ICSE Paper 2008

MATHEMATICS

SECTION A [40 MARKS]

(Answer all questions from this Section.)

Question 1.

(a) *The simple interest on a sum of money for 2 years at 4% per annum is ₹ 340. Find :*

    (i) *the sum of money and*

    (ii) *the compound interest on this sum for one year payable half yearly at the same rate.* [3]

(b) *If* $\dfrac{8a-5b}{8c-5d} = \dfrac{8a+5b}{8c+5d}$*, prove that* $\dfrac{a}{b} = \dfrac{c}{d}$ [3]

(c) *If* $(x-2)$ *is a factor of* $2x^3 - x^2 - px - 2$

    (i) *find the value of p.*

    (ii) *with the value of p, factorize the above expression completely.* [4]

Solution.

(a) (i) *Given :* S.I. = ₹ 340, T = 2Years, R = 4%, P = ?

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

or

$$P = \frac{\text{S.I.} \times 100}{R \times T} = \frac{340 \times 100}{4 \times 2}$$

$$= ₹ 4250 \qquad \textbf{Ans.}$$

(ii) *Given :* T = 1 Years = $1 \times 2 = 2$ times, R = 4%, $\Rightarrow$ R = 2% (for half yearly.)

$$\text{C.I.} = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$$

$$= 4250\left[\left(1 + \frac{2}{100}\right)^2 - 1\right]$$

$$= 4250\left[\frac{101}{2500}\right] = ₹\ 171{\cdot}70 \qquad \textbf{Ans.}$$

(b) *Given :*

$$\frac{8a + 5b}{8c + 5d} = \frac{8a - 5b}{8c - 5d}$$

$$\frac{8a + 5d}{8a - 5b} = \frac{8c + 5d}{8c - 5d} \qquad \text{(Apply alternendo)}$$

$$\frac{8a + 5b + 8a - 5b}{8a + 5b - 8a + 5b} = \frac{8c + 5d + 8c - 5d}{8c + 5d - 8c + 5d}$$

$$\text{(Apply componendo and dividendo rule)}$$

$$\frac{169}{10b} = \frac{16\,c}{10\,d}$$

$$\Rightarrow \qquad \frac{a}{b} = \frac{c}{d} \qquad \textbf{Proved}$$

(c) $(x-2)$ is a factor of $2x^3 - x^2 - px - 2$

(i) $\Rightarrow x = 2$ will satisfy this equation.

$\Rightarrow 2 \cdot 2^3 - 2^2 - 2p - 2 = 0$

$\Rightarrow 16 - 4 - 2p - 2 = 0$

$\Rightarrow 10 - 2p = 0$

$\Rightarrow 2p = 10$

$\Rightarrow p = 5$ **Ans.**

(ii) On dividing $2x^3 - x^2 - 5x - 2$ by $x - 2$, we get

$\Rightarrow (x-2)(2x^2 + 3x + 1)$

$\Rightarrow (x-2)(2x^2 + 2x + x + 1)$

$\Rightarrow (x-2)(2x(x+1) + 1(x+1))$

$\Rightarrow (x-2)(x+1)(2x+1)$ **Ans.**

$$\begin{array}{r} 2x^2 + 3x + 1 \\ x-2\overline{)\,2x^3 - x^2 - 5x - 2} \\ \underline{2x^3 - 4x^2} \\ 3x^2 - 5x - 2 \\ \underline{3x^2 - 6x} \\ x - 2 \\ \underline{x - 2} \\ \times \end{array}$$
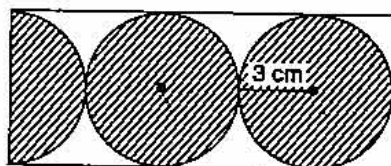
Question 2.

(a) *Solve the given inequation and graph the solution on the number line.*

$2y - 3 < y + 1 \le 4y + 7;\, y \in R.$ [3]

(b) *In the given figure, find the area of the unshaded portion within the rectangle.*

(Take $\pi = 3 \cdot 14$) [3]

(c) *A shopkeeper buys a camera at a discount of 20% from the wholesaler, the printed price of the camera being ₹ 1600 and the rate of sales tax is 6%. The shopkeeper sells it to the buyer at the printed price and charges tax at the same rate. Find :*

(i) *The price at which the camera can be bought.*

(ii) *The VAT (Value Added Tax) paid by the shopkeeper.* [4]

Solution.

(a) *Given :* $2y - 3 < y + 1 \le 4y + 7, y \in R$

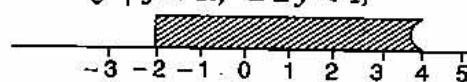| $2y - 3 < y + 1$	$y + 1 \le 4y + 7$
$\Rightarrow 2y - y < 3 + 1$	$4y - y \ge 1 - 7$
$\Rightarrow y < 4$	$3y \ge -6$
	$y \ge -2$

solution set : $\{y \mid y \in R, -2 \le y < 4\}$

(b) Length of rectangle = 15 cm

Breadth of rectangle = 6 cm.

