CBSE Class 11 Mathematics
Sample Papers 08 (2019-20)

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
iv. There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. Suppose f : [2, 2] $\rightarrow$R be defined by $f(x)=\begin{cases}-1 \;\; for & -2\leq x\leq 0\\ x-1\; for & 0\leq x\leq 2\end{cases}$, Then { x $\in$ [-2, 2]: x $\leq$ 0 and f (| x |) = x} =

   a. { - 1 }

   b. ϕ

   c. $\left\{-\frac{1}{2}\right\}$

   d. {0}

2. The number of diagonals that can be drawn by joining the vertices of an octagon is :

   a. 12

b. 20
c. 28
d. 48

3. The term independent of x in the expansion of $\left(2x-\frac{1}{2x^2}\right)^{12}$ is

a. ${}^{12}C_3 2^6$

b. $-{}^{12}C_5 2^2$

c. ${}^{12}C_6$

d. ${}^{12}C_4 2^4$

4. The total number of 4 digit odd numbers that can be formed using 0, 1, 2, 3, 5, and 7 are

a. 375
b. 720
c. 400

d. 520

5. Two functions $f: R \to R$ and g : $R \to R$are defined as follows :
$f\left(x\right)=\begin{Bmatrix} 0\left(xRational\right) \\ 1\left(xIrrational\right) \end{Bmatrix}, g\left(x\right)=\begin{Bmatrix} -1\left(xRational\right) \\ 0\left(xIrrational\right) \end{Bmatrix}$, then (gof)(e) + (fog)(π) =

a. 0

b. 1

c. 2

d. - 1

6. The inequality $2^n+1<n!$ is true for :

a. all n >1

b. none of these

c. all n

d. all n >3

7. An unbiased dice is rolled four times. The probability that the minimum number on any toss is not less than 3 is

a. $\frac{16}{81}$

b. $\frac{1}{81}$

c. $\frac{65}{81}$

d. $\frac{80}{81}$

8. A , B C and D are four points in spaces such that AB = BC = CD = DA . Then ABCD is a

a. nothing can be said

b. rectangle

c. rhombus

d. skew quadrilateral

9. Let A be set of 4 elements. From the set of all functions from A to A, a function is chosen at random. The chance that the selected function is an onto function is

a. 29/ 32

b. none of these

c. 1/ 64

d. 3/ 32

10. The coefficient of x^{12} in the expansion of $\left(3-\frac{x^3}{6}\right)^7$ is

a. $\frac{32}{48}$

b. $\frac{17}{14}$

c. $\frac{35}{48}$

d. $\frac{35}{42}$

11. Fill in the blanks:

    The real function f : R $\rightarrow$ R defined by f(x) = x, where x $\in$ R is called an ________ function.

12. Fill in the blanks:

    If n is odd, then number of terms in the binomial expansion of $[(x + a)^n + (x - a)^n]$ is ________.

13. Fill in the blanks:

    If 7 point lies on a circle, no. of chords that can be drawn by joining these points are ________.

14. Fill in the blanks:

    A line is parallel to xy-plane if all the points on the line have equal ________.

    OR

    Fill in the blanks:

    A line is parallel to xy-plane if all the points on the line have equal ________.

15. Fill in the blanks:

    The value of the limit $\lim_{x \to 4} \frac{4x+3}{x-2}$ is ________.

    OR

    Fill in the blanks:

    The value of the limit: $\lim_{x \to 3} x + 3$ is ________.

16. Let A = {3, 6, 12, 15, 18, 21), B = {4, 8,12,16, 20)

    Find: A - B

17. In how many ways, can 5 sportsmen be selected from a group of 10?

18. Solve $x^2 + 4 = 0$.

OR

Write the real and imaginary parts of the complex number $\frac{\sqrt{17}}{2} + \frac{2}{\sqrt{70}}i$.

19. Write the range of $y = \frac{|x-1|}{x-1}$.

20. Prove that n (n - 1) (n - 2)... (n - r + 1) = $\frac{n!}{(n-r)!}$

21. In a survey of 700 students in a college, 180 were listed as drinking Limca, 275 as drinking Miranda and 95 were listed as both drinking Limca as well as Miranda. Find how many students were drinking neither Limca nor Miranda.

OR

Let A = {1, 2, 4, 5}, B = {2, 3, 5, 6}, C = {4, 5, 6, 7}. Verify $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.

22. Three coins are tossed simultaneously. List the sample space for the event.

23. Using binomial theorem, expand: $\left(x + \frac{1}{y}\right)^{11}$.

24. A point moves, so that the sum of its distances from (ae, 0) and (-ae, 0) is 2a, prove that the equation to its locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $b^2 = a^2(1 - e^2)$.

OR

What is the value of y so that the line through (4, y) and (2, 7) is parallel to the line through (-1, 4) and (0, 6).

