Chapter 1. Irrational Numbers

Ex 1.1

Answer 1A.

$\frac{3}{5}$

$5 = 1 \times 5 = 2^0 \times 5^1$

i.e., 5 can be expressed as $2^m \times 5^n$.

$\therefore \frac{3}{5}$ has terminating decimal representation.

Answer 1B.

$\frac{5}{7}$

$7 = 1 \times 7$

i.e., 7 cannot be expressed as $2^m \times 5^n$.

$\therefore \frac{5}{7}$ does not have terminating decimal representation.

Answer 1C.

$\frac{25}{49}$

$49 = 7 \times 7$

i.e. 49 cannot be expressed as $2^m \times 5^n$.

Hence, $\frac{25}{49}$ does not have terminating decimal representation.

Answer 1D.

$\frac{37}{40}$

$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5^1$

i.e. 40 can be expressed as $2^m \times 5^n$.

Hence, $\frac{37}{40}$ has terminating decimal representation.

Answer 1E.

$\frac{57}{64}$

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 \times 5^0$

i.e. 64 can be expressed as $2^m \times 5^n$.

Hence, $\frac{57}{64}$ has terminating decimal representation.

Answer 1F.

$\frac{59}{75}$

$75 = 5 \times 5 \times 3 = 2^2 \times 3^1$

i.e. 75 cannot be expressed as $2^m \times 5^n$.

Hence, $\frac{59}{75}$ does not have terminating decimal representation.

Answer 1G.

$\frac{89}{125}$

$125 = 5 \times 5 \times 5 = 2^0 \times 5^3$

i.e. 125 can be expressed as $2^m \times 5^n$.

Hence, $\frac{89}{125}$ has terminating decimal representation.

Answer 1H.

$\frac{125}{213}$

$213 = 3 \times 71$

i.e. 213 cannot be expressed as $2^m \times 5^n$.

Hence, $\frac{125}{213}$ does not have terminating decimal representation.

Answer 1.

$\frac{147}{160}$

$160 = 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^5 \times 5^1$

i.e. 160 can be expressed as $2^m \times 5^n$.

Hence, $\frac{147}{160}$ has terminating decimal representation.

Answer 2A.

$$0.93 = \frac{93}{100}$$

Answer 2B.

$$4.56 = \frac{456}{100} = \frac{456 \div 4}{100 \div 4} = \frac{114}{25}$$

Answer 2C.

$$0.614 = \frac{614}{1000} = \frac{614 \div 2}{1000 \div 2} = \frac{307}{500}$$

Answer 2D.

$$21.025 = \frac{21025}{1000} = \frac{21025 \div 25}{1000 \div 25} = \frac{841}{40}$$

Answer 3.

(i) $\frac{3}{5}$

$\frac{3}{5} = 0.6$

(ii) $\frac{8}{11}$

$\frac{8}{11} = 0.72727272... = 0.\overline{72}$

(iii) $\frac{-2}{7}$

$\frac{-2}{7} = -0.285714285714... = -0.\overline{285714}$

(iv) $\frac{12}{21}$

$\frac{12}{21} = 0.571428571428... = 0.\overline{571428}$

(v) $\frac{13}{25}$

$\frac{13}{25} = 0.52$

(vi)

$\frac{2}{3} = 0.6666....... = 0.\dot{6}$

Answer 4A.

Let $x = 0.\dot{7}$

Then, $x = 0.7777.....$ ....(1)

Here, the number of digits recurring is only 1, so we multiply both sides of the equation (1) by 10.

$\therefore$ $10x = 10 \times 0.7777.... = 7.777.....$ ....(2)

On subtracting (1) from (2), we get

$9x = 7$

$\therefore x = \frac{7}{9}$

$\therefore 0.\dot{7} = \frac{7}{9}$

Answer 4B.

Let $x = 0.\overline{35}$

Then, $x = 0.353535.....$ ....(1)

Here, the number of digits recurring is 2, so we multiply both sides of the equation (1) by 100.

$\therefore$ $100x = 100 \times 0.353535.... = 35.3535.....$ ....(2)

On subtracting (1) from (2), we get

$99x = 35$

$\therefore x = \frac{35}{99}$

$\therefore 0.\overline{35} = \frac{35}{99}$

Answer 4C.

