PLAYING WITH NUMBERS

4
CHAPTER

CONTENTS

➢ GENERALISED FORM OF 2 DIGIT AND 3 DIGIT NUMBERS

(a) 2 digit number has the tens place and the units place

Eg. 45 = 4 × 10 + 5, 93 = 9 × 10 + 3

Eg. ab = 10a + b,   ba = 10b + a

(b) 3 digit number has the hundreds place, the tens place and the units place.

Eg. 393 = 3 × 100 + 9 × 10 + 3

Eg. 492 = 4 × 100 + 9 × 10 + 2

Eg. 102 = 1 × 100 + 0 × 10 + 2

Eg. abc = 100 a + 10 b + c

Eg. cba = 100 c + 10 b + a

Eg. The usual form of 10 × 7 + 8 and 10 × 5 + 7 are 78 and 57 respectively.

➢ REVERSING THE DIGITS

(a) **2 Digit Number :** If number is ab, a ≠ 0 then reverse is ba. The difference of number & its reverse is divisible by 9.

Eg. Reverse of 23 or 2×10 + 3 is 32 or 3×10 + 2

also 32 – 23 = 9 Its divisible by 9.

(b) **3 Digit Number :**

If number is abc or 100a + 10b + c, a ≠ 0 then the reverse is cba or 100c + 10b + a

If a > c then

abc – cba = (100a + 10b + c) – (100c + 10b + a)

= 99 (a – c)

If c > a then

cba – abc = (100c + 10b + a) – (100a + 10b + c)

= 99 (c – a)

That means difference of a 3 digit number and its reverse number is divisible by 99.

* We can make more numbers from given no. abc like bca, acb, bac, cab etc.

also abc + bca + cab = 111 (a + b + c)

= 37 × 3 (a + b + c)

∴ The number (abc + cab + bca) is divisible by 37, 3 and a + b + c.

eg. 927 :

927 + 279 + 792 = 3 × 37 (9 + 2 + 7)

= 3 × 37 × 18

1998 ÷ 3 = 666 = 18 × 37

1998 ÷ 37 = 54 = 18 × 3

1998 ÷ 18 = 111 = 3 × 37

Note : The first digit of a number can not be zero. eg. 29 is a two digit number but 029 is not a 3 digit no.

➢ FIND THE DIGITS

Ex.1 Find the value of x.

$$\begin{array}{r} 3\ \ 1\ \ x \\ +\ 1\ \ x\ \ 3 \\ \hline 5\ \ 0\ \ 1 \end{array}$$

Sol. In ones column addition of x, 3 gives 1

$\therefore$ x may be 8

If x = 8 then we get a number whose ones digit is 1 & remaining 1 makes 2 + x in II column

$\therefore$ 2 + 8 = 10

So 0 is tens digit of result and remaining 1 makes 5 of sum of III column.

$\therefore$ x = 8

Ex.2 Find the value of x, y

$$\begin{array}{r} x \\ +\ x \\ +\ x \\ \hline y\ x \end{array}$$

Sol. If x = 5 then 5 + 5 + 5 = 15

$\therefore$ y = 1, x = 5.

Ex.3 Find the value of x, y

$$\begin{array}{rrrrrr} & & 2 & 5 & x & 4 \\ + & & y & 5 & 2 & 8 \\ \hline & 1 & 2 & 1 & 0 & 2 \end{array}$$

Sol. x = 7, y = 9

Ex.4 Find the value of x

$$\begin{array}{rrrrr} & & 2 & x & 7 \\ & & 7 & 2 & x \\ + & & x & 7 & 2 \\ \hline & 1 & x & x & 2 \end{array}$$

Sol. x = 3

➤ PYTHAGOREAN TRIPLETS

If the square of a number is equal to sum of square other two numbers then these three numbers are called Pythagorean triplets.

eg. 3, 4, 5 here $5^2 = 3^2 + 4^2$

Other Pythagorean triplets are (5, 12, 13), (7, 24, 25), (6, 8, 10), (8, 15, 17) etc.

For any natural number m > 1,

we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$.

So, 2m, $m^2 - 1$ and $m^2 + 1$ forms a Pythagorean triplet.

Ex.5 Write a Pythagorean triplet whose smallest member is 8.

Sol. We can get Pythagorean triplet by using general from 2m, $m^2 - 1$, $m^2 + 1$.

Let us first take $m^2 - 1 = 8$

So, $m^2 = 8 + 1 = 9$

which gives m = 3

Therefore, 2m = 6

and $m^2 + 1 = 10$

The triplet is thus 6, 8, 10. But 8 is not the smallest member of this.

So, let us try 2m = 8

then m = 4

We get $m^2 - 1 = 16 - 1 = 15$

and $m^2 + 1 = 16 + 1 = 17$

The triplet is 8, 15, 17 with 8 as the smallest member.

Ex.6 Find a Pythagorean triplet in which one member is 12.

Sol. If we take $m^2 - 1 = 12$

Then, $m^2 = 12 + 1 = 13$

Then the value of m will not be an integer.

So, we try to take $m^2 + 1 = 12$.

Again $m^2 = 11$ will not give an integer value for m.

So, let us try 2m = 12

then m = 6

Thus, $m^2 - 1 = 36 - 1 = 35$

and $m^2 + 1 = 36 + 1 = 37$

Therefore, the required triplet is 12, 35, 37.

Note : All Pythagorean triplets may not be obtained using this form. For example another triplet 5, 12, 13 also has 12 as a member.

