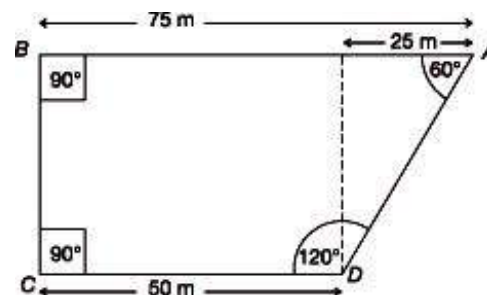


- Two circles touch externally. The sum of their areas is 130π sq. cm. and the distance between their centre is 14 cm. Find the radius of the circles.
- How many maximum circles of distance 2 cm can be cut from a rectangle sheet of $14 \text{ cm} \times 22\frac{4}{5} \text{ cm}$ dimensions.
- A calf is tied with a rope of length of length 6 m at the corner of a square grassy lawn of side 20m. If the length of the rope is increased by 5.5 m, find the increase in area of the grassy lawn in which the calf can graze.
- ABCD is a field in the shape of a



trapezium $\left| \begin{array}{l} D \\ C \end{array} \right.$ and $\angle ABC =$

90° , $\angle DAB =$

60° , $\angle BCD = 90^\circ$,

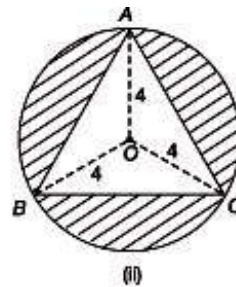
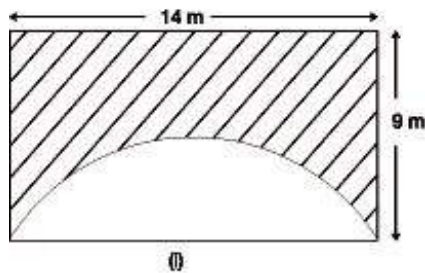
$\angle CDA = 120^\circ$ four sectors are formed with centres A, B, C and D.

The radius of each sector is 17.5 m.

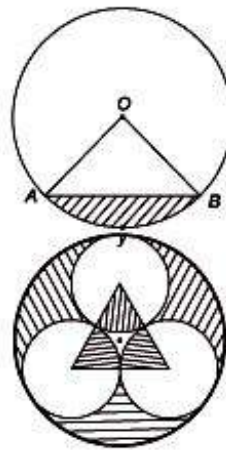
Find the (i) total area of the four sectors.

(ii) Area of remaining portion of the trapezium.

5. Find the area of the shaded region:



6. Two circles touch internally. The sum of their areas is $116 \pi \text{ cm}^2$ and distance between their centres is 6 cm. Find the radii of the circles.
7. A child prepare a poster on 'Save energy' on a square sheet whose each side measures 60 cm. At each corner of the sheet, she draws a quadrant of radius 17.5 cm in which she shows the ways to save energy. At the centre she draw a circle of diameter 21 cm and writes a slogan in it. Find the area of the remaining sheet.
- Write down the four ways by which the energy can be saved.
 - Write a slogan on 'Save Energy'.
 - Why do we need to save energy?
8. A bucket is raised from a well by means of a rope which is wound round a wheel of diameter 77 cm. If bucket ascends in 1 minute 28 seconds with a uniform speed of 1.1 m/sec. then calculate the number of complete revolutions the wheel makes in raising the bucket.
9. A survey was conducted in a particular area to find its most polluted region and it was found that the shaded region is the most polluted (in fig.) if radius of circular part that was surveyed is 14 m and the angle formed between the two radii is 60° . Find the area of polluted region.
- How is pollution Harmful?
 - What steps can be taken to reduce pollution in any region?
10. In figure three circles of radius 2 cm touch one another externally. These circles are circumscribed by a circle of radius R cm. find the value of R and the area of the shaded region.



Surface area and volume

Cube and cuboids : Solids like a book, a tile, a match box, an almirah, a room etc. are called cuboid. And like dice, ice cubes etc. are called cube.

For Cuboid length = l , breadth = b , height = h

$$\text{Volume} = l \times b \times h$$

$$\text{Lateral surface area} = 2h (l + b)$$

$$\begin{aligned} \text{Total surface area} &= 2 (lb + bh + hl) \\ &= \text{breadth} = \text{height} \end{aligned}$$

$$\text{In Cube length} = a$$

$$\text{Volume} = a^3$$

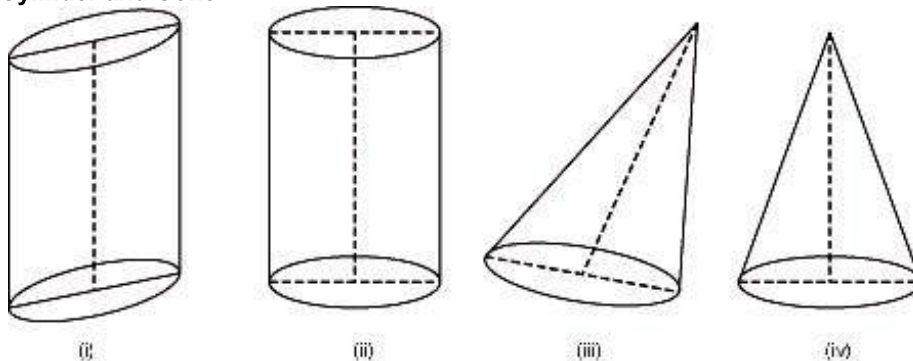
$$\text{Lateral surface area} = 4a^2$$

$$\text{Total surface area} = 6a^2$$

Cylinder: Solids like jars, circular pillars, circular pipes, gas jars, road rollers etc. are called cylinder.

Cone: Solids like conical tent, ice – cream cones, funnels etc. are called Cone.

Right circular Cylinder and Cone



In Fig (i) & (iii) the axis of cylinder and cone is not perpendicular to its base while in fig (ii) & (iv) the axis is perpendicular to the base. In fig (i) & (iii) cylinder and cone are not right circular.

In fig (ii) cylinder is right circular cylinder and in fig (iv) cone is right circular cone.

For right circular cylinder: base radius = r , height (length) = h Volume = $\pi r^2 h$ Curved surface area = $2\pi r h$



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$$\text{Total surface area} = 2\pi r (r + h)$$

For right circular hollow cylinder : external radius = R , internal radius = r , height = h

$$\text{Volume} = \pi(R^2 - r^2)h$$

$$\text{Curved surface area} = 2\pi(R + r)h$$

$$= 2\pi(R + r)h + 2\pi(R$$

$$\text{Total surface area} = 2\pi(R^2 - r^2)$$

For right circular Cone : base radius = r , height = h , slant height = l

$$l = \sqrt{r^2 + h^2}$$

$$l = \sqrt{r^2 + h^2}$$

$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$\text{Curved surface area} = \pi r l$$

$$\text{Total surface area} = \pi r (l + r)$$

ACTIVITY 1

Objective:

To verify the formula for the volume of a right circular cylinder in terms of its height h and radius of base (circle) is r .

Pre – requisite Knowledge:

Formula for volume of cuboid

Formula for circumference of a circle

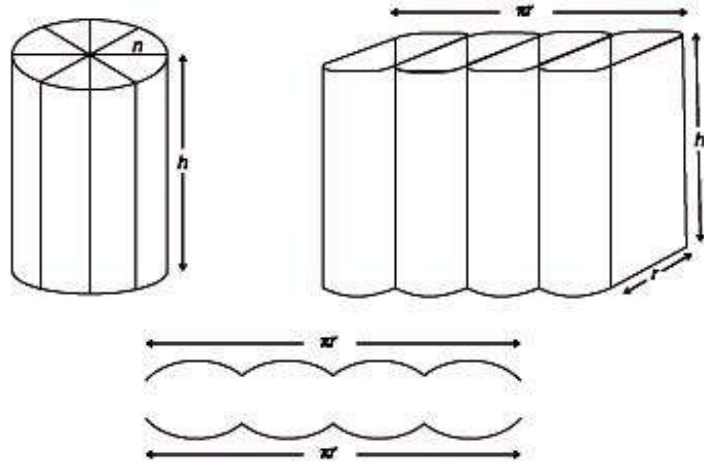
Material Required:

Plastic clay knife etc.

Procedure:

1. Make a cylinder of any dimensions using plastic clay. Let its height is h and radius of base circle is r .
2. Cut the cylinder into 8 equal sectorial section.
3. Place the segments alternatively to form a solid cuboid.





Observation

1. The circumference of the base of the cylinder which is circle is $2\pi r$.
2. The 8 segments approximately form a solid cuboid of height of h , breadth r and length πr .