Area of rectangle = 15×6

= 90 cm²

$$\text{Area of circle} = \pi r^2$$
$$= 3.14 \times 9$$
$$= 28.26 \text{ cm}^2.$$

$$\text{Area of shaded portion} = \text{Area of } 2\frac{1}{2} \text{ circle}$$
$$= 28.26 + 28.26 + 14.13$$
$$= 70.65 \text{ cm}^2.$$

$$\text{Area of unshaded portion in the rectangle} = \text{Area of the rectangle}$$
$$- \text{Area of shaded portion}$$
$$= 90 - 70.65$$
$$= 19.35 \text{ cm}^2 \qquad \textbf{Ans.}$$

(c) (i)
$$\text{Cost of camera for buyer} = \text{Printed Price} + \text{Sales tax on it}$$
$$= 1600 + \frac{6}{100} \times 1600$$
$$= ₹ 1696 \qquad \textbf{Ans.}$$

(ii)
$$\text{Discount on printed price} = \frac{20}{100} \times 1600 = ₹ 320.$$
$$\text{Cost Price of the camera} = 1600 - 320 = ₹ 1280.$$
$$\text{Sales tax} = \frac{6}{100} \times 1280 = ₹ 76.80$$
$$\text{Tax paid by shopkeeper} = \frac{6}{100} \times 1600 = ₹ 96.$$
$$\text{VAT paid by shopkeeper} = 96 - 76.80$$
$$= ₹ 19.20 \qquad \textbf{Ans.}$$

Question 3.

(a) *David opened a Recurring Deposit Account in a bank and deposited ₹ 300 per month for two years. If he received ₹ 7725 at the time of maturity, find the rate of interest per annum.* [3]

(b) *If* $\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3\begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$, *find the Matrix M.* [3]

(c) *Use a graph paper for this question. (Take 1 cm = 1 unit on both the axes). Plot the points A (–2, 0), B (4, 0), C (1, 4) and D (–2, 4).*

(i) *Draw the line of symmetry of △ ABC. Name it L_1.*

(ii) *Point D is reflected about the Line L_1 to get the image E. Write the coordinates of E.*

(iii) *Name the figure ABED.*

(iv) *Draw all the lines of symmetry of the figure ABED.* [4]

Solution.

(a) *Given :* Deposited per month (P) = ₹ 300, n = 2 Year = 24 months, Amount = ₹ 7725, R = ?

$$\text{Principal equivalent to 1 month} = P \times \frac{n(n+1)}{2} = \frac{300 \times 24 \times 25}{2}$$
$$= 300 \times 12 \times 25 = ₹ 90,000$$
$$A = P + SI$$
$$\Rightarrow \qquad 7725 = 24 \times 300 + \frac{90000 \times R \times 1}{12 \times 100}$$

$$\Rightarrow \qquad 525 = \frac{900 \times R}{12}$$

$$\Rightarrow \qquad R = \frac{525 \times 12}{900} = 7\%. \qquad \textbf{Ans.}$$

(b) *Given :*
$$\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix} + 2M = 3 \begin{bmatrix} 3 & 2 \\ 0 & -3 \end{bmatrix}$$
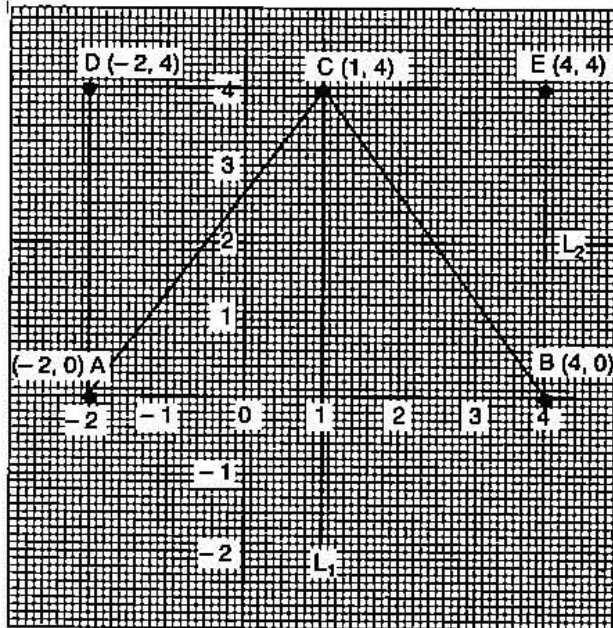
$$\Rightarrow \qquad 2M = \begin{bmatrix} 9 & 6 \\ 0 & -9 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

$$\Rightarrow \qquad = \begin{bmatrix} 8 & 2 \\ 2 & -12 \end{bmatrix}$$

$$M = \begin{bmatrix} 4 & 1 \\ 1 & -6 \end{bmatrix} \qquad \textbf{Ans.}$$

(c) **(i)**

(ii) Co-ordinate of E (4, 4)

(iii) ABED is rectangle.

