Numbers

INTRODUCTION

In Hindu Arabic System, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called *digits* to represent any number. This is the *decimal system* where we use the numbers 0 to 9. 0 is called *insignificant digit* whereas 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *significant digits*.

A group of figures denoting a number is called a *numeral*. For a given numeral, we start from extreme right as unit's place, ten's place, hundred's place and so on.

Illustration 1 We represent the number 309872546 as shown below:

Ten Crore 10^8	Crores 10^7	Ten Lacs (million) 10^6	Lac's 10^5	Ten Thousand 10^4	Thousand 10^3	Hundred 10^2	Ten's 10^1	Units 10^0
3	0	9	8	7	2	5	4	6

We read it as

'Thirty crores, ninety-eight lakhs, seventy-two thousands five hundred and forty-six.'

In this numeral:

The place value of 6 is $6 \times 1 = 6$.
The place value of 4 is $4 \times 10 = 40$.
The place value of 5 is $5 \times 100 = 500$.
The place value of 2 is $2 \times 1000 = 2000$ and so on.

The face value of a digit in a number is the value itself wherever it may be.

Thus, the face value of 7 in the above numeral is 7. The face value of 6 in the above numeral is 6 and so on.

NUMBER SYSTEM

Natural Numbers

Counting numbers 1, 2, 3, 4, 5, … are known as *natural numbers*.

The set of all natural numbers can be represented by

$$N = \{1, 2, 3, 4, 5, \ldots\}.$$

Whole Numbers

If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5, ... are called *whole numbers*.

The set of whole numbers can be represented by

$$W = \{0, 1, 2, 3, 4, 5, \ldots\}$$

Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

Integers

All counting numbers and their negatives including zero are known as *integers*.

The set of integers can be represented by

$$Z \text{ or, } I = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

Positive Integers

The set $I^+ = \{1, 2, 3, 4, \ldots\}$ is the set of all *positive integers*. Clearly, positive integers and natural numbers are synonyms.

Negative Integers

The set $I^- = \{-1, -2, -3, \ldots\}$ is the set of all *negative integers*. 0 is neither positive nor negative.

Non-negative Integers

The set $\{0, 1, 2, 3, \ldots\}$ is the set of all *non-negative integers*.

Rational Numbers

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are known as *rational numbers*, e.g., $\frac{4}{7}, \frac{3}{2}, -\frac{5}{8}, \frac{0}{1}, -\frac{2}{3}$, etc.

The set of all rational numbers is denoted by Q.

i.e., $Q = \left\{ x : x = \dfrac{p}{q} ; p, q \neq I, q \neq 0 \right\}$

Since every natural number 'a' can be written as $\dfrac{a}{1}$ s, so it is a rational number. Since 0 can be written as $\dfrac{0}{1}$ and every non-zero integer 'a' can be written as $\dfrac{a}{1}$, so it is also a rational number.

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimals or, non-terminating repeating decimals.

For example, $\dfrac{1}{5} = 0.2, \dfrac{1}{3} = 0.333 \ldots, \dfrac{22}{7} = 3.1428714287,$ $\dfrac{8}{44} = 0.181818 \ldots$, etc.

The recurring decimals have been given a short notation as

$$0.333\ldots = 0.\overline{3}$$
$$4.1555\ldots = 4.0\overline{5}$$
$$0.323232\ldots = 0.\overline{32}.$$

Irrational Numbers

Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as *irrational numbers*, e.g., $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$, etc.

Note that the exact value of π is not $\dfrac{22}{7} . \dfrac{22}{7}$ is rational while π is irrational number. $\dfrac{22}{7}$ is approximate value of π. Similarly, 3.14 is not an exact value of it.

Real Numbers

The rational and irrational numbers combined together are called *real numbers,* e.g., $\dfrac{13}{21}, \dfrac{2}{5}, -\dfrac{3}{7}, \sqrt{3}, 4 + \sqrt{2}$, etc. are real numbers.

The set of all real numbers is denoted by R.

Note that the sum, difference or, product of a rational and irrational number is irrational, e.g., $3 + \sqrt{2}, 4 - \sqrt{3}, \dfrac{2}{3} - \sqrt{5}, 4\sqrt{3}, -7\sqrt{5}$ are all irrational.

Even Numbers

All those numbers which are exactly divisible by 2 are called *even numbers*, e.g., 2, 6, 8, 10, etc., are even numbers.

Odd Numbers

All those numbers which are not exactly divisible by 2 are called *odd numbers*, e.g., 1, 3, 5, 7, etc., are odd numbers.

Prime Numbers

A natural number other than 1 is a *prime number* if it is divisible by 1 and itself only.

For example, each of the numbers 2, 3, 5, 7, etc., are prime numbers.

Composite Numbers

Natural numbers greater than 1 which are not prime are known as *composite numbers*.

For example, each of the numbers 4, 6, 8, 9, 12, etc., are composite numbers.

Notes:

1. The number 1 is neither a prime number nor a composite number.
2. The number 2 is the only even number which is prime.
3. Prime numbers up to 100 are:
   2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, i.e., 25 prime numbers between 1 and 100.
4. Two numbers which have only 1 as the common factor are called *co-primes* or, *relatively prime* to each other, e.g., 3 and 5 are co-primes.

Note that the numbers which are relatively prime need not necessarily be prime numbers, e.g., 16 and 17 are relatively prime although 16 is not a prime number.

ADDITION AND SUBTRACTION (SHORT-CUT METHODS)

The method is best illustrated with the help of following examples:

Illustration 2 $54321 - (9876 + 8967 + 7689) = ?$

Step 1 Add 1st column:

$$6 + 7 + 9 = 22$$

To obtain 1 at unit's place add 9 to make 31. In the answer, write 9 at unit's place and carry over 3.

Step 2 Add 2nd column:

$$3 + 7 + 6 + 8 = 24$$

$$\begin{array}{r} 54321 \\ \left.\begin{array}{r} 9876 \\ 8967 \\ 7689 \end{array}\right\} \\ \hline 27789 \\ \hline \end{array}$$

To obtain 2 at ten's place, add 8 to make 32. In the answer, write 8 at ten's place and carry over 3.

Step 3 Add 3rd column:

$$3 + 8 + 9 + 6 = 26$$

To obtain 3 at hundred's place, add 7 to make 33. In the answer, write 7 at hundred's place and carry over 3.

Step 4 Add 4th column:

$$3 + 9 + 8 + 7 = 27$$

To obtain 4 at thousand's place add 7 to make 34. In the answer, write 7 at thousand's place and carry over 3.

Step 5 Add 5th column:

To obtain 5 at ten-thousand's place, add 2 to make 5. In the answer, write 2 at ten-thousand's place.

$$\therefore \quad 54321 - (9876 + 8967 + 7689) = 27789$$

Multiplication (Short-cut Methods)

1. Multiplication of a given number by 9, 99, 999, etc., that is by $10^n - 1$.

Method: Put as many zeros to the right of the multiplicant as there are nines in the multiplier and from the result subtract the multiplicant and get the answer.

Illustration 3 Multiply

(*a*) 3893 by 99 (*b*) 4327 by 999

(*c*) 5863 by 9999

Solution:

(*a*) $3893 \times 99 = 389300 - 3893 = 385407$

(*b*) $4327 \times 999 = 4327000 - 4327 = 4322673$

(*c*) $5863 \times 9999 = 58630000 - 5863 = 58624137$

2. Multiplication of a given number by 11, 101, 1001, etc., that is, by $10^n + 1$.

Method: Place *n* zeros to the right of the multiplicant and then add the multiplicant to the number so obtained.

Illustration 4 Multiply

(*a*) 4782×11 (*b*) 9836×101

(*c*) 6538×1001

Solution:

(*a*) $4782 \times 11 = 47820 + 4782 = 52602$

(*b*) $9836 \times 101 = 983600 + 9836 = 993436$

(*c*) $6538 \times 1001 = 6538000 + 6538 = 6544538$

3. Multiplication of a given number by 15, 25, 35, etc.

Method: Double the multiplier and then multiply the multiplicant by this new number and finally divide the product by 2.

Illustration 5 Multiply

(*a*) 7054×15 (*b*) 3897×25

(*c*) 4563×35

Solution:

(*a*) $7054 \times 15 = \frac{1}{2}(7054 \times 30)$

$= \frac{1}{2}(211620) = 105810$

(*b*) $3897 \times 25 = \frac{1}{2}(3897 \times 50) = \frac{1}{2}(194850)$

$= 97425$

(*c*) $4536 \times 35 = \frac{1}{2}(4563 \times 70) = \frac{1}{2}(319410)$

$= 159705$

4. Multiplication of a given number by 5, 25, 125, 625, etc., that is, by a number which is some power of 5.

Method: Place as many zeros to the right of the multiplicant as is the power of 5 in the multiplier, then divide the number so obtained by 2 raised to the same power as is the power of 5.

Illustration 6 Multiply

(*a*) 3982×5 (*b*) 4739×25

(*c*) 7894×125 (*d*) 4863×625

Solution:

(*a*) $3982 \times 2 = \frac{39820}{2} = 19910$

(*b*) $4739 \times 25 = \frac{473900}{2^2} = \frac{473900}{4} = 118475$

(*c*) $7894 \times 125 = \frac{7894000}{2^3} = \frac{7894000}{8}$
$= 986750$

(*d*) $4863 \times 625 = \frac{48630000}{2^4} = \frac{48630000}{16}$
$= 3039375$

DISTRIBUTIVE LAWS

For any three numbers *a*, *b*, *c*, we have

(*a*) $a \times b + a \times c = a \times (b + c)$
(*b*) $a \times b - a \times c = a \times (b - c)$

Illustration 7 $438 \times 637 + 438 \times 367 = ?$

Solution: $438 \times 637 + 438 \times 367 = 438 \times (637 + 367)$
$= 438 \times 1000 = 438000$

Illustration 8 $674 \times 832 - 674 \times 632 = ?$

Solution: $674 \times 832 - 674 \times 632 = 674 \times (832 - 632)$
$= 674 \times 200 = 134800$

SQUARES (SHORT-CUT METHODS)

1. To square any number ending with 5.
Method: $(A5)^2 = A(A + 1)/25$

Illustration 9

(*a*) $(25)^2 = 2\ (2 + 1)/25 = 6/25 = 625$
(*b*) $(45)^2 = 4\ (4 + 1)/25 = 20/25 = 2025$
(*c*) $(85)^2 = 8\ (8 + 1)/25 = 72/25 = 7225$

2. To square a number in which every digit is one.
Method: Count the number of digits in the given number and start writing numbers in ascending order from one to this number and then in descending order up to one.

Illustration 10

(*a*) $11^2 = 121$ (*b*) $111^2 = 12321$
(*c*) $1111^2 = 1234321$
(*d*) $222^2 = 2^2\ (111)^2 = 4\ (12321) = 49284$
(*e*) $3333^2 = 3^2\ (1111)^2 = 9\ (1234321) = 11108889$

3. To square a number which is nearer to $10x$.
Method: Use the formula:
$$x^2 = (x^2 - y^2) + y^2 = (x + y)(x - y) + y^2$$

Illustration 11

(*a*) $(97)^2 = (97 + 3)\ (97 - 3) + 3^2$
$= 9400 + 9 = 9409$

(*b*) $(102)^2 = (102 - 2)\ (102 + 2) + 2^2$
$= 10400 + 4 = 10404$

(*c*) $(994)^2 = (994 + 6)\ (994 - 6) + 6^2$
$= 988000 + 36 = 988036$

(*d*) $(1005)^2 = (1005 - 5)\ (1005 + 5) + 5^2$
$= 1010000 + 25 = 1010025$

DIVISION

Division is repeated subtraction.

For example, when we divide 63289 by 43, it means 43 can be repeatedly subtracted 1471 times from 63289 and the remainder 36 is left.

```
              1471 ← Quotient
Divisor → 43) 63289  ← Dividend
              43
              202
              172
               308
               301
                79
                43
                36   ← Remainder
```

Dividend = (Divisor × Quotient) + Remainder

or, $\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

Illustration 12 On dividing 7865321 by a certain number, the quotient is 33612 and the remainder is 113. Find the divisor.

Solution: $\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}$

$= \frac{7865321 - 113}{33612} = \frac{7865208}{33612} = 234$

Illustration 13 A number when divided by 315 leaves remainder 46 and the value of quotient is 7. Find the number.

Solution: Number = (Divisor × Quotient) + Remainder

$= (315 \times 7) + 46 = 2205 + 46$

$= 2251$

Illustration 14 Find the least number of 5 digits which is exactly divisible by 632.

Solution: The least number of 5 digits is 10000. Dividing this number by 632, the remainder is 520. So, the required number = 10000 + (632 + 520) = 10112.

$$\begin{array}{r|l} & 15 \\ 632 & 10000 \\ & \underline{632} \\ & 3680 \\ & \underline{3160} \\ & 520 \end{array}$$

Illustration 15 Find the greatest number of 5 digits which is exactly divisible by 463.

Solution: The greatest number of 5 digits is 99999. Dividing this number by 463, the remainder is 454. So, the required number = 99999 – 454 = 99545.

$$\begin{array}{r|l} & 215 \\ 463 & 99999 \\ & \underline{926} \\ & 739 \\ & \underline{463} \\ & 2769 \\ & \underline{2315} \\ & 454 \end{array}$$

Illustration 16 Find the number nearest to 13700 which is exactly divisible by 235.

Solution: On dividing the number 13700 by 235, the remainder is 70. Therefore, the nearest number to 13700, which is exactly divisible by 235 = 13700 – 70 = 13630.

$$\begin{array}{r|l} & 58 \\ 235 & 13700 \\ & \underline{1175} \\ & 1950 \\ & \underline{1880} \\ & 70 \end{array}$$

TESTS OF DIVISIBILITY

1. **Divisibility by 2:** A number is divisible by 2 if the unit's digit is zero or divisible by 2.

   For example, 4, 12, 30, 18, 102, etc., are all divisible by 2.

2. **Divisibility by 3:** A number is divisible by 3 if the sum of digits in the number is divisible by 3.

   For example, the number 3792 is divisible by 3 since 3 + 7 + 9 + 2 = 21, which is divisible by 3.

3. **Divisibility by 4:** A number is divisible by 4 if the number formed by the last two digits (ten's digit and unit's digit) is divisible by 4 or are both zero.

   For example, the number 2616 is divisible by 4 since 16 is divisible by 4.

4. **Divisibility by 5:** A number is divisible by 5 if the unit's digit in the number is 0 or 5.

   For example, 13520, 7805, 640, 745, etc., are all divisible by 5.

5. **Divisibility by 6:** A number is divisible by 6 if the number is even and sum of its digits is divisible by 3.

   For example, the number 4518 is divisible by 6 since it is even and sum of its digits 4 + 5 + 1 + 8 = 18 is divisible by 3.

6. **Divisibility by 7:** The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7.

   For example, consider the number 448. On doubling the unit digit 8 of 448 we get 16. Then, 44 – 16 = 28. Since 28 is divisible by 7, 448 is divisible by 7.

7. **Divisibility by 8:** A number is divisible by 8, if the number formed by the last 3 digits is divisible by 8.

   For example, the number 41784 is divisible by 8 as the number formed by last three digits, i.e., 784 is divisible by 8.

8. **Divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.

   For example, the number 19044 is divisible by 9 as the sum of its digits 1 + 9 + 0 + 4 + 4 = 18 is divisible by 9.

9. **Divisibility by 10:** A number is divisible by 10, if it ends in zero.

   For example, the last digit of 580 is zero, therefore, 580 is divisible by 10.

10. **Divisibility by 11:** A number is divisible by 11, if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.

    For example, in the number 38797, the sum of the digits at odd places is 3 + 7 + 7 = 17 and the sum of the digits at even places is 8 + 9 = 17. The difference is 17 – 17 = 0, so the number is divisible by 11.

11. **Divisibility by 12:** A number is divisible by 12 if it is divisible by 3 and 4.

12. **Divisibility by 18:** An even number satisfying the divisibility test of 9 is divisible by 18.

13. **Divisibility by 25:** A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or the last two digits are zero.

    For example, the number 13675 is divisible by 25 as the number formed by the last two digits is 75, which is divisible by 25.

14. **Divisibility by 88:** A number is divisible by 88 if it is divisible by 11 and 8.

15. **Divisibility by 125:** A number is divisible by 125 if the number formed by the last three digits is divisible by 125 or the last three digits are zero.

