Coordinate Geometry

Selected NCERT Questions

1. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

Sol. In Fig. 6.4, let the point $P(-1, 6)$ divides the line joining $A(-3, 10)$ and $B(6, -8)$ in the ratio $k : 1$ then, the coordinates of P are $\left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right)$.

But, the coordinates of P are $(-1, 6)$. (Given)

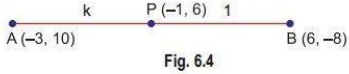


Fig. 6.4

$$\therefore \quad \frac{6k-3}{k+1} = -1 \quad \Rightarrow \quad 6k - 3 = -k - 1$$

$$\Rightarrow \quad 6k + k = 3 - 1 \quad \Rightarrow \quad 7k = 2 \quad \Rightarrow \quad k = \frac{2}{7}$$

Hence, the point P divides AB in the ratio $2 : 7$.

2. If $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$ find the value of x. Also, find the distances of QR and PR.

Sol. Since, point $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$.

Therefore, $QP = QR$

Squaring both sides, we have, $QP^2 = QR^2$

$\Rightarrow \quad (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$

$\Rightarrow \quad 25 + 16 = x^2 + 25$

$\Rightarrow \quad x^2 = 16 \qquad \therefore \quad x = \pm 4$

Thus, R is $(4, 6)$ or $(-4, 6)$.

Now, $QR = \sqrt{(4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$ units

or, $QR = \sqrt{(-4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$ units

and $PR = \sqrt{(4-5)^2 + (6+3)^2} = \sqrt{1+81} = \sqrt{82}$ units

or, $PR = \sqrt{(-4-5)^2 + (6+3)^2} = \sqrt{81+81} = 9\sqrt{2}$ units

3. Find the coordinates of the points which divide the line segment joining A (–2, 2) and B (2, 8) into four equal parts.

Sol. Let P, Q, R be the points that divide the line segment joining $A(-2, 2)$ and $B(2, 8)$ into four equal parts. (Fig. 6.5)

Fig. 6.5

Since, Q divides the line segment AB into two equal parts, *i.e.*, Q is the mid-point of AB.

$\therefore$ Coordinates of Q are $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$ *i.e.*, $(0, 5)$

Now, P divides AQ into two equal parts *i.e.*, P is the mid-point of AQ.

$\therefore$ Coordinates of P are $\left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$ *i.e.*, $\left(-1, \frac{7}{2}\right)$

Again, R is the mid-point of QB.

$\therefore$ Coordinates of R are $\left(\frac{0+2}{2}, \frac{5+8}{2}\right)$ *i.e.*, $\left(1, \frac{13}{2}\right)$.

4. Find the coordinates of a point A, where AB is the diameter of circle whose center is O (2, –3) and B (1, 4).

Sol. Let the coordinate of A be (x, y), $O(2, -3)$ be centre of circle and AB be diameter. Since, O is centre of AB.

By section formula

$$2 = \frac{1\times 1 + 1\times x}{1+1}, \quad -3 = \frac{1\times 4 + 1\times y}{1+1}$$

$\Rightarrow \quad 2 = \frac{1+x}{2}, \quad -3 = \frac{4+y}{2}$

$\Rightarrow \quad x = 4 - 1, \; y = -6 - 4$

$\Rightarrow \quad x = 3, \; y = -10$

Hence, coordinates of $A = (3, -10)$.

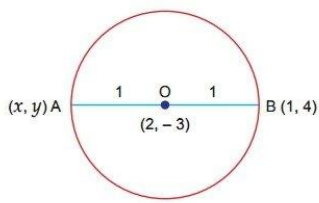


Fig. 6.6

5. In what ratio does the point (– 4, 6) divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$?

Sol. Let (– 4, 6) divide AB internally in the ratio $m_1 : m_2$. Using the section formula, we get

$$(-4, 6) = \left(\frac{3m_1 - 6m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right)$$

So, $$-4 = \frac{3m_1 - 6m_2}{m_1 + m_2} \text{ and } 6 = \frac{-8m_1 + 10m_2}{m_1 + m_2}$$

Now, $$-4 = \frac{3m_1 - 6m_2}{m_1 + m_2} \text{ gives}$$

$$-4m_1 - 4m_2 = 3m_1 - 6m_2$$

i.e., $$7m_1 = 2m_2$$

i.e., $$m_1 : m_2 = 2 : 7$$

Verification

Also, $$\frac{-8m_1 + 10m_2}{m_1 + m_2} = \frac{-8\frac{m_1}{m_2} + 10}{\frac{m_1}{m_2} + 1} \quad \text{(Dividing throughout by } m_2\text{)}$$

$$= \frac{-8 \times \frac{2}{7} + 10}{\frac{2}{7} + 1} = 6$$

Therefore, the point (– 4, 6) divides the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ in the ratio 2 : 7.

6. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Sol. Let A (5, – 2), B (6, 4) and C (7, – 2) be the vertices of a triangle.

Then we have,

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37} \text{ units}$$

$$AC = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{4} = 2 \text{ units}$$

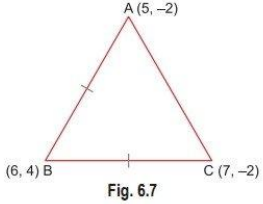


Fig. 6.7

Here, $AB = BC$

$\therefore$ ΔABC is an isosceles triangle.

7. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Sol. Let $A(1, 2)$, $B(4, y)$, $C(x, 6)$ and $D(3, 5)$ be the vertices of a parallelogram $ABCD$.

Since, the diagonals of a parallelogram bisect each other.

$$\therefore \quad \left(\frac{x+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$$

$$\Rightarrow \quad \frac{x+1}{2} = \frac{7}{2}$$

$$\Rightarrow \quad x + 1 = 7 \quad \text{or} \quad x = 6$$

$$\Rightarrow \quad 4 = \frac{5+y}{2} \quad \text{or} \quad 5 + y = 8 \quad \text{or} \quad y = 8 - 5 = 3$$

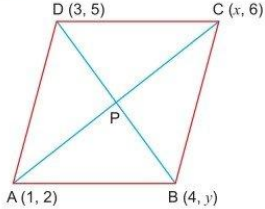


Fig. 6.8

Hence, $x = 6$ and $y = 3$.

8. Find the coordinates of the points of trisection of the line segment joining (4, – 1) and (– 2, – 3).

Sol. Let the given points be $A(4, -1)$ and $B(-2, -3)$ and points of trisection be P and Q.

Fig. 6.9

Let $AP = PQ = QB = k$

$PB = PQ + QB = k + k = 2k$

$AP : PB = k : 2k = 1 : 2$

Therefore, coordinates of P are

$$\left(\frac{1\times(-2)+2\times 4}{3}, \frac{1\times(-3)+2\times(-1)}{3}\right) = \left(2, -\frac{5}{3}\right)$$

Now, $AQ = AP + PQ = k + k = 2k$

$AQ : QB = 2k : k = 2 : 1$

and, coordinates of Q are

$$\left(\frac{2\times(-2)+1\times 4}{3}, \frac{2\times(-3)+1\times(-1)}{3}\right) = \left(0, -\frac{7}{3}\right)$$

Hence, points of trisection are $\left(2, -\frac{5}{3}\right)$ and $\left(0, -\frac{7}{3}\right)$.

9. Find the ratio in which the line segment joining $A(1, -5)$ and $B(-4, 5)$ is divided by the x-axis. Also find the coordinates of the point of division.