(iv) See figure (L_1 and L_2 are the line of symmetry of ractangle ABED)

Question 4.

(a) *Without using tables, evaluate :* $\dfrac{\sin 25°}{\sec 65°} + \dfrac{\cos 25°}{\csc 65°}$ **[3]**

(b) *In the alongside figure, AB is parallel to DC, $\angle BCE = 80°$ and $\angle BAC = 25°$. Find :*
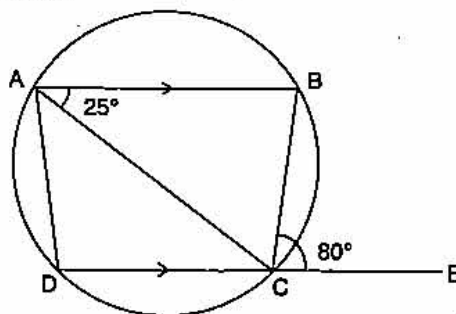
(i) $\angle CAD$,

(ii) $\angle CBD$,

(iii) $\angle ADC$. **[3]**

(c) *Mr. Dhoni has an account in the Union Bank of India. The following entries are from his pass book :*

Date	Particulars	Withdrawals (in ₹)	Deposits (in ₹)	Balance (in ₹)
Jan 3, 07	B/F	—	—	2642.00
Jan 16	To Self	640.00	—	2002.00
March 5	By Cash	—	850.00	2852.00
April 10	To Self	1130.00	—	1722.00
April 25	By Cheque	—	650.00	2372.00
June 15	By Cash	577.00	—	1795.00

Calculate the interest from January 2007 to June 2007 at the rate of 4% per annum. [4]

Solution.

(a) *Given :*
$$\frac{\sin 25°}{\sec 65°} + \frac{\cos 25°}{\operatorname{cosec} 65°}$$

$$= \sin 25° \cos 65° + \cos 25° \sin 65°$$

$$= \sin 25° \cos (90° - 25°) + \cos 25° \sin (90° - 25°)$$

$$= \sin 25° \sin 25° + \cos 25° \cos 25°$$

$$= \sin^2 25° + \cos^2 25° = 1 \qquad (\because \sin^2 \theta + \cos^2 \theta = 1) \textbf{ Ans.}$$

(b) (i) $\qquad \angle BCE = \angle BAD = 80°$

$\qquad\qquad\qquad$ ($\because$ ext. of cyclic quad. is equal to opp. int. angle.)

$\Rightarrow \qquad \angle CAD = \angle BAD - \angle BAC = 80° - 25° = 55° \qquad$ **Ans.**

(ii) $\qquad \angle CBD = \angle CAD \qquad$ ($\because$ Angle of the same segment are equal.)

$$= 55° \qquad\qquad \textbf{Ans.}$$

(iii) $\qquad\qquad AB \parallel DC$ (given)

$\because \qquad \angle BAD + \angle ADC = 180°$

$\Rightarrow \quad (25 + 55) + \angle ADC = 180°$

$$= 180 - 80 = 100° \qquad (\because ABCD \text{ is cyclic quad.})$$
$$\textbf{Ans.}$$

(c)

Minimum Balance for January = ₹	2,002
Minimum Balance for February = ₹	2,002
Minimum Balance for March = ₹	2,852
Minimum Balance for April = ₹	1,722
Minimum Balance for May = ₹	2,372
Minimum Balance for June = ₹	1,795
Total = ₹	12,745

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$= \frac{12745 \times 4 \times 1}{100 \times 12}$$

$$= ₹ 42.48 \qquad\qquad \textbf{Ans.}$$

SECTION B [40 Marks]

Answer any Four Questions in this Section.

Question 5.

(a) *A function in x is defined as :* *

$$f(x) = \frac{x+2}{2x-1}; x \in R \text{ and } x \neq \frac{1}{2},$$

Find : (i) $f(-3)$,

(ii) $f(x-1)$,

(iii) $x \text{ if } f(x) = 1$. [3]

(b) *Prove the identity :* $\dfrac{sin\,A}{1+cos\,A} = cosec\,A - cot\,A.$ [3]

(c) *If A = (-4, 3) and B = (8, -6)*

(i) *Find the length of AB.*

(ii) *In what ratio is the line joining AB, divided by the x-axis ?* [4]

Solution.

