Areas of Sectors formed at the Vertices of a Triangle

Objective

To verify that sum of areas of three sectors of the same radii 'r' formed at the vertices of any triangle is $\frac{1}{2}\pi r^2$ by paper cutting and pasting.
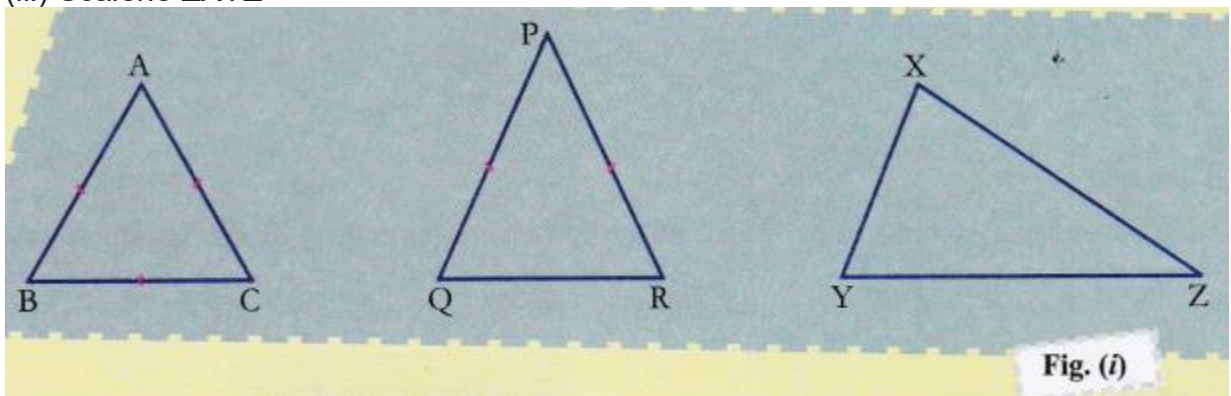
Prerequisite Knowledge

1. Concept of different types of triangles.
2. Definition of a sector.
3. Area of circle = πr^2, r → radius.
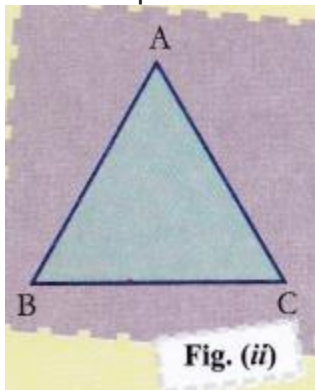
Materials Required

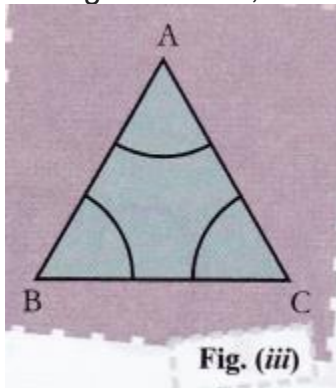Glazed paper, sketch pens, fevicol, a pair of scissors, pencil, geometry box.

Procedure

1. Draw three different types of triangles on a glazed paper as shown in fig. (i).
   (i) Equilateral ΔABC
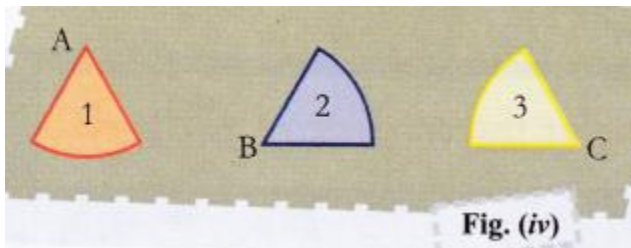   (ii) Isosceles ΔPQR
   (iii) Scalene ΔXYZ

Fig. (i)

2. Cut an equilateral ΔABC as shown in fig.(ii)

Fig. (ii)

3. Taking vertices A, B and C as centres of ΔABC, draw three sectors of same radii r.

Fig. (iii)

4. Cut these three sectors and marked them as 1,2,3 and fill different colours.

Fig. (iv)

5. Draw a straight line and mark any point 'O' on it. Place three sectors 1,2, 3 adjacent to each other so that the vertices A, B, C coincide with 'O' without leaving any gap as shown in fig. (v).

