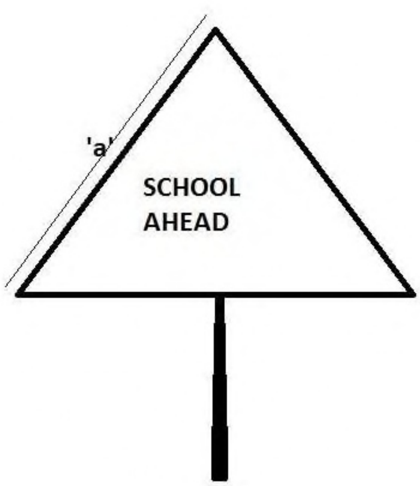
Chapter 12

Heron's formula

Exercise 12.1

Question: 1 A traffic signal board, indicating ‘SCHOOL AHEAD’, is an equilateral triangle with side ‘a’. Find the area of the signal board, using Heron’s formula. If its perimeter is 180 cm, what will be the area of the signal board?

Solution:



Given

Perimeter of Signal board = 180 cm

Length of the side of the equilateral triangle = a

The perimeter of the signal board = perimeter of the triangle

(Perimeter of Equilateral Triangle = $3 \times$ Side)

3 a = 180 cm

a = 60 cm

Therefore, each side of triangle = 60 cm.

Semi perimeter of the signal board (s) = Perimeter of board /2 = 180/2 = 90 cm

Using Heron's formula,

Area of the signal board = $\sqrt{s(s-a)(s-b)(s-c)}$

where, s = semi perimeter of signal board = 90 cm a = b = c = 60 cm Putting the value we

Area of signal board = $\sqrt{90\ (90-60)(90-60)(90-60)}$

Area of signal board = $\sqrt{90\ \times 30 \times 30 \times 30}$

Area of signal board = $100 \times 9\sqrt{3}$

Area of signal board = $900\sqrt{3}\ cm^2$

Method 2:

we know that,

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

where a = side of the equilateral triangle. putting the value of a = 60 cm we get,

Area = $\frac{\sqrt{3}}{4}(60)^2$

Area $= \frac{\sqrt{3}}{4} \times 3600$

Area $= 900\sqrt{3}\ cm^2$

Question: 2 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig. 12.9). The advertisements yield on earning of Rs 5000 per m2 per year. A company hired one of its walls for 3 months. How much rent did it pay?

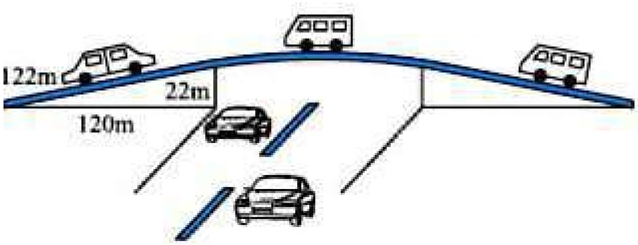


Fig. 12.9

Solution: As shown in the figure the sides of the triangular side walls of flyover are 122 m, 22 m, and 120 m.

Perimeter of the triangle = 122 + 22 + 120 = 264 m

Semi perimeter of triangle(s) = $\frac{264}{2}$

= 132m

Now,

Using Heron's formula

Area of the advertisement = $\sqrt{s(s-a)(s-b)(s-c)}$

where, s = semi perimeter of triangle and a, b, and c are the sides of the triangle.

= $\sqrt{132\ (132-122)(132-22)(132-120)}$

= $\sqrt{132 \times 10 \times 110 \times 12}$

= $\sqrt{132 \times 132 \times 10 \times 10}$

= $132 \times 10m^2$

$=1320m^2$

Rent of advertising per year = Rs. 5000 per m^2

Rent of one wall for 1 year = 1320 x 5000

Rent of one wall for 3 months = $\frac{1320 \times 5000 \times 3}{12}$

Rent of one wall for 3 months = Rs. 1650000

Question: 3 There is a slide in a park. One of its side walls has been painted in some color with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig. 12.10). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in color.