Volume of 3. cuboid

$$\begin{aligned}
 &= \text{length} \times \text{breadth} \times \text{height} \\
 &= \pi r \times r \times h \\
 &= \pi r^2 h
 \end{aligned}$$

3. Cuboid is formed from the cylinder so volume of right circular cylinder $= \pi r^2 h$

ACTIVITY 2

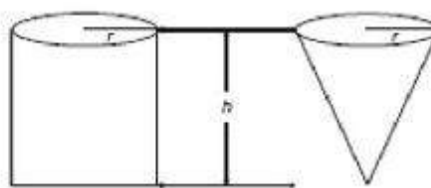
Objective:

To verify volume of cone is one third the volume of cylinder having same base radius and height.

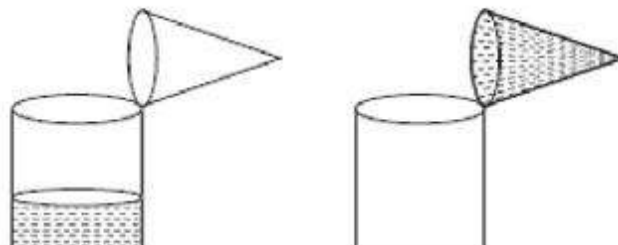
Material required:

Right circular cylinder and cone having same base radius (let r) and same height (h), clay (sand) Procedure :

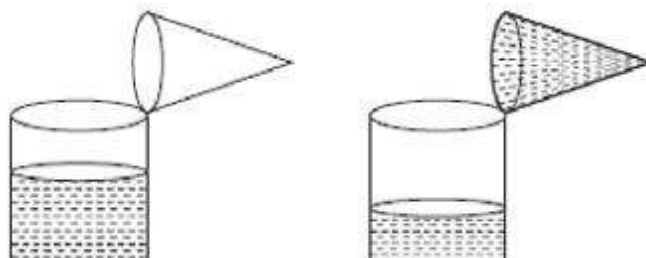
1. Take a hollow right circular cylinder and a hollow right circular cone of the same base radius and the same height.



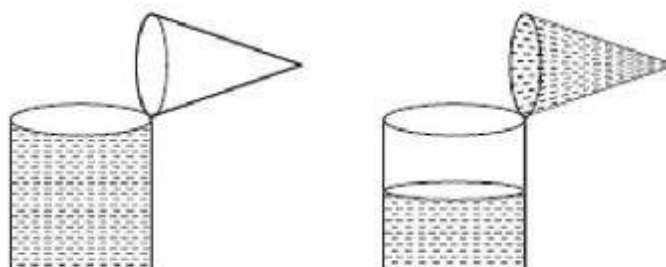
2. Fill the cone with sand fully and empty it into the cylinder



3. Again we fill up the cone with sand fully and empty it into the cylinder. We find the cylinder is still not full



4. Third time we fill the cone with sand fully and then empty it into the cylinder. We find the cylinder is fully packed with the sand.





We find that three times the volume of a cone, makes up the volume of cylinder. $3 \times \text{Volume of a cone} = \text{one}$

volume of right circular cylinder Volume of a cone = $\frac{1}{3}$ volume of a right circular cylinder

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

Sphere: Solids like cricket balls, football etc. are called spheres.

For Sphere :

Radius = r

$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$\text{Surface area} = 4\pi r^2$$

For hemisphere :

Radius = r

$$\text{Volume} = \frac{2}{3} \pi r^3$$

$$\text{Curved surface area} = 2\pi r^2$$

$$\text{Total surface area} = 3\pi r^2$$

ACTIVITY 3

Objective:

To give a suggestive demonstration of the formule for the volume of a sphere in terms of its radius.

Pre-requisite Knowledge:

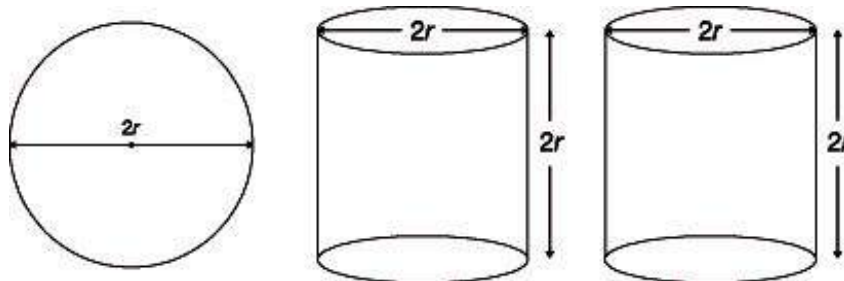
Volume of cylinder

Material Required:

A hollow sphere and two cylinders whose base diameter and height are equal to the diameter of the sphere, sand 186 Manual



Procedure:



1. Fill the hollow sphere with sand and empty into one of the cylinder.
2. Fill the hollow sphere second time with sand and empty it into the second cylinder.
3. Fill the hollow sphere third time with sand and empty it into remaining space of the two cylinders. Both the cylinders completely filled with sand.

Observation:

Total sand empties in three pouring fill both the cylinder completely.

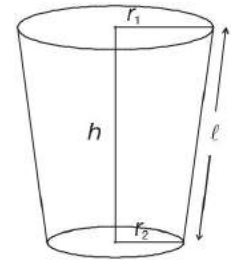
$$3 \text{ times the volume of sphere} = 2 \text{ times volume of cylinder} = 2 \times \pi r^2 h = 2 \times \pi r^2 \times 2r \text{ [here } h = 2r \text{]}$$

$$3 \times \text{volume of sphere} = 4\pi r^3$$

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

Frustum:

When a cone is cut by a plane parallel to the base of the cone then the portion between the plane and the base is called the frustum of the cone.



For a frustum of Cone :

Base radius = r_2

Top radius = r_1

Height = h Slant height = l

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

Volume

$$= \frac{\pi h}{3} (r_1^2 + r_1 r_2 + r_2^2)$$

$$\text{Lateral surface area} = \pi \ell (r_1 + r_2)$$

$$= \pi \ell (r_1 + r_2) + \pi (r_1^2 + r_2^2)$$

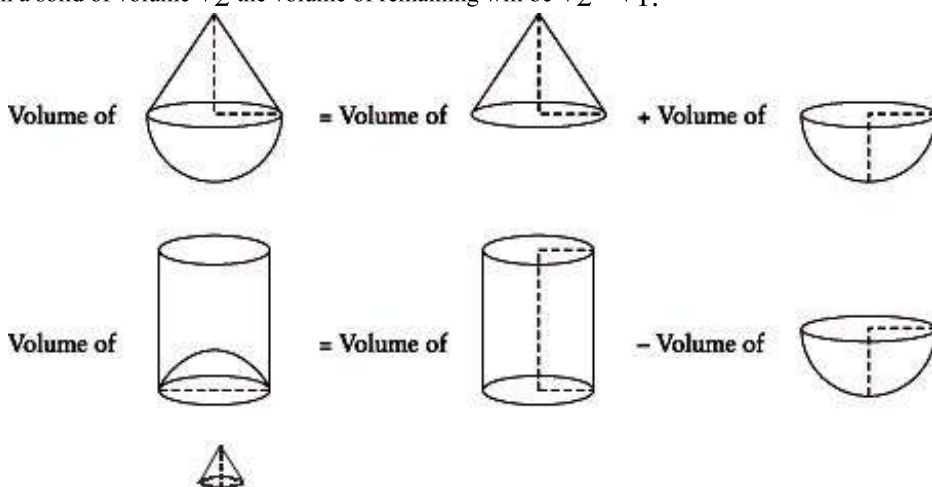
$$\text{Total surface area} = \pi \ell (r_1 + r_2) + \pi (r_1^2 + r_2^2)$$

Total surface area of bucket open from top

$$= \pi \ell (r_1 + r_2) + \pi r_2^2$$

Volume and surface area of combination of solids: There are many objects around us which are made up from combination of two or more solids. Sometimes one solid is taken out from other solid.

In calculating volume, if volumes of two solids are V_1 and V_2 and they are joined its volume will be $V_1 + V_2$. If a solid of volume V_1 is taken out from a solid of volume V_2 the volume of remaining will be $V_2 - V_1$.



If we consider a toy in which a cylinder is mounted on a hemisphere and is surmounted by



cone such that their flat surfaces are of same radii as shown in fig.

We wish to find the total surface area of this toy.

Total surface area of toy = Curved surface area of cone + Curved surface area of cylinder + Curved surface area of hemisphere.

For calculating total surface area of combined solid we calculate the surface area of portion which is visible.

In some case surface area is more/ less / equal than the sum of surface areas of solids from which it is made.