25. Write the truth value of each of the following bi conditional statements.

i. 3 < 2 if and only if 2 < 1.
ii. 3 + 5 > 7 if and only if 4 + 6 < 10.

26. Find the general solutions of the equation: tan 2x tan x = 1

27. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students who had only physics.

28. Let A be a non-empty set such that A $\times$ B = A $\times$ C. Show that B = C.

OR

Determine the domain and range of the following relations.

i. $R_1 = \left\{\left(x, \frac{1}{x}\right) : 0 < x < 6, x \in N\right\}$
ii. R_2 = {(x, x^2): x is prime number less than 10}

29. Find the derivative of the following functions from first principle
$f(x) = \cos\left(x - \frac{\pi}{8}\right)$

30. Solve $x^2 + x + \frac{1}{\sqrt{2}} = 0$

31. In drilling world's deepest hole, it was found that the temperature T in degree Celsius, x km below the surface of the earth was given by
T = 30 + 25 (x - 3), 3 < x < 15
At what depth will the temperature be between 200°C and 300°C?

OR

Solve 3x + 8 > 2 when
(i) x is integer
(ii) x is a real number

32. For every positive integer n, prove that $7^n - 3^n$ is divisible by 4.

33. If sin x = $\frac{3}{5}$, tan y = $\frac{1}{2}$ and $\frac{\pi}{2}$ < x < n < y < $\frac{3\pi}{2}$, find the value of 8 tan x - $\sqrt{5}$ sec y.

OR

Find the value of the expression: $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$

34. Find the sum of the first n terms of the series: 3 + 7 + 13 + 21 + 21 + ...

35. Find the equation of the hyperbola whose vertices are at (0 ± 7) and foci at $\left(0, \pm\frac{28}{3}\right)$.

OR

Find the equation of a circle concentric with the circle $2x^2 + 2y^2 + 8x + 10y - 39 = 0$ and having its area equal to 16π.

36. Calculate the mean and standard deviation for the following table, given the age distribution of a group of people:

Age:	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of persons:	3	51	122	141	130	51	2

Solution
Section A

1. (c) $\left\{-\frac{1}{2}\right\}$ **Explanation:**

   $\mathrm{f}:[-2,2] \rightarrow \mathrm{R}$ is defined by

   $\mathrm{f(x)} = \begin{cases} -1, & -2 \leqslant \mathrm{x} \leqslant 2 \\ \mathrm{x}-1, & 0 \leqslant \mathrm{x} \leqslant 2 \end{cases}$

   Let $\mathrm{x} \leqslant 0$ and $\mathrm{f}(|\mathrm{x}|) = \mathrm{x}$

   Now, $\mathrm{f}(|\mathrm{x}|) = \mathrm{x} \Rightarrow |\mathrm{x}| - 1 = \mathrm{x}$

   $\Rightarrow -\mathrm{x}-1 = \mathrm{x}\ [\because\ |\mathrm{x}| \geqslant 0]$

   $\Rightarrow -\mathrm{x}-1 = \mathrm{x}$ (as $\mathrm{x} \leqslant 0$)

   $\Rightarrow 2\mathrm{x} = -1 \Rightarrow \mathrm{x} = -\frac{1}{2}$

   $\therefore \{\mathrm{x} \in [-2,2] : \mathrm{x} \leqslant 0 \text{ and } \mathrm{f}\ (|\mathrm{x}| = \mathrm{x}\} = \left\{-\frac{1}{2}\right\}$

2. (b) 20 **Explanation:**

   We have octagon is an eight sided polygon which has 8 vertices.

   A diagonal is obtained by joining two points .

   Thus the number of diagonals obtained by joining any two points out of 8 is given by

   $8C_2 \quad -8 = \frac{8!}{2!(8-2)!} - 8 = \frac{1\times2\times3\times4\times5\times6\times7\times8}{1\times2\times1\times2\times3\times4\times5\times6} - 8 = \frac{7\times8}{1\times2} - 8 = 28 - 8 = 20$

3. (d) ${}^{12}C_4 2^4$ **Explanation:** We have the general term of $(x+a)^n$ is
   $T_{r+1} = ^n C_r \quad (x)^{n-r} a^r$
   Now Consider $\left(2x - \frac{1}{2x^2}\right)^{12}$
   Here $T_{r+1} = ^{12} C_r \quad (2x)^{12-r}\left(-\frac{1}{2x^2}\right)^r$

The term independent of x means index of x is 0.

Comparing the indices of x in x^0 and in T_{r+1}, we get $12 - r - 2r = 0 \Rightarrow r = \frac{12}{3} = 4$

Therefore the required term in $T_{4+1} = \quad T_5 = ^{12}C_4 \quad (2x)^{12-4}\left(-\frac{1}{2x^2}\right)^4 = ^{12}C_4 \quad 2^4$

4. (b) 720 **Explanation:**

We have to find the total number of four digit odd numbers formed using the digits 0,1,2,3,5,7

Since it is an odd number the last place (unit's place) can be filled by any of the odd numbers 1,3,5,7 in 4 different ways.