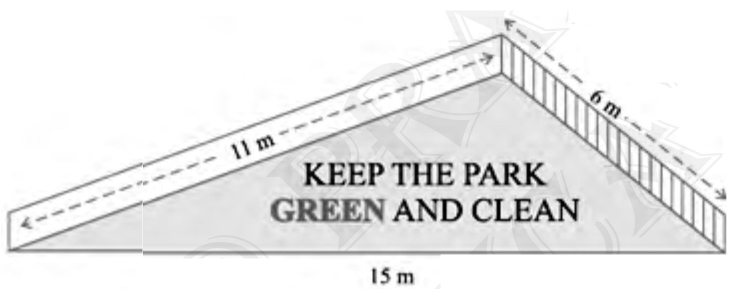


Fig. 12.10

Solution:

Given: The sides of the triangle are 15 m, 11 m and 6 m.

To find: Area painted in color

Explanation:

Perimeter of the triangular walls = sum of 3 sides

Perimeter = 15 + 11 + 6 = 32 m

Semi perimeter of triangular walls(s) = $\frac{32}{2}$

= 16 m

Using Heron's formula,

Area of the advertisement = Area of triangle =$\sqrt{s(s-a)(s-b)(s-c)}$

where, s = semi perimeter of the triangle a, b and c are the sides of the triangle Therefore for the given triangle,

Area $=\sqrt{16(16-15)(16-11)(16-6)}$

$= \sqrt{16 \times 1 \times 5 \times 10}$

$= \sqrt{16 \times 5 \times 10}$

$= \sqrt{800}\ m^2$

$= \sqrt{(20 \times 20 \times 2})$ (Taking 20 outside the square root, as a pair is present in square root, we get,)

$=20\sqrt{2}\ m^2$

Thus the area of painted color is $20\sqrt{2}\ m^2$

Question: 4 Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

Solution:

To find: Area of the triangle

Given:

Two sides of the triangle are 18 cm and 10 cm (Given)

Perimeter of the triangle = 42 cm

Perimeter of Triangle = Sum of sides of triangle

Let the third side of triangle be x, So

x + 18 + 10 = 42

Third side of triangle, x = 42 – (18 + 10)

= 14 cm

Semi perimeter of triangle(s) = $\frac{42}{2}$

= 21 cm

Using Heron's formula,

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{21(21-18)(21-10)(21-14)}$

= $\sqrt{21 \times 3 \times 11 \times 7}$

= $\sqrt{7 \times 3 \times 7 \times 3 \times 11}$

=$21\sqrt{11}\ cm^2$

Question: 5 Sides of a triangle are in the ratio of 12: 17: 25 and its perimeter is 540 cm. Find its area.

Solution: Let the common ratio of the sides of the triangle be x and sides are 12x, 17x and 25x

The perimeter of the triangle = 540 cm

Sum of sides of the triangle = 540 cm

12x + 17x + 25x = 540 cm

54 x = 540

x = 10

Sides of triangle:

12 x = 12×10 = 120 cm

17 x = 17×10 = 170 cm

25 x = 25×10 = 250 cm

Semi perimeter of triangle(s) = $\frac{540}{2}$

=270cm

Now,

Using Heron's formula,

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where, s = semi-perimeter of the triangle and a, b and c are the sides of the triangle

= $\sqrt{270(270-120)(270-170)(270-250)}$

= $\sqrt{270 \times 150 \times 100 \times 20}$

= √(81000000) cm^2

= 9000 cm^2

Area of triangle = 9000 cm^2

Question:.6 An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Solution: Length of equal sides of the triangle = 12cm

Perimeter of the triangle = 30 cm

Third side of triangle = 30 – (12 + 12)

= 6cm

Semi perimeter of triangle(s) = $\frac{30}{2}$

= 15cm

Now,

Using Heron's formula,

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

where, s = semi perimeter of the triangle

a, b and c are the sides of the triangle

= $\sqrt{15(15-12)(15-12)(15-6)}$

= $\sqrt{15 \times 3 \times 3 \times 9}$

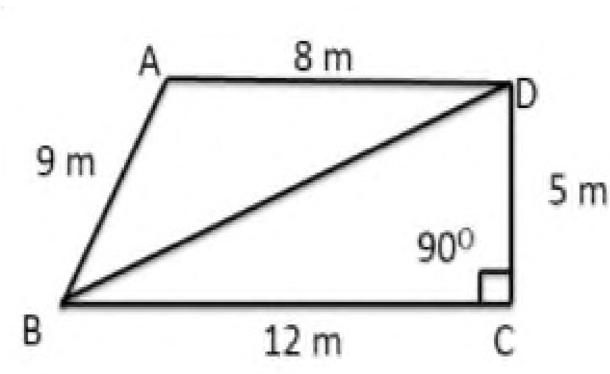
= $9\sqrt{15}\ cm^2$

Hence, the area of triangle is $9\sqrt{15}\ cm^2$

Exercise 12.2

Question: 1 A park, in the shape of a quadrilateral ABCD, has ∠ C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

Solution: The figure is given below:



The dimensions of the park as follows:

Angle C = 90°,

AB = 9m,

BC = 12m,

CD = 5m

And, AD = 8m

Now, BD is joined.