Sol. Let the required ratio be $k : 1$. Then, the coordinates of the point of division is

$$P\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right).$$

Since, this point lies on x-axis. Therefore its y-coordinate is zero.

i.e., $\frac{5k-5}{k+1} = 0$

$\Rightarrow$ $5k - 5 = 0$

$5k = 5$ or $k = \frac{5}{5} = 1$

Thus, the required ratio is 1 : 1 and the point of division is $P\left(\frac{-4\times 1+1}{1+1}, \frac{5\times 1-5}{1+1}\right)$

i.e., $P\left(-\frac{3}{2}, 0\right)$

10. Find the area of a rhombus if its vertices (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) are taken in order.

Sol. Let $A(3,0)$, $B(4, 5)$, $C(-1, 4)$ and $D(-2, -1)$ be the vertices of a rhombus.

Therefore, its diagonals

$AC = \sqrt{(-1-3)^2+(4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

and $BD = \sqrt{(-2-4)^2+(-1-5)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

$\therefore$ Area of rhombus $ABCD = \frac{1}{2} \times$ (Product of length of diagonals)

$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ sq units.

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. The co-ordinate of the point which is reflection of the point (–3, 5) in *X*-axis are **[*CBSE* 2020 (30/2/1)]**

(*a*) (3, 5) (*b*) (3, –5) (*c*) (–3, –5) (*d*) (–3, 5)

2. The distance between the points (0, 0) and $(a - b, a + b)$ is **[*CBSE* 2020 (30/C/1)]**

(*a*) $2\sqrt{ab}$ (*b*) $\sqrt{2a^2 + ab}$ (*c*) $2\sqrt{a^2 + b^2}$ (*d*) $\sqrt{2a^2 + 2b^2}$

3. The distance between the points $(a \cos \theta + b \sin \theta, 0)$ and $(0, a \sin \theta - b \cos \theta)$ is **[*CBSE* 2020 (30/1/1)]**

(*a*) $a^2 + b^2$ (*b*) $a^2 - b^2$ (*c*) $\sqrt{a^2 + b^2}$ (*d*) $\sqrt{a^2 - b^2}$

4. If the distance between the points (2, –2) and (–1, *x*) is 5, one of the values of *x* is **[*NCERT Exemplar*]**

(*a*) –2 (*b*) 2 (*c*) –1 (*d*) 1

5. The distance of the point *P* (2, 3) from the *x*-axis is **[*NCERT Exemplar*]**

(*a*) 2 units (*b*) 3 units (*c*) 1 units (*d*) 5 units

6. The coordinate of point *P* on *X*-axis equidistant from the points *A* (–1, 0) and *B* (5, 0) is **[*CBSE* 2020 (30/2/1)]**

(*a*) (2, 0) (*b*) (0, 2) (*c*) (3, 0) (*d*) (2, 2)

7. The distance between the points *A* (0, 6) and *B* (0, –2) is **[*NCERT Exemplar*]**

(*a*) 6 units (*b*) 8 units (*c*) 4 units (*d*) 2 units

8. The distance between two points, *M* and *N*, on a graph is given as $\sqrt{10^2 + 7^2}$. The coordinates of point *M* are (–4, 3). Given that the point *N* lies in the first quadrant, which of the following is true about the all possible *x* coordinates of point *N*? **[*Competency Based Question*]**

(*a*) They are multiple of 3. (*b*) They are multiple of 4.

(*c*) They are multiple of 5. (*d*) They are multiple of 6.

9. The coordinates of the point which is equidistant from the three vertices of the triangle shown in the given Fig. 6.10 are

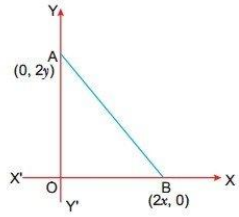


Fig. 6.10

(*a*) (x, y) (*b*) (y, x) (*c*) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (*d*) $\left(\frac{y}{2}, \frac{x}{2}\right)$

10. The end points of diameter of circle are (2, 4) and (–3, –1). The radius of the circle is

(*a*) $\frac{5\sqrt{2}}{2}$ units (*b*) $5\sqrt{2}$ units (*c*) $3\sqrt{2}$ units (*d*) $\frac{\pm 5\sqrt{2}}{2}$ units

11. Two poles are to be installed on an elevated road as shown in Fig. 6.11 The diagram also shows the starting and ending points of the road.

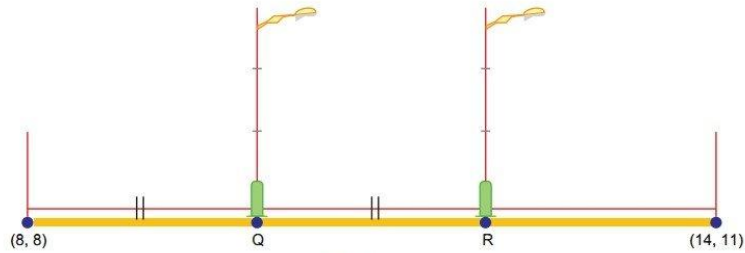


Fig. 6.11

Which of the following are the coordinates of the poles? [*Competency Based Question*]

(*a*) Q (10, 9) and R (12, 8) (*b*) Q (10, 9) and R (12, 10)

(*c*) Q (10, 8) and R (12, 11) (*d*) Q(–10, 9) and R (0, 11)

12. If (a, b) is the mid point of the line segment joining the points A (10, –6) and B $(k, 4)$ and $a - 2b = 18$, the values of k is [*CBSE* 2020 (30/3/1)]

(*a*) 30 (*b*) 22 (*c*) 4 (*d*) 40

13. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q (–6, 5) and R (–2, 3), then the value of a is [*NCERT Exemplar*]

(*a*) –4 (*b*) –12 (*c*) 12 (*d*) –6

14. A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively [*NCERT Exemplar*]

(*a*) (0, –5) and (2, 0) (*b*) (0, 10) and (–4, 0)

(*c*) (0, 4) and (–10, 0) (*d*) (0, –10) and (4, 0)

15. The points which lie on the perpendicular bisector of the line segment joining the points A (–2, –5), B (2, 5) is [*NCERT Exemplar*]

(*a*) (0, 0) (*b*) (0, 2) (*c*) (2, 0) (*d*) (–2, 0)

16. x-axis divides the join of (2, –3) and (5, 6) in the ratio [*CBSE Question Bank*]

(*a*) 1:2 (*b*) 2:1 (*c*) 2:5 (*d*) 5:2

17. The value of k for which the points A (0, 1), B (2, k) and C (4, –5) are collinear is [*CBSE* 2020 (30/3/1)]

(*a*) 2 (*b*) –2 (*c*) 0 (*d*) 4

18. The point on the x-axis which is equidistant from (– 4, 0) and (10, 0) is [*CBSE* 2020 (30/5/1)]

(*a*) (7, 0) (*b*) (5, 0) (*c*) (0, 0) (*d*) (3, 0)

19. It is being given that the points A (1, 2), B (0, 0) and C (a, b) are collinear. Which of the following relations between a and b is true? [*CBSE* 2020 (30/4/1)]

(*a*) $a = 2b$ (*b*) $2a = b$ (*c*) $a + b = 0$ (*d*) $a - b = 0$

20. The perpendicular bisector of the line segment joining the points A (1, 5) and B (4, 6) cuts the y-axis at [*NCERT Exemplar*]

(*a*) (0, 13) (*b*) (0, –13) (*c*) (0, 12) (*d*) (13, 0)

Answers

1. (*c*)	**2.** (*d*)	**3.** (*c*)	**4.** (*b*)	**5.** (*b*)	**6.** (*a*)	**7.** (*b*)
8. (*a*)	**9.** (*a*)	**10.** (*a*)	**11.** (*b*)	**12.** (*b*)	**13.** (*b*)	**14.** (*d*)
15. (*a*)	**16.** (*a*)	**17.** (*b*)	**18.** (*d*)	**19.** (*b*)	**20.** (*a*)	

Very Short Answer Questions

Each of the following questions are of 1 mark.