Since repetition is allowed the second and third places can be filled by any of the six given digits

Since it has to be a four digit number the first place can be filled by any of the five given digits other than zero in 5 ways

Hence all the four places can be filled in $4 \times 6 \times 6 \times 5 = 720$ ways

5. (d) - 1

Explanation:

$gof(e) \ + fog(\pi) \ = \ \mathrm{g}(\mathrm{f}(\mathrm{e})) \ + \ \mathrm{f}(\mathrm{g}(\pi))$

$= \ \mathrm{g}(1) \ + \mathrm{f}(-1) \ \ \{\because \ \text{e is irrational and } \pi \text{ is rational}\}$

$= \ -1 \ + 0 \ \ = \ -1$

6. (d) all n $>$ 3

Explanation:

When n = 1 we get $3 < 1$, and when n = 2 we get $5 < 2$,. when n = 3 $9 < 6$, which are inavlid inequations. Only when n = 4 we get $17 < 24$, which is valid.

7. (a) $\frac{16}{81}$ **Explanation:** Probability that the outcome of a single throw of a die is any one of 4, 3, 4, 5 and 5 is equal to $\frac{4}{6}=\frac{2}{3}$ of the die is rolled four times, the required probability is equal to
8. (a) nothing can be said

   Explanation: It can be square or rhombus(all sides are equal).Angle property must be mentioned.
9. (d) 3/ 32

   Explanation: No. of function from A to A = 4^4

   No. of onto function from A to A = 4!

   $\therefore$ Required probability $=\frac{4!}{4^4}=\frac{3}{32}$
10. (c) $\frac{35}{48}$ **Explanation:** We have the general term of $(x+a)^n$ is $T_{r+1}={}^nC_r \quad (x)^{n-r}a^r$

    Now consider $\left(3-\frac{x^3}{6}\right)^7$

    Here $T_{r+1}={}^7C_r \quad (3)^{7-r}\left(-\frac{x^3}{6}\right)^r$

    comparing the indices of x in x^{12} and in T_{r+1}, we get 3r = 12 $\Rightarrow$ r = 4

    Therefore the required term is $T_{4+1}= \quad T_4={}^7C_4 \quad (3)^{7-4}\left(-\frac{x^3}{6}\right)^4$

    $=35\times3^3\times\frac{x^{12}}{6^4}=\frac{35}{48}x^{12}$
11. identity
12. $\left(\frac{n+1}{2}\right)$
13. 21
14. z-coordinates

    OR

    z-coordinates
15. $\frac{19}{2}$

    OR

6

16. A - B = $\{x, x \in A : x \notin B\}$ = {3, 6, 15, 18, 21}

17. The required number of ways = $^{10}C_5$

$= \frac{10!}{5!(10-5)!} = \frac{10!}{5!5!} = \frac{10\times9\times8\times7\times6\times5!}{5!\times5\times4\times3\times2\times1}$

= 3$\times$2$\times$42 = 252 $\left[\because {}^nC_r = \frac{n!}{r!(n-r)!}\right]$

18. We have, $x^2 + 4 = 0$

$\Rightarrow x^2 - 4i^2 = 0$ [$\because i^2 = -1$]

$\Rightarrow x^2 - (2i)^2 = 0$

$\Rightarrow$ (x + 2i) (x - 2i) = 0

$\therefore$ x = 2i

or x = - 2i

Hence, the roots of the given equation are 2i and - 2i.

OR

Suppose, $z = \frac{\sqrt{17}}{2} + \frac{2}{\sqrt{70}} i$

Here, $Re(z)$ = $\frac{\sqrt{17}}{2}$

and $Im(z)$ = $\frac{2}{\sqrt{70}}$

19. Given, $y = \frac{|x-1|}{x-1}$

The value of y will be 1, if $x - 1$ > 0 and -1, if $x - 1$ < 0. Hence, the range is $\{-1, 1\}$.

20. LHS = n (n — 1) (n — 2)... (n — r +1)

$= \frac{n(n-1)(n-2)\ldots(n-(r-1)\}}{1} \times \frac{(n-r)!}{(n-r)!}$

[multiplying numerator and denominator by (n - r)!]

$= \frac{n!}{(n-r)!}$ = RHS

Hence proved.

21. Suppose U be the set of all surveyed students, A denotes the set of students drinking Limca and B be the set students drinking Miranda. It is given that $n(U) = 700$, $n(A) = 180$, $n(B) = 275$ and $n(A \cap B) = 95$.

We have to find $n(A' \cap B')$.