    For example, the number 5250 is divisible by 125 as 250 is divisible by 125.

Some Useful Short-cut Methods

1. Test to find whether a given number is a prime

Step 1 Select a least positive integer n such that $n^2 >$ given number.

Step 2 Test the divisibility of given number by every prime number less than n.

Step 3 The given number is prime only if it is not divisible by any of these primes.

Illustration 17 Investigate whether 571 is a prime number.

Solution: Since $(23)^2 = 529 < 571$ and $(24)^2 = 576 > 571$

$\therefore \quad n = 24$

Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Since 24 is divisible by 2, 571 is not a prime number

Illustration 18 Investigate whether 923 is a prime number.

Solution: Since $(30)^2 = 900 < 923$ and $(31)^2 = 961 > 923$

$\therefore \quad n = 31$

Prime numbers less than 31 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Since 923 is not divisible by any of these primes, therefore 923 is a prime number

2. The least number which when divided by d_1, d_2 and d_3 leaves the remainders r_1, r_2 and r_3, respectively, such that $(d_1 - r_1) = (d_2 - r_2) = (d_3 - r_3)$ is (LCM of d_1, d_2 and d_3) – $(d_1 - r_1)$ or $(d_2 - r_2)$ or $(d_3 - r_3)$.

Illustration 19 Find the least number which when divided by 9, 10 and 15 leaves the remainders 4, 5 and 10, respectively.

Solution: Here $9 - 4 = 10 - 5 = 15 - 10 = 5$

Also, L.C.M. (9, 10, 15) = 90

$\therefore$ the required least number $= 90 - 5 = 85$

3. A number on being divided by d_1 and d_2 successively leaves the remainders r_1 and r_2, respectively. If the number is divided by $d_1 \times d_2$, then the remainder is $(d_1 \times r_2 + r_1)$.

Illustration 20 A number on being divided by 10 and 11 successively leaves the remainders 5 and 7, respectively. Find the remainder when the same number is divided by 110.

Solution: The required remainder $= d_1 \times r_2 + r_1$

$= 10 \times 7 + 5 = 75$

4. To find the number of numbers divisible by a certain integer.

The method is best illustrated with the help of following example.

Illustration 21 How many numbers up to 532 are divisible by 15?

Solution: We divide 532 by 15.

$$532 = \underline{35} \times 15 + 7$$

The quotient obtained is the required number of numbers. Thus, there are 35 such numbers

Illustration 22 How many numbers up to 300 are divisible by 5 and 7 together?

Solution: L.C.M. of 5 and 7 = 35

We divide 300 by 35

$$300 = \underline{8} \times 35 + 20$$

Thus, there are 8 such numbers.

5. Two numbers when divided by a certain divisor give remainders r_1 and r_2. When their sum is divided by the same divisor, the remainder is r_3. The divisor is given by $r_1 + r_2 - r_3$.

Illustration 23 Two numbers when divided by a certain divisor give remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

Solution: The required divisor $= 437 + 298 - 236 = 499$

Practice Exercises

Difficulty Level-1 (Based on Memory)

1. A student was asked to divide a number by 6 and add 12 to the quotient. He, however, first added 12 to the number and then divided it by 6, getting 112 as the answer. The correct answer should have been:

(*a*) 122 (*b*) 118
(*c*) 114 (*d*) 124

[Based on MAT, 2004]

2. Which of the following integers is the square of an integer for every integer n?

(*a*) $n^2 + 1$ (*b*) $n^2 + n$
(*c*) $n^2 + 2n$ (*d*) $n^2 + 2n + 1$

[Based on MAT, 2004]

3. Given that $N = (521)^{125} \times (125)^{521}$, find the last two digits of N.

(*a*) 75 (*b*) 25
(*c*) 45 (*d*) None of these

4. The sum of the digits of a 3-digit number is subtracted from the number. The resulting number is always:

(*a*) Divisible by 6 (*b*) Not divisible by 6
(*c*) Divisible by 9 (*d*) Not divisible by 9

[Based on MAT, 2004]

5. The least number that must be subtracted from each of the numbers 14, 17, 34 and 42, so that the remainders may be proportional, is:

(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 7

[Based on MAT, 2003]

6. The highest power of 5 that is contained in $125^{125} - 25^{25}$ is:

(*a*) 25 (*b*) 50
(*c*) 75 (*d*) 125

7. Of the 120 people in the room, three-fifths are women. If two-thirds of the people are married, then what is the maximum number of women in the room who could be unmarried?

(*a*) 40 (*b*) 20
(*c*) 30 (*d*) 60

[Based on MAT, 2003]

8. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is:

(*a*) 3 (*b*) 2
(*c*) 1 (*d*) None of these

[Based on MAT, 2002]

9. A number of three digits in scale 7 when expressed in scale 9 has its digits reversed in order. The number is:

(*a*) 248 (*b*) 348
(*c*) 148 (*d*) 448

[Based on MAT, 2002]

10. For every positive real number:

$$\left[\frac{x}{2}\right] + \left[\frac{x+1}{2}\right] = \ldots$$

where ([]) is the greatest integer function.

(*a*) 0 (*b*) 1
(*c*) $[x + 1]$ (*d*) $[x]$

[Based on MAT, 2002]

11. How many 5-digit multiples of 11 are there, if the five digits are 3, 4, 5, 6 and 7 in the same order?

(*a*) 12 (*b*) 13
(*c*) 10 (*d*) None of these

[Based on MAT, 2002]

12. The smallest number by which 3600 can be divided to make it a perfect cube is:

(*a*) 9 (*b*) 50
(*c*) 300 (*d*) 450

[Based on MAT, 2002]

13. The least number having four digits which is a perfect square is:

(*a*) 1004 (*b*) 1016
(*c*) 1036 (*d*) None of these

[Based on MAT, 2002]

14. The remainder when 7^{84} is divided by 342 is:

(*a*) 0 (*b*) 1
(*c*) 49 (*d*) 341

[Based on MAT, 2001]

15. A 2-digit number is such that the product of the digits is 14. When 45 is added to the number, then the digits interchange their places. Find the number.

(*a*) 72 (*b*) 27
(*c*) 37 (*d*) 14

[Based on MAT, 2001]

16. In a division sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder is 48, then what is the dividend?

(*a*) 240 (*b*) 576
(*c*) 4800 (*d*) 4848

[Based on IIFT, 2003]

17. Which of the following integers has the most divisors?

(*a*) 88 (*b*) 91

(*c*) 99 (*d*) 101

[Based on SCMHRD Ent. Exam., 2003]

18. In which of the following pairs of numbers, it is true that their sum is 11 times their product?

(*a*) 1, 1/11 (*b*) 1, 1/10

(*c*) 1, 1/12 (*d*) 1, 10

[Based on SCMHRD, 2002]

19. If m, n, o, p and q are integers, then $m(n+o)(p-q)$ must be even when which of the following is even?

(*a*) $m+n$ (*b*) $n+p$

(*c*) m (*d*) p

[Based on REC Tiruchirapalli, 2002]

20. Which of the following numbers is exactly divisible by 99?

(*a*) 114345 (*b*) 135792

(*c*) 3572404 (*d*) 913464

[Based on MAT, 2005]

21. Of the three numbers, the sum of the first two is 45; the sum of the second and the third is 55 and the sum of the third and thrice the first is 90. The third number is:

(*a*) 20 (*b*) 25

(*c*) 30 (*d*) 35

[Based on MAT, 2005]

22. Two times a 2-digit number is 9 times the number obtained by reversing the digits and sum of the digits is 9. The number is:

(*a*) 72 (*b*) 54

(*c*) 63 (*d*) 81

[Based on MAT (Feb), 2010]

23. The sum of two numbers, one of which is one-third of the other is 36. The smaller number is:

(*a*) 6 (*b*) 7

(*c*) 8 (*d*) 9

[Based on MAT (Sept), 2009]

24. If such numbers which are divisible by 5 and also those which have 5 as one of the digits are eliminated from the numbers 1 to 60, how many numbers would remain?

(*a*) 40 (*b*) 47

(*c*) 53 (*d*) 45

[Based on MAT (May), 2009]

25. How many numbers are there between 500 and 600 in which 9 occurs only once?

(*a*) 19 (*b*) 18

(*c*) 20 (*d*) 21

[Based on MAT (Feb), 2009]

26. One of a group of swans, 7/2 times the square root of the number are playing on the shore of the pond. The two remaining are inside the pond. What is the total number of swans?

(*a*) 10 (*b*) 14

(*c*) 12 (*d*) 16

[Based on MAT (Dec), 2008]

27. A girl counted in the following way on the fingers of her left hand; she started by calling the thumb 1, the index finger 2, the middle finger 3, the ring finger 4, the little finger 5 and then reversed direction calling the ring figure 6, the middle finger 7 and so on. She counted upto 1994. She ended counting on which finger?

(*a*) The middle finger (*b*) The index finger

(*c*) The thumb (*d*) The ring finger

[Based on MAT (Sept), 2008]

28. An Army Commander wishing to draw up his 5180 men in the form of a solid square found that he had 4 men less. If he could get four more men and form the solid square, the number of men in the front row is:

(*a*) 72 (*b*) 68

(*c*) 78 (*d*) 82

[Based on MAT (Feb), 2008]

29. To win an election, a candidate needs three-fourths of the votes cast. If after two-thirds of the votes have been counted, a candidate has 5/6 of what he needs, then what part of the remaining votes does he still need?

(*a*) 1/8 (*b*) 7/12

(*c*) 1/4 (*d*) 3/8

[Based on MAT (Feb), 2008]

30. The sum of the place values of 3 in the number 503535 is:

(*a*) 3300 (*b*) 0.6

(*c*) 60 (*d*) 3030

[Based on MAT (Feb), 2008]

31. Find the whole number which when increased by 20 is equal to one-sixth times the new number:

(*a*) 7 (*b*) 5

(*c*) 3 (*d*) 4

[Based on MAT (Sept), 2007]

32. A number when divided by 765 leaves a remainder 42. What will be the remainder if the number is divided by 17?

(*a*) 8 (*b*) 5

(*c*) 7 (*d*) 6

[Based on MAT (Sept), 2007]

33. After being set up, a company manufactured 6000 scooters in the third year and 7000 scooters in the seventh year. Assuming that the production increases uniformly by a fixed number every year, what is the production in the tenth year?

(*a*) 7850 (*b*) 7650

(*c*) 7750 (*d*) 7950

[Based on MAT (May), 2006]

34. In a class, the number of girls is one less than the number of the boys. If the product of the number of boys and that of girls is 272, then the number of girls in the class is:

(*a*) 15 (*b*) 14

(*c*) 16 (*d*) 17

[Based on MAT (Feb), 2011]

35. A number of friends decided to go on a picnic and planned to spend ₹96 on eatables. Four of them, did not turn up. As a consequence, the remaining ones had to contribute ₹4 each extra. The number of those who attended the picnic was:

(*a*) 8 (*b*) 16

(*c*) 12 (*d*) 24

[Based on MAT (Feb), 2006]

36. A box of light bulbs contains 24 bulbs. A worker replaces 17 bulbs in the shipping department and 13 bulbs in the accounting department. How many boxes of bulbs did the worker use?

(*a*) 1 (*b*) $1\frac{1}{4}$

(*c*) $1\frac{3}{4}$ (*d*) 2

[Based on MAT (Sept), 2003]

37. Of the numbers 7, 9, 11, 13, 29, 33 how many are prime numbers?

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

[Based on MAT, 1998]

38. A three-digit number is selected such that it contains no zeros. Now this three-digit number is written beside itself to form the six-digit number. Its factor is:

(*a*) 5 (*b*) 11

(*c*) 4 (*d*) None of these

39. The sum of the digits of a three-digit number is 16. If the ten's digit of the number is three times the unit's digit and the unit's digit is one-fourth of the hundredth digit, then what is the number?

(*a*) 446 (*b*) 561

(*c*) 682 (*d*) 862

[Based on MAT, 1998]

40. If one-third of a number is 3 more than one-fourth of the number, then what is the number?

(*a*) 18 (*b*) 24

(*c*) 30 (*d*) 36

[Based on MAT, 1998]

41. What is the least fraction which when added to or subtracted from $\frac{29}{12}+\frac{15}{16}$ will make the result a whole number?

(*a*) $\frac{21}{38}$ (*b*) $\frac{31}{38}$

(*c*) $\frac{31}{48}$ (*d*) $\frac{17}{48}$

[Based on MAT, 1999]

42. The smallest perfect square that is divisible by 7! is:

(*a*) 44100 (*b*) 176400

(*c*) 705600 (*d*) 19600

[Based on IIFT, 2010]

43. How many two-digit numbers have their square as 1 more than a multiple of 24?

(*a*) 30 (*b*) 31

(*c*) 32 (*d*) 29

44. If x, y and z are consecutive negative integers, and if $x > y > z$, which of the following must be a positive integer?

(*a*) $x - yz$ (*b*) xyz

(*c*) $x + y + z$ (*d*) $(x - y)(y - z)$

[Based on MHT-CET, MBA, 2010]

45. Sum of three numbers is 132. First number is twice the second and third number is one-third of the first. Find the second number:

(*a*) 18 (*b*) 36

(*c*) 20 (*d*) 16

46. The number obtained by interchanging the two digits of a two digit number is less than the original number by 27. If the difference between the two digits of the number is 3, what is the original number?

(*a*) 74 (*b*) 63

(*c*) 85 (*d*) Cannot be determined

[Based on IRMA, 2009]

47. In certain games, each player scores either 2 points or 5 points. If n players score 2 points and m players score 5 points and the total number of points scored is 50, what is the least possible positive difference between n and m?

(*a*) 5 (*b*) 3

(*c*) 1 (*d*) 7

[Based on NMAT, 2005]

48. What least number must be added to 7231 so that the resulting number is exactly divisible by 5 and 9 together?

(*a*) 20 (*b*) 18

(*c*) 14 (*d*) 16

49. If the digit in the unit's place of a two-digit number is halved and the digit in the ten's place is doubled, the number thus obtained is equal to the number obtained by interchanging the digits. Which of the following is definitely true?

(*a*) Digit in the unit's place and the ten's place are equal.

(*b*) Digit in the unit's place is twice the digit in the ten's place.

(*c*) Sum of the digits is a two-digit number.

(*d*) Digit in the unit's place is half of the digit in the ten's place.

[Based on NMAT, 2005]

50. If m and n are two integers such that $m \times n = 64$, which of the following cannot be the value of $m + n$?

(*a*) 20 (*b*) 65

(*c*) 16 (*d*) 35 **[Based on ATMA, 2005]**

51. If the quotient is positive, which of the following must be true?

(*a*) $a > 0$ (*b*) $b > 0$

(*c*) $a - b > 0$ (*d*) $ab > 0$

[Based on ATMA, 2006]

52. If a positive integers n is divisible by both 5 and 7, n must also be divisible by which of the following?

I. 12 II. 35

III. 70

(*a*) None (*b*) II only

(*c*) I and II (*d*) II and III

[Based on ATMA, 2006]

53. A number consists of there digits whose sum is 10. The middle digit is equal to sum of the other two and the number will be increased by 99, if the final digit and the third digit are interchanged. The digit in the hundreds place is:

(*a*) 3 (*b*) 5

(*c*) 4 (*d*) 2

[Based on ATMA, 2006]

54. If x and y are any natural numbers, then which of the following is an odd number?

(*a*) $x^y + y^x (x - y)(x^y + x)$ (*b*) $x^y (x + y)(x^y + x)$

(*c*) $y^x (x^2 - y)(x^y - x)$ (*d*) None of these

[Based on ATMA, 2008]

55. a, b and c are positive integers divisible by 5, 3 and 12 respectively and p is a two-digit prime number, then which of the following statement (s) is/are TRUE?

I. Product of $abcp$ is zero.

II. $a + b + c + p$ is odd.

III. $(b^2 + c^2) - (p^2 - a^2)$ is odd.

IV. $a(p - c) + a(c + b)$ is divisible by 5.

(*a*) I and IV only (*b*) II and III only

(*c*) II and IV only (*d*) IV only

[Based on ATMA, 2008]

56. If x, y and z and positive integers such that x is a factor of y and x is a multiple of z, which of the following is NOT necessarily an integer?