1. Find the distance of a point p (x, y) from the origin. **[*CBSE* 2018 (30/1/1)]**

Sol.

3) Distance between (x,y) and $(0,0)$.

$\Rightarrow \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$= \sqrt{(x-0)^2+(y-0)^2}$

$= \sqrt{x^2+y^2}$.

The distance is $\sqrt{x^2+y^2}$.

[*Topper's Answer* 2018]

2. If the distance between the points $(4, k)$ and $(1, 0)$ is 5, then what can be the possible values of k? **[*CBSE Delhi* 2017]**

Sol. Using distance formula $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, we have

$\Rightarrow \quad \sqrt{(4-1)^2+(k-0)^2}=5 \quad \Rightarrow \quad 9+k^2=25$

$\Rightarrow \quad k^2=16 \quad \Rightarrow \quad k=\pm 4$

3. Find the coordinates of a point A, where AB is a diameter of the circle with centre $(-2, 2)$ and B is the point with coordinates $(3, 4)$. **[*CBSE* 2019 (30/1/2)]**

Sol. Let co-ordinate of point A be (x, y).

$\therefore \quad -2=\dfrac{x+3}{2} \quad \Rightarrow \quad x+3=-4 \quad \Rightarrow x=-7$

and $\quad 2=\dfrac{y+4}{2} \quad \Rightarrow \quad y+4=4 \quad \Rightarrow y=0$

$\therefore$ Co-ordinate of A are $(-7, 0)$.

4. Find the positive value of m, if the distance between the points A $(5, -3)$ and B $(13, m)$ is 10 units. **[*CBSE* 2019 (30/3/2)]**

Sol.

6. Given, $A = 5, -3$

$B = 13, m$

$AB = 10$ units

Using distance formula;

$\sqrt{(13-5)^2+(m+3)^2} = 10$

On squaring;

$8^2+(m+3)^2 = 100$

$\Rightarrow (m+3)^2 = 100-64$

$\Rightarrow \sqrt{(m+3)^2} = \sqrt{36}$

$\Rightarrow (m+3) = \pm 6$

Considering only positive value;

$m = 6-3$

$\Rightarrow \boxed{m=3}$

[*Topper's Answer* 2019]

5. **If the centroid of triangle formed by points $P\ (a, b)$, $Q\ (b, c)$ and $R\ (c, a)$ is at the origin, what is the value of $a + b + c$?**

Sol. Centroid of $\Delta PQR = \left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right)$

Given $\left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right) = (0, 0)$

$\Rightarrow \quad a + b + c = 0$

6. **The coordinates of the points P and Q are respectively $(4, -3)$ and $(-1, 7)$. Find the abscissa of a point R on the line segment PQ such that $\frac{PR}{PQ} = \frac{3}{5}$.** **[*Competency Based Question*]**

Sol.

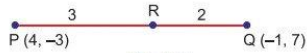


Fig. 6.12

$$\frac{PQ}{PR} = \frac{5}{3} \quad \Rightarrow \quad \frac{PR + RQ}{PR} = \frac{5}{3} \Rightarrow 1 + \frac{RQ}{PR} = \frac{5}{3} \quad \Rightarrow \quad \frac{RQ}{PR} = \frac{2}{3}$$

i.e., R divides PQ in the ratio $3 : 2$.

Abscissa of $R = \frac{3 \times (-1) + 2 \times 4}{3 + 2} = \frac{-3 + 8}{5} = 1$

Short Answer Questions–I

Each of the following questions are of 2 marks.

1. **Prove that the points (3, 0), (6, 4) and (–1, 3) are the vertices of a right isosceles triangle.** **[*CBSE (AI)* 2016]**

Sol.

θ) Let A = (3,0) ; B = (6,4) and C = (-1,3).

Applying distance formula –

$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ unit

$BC = \sqrt{(6+1)^2 + (4-3)^2} = \sqrt{7^2 + 1^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$ unit

$AC = \sqrt{(3+1)^2 + (0-3)^2} = \sqrt{25} = 5$ unit.

Since,

$AB = AC = 5$ unit

ΔABC is isosceles triangle.

Also, $AB^2 + AC^2$

$\Rightarrow 5^2 + 5^2$

$25 + 25 = 50 = BC^2 = (5\sqrt{2})^2 \Rightarrow AB^2 + AC^2 = BC^2$

Hence, by converse of Pythagoras Theorem,

$\Rightarrow \Delta ABC$ is right angled triangle.

[*Topper's Answer* 2016]

2. **If the point $A\ (0, 2)$ is equidistant from the points $B(3, p)$ and $C(p, 5)$, find p. Also find the length of AB.** **[*CBSE Delhi* 2014]**

Sol. Given that $A\ (0, 2)$ is equidistant from $B\ (3, p)$ and $C\ (p, 5)$.

$\therefore \qquad AB = AC$

$\Rightarrow \quad AB^2 = AC^2$

$\Rightarrow \quad (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow \quad 3^2 + p^2 + 4 - 4p = p^2 + 9 \qquad \Rightarrow \; 4 - 4p = 0$

$\Rightarrow \quad 4p = 4 \qquad \Rightarrow \; p = 1$

Length of $AB = \sqrt{(3-0)^2+(1-2)^2} = \sqrt{3^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$ units

3. If the point $P(x, y)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$. Prove that $bx = ay$. **[*CBSE (AI)* 2016]**

Sol. Given, $PA = PB$ or $(PA)^2 = (PB)^2$

$(a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$

$\Rightarrow \quad (a+b)^2 + x^2 - 2ax - 2bx + (b-a)^2 + y^2 - 2by + 2ay$

$= (a-b)^2 + x^2 - 2ax + 2bx + (a+b)^2 + y^2 - 2ay - 2by$

$\Rightarrow \quad 4ay = 4bx$ or $bx = ay$

Hence proved.

4. If the distances of $P(x, y)$ from $A(5, 1)$ and $B(-1, 5)$ are equal, then prove that $3x = 2y$. **[*CBSE (AI)* 2017]**

Sol.

8. $PA = PB$

$\therefore PA^2 = PB^2$

by distance formula,

$(5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$

$\Rightarrow (5-x)^2 + (1-y)^2 = (1+x)^2 + (5-y)^2$

$25 - 10x + x^2 + 1 - 2y + y^2 = 1 + 2x + x^2 + 25 - 10y + y^2$

$-10x - 2y = 2x - 10y$

$8y = 12x$

$4(2y) = 4(3x)$

$\therefore 3x = 2y$

Hence, proved.

[*Topper's Answer* 2017]

5. If two adjacent vertices of a parallelogram are (3, 2) and (– 1, 0) and the diagonals intersect at (2, – 5), then find the coordinates of the other two vertices. **[*CBSE (F)* 2017]**

Sol. Let other two coordinates are (x, y) and (x', y').