Now, $n(A' \cap B') = n(AB)' = n(U) - n(A \cup B)$

= n(U) - [n (A)+ n (B) - n (A⋂B)]

$\Rightarrow$ n(A'⋂B') $= 700 - (180 + 275 - 95)$

$= 700 - 360$

$= 340.$

OR

A= {1, 2, 4, 5}, B = {2,3,5,6}, C = {4, 5, 6, 7}

$B\Delta C$ = (B - C) $\cup (C - B) = \{2,3\} \cup \{4,7\}$= {2, 3, 4, 7}

$A \cap (B\Delta C)$= {2, 4} ....(i)

$(A \cap B)$= {2, 5}

$(A \cap C)$= {4, 5}

$(A \cap B)\Delta(A \cap C) = [(A \cap B) - (A \cap C)] \cup [(A \cap C) - (A \cap B)]$

$(A \cap B)\Delta(A \cap C) = \{2\} \cup \{4\}$ = {2,4}......(ii)

From eq. (i) and eq. (ii), we get

$A \cap (B\Delta C) = (A \cap B)\Delta(A \cap C)$

22.

1st coin	H	H	H	T	H	T	T	T
2nd coin	H	H	T	H	T	H	T	T
3rd coin	H	T	H	H	T	T	H	T

Sample Space, S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

23. We have,

$\left(x + \frac{1}{y}\right)^{11} = ^{11}\mathrm{C}_0 x^{11}\left(\frac{1}{y}\right)^0 + ^{11}\mathrm{C}_1 x^{10}\left(\frac{1}{y}\right) + ^{11}\mathrm{C}_2 x^9\left(\frac{1}{y}\right)^2 + ^{11}\mathrm{C}_3 x^8\left(\frac{1}{y}\right)^3$

$+ ^{11}C_4 x^7\left(\frac{1}{y}\right)^4 + ^{11}C_5 x^6\left(\frac{1}{y}\right)^5 + ^{11}C_6 x^5\left(\frac{1}{y}\right)^6 + ^{11}C_7 x^4\left(\frac{1}{y}\right)^7 + ^{11}C_8 x^3\left(\frac{1}{y}\right)^8$ +

$^{11}\mathrm{C}_9 x^2\left(\frac{1}{y}\right)^9 + ^{11}\mathrm{C}_{10} x\left(\frac{1}{y}\right)^{10} + ^{11}\mathrm{C}_{11}\left(\frac{1}{y}\right)^{11}$

= $x^{11} + 11\frac{x^{10}}{y} + 55\frac{x^9}{y^2} + 165\frac{x^8}{y^3} + 330\frac{x^7}{y^4} + 462\frac{x^6}{y^5} + 462\frac{x^5}{y^6}$

$+ \frac{330x^4}{y^7} + \frac{165x^3}{y^8} + \frac{55x^2}{y^9} + \frac{11x}{y^{10}} + \frac{1}{y^{11}}$

24. Let, P(h, k) be the moving point such that the sum of its distances from A(ae, 0) and B(-ae, 0) is 2a.

Then, PA + PB = 2a

$\Rightarrow \sqrt{(h-ae)^2+(k-0)^2}+\sqrt{(h+ae)^2+(k-0)^2}=2a$

$\left[\because \text{ distance } = \sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}\right]$

$\Rightarrow \quad \sqrt{(h-ae)^2+k^2}=2a-\sqrt{(h+ae)^2+k^2}$

$\Rightarrow$ $(h - ae)^2 + k^2 = 4a^2 + (h + ae)^2 + k^2 -4a\sqrt{(h+ae)^2+k^2}$ [squaring on both sides]

$\Rightarrow \quad -4aeh-4a^2=-4a\sqrt{(b+ae)^2+k^2}$

$\Rightarrow \quad (eh+a)=\sqrt{(h+ae)^2+k^2}$

$\Rightarrow$ $(eh + a)^2 = (h + ae)^2 + k^2$ [again, squaring on both sides]

$\Rightarrow$ $e^2 h^2 + 2aeh + a^2 = h^2 + a^2 e^2 + 2aeh + k^2$

$\Rightarrow$ $h^2 (1 - e^2) + k^2 = a^2 (1 - e^2)$

$\Rightarrow \quad \frac{h^2}{a^2}+\frac{k^2}{a^2(1-e^2)}=1$

Hence, locus of point P (h, k) is

$\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1$

or $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $b^2 = a^2 (1 - e^2)$

OR

Here, m_1= Slope of line through (4, y) and (2, 7)

$=\frac{7-y}{2-4}=\frac{7-y}{-2}$

and m_2 = Slope of line through (-1, 4) and (0, 6)

$=\frac{6-4}{0+1}=2$

We know that, slope of two parallel lines are equal.

$\therefore$ $m_1 = m_2$

$\Rightarrow \quad \frac{7-y}{-2}=2$

$\Rightarrow$ -y = -4 - 7

$\therefore$ y = 11

25. i. Let p : 3 < 2 and q: 2 < 1.

