 - 0 + c$$

$$\Rightarrow c = 1$$



$$y = \frac{1}{2} \left\{ \log|x+1| + \frac{3}{2} \log(x^2+1) - \tan^{-1} x \right\} + 1$$

#### 49. Question

Solve  $\frac{dy}{dx} = y \tan x$ , given that  $y = 1$  when  $x = 0$ .

#### Answer

we have,  $\frac{dy}{dx} = y \tan x$ ,

given that:  $y=1$  when  $x=0$

$$\Rightarrow \frac{dy}{dx} = y \tan x$$

$$\Rightarrow \frac{dy}{y} = \tan x \, dx$$

$$\Rightarrow \log y = \log \sec x + c$$

$$\Rightarrow 0 = 0 + c$$

$\Rightarrow y \cos x = 1$  is the particular solution...

#### 50. Question

Solve  $\frac{dy}{dx} = y^2 \tan 2x$ , given that  $y = 2$  when  $x = 0$ .

#### Answer

we have:  $\frac{dy}{dx} = y^2 \tan 2x$ ,

Given that,  $y=2$  when  $x=0$

$$\Rightarrow \frac{dy}{y^2} = \tan 2x \, dx$$

$$\Rightarrow \int \frac{dy}{y^2} = \int \tan 2x \, dx \text{ ...integrating both sides}$$

$$\Rightarrow -\frac{1}{y} = \frac{\log(\sec 2x)}{2}$$

$$\Rightarrow -\frac{1}{2} = 0 + c$$

$$\Rightarrow c = -\frac{1}{2}$$

$\Rightarrow y(1 + \log \cos 2x) = 2$  ...is the particular solution

#### 51. Question

Solve  $\frac{dy}{dx} = y \cot 2x$ , given that  $y = 2$  when  $x = \frac{\pi}{4}$ .

#### Answer

we have  $\frac{dy}{dx} = y \cot 2x$ ,

Given that,  $y=2$  when  $x=\frac{\pi}{4}$

$$\Rightarrow \frac{dy}{y} = y \cot 2x$$

$$\Rightarrow \frac{dy}{y} = \cot 2x dx$$

$$\Rightarrow \int \frac{dy}{y} = \int \cot 2x dx$$

$$\Rightarrow \log y = -\frac{\log(\sin 2x)}{2} + c$$

$$\Rightarrow \log 2 = 0 + c$$

$$\Rightarrow \text{Thus, } c = \log 2$$

$$\text{The particular solution is :- } \log \frac{y}{\sqrt{\sin 2x}} = \log 2$$

$$\therefore y = 2\sqrt{\sin 2x}$$

## 52. Question

Solve  $(1 + x^2) \sec^2 y dy + 2x \tan y dx = 0$ , given that  $y = \frac{\pi}{4}$  when  $x = 1$ .

### Answer

we have,  $(1 + x^2) \sec^2 y dy + 2x \tan y dx = 0$ ,

Given that,  $y = \frac{\pi}{4}$  when  $x=1$

$$\Rightarrow (1 + x^2) \sec^2 y dy + 2x \tan y dx = 0$$

$$\Rightarrow \frac{\sec^2 y}{\tan y} dy + \frac{2x}{1+x^2} dx = 0$$

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy + \int \frac{2x}{1+x^2} dx = 0$$

$$\Rightarrow \log \tan y + \log(1 + x^2) = \log c$$

$$\text{For } y = \frac{\pi}{4}, x = 1$$

$$\text{We have, } 0 + \log 2 = \log c,$$

$$c = 2,$$

Hence the required particular solution is:-

$$\therefore \tan y(1 + x^2) = 2$$

## 53. Question

Find the equation of the curve passing through the point  $\left(0, \frac{\pi}{4}\right)$  whose differential equation is  $\sin x \cos y dx + \cos x \sin y dy = 0$ .