Fig. (v)
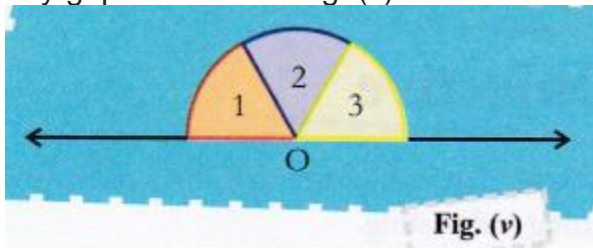
6. The same process (steps 1-5) can be taken up with isosceles triangle and scalene triangle fig. (i).

Observation
The shape formed on the straight line is a semi circle

∴ area of circle = πr²

∴ area of semicircle = $\frac{1}{2}$ πr²

Result

It is verified that the sum of areas of three sectors of same radii 'r' formed at the vertices of any triangle is $\frac{1}{2}$ πr².

Learning Outcome

The students are able to understand the concept of this activity through paper cutting. It is clear to them that a semicircle is always obtained, whatever be the type of a triangle.
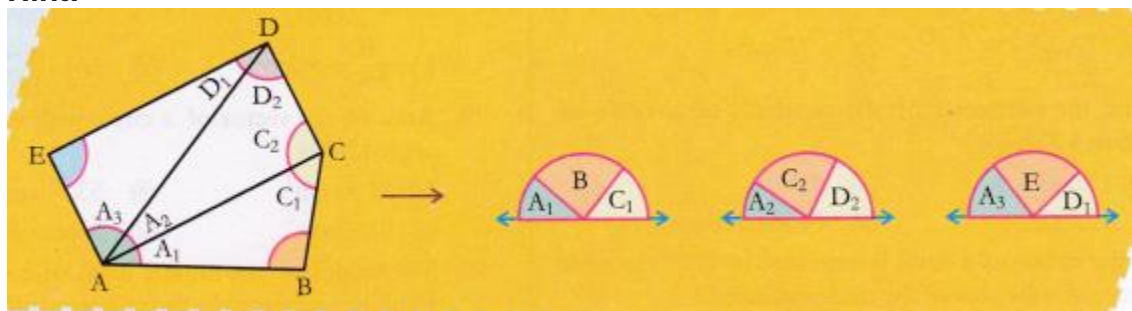
Activity Time

1. Find the sum of areas of four sectors of same radii 'r' formed at the vertices (as centre) of any quadrilateral through activity.
   Hint:

(Cut the sectors from vertices and arrange them as shown)
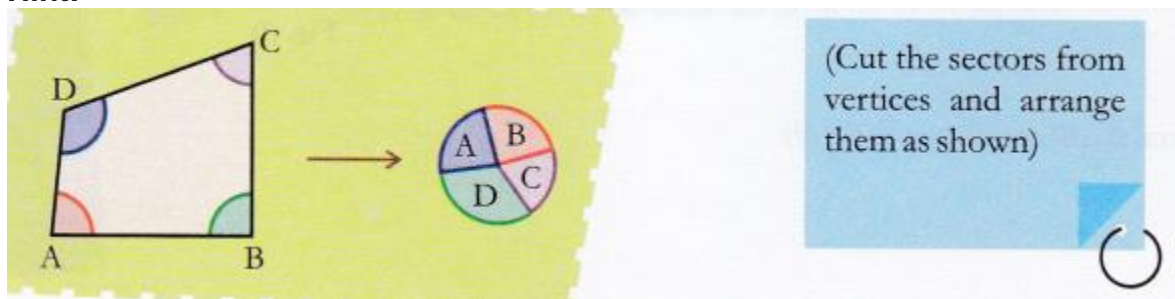
2. Find the sum of areas of five sectors of same radii 'r' formed at the vertices (as centre) of any pentagon through activity.
   Hint:

Viva Voce

Question 1.