Here, the given statement is p $\Leftrightarrow$ q.

Also, both p and q are false and therefore, p $\Leftrightarrow$ q is true.

Hence, the truth value of the given statement is T.

ii. Let p : 3 + 5 > 7 and q : 4 + 6 < 10.

Here, the given statement is p $\Leftrightarrow$ q.

Also, p is true and q is false and therefore, p $\Leftrightarrow$ q is false.

Hence, the truth value of the given statement is F.

26. We have,

tan 2x tan x = 1

$\Rightarrow \quad \tan 2x = \frac{1}{\tan x}$

$\Rightarrow$ tan 2x = cot x

$\Rightarrow$ tan 2x = tan $\left(\frac{\pi}{2} - x\right)$

$\Rightarrow \quad 2x = n\pi + \frac{\pi}{2} - x, n \in Z$

$\Rightarrow \quad 3x = n\pi + \frac{\pi}{2}, n \in Z$

$\Rightarrow \quad x = \frac{n\pi}{3} + \frac{\pi}{6}, n \in Z$

27. Let M be the set of students who had taken mathematics, P be the set of students who had taken physics and C be the set of students who had taken chemistry.

Here n(U) = 25, n(M) = 15, n(P) = 12, n(C) = 11, $n(M \cap C) = 5,$

$n(M \cap P) = 9, n(P \cap C) = 4, n(M \cap P \cap C) = 3,$

From the Venn diagram, we have

n(M) = a + b + d + e = 15

n(P) = b + c + e + f = 12

n(C) = d + e + f + g = 11

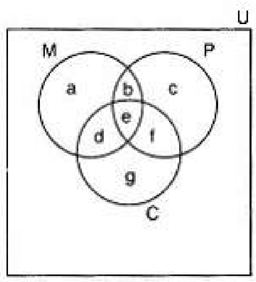
$n(M \cap C) = d + e = 5,$

$n(M \cap P) = b + e = 9,$

$n(P \cap C) = e + f = 4$

$n(M \cap P \cap C) = e = 3$

Now e = 3



$d+e=5 \Rightarrow d+3=5 \Rightarrow d=5-3 \Rightarrow d=2$

b +e = 9 $\Rightarrow$b + 3 = 9 $\Rightarrow$b = 9 - 3 $\Rightarrow$ b = 6

e + f = 4 $\Rightarrow$ 3 + f = 4 $\Rightarrow$ f = 4 - 3 $\Rightarrow$ f = 1

a +b + d + e = 15 $\Rightarrow$ a + 6 + 2 + 3 = 15 $\Rightarrow$ a = 15 - 14 = 4

b + c + e + f = 12 $\Rightarrow$ 6 + c + 3 + 1 = 12 $\Rightarrow$ c = 12 - 10 = 2

d + e + f + g = 11 $\Rightarrow$ 2 + 3+ 1+ g = 11 $\Rightarrow$g = 11 - 6 = 5

$\therefore$ c = 2

28. Suppose b be an arbitrary element of B.

$\therefore (a, b) \in A \times B$ for all $a \in A$.

$\Rightarrow (a, b) \in A \times C$ for all $a \in A$ $[\because A \times B = A \times C]$

$\Rightarrow \quad b \in C$

$\therefore b \in B \Rightarrow b \in C$

$\therefore \quad B \subseteq C$ ...... (i)

Suppose c be an arbitrary element of C.

Then, $(a, c) \in A \times C$ for all $a \in A$.

$\Rightarrow \quad (a, c) \in A \times B$ for all $a \in A$

$\Rightarrow \quad c \in B$ ..... (ii)

Thus, $c \in C \Rightarrow c \in B$

$\therefore \quad C \subseteq B$

From equation (i) and equation (ii), $B = C$

OR

i. $R_1 = \left\{\left(x, \frac{1}{x}\right) : 0 < x < 6, x \in N\right\}$
$= \left\{\left(x, \frac{1}{x}\right) : x = 1, 2, 3, 4, 5\right\}$
$= \left\{(1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right), \left(5, \frac{1}{5}\right)\right\}$

$\therefore$ domain of R_1 = $\{1, 2, 3, 4, 5\}$

and range of $R_1 = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

ii. R_2 = $\{(x, x^2)$: x is prime number less than 10$\}$

= $\{(x, x^2) : x = 2, 3, 5, 7\}$

= $\{(2, 4), (3, 9), (5, 25), (7, 49)\}$

Hence, domain of R_2 = $\{2, 3, 5, 7\}$

and range of R_2 = $\{4, 9, 25, 49\}$

29. Here $f(x) = \cos\left(x - \frac{\pi}{8}\right)$

$f(x) = \cos\left(x - \frac{\pi}{8}\right)$

Then f (x + h) = $\cos\left(x + h - \frac{\pi}{8}\right)$

We know that $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h \to 0} \frac{\cos\left(x+h-\frac{\pi}{8}\right) - \cos\left(x-\frac{\pi}{8}\right)}{h}$