$\because$ O is mid point of AC and BD.

$\therefore \quad 2 = \dfrac{x+3}{2}$

$\Rightarrow \quad x = 1$

and, $\quad -5 = \dfrac{2+y}{2}$

$\Rightarrow \quad y = -12$

Again, $\quad \dfrac{-1+x'}{2} = 2$

$\Rightarrow \quad x' = 5$

and $\quad \dfrac{0+y'}{2} = -5 \quad \Rightarrow \quad y' = -10$

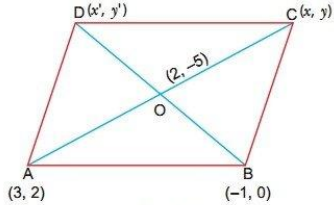


Fig. 6.13

Hence, co-ordinates are (1, –12) and (5, –10).

6. Point P and Q trisect the line segment joining the points A (–2, 0) and B (0, 8), such that P is nearer to A. Find the co-ordinates of points P and Q. **[*CBSE* 2019(30/3/1)]**

Sol. P divides AB in the ratio 1 : 2

$\therefore$ Coordinates of P are $\left(\frac{0-4}{3}, \frac{8+0}{3}\right) = \left(\frac{-4}{3}, \frac{8}{3}\right)$

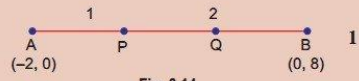


Fig. 6.14

1

Q divides AB in the ratio 2 : 1

$\therefore$ Coordinates of Q are $\left(\frac{0-2}{3}, \frac{16+0}{3}\right) = \left(\frac{-2}{3}, \frac{16}{3}\right)$ 1

[*CBSE Marking Scheme* 2019 (30/3/1)]

7. If the point C (–1, 2) divides internally the line segment joining the points $A(2, 5)$ and $B(x, y)$ in the ratio of 3 : 4, find the value of $x^2 + y^2$. **[*CBSE (F)* 2016]**

Sol. $\because$ C (–1, 2) divides AB in the ratio 3 : 4.

$\therefore$ $\frac{3x + 4(2)}{7} = -1 \Rightarrow x = -5$

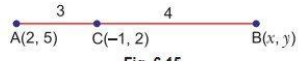


Fig. 6.15

and $\frac{3y + 4(5)}{7} = 2 \Rightarrow y = -2$

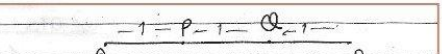
$\therefore$ $x^2 + y^2 = (-5)^2 + (-2)^2 = 29$

8. Let P and Q be the points of trisection of the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ such that P is nearer to A. Find the coordinates of P and Q. **[*CBSE (AI)* 2016]**

Sol.

9) We have –

Line AB, joining points A(2, –2) and B(–7, 4)



P & Q are points of trisection, then,

P ~~divides~~ divides AB in ratio 1:2 & Q in ratio 2:1

By section formula –

$x = \frac{mx_2 + nx_1}{m+n}$ & $y = \frac{my_2 + ny_1}{m+n}$

$P(x, y) = \frac{1 \times -7 + 2 \times 2}{3}$ and $y = \frac{1 \times 4 + 2 \times -2}{3}$

$P(x, y) = \frac{-7+4}{3}$ and $\frac{4-4}{3}$

$\boxed{P(x, y) = (-1, 0)}$

$Q(x', y') = \frac{2 \times -7 + 1 \times 2}{3}$ and $y = \frac{2 \times 4 + 1 \times -2}{3}$

$Q(x', y') = \frac{-14+2}{3}$ and $\frac{8-2}{3}$

$\boxed{Q(x'y') = (-4, 2)}$

[*Topper's Answer* 2016]

9. Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m. **[*CBSE* 2018 (30/1/1)]**

Sol.

Ex 9) Points A(2,3), B(6,-3) divided by P(4,m).

Let the ratio be k:1.

By Seg: section formula,

$P(4,m) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$(4,m) = \left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right)$

$\Rightarrow \frac{6k+2}{k+1} = 4$

$6k+2 = 4k+4$

$2k = 2$

$k = 1$. → The ratio is 1:1.

Now, $m = \frac{-3k+3}{k+1}$

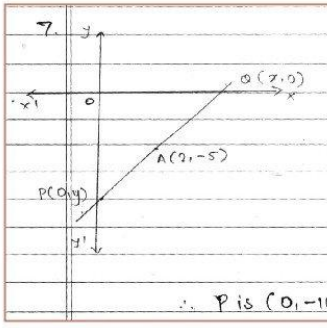
[*Topper's Answer* 2018]

$m = \frac{-3+3}{1+1}$

$m = 0$. → Value of m is 0, the point is P(4,0).

10. A line intersects the Y-axis and X-axis at the points P and Q respectively. If $(2, -5)$ is the mid-point of PQ, then find the co-ordinates of P and Q. **[*CBSE (AI)* 2017)]**

Sol.



Let coordinate of P be (0,y) and of Q be (x,0).

A(2,-5) is mid point of PQ.

by section formula,

$(2,-5) = \left(\frac{0+x}{2}, \frac{y+0}{2}\right)$

$2 = \frac{x}{2}$ and $-5 = \frac{y}{2}$

$\therefore x = 4$ and $y = -10$.

$\therefore$ P is (0,-10) and Q is (4,0) [*Topper's Answer* 2017]

Short Answer Questions–II

Each of the following questions are of* 3 *marks.

1. Find the coordinates of a point on the x-axis which is equidistant from the points $A(2, -5)$ and $B(-2, 9)$. **[*CBSE Delhi* 2017 (*C*)]**

Sol. Let $P(x, 0)$ be any point on x-axis.

Now, $P(x, 0)$ is equidistant from point $A(2, -5)$ and $B(-2, 9)$.

$\therefore \quad AP = BP$

$\Rightarrow \quad \sqrt{(x-2)^2+(0+5)^2} = \sqrt{(x+2)^2+(0-9)^2}$

Squaring both sides, we have

$$(x-2)^2 + 25 = (x+2)^2 + 81$$

$\Rightarrow \quad x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81 \quad \Rightarrow \quad -8x = 56$

$\therefore \quad x = \dfrac{56}{-8} = -7$

$\therefore$ The point on the x-axis equidistant from given points is $(-7, 0)$.

2. In Fig. 6.16, ΔABC is an equilateral triangle of side 3 units. Find the coordinates of the other two vertices. **[*Competency Based Question*] [*CBSE (F)* 2017]**

Sol. Since $AB = BC = AC = 3$ units

$\therefore$ Co-ordinates of B are $(5, 0)$.

Let co-ordinates of C be (x, y).

$AC^2 = BC^2$ [$\because$ ΔABC is an equilateral triangle]

Using distance formula

$(x-2)^2 + (y-0)^2 = (x-5)^2 + (y-0)^2$

$\Rightarrow \quad x^2 + 4 - 4x + y^2 = x^2 + 25 - 10x + y^2$

$6x = 21$

$\Rightarrow \quad x = \dfrac{21}{6} = \dfrac{7}{2}$

Again $(x-2)^2 + (y-0)^2 = 9$

$\Rightarrow \quad \left(\dfrac{7}{2} - 2\right)^2 + y^2 = 9$

$\Rightarrow \quad \left(\dfrac{3}{2}\right)^2 + y^2 = 9 \quad \Rightarrow \quad y^2 = 9 - \dfrac{9}{4}$

$\Rightarrow \quad y^2 = \dfrac{27}{4} \quad \Rightarrow \quad y = \dfrac{3\sqrt{3}}{2}$ (+ve sign to be taken)

Hence, co-ordinate of $C\left(\dfrac{7}{2}, \dfrac{3\sqrt{3}}{2}\right)$.