(b)
$$L.H.S. = \frac{sin\,A}{1+cos\,A}$$

$$= \frac{sin\,A}{1+cos\,A} \times \frac{1-cos\,A}{1-cos\,A}$$

$$= \frac{sin\,A\,(1-cos\,A)}{1-cos^2\,A}$$

$$= \frac{sin\,A\,(1-cos\,A)}{sin^2\,A}$$

$$\Rightarrow \qquad = \frac{1}{sin\,A} - \frac{cos\,A}{sin\,A}$$

$$= cosec\,A - cot\,A = R.H.S. \qquad \textbf{Proved}$$

(c) (i) *Given :* $A = (-4, 3), B = (8, -6)$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8+4)^2 + (-6-3)^2}$$

$$= \sqrt{144 + 81}$$

$$= \sqrt{225} = 15. \qquad \textbf{Ans.}$$

(ii) Let any point on x-axis will be $P\,(x, 0)$.

$$x = \frac{mx_1 + nx_2}{m+n},$$

$$y = \frac{my_1 + ny_2}{m+n}$$

$$0 = \frac{m \cdot 3 + n \cdot (-6)}{m+n}$$

$$\Rightarrow \qquad 3m = 6n$$

$$\Rightarrow \qquad \frac{m}{n} = \frac{6}{3} = \frac{2}{1}$$

The ratio will be 2 : 1. **Ans.**

Question 6.

(a) *Solve the following quadratic equation for x and give your answer correct to two decimal places :*
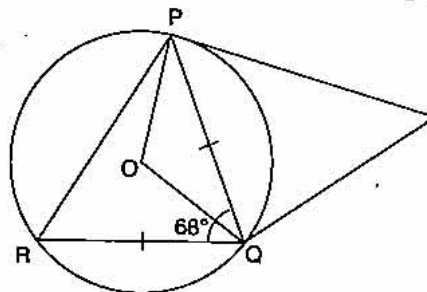
$$5x\,(x+2) = 3 \qquad\qquad \text{[3]}$$

(b) *In the figure given alongside PQ = QR, ∠*

RQP = 68°, PC and CQ are tangents to the

circle with centre O. Calculate the values of :

   (i)   ∠QOP,

   (ii) ∠QCP.   **[3]**

(c) *A company with 4000 shares of nominal value of ₹ 110 each declares an annual dividend of 15%. Calculate :*

   (i)   *The total amount of dividend paid by the company.*

   (ii) *The annual income of Shah Rukh who holds 88 shares in the company.*

   (iii) *If he received only 10% on his investment, find the price Shah Rukh paid for each share.*   **[4]**

Solution.

(a) *Given :*
$$5x\,(x+2) = 3$$
$$5x^2 + 10x - 3 = 0$$

Camparing with $ax^2 + bx + c = 0$, we get $a = 5, b = 10$ and $c = -3$.

We know that
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{(10)^2 - 4 \times 5 \times (-3)}}{2 \times 5}$$

$$= \frac{-10 \pm \sqrt{100 + 60}}{10}$$

$$= \frac{-10 \pm 4\sqrt{10}}{10} = \frac{-5 \pm 2\sqrt{10}}{5}$$

$$= \frac{-5 \pm 2 \times 3 \cdot 16}{5} = \frac{-5 \pm 6 \cdot 32}{5}$$

Hence $x = 0 \cdot 26$ or $-2 \cdot 26.$ **Ans.**

(b) *Given :* PQ = QR, ∠ RQP = 68°

   (i)   In Δ PQR,   PQ = RQ

∴   ∠ PRQ = ∠ QPR

⇒   ∠ PRQ + ∠ QPR + 68° = 180°   [sum of the angle of a Δ is 180°]

⇒   2 ∠ PRQ = 180 − 68 = 112°

⇒   ∠ PRQ = 56°

∴   ∠ QOP = 2 × ∠ PRQ

                                [angle at centre of the circle is twice the angle of at the remaining circumference]

                        = 2 × 56 = 112°   **Ans.**

(ii)
$$\angle PQC = \angle PRQ$$
$$\angle QPC = \angle PRQ$$
$$\Rightarrow \quad \angle PQC = \angle QPC = 56°$$
$$\Rightarrow \quad \angle PQC + \angle QPC + \angle PCQ = 180°$$
$$\Rightarrow \quad 56° + 56° + \angle PCQ = 180°$$
$$\Rightarrow \quad \angle QCP = 180° - 56° - 56°$$
$$= 68° \qquad \textbf{Ans.}$$

(c) *Given* : No. of shares = 4000, NV = ₹ 110, Dividend = 15%

(i)
$$\text{Dividend on 1 share} = \frac{15}{100} \times 110$$
$$= ₹ 16·50$$
$$\text{Total dividend} = 4000 \times 16·50$$
$$= ₹ 66000 \qquad \textbf{Ans.}$$

(ii)
$$\text{Income on 88 shares} = 88 \times 16·50$$
$$= ₹ 1,452$$
$$\text{Annual income of Shah Rukh} = ₹ 1,452. \qquad \textbf{Ans.}$$

(iii)
$$\text{Let his investment} = x$$
$$\frac{10}{100} \times x = ₹ 1452$$
$$x = ₹ 14520$$
$$\therefore \quad \text{Price for each share} = \frac{14520}{88} = ₹ 165. \qquad \textbf{Ans.}$$

Question 7.