(*a*) $\frac{xy}{}$ (*b*) $\frac{y+z}{x}$

(*c*) $\frac{yz}{x}$ (*d*) $\frac{x+y}{z}$

[Based on ATMA, 2008]

57. If a is a positive integer and if the unit's digit of a^2 is 9 and $(a + 1)^2$ is 4, what is the unit's digit of $(a + 2)^2$?

(*a*) 1 (*b*) 3

(*c*) 5 (*d*) 14

[Based on ATMA, 2008]

58. When 10 is divided by the positive integer n, the remainder is $n - 4$. Which of the following could be the value of n?

(*a*) 3 (*b*) 4

(*c*) 7 (*d*) 12

[Based on ATMA, 2008]

59. $4^{109} + 6^{109}$ is divided by 25, the remainder is:

(*a*) 20 (*b*) 10

(*c*) 5 (*d*) 0

[Based on JMET, 2006]

60. What is the digit in the units place of 102^{51}?

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 8

[Based on JMET, 2006]

61. Find the least number which must be subtracted from 9269 so that resulting number is exactly divisible by 73:

(*a*) 17 (*b*) 57

(*c*) 71 (*d*) 63

62. Find the least number which must be added to 15463 so that the resulting number is exactly divisible by 107?

(*a*) 52 (*b*) 71

(*c*) 55 (*d*) 19

63. If a, b, c, d and e are real numbers such that $a + b < c + d$, $b + c < d + e$, $c + d < e + a$ and $d + e < a + b$, then:

(*a*) the largest number is a and the smallest is b.

(*b*) the largest number is a and the smallest is c.

(*c*) the largest number is e and the smallest is c.

(*d*) the largest number is c and the smallest is b.

[Based on GBO, Delhi University, 2011]

64. Let $2^{x+y} = 10$, $2^{y+z} = 20$ and $2^{z+x} = 30$ where x, y and z are any three real numbers. The value of $2x$ is:

(*a*) $\frac{3}{2}$ (*b*) $\sqrt{15}$

(*c*) $\frac{\sqrt{6}}{2}$ (*d*) 15

[Based on GBO, Delhi University, 2011]

65. What is the number just more than 5000 which is exactly divisible by 73?

(*a*) 5001 (*b*) 5009

(*c*) 5037 (*d*) 5027

66. The sum of two numbers is 100 and their difference is 37. The difference of their squares is:

(*a*) 37 (*b*) 100

(*c*) 63 (*d*) 3700

67. The number of times 79 be subtracted from 50000, so that the remainder be 43759 is:

(*a*) 69 (*b*) 79

(*c*) 59 (*d*) None of these

68. The nearest figure to 58701 which is divisible by 567 is:

(*a*) 58968 (*b*) 58434

(*c*) 58401 (*d*) None of these

69. The number of five figures to be added to a number of four fives to obtain the least number of six figures exactly divisible by 357 is:

(*a*) 94762 (*b*) 94802

(*c*) 94485 (*d*) None of these

70. The least value to be given to * so that the number 5 * 3457 is divisible by 11 is:

(*a*) 2 (*b*) 3

(*c*) 0 (*d*) 4

Difficulty Level-2 (Based on Memory)

1. Let a, b, c, d be the four integers such that $a + b + c + d = 4m + 1$, where m is a positive integer. Given m, which one of the following is necessarily true?

(*a*) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$

(*b*) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$

(*c*) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$

(*d*) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$.

[Based on CAT, 2003]

2. How many three-digit positive integers with digits x, y and z in the hundred's, ten's and unit's place, respectively, exist such that $x < y$, $z < y$ and $x \neq 0$?

(*a*) 245 (*b*) 285

(*c*) 240 (*d*) 320

[Based on CAT, 2003]

3. The number of positive integers n in the range $12 \leq n \leq 40$ such that the product $(n - 1)(n - 2) \ldots 3 \times 2 \times 1$ is not divisible by n is:

(*a*) 5 (*b*) 7

(*c*) 13 (*d*) 14

4. Let x and y be positive integers such that x is prime and y is composite. Then,

(*a*) $y - x$ cannot be an even integer.

(*b*) xy cannot be an even integer.

(*c*) $\frac{(x+y)}{x}$ cannot be an even integer.

(*d*) None of these **[Based on CAT, 2004]**

5. If a, $a + 2$ and $a + 4$ are prime numbers, then the number of possible solutions for a is:

(*a*) One (*b*) Two

(*c*) Three (*d*) None of these

[Based on CAT, 2004]

6. What is the remainder when 4^{96} is divided by 6?

(*a*) 0 (*b*) 2

(*c*) 3 (*d*) 4

[Based on CAT, 2004]

7. The remainder when 5^{163} is divided by 1000 is:

(*a*) 125 (*b*) 625

(*c*) 25 (*d*) None of these

8. If the sum of n consecutive integers is 0, which of the following must be true?

I. n is an even number.

II. n is an odd number.

III. The average of the n integers is 0.

(*a*) I only (*b*) II only

(*c*) III only (*d*) II and III

9. A player holds 13 cards of four suits of which seven are black and six are red. There are twice as many diamonds as spades and twice as many hearts as diamonds. How many clubs does he hold?

(*a*) 4 (*b*) 5

(*c*) 6 (*d*) 7

[Based on FMS Delhi, 2004]

10. In three coloured boxes: red, green and blue, 108 balls are placed. There are twice as many in the green and red boxes combined as they are in the blue box and twice

as many in the blue box as they are in the red box. How many balls are there in the green box?

(*a*) 18 (*b*) 36
(*c*) 45 (*d*) None of these

[Based on FMS Delhi, 2004]

11. If $a = 1^2, b = 2^3, c = 3^4 \ldots z = 26^{27}$. In the product of all the alphabets, how many zeros exist in the end?

(*a*) 100 (*b*) 104
(*c*) 80 (*d*) 106

[Based on FMS Delhi, 2004]

12. The unit's digit of a two-digit number is one more than the digit at ten's place. If the number is more than five times of the sum of the digits of the number, then find the sum of all such possible numbers:

(*a*) 246 (*b*) 275
(*c*) 290 (*d*) 301

[Based on FMS Delhi, 2004]

13. Let $20 \times 21 \times 22 \times \ldots \times 30 = A$. If A is divisible by 10^x, then find the maximum value of x:

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

[Based on FMS Delhi, 2004]

14. A student was asked to find the sum of all the prime numbers between 10 to 40. He found the sum as 180. Which of the following statements is true?

(*a*) He missed one prime number between 10 and 20.
(*b*) He missed one prime number between 20 and 30.
(*c*) He added one extra prime number between 10 and 20.
(*d*) None of these

[Based on FMS Delhi, 2004]

15. $\sqrt{-1}$ is not defined but it is denoted by i. Clearly, i is not a real number, so it is called imaginary number. Now find $\sum_{n=1}^{100} (i)^n$:

(*a*) i (*b*) 1
(*c*) –1 (*d*) 0

[Based on FMS Delhi, 2004]

16. $(a + b + c + d + e)/(v + w + x + y + z) = N$, where a, b, c, d, e are five consecutive even integers and v, w, x, y, z are five consecutive odd integers. If $v = a + 1$ and n represent natural numbers, then which of the following is the most suitable value of N?

(*a*) $(n + 4)/(n + 5)$ (*b*) $(n + 3)/(n + 4)$
(*c*) $(n + 2)/(n + 3)$ (*d*) $(n + 2)/(n + 2.5)$

[Based on FMS Delhi, 2004]

17. Manu and Tanu are playing mathematical puzzles. Manu asks Tanu: 'which whole numbers, greater than one, divide evenly all the nine numbers, i.e., 111, 222, 333, 444, 555, 666, 777, 888, 999?' Tanu immediately gave the desired answer. It was:

(*a*) 7, 37, 111 (*b*) 3, 37, 111
(*c*) 9, 37, 111 (*d*) 9, 13, 111

18. The smallest prime number that is the fifth term of an increasing arithmetic sequence for which all four preceding terms are also prime:

(*a*) 17 (*b*) 37
(*c*) 29 (*d*) 53

19. When $10^{12} - 1$ is divided by 111, the quotient is:

(*a*) 9009009 (*b*) 9000009
(*c*) 9009009009 (*d*) 9000000009

20. A number N is defined as the addition of 4 different integers. Each of the four numbers gives a remainder zero when divided by four. The first of the four numbers defined as A is known to be as 4^{61}. The other three numbers arranged in the increasing order and defined as B, C and D are each 4 times more than the previous number. Thus, the number $B = 4 \times A$, similarly $C = 4 \times B$ and also $D = 4 \times C$. Thus the number N so formed is perfectly divisible by:

(*a*) 11 (*b*) 10
(*c*) 3 (*d*) 13

21. Which of the following is a prime number?

(*a*) 889 (*b*) 997
(*c*) 899 (*d*) 1147

[Based on FMS Delhi, 2004]

22. A cube is cut into n identical pieces. If it can be done so in only one way, then which of the following could be the value of n?

(*a*) 179 (*b*) 203
(*c*) 143 (*d*) 267

[Based on IIT Joint Man. Ent. Test, 2004]

23. A gardener has to plant trees in rows containing equal number of trees. If he plants in rows of 6, 8, 10 or 12, then five trees are left unplanted. But if he plants in rows of 13 trees each, then no tree is left. What is the number of trees that the gardener plants?

(*a*) 485 (*b*) 725
(*c*) 845 (*d*) None of these

[Based on IIT Joint Man. Ent. Test, 2004]

24. I think of a number. I double the number, add 6 and multiply the result by 10. I now divide by 20 and subtract the number I first thought of. The result is:

(*a*) Depends upon the number thought
(*b*) 1
(*c*) 2
(*d*) 3

25. Consider a 99-digit number created by writing side by side the first fifty four natural numbers as follows:

1 2 3 4 5 6 7 8 9 10 11 12 13 _ _ _ _ 53 54

the above number when divided by 8 will leave a remainder:

(*a*) 6 (*b*) 4
(*c*) 2 (*d*) 0

26. The denominator of a rational number is 3 more than its numerator. If the numerator is increased by 7 and the denominator is decreased by 2, we obtain 2. The rational number is:

(*a*) $\frac{1}{4}$ (*b*) $\frac{5}{8}$
(*c*) $\frac{7}{10}$ (*d*) $\frac{8}{11}$

[Based on FMS Delhi, 2003]

27. A teacher gave the simple multiplication exercise to the kids. But one kid reversed the digits of both the numbers and carried out the multiplication and found that the product was exactly the same as the one expected by the teacher. Only one of the following pairs of numbers will fit in the description of the exercise. Which one is that?

(*a*) 14, 42 (*b*) 42, 28
(*c*) 19, 63 (*d*) 13, 62

28. Find the remainder when $(11^{17^{15}} + 13^{11^{15}})$ is divided by 7:

(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

29. What is the smallest value of n for which $(n^{13} - n)(5^{2n} - 1)$ is divisible by 169?

(*a*) 5 (*b*) 4
(*c*) 1 (*d*) 2

30. If a number is divided by 2 the remainder is 1. If it is divided by 3 the remainder is 2. What is the remainder when the number is divided by 6?

(*a*) 0 (*b*) 1
(*c*) 4 (*d*) 5

31. If there are 10 positive real numbers $n_1 < n_2 < n_3 \ldots < n_{10} \ldots$. How many triplets of these numbers (n_1, n_2, n_3), (n_2, n_3, n_4) … can be generated such that in each triplet the first number is always less than the second number, and the second number is always less than the third number?

(*a*) 45 (*b*) 90
(*c*) 120 (*d*) 180

[Based on CAT, 2002]

32. Number S is obtained by squaring the sum of digits of a two-digit number D. If difference between S and D is 27, then the two digit number D is:

(*a*) 24 (*b*) 54
(*c*) 34 (*d*) 45

[Based on CAT, 2002]

33. The owner of a local jewellery store hired 3 watchmen to guard his diamonds, but a thief still got in and stole some diamonds. On the way out, the thief met each watchman, one at a time. To each he gave half of the diamonds he had then, and 2 more besides. He escaped with one diamond. How many did he steal originally?

(*a*) 40 (*b*) 36
(*c*) 25 (*d*) None of these

[Based on CAT, 2002]

34. A rich merchant had collected many gold coins. He did not want anybody to know about him. One day, his wife asked, 'how many gold coins do we have?' After pausing a moment, he replied, 'well! if I divide the coins into two unequal numbers, then 48 times the difference between the two numbers equals the difference between the squares of the two numbers. 'The wife looked puzzled. Can you help the merchant's wife by finding out how many gold coins the merchant has?

(*a*) 96 (*b*) 53
(*c*) 43 (*d*) None of these

[Based on CAT, 2002]

35. A child was asked to add first few natural numbers (that is 1 + 2 + 3 + ...) so long his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong, the child discovered he had missed one number in the sequence during addition. The number he missed was:

(*a*) Less than 10 (*b*) 10
(*c*) 15 (*d*) More than 15

[Based on CAT, 2002]

36. When 2^{256} is divided by 17, the remainder would be:

(*a*) 1 (*b*) 16
(*c*) 14 (*d*) None of these

[Based on CAT, 2002]

37. After the division of a number successively by 3, 4 and 7, the remainders obtained are 2, 1 and 4, respectively. What will be the remainder if 84 divides the same number?

(*a*) 80 (*b*) 75
(*c*) 41 (*d*) 53

[Based on CAT, 2002]

38. $7^{6n} - 6^{6n}$, where n is an integer > 0, is divisible by:

(*a*) 13 (*b*) 127
(*c*) 559 (*d*) None of these

[Based on CAT, 2002]

39. If $x^2 < 51$ and $y^2 < 21$ and x and y are integers, then which of the following is the least number which when divided by the least value of x and least value of y gives a negative quotient?

(*a*) 28 (*b*) 56

(*c*) –28 (*d*) –56

40. What is the product of remainders when 6^4 is divided by 2^4 and 7^5 is divided by 14^2?

(*a*) 7 (*b*) 5

(*c*) 0 (*d*) 4

41. Of 128 boxes of oranges, each box contains at least 120 and at most 144 oranges. The number of boxes containing the same number of oranges is at least:

(*a*) 5 (*b*) 103

(*c*) 6 (*d*) None of these

[Based on CAT, 2001]

42. In a four-digit number, the sum of the first two digits is equal to that of the last two digits. The sum of the first and last digits is equal to the third digit. Finally, the sum of the second and fourth digits is twice the sum of the other two digits. What is the third digit of the number?

(*a*) 5 (*b*) 8

(*c*) 1 (*d*) 4

[Based on CAT, 2001]

43. Anita had to do a multiplication. Instead of taking 35 as one of the multipliers, she took 53. As a result, the product went up by 540. What is the new product?

(*a*) 1050 (*b*) 540

(*c*) 1440 (*d*) 1590

[Based on CAT, 2001]

44. m is the smallest positive integer such that for any integer $n \le m$, the quantity $n^3 - 7n^2 + 11n - 5$ is positive. What is the value of m?

(*a*) 4 (*b*) 5

(*c*) 8 (*d*) None of these

[Based on CAT, 2001]

45. Three friends, returning from a movie, stopped to eat at a restaurant. After dinner, they paid their bill and noticed a bowl of mints at the front counter. Sita took one-third of the mints, but returned four because she had a monetary pang of guilt. Fatima then took one-fourth of what was left but returned three for similar reasons. Eswari then took half of the remainder but threw two back into the bowl. The bowl had only 17 mints left when the raid was over. How many mints were originally in the bowl?

(*a*) 38 (*b*) 31

(*c*) 41 (*d*) None of these

[Based on CAT, 2001]

46. In a number system, the product of 44 and 11 is 1034. The number 3111 of this system, when converted to the decimal number system, becomes:

(*a*) 406 (*b*) 1086

(*c*) 213 (*d*) 691

[Based on CAT, 2001]

47. A set of consecutive positive integers beginning with 1 is written on the blackboard. A student came along and erased one number. The average of the remaining numbers is $35\frac{7}{77}$. What was the number erased?

(*a*) 7 (*b*) 8

(*c*) 9 (*d*) None of these

[Based on CAT, 2001]

48. Let D be a recurring decimal of the form $D = 0.\,a_1a_2a_1a_2a_1a_2$ ..., where digits a_1 and a_2 lie between 0 and 9. Further, at most one of them is zero. Which of the following numbers necessarily produces an integer, when multiplied by D?