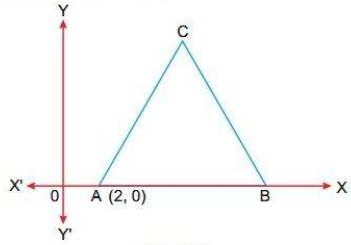


Fig. 6.16

3. If the point $P(k-1, 2)$ is equidistant from the points $A(3, k)$ and $B(k, 5)$, find the values of k. **[*CBSE (AI)* 2014]**

Sol. Since P is equidistant from A and B,

$AP = BP$ or $AP^2 = BP^2$

$\Rightarrow \quad [3 - (k-1)]^2 + (k-2)^2 = [k - (k-1)]^2 + (5-2)^2$

$\Rightarrow \quad (3 - k + 1)^2 + (k-2)^2 = (k - k + 1)^2 + (3)^2$

$\Rightarrow \quad (4-k)^2 + (k-2)^2 = (1)^2 + (3)^2$

$\Rightarrow \quad 16 + k^2 - 8k + k^2 + 4 - 4k = 1 + 9$

$\Rightarrow \quad 2k^2 - 12k + 20 = 10 \quad \Rightarrow \quad k^2 - 6k + 10 = 5$

$\Rightarrow \quad k^2 - 6k + 5 = 0 \quad \Rightarrow \quad k^2 - 5k - k + 5 = 0$

$\Rightarrow \quad k(k-5) - 1(k-5) = 0 \quad \Rightarrow \quad k = 1$ or $k = 5$

4. If the mid-point of the line segment joining the points $A(3, 4)$ and $B(k, 6)$ is $P(x, y)$ and $x + y - 10 = 0$, find the value k. **[*CBSE* 2020 (30/2/1)]**

Sol.

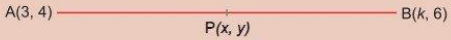


Fig. 6.17

$x = \dfrac{3+k}{2}, \quad y = 5$ ½+½

$x + y - 10 = 0 \quad \Rightarrow \quad \dfrac{3+k}{2} + 5 - 10 = 0$ 1

$\Rightarrow \quad k = 7$ 1

[*CBSE Marking Scheme* 2020 (30/2/1)]

5. In what ratio does the point $P(-4, y)$ divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$ if it lies on AB? Hence find the value of y. **[*CBSE* 2020 (30/3/1)]**

Sol. Let $k : 1$ be the required ratio.

$\therefore$ Using section formula

$$x = \frac{kx_2 + x_1}{k+1} \quad \Rightarrow \quad -4 = \frac{k \times 3 + 1 \times (-6)}{k+1}$$

$$\Rightarrow \quad -4k - 4 = 3k - 6 \quad \Rightarrow \quad -4 + 6 = 3k + 4k$$

$$\Rightarrow \quad 2 = 7k \quad \Rightarrow \quad k = \frac{2}{7}$$

$\therefore$ Ratio be $2 : 7$.

Now, $$y = \frac{k \times (-8) + 1 \times 10}{k+1} = \frac{\frac{2}{7} \times (-8) + 10}{\frac{2}{7} + 1}$$

$$= \frac{-16 + 70}{9} = \frac{54}{9} = 6$$

$\therefore$ $y = 6$

6. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the point $P(2, -2)$ and $Q(3, 7)$? Also find the value of y. **[*CBSE* 2017 (30/3)]**

Sol.

14.

m : n

P(2, −2) (24/11, y) Q(3, 7)

Here $x_1 = 2$, $y_1 = -2$

$x_2 = 3$, $y_2 = 7$

Using section formula,

$$\left(\frac{24}{11}, y\right) = \left(\frac{3m + 2n}{m+n}, \frac{7m - 2n}{m+n}\right) \quad -①$$

$$\Rightarrow \quad \frac{24}{11} = \frac{3m + 2n}{m+n}$$

$$24m + 24n = 33m + 22n$$

$$2n = 9m$$

$$\frac{2}{9} = \frac{m}{n}$$

∴ The given point divides the line segment in ratio 2 : 9.

Taking $m = 2$ and $n = 9$,

$$y = \frac{7m - 2n}{m+n} \quad \text{(from ①)}$$

$$y = \frac{7(2) - 2(9)}{2+9}$$

$$y = \frac{14 - 18}{11}$$

$$\boxed{y = \frac{-4}{11}}$$

[*Topper's Answer* 2017]

7. Find the coordinates of the points of trisection of the line segment joining the points (3, –1) and (6, 8). **[*CBSE* 2020 (30/4/1)]**

Sol. **Case I:** If C and D trisect AB then C divides AB in the ratio 1 : 2. ½

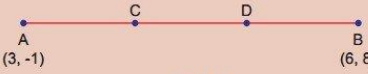


Fig. 6.18

Co-ordinates of C : $x = \frac{1\times 6 + 2\times 3}{3} = 4$ ½

and $y = \frac{1\times 8 + 2\times(-1)}{3} = 2$ ½

$\therefore$ Co-ordinates of point C be (4, 2).

Case II: If D divides AB in the ratio 2 : 1, then ½

Co-ordinates of D: $x' = \frac{2\times 6 + 1\times 3}{3} = 5$ ½

and $y' = \frac{2\times 8 + 1\times(-1)}{3} = 5$ ½

$\therefore$ Co-ordinates of D (5, 5). **[*CBSE Marking Scheme* 2020 (30/4/1)]**

Long Answer Questions

Each of the following questions are of* 5 *marks.

1. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, – 3) and (3, 4). Find its centroid.

Sol. Let $P(1, 1)$, $Q(2, -3)$, $R(3, 4)$ be the mid-points of sides AB, BC and CA respectively, of triangle ABC. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of triangle ABC. Then, P is the mid-point of AB.

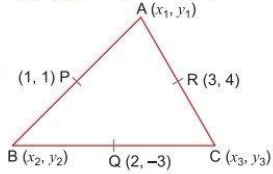


Fig. 6.19

$\Rightarrow \quad \frac{x_1 + x_2}{2} = 1, \quad \frac{y_1 + y_2}{2} = 1$

$\Rightarrow \quad x_1 + x_2 = 2 \quad \text{and} \quad y_1 + y_2 = 2 \qquad ...(i)$

Q is the mid-point of BC

$\Rightarrow \quad \frac{x_2 + x_3}{2} = 2, \quad \frac{y_2 + y_3}{2} = -3$

$\Rightarrow \quad x_2 + x_3 = 4 \quad \text{and} \quad y_2 + y_3 = -6 \qquad ...(ii)$

R is the mid-point of AC

$\Rightarrow \quad \frac{x_1 + x_3}{2} = 3 \quad \text{and} \quad \frac{y_1 + y_3}{2} = 4$

$\Rightarrow \quad x_1 + x_3 = 6 \quad \text{and} \quad y_1 + y_3 = 8 \qquad ...(iii)$

From (i), (ii) and (iii), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6$

and $y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2 - 6 + 8$

$\Rightarrow \quad x_1 + x_2 + x_3 = 6 \quad \text{and} \quad y_1 + y_2 + y_3 = 2 \qquad ...(iv)$