(a) *The income of Mr. Bachhan was as follows :* **

- *Basic Salary* : *₹ 20,000 per month*
- *Dearness Allowance* : *₹ 12,000 per month*
- *Interest from Bank* : *₹ 16,000 for the whole year.*

Savings :
- *Contribution towards Provident Fund* : *15% of Basic salary*
- *National Savings Certificate* : *₹ 40,000*
- *Contribution towards LIC premium* : *₹ 30,000 per year*

Donations
- *To National Defence Fund* : *₹ 12,000 (eligible for 100% exemption)*

If a sum of ₹ 3,000 was deducted every month towards Income tax from his salary for the first 11 months of the year, calculate the tax Mr. Bachhan has to pay in the last month of the financial year : **[6]**

Tax slab :

Upto ₹ 1,00,000 : *No tax.*

From ₹ 1,00,001 to ₹ 1,50,000 : *10% of the income exceeding ₹ 1,00,000*

From ₹ 1,50,001 to ₹ 2,50,000 : *₹ 5,000+ 20% of the income exceeding ₹ 1,50,000*

Above ₹ 2,50,000 : *₹ 25,000 + 30% of the income exceeding ₹ 2,50,000*

Deductions against savings : *Upto a maximum amount of ₹ 1,00,000*

Education Cess : *2% of the tax payable.*

(b) *A vertical pole and a vertical tower are on the same level ground. From the top of the pole the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. Find the height of the tower if the height of the pole is 20 m.* **[4]**

Solution.

(b) Let h be the height of tower and x be the distance between tower and pole.

In rt. $\triangle$ BCE, $\qquad \tan 30° = \dfrac{20}{x}$

$\Rightarrow \qquad\qquad \dfrac{1}{\sqrt{3}} = \dfrac{20}{x}$

$\Rightarrow \qquad\qquad x = 20\sqrt{3}$ m

In rt. $\triangle$ ECA, $\qquad \tan 60° = \dfrac{h-20}{x}$

$\Rightarrow \qquad\qquad \dfrac{\sqrt{3}}{1} = \dfrac{h-20}{x}$

$\Rightarrow \qquad\qquad h-20 = x\sqrt{3}$

$\Rightarrow \qquad\qquad h-20 = 20\sqrt{3} \times \sqrt{3} = 60$ m

$\Rightarrow \qquad\qquad h = 80$ m. **Ans.**
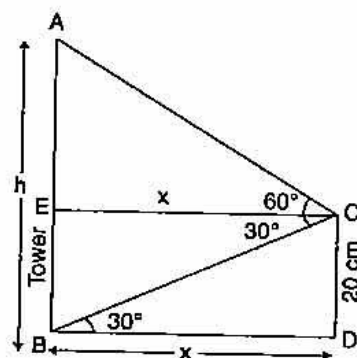
Question 8.

(a) *Find the H.C.F. of the given polynomials :* ******

$$x^2 - \dfrac{1}{a^2} \text{ and } x^2 + \dfrac{2x}{a} + \dfrac{1}{a^2}$$ **[3]**

(b) *Using a ruler and a pair of compasses only, construct :*

(i) *A triangle ABC, given AB = 4 cm, BC = 6 cm and $\angle ABC = 90°$.*

(ii) *A circle which passes through the points A, B and C and mark its centre as O.* **[3]**

**(c)** *Points A and B have coordinates (7, –3) and (1, 9) respectively. Find :*

(i) *The slope of AB.*

(ii) *The equation of the perpendicular bisector of the line segment AB.*
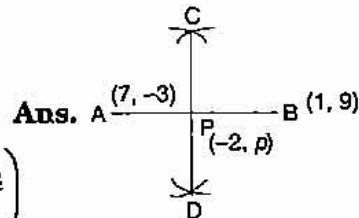
(iii) *The value of 'p' if (–2, p) lies on it.* **[4]**

Solution.

(b) Steps of Construction :

(1) Draw side BAC, 6 cm.

(2) Draw a $\angle$ B = 90°, and cut AB = 4 cm.

(3) Meet AC.

(4) Draw Bisector of BC and AB. Which meet at point 'O'.

(5) Now draw a circle as centre 'O'.