(*a*) 18 (*b*) 108

(*c*) 198 (*d*) 288

[Based on CAT, 2000]

49. What is the value of the following expression?

$$\left(\frac{1}{(2^2-1)}\right)+\left(\frac{1}{(4^2-1)}\right)+\left(\frac{1}{(6^2-1)}\right)+\left(\frac{1}{(20^2-1)}\right)$$

(*a*) $\frac{9}{19}$ (*b*) $\frac{10}{19}$

(*c*) $\frac{10}{21}$ (*d*) $\frac{11}{21}$

[Based on CAT, 2000]

50. Consider a sequence of seven consecutive integers. The average of the first five integers is n. The average of all the seven integers is:

(*a*) n

(*b*) $n + 1$

(*c*) $k \times n$, where k is a function of n

(*d*) $n + \left(\frac{2}{7}\right)$

[Based on CAT, 2000]

51. Let $N = 1421 \times 1423 \times 1425$. What is the remainder when N is divided by 12?

(*a*) 0 (*b*) 9

(*c*) 3 (*d*) 6

[Based on CAT, 2000]

52. The integers 34041 and 32506, when divided by a three-digit integer n, leave the same remainder. What is the value of n?

(*a*) 289 (*b*) 367

(*c*) 453 (*d*) 307

[Based on CAT, 2000]

53. $f(a, b, c) = a + b + c$ and $g(a, b, c) = a \times b \times c$.

Then, how many such integer triplets a, b, c are there for which $f(a, b, c) = g(a, b, c)$? (a, b, c are all distinct).

(*a*) 0 (*b*) Only 1

(*c*) 2 (*d*) More than 2

54. Let $N = 55^3 + 17^3 - 72^3$. N is divisible by:

(*a*) Both 7 and 13 (*b*) Both 3 and 13

(*c*) Both 17 and 7 (*d*) Both 3 and 17

[Based on CAT, 2000]

55. Which of the following numbers has maximum factors?

(*a*) 36 (*b*) 76

(*c*) 82 (*d*) 191

56. Which of the following numbers has minimum factors?

(*a*) 58 (*b*) 88

(*c*) 137 (*d*) 184

57. From 1–90 how many numbers end in 4?

(*a*) 25 per cent (*b*) 30 per cent

(*c*) 20 per cent (*d*) 10 per cent

58. From 10–99 both inclusive how many numbers have their unit digit smaller than the other digit?

(*a*) 90 (*b*) 45

(*c*) 32 (*d*) 26

59. If $x = \sqrt{\frac{5}{2}+\sqrt{\frac{13}{4}+6\sqrt{\frac{5}{2}+\sqrt{\frac{13}{4}+6\sqrt{\dots \text{ to infinite terms,}}}}}}$

then $x =$

(*a*) $\frac{3+\sqrt{2}}{2}$ (*b*) $\frac{3+\sqrt{5}}{2}$

(*c*) $\frac{2\sqrt{5}}{3}$ (*d*) $\frac{3\sqrt{5}+1}{2}$

60. The number of people in a row is equal to the number of rows in a playground. If total number of people in the playground is 19044, find the number of rows:

(*a*) 128 (*b*) 138

(*c*) 148 (*d*) 158

61. Let R be the remainder when $35n + 1$ is divided by 7. Which of the following statements are true?

I. $R = 4$, when n is even.

II. $R = 5$, when n is even.

III. $R = 6$, when n is odd.

IV. $R = 3$, when n is odd.

(*a*) I and III (*b*) II and III

(*c*) II and IV (*d*) I and IV

62. If $2x - 1$ is an odd number and $3y - 1$ is an even number, which of the following is/are necessarily even?

I. $x^2 - 2y + 2$

II. $y^2 - 2x + 3$

III. $4x^2 - y - 1$

(*a*) I only (*b*) II only

(*c*) I and II (*d*) II and III

63. Which of the following statements is/are true?

I. $n^p - n$ is divisible by p where n and p are integers.

II. $n^p - n$ is divisible by p where n is a whole number and p is a natural number.

III. $n^p - n$ is divisible by p where n is an integer and p is a prime number.

(*a*) Only I (*b*) Only II

(*c*) Only III (*d*) I and III

64. A two-digit number is four times the sum of the two digits. If the digits are reversed, the number so obtained is 18 more than the original number. What is the original number?

(*a*) 36 (*b*) 24

(*c*) 48 (*d*) None of these

65. $\left(\left(\left(\left(2^{1-\frac{1}{2}}\right)^{1-\frac{1}{3}}\right)\right)^{1-\frac{1}{4}}\right)^{\dots(n \text{ terms})}$ is equal to

(*a*) $(2)^{\frac{1}{n+1}}$ (*b*) 2^n

(*c*) $2^{\frac{n}{n+1}}$ (*d*) $2^{\log n}$

66. The number 311311311311311311311 is:

(*a*) Divisible by 3 but not by 11

(*b*) Divisible by 11 but not by 3

(*c*) Divisible by both 3 and 11

(*d*) Neither divisible by 3 nor by 11

[Based on SNAP, 2007]

67. If $p = 23^n + 1$, then which of the following is correct about p?

(*a*) p is always divisible by 24.

(*b*) p is never divisible by 24.

(*c*) p is always divisible by 22.

(*d*) p is never divisible by 22.

68. A three-digit number $4a3$ is added to another three-digit number 984 to give the four-digit number $13b7$, which is divisible by 11. Then, $(a + b)$ is:

(*a*) 10 (*b*) 11

(*c*) 12 (*d*) 15

[Based on FMS (MS), 2006]

69. A three-digit number has, from left to right, the digits h, t and u with $h > u$. When the number with the digits reversed is subtracted from the original number, the units' digit in the difference is 4. The next two digits, from right to left, are:

(*a*) 5 and 9 (*b*) 9 and 5

(*c*) 5 and 4 (*d*) 4 and 5

[Based on FMS, 2011]

70. In our number system the base is ten. If the base were changed to four, you would count as follows:

1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30,

The twentieth number would be:

(*a*) 110 (*b*) 104

(*c*) 44 (*d*) 38

[Based on FMS, 2011]

71. If the square of a number of two digits is decreased by the square of the number formed by reversing the digits, then the result is not always divisible by:

(*a*) 9

(*b*) the product of the digits.

(*c*) the sum of the digits.

(*d*) the difference of the digits.

[Based on FMS, 2011]

72. If the digit 1 is placed after a two-digit number whose ten's digit is t, and unit's digit is u, the new number is:

(*a*) $10t + u + 1$

(*b*) $100t + 10u + 1$

(*c*) $1000t + 10u + 1$

(*d*) $t + u + 1$

[Based on FMS, 2011]

73. A number n is said to be perfect, if the sum of all its divisors (excluding n itself) is equal to n. An example of perfect number is:

(*a*) 9 (*b*) 15

(*c*) 21 (*d*) 6

[Based on XAT, 2006]

74. For how many integers n, $\frac{n}{20-n}$ is the square of an integer?

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

[Based on XAT, 2007]

75. Let p be any positive integer and $2x + p = 2y$, $p + y = x$ and $x + y = z$. For what value of p would $x + y + z$ attain its maximum value?

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

[Based on XAT, 2007]

76. Let S be the set of rational numbers with the following properties:

I. $\frac{1}{2} \in S$;

II. If $x \neq S$, then both $\frac{1}{x+1} \in S$ and $\frac{x}{x+1} \in S$

Which of the following is true?

(*a*) S contains all rational numbers in the interval $0 < x < 1$.

(*b*) S contains all rational numbers in the interval $-1 < x < 1$.

(*c*) S contains all rational numbers in the interval $-1 < x < 0$.

(*d*) S contains all rational numbers in the interval $-1 < x < \infty$.

[Based on XAT, 2007]

77. We define a function f on the integers $f(x) = x/10$, if x is divisible by 10, and $f(x) = x + 1$ if x is not divisible by 10. If $A_0 = 1994$ and $A_{n+1} = f(A_n)$, what is the smallest n such that $A_n = 2$?

(*a*) 9 (*b*) 18

(*c*) 128 (*d*) 1993

[Based on XAT, 2007]

78. Four digits of the number 29138576 are omitted so that the result is as large as possible. The largest omitted digit is:

(*a*) 9 (*b*) 8

(*c*) 7 (*d*) 5

[Based on XAT, 2008]

Directions (Q. 79): *The question given below is followed by two statements labelled as I and II. You have to decide if these statements are sufficient to conclusively answer the question. Choose the appropriate answer from options given below:*

(*a*) If Statement I alone is sufficient to answer the question.

(*b*) If Statement II alone is sufficient to answer the question.

(*c*) If Statement I and Statement II together are sufficient but neither of the two alone is sufficient to answer the question.

(*d*) If either Statement I or Statement II alone is sufficient to answer the question.

(*e*) Both Statement I and Statement II are insufficient to answer the question.

79. A, B, C, D, E and F are six integers such that $E < F$, $B > A$, $A < D < B$. C is the greatest integer?

I. $E + B < A + D$

II. $D < F$

[Based on XAT, 2008]

80. If x and y are real numbers, then the minimum value of $x^2 + 4xy + 6y^2 - 4y + 4$ is:

(*a*) –4 (*b*) 0

(*c*) 2 (*d*) 4

[Based on XAT, 2010]

81. Let X be a four-digit positive integer such that the unit digit of X is prime and the product of all digits of X is also prime. How many such integers are possible?

(*a*) 4 (*b*) 8

(*c*) 12 (*d*) 24

[Based on XAT, 2010]

82. The micro manometer in a certain factory can measure the pressure inside the gas chamber from 1 unit to 999999 units. Lately this instrument has not been working properly. The problem with the instrument is that it always skips the digit 5 and moves directly from 4 to 6. What is the actual pressure inside the gas chamber if the micro manometer displays 003016?

(*a*) 2201 (*b*) 2202

(*c*) 2600 (*d*) 2960

[Based on XAT, 2011]

83. Let $a_n = 1111111 \ldots 1$, where 1 occurs n number of times. Then,

I. a_{741} is not a prime.

II. a_{534} is not a prime.

III. a_{123} is not a prime.

IV. a_{77} is not a prime.

(*a*) (I) is correct

(*b*) (I) and (II) are correct

(*c*) (II) and (III) are correct

(*d*) All of these are correct

[Based on XAT, 2011]

84. What is the least value of K so that the number 6735K1 is divisible by 9?

(*a*) 5 (*b*) 7

(*c*) 4 (*d*) 3

85. What is the remainder when $7^{74} - 5^{74}$ is divided by 4?

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) None of these

[Based on CAT, 2009]

86. The sum of the number of factors of the number N and N^2 is 34. How many such distinct numbers $N < 150$ exist?

(*a*) 6 (*b*) 2

(*c*) 4 (*d*) 3

[Based on CAT, 2010]

87. The values of the numbers 2^{2004} and 5^{2004} are written one after another. How many digits are there in all?

(*a*) 4008 (*b*) 2003

(*c*) 2004 (*d*) None of these

[Based on CAT, 2011]

88. Let $P = \{2, 3, 4, \ldots, 100\}$ and $Q = \{101, 102, 103, \ldots, 200\}$. How many elements of Q are there such that they do not have element of P as a factor?

(*a*) 20 (*b*) 24

(*c*) 23 (*d*) 21

[Based on CAT, 2012]

89. What is the sum of all the two-digit numbers that leave a remainder of 6 when divided by 8?

(*a*) 612 (*b*) 594

(*c*) 324 (*d*) 872

[Based on CAT, 2012]

90. Find the remainder of 2^{1040} divided by 131.

(*a*) 1 (*b*) 3

(*c*) 5 (*d*) 7

[Based on CAT, 2012]

91. If p is a prime number, $p > 3$, and let x be the product of positive numbers 1, 2, 3,...., $(p - 1)$, then consider the following statements:

I. x is a composite number divisible by p.

II. x is a composite number not divisible by p but some prime number greater than p may divide x.

III. x is not divisible by any prime number $(p - 2)$.

IV. All prime numbers less than $(p - 1)$ divide$x$

which of the following statement(s) is/are correct.

(*a*) I and II are correct

(*b*) II and III are correct

(*c*) III and IV are correct

(*d*) IV alone is correct

[Based on CAT, 2013]

92. A three-digit number which on being subtracted from another three-digit number consisting of the same digits in reverse order gives 594. The minimum possible sum of all the three digits of this number is:

(*a*) 6 (*b*) 7
(*c*) 8 (*d*) Cannot be determined

[Based on CAT, 2013]

93. If $22^3 + 23^3 + 24^3 + \ldots + 87^3 + 88^3$ is divided by 110, then the remainder will be:

(*a*) 55 (*b*) 1
(*c*) 0 (*d*) 44

[Based on CAT, 2013]

94. If p, q, r and s are positive real numbers such that $p + q + r + s = 2$, then $m = (p + q)(r + s)$ satisfies the relation:

(*a*) $0 \le m \le 1$

(*b*) $1 \le m \le 2$

(*c*) $2 \le m \le 3$

(*d*) $3 \le m \le 4$

95. Consider four digit numbers for which the first two digits and the last two digits are equal. How many such numbers are perfect squares?

(*a*) 2 (*b*) 4
(*c*) 0 (*d*) 1

[Based on CAT, 2007]

96. A confused bank teller transposed the rupees and paise when he cashed a cheque for Shailaja, giving her rupees instead of paise and paise instead of rupees. After buying a toffee for 50 paise, Shailaja noticed that she was left with exactly three times as much as the amount on the cheque. Which of the following is a valid statement about the cheque amount?

(*a*) Over ₹7 but less than ₹8

(*b*) Over ₹22 but less than ₹23

(*c*) Over ₹18 but less than ₹19

(*d*) Over ₹4 but less than ₹5

[Based on CAT, 2007]

97. A set of consecutive positive integers beginning with 1 is written on the blackboard. A student came along and erased one number. The average of the remaining numbers is $35\frac{7}{17}$. What was the number erased?

(*a*) 7 (*b*) 8
(*c*) 9 (*d*) None of these

[Based on CAT, 2001]

98. Three friends, returning from a movie, stopped to eat at a restaurant. After dinner, they paid bill and noticed a bowl of mints at the front counter. Sita took 1/3 of the mints, but returned four because she had a monetary pang of guilt. Fatima then took 1/4 of what was left but returned three for similar reasons. Eswari then took half of the remainder but threw two back into the bowl. The bowl had only 17 mints left when the raid was over. How many mints were originally in the bowl?

(*a*) 38 (*b*) 31
(*c*) 41 (*d*) None of these

[Based on CAT, 2001]

99. *ABCDEFGH* is a regular octagon. *A* and *E* are opposite vertices of octagon. A frog starts jumping from vertex to vertex, beginning from *A*. From any vertex of the octagon except *E*, it may jump to either of the two adjacent vertices. When it reaches *E*, the frog stops and stays there. Let, a_n be the number of distinct paths of exactly n jumps ending in *E*. Then, what is the value of a_{2n-1}?

(*a*) 0 (*b*) 4
(*c*) 2_{n-1} (*d*) None of these

[Based on CAT, 2000]

100. Let $N = 55^3 + 17^3 - 72^3$. N is divisible by:

(*a*) Both 7 and 13 (*b*) Both 3 and 13
(*c*) Both 17 and 7 (*d*) Both 3 and 17

[Based on CAT, 2000]

101. The integers 34041 and 32506, when divided by a three-digit integer n, leave the same remainder. What is the value of n?

(*a*) 289 (*b*) 367
(*c*) 453 (*d*) 307

[Based on CAT, 2000]

102. Let D be a recurring decimal of the form $D = 0.\ a_1\ a_2\ a_1\ a_2\ a_1\ a_2 \ldots$, where digits a_1 and a_2 lie between 0 and 9. Further, at most one of them is zero. Which of the following numbers necessarily produces an integer, when multiplied by D?

(*a*) 18 (*b*) 108
(*c*) 198 (*d*) 288

[Based on CAT, 2000]

103. If $n = 1 + x$, where x is the product of four consecutive positive integers, then which of the following is/are true?

A. n is odd

B. n is prime

C. n is a perfect square

(*a*) A and C only (*b*) A and B only
(*c*) A only (*d*) None of these

[Based on CAT, 1999]

104. The remainder when 7^{84} is divided by 342 is:

(*a*) 0 (*b*) 1

(*c*) 49 (*d*) 341

[Based on CAT, 1999]

105. Let *a*, *b*, *c* be distinct digits. Consider a two-digit number '*ab*' and a three-digit number '*ccb*', defined under the usual decimal number system, if $ab^2 = ccb > 300$, then the value of *b* is:

(*a*) 1 (*b*) 0

(*c*) 5 (*d*) 6

[Based on CAT, 1999]

106. A hundred digit number is formed by writing first 54 natural numbers in front of each other as 12345678910111213 . . . . Find the remainder when this number is divided by 8.