The coordinates of the centroid of ΔABC are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) = \left(\frac{6}{3}, \frac{2}{3}\right) = \left(2, \frac{2}{3}\right) \quad \text{[Using } (iv)\text{]}$$

2. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the coordinates of the point of intersection. **[CBSE 2019 (30/2/1)]**

Sol. Let the line $x - 3y = 0$ intersect the segment joining $A(-2, -5)$ and $B(6, 3)$ in the ratio $k : 1$

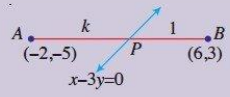


Fig. 6.20

$\therefore$ Coordinates of P are $\left(\frac{6k-2}{k+1}, \frac{3k-5}{k+1}\right)$

P lies on $x - 3y = 0 \Rightarrow \frac{6k-2}{k+1} = 3\left(\frac{3k-5}{k+1}\right) \Rightarrow k = \frac{13}{3}$

$\therefore$ Ratio is $13 : 3$

$\Rightarrow$ Coordinates of P are $\left(\frac{9}{2}, \frac{3}{2}\right)$

[CBSE Marking Scheme 2019 (30/2/1)]

3. (i) Find the ratio in which the y-axis divides the line segment joining the points $(6, -4)$ and $(-2, -7)$. Also find the point of intersection.

(ii) Show that the points $(7, 10)$, $(-2, 5)$ and $(3, -4)$ are vertices of an isosceles right triangle. **[CBSE 2020 (30/5/1)]**

Sol. (i) Let the point $P(0, y)$ on y-axis divides the line segment AB in $k : 1$

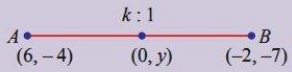


Fig. 6.21

$\therefore \quad 0 = \frac{-2k+6}{k+1} \Rightarrow k = 3 \ \therefore$ Ratio is $3 : 1$

Also, $y = \frac{3(-7)+1(-4)}{3+1} = \frac{-25}{4} \ \therefore$ Point of intersection is $\left(0, \frac{-25}{4}\right)$

(ii) Let the points be $A(7, 10)$, $B(-2, 5)$ and $C(3, -4)$

$$AB = \sqrt{(-2-7)^2 + (5-10)^2} = \sqrt{106}$$

$$BC = \sqrt{(3+2)^2 + (-4-5)^2} = \sqrt{106}$$

$$AC = \sqrt{(3-7)^2 + (-4-10)^2} = \sqrt{212}$$

$AB = BC$ and $AC^2 = AB^2 + BC^2$

Hence ABC is isosceles right triangle.

[CBSE Marking Scheme 2020 (30/5/1)]

4. Point P divides the line segment joining the points $A(2,1)$ and $B(5,-8)$ such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k. **[CBSE 2019 (30/5/1)]**

Sol. Let the point P be (x, y) which divides AB such that

$$\frac{AP}{AB} = \frac{1}{3} \Rightarrow \frac{AB}{AP} = \frac{3}{1} \Rightarrow \frac{AB}{AP} - 1 = \frac{3}{1} - 1$$

$$\Rightarrow \quad \frac{AB - AP}{AP} = \frac{2}{1} \quad \Rightarrow \quad \frac{PB}{AP} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{AP}{BP} = \frac{1}{2} \Rightarrow AP : BP = 1 : 2$$

$\therefore$ Using section formula, we have

$$x = \frac{1 \times 5 + 2 \times 2}{1 + 2} = \frac{9}{3} = 3$$

$$y = \frac{1 \times (-8) + 2 \times 1}{1 + 2} = \frac{-8 + 2}{3} = \frac{-6}{3} = -2$$

$\therefore$ Co-ordinate of P be $(3, -2)$.

Now, $P(3, -2)$ lies on the line $2x - y + k = 0$.

$\therefore \quad 2 \times 3 - (-2) + k = 0$

$\Rightarrow \quad 6 + 2 + k = 0 \quad \Rightarrow \quad k = -8$

Case Study-based Questions

Each of the following questions are of 4 marks.

1. Read the following and answer any four questions from (*i*) to (*v*).

In order to conduct Sports Day activities in your School, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground $ABCD$, 100 flowerpots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs 1/4 th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5th distance AD on the eighth line and posts a red flag. **[*Competency Based Question*]**

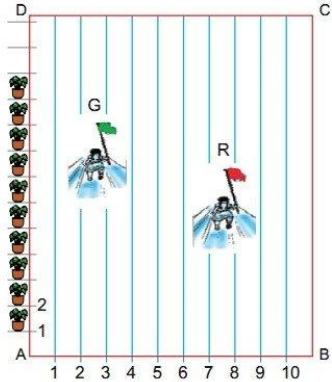


Fig. 6.22

(*i*) The position of green flag is

(*a*) (2, 25) (*b*) (2, 0.25) (*c*) (25, 2) (*d*) (0, –25)

(*ii*) The position of red flag is

(*a*) (8, 0) (*b*) (20, 8) (*c*) (8, 20) (*d*) (8, 0.2)

(*iii*) What is the distance between both the flags?

(*a*) $\sqrt{41}$ (*b*) $\sqrt{11}$ (*c*) $\sqrt{61}$ (*d*) $\sqrt{51}$

(*iv*) If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

(*a*) (5, 22.5) (*b*) (10, 22) (*c*) (2, 8.5) (*d*) (2.5, 20)

(*v*) If Joy has to post a flag at one-fourth distance from green flag in the line segment joining the green and red flags, then where should he post his flag?

(*a*) (3.2, 24) (*b*) (0.5, 12.5) (*c*) (2.25, 8.5) (*d*) (25, 20)

Sol. (*i*) Niharika runs $\frac{1}{4}$th the distance AD on the second line, post a green flag.

$\therefore$ Position of the green flag is $\left(2, \frac{1}{4}\times 100\right)$, *i.e.*, (2, 25).

$\therefore$ Option (*a*) is correct.

(*ii*) Preet runs $\frac{1}{5}$th distance AD on the eighth line and posts a red flag.

$\therefore$ Position of the red flag is $\left(8, \frac{1}{5}\times 100\right)$, *i.e.*, (8, 20).

Its position is (8, 20).

$\therefore$ Option (*c*) is correct.

(*iii*) Distance between green flag and red flag is

$$= \sqrt{(20-25)^2 + (8-2)^2} = \sqrt{25+36} = \sqrt{61} \text{ units}$$

$\therefore$ Option (*c*) is correct.

(*iv*) Position of the blue flag $= \left(\frac{2+8}{2}, \frac{25+20}{2}\right) = (5, 22.5)$

$\therefore$ Option (*a*) is correct.

(*v*) Let $P(x, y)$ be the position of a flag posted by Joy using section formula, we have

$$x = \frac{1\times 8 + 4\times 2}{1+4} = \frac{16}{5} = 3.2$$

$$y = \frac{1\times 20 + 4\times 25}{1+4} = \frac{120}{5} = 24$$

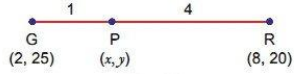
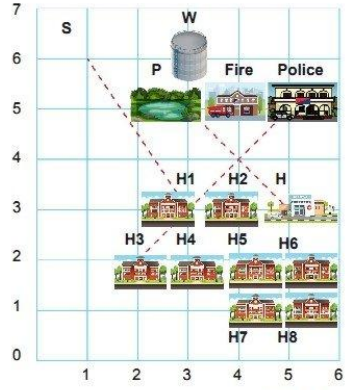


Fig. 6.23

$\therefore$ Position of the flag posted by Joy is (3.2, 24).