(6) This circle passes through the point A, B, and 'C'.
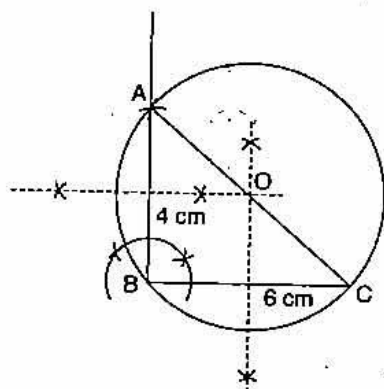
(c) (i) Slope of AB (m_1) = $\dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \dfrac{9+3}{1-7} = \dfrac{12}{-6} = -2 \qquad \textbf{Ans.}$$

(ii) The mid point of line AB = $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$$= \left(\dfrac{7+1}{2}, \dfrac{-3+9}{2}\right) = (4, 3)$$

Slope of perpendicular bisector of AB

$$m_2 = \dfrac{-1}{m_1} = \dfrac{-1}{-2} = \dfrac{1}{2}$$

Equation of perpendicular bisector

$$y - y_1 = m\,(x - x_1)$$

$\Rightarrow \qquad\qquad y - 3 = \dfrac{1}{2}(x - 4)$

$\Rightarrow \qquad\qquad 2y - 6 = x - 4$

$\Rightarrow \qquad x - 2y + 2 = 0 \qquad\qquad \textbf{Ans.}$

(iii) Given point $(-2, p)$ lies on the equation $x - 2y + 2 = 0$

$$-2 - 2p + 2 = 0$$

$\Rightarrow \qquad\qquad -2p = 0$

$\Rightarrow \qquad\qquad p = 0 \qquad\qquad \textbf{Ans.}$

Question 9.

(a) Given $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ and $BA = C^2$.

Find the values of p and q. [3]

(b) In $\triangle ABC$, $AP : PB = 2 : 3$. PO is parallel to BC and is extended to Q so that CQ is parallel to BA. Find :

(i) Area $\triangle APO$: area $\triangle ABC$.
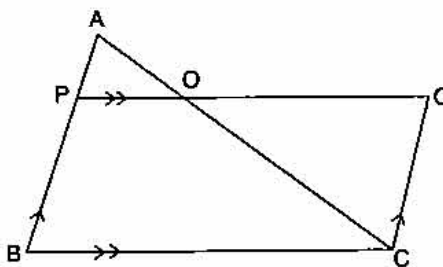
(ii) Area $\triangle APO$: area $\triangle CQO$. [3]

(c) The volume of a conical tent is 1232 m^3 and the area of the bare floor is 154 m^2. Calculate the :

(i) Radius of the floor.

(ii) Height of the tent.

(iii) Length of the canvas required to cover this conical tent if its width is 2 m.

[4]

Solution.

(a) Given : $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

$$\therefore \quad BA = C^2$$

$$\Rightarrow \qquad \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 \times p + (-q) \times 0 & 0 \times 0 + (-q) \times 2 \\ 1 \times p + 0 \times 0 & 1 \times 0 + 0 \times 2 \end{bmatrix} = \begin{bmatrix} 2 \times 2 + (-2) \times 2 & 2 \times (-2) + (-2) \times 2 \\ 2 \times 2 + \ 2 \times 2 & 2 \times (-2) + \ 2 \times 2 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 0 & -2q \\ p & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$$\Rightarrow \qquad\qquad\qquad p = 8, q = 4 \qquad\qquad \textbf{Ans.}$$

(b) (i)
$$\frac{AP}{PB} = \frac{2}{3} = \frac{AO}{OC}$$

$$\frac{AP}{AP + PB} = \frac{2}{2 + 3}$$

$$\Rightarrow \qquad \frac{AP}{AB} = \frac{2}{5} \qquad\qquad \text{...(1)}$$

PO is parallel to BC and CQ is parallel to BA.

So, PBCQ is a parallelogram.

$$\Rightarrow \qquad\qquad PB = CQ$$

$$\Rightarrow \qquad\qquad \frac{AP}{PB} = \frac{2}{3} = \frac{AP}{CQ}$$

In $\triangle$ APO and $\triangle$ ABC,

$$\angle APO = \angle ABC \qquad\qquad (\because PO \parallel BC)$$

$$\angle A = \angle A$$

$$\Rightarrow \qquad \triangle APO \sim \triangle ABC$$

$$\frac{AP}{AB} = \frac{AO}{AC} = \frac{2}{5}$$

$$\Rightarrow \qquad \frac{\text{area of } \triangle APO}{\text{area of } \triangle ABC} = \frac{AP^2}{AB^2}$$

[Ratio between the areas of two similar $\triangle$'s is equal to the ratio between the squares of their corresponding sides]

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25} \qquad\qquad \textbf{Ans.}$$

(ii) In $\triangle$ APO and $\triangle$ CQO, $\quad \angle AOP = \angle COQ \qquad$ (vertically opp.)

$$\angle OAP = \angle OCQ$$

$$\Rightarrow \qquad\qquad \triangle AOP \sim \triangle COQ \qquad \text{(By A.A axiom)}$$

So $\qquad \dfrac{\text{area of } \triangle APO}{\text{area of } \triangle CQO} = \dfrac{AP^2}{CQ^2} = \left(\dfrac{2}{3}\right)^2 = \dfrac{4}{9} \qquad$ **Ans.**

(c) *Given* : Volume (V) = 1232 m, Area of base = 154 m^2

Let r be the radius and h be the height.