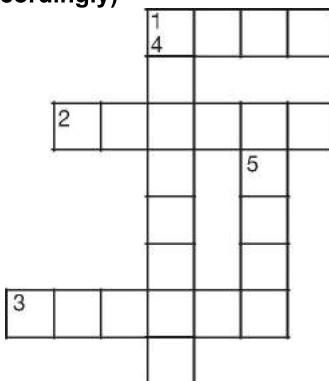
Teacher can give the following project in the classs (forming a gp of 5 – 6 students)

1. Collect any five objects from their surrounding whose shape is cuboid and five object whose shape is cube and then find their curved surface area, total surface area and volume.
2. Collect any five objects from their surrounding whose shape is in the form of right circular cylinder and five objects whose shape is in the form of right circular cone and then find their curved surface area. Total surface area and volume.
3. Collect any three objects each from their surrounding whose shape is in the form of sphere, hemisphere and frustum of a right circular cone and then find their curved surface area, total surface area and volume.
4. Make a net of cuboid, whose dimensions are 3, 4, 5 cm. Make a net of cube whose side is 5 cm and make a net of cylinder whose radius of base is 4 cm and its height is 7 cm.
5. Collect any four objects from their surrounding which are in the shape of combination of two or more solids. Find their curved surface area, total surface area and volume.
6. Form any solid object (like cube, cuboid, sphere etc.) from the clay find its total surface are and volume. From the same clay form other solid object and find its total surface area and volume and check both have same surface area and volume or different.



WORKSHEET 1

FORMATIVE



Cross word Puzzle (Fill in the blanks accordingly)



**Across**

1.  is called a right circular
2. If ℓ , b , h denote the sides of a cuboid then $\ell \times b \times h$ is called its
3.  is called hemi-

Down

4.  is called a right circular
5.  is called a (here $\ell = b = h$)

WORKSHEET 2

FORMATIVE

- A solid sphere of radius r is melted and cast into the shape of a solid cone of height r , then radius of the base of this cone will be
(i) r (ii) $2r$ (iii) $r/2$ (iv) $3r$
- If h , c , and v respectively are the height, the curved surface and volume of cone then the value of $3\pi v h^3 - c^2 h^2 + 9v^2$ is
(i) 2.0 (ii) 1.5 (iii) 1.0 (iv) 0
- A cylinder, a cone and a hemisphere have same radius of base and same height then ratio in their volume is
(i) 1 : 2 : 3 (ii) 3 : 1 : 2 (iii) 2 : 1 : 3 (iv) $2/3$: $1/3$: 1
- Radius of circular ends of a frustum of a cone are 9 cm and 4 cm and its height is 12 cm. Its slant height will be
(i) 13 cm (ii) 14 cm (iii) 15 cm (iv) 16 cm.
- Radius of a sphere is increased by 10 % then its volume is increased by
(i) 10 % (ii) 1000 % (iii) 21 % (iv) 33.1 %
- Ratio of volume of two spheres is 8 : 27. If their radius is r and R then $r : (R - r)$ is
(i) 9 : 4 (ii) 3 : 2 (iii) 3 : 1 (iv) 2 : 1

7. Three cubes whose edges are 6 cm, 8 cm, and 10 cm respectively are melted without any loss of metal into a single cube. The edge of resulting cube is
(i) 8 cm (b) 10 cm (c) 12 cm (d) 13 cm
8. Each edge of a cube is increased by 50%. Then the percentage increase in its surface area is
(i) 125 % (b) 150 % (c) 175 % (d) 180 %
9. Two cones A and B have their base radii in the ratio of 4 : 3 and their heights in the ration 3 : 4. The ratio of volume of cone A to cone B is
(i) 4 : 3 (b) 3 : 4 (c) 2 : 3 (d) 1 : 2
10. How many small spheres of radius 2 cm. can be formed by melting a ball of iron of radius 4 cm
?
(i) 4 (b) 16 (c) 8 (d) 12

WORKSHEET 3

FORMATIVE

Fill in the blanks:

1. The total surface area of a cuboid of dimension $2a \times a \times b$ is
2. If the radius of a sphere is halved, its volume becomes.....times the volume of original sphere.
3. The total surface are of a hemisphere whose diameter a is
4. The radius of base of a cylinder is 7 cm and its volume is 770 cm^3 . The height of the cylinder is.....
5. If the height of cone is equal to the diameter of its base, the volume of cone is.....
6. The radii of the circular ends of a solid frustum of a cone are 33 cm and 27 cm and it slant height is 100 cm. Then its curved surface area is.....
7. The height of a circular cone is 12 cm and the radius of its base is 4.5 cm. Then the slant height of cone is.....
8. Volume of a spherical shell whose external and internal radii are r_1 and r_2 will be
9. A Surahi is the combination of andshape.
10. A cuboid has a volume of 64000 cm^3 . If the ratio of its sides are 1 : 2 : 4, then the larger side is

WORKSHEET 4

FORMATIVE

Complete the following table:

Shape	Dimensions	C.S.A.	T.S.A.	Volume
Cube	Side = a	$4a^2$		
Cuboid	Length = ℓ Breadth = b Height = h			ℓbh
Right circular cylinder	Radius of Base = r Height = h		$2\pi r(r + h)$	
Right circular cone	Radius of Base = r Height = h Slant height = ℓ	$\pi r\ell$		
Sphere	Radius = r	$4\pi r^2$		
Hemisphere (solid)	Radius = r			$\frac{2}{3}\pi r^3$
Frustum of a cone	Radius of two circular ends = r_1 and r_2 Height = h Slant height = ℓ	$\pi\ell(r_1 + r_2)$		

WORKSHEET 5

SUMMATIVE

Short answer questions:

1. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 cm. Then find the length of wire.
2. The rain water from a roof $22\text{ m} \times 20\text{ m}$ drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the vessel is just full. Find the rain fall in cm.

3. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 Km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?
4. A cuboidal metal plate of 1 cm thickness, 9 cm breadth and 81 cm length is melted into a cube. Find the total surface area of the cube.
5. The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.
6. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.
7. A cylindrical boiler 2 m high is 3.5 m in diameter. It has a hemispherical lid. Find the interior volume of the boiler.
8. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter, hemisphere can have? Find the surface area of the solid.
9. From a solid cylinder of height 7 cm and base diameter 12 cm, a conical cavity of same height and same base diameter is

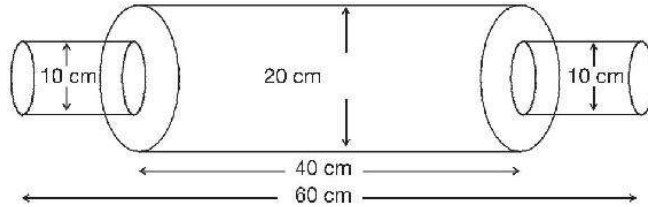
hollowed out. Find the total surface area of the remaining solid.

10. Three cubes of a metal whose edges in the ratio 3 : 4 : 5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

✓

Long Questions:

1. An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height be 22 cm, diameter of the cylindrical portion be 8 cm and the diameter of the top of the funnel be 18 cm, find the area of the tin required to make the funnel.
2. The storage oil tanker consists of a cylindrical portion 7 m in diameter with two hemispherical ends of the same diameter. The oil tanker is lying horizontally. If the total length of the tanker is 20 m, then find the capacity and total surface area of the container.
3. A model is made by joining 3 cylindrical pieces of wood as shown in figure. Find the cost of painting it at the rate of 50 paise per cm^2 .



4. A bucket is in the form of a frustum of a cone whose radii of bottom and top are 7 cm and 28 cm respectively. If the capacity of the bucket is 21560 cm^3 . Find the whole surface area of the bucket?



5. A circus tent is made of canvas and is in the form of a right circular cylinder and a right circular cone above it. The diameter and height of the cylindrical part of the tent are 126 m and 5 m respectively. The total height of the tent is 21 m. Find the total cost of the tent if the canvas used costs ₹ 12 per m^2 .

Value based/ Multidisciplinary Questions:

1. A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of bucket, if the cost of metal sheet used is ₹ 15 per 100 cm^2 . After some time a rust is shown on the metal sheet. From which metal the bucket is made and how can we prevent it from rusting.

2. A manufacturer involved ten children in colouring playing top (lattu) which is shaped like a cone surmounted by a hemisphere. The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area they had to paint if 50 playing tops were given to them.

(a) How is child labour an abuse for the society?

(b) What steps can be taken to abolish child labour?

3. A night camp was organised for class X students for two days and their accommodation was planned in tents. Each tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height

of the top is 2.8 m find the area of the canvas used for making the tent. Also find the cost of the canvas of the tent at the rate of

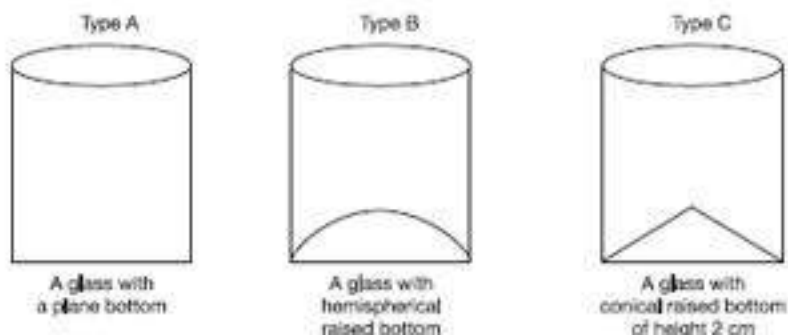
₹ 500 per m^2 . Is camping helpful to students in their development? Justify your answer.