(*a*) 1 (*b*) 7

(*c*) 2 (*d*) 0

[Based on CAT, 1998]

107. A certain number when divided by 899 leaves the remainder 63. Find the remainder when the same is divided by 29.

(*a*) 5 (*b*) 4

(*c*) 1 (*d*) Cannot be determined

[Based on CAT, 1998]

108. What is the digit in the unit's place of 2^{51}?

(*a*) 2 (*b*) 8

(*c*) 1 (*d*) 4

[Based on CAT, 1998]

109. n^3 is odd. Which of the following statement(s) is (are) true?

A. *n* is odd

B. n^2 is odd

C. n^2 is even

(*a*) A only (*b*) B only

(*c*) A and B only (*d*) A and C only

[Based on CAT, 1998]

110. Which of the following is true?

(*a*) $7^{3^2} = (7^3)^2$ (*b*) $7^{3^2} > (7^3)^2$

(*c*) $7^{3^2} < (7^3)^2$ (*d*) None of these

[Based on CAT, 1997]

111. If *m* and *n* are integers divisible by 5, which of the following is not necessarily true?

(*a*) $(m - n)$ is divisible by 5

(*b*) $(m^2 - n^2)$ is divisible by 25

(*c*) $(m + n)$ is divisible by 10

(*d*) None of these

[Based on CAT, 1997]

112. *P* and *Q* are two positive integers such that $PQ = 64$. Which of the following cannot be the value of $P + Q$?

(*a*) 20 (*b*) 65

(*c*) 16 (*d*) 35

[Based on CAT, 1997]

113. If *n* is an integer, how many values of *n* will give an integral value of $\frac{(16n^2 + 7n + 6)}{n}$?

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) None of these

[Based on CAT, 1997]

114. If *n* is any odd number greater than 1, then $n(n^2 - 1)$ is:

(*a*) divisible by 96 always

(*b*) divisible by 48 always

(*c*) divisible by 24 always

(*d*) None of these

[Based on CAT, 1996]

115. If a number 774958*A*96*B* is to be divisible by 8 and 9, the respective values of *A* and *B* will be:

(*a*) 7 and 8 (*b*) 8 and 0

(*c*) 5 and 8 (*d*) None of these

[Based on CAT, 1996]

116. Three consecutive positive even numbers are such that thrice the first number exceeds double the third by 2, the third number is:

(*a*) 10 (*b*) 14

(*c*) 16 (*d*) 12

[Based on CAT, 1995]

117. The remainder obtained when a prime number greater than 6 is divided by 6 is:

(*a*) 1 or 3 (*b*) 1 or 5

(*c*) 3 or 5 (*d*) 4 or 5

[Based on CAT, 1995]

118. The value of $\frac{55^3 + 45^3}{55^2 - 55 \times 45 + 45^2}$ is:

(*a*) 100 (*b*) 105

(*c*) 125 (*d*) 75

[Based on CAT, 1995]

119. The number of positive integers not greater than 100, which are not divisible by 2, 3 or 5, is:

(*a*) 26 (*b*) 18

(*c*) 31 (*d*) None of these

[Based on CAT, 1993]

120. What are the last two digits of 7^{2008}?

(*a*) 21 (*b*) 61

(*c*) 01 (*d*) 41

[Based on CAT, 2008]

121. After distributing the sweets equally among 25 children, 8 sweets remain. Had the number of children been 28, 22 sweets would have been left after equal distribution. What was the total number of sweets?

(*a*) 328 (*b*) 348

(*c*) 358 (*d*) Data inadequate

[Based on SNAP, 2013]

122. Consider four natural numbers: x, y, $x + y$ and $x - y$. Two statements are provided below:

I. All four numbers are prime numbers.

II. The arithmetic mean of the numbers is greater than 4.

Which of the following statements would be sufficient to examine the sum of the four numbers?

(*a*) Statement I

(*b*) Statement II

(*c*) Statement I and Statement II

(*e*) Either Statement I or Statement II

123. How many whole numbers between 100 and 800 contain the digit 2?

(*a*) 200 (*b*) 214

(*c*) 220 (*d*) 240

(*e*) 248

[Based on XAT, 2013]

124. p, q and r are three non negative integers such that $p + q + r = 10$. The maximum value of $pq + qr + pr + pqr$ is:

(*a*) ≥ 40 and < 50 (*b*) ≥ 50 and < 60

(*c*) ≥ 60 and < 70 (*d*) ≥ 70 and < 80

[Based on XAT, 2013]

Answer Keys

Difficulty Level-1

1. (*a*)	**2.** (*d*)	**3.** (*b*)	**4.** (*c*)	**5.** (*c*)	**6.** (*b*)	**7.** (*a*)	**8.** (*d*)	**9.** (*a*)	**10.** (*d*)	**11.** (*a*)	**12.** (*d*)	**13.** (*d*)
14. (*b*)	**15.** (*b*)	**16.** (*d*)	**17.** (*a*)	**18.** (*b*)	**19.** (*c*)	**20.** (*a*)	**21.** (*c*)	**22.** (*d*)	**23.** (*d*)	**24.** (*a*)	**25.** (*b*)	**26.** (*d*)
27. (*d*)	**28.** (*a*)	**29.** (*b*)	**30.** (*d*)	**31.** (*d*)	**32.** (*a*)	**33.** (*c*)	**34.** (*c*)	**35.** (*a*)	**36.** (*d*)	**37.** (*b*)	**38.** (*b*)	**39.** (*d*)
40. (*d*)	**41.** (*d*)	**42.** (*b*)	**43.** (*a*)	**44.** (*d*)	**45.** (*b*)	**46.** (*d*)	**47.** (*b*)	**48.** (*c*)	**49.** (*b*)	**50.** (*d*)	**51.** (*d*)	**52.** (*b*)
53. (*d*)	**54.** (*a*)	**55.** (*d*)	**56.** (*b*)	**57.** (*c*)	**58.** (*c*)	**59.** (*c*)	**60.** (*d*)	**61.** (*c*)	**62.** (*a*)	**63.** (*a*)	**64.** (*b*)	**65.** (*c*)
66. (*d*)	**67.** (*b*)	**68.** (*a*)	**69.** (*a*)	**70.** (*a*)								

Difficulty Level-2

1. (*b*)	**2.** (*c*)	**3.** (*b*)	**4.** (*d*)	**5.** (*a*)	**6.** (*d*)	**7.** (*a*)	**8.** (*d*)	**9.** (*c*)	**10.** (*d*)	**11.** (*d*)	**12.** (*c*)	**13.** (*b*)
14. (*d*)	**15.** (*d*)	**16.** (*d*)	**17.** (*b*)	**18.** (*c*)	**19.** (*c*)	**20.** (*b*)	**21.** (*b*)	**22.** (*a*)	**23.** (*c*)	**24.** (*d*)	**25.** (*c*)	**26.** (*b*)
27. (*d*)	**28.** (*b*)	**29.** (*d*)	**30.** (*d*)	**31.** (*c*)	**32.** (*b*)	**33.** (*b*)	**34.** (*d*)	**35.** (*d*)	**36.** (*a*)	**37.** (*d*)	**38.** (*b*)	**39.** (*a*)
40. (*c*)	**41.** (*a*)	**42.** (*a*)	**43.** (*d*)	**44.** (*d*)	**45.** (*d*)	**46.** (*a*)	**47.** (*a*)	**48.** (*c*)	**49.** (*c*)	**50.** (*b*)	**51.** (*c*)	**52.** (*d*)
53. (*d*)	**54.** (*d*)	**55.** (*a*)	**56.** (*c*)	**57.** (*d*)	**58.** (*b*)	**59.** (*b*)	**60.** (*b*)	**61.** (*a*)	**62.** (*d*)	**63.** (*c*)	**64.** (*b*)	**65.** (*a*)
66. (*d*)	**67.** (*d*)	**68.** (*a*)	**69.** (*b*)	**70.** (*a*)	**71.** (*b*)	**72.** (*b*)	**73.** (*d*)	**74.** (*c*)	**75.** (*a*)	**76.** (*a*)	**77.** (*a*)	**78.** (*d*)
79. (*a*)	**80.** (*c*)	**81.** (*a*)	**82.** (*a*)	**83.** (*d*)	**84.** (*a*)	**85.** (*a*)	**86.** (*b*)	**87.** (*d*)	**88.** (*d*)	**89.** (*b*)	**90.** (*a*)	**91.** (*d*)
92. (*c*)	**93.** (*a*)	**94.** (*a*)	**95.** (*d*)	**96.** (*c*)	**97.** (*a*)	**98.** (*d*)	**99.** (*d*)	**100.** (*d*)	**101.** (*d*)	**102.** (*c*)	**103.** (*a*)	**104.** (*b*)
105. (*a*)	**106.** (*a*)	**107.** (*a*)	**108.** (*b*)	**109.** (*c*)	**110.** (*b*)	**111.** (*c*)	**112.** (*d*)	**113.** (*c*)	**114.** (*c*)	**115.** (*b*)	**116.** (*b*)	**117.** (*b*)
118. (*a*)	**119.** (*a*)	**120.** (*c*)	**121.** (*c*)	**122.** (*a*)	**123.** (*b*)	**124.** (*c*)						

Explanatory Answers

Difficulty Level-1

1. (*a*) Let x be the number,

$\therefore \quad (x+12)\div 6 = 112 \Rightarrow \dfrac{x+12}{6} = 112$

$\Rightarrow \quad x = 112\times 6 - 12$

$\Rightarrow \quad x = 672 - 12 = 660$

$\therefore$ Correct answer $= \dfrac{x+12}{6}$

$= \dfrac{660}{6} + 12 = 110 + 12 = 122.$

2. (*d*) $(n+1)^2 = n^2 + 2n + 1.$

3. (*b*) Last 2 digits of $(125)^{521}$ will be 25.

To find the last two digits of $(521)^{125}$, we need to consider $(21)^{125}$ only.

The last 2 digits for different powers of 21 are:

$(21)^1 \rightarrow 21$
$(21)^2 \rightarrow 41$
$(21)^3 \rightarrow 61$
$(21)^4 \rightarrow 81$
$(21)^5 \rightarrow 01$
$(21)^6 \rightarrow 21$

It is a cycle of 5 for the last two digits.

So, 125 being divisible by 5, the last 2 digits of $(521)^{125}$ will be 01.

Thus, $25\times 01 = 25$

Therefore, last two digits of N are 25.

4. (*c*) Let the three-digit number be $100x + 10y + z$

$\therefore \quad (100x + 10y + z) - (x + y + z) = 99x + 9y$

$= 9(11x + y)$

which is always divisible by 9.

5. (*c*) Let x must be subtracted from 14, 17, 34 and 42 such that

$(14 - x)(42 - x) = (17 - x)(34 - x) \Rightarrow x = 2.$

6. (*b*) $125^{125} - 25^{25} = 5^{375} - 5^{50} = 5^{50}(5^{325} - 1).$

Now $5^{325} - 1$ is not divisible by 5. Hence, the highest power of 5 that is contained in the given expression is 50.

7. (*a*) No. of women in the room $= \dfrac{2}{5}\times 120 = 72$

No. of married people $= \dfrac{2}{5}\times 120 = 80$

No. of unmarried people = 40

No. of men in the room = 48

If all the men are supposed to be married, then number of married women could be $80 - 48 = 32$

$\therefore$ Maximum number of unmarried women could be

$72 - 32 = 40.$

8. (*d*) $x = 2 + 2^{2/3} + 2^{1/3} \Rightarrow x > 2$

For, $x = 2,\ x^3 - 6x^2 + 6x = -4$

For, $x = 3,\ x^3 - 6x^2 + 6x = -9$

$\therefore \quad x^3 - 6x^2 + 6x < 0.$

9. (*a*) 248 in the scale of 7 is written as 503. In scale 9, it is written as 305.

10. (*d*) Given expression $= \dfrac{x}{2} + \dfrac{x+1}{2}$

$= \dfrac{2x+1}{2} = x + \dfrac{1}{2} = [x].$

11. (*a*) 5 3 6 4 7 is a multiple of 11 because the difference of the sum of the digits in the odd places and the sum of the digits in the even places is divisible by 11, because

$(5 + 6 + 7) - (3 + 4) = 11$

$\therefore$ Total number of five-digit multiples of 11

= 3! (Permutation of 5, 6 and 7 in the odd places)
$\times$ 2! (Permutation of 3 and 4 in the even places)
$= 6\times 2 = 12.$

12. (*d*) $\dfrac{3600}{450} = 8 = 2^3.$

13. (*d*) 1024.

14. (*b*) $7^3 = 343$, when divided by 342, leaves a remainder of 1

$7^4 = 2401$, when divided by 342, leaves a remainder of 7

$7^5 = 16807$, when divided by 342, leaves a remainder of 49

$7^6 = 117649$, when divided by 342, leaves a remainder of 1

And so on.

$\therefore \ 7^{84}$, when divided by 342, will leave a remainder of 1.

15. (*b*) Let the digits be a and b such that the number is $10a + b$

$\therefore \quad ab = 14$ and $10a + b + 45$

$= 10b + a$

i.e., $9a - 9b = -45$

i.e., $a - b = -5$

$\therefore \quad (a+b)^2 = (a-b)^2 + 4ab = 81$

$\Rightarrow \quad a+b = 9$

$\Rightarrow \quad a = 2, b = 7$

$\therefore$ The number is 27.

16. (*d*) Divisor $= 12 \times$ Quotient

Divisor $= 5 \times$ Remainder

Remainder $= 48$

$\Rightarrow$ Divisor $= 240$,

$\therefore$ Quotient $= 20$

Hence,

Dividend $= 240 \times 20 + 48 = 4848$.

17. (*a*) Divisors of 88 are 2, 4, 8, 11, 22, 44

Divisors of 91 are 7 and 13

Divisors of 99 are 3, 9, 11, 33.

18. (*b*) $1 + \frac{1}{10} = \frac{11}{10}$

$1 \times \frac{1}{10} = \frac{1}{10}$

$\therefore$ Sum $= \frac{11}{10} = 11 \times \frac{1}{10} = 11 \times$ Product.

19. (*c*) $m(n+0)(p-q)$ is even $\Rightarrow m$ must be even.

20. (*a*) A number divisible by 99 must be divisible by 9 as well as 11.

$\therefore$ 114345 is divisible by both.

21. (*c*) Let the numbers be x, y and z.

$x + y = 45,\ y + z = 55$ and $3x + z = 90$

$y = 45 - x,$

$z = 55 - y = 55 - (45 - x) = 10 + x$

$\therefore \quad 3x + 10 + x = 90$

or, $\quad x = 20$

$y = 45 - 20 = 25$

$z = 10 + 20 = 30$

$\therefore$ Third number is 30.

22. (*d*) Let the two-digit number $= xy$

$\therefore \quad 2(10x + y) = 9(10y + x)$

$\Rightarrow \quad 88y - 11x = 0 \qquad (1)$

Also, $\quad x + y = 9 \qquad (2)$

Solving Eqs. (1) and (2), we get

$x = 8$ and $y = 1$

So, the number is 81.

23. (*d*) Let the numbers be $3x$ and x.

$3x + x = 36$

$\Rightarrow \quad 4x = 36$

$\Rightarrow \quad x = 9.$

24. (*a*) Eliminated numbers are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 51, ..., 60

So, total eliminated numbers are 20

$\therefore$ 40 numbers would remain.

25. (*b*) Required numbers are 509, 519, 529, 539, 549, 559, 569, 579, 589, 590, 591, 592, ..., 1598.

26. (*d*) Let the total number of swans be x.

The number of swans playing on shore $= \frac{7}{2}\sqrt{x}$

Number of remaining swans $= 2$

$\therefore \quad x = \frac{7}{2}\sqrt{x} + 2$

$\Rightarrow \quad (x-2) = \frac{7}{2}\sqrt{x}$

27. (*d*)

Thumb	Index	Middle	Ring	Little
1	**2**	**3**	**4**	**6**
9 ←	8	7	6	
	10	11	12	13
17 ←	16	15	14	
	18	19	20	21
25 ←	24	23	22	
	26	27	28	29
33 ←	32	31	30	

From the above counting pattern, we find that every multiple of 8 comes on index finger and moves towards thumb therefore, the last multiple of 8 which appears on index finger will be $\frac{1994}{8} \Rightarrow 1992$

Hence, 1994 will be on ring finger.