(i) $\qquad\qquad$ Area $= \pi r^2 = 154$

$$\frac{22}{7} r^2 = 154$$

$$r^2 = 49 \qquad\qquad \Rightarrow \qquad r = 7 \text{ m} \qquad \textbf{Ans.}$$

(ii) $\qquad$ Volume (V) $= \dfrac{1}{3}\pi r^2 h = 1232$

$$\Rightarrow \qquad \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times h = 1232$$

$$\Rightarrow \qquad h = \frac{1232 \times 3}{22 \times 7} = 24 \text{ m.} \qquad \textbf{Ans.}$$

(iii) Let l be the slant height of the conical tent, then

$$l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 7^2} = 25\text{m}$$
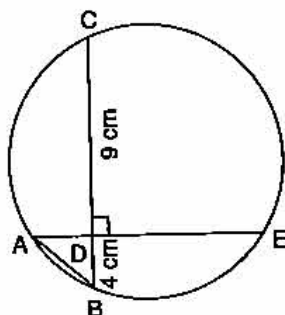
Area of canvas required = Curved surface area of the tent

$\Rightarrow$ Length $\times$ width of canvas = $\pi r l$ (for tent)

$$\Rightarrow \qquad \text{Length of canvas} \times 2 = \frac{22}{7} \times 7 \times 25$$

$$\text{Length of canvas} = \frac{550}{2} = 275 \text{ m.} \qquad \textbf{Ans.}$$

Question 10.

(a) *In the given figure, AE and BC intersect each other at point D. If $\angle CDE = 90°$, AB = 5 cm, BD = 4 cm and CD = 9 cm, find DE.* [3]

(b) *A straight line AB is 8 cm long. Locate by construction the locus of a point which is :*

(i) *Equidistant from A and B.*

(ii) *Always 4 cm from the line AB.*

(iii) *Mark two points X and Y, which are 4 cm from AB and equidistant from A and B, Name the figure AXBY.* [3]

(c) *Some students planned a picnic. The budget for the food was ₹ 480. As eight of them failed to join the party, the cost of the food for each member increased by ₹ 10. Find how many students went for the picnic.* [4]

Solution.

(a) *Given :* $\angle CDE = 90°$, AB = 5 cm, BD = 4 cm, CD = 9 cm, DE = ?

$$AD = \sqrt{AB^2 - DB^2} = \sqrt{25 - 16} = 3$$

$DA \times DE = DB \times DC$ (Product of the length of their segment is equal.)

$$\Rightarrow \qquad 3 \times DE = 4 \times 9$$

$$\Rightarrow \qquad DE = 12 \text{ cm} \qquad \textbf{Ans.}$$

(b) **Steps of Construction :**

(i) Draw AB is 8 cm, Draw PQ the perpen-dicular bisector of AB.

(ii) Draw CD and EF both parallel to AB and each at a distance of 4 cm from AB.

(iii) AXBY is a square.
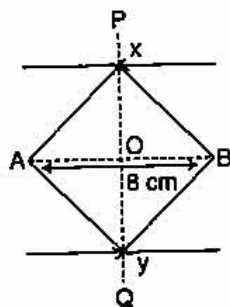
(c) Let the no. of students planned a picnic = x.

$$\text{Budget for food} = ₹480$$

$$\text{Budget for each student} = \frac{480}{x}$$

If eight student failed to join the party, the cost for each student $= \dfrac{480}{x-8}$

$$\frac{480}{x-8} - \frac{480}{x} = 10$$

$$\frac{480x - 480(x-8)}{x(x-8)} = 10$$

$\Rightarrow \qquad \dfrac{480\,x - 480\,x + 3840}{x(x-8)} = 10$

$\Rightarrow \qquad\qquad x^2 - 8x - 384 = 0$

$\Rightarrow \qquad\quad x^2 - 24x + 16x - 384 = 0$

$\qquad\qquad x(x-24) + 16(x-24) = 0$

$\Rightarrow \qquad\qquad (x-24)(x+16) = 0$

$\Rightarrow \qquad\qquad\qquad x = 24, -16$

But the number of students can not be negative.

$$\therefore \qquad\qquad x = 24$$

∴ The number of student who went for picnic = $x - 8 = 24 - 8 = 16$ **Ans.**

Question 11.