Let $x = 0.\overline{89}$

Then, $x = 0.898989.....$ ....(1)

Here, the number of digits recurring is 2, so we multiply both sides of the equation (1) by 100.

$\therefore$ $100x = 100 \times 0.898989.... = 89.8989.....$ ....(2)

On subtracting (1) from (2), we get

$99x = 89$

$\therefore x = \frac{89}{99}$

$\therefore 0.\overline{89} = \frac{89}{99}$

Answer 4D.

Let $x = 0.\overline{057}$

Then, $x = 0.057057.....$ ....(1)

Here, the number of digits recurring is 3, so we multiply both sides of the equation (1) by 1000.

$\therefore$ $1000x = 1000 \times 0.057057.... = 57.057.....$ ....(2)

On subtracting (1) from (2), we get

$999x = 57$

$\therefore \ x = \frac{57}{999} = \frac{19}{333}$

$\therefore \ 0.\overline{057} = \frac{19}{333}$

Answer 4E.

Let $x = 0.\overline{763}$

Then, $x = 0.763763.....$ ....(1)

Here, the number of digits recurring is 3, so we multiply both sides of the equation (1) by 1000.

$\therefore$ $1000x = 1000 \times 0.763763.... = 763.763.....$ ....(2)

On subtracting (1) from (2), we get

$999x = 763$

$\therefore \ x = \frac{763}{999}$

$\therefore \ 0.\overline{763} = \frac{763}{999}$

Answer 4F.

Let $x = 2.\overline{67}$

Then, $x = 2.676767.....$ ....(1)

Here, the number of digits recurring is 2, so we multiply both sides of the equation (1) by 100.

$\therefore$ $100x = 100 \times 2.676767.... = 267.8989.....$ ....(2)

On subtracting (1) from (2), we get

$99x = 265$

$\therefore \ x = \frac{265}{99}$

$\therefore \ 2.\overline{67} = \frac{265}{99}$

Answer 4G.

Let $x = 4.6\overline{724} = 4.6724724...$

Here, only numbers 724 is being repeated, so first we need to remove 6 which proceeds 724.

We multiply by 10 so that only the recurring digits remain after decimal.

$\therefore 10x = 46.724724....$ ....(1)

The number of digits recurring in equation (1) is 3, so we multiply both sides of the equation (1) by 1000.

$\therefore 10000x = 1000 \times 46.724724.... = 46724.724.....$ ....(2)

On subtracting (1) from (2), we get

$9990x = 46678$

$\therefore x = \frac{46678}{9990} = \frac{23339}{4995}$

$\therefore 4.6\overline{724} = \frac{763}{999} = \frac{23339}{4995}$

Answer 4H.

Let $x = 0.0\overline{17} = 0.01717..$

Here, only numbers 17 is being repeated, so first we need to remove 0 which proceeds 17.

We multiply by 10 so that only the recurring digits remain after decimal.

$\therefore 10x = 0.1717....$ ....(1)

The number of digits recurring in equation (1) is 2, so we multiply both sides of the equation (1) by 100.

$\therefore 1000x = 100 \times 0.1717.... = 17.1717.....$ ....(2)

On subtracting (1) from (2), we get

$990x = 17$

$\therefore x = \frac{17}{990}$

$\therefore 0.0\overline{17} = \frac{17}{990}$

Answer 4I.

Let $x = 17.02\overline{7} = 17.027777..$

Here, only number 7 is being repeated, so first we need to remove 02 which proceeds 7.

We multiply by 100 so that only the recurring digits remain after decimal.

$\therefore 100x = 1702.7777....$ ....(1)

The number of digits recurring in equation (1) is 1, so we multiply both sides of the equation (1) by 10.

$\therefore 1000x = 10 \times 1702.7777.... = 17027.7774.....$ ....(2)

On subtracting (1) from (2), we get

$900x = 15325$

$\therefore x = \frac{15325}{900} = \frac{613}{36}$

$\therefore 17.02\overline{7} = \frac{613}{36}$

Answer 5A.

A rational number lying between $\frac{2}{5}$ and $\frac{3}{4}$

$$= \frac{\frac{2}{5} + \frac{3}{4}}{2}$$

$$= \frac{\frac{8 + 15}{20}}{2}$$

$$= \frac{\frac{23}{20}}{2}$$

$$= \frac{23}{40}$$

Answer 5B.