$= \lim_{h \to 0} \frac{-2\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{h} = \lim_{h \to 0} \frac{-\sin\left(x-\frac{\pi}{8}+\frac{h}{2}\right)\cdot\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$

$= -\sin\left(x - \frac{\pi}{8}\right)$

30. Here $x^2 + x + \frac{1}{\sqrt{2}} = 0$

Comparing the given quadratic equation with $ax^2 + bx + c = 0$ we have

a = 1, b =1 and $c = \frac{1}{\sqrt{2}}$

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$\therefore x = \frac{-1 \pm \sqrt{1 - 2\sqrt{2}}}{2} = \frac{-1 \pm \sqrt{(2\sqrt{2}-1)i}}{2}$

Thus, $x = \frac{-1 + \sqrt{(2\sqrt{2}-1)i}}{2}$samp;$x = \frac{-1 - \sqrt{(2\sqrt{2}-1)i}}{2}$

31. Here T = 30 + 25 (x - 3), 3 < x < 15

Now 200 < 30 + 25 (x - 3) < 300

$\Rightarrow 170 < 25(x - 3) < 270$

$\Rightarrow \frac{170}{25} < (x - 3) < \frac{270}{25}$

$\Rightarrow 6.8 < (x - 3) < 10.8$

$\Rightarrow 6.8 + 3 < x < 10.8 + 3$

$\Rightarrow 9.8 < x < 13.8$

Thus required depth will be between 9.8 km and 13.8 km.

OR

Here 3x + 8 > 2

$\Rightarrow 3x > 2 - 8 \Rightarrow 3x > -6$

Dividing both sides by 3, we have

x > -2

(i) When x is an integer then values of x that make the statement true are -1, 0, 1, 2, 3, . . . The solution set of inequality is {-1, 0, 1, 2, 3, . . . }

(ii) When x is a real number. The solution set of inequality is $x \in (-2, \infty)$

32. **Step 1:** Let $P(n) : 7^n - 3^n$ is divisible by 4.

    Step 2: For n = 1, we have

    $P(1) : 7^1 - 3^1$ [put n = 1]

    $= 4$, which is divisible by 4.

    Thus, P(n) is true for n = 1.

    Step 3: Let P(k) be true for some natural number k, i.e.,

    $P(k) : 7^k - 3^k$ is divisible by 4.

    So, we can write, $7^k - 3^k = 4d$, where d ∈ N

    Step 4: Now, consider P(k + 1): $7^{k+1} - 3^{k+1}$

    $= 7^{k+1} - 7.\ 3^k + 7.3^k - 3^{k+1}$ [adding and subtracting $7 .3^k$]

    $= 7^k.7 - 7.3^k + 7.3^k - 3^k.3$

    $= 7(7^k - 3^k) + 3^k(7 - 3)$

    $= 7(4d) + 3^k(4)$ [using Eq. (i)]

    $= 4(7d + 3^k)$, which is divisible by 4.

    Thus, $7^{k+1} - 3^{k+1}$ is divisible by 4.

    So, P(k + 1) is true whenever P(k) is true.

    Hence, by Principle of Mathematical Induction, the statement is true for every positive integer n ∈ N.

33. We have,

    Sinθ = $\frac{3}{5}$, tanϕ = $\frac{1}{2}$ and $\frac{\pi}{2}$ < θ < π < $\frac{3\pi}{2}$

    $\Rightarrow \theta$ lies in the second quadrant and θ lies in the third quadrant.

    Now, $\sin^2\theta + \cos^2\theta = 1$

    $\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

    $\Rightarrow \cos\theta = \pm\sqrt{1 - \sin^2\theta}$

In the 2^{π} quadrant cosθ is negative and tanθ is also negative

$\therefore$ cosθ = -$\sqrt{1-\sin^2\theta}$

= -$\sqrt{1-\left(\frac{3}{5}\right)^2}$

= - $\sqrt{1-\frac{9}{25}}$

= - $\sqrt{\frac{16}{25}}$

= - $\frac{4}{5}$

$\Rightarrow$ cosθ = - $\frac{4}{5}$

and, tanθ = $\frac{\sin\theta}{\cos\theta}$ = $\frac{\frac{3}{5}}{\frac{-4}{5}}$ = -$\frac{3}{4}$ ....(i)