What is the angle subtended by a circle at centre ?
Answer:
360°

Question 2.
What is the sum of angles of four sectors of same radii 'r' formed at the vertices (as centre) at any quadrilateral ?
Answer:
360°

Question 3.
If the perimeter of semicircle is 12 cm, find its radius.
Answer:
$\frac{7}{3}$ cm

Question 4.
What is the area of a semicircle of radius 2 cm ?
Answer:
2π

Question 5.
Define a segment of a circle.
Answer:
A chord divides a circle in two parts each of which is called a segment of a circle.

Question 6.
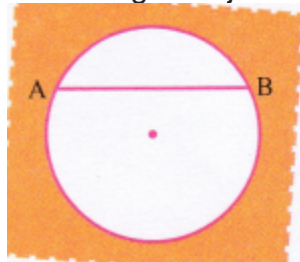What is the difference between a sector and a segment of a circle ?
Answer:
A sector of a circle is formed by an arc and two radii of the circle, while a segment is one of its part in which a chord divides a circle.

Question 7.
Define chords.
Answer:
A line segment joining two points on the circumference of a circle is known as a chord.

Question 8.
"A tangent intersects the circle in more than one point". Is the statement true or false ?
Answer:
False

Multiple Choice Questions

Question 1.
If the area of a semicircle is 121 cm², find its radius.
(a) √77 cm
(b) √76 cm
(c) √74 cm
(d) None of these

Question 2.
If the perimeter and the area of a circle are numerically equal, then the radius of the circle is
(a) π units
(b) 2 units
(c) 4 units
(d) 5 units

Question 3.
Area of a sector of angle P (in degree) of a circle witl radius R is
(a) $\frac{P}{180^0} \times 2\pi R$
(b) $\frac{P}{180^0} \times 2\pi R^2$
(c) $\frac{P}{360^0} \times 2\pi R$
(d) $\frac{P}{720^0} \times 2\pi R^2$

Question 4.
Area of the sector of a circle with radius 4 cm and angle 30° is
(a) 4.91 cm
(b) 14.9 cm
(c) 4.19 cm
(d) 94.1cm

Question 5.
Find the area of a quadrant of circle whose circumference is 22 cm.
(a) $\frac{77}{8}$ cm²
(b) $\frac{76}{8}$ cm²
(c) $\frac{77}{2}$ cm²
(d) $\frac{77}{4}$ cm²

Question 6.
Find the perimeter of the quadrant of a circle of radius 4.2 cm.
(d) 15 cm
(b) 8.4 cm

(c) 12.6 cm
(d) None of these

Question 7.
If the radius of a circle is increased by 100% by what percent is the area of the circle increased ?
(a) 200%
(b) 300%
(c) 400%
(d) None of these

Question 8.
The perimeter of a sector with radius r and angle θ of a circle is given by
(a) $d + \frac{2\pi r\theta}{180^0}$
(b) $r + \frac{\pi r\theta}{180^0}$
(c) $\frac{\theta}{360^0} \times 2\pi r$
(d) $2r + \frac{\pi r\theta}{180^0}$

Question 9.
Area of the sector of a circle with radius 7 cm and angle 120° is
(a) 51.33 cm²
(b) 53.11 cm²
(c) 53.13 cm²
(d) Noneofthese

Question 10.
The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.
(a) $\frac{145}{3}$ cm²
(b) $\frac{514}{3}$ cm²
(c) $\frac{451}{3}$ cm²
(d) $\frac{154}{3}$ cm²

Answers

1. (a)
2. (b)
3. (d)
4. (c)
5. (a)
6. (a)
7. (b)
8. (d)
9. (a)
10. (d)