28. (*a*) Total number of men $= 5180 + 4 = 5184$

$\therefore$ Number of men in first row $= \sqrt{5184} = 72$.

29. (*b*) Let total number of votes cast be x.

Total number of counted votes $= \frac{2}{3}x$

Votes that candidate got $= \frac{5}{6} \times \frac{2}{3}x = \frac{5}{9}x$

Votes still need to win $= \frac{3}{4}x - \frac{5}{9}x = \frac{7}{36}x$

Remaining uncounted votes $= \frac{1}{3}x$

$\therefore$ Required part $= \frac{7}{36} \times \frac{3}{1} = \frac{7}{12}$.

30. (*d*) Required sum $= 3000 + 30 = 3030$.

31. (*d*) Let the whole number be x.

$\therefore \quad x = \frac{1}{6}(x + 20)$

$\Rightarrow \quad 6x = x + 20$

$\Rightarrow \quad 5x = 20$

$\Rightarrow \quad x = 4.$

32. (*a*) Let the number be $(765x + 42)$

When this number is divided by 17, then quotient will be $(45x + 2)$ and remainder will be 8

33. (*c*) Production in third year = 6000

Production in seventh year = 7000

$\therefore$ Production in fourth year = 1000

i.e., Production increases @ 250 scooters every year.

$\therefore$ Production in tenth year

$$= (7000 + 250 \times 3) = 7750.$$

34. (*c*) Let the number of girls and boys be x and y.

Then, $x - 1 = y$

and $xy = 272$

$\Rightarrow x(x-1) = 272$

$\Rightarrow x^2 - x - 272 = 0$

$\Rightarrow (x+17)(x-16) = 0$

$\Rightarrow x = 16.$

35. (*a*) Let there were x friends, then contribution of one friend $= \frac{96}{x}$

If four friends have left, then contribution of each friend $= \frac{96}{x-4}$

$$\therefore \frac{96}{x-4} - \frac{96}{x} = 4 \Rightarrow x = 12$$

Hence, number of friends who attended the picnic

$$= 12 - 4 = 8.$$

36. (*d*) Number of boxes used

$$= \frac{17+13}{24} = \frac{30}{24} = \frac{5}{4} = 1\frac{1}{4}$$

Since, the number of boxes used should be a whole number, hence the number of boxes used is 2.

37. (*b*) There are four prime numbers, *viz*., 7, 1 1, 13, 29.

38. (*b*) Let the number be abc; so the six-digit number is abcabc. Now, the sum of alternate digits is:

(i) $a + c + b$

(ii) $b + a + c$

Both being equal, the six-digit number is definitely divisible by 11.

39. (*d*) Let x, y and z be the digits at the hundredth place, ten's place and unit's place respectively.

$\therefore \quad x + y + z = 16 \quad (1)$

$y = 3z \quad (2)$

$z = \frac{1}{4}x \quad (3)$

$\therefore \quad (2) \Rightarrow y = \frac{3}{4}x \quad (4)$

Using (3) and (4) in (2), we get

$$x + \frac{3}{4}x + \frac{1}{4}x = 16$$

$\Rightarrow x = 8$

$\therefore y = 6, z = 2$

Hence, the number is 862.

40. (*d*) $\frac{1}{3}K = \frac{1}{4}K + 3$

$\Rightarrow K = 36.$

41. (*d*) $\frac{29}{12} + \frac{15}{16} = \frac{116+45}{48} = \frac{161}{48} = 3\frac{17}{48}$

42. (*b*) $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 2^4 \times 3^2 \times 5^1 \times 7^1$

Thus, the least perfect square which is divisible by 7! should be $(2^4 \times 3^2 \times 5^1 \times 7^1)(5^1 \times 7^1)$

i.e., $5040 \times 35 = 176400.$

43. (*a*) If the square of any natural number n leaves a remainder of 1 when divided by 24, that natural number must be of the form $6p \pm 1$ (since n must be divisible by neither 2 nor 3) where p is a natural number.

$\therefore$ the two digit numbers must be of the form $6p \pm 1$,

There are 15 two-digit numbers in the form $6p + 1$ and the same number of two digit numbers in the form $6p - 1$.

$\therefore$ a total of 30 two-digit numbers satisfy the given condition.

44. (*d*) If we put consecutive negative integers as $x = -1$, $y = -2$ and $z = -3$, then from option (*d*),

$(-1+2)(-2+3) = 1 \times 1$

$= 1$ (Positive integer).

45. (*b*) Let the second number be $3x$, so that the first number is $6x$ and the third one is $2x$.

$\therefore \quad 6x + 3x + 2x = 132$

$\Rightarrow \quad 11x = 132$ or, $x = 12$

Second number $= 3x = 3 \times 12 = 36.$

46. (*d*) Let the unit digit be y and tens digit be x.

$\therefore$ The number $= 10x + y$

On interchanging the digits, the number $= 10y + x$

$\therefore \quad 10x + y - 10y - x = 27$

$\Rightarrow \quad x - y = 3$

(already given in the question)

Now, $y \neq 0$ and the set of digits satisfying the condition are (9, 6), (8, 5), (7, 4), (6, 3), (5, 2), (4, 1)

$\therefore$ We can not reach on a distinct answer.

47. (*b*) $2n + 5m = 50$

$\therefore$ Possible value of n and m are:

(25, 0), (10, 6), (20, 5), (15, 4), (5, 8)

Hence, least difference between 5 and 8 is 3.

48. (*c*) Divide 7231 by 45, the remainder is 31

$\therefore$ Required number = 45 – 31 = 14.

49. (*b*) Let a two-digit number = 48

When unit digit is halved = 4

Ten's digit is doubled = 8

$\therefore$ Number = 84

Hence, digit in the unit's place is twice the digit in the ten's place.

50. (*d*) According to question

$16 \times 4 = 64$	$64 \times 1 = 64$	$8 \times 8 = 64$	$7 \times 5 = 35$
$(16+4) = 20$	$(64+1) = 65$	$(8+8) = 16$	$(7+5) = 12$

51. (*d*) $ab > 0$ because a and b both are positive.

52. (*b*) n must be divisible by 35.

53. (*d*) Let the number be 253

Which unit place is 2

$\therefore$ Digit at 100 place of original number is 2.

54. (*a*) x and y are natural numbers

We know that for any natural number p,

$p^n + p$ is even

and, $p^n - p$ is even

When, we multiply an even number to any natural number the resultent number is even.

55. (*d*) (I) Product of 4 positive numbers cannot be zero.

(II) a can be odd or even, b can be odd or even, c is even, p is odd. We cannot definitely say that $a + b + c + p$ is odd.

(III) $(b^2 + c^2) - (p^2 - a^2)$, here $b^2 + c^2$ can be odd or even, $(p^2 - a^2)$ can be odd or even.

(IV) $a\,(p - c) + a\,(c + b) = a\,[p - c + c + b]$

Where a is divisible by 5

So, $a\,(p - c) + a\,(c + b)$ will be divisible by 5

So, only (IV) is correct.

56. (*b*) x is a factor of y

$\therefore$ $y = ax$ (Suppose)

x is a multiple of z

$\therefore$ $x = bz$ (Suppose)

(*a*) $\frac{xy}{z}$ = by, it is an integer

(*b*) $\frac{y+z}{x} = \frac{ax+z}{x}$, it is not an integer

(*c*) $\frac{yz}{x} = az$, it is an integer

(*d*) $\frac{x+y}{z} = \frac{bz+abz}{z} = (b + ab)$, it is an integer.

57. (*c*) Given that unit digit of $a^2 = 9$

and, $(a + 1)^2 = 4$

i.e., unit digit of a must be 3

$\therefore$ Unit digit of $(a + 2)^2$

$\Rightarrow$ $(3 + 2)^2 = 5^2$

$\Rightarrow$ $= 25$

i.e., 5.

58. (*c*) After dividing 10 by 7,

we get remainder $n - 4$

i.e., $7 - 4 = 3$.

59. (*c*) We see that $4^2 + 6^2 = 52$ when divided by 25, remainder is 2.

$4^3 + 6^3 = 280$, divide by 25, remainder is 5

$4^4 + 6^4 = 1552$, divide by 25, remainder is 2

When taking m odd, the remainder is 5

When taking m even, the remainder is 2

Hence, remainder = 5.

60. (*d*) Unit's digit in 102 is 2.

The digit in the unit's place of 102^{51} will be same as in

2^{51} or, $2^3 = 8$. $[\because 51 = 4.12 + 3]$

61. (*c*) Divide 9269 by 73, the remainder is 71

$\therefore$ 71 is the required least number.

62. (*a*) Divide 15463 by 107, the remainder is 55, therefore, the number to be added = 107 – 55 = 52.

63. (*a*) $a + b < c + d$ (1)

$b + c < d + e$ (2)

$c + d < e + a$ (3)

$d + e < a + b$ (4)

From (1) and (4),

$$a + b + d + e < c + d + a + b$$

$\Rightarrow$ $e < c$

From (2) and (4),

$$b + c + d + e < d + e + a + b$$

$\Rightarrow$ $c < a$

From (1) and (3),

$$a + b + c + d < c + d + e + a$$

$\Rightarrow$ $b < e$.

64. (*b*)
$$z^{x+y} = 10,\ z^{y+z} = 20$$
$$2^{x+z} = 30$$
$$\Rightarrow\ 2^{z+y} \times 2^{y+z} \times 2^{z+x} = 10 \times 20 \times 30 = 6000$$
$$\Rightarrow\ 2^{2(x+y+z)} = 6000$$
$$\Rightarrow\ 2^{2(y+z)} = 400$$
$$\Rightarrow\ 2^{2(y+y+z-y-z)} = \frac{6000}{400} = 15$$
$$\Rightarrow\ 2^{2x} = 5$$
$$\Rightarrow\ 2^{x} = \sqrt{15}$$

65. (*c*) Dividing 5000 by 73, the remainder is 36. The number greater than 5000 is obtained by adding to 5000 the difference of divisor and the remainder.

∴ The required number

$= 5000 + (73 - 36)$

$= 5037.$

66. (*d*) Let the numbers be a and b.

Then, $a + b = 100$ and, $a - b = 37$

∴ $a^2 - b^2 = (a + b)(a - b)$

$= 100 \times 37 = 3700.$

67. (*b*) $50000 = 79 \times \text{quotient} + 43759$

∴ $50000 - 43759 = 79 \times \text{quotient}$

or, $6241 = 79 \times \text{quotient}$

∴ Required number of times $= \frac{6241}{79} = 79.$

68. (*a*) On dividing 58701 by 567

Remainder $= 300 > \frac{1}{2}(567)$

∴ Integer nearest to 58701 and divisible by 567

$= 58701 + (567 - 300)$

$= 58701 + 267 = 58968.$

69. (*a*) The least no. of six figures is 100000.

On dividing 100000 by 357, remainder = 40

∴ Least number of six figures which is divisible by

$357 = 100000 + (357 - 40)$

$= 100317$

∴ Required number $= 100317 - 5555 = 94762.$

70. (*a*) Let the least value to be given to * be x

Then, $x + 4 + 7 = 5 + 3 + 5$

$x = 2$

Difficulty Level-2

1. (*b*) a, b, c and d are four integers such that $a + b + c + d = 4m + 1$.

Minimum possible value of $a^2 + b^2 + c^2 + d^2$ is when a, b, c and d are as close to each other as possible. Since RHS is not the multiple of 4, *as*, b, c and d can not be equal to m.

Hence the numbers may be of the form, m, m, m and $m + 1$.

∴ $a^2 + b^2 + c^2 + d^2 = 4m^2 + 2m + 1.$

2. (*c*) We have to find the number of three-digit numbers in which the digit at ten's place is greater than the digit at unit's and hundred's places. That is,

Hundred	Ten	Unit
x	y	z

$x < y > z$

The following chart shows the number of ways in which it can be formed.

Number of ways in which unit's place, i.e., x can be filled	Digit at ten's place, i.e., y	Number of ways in which unit's place, i.e., y can be filled
1 (i.e., 1)	2	2 (i.e., 0, 1)
2 (i.e., 1, 2)	3	3 (i.e., 0, 1, 2)
...	...	...
8 (i.e., 1, 2, 3, ... 8)	9	9 (i.e., 0, 1, 2, ...9)

∴ Total no. of possible three-digit numbers

$= (1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + \ldots + (7 \times 8) + (8 \times 9) = 240.$

3. (*b*) Product $(n - 1)(n - 2) \ldots 3 \times 2 \times 1$ is not divisible by n if n is 4 or a prime number.

We have to find the number of primes in

$12 \le n \le 40.$

i.e., {13, 17, 19, 23, 29, 31, 37}

∴ No. of positive integers in the range $12 \le n \le 40$ is 7.

4. (*d*) Take any arbitrary value of x and y

Let, $x = 2$ (prime number)

$y = 50$ (composite number)

Going through the options,

(*a*), (*b*) and (*c*) are wrong because $y - x$, xy and $\frac{x + y}{x}$ are even integers for $x = 2$ and $y = 50$

∴ None of the statements are true.

5. (*a*) The set of prime numbers 3, 5, 7 is the only set which satisfies the given condition.

6. (*d*) If 4^2 is divided by 6, remainder is 4

If 4^3 is divided by 6, remainder is 4

If 4^4 is divided by 6, remainder is 4

... ...

If 4^{96} is divided by 6, remainder is 4.

7. (*a*) After 5^4, the remainder left when 5^n is divided by 1000 is 125 when n is odd and 625 when n is even. Hence, the remainder is 125.

8. (*d*) For every integer a, $a + (-a) = 0$. Therefore, by pairing 1 with –1, 2 with –2, and so on, one can see that in order for the sum to be zero, a list of consecutive integers must contain the same number of positive integers as negative integers, in addition to the integer '0'. Therefore, the list has an odd number of consecutive integers and their average will also be 0.

9. (*c*) No. of Spades = 1

No. of Diamonds = 2

No. of Hearts = 4

No. of Clubs = 6.

10. (*d*) No. of balls in Red Box = 18

No. of balls in Blue Box = 36

No. of balls in Green Box = 54.

11. (*d*) The given product contains 5^{106} and $2x$

where, $x > 106$

$\therefore$ There will be 106 zeroes in the product, because zero will come only by multiplying 2 and 5.

12. (*c*) Such numbers are 56, 67, 78 and 89

Sum of these numbers = 290.

13. (*b*)

$$20 \times 21 \times 22 \times 23 \times 24 = 5100480$$
$$25 \times 26 \times 27 \times 28 = 491400$$
$$29 \times 30 = 870.$$

14. (*d*) Sum of the prime numbers between 10 and 40 = 11 + 13 + 17 + 19 + 23 + 29 + 31 + 37 = 180.

15. (*d*)

$$\sum_{n=1}^{100} i^n = (i + i^2 + i^3 + i^4) + (i^5 + i^6 + i^7 + i^8) + \ldots + (i^{97} + i^{98} + i^{99} + i^{100})$$
$$= (i - 1 - i + 1) + (i - 1 - i + 1) + \ldots$$
$$= 0 + 0 + \ldots + 0 = 0.$$

16. (*d*) Let the five consecutive even numbers $2n$, $2n + 2$, $2n + 4$, $2n + 6$, $2n + 8$ be respectively equal to a, b, c, d and e, where n is a natural number.

Then, v, w, x, y and z are equal to $2n + 1$, $2n + 3$, $2n + 5$, $2n + 7$, $2n + 9$.

$$\Rightarrow \quad N = \frac{2n + 2n + 2 + 2n + 4 + 2n + 6 + 2n + 8}{2n + 1 + 2n + 3 + 2n + 5 + 2n + 7 + 2n + 9}$$

$$= \frac{10n + 20}{10n + 25} = \frac{n + 2}{n + 2.5}.$$

17. (*b*) For the number to be divisible by 3, the sum of the digits of a number should be divisible by 3. Also, for the number to be divisible by 9, the sum of the digits of a number should be divisible by 9. Hence options (*c*) and (*d*) are ruled out as all the given numbers are not divisible by 9 (because the sum of their digits is not divisible by 9). Option (*b*) is the answer as 3 and 37 are factors of 111 and 111 is the divisor of all the given numbers.