$\therefore$ Option (*a*) is correct.

2. Shown below is a town plan on a coordinate grid, where 1 unit = 1 km. Consider the co-ordinates of each building to be the point of intersection of the respective grid lines.



S	-	School
W	-	Water Tank
P	-	Pond
Fire	-	Fire Station
Police	-	Police Station
H	-	Hospital
H1	-	House 1
H2	-	House 2
H3	-	House 3
H4	-	House 4
H5	-	House 5
H6	-	House 6
H7	-	House 7
H8	-	House 8

Fig. 6.24

Based on the above information answer the following questions.

(*i*) What is the distance between the School and House 1 along the path shown?

(*ii*) What is the ratio in which House 1 divides the path joining house 3 and the police station?

Sol. (*i*) Coordinates of school are (1, 6).

Coordinates of House 1 are (3, 3).

$\therefore$ Distance between school and House 1, is

$$\sqrt{(3-1)^2 + (3-6)^2} = \sqrt{4+9} = \sqrt{13} \text{ km}$$

(*ii*) Coordinates of House 1 (H_1) are (3, 3).

Fig. 6.25

House 3 (H_3) coordinates are (2, 2).

Coordinates of police station are (5, 5).

Let H_1 divides path joining H_3 and police station in the ratio $k : 1$.

By section formula we have

$$3 = \frac{k(5) + 1(2)}{k+1}$$

$$\Rightarrow \quad 3k + 3 = 5k + 2$$

$$\Rightarrow \quad 3 - 2 = 5k - 3k$$

$$\Rightarrow \quad 2k = 1 \Rightarrow k = \frac{1}{2}$$

$$\therefore k : 1 = \frac{1}{2} : 1 = 1 : 2$$

PROFICIENCY EXERCISE

■ Objective Type Questions: [1 mark each]

1. Choose and write the correct option in each of the following questions.

(*i*) Three points lie on a vertical line. Which of the following could be those points?

[*Competency Based Question*]

(*a*) (– 8, 3), (– 8, 8), (8, 7) (*b*) (–8, 7), (–8, –8), (–8, –100)

(*c*) (4, 3), (5, 3), (–12, 3) (*d*) (0, 4), (4, 0), (0, 0)

(*ii*) The distance of the point P (–6, 8) from the origin is

(*a*) 8 units (*b*) $2\sqrt{7}$ units (*c*) 10 units (*d*) 6 units

(*iii*) A point G divides a line segment in the ratio 3 : 7. The segment starts at the origin and ends at a point K having 20 as its abscissa and 40 as its ordinate. Given that G is closer to the origin than to point K,

Which of the following are the coordinates of point G?

(*a*) (6, 12) (*b*) (12, 6) (*c*) (14, 28) (*d*) (28, 14)

(*iv*) The ratio in which x-axis divides the line segment joining $A(2, -3)$ and B (5, 6) is

(*a*) 3 : 5 (*b*) 1 : 2 (*c*) 2 : 1 (*d*) 2 : 3

(*v*) Which of the following are the coordinates of the intersection points of the diagonals of the rectangle $ABCD$ with vertices $A(0, 3)$, $B(3, 0)$, $C(1, -2)$ and $D(-2, 1)$?

(*a*) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (*b*) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (*c*) (1.5, 1.5) (*d*) (2, –1)

■ Very Short Answer Questions: [1 mark each]

2. Find the values of x for which the distance between the points $A(x, 2)$ and $B(9, 8)$ is 10 units.
[*CBSE* 2019 (30/5/3)]

3. Find the value(s) of x, if the distance between the points $A(0, 0)$ and $B(x, -4)$ is 5 units.
[*CBSE* 2019 (30/3/1)]

4. $A(5, 1)$; $B(1, 5)$ and $C(-3, -1)$ are the vertices of ΔABC. Find the length of median AD.
[*CBSE* 2018 (*C*) (30/1)]

5. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, then find the value of a. **[*NCERT Exemplar*]**

6. A line intersects the y-axis and x-axis at the points P and Q respectively. If $(2, -5)$ is the mid-point of PQ, then find the coordinates of P and Q. **[*NCERT Exemplar*, *CBSE (AI)* 2017]**

7. Where does the perpendicular bisector of the line segment joining the points $A(1, 5)$ and $B(4, 6)$ cut the y-axis? **[*NCERT Exemplar*]**

8. In what ratio is the line segment joining the points $P(3, -6)$ and $Q(5, 3)$ divided by x-axis?
[*CBSE* 2019 (30/1/1)]

9. The point which divides the line segment joining the points $(7, -6)$ and $(3, 4)$ in ratio $1 : 2$ internally lies in which quadrant? **[*Competency Based Question*] [*NCERT Exemplar*]**

10. If the points $(k, 2k)$, $(3k, 3k)$ and $(3, 1)$ are collinear, then find the values of k.

11. If points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear, then find the value of $\frac{1}{a} + \frac{1}{b}$.
[*Competency Based Question*]

12. If the centroid of a triangle formed by the points (a, b), (b, c) and (c, a) is at the origin, then find the value of $a^3 + b^3 + c^3$.

■ Short Answer Questions–I: [2 marks each]

13. Find the linear relation between x and y such that $P(x, y)$ is equidistant from the points $A(1, 4)$ and $B(-1, 2)$. **[CBSE 2018 (C) (30/1)]**

14. Prove that the points $(3, 0)$, $(6, 4)$ and $(-1, 3)$ are the vertices of a right angled isosceles triangle. **[CBSE (AI) 2016]**

15. Name the type of triangle ABC formed by the points $A(\sqrt{2}, \sqrt{2})$, $B(-\sqrt{2}, -\sqrt{2})$ and $C(-\sqrt{6}, \sqrt{6})$.

16. Find a point which is equidistant from the points $P(-5, 4)$ and $Q(-1, 6)$. How many such points are there?

17. If $A(-2, 2)$, $B(5, 2)$ and $C(k, 8)$ are the vertices of a right-angled triangle ABC with $\angle B = 90°$, then find the value of k. **[CBSE 2019 (C) (30/1/2)]**

18. Points $A(3, 1)$, $B(5, 1)$, $C(a, b)$ and $D(4, 3)$ are vertices of a parallelogram $ABCD$. Find the values of a and b. **[CBSE 2019 (30/3/1)]**

19. If the mid-point of a segment joining $A\left(\frac{x}{2}, \frac{y+1}{2}\right)$ and $B(x + 1, y - 3)$ is $C(5, -2)$, find x, y.

20. The point R divides the line segment AB, where $A(-4, 0)$ and $B(0, 6)$ such that $AR = \frac{3}{4} AB$. Find the coordinates of R. **[CBSE 2019 (30/4/2)]**

21. Find the relation between x and y such that the points (x, y), $(1, 2)$ and $(7, 0)$ are collinear. **[CBSE 2019 (C) (30/1/1)]**

22. The vertices of a triangle are $(a, b - c)$, $(b, c - a)$ and $(c, a - b)$. Prove that its centroid lies on x-axis.

■ Short Answer Questions–II: [3 marks each]

23. Show that (a, a), $(-a, -a)$, and $(-\sqrt{3}a, \sqrt{3}a)$ are vertices of an equilateral triangle. **[CBSE 2019 (C) (30/1/1)]**

24. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point $(7, -4)$. How many such points are there?