A rational number lying between $\frac{3}{4}$ and $\frac{5}{7}$

$$= \frac{\frac{3}{4} + \frac{5}{7}}{2}$$

$$= \frac{\frac{21 + 20}{28}}{2}$$

$$= \frac{\frac{41}{28}}{2}$$

$$= \frac{41}{56}$$

Answer 5C.

A rational number lying between $\frac{4}{3}$ and $\frac{7}{5}$

$$= \frac{\frac{4}{3} + \frac{7}{5}}{2}$$

$$= \frac{\frac{20 + 21}{15}}{2}$$

$$= \frac{\frac{41}{15}}{2}$$

$$= \frac{41}{30}$$

Answer 5D.

A rational number lying between $\frac{5}{9}$ and $\frac{6}{7}$

$= \dfrac{\frac{5}{9}+\frac{6}{7}}{2}$

$= \dfrac{\frac{35+54}{63}}{2}$

$= \dfrac{\frac{89}{63}}{2}$

$= \dfrac{89}{126}$

Answer 6A.

A rational number lying between 3 and 4

$= \dfrac{3+4}{2}$

$= \dfrac{7}{2}$

$= 3.5$

Answer 6B.

A rational number lying between 7.6 and 7.7

$= \dfrac{7.6+7.7}{2}$

$= \dfrac{15.3}{2}$

$= 7.65$

Answer 6C.

A rational number lying between 8 and 8.04

$= \dfrac{8+8.04}{2}$

$= \dfrac{16.04}{2}$

$= 8.02$

Answer 6D.

A rational number lying between 101 and 102

$= \dfrac{101+102}{2}$

$= \dfrac{203}{2}$

$= 101.5$

Answer 7A.

A rational number lying between 0 and 1 $= \dfrac{0+1}{2} = \dfrac{1}{2}$

A rational number lying between 0 and $\dfrac{1}{2} = \dfrac{0+\frac{1}{2}}{2} = \dfrac{1}{4}$

A rational number lying between 0 and $\dfrac{1}{4} = \dfrac{0+\frac{1}{4}}{2} = \dfrac{1}{8}$

$0 < \dfrac{1}{8} < \dfrac{1}{4} < \dfrac{1}{2} < 1$

Hence, three rational numbers between 0 and 1 are

$\dfrac{1}{8}, \dfrac{1}{4}$ and $\dfrac{1}{2}$.

Answer 7B.

A rational number lying between 6 and 7 $= \dfrac{6+7}{2} = \dfrac{13}{2}$

A rational number lying between 6 and $\dfrac{13}{2} = \dfrac{6+\frac{13}{2}}{2} = \dfrac{\frac{25}{2}}{2} = \dfrac{25}{4}$

A rational number lying between $\dfrac{13}{2}$ and 7 $= \dfrac{\frac{13}{2}+7}{2} = \dfrac{\frac{27}{2}}{2} = \dfrac{27}{4}$

$6 < \dfrac{25}{4} < \dfrac{13}{2} < \dfrac{27}{4} < 7$

Hence, three rational numbers between 6 and 7 are

$\dfrac{25}{4}, \dfrac{13}{2}$ and $\dfrac{27}{4}$.

Answer 7C.

A rational number lying between -3 and $3 = \frac{-3+3}{2} = \frac{0}{2} = 0$

A rational number lying between -3 and $0 = \frac{-3+0}{2} = -\frac{3}{2}$

A rational number lying between 0 and $3 = \frac{0+3}{2} = \frac{3}{2}$

$-3 < -\frac{3}{2} < 0 < \frac{3}{2} < 3$

Hence, three rational numbers between -3 and 3 are

$-\frac{3}{2}$, 0 and $\frac{3}{2}$.

Answer 7D.

A rational number lying between -5 and $-4 = \frac{-5+(-4)}{2} = -\frac{9}{2}$

A rational number lying between -5 and $-\frac{9}{2} = \frac{-5+\left(-\frac{9}{2}\right)}{2} = \frac{-\frac{19}{2}}{2} = -\frac{19}{4}$

A rational number lying between $-\frac{9}{2}$ and $-4 = \frac{-\frac{9}{2}+(-4)}{2} = \frac{-\frac{17}{2}}{2} = -\frac{17}{4}$

$-5 < -\frac{19}{4} < -\frac{9}{2} < -\frac{17}{4} < -4$

Hence, three rational numbers between -5 and -4 are

$-\frac{19}{4}$, $-\frac{9}{2}$ and $-\frac{17}{4}$.