4. Five containers shaped like a right circular cylinder having diameter 12 cm and height 15 cm are full of ice-cream. This ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top and is to be distributed to the children in an orphanage. Find the number of such cones which can be filled with ice-cream. Which value are being reflected by such an action?

5. Sarita donates some part of his monthly income on differently abled children. In a particular month she wishes to donate toys for the children. Each toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy also find the cost of 50 such toys if the cost of material used in the toy is

₹ 5 per 100 cm^2 and the cost of making is ₹ 10 per toy use $\frac{22}{7}$. What value of Sarita is reflected here? Justify your answer? Is anything should be kept in mind for making toys for the age group of 0 to 3 years children?

6. In fig, Neeraj a juice seller has set up his juice shop where he has three types of glasses of inner diameter 5 cm to serve the customers. The height of the glasses is 12 cm. (Use $\pi = 3.14$)



He decided to serve the customer in 'A' type of glass

- (i) Find the volume of glass of type A
 - (ii) Which glass has the minimum capacity?
 - (iii) Which mathematical concept is used in above problem?
 - (iv) By choosing a glass of type A which value is depicted by Juice seller Neeraj?
7. A candle is cylindrical at one end and a perfect conical at the other end. The diameter of the candle is 3 cm and the overall height is 8 cm. If the slant height of the conical portion is 2.5 cm, find the volume of the wax of the candle. If the mass of the wax is 1.75 gm/cm^3 . Find the mass of the candle. Name any two sources of light used at home other than candle.
8. Haneef is processing the sugarcane juice to make the molasses, which is poured into moulds in the shape of a frustum of a cone having the diameters of its two circular faces as 30 cm and 35 cm and the vertical height of the mould is 14 cm. If each cm^3 of molasses has mass about 1.2 gm find the mass of the molasses that can be poured into each mould. Name the disease in which doctor suggest not to take sugarcane juice.
9. A lead pencil consists of a cylinder of wood with solid cylinder of lead filled into it. The diameter of the pencil is 7 mm; the diameter of the lead is 1 mm and the length of the pencil is 10 cm. Calculate the weight of the whole pencil if the specific gravity of the wood is 0.7 gm/cm^3 and that of the lead is 2.1 gm/cm^3 . Name the substance from which lead of the pencil is made.
10. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 Km/h . How much area will it irrigate in 30 minutes if 8 cm of standing water is needed. Name two more sources of irrigation used by the farmers in India.



MISCONCEPTIONS (COMMON ERRORS COMMITTED BY STUDENTS)

1. Computational errors are very common.
2. In writing units of surface area and volume.
3. In problems of combination of solids, when problem is solved without drawing the figure, total surface area of solid is taken as sum of areas of individual solids.
4. By taking out a solid from another solid, the surface area of the remaining part is taken as difference of the surface areas of the two.
5. When one solid is melted to form other solids, most of students find their volumes independently, it takes more time and often committed error, In it two volumes in the form of formula should be equated to minimise errors.
6. Diameter is taken as radius.
7. Student mix up formulae.

Reminders

Lot of emphasis need to be given to careful reading, analysis and interpretation of the statement of questions.

Historical Background of Heron's Formula

Heron was born in about 10 AD possibly in Alexandria in Egypt. He worked in applied mathematics. His works on mathematical and physical subjects are so numerous and varied that he is considered to be an encyclopedic writer in these fields. His geometrical works deal largely with problems on mensuration written in three books. Book I deals with the area of square, rectangles, triangles, trapezoids (trapezia), various other specialised quadrilaterals, the regular polygons, circles, surfaces of cylinder, cones, sphere etc. In this book, Heron has derived the famous formula for the area of triangle in terms of its three sides.

The formula gives by Heron about the area of triangle is also known as Heron's Formula.

It is stated as



$$\text{Area of triangle} = \sqrt{S(S-a)(S-b)(S-c)}$$

where a , b , c are the sides of the triangle and S = semi perimeter of the

$$\text{triangle} = \frac{a+b+c}{2}$$

2



Heron

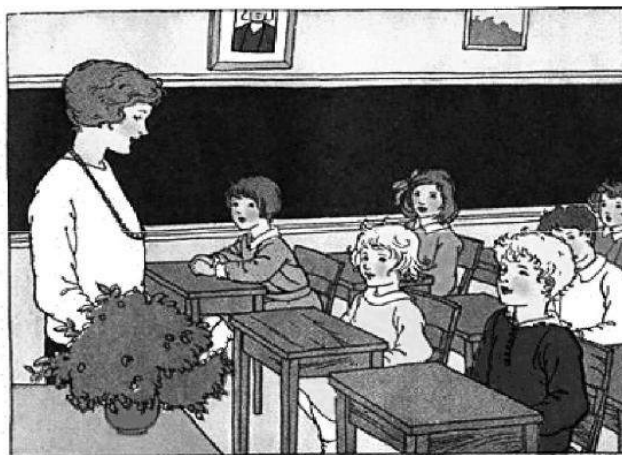
6

COORDINATE GEOMETRY

INTRODUCTION

How do you locate the position of your seat in a cinema hall? How do you locate your seat in a train?

Imagine a standard classroom situation. Students are sitting in rows and columns with reference to the teacher. Ask the students to speak about their location in terms of row number and column numbers. You may ask a student sitting at a particular location to stand up, wave your hand, clap three times etc. This would help them to create an insight of importance of their place as unique location in the classroom as well as gear up for learning the concept of coordinate geometry.



Visit the website <http://funbasedlearning.com/algebra/graphing/points/default.htm> Play the game Graph mole.



Coordinate Geometry

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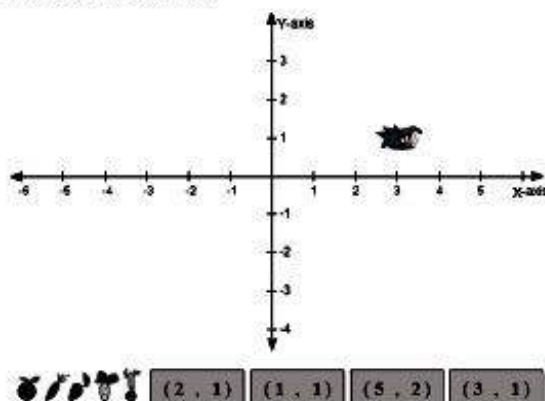
This is the easy version of Graph Mole, a fun little Algebra game that teaches you how to plot points on a Cartesian coordinate plane.

If you have never graphed Cartesian coordinate before, click on the “introduction” button above to see a funny interactive story that teaches you how to do it.

If you already know how to graph Cartesian coordinates, click on the “Play Game” button above to play the easy version of the Graph Mole game.

Once you’re mastered the easy version, try the *medium version of Graph Mole* or the *hard version of Graph Mole*. Can you bank the mole before he gets all the vegetables?

Medium Version of Graph Mole



KEY CONCEPTS

1. To learn that the points on a coordinate grid, i.e Cartesian plane are identified by ordered pair (x, y)
2. To learn to plot points on the coordinate grid/cartesian plane.
3. To learn to find distance between two points using distance formula.
4. To learn to find the formula for dividing a line segment internally in the given ratio.
5. To learn to find area of a triangle using vertices.

Mathematical Terms (Vocabulary)

- | | |
|---------------------|---------------|
| 1. Coordinate Grid. | Cartesian |
| 2. System. | 2. System. |
| 3. x-axis | 4. y-axis |
| 5. Abscissa | 6. Ordinate |
| 7. Vertical | 8. Horizontal |
| 9. Ordered pair | 10. Quadrant |

- | | |
|----------------------|---------------------------|
| 11. Origin | 12. Pythagoras Theorem |
| 13. Plotting points | 14. Point of intersection |
| 15. Triangle | 16. Area of Triangle |
| 17. Distance Formula | 18. Midpoint |
| 19. Section Formula | 20. Trisection |
| 21. Median | 22. Centroid |
| 23. Equidistant | 24. Circum centre |
| 25. Circum radius | |

ACTIVITY 1

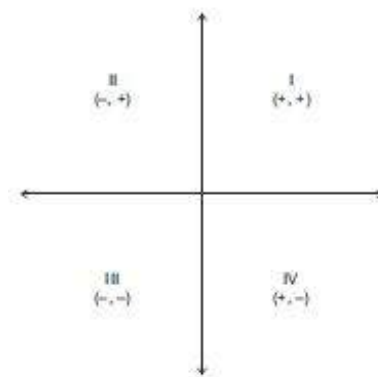
Students will explore situations in which to find a point we are required to describe its position with reference to more than one line.

Students will be introduced to Cartesian plane.

They will learn new terms x -axis, y -axis, ordered pair, abscissa and ordinate. They will learn the concept that a point is represented as an ordered pair (x, y) in a plane.

They will observe that every point in a plane has a x value and a y value where x is the distance of point from the y -axis and y is the distance of point from the x -axis.