Thus quadrilateral ABCD can now be divided into two triangles ABD and BCD

Area of quadrilateral ABCD = Area of triangle ABD + Area of triangle BCD

Pythagoras Theorem: It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides.

In ΔBCD by Pythagoras theorem,

$BD^2 = BC^2 + CD^2$

$BD^2 = 122 + 5^2$

$BD^2 = 169$

BD = 13m

As triangle BCD is right angled triangle,

Area of $\Delta BCD = \frac{1}{2} \times base \times height$

Area of $\Delta BCD = \frac{1}{2} \times 12 \times 5 = 30m^2$

Now, Semi perimeter of triangle (ABD) = $\frac{(8+9+13)}{2}$

$= \frac{30}{2} = 15$m

Using Heron's formula,

ar (ABD) = $\sqrt{s(s-a)(s-b)(s-c)}$

where, s = semi-perimeter of the triangle and a, b, and c are the sides of the triangle.

$= \sqrt{15(15-13)(15-8)(15-9)}$

$= \sqrt{15 \times 2 \times 7 \times 6}$

$= \sqrt{6 \times 6 \times 5 \times 7}$

$= 6\sqrt{35}m^2$

	5.91
5	$\overline{35}\ \overline{00}$
	25
109	10 00
	9 81
1181	1900
	1181

$= 6 \times 5.91 = 35.46\ m^2$

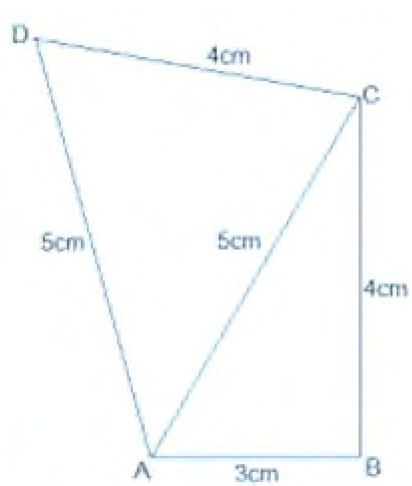
Area of quadrilateral ABCD = 35.46 + 30

Area of quadrilateral ABCD = $65.46\ m^2$

Thus, the park acquires an area of $65.5\ m^2$(approx).

Question: 2 Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Solution:



Given: AB = 3 cm, BC = 4 cm, CD = 4 cm, DA =5 cm and AC = 5 cm

To find: Area of quadrilateral ABCD.

Now we can divide the quadrilateral into two separate triangles and then calculate their area. Now, in Δ ABC,

Using Pythagoras theorem,

$AC^2 = BC^2 + AB^2$

$5^2 = 3^2 + 4^2$

25= 16 + 9

25 = 25

Thus,

Triangle ΔABC is right angled triangle and since AB and BC are smaller sides so they work as legs.

And we know that,

Area of right angles triangle $= \frac{1}{2} \times base \times height$

ar(ABC) $= \frac{1}{2} \times 3 \times 4 = 6\ cm^2$

Now,

Perimeter of = 5 + 5 + 4

Semi perimeter of Δ ACD = $\frac{5+5+4}{2}$

$= \frac{14}{2}$

= 7 cm

Using Heron's formula,

Area of Δ ACD = $\sqrt{s(s-a)(s-b)(s-c)}$

where, s = semi perimeter of the triangle and a, b, and c are the sides of the triangle

$= \sqrt{7(7-5)(7-5)(7-4)}$

$= \sqrt{2 \times 2 \times 7 \times 3}$

$= 2\sqrt{21}$ cm^2

Take = $\sqrt{21}$ = 4.58

= 9.16 cm^2 (approx)