Now, sec$^2\phi$ - tan$^2\phi$ = 1

$\Rightarrow$ sec$^2\phi$ = 1 + tan$^2\phi$

$\Rightarrow$ secϕ = $\pm\sqrt{1+\tan^2\phi}$

In the third quadrant, secϕ is negative

$\therefore$ secϕ = - $\sqrt{1+\left(\frac{1}{2}\right)^2}$

= -$\sqrt{1+\frac{1}{4}}$

= - $\sqrt{\frac{5}{4}}$

$\Rightarrow$ secϕ = - $\frac{\sqrt{5}}{2}$ .....(ii)

$\therefore$ 8 tanϕ - $\sqrt{5}$ secϕ

= 8 $\times\left(\frac{-3}{4}\right)-\sqrt{5}\times\left(-\frac{\sqrt{5}}{2}\right)$[by equations (i) and (ii)]

= - 2 $\times$ 3 + $\frac{5}{2}$

= - 6 + $\frac{5}{2}$

= $\frac{-12+5}{2}$

= $\frac{-7}{2}$

$\therefore$ 8 tanθ - $\sqrt{5}$ secϕ = - $\frac{7}{2}$

OR

$cos^4\ \frac{\pi}{8}\ + cos^4\ \frac{3\pi}{8} + cos^4\ \frac{5\pi}{8}\ + cos^4\ \frac{7\pi}{8}$

$= cos^4\ \frac{\pi}{8}\ + cos^4\ \frac{3\pi}{8}\ + cos^4\left(\frac{\pi}{2}+\frac{\pi}{8}\right) + cos^4\left(\frac{\pi}{2}+\frac{3\pi}{8}\right)$

$= cos^4\ \frac{\pi}{8}\ + cos^4\ \frac{3\pi}{8}\ + sin^4\ \frac{\pi}{8}\ + sin^4\ \frac{3\pi}{8}$ [$\because$ cos $\left(\frac{\pi}{2}+\theta\right)$ = - sin θ]

$= (\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8}) + (\cos^4 \frac{3\pi}{8} + \sin^4 \frac{3\pi}{8})$

$= (\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8} + 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} - 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8}) + (\cos^4 \frac{3\pi}{8} + \sin^4 \frac{3\pi}{8} + 2 \sin^2 \frac{3\pi}{8} \cos^2 \frac{3\pi}{8} - 2 \sin^2 \frac{3\pi}{8} \cos^2 \frac{3\pi}{8})$

$= (\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8})^2 - 2 \sin^2 \frac{\pi}{8} \cos^2 \frac{\pi}{8} + (\cos^2 \frac{3\pi}{8} + \sin^2 \frac{3\pi}{8})^2 - 2 \sin^2 \frac{3\pi}{8} \cos^2 \frac{3\pi}{8}$

[$\because a^4 + b^4 = (a^2 + b^2) - 2a^2b^2$]

$= 1 - \frac{1}{2} (2 \sin \frac{\pi}{8} \cos \frac{\pi}{8})^2 + 1 - \frac{1}{2} (2 \sin \frac{3\pi}{8} \cos \frac{3\pi}{8})^2$

[$\because sin^2\theta + cos^2\theta = 1$]

$= 2 - \frac{1}{2} (\sin 2 \times \frac{\pi}{8})^2 - \frac{1}{2} (\sin 2 \times \frac{3\pi}{8})^2$ [$\because \sin 2x = 2 \sin x \cos x$]

$= 2 - \frac{1}{2} \sin^2 \frac{\pi}{4} - \frac{1}{2} \sin^2 \frac{3\pi}{4}$

$= 2 - \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 - \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2$

[$\because \sin \frac{3\pi}{4} = \sin \left(\pi - \frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$]

$= 2 - \frac{1}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{1}{2}$

$= 2 - \frac{1}{4} - \frac{1}{4} = 2 - \frac{1}{2} = \frac{3}{2}$

34. Given: $S_n = 3 + 7 + 13 + 21 + 31 + \ldots\ldots + a_{n-1} + a_n$..........(i)

Also $S_n = 3 + 7 + 13 + 21 + 31 + \ldots\ldots + a_{n-2} + a_{n-1} + a_n$ ....(ii)

Subtracting eq. (i) from eq. (ii), 0 = 3 + (4 + 6 + 8 + 10 + ... up to (n - 1) terms) - a_n

$\Rightarrow a_n = 3 + \frac{n-1}{2}[2 \times 4 + (n-2) \times 2]$

$\Rightarrow a_n = 3 + \frac{n-1}{2}[8 + 2n - 4]$

$\Rightarrow a_n = 3 + (n - 1)(n + 2)$

$\Rightarrow an = 3 + n^2 + n - 2$

$\Rightarrow a_n = n^2 + n + 1$

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} (k^2 + k + 1)$$

$= (1^2 + 1 + 1) + (2^2 + 2 + 1) + (3^2 + 3 + 1) + \ldots\ldots + (n^2 + n + 1)$

= (1^2 + 2^2 + 3^2 + ... + n^2) + (1 + 2 + 3 + ... + n) + n

$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + n$

$= n\left[\frac{2n^2+3n+1+3n+3+6}{6}\right]$

$= n\left[\frac{2n^2+6n+10}{6}\right]$

$= \frac{n}{3}\left(n^2 + 3n + 5\right)$

35. Since, the vertices are on y-axis, so let the equation of the required hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2}$ = - 1 ....(i)

The coordinate of its vertices and foci are (0, $\pm$ b) nad (0, $\pm$ be) respectively.