### Answer

we have,  $\sin x \cos y dx + \cos x \sin y dy = 0$

$$\Rightarrow \sin x \cos y dx + \cos x \sin y dy = 0$$

$$\Rightarrow \tan x dx + \tan y dy = 0$$

$$\Rightarrow \log \sec x + \log \sec y = \log c$$

$$\Rightarrow \sec x \sec y = c$$

Given that, coordinates of point,  $(0, \frac{\pi}{4})$

$$\Rightarrow c = \sqrt{2}$$

$$\Rightarrow \sec y = \sqrt{2} \cos x$$

$\therefore y = \cos^{-1}(\frac{1}{\sqrt{2}} \sec x)$  ...is the required particular solution

#### 54. Question

Find the equation of a curve which passes through the origin and whose differential equation is

$$\frac{dy}{dx} = e^x \sin x.$$

#### Answer

Given,  $\frac{dy}{dx} = e^x \sin x$

$$dy = e^x \sin x dx$$

$$\Rightarrow \int dy = \int e^x \sin x dx$$

$$\left[ \right] \left[ \right] \left[ \right] \text{Let } I = \int e^x \sin x dx$$

$$\Rightarrow I = \int e^x dx \sin x - \int (\int e^x dx) \cdot \left(\frac{d}{dx} \sin x\right) dx$$

$$\Rightarrow I = e^x \sin x - \int e^x \cos x dx$$

$$\Rightarrow I = e^x \sin x - \int e^x dx \cos x - \int (\int e^x dx) \cdot \left(\frac{d}{dx} \cos x\right) dx$$

$$\Rightarrow I = e^x \sin x - e^x \cos x - \int e^x \sin x dx$$

$$\Rightarrow 2I = e^x \sin x - e^x \cos x$$

$$\Rightarrow I = \frac{e^x \sin x - e^x \cos x}{2} + c$$

$$\therefore y = \frac{e^x \sin x - e^x \cos x}{2} + c$$

For the curve passes through (0,0)

We have,  $c = \frac{1}{2}$

$$\therefore 2y - e^x \sin x + e^x \cos x = 1$$

#### 55. Question

A curve passes through the point (0, -2) and at any point (x, y) of the curve, the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the point. Find the equation of the curve.

#### Answer

Given that the product of slope of tangent and y coordinate equals the x-coordinate i.e.,  $y \frac{dy}{dx} = x$

We have,  $y dy = x dx$

$$\Rightarrow \int y dy = \int x dx$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$$

For the curve passes through (0, -2), we get  $c = 2$ ,

Thus, the required particular solution is:-

$$\therefore y^2 = x^2 + 4$$

### 56. Question

A curve passes through the point (-1, 1) and at any point (x, y) of the curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (-4, -3). Find the equation of the curve.

### Answer

$$\text{Given : } \frac{dy}{dx} = \frac{2(y+3)}{x+4}$$

$$\Rightarrow \frac{dy}{y+3} = \frac{2dx}{x+4}$$

$$\Rightarrow \int \frac{dy}{y+3} = 2 \int \frac{dx}{x+4}$$

$$\Rightarrow \log(y+3) = 2\log(x+4) + c$$

The curve passes through (-2, 1) we have,

$$c = 0,$$

$$\therefore y+3 = (x+4)^2$$

### 57. Question

In a bank, principal increases at the rate of r% per annum. Find the value of r if ₹ 100 double itself in 10 years.

(Given  $\log_e 2 = 0.6931$ )

### Answer

$$\text{Given: } \frac{dp}{dt} = \left(\frac{r}{100}\right) \times p$$

Here, p is the principal, r is the rate of interest per annum and t is the time in years.

Solving the differential equation we get,

$$\frac{dp}{p} = \left(\frac{r}{100}\right) dt$$

$$\Rightarrow \int \frac{dp}{p} = \int \frac{r}{100} dt$$

$$\Rightarrow \log p = \frac{rt}{100} + c$$

$$\Rightarrow p = e^{\frac{rt}{100} + c}$$

As it is given that the principal doubles itself in 10 years, so

Let the initial interest be p<sub>1</sub> (for t = 0), after 10 years p<sub>1</sub> becomes 2p<sub>1</sub>.

Thus,  $p_1 = e^c$  for (t = 0) ... (i)

$$p = 2p_1 = e^{\frac{r(10)}{100}} \cdot e^c \dots (ii)$$

Substituting (i) in (ii), we get,

$$\Rightarrow 2p_1 = e^{\frac{r}{10}} \cdot p_1$$

$$\Rightarrow 2 = e^{\frac{r}{10}}$$

$$\Rightarrow \log 2 = \frac{r}{10}$$

$$\Rightarrow r = 10 \log 2$$

$$\Rightarrow r = 6.931$$

$\therefore$  Rate of interest = 6.931

### 58. Question

In a bank, principal increases at the rate of 5% per annum. An amount of ` 1000 is deposited in the bank. How much will it worth after 10 years?