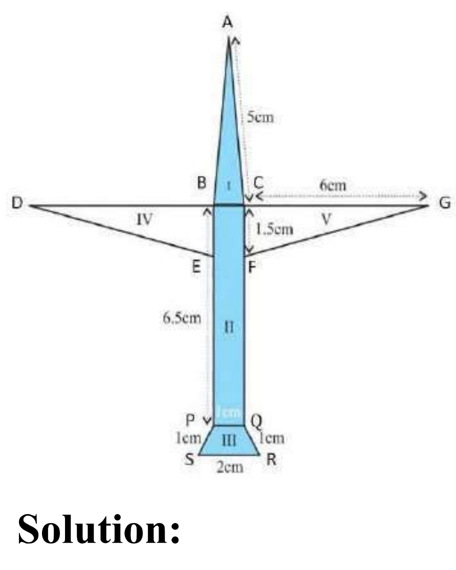
Area of quadrilateral ABCD = Area of ΔABC + Area of Δ ACD

= 6 + 9.16

= 15.16 cm^2

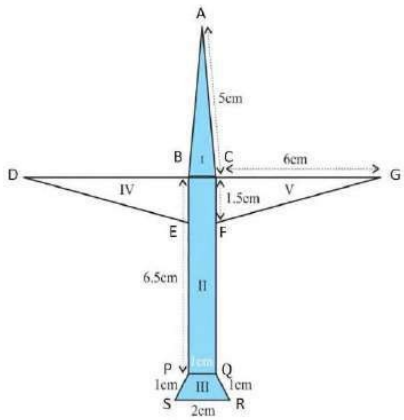
By rounding off = 15.2 cm^2 (approx)

Question: 3 Radha made a picture of an aero plane with colored paper as shown in Fig 12.15. Find the total area of the paper used.

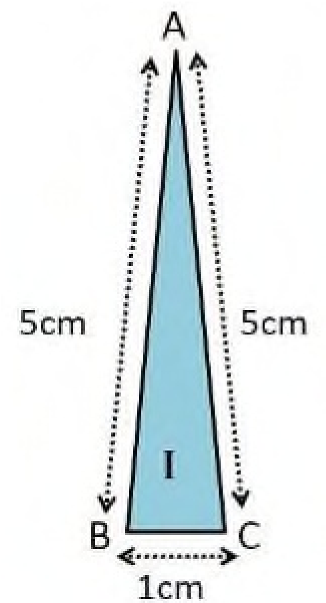


Solution:

The figure of the questions is:



Now to find area:



Sides of triangular section I = 5cm, 1cm and 5cm

Perimeter of the triangular section I = 5 + 5 + 1 = 11cm

Semi perimeter, s = $\frac{perimeter\ of\ triangular\ section\ I}{2} = \frac{11}{2}$ = 5.5cm

We use Heron's formula to find the area of the section I,

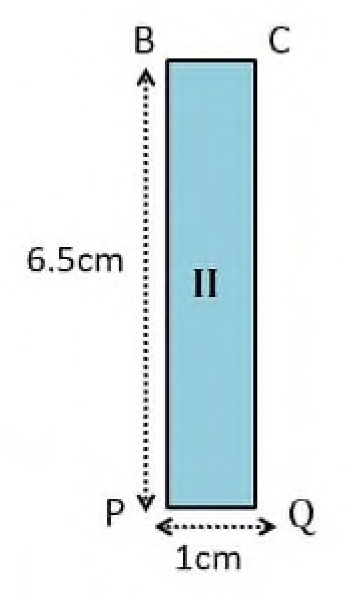
Area of Section I = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$