Introducing Quadrants



The coordinate plane is divided into four quadrants. Each quadrant is a specific region in the coordinate plane. The region in which $x > 0$ and $y > 0$ is Quadrant I. The region in which $x < 0$ and $y > 0$ is quadrant II. The region in which $x < 0$ and $y < 0$ is quadrant III. The region in which $x > 0$ and $y < 0$ is quadrant IV.

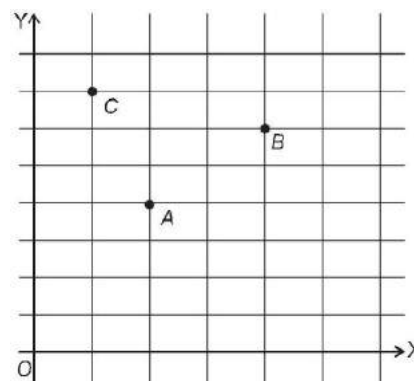
For example, the point $(2, -3)$ lies in quadrant IV with an x -coordinate 2 units to the right of the y -axis and a y coordinate 3 units below the axis. This is how the coordinates of a point specify its exact location. The coordinates of the origin by definition $(0, 0)$.

http://hotmath.com/hotmath_help/games/ctf/ctf-hotmath.scof.

<http://www.athena.bham.org.uk/old/coordinates.htm>.

http://hotmath.com/hotmath_help/games/ctf/ctf_hotmath.scof.

Coordinate system on the right to help answer the following questions:



(a) Give the coordinates for point B .

(b) How far is B from the vertical or y -axis?

(c) How far is B from the horizontal or x -axis?

(d) If the x -coordinate of a point is defined as the distance from the vertical or y -axis define y -coordinate (e) What is the value of the y -coordinate for point C ?

Scale: x -axis 1 unit = 1 cm

y -axis 1 unit = 1 cm

WORKSHEET 1

(location of a point, distance from x -axis and y -axis) Q1. Mark the location of these points in the coordinate plane. What do you say about their location?

S. No.	Ordered Pair	Abscissa	Ordinate	Position Quadrant/axis	Distance of point from the x -axis	Distance of point from the y -axis
1	(1, 2)					
2	(-2, 1)					
3	(4, 3)					
4	(-4, 2)					
5	(2, -6)					
6	(-1, 0)					
7	(0, 3)					
8	(0, -2)					
9	(4, 0)					
10	(5, 3)					

11	(-2, 5)					
12	(-4, -3)					
13	(-2, -2)					
14						
15	(2, -5)					
16	(1, 3)					
17	(6, 0)					
18	(-5, 0)					
19	(0, 1)					
20	(0, -4)					

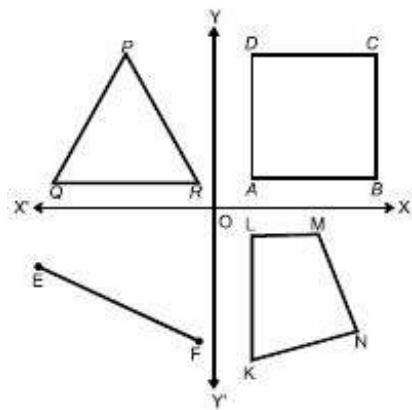
Q 2. Plot them in coordinate Grid (Graph Paper).

Enrichment Activity 1: Find my new location Plot a square $ABCD$ in I quadrant.

Plot any triangle PQR in II quadrant.

Plot any line segment EF in III quadrant.

Plot any quadrilateral $KLMN$ in IV quadrant.

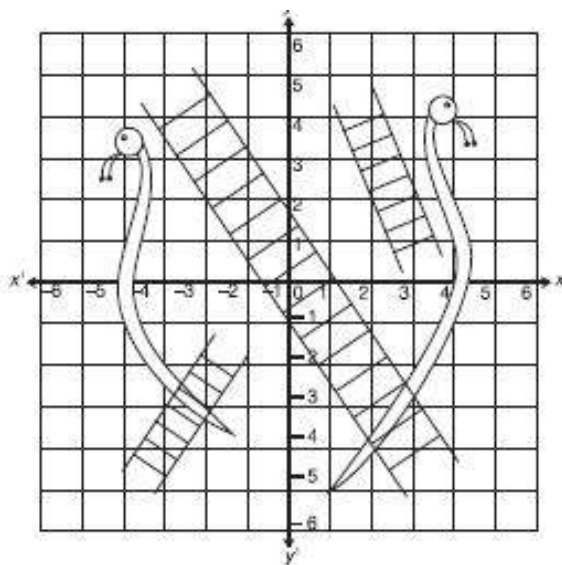


- (i) Find the coordinates of all the points.
 - (ii) Consider all the points and move them 2 steps towards their right. Record the new coordinates write your observation.
- If (x, y) is a general point and moved to 2 steps towards the right. What will be the new coordinates.
- (iii) Explore the other cases when the given points are moved
 - (a) Two steps towards their left.
 - (b) two steps vertically upwards.
 - (c) two steps vertically downwards.

ACTIVITY 2

To reinforce the plotting of points in two dimensional Cartesian System.

The task may be performed in the classroom. Students can be given the photo copies of the game sheet and are asked to work in pairs and play the game.



No. of players 2.

Material required

Coordinate Snake ladder Game sheet.

2 Red and 2 Blue dice (numbers on one dice of each colour as 1 to 6, numbers on another dice of each colour as -1 to -6).

Board piece for each player to locate his/her position.

Game direction

To start with, first player has to choose a red dice (randomly out of two red dice) and a blue dice (randomly) out of two blue dice). Red is for x-axis and blue is for y-axis.

These the player has to throw the two dice simultaneously, locate the coordinates on the game sheet (suppose he gets -2 on red and 3 on blue dice, then this will correspond to $(-2, 3)$ on game sheet and put his/her board piece at that position.

The second player will move in the same way.

Snakes and ladder will act as in normal snake ladder game, snake will bring the position down and ladder will give a lift. The game ends when one of the player reaches (-6, 6) first.

ACTIVITY 3

Distance formula

How to find the distance between two points.

Take two points on the same horizontal line and find the distance between them. Take two points on the same vertical line and find the distance between them.

Observation

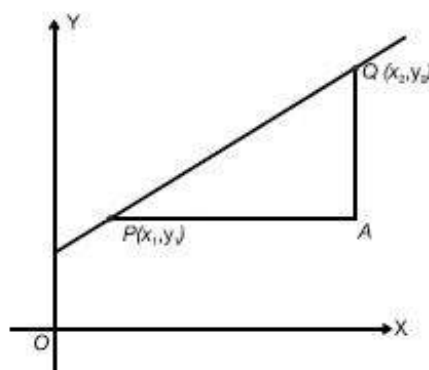
Along a single number line, finding the distance between two points is a matter of computing the difference between the points. It is usually defined as the absolute value of the difference between two points.

Now take any two points in the coordinate plane. How will you find the distance between them.

[Hint: Pythagoras Theorem]

Note: The distance between two points is identical with the length of hypotenuse of a right triangle.

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



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WORKSHEET 2

1. Q The distance between the points (2, 0) and (-2, 0) is _____.
2. Q The distance between the points (a, b) and (-a, -b) is _____.
3. Q The distance of P(2, 7) from origin is _____.
4. Q The value of a for which the distance between (a, 3) and (4, 2) is 37 is _____.

Q 5. Show that the point (3, -4) and (8, 1) are equidistant from the point (7, -3)

ACTIVITY 4

Discussion Points

1. When do three points A(x₁, y₁), B(x₂, y₂) and C(x₃, y₃) will form (a) a triangle (b) a right triangle, (c) an isosceles triangle (d) an equilateral triangle (e) an isosceles right angled triangle.
2. When do four points A(x₁, y₁), B(x₂, y₂), C(x₃, y₃) and D(x₄, y₄) will form a

(a) parallelogram
(d) square

(b) rhombus
(e) kite

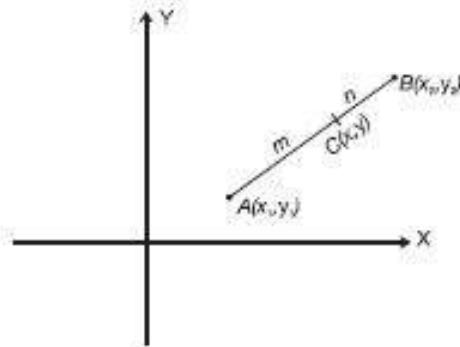
(c)
rectangle

ACTIVITY 5

Section Formula – Discussion points

1. To find the co-ordinates of a point dividing the line segment $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ internally.

$$C(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$



2. To find the coordinates of midpoint of a line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$.

3. To find the coordinates of centroid of triangle $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Note: (Recall definition of median and property of centroid. (Centroid divides every median in the ratio 2: 1.

4. To find the ratio in which a point $C(x, y)$ divides a line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$.

To

5. find the ratio in which the x-axis divides a line segment joining $A(x_1, y_1)$, $B(x_2, y_2)$.