➢ DIVISIBILITY TEST

No.	Divisibility Test
2	Unit digit should be 0 or even.
3	The sum of digits of no. should be divisible by 3.
4	The no. formed by last 2 digits of given no. should be divisible by 4.
5	Unit digit should be 0 or 5.
6	No. should be divisible by 2 & 3 both.
7	No. without ones – 2(ones) = no. should divisible by 7.
8	The number formed by last 3 digits of given no. should be divisible by 8.
9	Sum of digits of given no. should be divisible by 9.
11	The difference between sums of the digits at even & at odd places should be zero or multiple of 11.
13	No. without ones +4 (ones digit) = No. should be divisible by 13.
25	Last 2 digit of the number should be 00, 25, 50 or 75.

Ex.7 Check 119 and 329 is divisible by 7 or not.

Sol. (i) $11 - 2(9) = -7$, it is divisible by 7

$\therefore$ 119 is divisible by 7

(ii) $32 - 2(9) = 32 - 18 = 14$ is divisible by 7

$\therefore$ 329 is divisible by 7

Ex.8 Check 611 is divisible by 13 or not.

Sol. $61 + 4(1) = 61 + 4 = 65$

here 65 is divisible by 13

$\therefore$ 611 is divisible by 13

EXERCISE

Q.1 Find the other two numbers for each of the numbers given below, making the three numbers Pythagorean triplets.

(a) 6 (b) 15 (c) 50 (d) 3

Q.2 Without adding, find the value of the following -

(a) 1 + 3 + 5

(b) 1 + 3 + 5 + 7 + 9 + 11

(c) 1 + 3 + 5 + 7 + 9

(d) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17

Q.3 Find the cube roots of the following numbers by successive subtraction of numbers :
1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397, ...

(a) 125 (b) 343 (c) 1728 (d) 512

(e) 1331

Q.4 Using the method of successive subtraction, examine if the following numbers are perfect cubes. If not, find the smallest number which must be subtracted from the numbers so as to make them perfect cubes. Also, find their cube roots.

(a) 70 (b) 221 (c) 735 (d) 1011

(e) 349

Q.5 Solve and find values of a, b, c

(a) 4a + 3(6 – 2) + 25 ÷ 5 = 21

(b) (15 ÷ 5) + 3 × 4 – b = 17

(c) a(18 + 3) + 4 × 5 ÷ 2 – 7 = 45

(d) 2 × 3 + 14 ÷ 7 + 6 – 7c = 35

(e) $48 \div 12 \times \left(\frac{9}{8} \text{of} \frac{4}{3} \div \frac{3}{4} \text{of} \frac{2}{3} + a\right) = 6$

(f) 10 – [9 – {8 – (7 – 6)}] – c = 3

(Ques. Q.6 to Q.9) Find a, b, c in the following.

Q.6 (a) 7a + 43b + c = 518, where a, b, c are in the units place and c < a < b.

(b) a36 + b8 + c = 317, where a is in the hundred digit, b is the tens digit and c is the ones digit.

Q.7 a38 + b3 + 5c = 745

Q.8 a96 – 43c + 402 – b2 = 814

Q.9 a62 – 473 + 2b6 – 105 + 43c = 1106

Q.10 Fill in the blanks.

(a) The square of any natural number n can be written as the sum of _____ odd numbers.

(b) When divided by 3, a perfect square leaves a remainder of ______ or _____.

Q.11 Investigate the patterns.

$1^3 + 2^3$

$1^3 + 2^3 + 3^3$

Q.12 Create pattern.

Investigate what is

1 × 2 × 3 × 4 + 1

2 × 3 × 4 × 5 + 1

3 × 4 × 5 × 6 + 1

Using this find value of a, b, c, d if

a × b × c × d + 1 = 1 6 8 1

Q.13 Find the values of unknowns.

(a)
```
   2 5 x 4
 + y 5 2 8
 ---------
 1 2 1 0 2
```

(b)
```
     4 p q
 +   7 6 8
 ---------
   1 r 2 0
```

(c)
```
     b 4 a
   – 6 8 5
   -------
     2 c 8
```

(d)
```
   2 a 4 2
 ×     2 a
 ---------
   1 b 8 b 2
   5 2 8 4 0
 -----------
   a 8 a 9 2
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ANSWER KEY

EXERCISE

1. (a) 8, 10 (b) 8, 17 (c) 40, 30 (d) 4, 5

2. (a) 9 (b) 36 (c) 25 (d) 81

3. (a) 5 (b) 7 (c) 12 (d) 8 (e) 11

4. (a) 6, 4 (b) 5, 6 (c) 6, 9 (d) 11, 10 (e) 6, 7

5. (a) 1 (b) –2 (c) 2 (d) – 3 (e) $\frac{-7}{3}$ (f) 5

6. (a) a = 5, b = 9, c = 4 or a = 6, b = 8, c = 4 (b) a = 2, b = 7, c = 3

7. a = 6, b = 5, c = 4

8. a = 8, b = 6, c = 2

9. a = 9, b = 8, c = 6

10. (a) n (b) 0, 1

11. $1^3 + 2^3 = 9 = 3^3$; $1^3 + 2^3 + 3^3 = 36 = 6^2$

12. a = 5, b = 6, c = 7, d = 8

13. (a) x = 7, y = 9 (b) q = 2, p = 5, r = 2 (c) a = 3, c = 5, b = 8 (d) a= 6, b = 5