= $\sqrt{5.5 \times 0.5 \times 0.5 \times 4.5}$

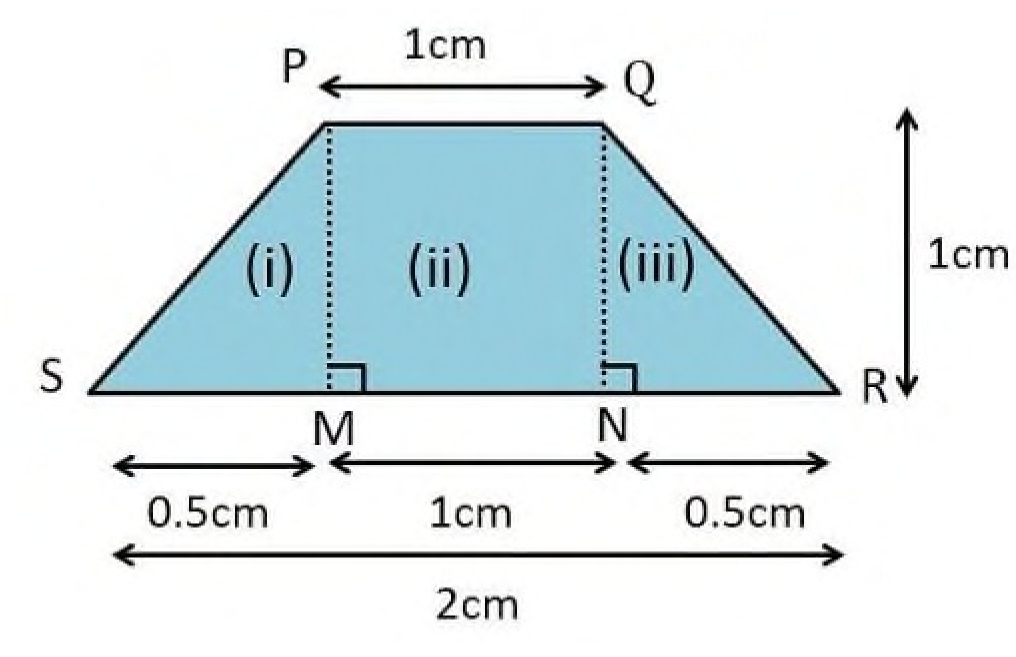
$= \sqrt{6.1875}\ cm^2$

Area of Section I = 2.48 cm^2 (approx.) ~ 2.5 cm^2



Length of the sides of the rectangle of section II = 6.5 cm and 1 cm

Area of section II = 6.5×1 = 6.5cm^2

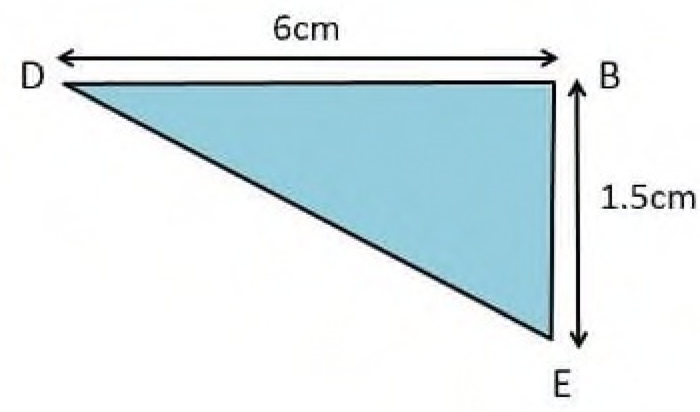


Section III is an isosceles trapezium which is divided into two right triangles and one rectangle.

Now, Area of trapezium

= Area (i) + Area (ii) + Area (iii)

= $1/2 \times 0.5 \times 1 + 1 \times 1 + 1/2 \times 0.5 \times 1 = 1.5\ cm^2$



Section IV and V are congruent right-angled triangles with height 1.5cm and base 6cm.

now area of triangle,

$= \frac{1}{2} \times$ base$\times$ height = $= \frac{1}{2} \times 1.5 \times 6$

$= 4.5\ cm^2$

Area of region IV and V = $2 \times\ 4.5\ cm^2 = 9cm^2$

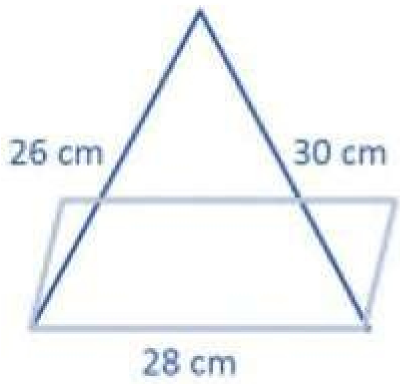
Thus, total area is 2.5 + 6.5 + 9 + 1.5 = 19.5 sq cm

Question: 4 A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Solution:

Given,

Area of the parallelogram and triangle are equal



Sides of triangle are 26 cm, 28 cm and 30 cm.

Semi perimeter of triangle ,s = $\frac{26+28+30}{2}$

S = $\frac{84}{2}$

= 42 cm

Using Heron's formula,

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{42(42-26)(42-28)(42-30)}$

= $\sqrt{42 \times 16 \times 14 \times 12}$

= $336 cm^2$

= Let the height of parallelogram be h,

As parallelogram and triangle having same base and same height have equal areas.