Answer 8A.

Since, $\frac{2}{5} < \frac{2}{3}$

Let $a = \frac{2}{5}$, $b = \frac{2}{3}$ and $n = 5$

$$\therefore\ d = \frac{b - a}{n + 1} = \frac{\frac{2}{3} - \frac{2}{5}}{5 + 1} = \frac{\frac{10 - 6}{15}}{6} = \frac{4}{90} = \frac{2}{45}$$

Hence, required rational numbers are:

$$a + d = \frac{2}{5} + \frac{2}{45} = \frac{18 + 2}{45} = \frac{20}{45} = \frac{4}{9}$$

$$a + 2d = \frac{2}{5} + 2 \times \frac{2}{45} = \frac{2}{5} + \frac{4}{45} = \frac{18 + 4}{45} = \frac{22}{45}$$

$$a + 3d = \frac{2}{5} + 3 \times \frac{2}{45} = \frac{2}{5} + \frac{2}{15} = \frac{6 + 2}{15} = \frac{8}{15}$$

$$a + 4d = \frac{2}{5} + 4 \times \frac{2}{45} = \frac{2}{5} + \frac{8}{45} = \frac{18 + 8}{45} = \frac{26}{45}$$

$$a + 5d = \frac{2}{5} + 5 \times \frac{2}{45} = \frac{2}{5} + \frac{2}{9} = \frac{18 + 10}{45} = \frac{28}{45}$$

Thus, five rational numbers between $\frac{2}{5}$ and $\frac{2}{3}$ are

$\frac{4}{9}, \frac{22}{45}, \frac{8}{15}, \frac{26}{45}$ and $\frac{28}{45}$.

Answer 8B.

Since, $-\frac{3}{4} < -\frac{2}{5}$

Let $a = -\frac{2}{5}$, $b = -\frac{3}{4}$ and $n = 5$

$$\therefore\ d = \frac{b-a}{n+1} = \frac{-\frac{3}{4}-\left(-\frac{2}{5}\right)}{5+1} = \frac{\frac{-3}{4}+\frac{2}{5}}{6} = \frac{\frac{-15+8}{20}}{6} = -\frac{7}{120}$$

Hence, required rational numbers are:

$$a+d = -\frac{2}{5}+\left(-\frac{7}{120}\right) = -\frac{2}{5}-\frac{7}{120} = \frac{-48-7}{120} = -\frac{55}{120} = -\frac{11}{24}$$

$$a+2d = \frac{2}{5}+2\times\left(-\frac{7}{120}\right) = \frac{2}{5}+\frac{4}{45} = \frac{18+4}{45} = \frac{22}{45}$$

$$a+3d = \frac{2}{5}+3\times\left(-\frac{7}{120}\right) = \frac{2}{5}+\frac{2}{15} = \frac{6+2}{15} = \frac{8}{15}$$

$$a+4d = \frac{2}{5}+4\times\left(-\frac{7}{120}\right) = \frac{2}{5}+\frac{8}{45} = \frac{18+8}{45} = \frac{26}{45}$$

$$a+5d = \frac{2}{5}+5\times\left(-\frac{7}{120}\right) = \frac{2}{5}+\frac{2}{9} = \frac{18+10}{45} = \frac{28}{45}$$

Thus, five rational numbers between $\frac{2}{5}$ and $\frac{2}{3}$ are

$\frac{4}{9}, \frac{22}{45}, \frac{8}{15}, \frac{26}{45}$ and $\frac{28}{45}$.

Answer 9A.

Given numbers: $\frac{6}{7}, \frac{9}{14}$ and $\frac{23}{28}$

The L.C.M. of 7, 14 and 28 is 28.

Thus, numbers are:

$$\frac{6}{7} = \frac{6\times4}{7\times4} = \frac{24}{28};\ \frac{9}{14} = \frac{9\times2}{14\times2} = \frac{18}{28} \text{ and } \frac{23}{28}.$$

Since $24 > 23 > 18$, we have $\frac{6}{7} > \frac{23}{28} > \frac{9}{14}$.