$\therefore$ b = 7 [$\because$ vertices = (0, $\pm$ 7)]

$\Rightarrow$ b^2 = 49

and,

be = $\frac{28}{3}$ $\left[\because Foci = \left(0, \pm\frac{28}{3}\right)\right]$

$\Rightarrow$ 7 $\times$ e = $\frac{28}{3}$

$\Rightarrow$ e = $\frac{4}{3}$

$\Rightarrow$ e^2 = $\frac{16}{9}$

Now,

a^2 = b^2 (e^2 - 1)

$\Rightarrow$ a^2 = 49 $\left(\frac{16}{9} - 1\right)$

$\Rightarrow$ a^2 = 49 $\times$ $\frac{7}{9}$

$\Rightarrow$ a^2 = $\frac{343}{9}$

Putting a^2 = $\frac{343}{9}$ and b^2 = 49 in equation (i), we get

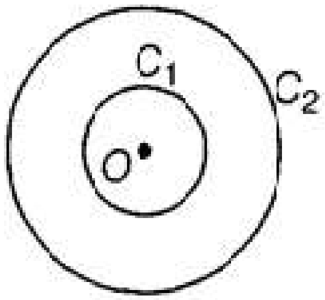
$\frac{\frac{x^2}{343}}{9} - \frac{y^2}{49}$ = -1

$\frac{9x^2}{343} - \frac{y^2}{49}$ = - 1

This is the equation of the required hyperbola.

OR

Here, given equation of circle (C_1) is $2x^2 + 2y^2 + 8x + 10y - 39 = 0$



$\Rightarrow x^2 + y^2 + 4x + 5y - \frac{39}{2} = 0$ [dividing both sides by 2]

On adding 4 and $\frac{25}{4}$ both sides to make perfect squares, we get

$(x^2 + 4x + 4) + (y^2 + 5y + \frac{25}{4}) = \frac{39}{2} + 4 + \frac{25}{4}$

$\Rightarrow (x + 2)^2 + (y + \frac{5}{2})^2 = \frac{78+16+25}{4}$

$\Rightarrow (x + 2)^2 + (y + \frac{5}{2})^2 = \frac{119}{4}$

On comparing with $(x - h)^2 + (y - k)^2 = r^2$, we get,

Centre is $(- 2, - \frac{5}{2})$

Area of required circle (C_2) = 16π

$\Rightarrow \pi r^2 = 16\pi \Rightarrow r^2 = 16$

Hence, equation of circle (C_2) having centre $(- 2, - \frac{5}{2})$ and radius 4 is,

$(x + 2)^2 + (y + \frac{5}{2})^2 = 16$

$\Rightarrow x^2 + 4x + 4 + y^2 + 5y + \frac{25}{4} = 16$

$\Rightarrow 4x^2 + 16x + 16 + 4y^2 + 20y + 25 = 64$

$\Rightarrow 4x^2 + 4y^2 + 16x + 20y - 23 = 0$

36. Here A = 55, h = 10

Calculation of Mean and Standard Deviation

Age	mid-values (x_i)	Number of persons (f_i)	$u_i = \frac{x_i - 55}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
20-30	25	3	-3	-9	9	27
30-	35	51	-2	-102	4	204

40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
45-50	45	122	-1	-122	1	122
50-60	55	141	0	0	0	0
60-70	65	130	1	130	1	130
70-80	75	51	2	102	4	204
80-90	85	2	3	6	9	18
		$N = \Sigma f_i = 500$		$\Sigma f_i u_i = 5$		$\Sigma f_i u_i^2 = 705$

Here, $N = \Sigma f_i = 500$, $\Sigma f_i u_i = 5$, $\Sigma f_i u_i^2 = 705$

$\therefore \overline{X} = A + h\left(\frac{1}{N}\Sigma f_i u_i\right) = 55 + 10\left(\frac{5}{500}\right) = 55.1$

and, $\sigma^2 = h^2\left\{\left(\frac{1}{N}\Sigma f_i u_i^2\right) - \left(\frac{1}{N}\Sigma f_i u_i\right)^2\right\}$

$\Rightarrow \sigma^2 = 100\left\{\frac{705}{500} - \left(\frac{5}{500}\right)^2\right\} = 100 \times 1.4099 = 140.99$

$\Rightarrow$ Standard Deviation, $\sigma = \sqrt{140.99} = 11.8739$