∴ Area of parallelogram = Area of triangle

28 × h = 336

$h = \frac{336}{28}$

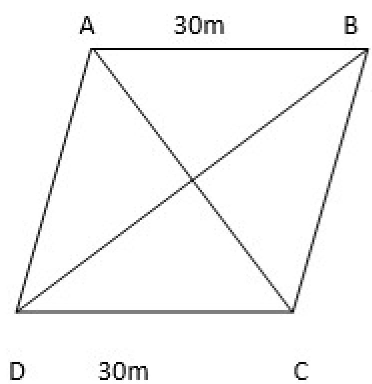
h =12 cm

Hence, the height of the parallelogram is 12 cm.

Question: 5 A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Solution:

Diagonals divide the rhombus ABCD into two congruent triangles of equal area.



Semi perimeter of $\Delta ABC = \frac{30+30+48}{2} =$ 54m

Using Heron's formula,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$\sqrt{54(54-30)(54-30)(54-48)}$

$= \sqrt{54 \times 24 \times 24 \times 6}$

$= 432m^2$

Area of field = 2 x area of field ΔABC

$= 2 \times 432$

$= 864\ m^2$

Thus,

Area of grass field which each cow will be getting = $\frac{864}{18} = 48m^2$

Question:6 An umbrella is made by stitching 10 triangular pieces of cloth of two different colors (see Fig. 12.16), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each color is required for the umbrella?

Solution: To Find: Total Amount of cloth required

Given: Umbrella is made up of 10 triangular pieces of sides 50, 50 and 20 cm

Perimeter of each triangular piece of cloth = 50 + 50 + 20

Semi perimeter of each triangular piece of cloth = $\frac{50 + 50 + 20}{2}$

$= \frac{120}{2}$

= 60 cm

Using Heron's formula,

Area of one triangular piece = $\sqrt{s(s-a)(s-b)(s-c)}$

where, as is the semi-perimeter of the triangle and a, b and c are the sides of the triangle.

= $\sqrt{60(60-50)(60-50)(60-20)}$

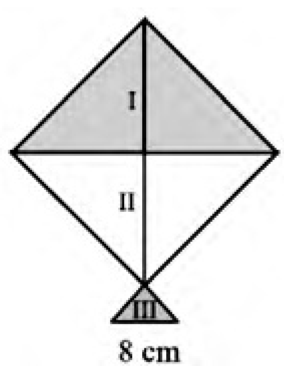
= $\sqrt{60 \times 10 \times 10 \times 40}$

= 200 $\sqrt{6} cm^2$

Area of piece of cloth of one colour = 5 x 200 √6

= 1000$\sqrt{6} cm^2$

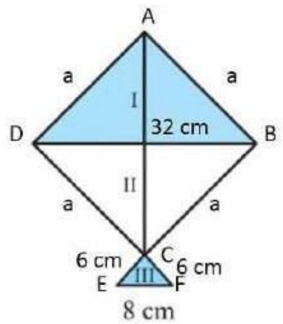
Question:7 A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Fig. 12.17. How much paper of each shade has been used in it?



Solution:

For I and II section:

For calculating the area of the square, Let the length of side = x cm



Each Interior Angle of square = 90º Pythagoras Theorem: Square of Hypotenuse equals to the sum of squares of other two sides

Now by Pythagoras theorem, $x^2 + x^2 = 3^2 \text{ x } 3^2$

$2\ x^2 = 3^2 \text{ x } 3^2$

$x^2 = 16 \text{ x } 32 = 512\ cm^2$

Area of Square = $(\text{side})^2$

And this will be the area of Square.

Area of square = 512 cm^2

Area of section I = Area of section II = $\frac{1}{2}$ Area of square

Now, we need half of this area,

So half of the area of square = 256 cm^2

For the III section

Length of the sides of triangle = 6cm, 6cm and 8cm

Perimeter of triangle = 6 + 6 + 8 = 20 cm

Semi-Perimeter of Triangle, s = 10 cm

Using Heron's formula,

Area of the III triangular piece = $\sqrt{s(s-a)(s-b)(s-c)}$

where, s = semi-perimeter and a, b, and c are the sides of the triangle

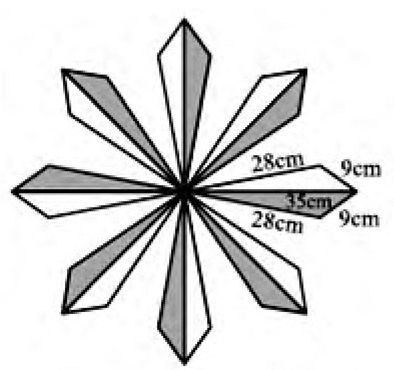
Area of Triangle = $\sqrt{10(10-6)(10-6)(10-8)}$