25. Find the coordinates of the point on the y-axis which is equidistant from the points $A(5, 3)$ and $B(1, -5)$. **[CBSE Delhi 2017 (C)]**

26. Show that $(1, -1)$ is the centre of the circle circumscribing the triangle whose vertices are $(4, 3)$, $(-2, 3)$ and $(6, -1)$.

27. If $R(x, y)$ is a point on the line segment joining the points $P(a, b)$ and $Q(b, a)$, then prove that $x + y = a + b$.

28. If $A(-2, 1)$, $B(a, 0)$, $C(4, b)$ and $D(1, 2)$ are vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides. **[CBSE 2018 (30/1)]**

29. Point A lies on the line segment XY joining $X(6, -6)$ and $Y(-4, -1)$ in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point A also lies on the line $3x + k(y + 1) = 0$, find the value of k. **[CBSE 2019 (30/2/2)]**

30. In what ratio does the x-axis divide the line segment joining the points $(-4, -6)$ and $(-1, 7)$? Find the coordinates of the point of division.

31. In what ratio does the point $P(-4, y)$ divide the line segment joining the point $A(-6, 10)$ and $B(3, -8)$? Hence find the value of y. **[CBSE 2019 (30/4/2)]**

32. Find the value of p for which the points $(-5, 1)$, $(1, p)$ and $(4, -2)$ are collinear. **[CBSE 2019 (30/4/2)]**

33. If the points P (–3, 9), Q (a, b) and R (4, –5) are collinear and $a + b = 1$, find the values of a and b. **[*CBSE Delhi* 2014]**

■ **Long Answer Questions:** **[5 marks each]**

34. Find the centre of a circle passing through the points (6, –6), (3, –7) and (3, 3).

35. The points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of ΔABC.

(*i*) The median from A meets BC at D. Find the coordinates of the point D.

(*ii*) Find the coordinates of the point P on AD such that $AP : PD = 2 : 1$.

(*iii*) Find the coordinates of points Q and R on medians BE and CF respectively such that $BQ : QE = 2 : 1$ and $CR : RF = 2 : 1$.

(*iv*) What are the coordinates of the centroid of the triangle ABC? **[*NCERT Exemplar*]**

36. The mid-point P of the line segment joining the points A (–10, 4) and B (–2, 0) lies on the line segment joining the points C (–9, –4) and D (–4, y). Find the ratio in which P divides CD. Also find the value of y. **[*CBSE (F)* 2014]**

Answers

1. (*i*) (*b*) (*ii*) (*c*) (*iii*) (*a*) (*iv*) (*b*) (*v*) (*a*) (*vi*) (*c*)

2. $x = 17,\ x = 1$ **3.** $x = \pm 3$ **4.** $\sqrt{37}$ **5.** $a = -12$

6. (0, –10) and (4, 0) **7.** (0, 13) **8.** 2 : 1 **9.** IVth **10.** $k = -\frac{1}{3}$ or 0

11. 1 **12.** $3abc$ **13.** $x + y - 3 = 0$ **15.** Equilateral

16. (–3, 5), infinitely many **17.** $k = 5$ **18.** $a = 6, b = 3$ **19.** $x = 6, y = -1$

20. $R\left(-1, \frac{9}{2}\right)$ **21.** $x + 3y - 7 = 0$ **22.** (3, 1) **24.** (9, 0), (5, 0); 2 points

25. $0, \frac{1}{2}$ **28.** $a = 1, b = 1$; $\sqrt{10}$ units **29.** $k = 2$ **30.** 6 : 7; $\left(\frac{-34}{13}, 0\right)$

31. 2 : 7; $y = 6$ **32.** $p = -1$ **33.** $a = 2, b = -1$ **34.** (3, –2)

35. (*i*) $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$ (*ii*) $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

(*iii*) $Q\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right), R\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$ (*iv*) $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

36. 3 : 2, $y = 6$

Self-Assessment

Time allowed: 1 hour Max. marks: 40

SECTION A

1. Choose and write the correct option in the following questions. **(3 × 1 = 3)**

(*i*) Komal was asked to plot a point 10 units on the left of the origin and other points 4 units directly above the origin. Which of the following are the two points?

(*a*) (10, 0) and (0, 4) (*b*) (–10, 0) and (0, 4)

(*c*) (10, 0) and (0, –4) (*d*) (–10, 0) and (4, 0)

(*ii*) The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(*a*) 5 units (*b*) 12 units (*c*) 11 units (*d*) 14 units

(*iii*) If the points $(1, x)$, (5, 2) and (9, 5) are collinear then value of x is

(*a*) $\frac{5}{2}$ (*b*) $-\frac{5}{2}$ (*c*) -1 (*d*) 1

2. Solve the following questions. **(2 × 1 = 2)**

(*i*) Find the value of 'a' so that the point $(3, a)$ lies on the line represented by $2x - 3y = 5$. **[*CBSE* 2019 (30/5/1)]**

(*ii*) What is the ratio in which the point $P\left(\frac{-2}{5}, 6\right)$ divides the line joining A (– 4, 3) and B (2, 8)?

SECTION B

■ **Solve the following questions.** **(4 × 2 = 8)**

3. What is the distance between the points A $(5\cos\theta, 0)$ and B $(0, 5\sin\theta)$?

4. Examine whether the points (1, – 1), (–5, 7) and (2, 5) are equidistant from the point (–2, 3)?

5. Prove that the points (2, –2), (–2, 1) and (5, 2) are the vertices of a right angled triangle.

6. Find the value of k for which the points (–5, 1), $(1, k)$ and (4, –2) are collinear. **[*CBSE Delhi* 2017 (*C*)]**

■ **Solve the following questions.** **(4 × 3 = 12)**

7. Find the ratio in which the line segment joining A (1, –5) and B (–4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

8. If $a \neq b \neq 0$, prove that the points (a, a^2), (b, b^2), (0, 0) will not be collinear.

9. If $ABCD$ be a rectangle and P be any point in the plane of the rectangle, then prove that

$$PA^2 + PC^2 = PB^2 + PD^2$$

[**Hint:** Take A as the origin and AB and AD as x and y-axis respectively. Let $AB = a$, $AD = b$]

10. The two opposite vertices of a square are (–1, 2) and (3, 2). Find the coordinates of other two vertices.

■ **Solve the following questions.** **(3 × 5 = 15)**

11. If the distance of $P(x, y)$ from $A(6, 1)$ and $B(-1, 6)$ are equal then prove that $7x = 5y$.

12. The base AB of two equilateral triangles ABC and ABC' with side $2a$ lies along the x-axis, such that the mid point of AB is at the origin. Find coordinates of vertices C and C' of the triangles.

13. Show that the points A (1, 0), B (5, 3), C (2, 7) and D (–2, 4) are the vertices of a rhombus.

Answers

1. (*i*) (*b*) (*ii*) (*b*) (*iii*) (*c*) **2.** (*i*) $a = \frac{1}{3}$ (*ii*) 3 : 2

3. 5 units **4.** No **6.** $k = -1$ **7.** 1 : 1; $\left(-\frac{3}{2}, 0\right)$

10. (1, 0); (1, 4) **12.** $C(0, \sqrt{3}a)$, $C'(0, -\sqrt{3}a)$