Hence, the greatest rational number is $\frac{6}{7}$ and

the smallest rational number is $\frac{9}{14}$.

Answer 9B.

Given numbers: $\frac{-2}{3}, \frac{-7}{9}$ and $\frac{-5}{6}$

The L.C.M. of 3, 9 and 6 is 18.

Thus, numbers are:

$\frac{-2}{3} = \frac{-2 \times 6}{3 \times 6} = \frac{-12}{18}; \frac{-7}{9} = \frac{-7 \times 2}{9 \times 2} = \frac{-14}{18}; \frac{-5}{6} = \frac{-5 \times 3}{6 \times 3} = \frac{-15}{18}$

Since $-12 > -14 > -15$, we have $\frac{-2}{3} > \frac{-7}{9} > \frac{-5}{6}$.

Hence, the greatest rational number is $\frac{-2}{3}$ and

the smallest rational number is $\frac{-5}{6}$.

Answer 10A.

Given numbers: $\frac{4}{5}, \frac{6}{7}$ and $\frac{7}{10}$

The L.C.M. of 5, 7 and 10 is 70.

Thus, numbers are:

$\frac{4}{5} = \frac{4 \times 14}{5 \times 14} = \frac{56}{70}; \frac{6}{7} = \frac{6 \times 10}{7 \times 10} = \frac{60}{70}$ and $\frac{7}{10} = \frac{7 \times 7}{10 \times 7} = \frac{49}{70}$.

Since $49 < 56 < 60$, we have $\frac{7}{10} < \frac{4}{5} < \frac{6}{7}$.

Answer 10B.

Given numbers: $\frac{-7}{12}, \frac{-3}{10}$ and $\frac{-2}{5}$

The L.C.M. of 12, 10 and 5 is 60.

Thus, numbers are:

$\frac{-7}{12} = \frac{-7 \times 5}{12 \times 5} = \frac{-35}{60}; \frac{-3}{10} = \frac{-3 \times 6}{10 \times 6} = \frac{-18}{60}; \frac{-2}{5} = \frac{-2 \times 10}{5 \times 10} = \frac{-20}{60}$

Since $-35 < -20 < -18$, we have $\frac{-7}{12} < \frac{-2}{5} < \frac{-3}{10}$.

Answer 10C.

Given numbers: $\frac{10}{9}$, $\frac{13}{12}$ and $\frac{19}{18}$

The L.C.M. of 9, 12 and 18 is 36.

Thus, numbers are:

$$\frac{10}{9} = \frac{10 \times 4}{9 \times 4} = \frac{40}{36}; \frac{13}{12} = \frac{13 \times 3}{12 \times 3} = \frac{39}{36} \text{ and } \frac{19}{18} = \frac{19 \times 2}{18 \times 2} = \frac{38}{36}.$$

Since $38 < 39 < 40$, we have $\frac{19}{18} < \frac{13}{12} < \frac{10}{9}$.

Answer 10D.

Given numbers: $\frac{7}{4}$, $\frac{-6}{5}$ and $\frac{-5}{2}$

The L.C.M. of 4, 5 and 2 is 20.

Thus, numbers are:

$$\frac{7}{4} = \frac{7 \times 5}{4 \times 5} = \frac{35}{20}; \frac{-6}{5} = \frac{-6 \times 4}{5 \times 4} = \frac{-36}{20} \text{ and } \frac{-5}{2} = \frac{-5 \times 10}{2 \times 10} = \frac{-50}{20}$$

Since $-50 < -36 < 35$, we have $\frac{-5}{2} < \frac{-6}{5} < \frac{7}{4}$.

Answer 11A.

Given numbers: $\frac{7}{13}$, $\frac{8}{15}$ and $\frac{3}{5}$

The L.C.M. of 13, 15 and 5 is 195.

Thus, numbers are:

$$\frac{7}{13} = \frac{7 \times 15}{13 \times 15} = \frac{105}{195}, \frac{8}{15} = \frac{8 \times 13}{15 \times 13} = \frac{104}{195}; \frac{3}{5} = \frac{3 \times 39}{5 \times 39} = \frac{117}{195}$$

Since $117 > 105 > 104$, we have $\frac{3}{5} > \frac{7}{13} > \frac{8}{15}$.