(Given  $e^{0.5} = 1.648$ )

### Answer

Given: rate of interest = 5%

P(initial) = Rs 1000

And,

$$\frac{dp}{dt} = \frac{5}{100} \times p$$

$$\Rightarrow \frac{dp}{p} = \frac{5}{100} dt$$

$$\Rightarrow \int \frac{dp}{p} = \int \frac{5}{100} dt$$

$$\Rightarrow \log p = \frac{5t}{100} + c$$

$$\Rightarrow p = e^{\frac{5t}{100} + c}$$

For  $t = 0$ , we have  $p = 1000$

$$1000 = e^c$$

For  $t = 10$  years we have,  $p = e^{\frac{50}{100}} \cdot 1000$

$$p = 1000e^{1/2}$$

$$p = 1648$$

Thus, principal is Rs1648 for  $t = 10$  years.

### 59. Question

The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of the balloon after  $t$  seconds.

### Answer

Given:

$$\text{Volume } V = \frac{4\pi r^3}{3}$$

$$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dV}{dt} = k \text{ (constant)}$$

$$4\pi r^2 \frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2 dr = k dt$$

$$\Rightarrow \int 4\pi r^2 dr = \int k dt$$

$$\Rightarrow \frac{4\pi r^3}{3} = kt + c$$

For  $t = 0$ ,  $r = 3$  and for  $t = 3$ ,  $r = 6$ , So, we have,

$$\Rightarrow \frac{4\pi(3)^3}{3} = 0 + c$$

$$\Rightarrow c = 36\pi$$

$$\frac{4\pi(6)^3}{3} = k \cdot (3) + 36\pi$$

$$\Rightarrow k = 84\pi$$

So after  $t$  seconds the radius of the balloon will be,

$$\Rightarrow \frac{4\pi r^3}{3} = 84\pi t + 36\pi$$

$$\Rightarrow 4\pi r^3 = 252\pi t + 108\pi$$

$$\Rightarrow r^3 = \frac{252\pi t + 108\pi}{4\pi}$$

$$\Rightarrow r^3 = 63t + 27$$

$$\Rightarrow r = \sqrt[3]{63t + 27}$$

Hence, radius of the balloon as a function of time is

$$\therefore r = (63t + 27)^{1/3}$$

#### 60. Question

In a culture the bacteria count is 100000. The number is increased by 10% in 2 hours. In how many hours will the count reach 200000, if the rate of growth of bacteria is proportional to the number present?

#### Answer

Let  $y$  be the bacteria count, then, we have,

rate of growth of bacteria is proportional to the number present

$$\text{So, } \frac{dy}{dt} = cy$$

Where  $c$  is a constant,

Then, solving the equation we have,

$$\frac{dy}{y} = c dt$$

$$\int \frac{dy}{y} = \int c dt$$

$$\log y = ct + k$$

Where  $k$  is constant of integration

$$y = e^{ct+k}$$

And we have for  $t = 0$ ,  $y = 10000$ ,

$$10000 = e^k \dots (i)$$

For  $t = 2$  hrs,  $y$  is increased by 10% i. e.  $y = 110000$

$$110000 = e^{c(2)} \cdot e^k$$

$$\Rightarrow 110000 = e^{2c} \cdot (100000) \text{ from (i)}$$

$$\Rightarrow e^{2c} = 1.1$$

$$\Rightarrow e^c = \sqrt{1.1}$$

$$\Rightarrow c = \frac{1}{2} \log\left(\frac{11}{10}\right)$$

When  $y = 200000$ , we have,

$$200000 = e^{ct} \cdot 100000$$

$$\Rightarrow e^{ct} = 2$$

$$\Rightarrow (e^c)^t = 2$$

$$\Rightarrow tc = \log 2$$

$$\Rightarrow t = \frac{2 \log 2}{\log \frac{11}{10}}$$

Hence,  $t = \frac{2 \log 2}{\log \frac{11}{10}}$