6. To find the ratio in which the y-axis divide a line segment joining $A(x_1, y_1)$, $B(x_2, y_2)$.

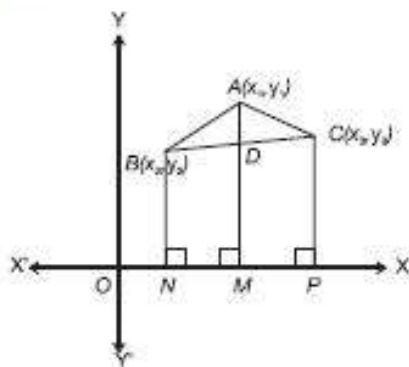
To

7. find the ratio in which the line axis divides a line segment joining $A(x_1, y_1)$, $B(x_2, y_2)$.

ACTIVITY 6

Area of a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Consider the following diagram.



In terms of (x_1, y_1) , (x_2, y_2) and (x_3, y_3) .

AM, BN and CP are perpendicular to _____.

AM, BN and CP are _____ to each other.

$\text{ar}(\triangle ABC) = \text{area trap}(AMNB) + \text{area trap}(AMPC) - \text{area trap}(BNPC)$

WORKSHEET 3

Recapitulation Worksheet

- Distance between two points $(3, 5)$ and $(-7, 10)$ is _____.
- Which point is nearer to origin $P(2, 7)$ or $Q(-2, 5)$?
- The coordinates of a point on the x-axis are of the form _____.
- The coordinates of a point on the y-axis are of the form _____.
- Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ will represent vertices of an equilateral triangle if _____.
- Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ will represent vertices of an isosceles triangle if _____.
- Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ will represent vertices of a right triangle if _____.

8. Four points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ will represent vertices of a square if

_____.

All sides are equal

All sides are equal and diagonals are equal

Diagonals are equal and bisect each other at right angles. Any two consecutive sides are equal.

9. Four points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ will represent vertices of a rectangle if _____.

10. Four points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ will represent vertices of a parallelogram if _____.

$AB = BC$

$AB = CD$ and $BC = DA$ $AB = BC = CD = DA$

$AB = CD$ and $BC = DA$ and $AC = BD$.

11. Four points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ will represent vertices of a rhombus if _____.

$AB = BC = CD = DA$.

$AB = BC = CD = DA$ and $AC = BD$ $AC = BD$

$AB = BC$

12. Do the points (3, 2), (3, 0), and (3, -5) form a triangle, Yes

No

13. The area of triangle formed by $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by _____.

14. When three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ collinear, then area of triangle formed by them is _____.

15. The coordinates of centroid of triangle formed by $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by _____.

WORKSHEET 4

Multiple Choice Question

- P is a point on x -axis at a distance of 5 units from y -axis to its right. The coordinate of P are
 (a) (5, 0) (b) (0, 5) (c) (5, 5) (d) (-5, 5)
- The distance of point P (4, -3) from the origin is
 (a) 1 unit (b) 7 units (c) 5 units (d) 3 units
- If P (-1, 1) is the midpoint of the line segment joining A (-3, q) and B (1, q + 4), then the value of q is
 (a) 1 (b) -1 (c) 2 (d) 0
- If centroid formed by A (-1, y), B (5, 2) and C (x, 4) is G (0, 3). Then value of (x, y) is
 (a) (-4, 3) (b) (4, 15) (c) (-4, -15) (d) none of these
- If the distance between the points (4, P) and (1, 0) is 5 then P =
 (a) ± 4 (b) 4 (c) -4 (d) 0
- The points A (0, -2), B (3, 1), C (0, 4) and D (-3, 1) are the vertices of a
 (a) rhombus (b) rectangle (c) square (d) parallelogram
- The area of triangle OAB, the coordinates of the points A (2, 0), B (0, -14) and O origin, is
 (a) 11 sq. units (b) 18 sq. units (c) 28 sq. units (d) 14 sq. units
- The distance between the line $2x + 4 = 0$ and $x - 5 = 0$ is
 (a) 9 units (b) 1 unit (c) 5 units (d) 7 units

9. AOBC is a rectangle whose three vertices are A (0, 3), O (0, 0), B (5, 0). The length of its diagonal is:
 (a) 5 units (b) 3 units (c) 34 units (d) 4 units
10. If PQ is the diameter of the circle whose centre is (2, -3) and Q is (1, 4), then the coordinates of P are
 (a) (10, -3) (b) (3, 10) (c) (-3, 10) (d) (3, -10)
11. The distance between the points $(\sqrt{2}, 2)$ and $(-\sqrt{2}, 2)$ is
 (a) $\sqrt{2}$ units (b) 2 units (c) 6 units (d) $3\sqrt{2}$ units
12. The perimeter of triangle formed by the points (0, 0), (3, 0) and (0, 3) is
 (a) 9 units (b) 6 units (c) $9 + \sqrt{2}$ units (d) $6 + 3\sqrt{2}$ units
13. The points on the y-axis which are at a distance of 10 units from the point (8, 8) are
 (a) (0, -2) and (0, -14) (b) (0, 2) and (0, 14)
 (c) (0, 2) and (0, -14) (d) (0, 14) and (0, -2)
14. The midpoints of the line segment PQ is (2, -4). Then the coordinates of P and Q can be
 (a) (0, 4) and (0, -8) (b) (8, 0) and (-4, -8)
 (c) (8, 0) and (0, -4) (d) (4, 0) and (-4, 0)
15. If the centroid of the triangle formed by the points (p, q) (q, r) and (r, p) is the origin, then $p^3 + q^3 + r^3$ is equal to
 (a) pqr (b) 0 (c) $p + q + r$ (d) $3 pqr$

WORKSHEET 5

- Q 1. Do the points (0, 8), (3, 0) and (-3, 0) form a triangle? If so, name the type of triangle formed.
- Q 2. Check whether the points (-1, 0), (4, 0), (4, 5) and (-1, 5) will form a square or not? Justify your answer.
- Q 3. Find the relation between x and y such that the point (x, y) is equidistant from the points (4, 6) and (3, 8) Q 4. Find a point on the y-axis which is equidistant from the points A (3, 2) and B (1, 7) Q 5. Check whether the points (0, 3), (3, 1) and (4.5, 0) are collinear or not?
- Q 6. Find the point on the x-axis which is equidistant from the points (2, 3) and (1, 5).
- Q 7. Name the type of quadrilateral formed by joining the points (1, 2), (6, 2), (8, 5) and (3, 5) **208** Manual for Effective Learning
 In Mathematics In Secondary Level
- Q 8. Find the values of y for which the distance between the points A (2, -3) and B (10, y) is 10 units.
- Q 9. Check whether the points (-3, 5), (-3, 2) and (0, 2) are the vertices of an isosceles triangle.
- Q 10. Find the perimeter of a triangle whose vertices are (0, 4), (0, 0) and (3, 0)
- Q Find
11. the ratio in which the line segment joining (6, 4) and (1, 7) is divided by the x-axis.
- Q Find
12. the ratio in which the line segment joining (6, 4) and (1, 7) is divided by the y-axis.
- Q Find
13. the ratio in which the line segment joining (6, 4), (1, -7) is divided by the line $x + y = 2$.
- Q 14. The vertices of a triangle are (a, b-c), (b, c-a) and (c, a-b). Prove that the centroid lies on the x-axis.
- Q 15. If the points (a, 0), (0, b) and (1, 1) are collinear, show that $\frac{1}{a} + \frac{1}{b} = 1$
- Q 16. The coordinates of the midpoint of the line joining the points (3p, 4) and (-2, 2q) is (5, p). Find p and q.
- Q 17. Three vertices of a parallelogram are (1, 2), (1, 0) and (4, 0). Find the fourth vertex.
- Q 18. Find the lengths of medians of triangle whose vertices are (1, -1), (0, 4) and (-5, 3).
- Q Find the ratio in which the points (11, 15) divides the segment joining the points (15, 5) and (9, 20)
- Q Find the ratio in which the point P (m, 6) divides the segment joining the points (-4, 3)

and (2, 8). Also find the value of m.

THOUGHT PROVOKING QUESTION

HOTS

- Q If the point P(x, y) is equidistant from the points A(5, 1) and P(-1, 5). Prove that

$$3x = 2y.$$

- Q Find the distance between the points $(a \cos \theta + b \sin \theta, 0)$ and $(0, a \sin \theta - b \cos \theta)$.

$$[Ans. \sqrt{a^2 + b^2}]$$

- Q Find the coordinates of centre of the circle passing through the points (0, 0), (-2, 1) and (-3, 2). Also find its radius.

$$[Ans. (3/2, 11/2), \frac{1}{2} \sqrt{130} \text{ units}]$$



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- Q 4. The line segment P(6, 3) to Q(-1, -4) is doubled in length by having half its length added to each end. Find the coordinates of new ends.

$$[Ans. (19/2, 13/2), (-9/2, -15/2)]$$

- Q 5. The opposite angular points of a square are (2, 0) and (5, 1). Find the remaining vertices.