Answer 11B.

Given numbers: $\frac{4}{3}$, $\frac{-14}{5}$ and $\frac{17}{15}$

The L.C.M. of 3 and 5 is 15.

Thus, numbers are:

$$\frac{4}{3} = \frac{4 \times 5}{3 \times 5} = \frac{20}{15}, \frac{-14}{5} = \frac{-14 \times 3}{5 \times 3} = \frac{-42}{15}, \frac{17}{15}$$

Since $20 > 17 > -42$, we have $\frac{4}{3} > \frac{17}{5} > \frac{-14}{5}$.

Answer 11C.

Given numbers: $\frac{-7}{10}, \frac{-8}{15}$ and $\frac{-11}{30}$

The L.C.M. of 10, 15 and 30 is 30.

Thus, numbers are:

$\frac{-7}{10}=\frac{-7\times 3}{10\times 3}=\frac{-21}{30}, \frac{-8}{15}=\frac{-8\times 2}{15\times 2}=\frac{-16}{30}, \frac{-11}{30}$

Since $-11>-16>-21$, we have $\frac{-11}{30}>\frac{-8}{15}>\frac{-7}{10}$.

Answer 11D.

Given numbers: $\frac{-3}{8}, \frac{2}{5}$ and $\frac{-1}{3}$

The L.C.M. of 8, 5 and 3 is 120.

Thus, numbers are:

$\frac{-3}{8}=\frac{-3\times 15}{8\times 15}=\frac{-45}{120}, \frac{2}{5}=\frac{2\times 24}{5\times 24}=\frac{48}{120}, \frac{-1}{3}=\frac{-1\times 40}{3\times 40}=\frac{-40}{120}$

Since $48>-40>-45$, we have $\frac{2}{5}>\frac{-1}{3}>\frac{-3}{8}$.

Answer 12A.

Let $x = 2.6\bar{5} = 2.6555....$

$\Rightarrow 10x = 26.\bar{5}$ ....(i)

$\Rightarrow 100x = 265.\bar{5}$ ....(ii)

Subtracting (i) from (ii),

$90x = 239$

$\Rightarrow x = \frac{239}{90}$

Let $y = 1.\overline{25}$ ....(iii)

$\Rightarrow 100y = 125.\overline{25}$ ....(iv)

Subtracting (iii) from (iv),

$99y = 124$

$\Rightarrow y = \frac{124}{99}$

$\therefore 2.6\bar{5} + 1.\overline{25} = x + y$

$$= \frac{239}{90} + \frac{124}{99}$$

$$= \frac{239 \times 11 + 124 \times 10}{990}$$

$$= \frac{2629 + 1240}{990}$$

$$= \frac{3869}{990}$$

$$= 3.9\overline{08}$$

Answer 12B.

Let $x = 1.\overline{32}$ ....(i)

$\Rightarrow 100x = 132.\overline{32}$ ....(ii)

Subtracting (i) from (ii),

$99x = 131$

$\Rightarrow x = \dfrac{131}{99}$

Let $y = 0.9\overline{1}$

$\Rightarrow 10y = 9.\overline{1}$ ....(iii)

$\Rightarrow 100y = 91.\overline{1}$ ....(iv)

Subtracting (iii) from (iv),

$90y = 82$

$\Rightarrow y = \dfrac{82}{90} = \dfrac{41}{45}$

$\therefore 1.\overline{32} - 0.9\overline{1} = x - y$

$$= \frac{131}{99} - \frac{41}{45}$$

$$= \frac{131 \times 5 - 41 \times 11}{495}$$

$$= \frac{655 - 451}{495}$$

$$= \frac{204}{495}$$

$$= 0.4\overline{12}$$

Answer 12C.

Let $x = 2.\overline{12}$ ....(i)

$\Rightarrow 100x = 212.\overline{12}$ ....(ii)

Subtracting (i) from (ii),

$99x = 210$

$\Rightarrow x = \frac{210}{99} = \frac{70}{33}$

Let $y = 0.4\overline{5}$

$\Rightarrow 10y = 4.\overline{5}$ ....(iii)

$\Rightarrow 100y = 45.\overline{5}$ ....(iv)

Subtracting (iii) from (iv),

$90y = 41$

$\Rightarrow y = \frac{41}{90}$

$\therefore 2.\overline{12} - 0.4\overline{5} = x - y$

$= \frac{70}{33} - \frac{41}{90}$

$= \frac{70 \times 30 - 41 \times 11}{990}$

$= \frac{2100 - 451}{990}$

$= \frac{1649}{990}$

$= 1.6\overline{65}$

Answer 12D.