$= \sqrt{4 \times 10 \times 2 \times 4}$

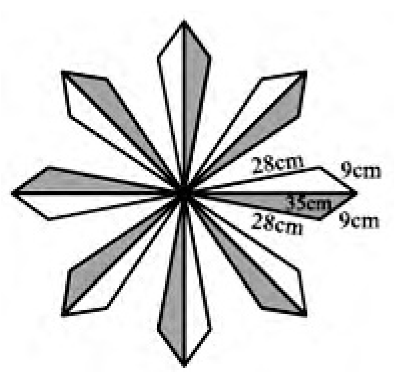
Area of Triangle = $8\sqrt{5}\ cm^2$

$= 17.92\ cm^2$

Question:8 A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Fig. 12.18). Find the cost of polishing the tiles at the rate of 50p per cm^2



Solution:



So, we need to find the area of all 16 tiles or we can say that Area of 16 triangles.

Perimeter of each triangular shaped tiles = 28 + 9 + 35

Semi perimeter of each triangular shaped tiles, s = $\frac{28+9+35}{2}$

S = $\frac{72}{2}$

s = 36 cm

Now,

Using Heron's formula,

Area of each triangular shape= $\sqrt{s(s-a)(s-b)(s-c)}$

where s = semi perimeter of triangle and a, b, c are the sides of the triangle

Area of 1 triangular shape = $\sqrt{36\,(36-28)(36-9)(36-35)}$

=$\sqrt{36 \times 8 \times 1 \times 27}$

Area of 1 tile = $36\sqrt{6}\ cm^2$

Total area of 16 tiles = $16 \times 36\sqrt{6}$ cm^2 = $576\sqrt{6}$ cm^2

Total Area of 16 tiles = 576×2.45 cm^2 [$\sqrt{6}$= 2.45]

Total Area of 16 tiles = 1411.2 cm^2

Rate of polishing tiles = 50 p per cm^2

Hence,

The total cost of polishing tiles = 50 × 1411.2

= 70560 paise As 1 paisa = $\frac{1}{100}$ Rs= RS. 705.60

Question:9 A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Solution:

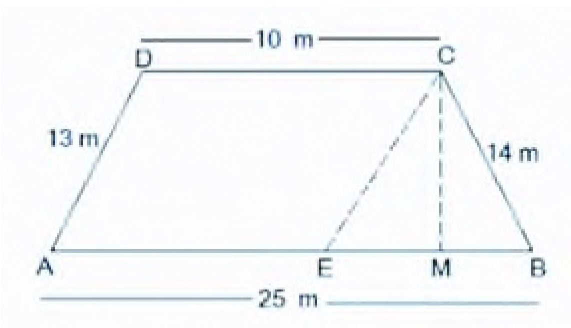
Let ABCD be the given trapezium whose

parallel sides are:

AB = 25m and CD = 10m

And,

Non-parallel sides are AD = 13m and BC = 14m



Now,

CM perpendicular to AB and CE ∥ AD

In ΔBCE

BC = 14m, CE = AD = 13m and

BE = AB – AE

= 25- 10

= 15m

Semi perimeter of $\Delta BCE = \frac{15+13+!4}{2}$

$= \frac{42}{2}$

= 21cm

Now,

Using Heron's formula,

Area of $\Delta BCE = \sqrt{s(s-a)(s-b)(s-c)}$

where, s = semi perimeter, a, b and c are the sides of the triangle.

$= \sqrt{21(21-15)(21-13)(21-14)}$

$= \sqrt{21 \times 8 \times 6 \times 7}$

$= 84m^2$

Now from Δ CEB, Area of triangle = 1/2 x base x height

For CEB, CM will be the height and 15 cm will be base of the triangle.

And we calculated the area from above. So,

$\frac{1}{2} \times 15 \times CM = 84$

$CM = \frac{56}{5} m$

Area of parallelogram AECD = Base ×Altitude

= AE ×CM

$= 10 \times \frac{56}{5}$

= 112 m^2

Area of trapezium ABCD = Area of AECD + Area of triangle BCE

= (112 + 84) m^2

= 196 m^2