18. (*c*) The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc.

The numbers 5, 11, 17, 23, 29 form an increasing sequence for which 29 is the fifth term.

19. (*c*)

$$10^{12} - 1 = (10^6 - 1)(10^6 + 1)$$
$$= (10^3 - 1)(10^3 + 1)(10^6 + 1)$$
$$= 999 \times 1001 \times 1000001$$

$$\text{Therefore, } = \frac{10^{12} - 1}{111} = \frac{999 \times 1001 \times 1000001}{111}$$
$$= 9 \times 1001 \times 1000001$$
$$= 9009009009.$$

20. (*b*) $N = 4^{61} + 4^{62} + 4^{63} + 4^{64}$

$$= 4^{61}(1 + 4 + 16 + 64) = 4^{61} \times 85$$
$$= 4^{61} \times 5 \times 17 = 4^{60} \times 4 \times 5 \times 17$$
$$= 4^{60} \times 2 \times 17 \times 10$$

Hence, it is divisible by 10.

21. (*b*)

$$889 = 7 \times 127$$
$$899 = 29 \times 31$$
$$1147 = 31 \times 437.$$

22. (*a*) 179 is a prime number.

$$203 = 7 \times 29$$
$$143 = 11 \times 13$$
$$267 = 3 \times 89.$$

23. (*c*) Multiple of 120 + 5, which is divisible by 13.

24. (*d*) Let n be the number.

Then, the result is

$$= \left[\frac{\{(2n + 6)10\}}{20}\right] - x = \frac{2n + 6 - 2n}{2} = 3.$$

25. (*c*) By the rules of divisibility, we know that any number is divisible 8, if the last three digits of the number is also divisible by 8.

In the given number last three digits are 354. So, the remainder is 2.

26. (*b*) Let the rational number be $\frac{p}{q}$

$$\therefore \quad q = p + 3$$

$$\therefore \quad \frac{p + 7}{p + 3 - 2} = 2 \Rightarrow p + 7 = 2p + 2$$

$$\Rightarrow \quad p = 5$$

$$\Rightarrow \text{Given rational number} = \frac{5}{8}.$$

27. (*d*) Let the two numbers be ab and xy.

$\therefore\ (100a+b)\times(100x+y)=(100b+a)+(100y+x)$

$\Rightarrow 10000ax+100ay+100bx+by$

$= 10000by+100bx+100ay+ax$

$\Rightarrow\quad 9999ax = 9999by$

$\Rightarrow\quad ax = by$

Now, check from the options

For option (*d*): $a=1, b=3, x=6, y=2$

$\therefore\quad ax = 1\times 6 = 6$ and $by = 3\times 2 = 6$

Hence, $ax = by$.

28. (*b*) When 17^{15} is divided by 6

$\dfrac{(18-1)^{15}}{6}$, remainder = 5

$\therefore$ 1715 can be written as 6K + 5

$$\therefore\ \frac{11^{6K+5}}{7} = \frac{(7+4)^{6K+5}}{7} = \frac{4^{6K+5}}{7}$$

$$= \frac{16\times(4^3)^{2K+1}}{7} = \frac{16\times(63+1)^{2K+1}}{7}$$

Remainder = 2 × 1 = 2

$$\frac{13^{11^{15}}}{7} = \frac{(14-1)^{odd}}{7} \Rightarrow \text{Remainder} = 6$$

$\therefore$ Remainder when 111715 + 131715 is divided by 7 is 1.

29. (*d*) $n^{13}-n$ is divisible by 13 for all $n \in$ whole numbers $52n-1$ is divisible by 13 for even n.

The smallest even number is 2.

$\therefore$ When $n=2$, the expression is divisible by 169.

30. (*d*) When a number is divided by 6 possible remainders are 1, 2, 3, 4, 5 ($x = 6y$ + remainder). But only odd numbers are possible as with even numbers the remainder when divided by 2 would be 0.

Of 1, 3, 5 only for 5, division by 3 has remainder 2.

$\therefore$ Remainder when divided by 6 = 5.

31. (*c*) Total possible arrangements = 10 × 9 × 8

Now three numbers can be arranged among themselves in = 3! ways = 6 ways.

Given condition is satisfied by only 1 out of 6 ways.

Hence, required number of arrangements.

$$= \frac{10\times 9\times 8}{6} = 120.$$

32. (*b*) Check choices

$54 \Rightarrow S = (5+4)^2 = 81$

$\Rightarrow S - D = 81 - 54 = 27.$

33. (*b*) Escaped with 1

Before 3rd watchman, he had (1 + 2) × 2 = 6

Before 2nd watchman, he had (6 + 2) × 2 = 16

Before 1st watchman, he had (16 + 2) × 2 = 36.

34. (*d*) Let the no. of gold coins $= x + y$

$48(x-y) = x^2 - y^2$

$\Rightarrow 48(x-y) = (x-y)(x+y)$

$\Rightarrow\quad x + y = 48.$

35. (*d*) $$575 = \frac{n^2+n}{2} - x$$

$\Rightarrow\quad 1150 = n^2 + n - 2x$

For, $n = 34$,

$40 = 2x$

$\therefore\quad x = 20.$

36. (*a*) $(2^4)^{64} = (17-1)^{64} = 17n + (-1)^{64}$

$= 17n + 1$

Hence, remainder = 1.

37. (*d*) $3\{4(7x+4)+1\}+2 = 84x+53$

Therefore, remainder is 53.

38. (*b*) $7^{6n} - 6^{6n}$

Put $n = 1$

$7^6 - 6^6 = (7^3-6^3)(7^3+6^3)$

This is a multiple of $7^3 - 6^3 = 127$.

39. (*a*) Here the least value of $x = \sqrt{49} = -7$

and the least value of $y = \sqrt{16} = -4$

So, the least number here which when divided by –7 and –4 gives a negative quotient in each case is 28 since $\frac{28}{-7} = -4$ and $\frac{28}{-4} = -7$.

40. (*c*) Since 64 ÷ 24 = 1296 ÷ 16 = 81 and remainder 0.

So, we need not calculate the remainder in second case as the product will be 0.

41. (*a*) Since he has to put minimum 120 oranges and maximum 144 oranges, i.e., 25 oranges need to be filled in 128 boxes with same number of oranges in the boxes.

Therefore, total 125 = 25 × 5 oranges could be filled in the boxes, i.e., 25 in each of the 5 boxes which would be the minimum and have the same number of oranges.

Hence, the answer is 5.

42. (*a*) Let the 4-digit number be *abcd*.

Then, $a + b = c + d$ (1)

$b + d = 2(a + c)$ (2)

and, $a + d = c$ (3)

From Eqs. (1) and (3), $b = 2d$

From Eqs. (1) and (2), $3b = 4c + d$

$\Rightarrow \quad 3(2d) = 4c + d$

$\Rightarrow \quad 5d = 4c$

$\Rightarrow \quad c = \frac{5}{4}d$

Now d can be 4 or 8. But if $d = 8$, then $c = 10$ is not possible. So, $d = 4$, which gives $c = 5$.

43. (*d*) Let the number be x

Increase in product $= 53x - 35x = 18x$

$\Rightarrow \quad 18x = 540 \Rightarrow x = 30$

Raised product $= 53 \times 30 = 1590$.

44. (*d*) Let, $\quad y = n^3 - 7n^2 + 11n - 5$

At $\quad n = 1, y = 0$

$\therefore \ (n-1)(n^2 - 6n + 5) = (n-1)^2 (n-5)$

Now, $(n-1)^2$ is always positive.

Also, for $n < 5$, the expression gives a negative quantity. Therefore, the least value of n will be 6. Hence, $m = 6$.

45. (*d*) Let there be x mints originally in the bowl.

Sita took $\frac{1}{3}$, but returned 4

So, now the bowl has $\frac{2}{3}x + 4$ mints.

Fatima took $\frac{1}{4}$ of remainder, but returned 3

So, the bowl has $\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 3$ mints.

Eswari took half of remainder that is $\frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 3\right]$. She returns 2, so the bowl now has $\frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x + 4\right) + 3\right] + 2 = 17 \Rightarrow x = 48$.

46. (*a*) The product of 44 and 11 is 484

Here, $3x^3 + 4x^2 + 1x^1 + 4 \times x^0 = 484$

$\Rightarrow \quad 3x^3 + 4x^2 + x = 480$

This equation is satisfied only when $x = 5$.

In decimal system, the number 3111 can be written as 406.

47. (*a*) Let the highest number be n.

Then, $\dfrac{\frac{n(n+1)}{2} - x}{(n-1)} = 35\frac{7}{77} = \frac{602}{17}$,

where x is the number erased.

Hence, $n = 69$ and $x = 7$ satisfy the above conditions.

48. (*c*) $99 \times D = a_1 a_2$. Hence, $D = \frac{a_1 a_2}{99}$. So, D must be multiplied by 198 as 198 is a multiple of 99.

49. (*c*) $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{19.21}$

$= \frac{1}{2}\left(1 - \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) + \ldots \frac{1}{2}\left(\frac{1}{19} - \frac{1}{21}\right)$

$= \frac{1}{2} - \frac{1}{42} = \frac{(21-1)}{42} = \frac{20}{42} = \frac{10}{21} = \frac{10}{21}$.

50. (*b*) Use any 7 consecutive numbers to check the answers.

$n = \frac{(1+2+3+4+5)}{5} = 3$, average of 7 integers is

$k = \frac{(1+2+3+4+5)}{7} = 4$. So $k = n + 1$.

51. (*c*) $N = 1421 \times 1423 \times 1425$. When divided by 12, it shall look like $\frac{[(1416+5)\times(1416+7)\times(1416+9)]}{12}$.

Now the remainder will be governed by the term $5 \times 7 \times 9$, which when divided by 12 leaves the remainder 3.

52. (*d*) Let r be the remainder. Then, $34041 - r$ and $32506 - r$ are perfectly divisible by n. Hence, their difference should also be divisible by the same.

$(34041 - r) - (32506 - r) = 1535$

which is divisible by only 307.

53. (*d*) Any triplet of the form $(-n, 0, n)$ satisfies the given condition, e.g., $(-2, 0, 2)$.

54. (*d*) N can be written either $(54+1)^3 + (18-1)^3 - 72^3$ or $(51+4)^3 + 17^3 - (68+4)^3$.

The first form is divisible by 3, and the second by 17.

55. (*a*) $\quad 36 = 2 \times 2 \times 3 \times 3$

Hence, divisors of

36 = 1, 2, 3, 4, 6, 9, 12, 18, 36, i.e., 9 in all.

$76 = 2 \times 2 \times 19$

Hence, divisors of

76 = 1, 2, 4, 19, 38, 76, i.e., 6 in all.

$82 = 2 \times 41$

Hence, divisors of

82 = 1, 2, 41, 82, i.e., 4 in all

$191 = 1 \times 191$

Hence, divisors of

191 = 1, 191, i.e., 2 in all.

56. (*c*) $\quad 58 = 2 \times 29$

Hence, divisors of

58 = 1, 2, 29, 58, i.e., 4 in all

$88 = 2 \times 2 \times 2 \times 11$

Hence, divisors of

88 = 1, 2, 4, 8, 11, 22, 44, 88, i.e., 8 in all

$137 = 1 \times 137$

Hence, divisors of

$137 = 1, 137$, i.e., 2 in all

Hence, divisors of

$184 = 1, 2, 4, 8, 23, 46, 92, 184$, i.e., 8 in all.

57. (*d*) Total number of numbers, which end with $4 = 9$

Total numbers from 1 to $90 = 90$

Therefore, required percentage $= \frac{9}{90} = 10\%$

58. (*b*) There are $99 - 10 + 1 = 90$ two digit numbers in all. We can have 0–9 digits at unit's place. For 0 in unit's place we can have 1–9 digits at tens place, i.e., we have 9 choices. For 1 in unit's place we have 8 choices and so on. Hence, total numbers satisfying given condition $= 9 + 8 + \ldots + 1 = 45$.

59. (*b*) Squaring both sides of the given equation

$$x^2 = \frac{5}{2} + \sqrt{\frac{13}{4} + 6x}$$

$$\Rightarrow \quad x^2 - \frac{5}{2} = \sqrt{\frac{13}{4} + 6x}$$

$$\Rightarrow \quad \left(x^2 - \frac{5}{2}\right)^2 = \frac{13}{4} + 6x$$

(Squaring both sides again)

Going by the choices, only $x = \frac{3 + \sqrt{5}}{2}$ satisfies the equation above.

60. (*b*) Assume the number of rows be n.

Then, $n \times n = 19044$

or, $n = 138$.

61. (*a*) $n = 0 \Rightarrow 3^{5^0} + 1 = 4, n = 1 \Rightarrow 3^{5^1} + 1 = 244$

The remainders can be seen to be $R = 4$, when $n = 0$, i.e., even and $R = 6$ when $n = 1$, i.e., odd. Therefore, I and III are true.

62. (*d*) $2x - 1$ is an odd number.

$\Rightarrow x$ can be either odd or even.

$3y - 1$ is an even number.

$\Rightarrow y$ is an odd number.

I. In $x^2 - 2y + 2$, $2y$ is even, but x^2 can be either odd or even, so we can not say whether $x^2 - 2y + 2$ is odd or even.

II. In $y^2 - 2x + 3$, y^2 is odd, $2x$ is even and 3 is odd $\Rightarrow y^2 - 2x + 3$ is even.

III. In $4x^2 - y - 1$, $4x^2$ is even, y is odd and 1 is odd $\Rightarrow 4x^2 - y - 1$ is even.

63. (*c*) For any integer n, $n^3 - n$ is divisible by 3, $n^5 - n$ is divisible by 5, $n^{11} - n$ is divisible by 11 but $n^4 - n$ is not necessarily divisible by 4. Thus, statement III is true.

64. (*b*) All the options satisfy the first condition. So, testing the options for second condition, only option (*b*), i.e., 24 satisfies the second condition, i.e., $24 + 18 = 42$.

65. (*a*) The given expression $= 2^{\frac{1}{2} \times \frac{2}{3} \times \ldots \frac{n}{n+1}} = (2)^{\frac{1}{n+1}}$

66. (*d*) The number is neither divisible by 3 nor by 11.

67. (*d*) p may or may not be divisible by 24. But, p is never divisible by 22 because

$$23^n + 1 = (22 + 1)^n + 1 = 22k + 2.$$

68. (*a*)

$$\begin{array}{r} 4\,a\,3 \\ 9\,8\,4 \\ \hline 13\,b\,7 \end{array}$$

As $13\,b\,7$ is divisible by 11

$\therefore \quad a = 1$ and, $b = 9$

$\therefore \quad a + b = 1 + 9 = 10$.

69. (*b*) Given

$$\begin{array}{rrr} h & t & u \\ -\ u & t & h \\ \hline - & - & 4 \end{array}$$

The difference between a three-digit number and its reverse is always a multiple of 99. The only multiple of 99 and less than 1000 that ends in 4 is 594. Thus, the remaining two-digits in that order are 9 and 5.

70. (*a*) In base 4, the 20th number will be

$$= 4^2\,(1) + 4^1\,(1) + 4^0\,(0) = 110.$$

71. (*b*) Let the two-digit numbers $= xy$

The square of $\quad xy = (10x + y)^2$

The square of the number formed by reversing the digits of $\quad xy = (10y + x)^2$

$$(10x + y)^2 - (10y + x)^2 = 99\,(x^2 - y^2)$$

$$= 99\,(x - y)\,(x + y)$$

Thus, it will always be divisible by 9, the sum of the digits as well as the difference of the digits. But, it is not divisible by their product xy.

72. (*b*) The value of the three-digit number $tu\,1$

$$= 100t + 10u + 1.$$

73. (*d*) Factors of 6 are 1, 2, 3. Now $1 + 2 + 3 = 6$.

74. (*c*) By hit and trial, we see that $n = 10$ and $n = 16$ satisfy the conditions.

75. (*a*) Given, $\quad 2x + p = 2y, p + y = x$

and, $\quad x + y = z$

$\Rightarrow \quad x + y + z = 2z = 2(x + y)$

So, $\quad p = x - y = 2(y - x)$

The condition is satisfied only when $x = y$

Then, $\quad p = 0$

76. (*a*) We know that for any rational number,

$$\frac{1}{x + 1} < 1$$

and, $\dfrac{x}{x+1} < 1$

Hence, (*a*) is the correct answer.