Let $x = 1.3\overline{5}$

$\Rightarrow 10x = 13.\overline{5}$ ....(i)

$\Rightarrow 100x = 135.\overline{5}$ ....(ii)

Subtracting (i) from (ii),

$90x = 122$

$\Rightarrow x = \frac{122}{90} = \frac{61}{45}$

Let $y = 1.\overline{5}$ ....(iii)

$\Rightarrow 10y = 15.\overline{5}$ ....(iv)

$\Rightarrow 9y = 14$

$\Rightarrow y = \frac{14}{9}$

$\therefore 1.3\overline{5} + 1.\overline{5} = x + y$

$= \frac{61}{45} + \frac{14}{9}$

$= \frac{61 \times 1 + 14 \times 5}{45}$

$= \frac{61 + 70}{45}$

$= \frac{131}{45}$

$= 2.9\overline{1}$

Ex 1.2

Answer 1A.

$$(3+\sqrt{3})^2$$
$$= (3)^2 + \left(\sqrt{3}\right)^2 + 2\times 3\times \sqrt{3}$$
$$= 9+3+6\sqrt{3}$$
$$= 12+6\sqrt{3},\ \text{which is irrational}$$

Answer 1B.

$$(5-\sqrt{5})^2$$
$$= (5)^2 + \left(\sqrt{5}\right)^2 - 2\times 5\times \sqrt{5}$$
$$= 25+5-10\sqrt{5}$$
$$= 30-10\sqrt{5},\ \text{which is irrational}$$

Answer 1C.

$$(2+\sqrt{2})(2-\sqrt{2})$$
$$= (2)^2 - \left(\sqrt{2}\right)^2$$
$$= 4-2$$
$$= 2,\ \text{which is rational}$$

Answer 1D.

$$\left(\frac{\sqrt{5}}{3\sqrt{2}}\right)^2 = \frac{5}{9\times 2} = \frac{5}{18},\ \text{which is rational}$$

Answer 2A.

$$\left(3\sqrt{2}\right)^2 = 9\times 2 = 18,\ \text{which is rational}$$

Answer 2B.

$\left(3+\sqrt{2}\right)^2$

$= (3)^2 + \left(\sqrt{2}\right)^2 + 2 \times 3 \times \sqrt{2}$

$= 9 + 2 + 6\sqrt{2}$

$= 11 + 6\sqrt{2}$, which is irrational

Answer 2C.

$\left(\frac{3\sqrt{2}}{2}\right)^2$

$= \frac{9 \times 2}{4}$

$= \frac{9}{2}$, which is rational

Answer 2D.

$\left(\sqrt{2}+\sqrt{3}\right)^2$

$= \left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2 + 2 \times \sqrt{2} \times \sqrt{3}$

$= 2 + 3 + 2\sqrt{6}$

$= 5 + 2\sqrt{6}$, which is irrational

Answer 3.

	2.23606...
2	5.0000000000...
	−4
42	100
	− 84
443	1600
	− 1329
4466	27100
	− 26796
447206	3040000
	− 2683236
	356764 ...

Clearly, $\sqrt{5} = 2.23606......$; which is an irrational number.

Hence, $\sqrt{5}$ is an irrational number.

Answer 4.

Let $\sqrt{7}$ be a rational number.

$\therefore \sqrt{7} = \frac{a}{b}$

$\Rightarrow 7 = \frac{a^2}{b^2}$

$\Rightarrow a^2 = 7b^2$

Since a^2 is divisible by 7, a is also divisible by 7. ....(I)

Let $a = 7c$

$\Rightarrow a^2 = 49c^2$

$\Rightarrow 7b^2 = 49c^2$

$\Rightarrow b^2 = 7c^2$

Since b^2 is diviisble by 7, b is also divisible by 7. ....(II)

From (I) and (II), we get a and b both divisible by 7.

i.e., a and b have a common factor 7.