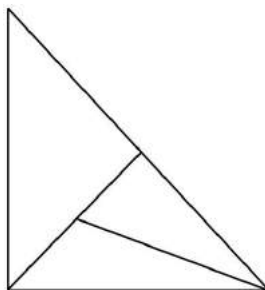
$$[Ans. (4, 1) \text{ and } (3, 2)]$$

- Q 6. Two points A(3, 4) and B(5, -2) are given. Find the point P such that PA = PB and area of ABP = 20 sq. units.

$$[Ans. (10, 3) \text{ or } (-2, -1)]$$

- Q 7. In Fig. P divides AC in the ratio 1 : 2 and Q divides BP in the ratio 3 : 1. Find the area of BQC and the area of PQC.

A(-1, -2)



P

Q

- B(0, 3) C(4, 2) Q 8. If A and B are points (2, -1) and (-3, -2) resp. then find the coordinates of P such that AP

$$= \frac{5}{7} AB.$$

- Q 9. ABCD is a parallelogram with vertices A(x₁, y₁), B(x₂, y₂) and C(x₃, y₃). Find the coordinates of the fourth vertex D in terms of x₁, x₂, x₃, y₁, y₂ and y₃.

- Q 10. Find the condition so that the point (x, y) lies on the line joining (3, 4) and (-5, 6).

MISCONCEPTION/COMMON ERRORS

1. Questions based on distance formula

(i) While substituting the coordinate points sometimes students do not take care of order of (x, y) i.e. x as x-coordinate and y as y-coordinate.

$$(x, y) \neq (y, x)$$

- (ii) For finding distance between P(-3, 2) and Q(-5, 3)

$$PQ = \sqrt{(-5 - (-3))^2 + (3 - 2)^2}$$

In place of this student sometimes take

$$PQ = \sqrt{(-5 - 3)^2 + (3 - 2)^2}$$

(iii) In case of proving four given points are vertices of a square, they forget to prove two diagonals are also equal.

2. Students commit errors in interpreting the conditions like

→ Line segment is bisected

→ Line segment is trisected

3. The
takecoordinate of centroid as $\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$ of in place

$$\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$$

SUGGESTED PROJECT WORK (CO-ORDINATE GEOMETRY)

Project 1:

Dilation: A transformation in which a polygon is enlarged or reduced by a given factor around a given centre point. It is like zooming in or out. The centre point is point of reference.

Scale factor: The amount by which the image grows or shrinks is called the scale factor.

Activity: Draw any quadrilateral of your choice on a graph paper. Write the coordinates of all the points. [With scale factor two and one of the Vertices as point of reference. Write the new coordinates. Find the area before scale factor and after it. Compare the ratio. What do you observe.

Try it with other polygons and generalise the observation.

Project 2:

To understand and appreciate the use of coordinate geometry in computer application (MS-paint).

Students will working group of 4 to 5 and make drawings of objects using MS-paint. They will be asked to observe the use of coordinate geometry while making drawings and sketches.

Project 3:

To explore and appreciate the use of coordinate geometry in the real world.

Students will work in group and make a map of important land mark of their surroundings from home to school. They will mark the coordinates of the important land marks by taking a point of reference. The project can be extended by taking the actual coordinates using Google Map and distances between the landmarks can be calculated and verified.

Historical Background

Coordinate system or Cartesian coordinate system used to uniquely determine a point in two or three dimensional space by its distance from the origin of the coordinate system.

It gained its name from a French mathematician and philosopher Rene Descartes (1596-1650), most famously known for his work known merging algebra and geometry into algebraic geometry. He developed ideas about this system in his book–Discourse on Method (published in 1637), to which an appendix. The Geometry (La Geometrie) was added trying to mathematically show the application of his philosophy. The book was also famous for the quote ‘Je pense, donc je suis’ (I think, therefore I am).

Visit the suggested link and read the quotes by Rene Descartes Source (<http://www.groups.dsc.st-and.ac.uk/history/quotations/Descartes.html>) Each problem that I solved became a rule which served afterwards to solve other problems.

Discours **de La Mithede** Cogito Ergo Sum “ I think, therefore I am.”

Discours **de La Mitho de** I hope that positivity will judge me kindly, not only as to the things which I have explained, but also to those which I have intentionally omitted so as to leave to others pleasure of discovery.

Le Ge’ome’trie.

Perfect members like perfect men are vey rare.

Quoted in H Eve mathematical circles squared (Boston 1972).

Students may be asked to share their views on these quote.

How did Rene’ Descarte develops the coordinate plane system?

STATISTICS

INTRODUCTION

Everyday we come across a wide variety of information in the form of figures, tables, graphs, etc through newspapers, radio, television etc. These numerical figures may be about imports and exports of a country, per capita income, stock exchange sensex rates, minimum and maximum temperatures, batting and bowling averages of a cricket team, literacy rate among the people, expenditures in various sectors of a five year plan, different surveys carried out by different agencies on health etc.

These facts and figures, numerical or otherwise, which are collected with a definite purpose, are called data. Every facet of life uses/utilises data in one form or the other. So, it is essential to know how to extract meaningful information from such data. This extraction of meaningful information is studied in a branch of mathematics called statistics. In fact, it is the branch of mathematics in which facts and information are collected, sorted, displayed and analysed. Statistics are used to make decisions and predict what may happen in the future.

KEY CONCEPTS

Data

Facts or figures, which are numerical or otherwise collected with a definite purpose.

Types of Data:

Primary Data: Data which an investigator collects for the first time for his own purpose. Data regarding weights of students of a school by a school medical attendant is an example of primary data.

Secondary Data: Data which the investigator obtains from some other source, agency or office for his own purpose.

Data collected from newspapers regarding minimum/maximum temperatures is an example of secondary data.

Raw or Ungrouped Data: The data obtained in original form and presented ungrouped without any re-arrangement or condensed form. **An Array:** The Presentation of a data in ascending or descending order of magnitude.

Grouped Data: Rearrangement or condensed form of data into classes or groups.

Range of Data: Difference between the highest and lowest values in the data.

Frequency: The number of times an observation occurs in data.

Class Interval: Each group in which the observations/values of a data are condensed e.g. 21-30, 31-40 etc, then 21-30 is called the class interval. In a class 21-30, then the class size is the difference between the upper class limit and the lower class limit i.e. $30.5 - 20.5 = 10$. The class interval is also known as class width or class size.

Class limits: Values by which each class interval is bounded. Value on the left is called lower limit and value on the right is called upper limit. In a class interval of 21-30, then 21 and 30 are called class limits.

Lower class limits: In the class interval 21-30, the lower class limit is 21

Upper class limit: in the class interval 21-30, the upper class limit is 30

Class size: Difference between the upper limit and the lower limit.

Class mark of a class interval: Mid value of a class interval $= (\text{lower limit} + \text{upper limit})/2$

In a class interval 21-30, the class mark is the average of 21 and 30 i.e. $(21+30)/2 = 25.5$

Cumulative Frequency of a class: Total of frequencies of a particular class and of all classes prior to that class.

Bar Graph: A pictorial representation of data in which usually bars of uniform width are drawn with equal spacing between them on one axis and values of variable (frequencies) are shown on other axis.

Histogram: A pictorial representation like bar graph with no space between the bars. It is used for continuous grouped frequency distribution.

Frequency Polygon: A graphical representation of grouped frequency distribution in which the values of the frequencies are marked against the class mark of the intervals and the points are joined by line segments.

Central Tendency: A single quantity which enables us to know the average characteristics of the data under consideration. Use of central tendency is a technique to analyse the data.

Various Measures of central tendency:

The arithmetic mean/the mean/Average The median

The mode

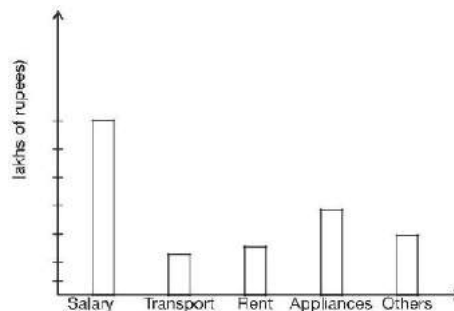
Mean: It is the ratio of the sum of all values of the variable and the number of observations.**Median:** It is the middle most value of arrayed data. It divides the arrayed data into two equal parts.**Mode:** It is the most frequently occurring value amongst the given values of the variate in the data or it is an observation with the maximum frequency in the given data.**LEARNING-TEACHING STRATEGIES****Bar Graph**

In a bar graph, data are represented using bars (rectangles) of equal width. The width itself is not significant, but all the bars should be of same width. Sometimes, the bars may be just thick lines. Usually, the bars are separated by gaps of equal width. The height of a bar is taken on the basis of corresponding value of the variable with a suitable scale.

The expenditures (in lakhs of rupees) of a company under different heads is given below:

<i>Head</i>	<i>Expenditure (in lakhs of rupees)</i>
Salary of employees	50
Transport	10
Rent	12
Appliances	25
Others	15

Draw a bar graph to represent the above data.