(a) *The weight of 50 apples were recorded as given below. Calculate the mean weight, to the nearest gram, by the Step Deviation Method.* **[5]**

Weight in grams	80–85	85–90	90–95	95–100	100–105	105–110	110–115
No. of apples	5	8	10	12	8	4	3

(b) *Using a graph paper, draw an ogive for the following distribution which shows the marks obtained in the General Knowledge paper by 100 students.*

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of students	5	10	20	25	15	12	9	4

Use the ogive to estimate :

(i) *The median.*

(ii) *The number of students who score marks above 65.* **[5]**

Solution.

(a)

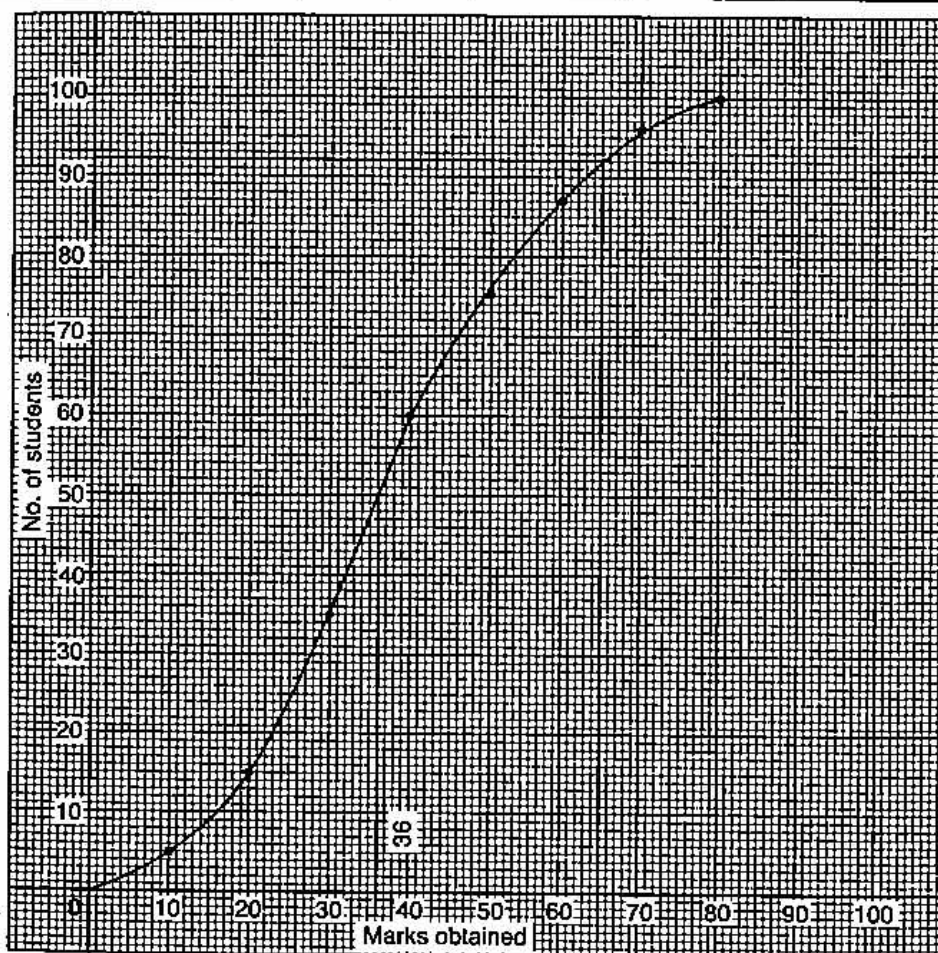| Weight	No. of apples (f_i)	Mid Value (x)	$u_i = \dfrac{x-A}{C}$	$f_i u_i$
80–85	5	82·5	–3	–15
85–90	8	87·5	–2	–16
90–95	10	92·5	–1	–10
95–100	12	97·5	0	0
100–105	8	102·5	1	8
105–110	4	107·5	2	8
110–115	3	112·5	3	9
	$\Sigma f_i = 50$			$\Sigma f_i u_i = -16$

Taking assumed mean $(A) = 97.5$, $C = 5$

$$\text{Mean} = A + C \times \frac{\Sigma f_i\, u_i}{\Sigma f_i} = 97.5 + 5 \times \frac{-16}{50}$$

$$= 97.5 - 1.6 = 95.9 = 96 \qquad \textbf{Ans.}$$

(b)

Marks	No. of students	c.f.
0–10	5	5
10–20	10	15
20–30	20	35
30–40	25	60
40–50	15	75
50–60	12	87
60–70	9	96
70–80	4	100
	100	

Here $n = 100$ which is even.

(i)
$$\text{Median} = \left(\frac{n}{2}\right)^{\text{th}} \text{term} = 50^{\text{th}} \text{term}$$

From the graph, we get

The required median $= 36$

(ii) The number of student who score above $65 = 100 - 92 = 8$. **Ans.**