77. (*a*) $A_0 = 1994$, which is not divisible by 10. Hence, $f(A_0) = A_0 + 1 = 1995$. Since, $A_{m+1} = f(A_m) \Rightarrow A_1 = f(A_0) = 1995$, similarly $A_2 = 1996$, $A_3 = 1997$, $A_4 = 1998$, $A_5 = 1999$, $A_6 = 2000$, which is divisible by 10. Hence, $f(A_6) = \dfrac{2000}{10} = 200 = A_7$ similarly $A_8 = 20$ and $A_9 = 2$.

78. (*d*) Four digits of the number 29138576 are omitted so that result is large.

$\therefore$ Omitted digits are 1, 2, 3, 5

Hence, the largest omitted digit is 5.

79. (*a*) $A < B$

$A < D < B$, C is the greatest integer.

$\therefore$ With the help of 1st statement $E + B < A + D$, the result can be obtained.

80. (*c*) $x^2 + 4xy + 6y^2 - 4y + 4$

$$= (x)^2 + 2\ 2x2y + (2y)^2 + (\sqrt{2}y)^2$$
$$- 2\sqrt{2}y\sqrt{2} + (\sqrt{2})^2 + 2$$
$$= (x + 2y)^2 + (\sqrt{2}y - \sqrt{2})^2 + 2$$

Now, on putting the value of $x = -2$ and $y = 1$, we get the minimum value of expression.

81. (*a*) Numbers can be 1112, 1113, 1115, 1117.

82. (*a*) The metre skips all the numbers in which there is a 5. From 0000 to 0099, 5 occurs 10 times in the tens place and 10 times in the units place, (which includes the number 55).

$\therefore$ It occurs in a total of 10 + 10 – 1 numbers, i.e., 19 numbers. Similarly, from 0100 to 0199, from 0200 to 0299, 0300 to 0399 from 0400 to 0499, 0600 to 0699, ... 0900 to 0999. It occurs in 8 (19) numbers. From 0500 to 0599, there are 100 numbers. The micromanometer reading could change from 0499 to 0600.

Total number of numbers skipped from 0000 to 0999

$$= 19\ (9) + 100 = 271$$

Similarly, from 1000 to 1999 and from 2000 to 2999, 271 + 271 numbers are skipped. Finally, 3005 and 3015 are also skipped.

$\therefore$ Total number of skips = 271 (3) + 2 = 815

$\therefore$ Actual pressure = 3016 – 815 = 2201.

83. (*d*) I has 741 ones, II has 534 ones and III has 123 ones. Sum of the digits of each of I, II, and III is divisible by 3.

$\therefore$ I, II, III are all divisible by 3 and hence not prime. Choice (*d*) follows, since only this supports that all these three are not prime.

84. (*a*) $6 + 7 + 3 + 5 + K + 1 = 22 + K$

The least number greater than 22 and divisible by 9 is 27.

$\therefore \quad 27 = 22 + K$

$\Rightarrow \quad K = 5.$

85. (*a*) We know that,

$a^n - b^n$ is divisible by $(a + b)$ and $(a - b)$ if n is even.

Therefore, $7^{74} - 5^{74}$ is divisible by 12 and 2, and as a result by 4.

86. (*b*) We know that number of factors in a perfect square is always even. So factors in N^2 is an even number.

Now, factors of $N^2 = 34$ – an even number
= even number

So, N is a perfect square too.

Now to find out the actual values of N, we consider that if the factors of N are of form $a^x \times b^y$ then factors of N^2 will be $a^{2x} \times b^{2y}$

So, sum of factors will be $(x + 1)(y + 1) + (2x + 1)(2y + 1) = 34$. Only value which satisfies this equation is $x = 2$, $y = 2$.

(x, y are obviously positive integers)

For $N < 150$, we have only $N = 36$ and $N = 100$ (check for perfect squares)

So, the answer is 2.

87. (*d*) Total number of digits when 2^1 and 5^1 are written side by side

$$(25) = (1 + 1)$$

Total numbers of digits when 2^2 and 5^2 are written one after another

$$(425) = (2 + 1)$$

Total number of digits when 2^3 and 5^3 are written one after another

$$(8125) = (3 + 1) \text{ and so on}$$

Therefore, the total number of digits when 2^{2004} and 5^{2004} are written one after another

$$2004 + 1 = 2005$$

88. (*d*) We have to find the number of prime numbers from 101 to 200, which is 21.

89. (*b*) The numbers are of the from $8n + 6$

Therefore, sum of all such numbers is

$$\sum_{n=1}^{11}(8n + 6) = 6(11) + 8\left(\frac{11}{2}\right)(12) = 594.$$

90. (*a*) We have

$$\frac{2^{1040}}{131} = \frac{(2^8)^{130}}{131} = \frac{(256)^{130}}{131}$$

The remainder is 1.

Shortcut: Whenever a^n is divided by $(n + 1)$, where $n + 1$ is prime and relatively prime to a, the remainder is always 1.

91. (*d*) Clearly Statements I and II are wrong, since when p is prime number so it does not have any factor. Therefore, $x = 1.2....(p - 1)$ is not divisible by p or any prime number greater than p. Statement III is wrong, as 1.2.3.4.5.6 is divisible by 5.

But Statement IV is correct.

92. (*c*) Let x, y and z be the hundredth, tens and unit digits of the original number.

We are given,

$(100z + 10y + x) - (100x + 10y + z) = 594$

$\Rightarrow 99(z - x) = 594 \Rightarrow (z - x) = 6$

So, the possible values of (x, z) are (1, 7), (2, 8) and (3, 9)

As the ten digits can have any values from 0, 1, 2,..., 9,

$\therefore$ Minimum values for their sum $= x + y + z = 1 + 0 + 7 = 8$.

93. (*a*) We have

$22^3 + 23^3 + 24^3 + \quad + 87^3 + 88^{33}$

$= (22^3 + 88^3) + (23^3 + 87^3) + (24^3 + 86^3) + \quad + (54^3 + 56^3) + 55^3$

Now, all the terms except 55^3 is divisible by 110

[Shortcut: $a^n + b^n$ is divisible by $(a + b)$ when n is an odd number.]

Therefore, the required remainder when the given expression is divided by 110 is 55.

94. (*a*) Let $p + q = \alpha$ and $r + s = \beta$

Given: $p + q + r + s = 2$

So, $\alpha + \beta = 2$ and $\alpha\,\beta > 0$

Since AM $\geq$ GM

$$\Rightarrow \frac{\alpha+\beta}{2} \geq \sqrt{\alpha\beta}$$

$$\Rightarrow 1 \geq \sqrt{\alpha\beta}$$

On squaring both sides, we get

$$1 \geq \sqrt{\alpha\beta}$$

$$\Rightarrow \alpha\beta = m \leq 1$$

$$\therefore 0 \leq m \leq 1$$

95. (*d*) Any four digit number in which the first two digits and last two digits are equal will be of the form $11 \times (100 + b)$ i.e., it will be a multiple of 11 like 1122, 3366, 2244, ... Now, let the required, number be *aabb*. Since, *aabb* is a perfect square, the only pairs, of values a and b, that satisfy the above mentioned condition is $a = 7$ and $b = 4$. Hence, 7744 is a perfect square.

96. (*c*) Suppose the cheque for Shailaja is of ₹X and Y paise

It is given that

$$3 \times (100X + Y) = (100Y + X) - 50$$

$$300X + 3Y = 100Y + X - 50$$

$$299X = 97Y - 50$$

$$\therefore Y = \frac{299X + 50}{97}.$$

Now, the value of Y should be an integer. For $X = 18$, Y is an integer 56. Hence, option (c) is the correct choice.

97. (*a*) Let the last number of the series be n and number erased be x, then

$$\frac{\frac{n(n+1)}{2} - x}{n-1} = \frac{602}{17}$$

$$\Rightarrow \frac{n(n-1) - 2x}{2(n-1)} = \frac{602}{17}$$

From the options, we find that $x = 7$, n is an integer i.e., 69.

98. (*d*) Let x mints were originally in the bowl.

Number of mints before Eswari took $= \left(x - \frac{x}{2}\right) + 2 = 17$

$\Rightarrow \quad x = 30$

Number of mint before Fatima took $= \left(x - \frac{x}{4}\right) + 3 = 30$

$\Rightarrow \quad x = 36$

Number of mint before Sita took $= \left(x - \frac{x}{3}\right) + 4 = 36$

$\Rightarrow \quad x = 48$

Hence, there were 48 mints originally.

99. (*d*) The frog can move either clockwise or anticlockwise in order to reach point E. In any case number of jumps required is 4.

For, $n = 4$, $a_{2n-1} = a_{8-1} = 7$.

100. (*d*) $N = 55^3 + 17^3 - 72^3 = (54 + 1)^3 + (18 - 1)^3 - 72^3$

$= (51 + 4)^3 + 17^3 - (68 + 4)^3$

These two different forms of given expressions are divisible by 3 and 17 both.

101. (*d*) Let the common remainder be x.

Then, the numbers $(34041 - x)$ and $(32506 - x)$ would be completely divisible by n. Hence, the difference of the numbers i.e., $(34041 - x)$ and $(32506 - x)$ will also be divisible by n or $(34041 - x - 32506 + x) = 1535$ will also be divisible by n.

Now, using options, we find that 1535 is divisible by 307.

102. (*c*) $D = 0.\overline{a_1a_2}$

$100D = a_1a_2 \cdot \overline{a_1a_2}$

$\therefore \quad 99D = a_1a_2 \quad \Rightarrow \quad D = \frac{a_1a_2}{99}.$

Required number should be the multiple of 99.

Hence, 198 is the required number.

103. (*a*) We have,

$$x = 1 \times 2 \times 3 \times 4 = 24$$

(We have taken the four consecutive integers to be 1, 2, 3 and 4)

$\therefore\ n = 1 + 24 = 25$, we find that n is odd and a perfect square. This is true for any set of four consecutive positive integers.

104. (*b*) According to the remainder theorem, the following expression will have the same remainder,

$$\frac{(7)^{84}}{342} \text{ or } \frac{(7^3)^{28}}{342} \text{ or } \frac{(343)^{28}}{(342)} \Rightarrow \text{Remainder} = 1.$$

105. (*a*) $(ab)^2 = ccb$. The greatest possible value of 'ab' to be 31. Since $(31)^2 = 961$ and $ccb > 300$, $300 < ccb < 961$, so $18 < ab < 31$. So, the possible value of ab that satisfies $(ab)^2 = ccb$ is 21. So, $(21)^2 = 441$

$$\therefore\ a = 2, b = 1 \text{ and } c = 4.$$

106. (*a*) We know that any number is divisible by 8, if the number formed by the last three digits is divisible by 8. And the same rule will be applicable to find the remainder.

Now, the last three digits in the hundred digit number of the form 1234567891011121314…is 545. Therefore, the remainder when 545 is divided by 8 is 1.

107. (*a*) Dividend = Divisor × Quotient + Remainder

$$= 899Q + 63$$

Dividend $= 29 \times 31Q + 29 \times 2 + 5 = 29(31Q + 2) + 5$.

108. (*b*) Unit digit in $(2)^4 = 6, (2)^8 = 6, (2)^{16} = 6$. Hence, 2 has a cyclicity of four. Hence, unit digit in $(2)^{48} = 6$

Therefore, unit digit in $(2)^{51} = (2)^{48} \times (2)^3 = 6 \times 8 \Rightarrow 8$

109. (*c*) If n^3 is odd, then n and n^2 will be odds. It can be checked for any odd integer. If $n = 3, n^2 = 9, n^3 = 27$.

110. (*b*) $7^{3^2} = 7^9$ and $(7^3)^2 = 7^6$. Hence, clearly $7^9 > 7^6$.

111. (*c*) Let $m = 10$ and $n = 5$, then $(m - n) = (10 - 5) = 5$, which is divisible by 5

$(m^2 - n^2) = (100 - 25) = 75$, which is divisible by 5

$(m + n) = (10 + 5) = 15$, which is divisible by 10.

112. (*d*) (P, Q) may be any of the following:

$(1, 64), (2, 32), (4, 16), (8, 8)$. Hence, $P + Q$ cannot be 35.

113. (*c*) $\dfrac{(16n^2 + 7n + 6)}{n} = 16n + 7 + \left(\dfrac{6}{n}\right)$.

Since n is an integer, hence for the entire expression to become an integer $\left(\dfrac{6}{n}\right)$ should be an integer. And $\left(\dfrac{6}{n}\right)$ can be integer for $n = 1, 2, 3, 6$. Hence, n will have four values.

114. (*c*) Let us solve the question for any two odd numbers greater then 1 i.e., 3 and 5 then

$$n(n^2 - 1) \text{ for } n = 3 = 3 \times 8 = 24$$
$$n(n^2 - 1) \text{ for } n = 5 = 5 \times 24 = 120$$

From the options, we find that both the numbers are divisible by 24.

115. (*b*) The number $77958A96B$ is divisible by 8 if $96B$ divisible by 8. And $96B$ is divisible by 8 if B is either 0 or 8

Now to make the same number divisible by 9, sum of all the digits should be divisible by 9. Hence, $(55 + A + B)$ is divisible by 9 if $(A + B)$ is either 0 or 8 $\Rightarrow$ either $A = 0$ and $B = 8$ or $A = 8$ and $B = 0$

Since, the number is divisible by both A and B. Hence, A and B may take either values i.e., 8 or 0

116. (*b*) Let the three even numbers be $(x - 2), x, (x + 2)$

Then, we are given

$$3(x - 2) - 2(x + 2) = 2$$
$$\Rightarrow\ 3x - 6 - 2x - 4 = 2$$
$$\Rightarrow\ x = 12$$

$\therefore$ The third number $= (12 + 2) = 14$.

117. (*b*) Let us solve the question for some prime numbers greater than 6 i.e., 7, 11, 13 and 7. If these numbers are divided by 6, the remainder is always either 1 or 5.

118. (*a*) $\dfrac{55^3 + 45^3}{55^2 - 55 \times 45 + 45^2}$

$$= \frac{(55 + 45)(55^2 - 55 \times 45 + 45^2)}{(55^2 - 55 \times 45 + 45^2)}$$
$$= (55 + 45)$$
$$= 100.$$

119. (*a*) There are 50 odd numbers less than 100 which are not divisible by 2.

Out of these 50, 17 numbers are divisible by 3

Out of remaining, 7 numbers are divisible by 5

Hence, numbers which are not divisible by 2, 3, and 5 $= (50 - 17 - 7) = 26$

120. (*c*) The last two digits of the number in the expansion is $(7)^4 = 01(2401)$ and if the power of 7 is any multiple of 4, the last two digits will not change

i.e., $(7)^4 = 2401 \Rightarrow 01$

$$(7)^8 = 5764801 \Rightarrow 01$$

Since, power of 7, i.e., 2008 is a multiple of 4, the last two digits of $(7)^{2008}$ will be 01.

121. (*c*) Let the total number of sweets be $(25x+8)$

Then $(25x+8)-22$ is divisible by 28

$\Leftrightarrow (25x-14)$ is divisible by 28 $\Leftrightarrow 28x-(3x+14)$ is divisible by 28

$\Leftrightarrow (3x+14)$ is divisible by 28 $\Leftrightarrow x=14$

$\therefore$ Total number of sweets $=(25\times 14+8)=358$

122. (*a*) Considering Statement I:

x	y	$x+y$	$x-y$
5	2	7	3

Since this the only possible solution, Statement I is sufficient.

Considering Statement II:

x	y	$x+y$	$x-y$
5	2	8	2
6	4	10	2

Since no unique solution is possible, Statement II is not sufficient.

123. (*b*) Between 100 and 199, there will be 19 numbers which contain '2'. They are as follows:

100, 112, 120–129 (10 numbers), 132, 142, 152, 162, 172, 182, 192

Similar would be the case for 300 – 339, 400 – 499, 500 – 599, 600 – 699

For 200–299, all 100 numbers will have 2

$\therefore$ Total number of numbers containing '2' $= 19\times 6+100=114+100=214$.

124. (*c*) As p, q, r are non-negative integers, the maximum will be achieved when the value of each variable is close to each other.

i.e., p, q are 3, 3, 4 (not necessarily in the same order).

Hence the value of

$pq+qr+pr+pqr=3\times 3+3\times 4+3\times 4+3\times 3\times 4$
$=9+12+12+36=69.$