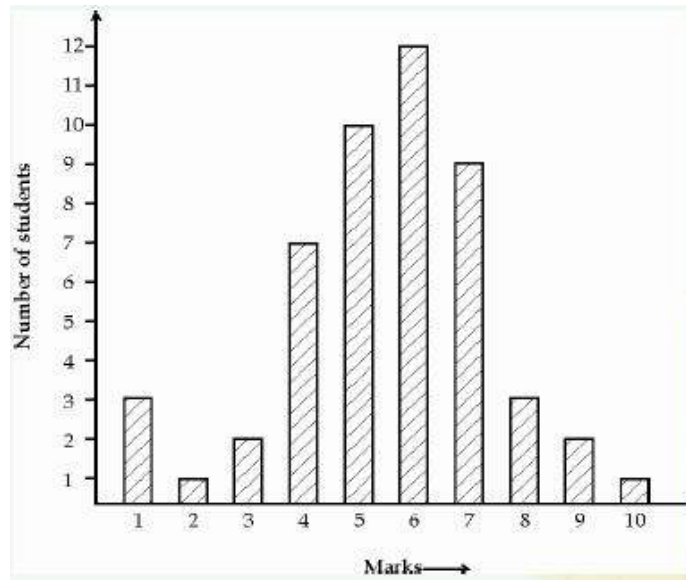


We draw two perpendicular lines intersecting each other. Along the horizontal axis, we represent “Heads” and along the vertical axis, we represent the expenditure” (in lakhs of rupees). For each head, we draw a bar (rectangle) of length corresponding to value of the variable (here it is expenditure) using a suitable scale (as shown in the figure).

employees
Expenditure (in
Head →

WORKSHEET 1**Bar Graph**

Given below is a bar graph of marks obtained by students in mathematics test:



Read the graph and answer the following questions:

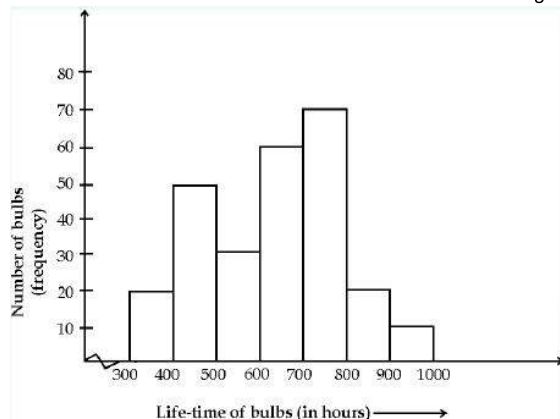
- How many students appeared in the test?
- How many students have obtained the maximum number of marks?
- How many students have obtained marks below 5?
- How many students have obtained 10 marks?

Histogram: This is a form of representation like the bar graph, but it is used for continuous class intervals.

WORKSHEET 2

Histogram

Following is a histogram showing life-time of bulbs: 216 Manual for Effective Learning In Mathematics In Secondary Level



Read the histogram and answer the following questions:

- Find the number of bulbs tested?
- How many bulbs have their life time less than 500 hours?
- How many bulbs have their life time at least 400 hours?
- How many bulbs have their life time equal to 800 hours but less than 900 hours?

Frequency Polygon

A frequency polygon can be obtained by joining the mid points of the tops of the rectangles, in an order, of the corresponding histogram by line segments. Frequency polygons can also be drawn independently without drawing histograms.

WORKSHEET 3

Frequency Polygon

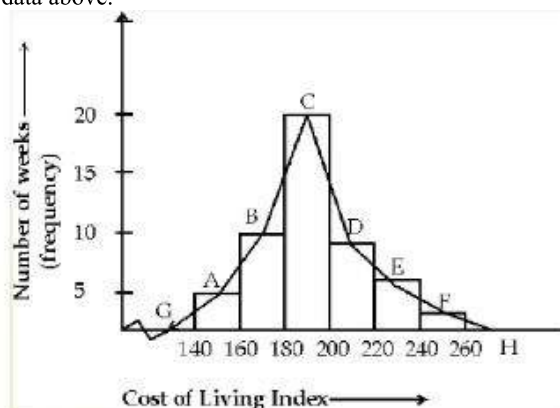
Following are the data regarding cost of living index in a city for 52 weeks:

Cost of Living Index	Number of Weeks
140-160	5
160-180	10

180-200	20
200-220	9

220-240	6
240-260	2

Draw a frequency polygon for the data above.



Mean: It is ratio of the sum of all values of the variable and the number of observations.

Median: It is the middle most value of arranged data. It divides the arranged data into two equal parts.

Mode: It is most frequently occurring value amongst the given value of variant in the data.

EXPLORATORY TASK 1

Concept: Mean

1. If a number is added to each term of the given data, then what will be the change in the mean?
2. If a number is subtracted from each term of the given data, then what will be the change in the mean?
3. If a number is multiplied with each term of the given data, then what will be the change in the mean?

EXPLORATORY TASK 2

Concept:

Mean

First row	1	3	5	7	9	11	13
Second row	2	4	6	8	10	12	14

1. What is the mean of the top row?

2. How did you work it out?
3. What is the mean of the second row?
4. Compare the two rows, what has happened to each number?
5. What has happened to the range of each row?
6. What would happen to the mean and range if the second row was increased by two, by three?
7. What would happen to the mean and range if the second row was multiplied by two, or three?

WORKSHEET 3

Concept: Mean, Median, Mode

Activity

Suppose we are told that the median of five numbers is 5, the mode is 1, and the mean is 4. How could we find the five numbers?

We can start by drawing a blank for each of the values. Then we'll try to fill them in, putting them in ascending order as we go.

Now, what do we know about the numbers? We know that "the median of five numbers is 5." The median is the middle number when arranged in ascending order, so let's put it there.

What else do we know? We're told that "the mode is 1." That means that 1 has to appear more than any other number. Since 1 is less than 5, all 1's will have to go to the left of the 5. We know we need at least two of them (otherwise, 5 would be a mode as well), so both blanks on the left will have to be 1's. Putting them in, we have: Now the last clue is, "the mean is 4." The mean is the "average" of the numbers. It is computed by taking the sum of the numbers and dividing it by the number of numbers.

Algebraically, if we call our values A, B, C, D and E, we would write: $M = (A+B+C+D+E) / 5$

Since we already know the first three numbers, let's plug them in for A, B, and C. We also know that the mean is 4, so we'll plug that in, too: $4 = (1+1+5+D+E) / 5$

$$4 = (7+D+E) / 5$$

Now let's solve for D+E, the two numbers we don't know: $4 = (7+D+E) / 5$

$$4 \times 5 = (7+D+E)$$

$$20 = 7+D+E$$

$$20 - 7 = D+E$$

$$D + E = 13$$

So the sum of the last two numbers must be 13. We also know that each of those two numbers must be greater than 5. What two numbers will work? Only 6 and 7 (can you think of WHY only 6 and 7 work?) So our five numbers must be: 1 1 5 6 7

To check; median = 5 (check), mode = 1 (check), mean is:

$$\begin{aligned} M &= (1+1+5+6+7) / 5 \\ &= 20 / 5 \\ &= 4 \end{aligned}$$

WORKSHEET 4

Multiple Choice Questions:

1. The median of 10, 12, 14, 16, 18, 20 is
(a) 12 (b) 14 (c) 15 (d) 16
2. If the mode of 12, 16, 19, 16, x, 12, 16, 19, 12 is 16, then the value of x is
(a) 12 (b) 16 (c) 19 (d) 18
3. The class mark of the class 120-150 is
(a) 120 (b) 130 (c) 135 (d) 150
4. The class mark of a class is 10 and its class width is 6. The lower limit of the class is
(a) 5 (b) 7 (c) 8 (d) 10
5. In a frequency distribution, the class width is 4 and the lower limit of first class is 10. If there are six classes, the upper limit of last class is
(a) 22 (b) 26 (c) 30 (d) 34
6. The range of the data 14, 27, 29, 61, 45, 15, 9, 18 is
(a) 61 (b) 52 (c) 47 (d) 53
7. The class marks of a distribution are 15, 20, 25,, 45. The class corresponding to 45 is
(a) $12.5 - 17.5$ (b) $22.5 - 27.5$ (c) $42.5 - 47.5$ (d) None of these
8. The mean of first ten multiples of 7 is

- (a) 35.0 (b) 36.5 (c) 38.5 (d) 39.2

9. The mean of $x + 3$, $x - 2$, $x + 5$, $x + 7$ and $x + 72$ is

- (a) $x + 5$ (b) $x + 2$ (c) $x + 3$ (d) $x + 7$

10. The mean of 10 observations is 42. If each observation in the data is decreased by 12, the new mean of the data is

- (a) 12 (b) 15 (c) 30 (d) 54

11. The mean of 10 numbers is 15 and that of another 20 number is 24 then the mean of all 30 observations is