This contracdicts our assumption that $\frac{a}{b}$ is rational.

i.e. a and b do not have any common factor other than unity (1).

$\Rightarrow \frac{a}{b}$ is not rational

$\Rightarrow \sqrt{7}$ is not rational, i.e. $\sqrt{7}$ is irrational.

Answer 5A.

$\left(\sqrt{3}+5\right)$ and $\left(\sqrt{5}-3\right)$ are irrational numbers whose sum is irrational.

Thus, we have

$\left(\sqrt{3}+5\right)+\left(\sqrt{5}-3\right)$

$= \sqrt{3}+5+\sqrt{5}-3$

$= \sqrt{3}+\sqrt{5}+2$, which is irrational.

Answer 5B.

$\left(\sqrt{3}+5\right)$ and $\left(4-\sqrt{3}\right)$ are two irrational numbers whose sum is rational.

Thus, we have

$\left(\sqrt{3}+5\right)+\left(4-\sqrt{3}\right)$

$= \sqrt{3}+5+4-\sqrt{3}$

$= 9$, which is a rational number.

Answer 5C.

$\left(\sqrt{3}+2\right)$ and $\left(\sqrt{2}-3\right)$ are irrational numbers whose difference is irrational.

Thus, we have

$\left(\sqrt{3}+2\right)-\left(\sqrt{2}-3\right)$

$=\sqrt{3}+2-\sqrt{2}+3$

$=\sqrt{3}-\sqrt{2}+5,$ which is irrational.

Answer 5D.

$\left(\sqrt{5}-3\right)$ and $\left(\sqrt{5}+3\right)$ are irrational numbers whose difference is rational.

Thus, we have

$\left(\sqrt{5}-3\right)-\left(\sqrt{5}+3\right)$

$=\sqrt{5}-3-\sqrt{5}-3$

$=-6,$ which is a rational number.

Answer 5E.

Consider two irrational numbers $\left(5+\sqrt{2}\right)$ and $\left(\sqrt{5}-2\right)$.

Thus, we have

$\left(5+\sqrt{2}\right)\left(\sqrt{5}-2\right)$

$=5\left(\sqrt{5}-2\right)+\sqrt{2}\left(\sqrt{5}-2\right)$

$=5\sqrt{5}-10+\sqrt{10}-2\sqrt{2},$ which is irrational

Answer 5F.

$\left(\sqrt{3}+\sqrt{2}\right)$ and $\left(\sqrt{3}-\sqrt{2}\right)$ are irrational numbers whose product is rational.

Thus, we have

$\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)=\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2=3-2=1,$ which is a rational number.

Answer 6A.

$\sqrt[4]{12} = 12^{\frac{1}{4}}$ has power $\frac{1}{4}$

$\sqrt[3]{15} = 15^{\frac{1}{3}}$ has power $\frac{1}{3}$

Now, L.C.M. of 4 and 3 = 12

$\sqrt[4]{12} = 12^{\frac{1}{4}} = 12^{\frac{3}{12}} = \left(12^3\right)^{\frac{1}{12}} = (1728)^{\frac{1}{12}}$

$\sqrt[3]{15} = 15^{\frac{1}{3}} = 15^{\frac{4}{12}} = \left(15^4\right)^{\frac{1}{12}} = (50625)^{\frac{1}{12}}$

Since $1728 < 50625$, we have $(1728)^{\frac{1}{12}} < (50625)^{\frac{1}{12}}$.

Hence, $\sqrt[4]{12} < \sqrt[3]{15}$.

Answer 6B.

$\sqrt[3]{48} = 48^{\frac{1}{3}}$ has power $\frac{1}{3}$

$\sqrt{36} = 6$

Now, L.C.M. of 3 and 1 = 3

$\sqrt[3]{48} = 48^{\frac{1}{3}}$

$\sqrt{36} = 6 = 6^{\frac{3}{3}} = \left(6^3\right)^{\frac{1}{3}} = 216^{\frac{1}{3}}$

Since $48 < 216$, we have $48^{\frac{1}{3}} < 216^{\frac{1}{3}}$.

Hence, $\sqrt[3]{48} < \sqrt{36}$.

Answer 7A.

$2\sqrt{5} = \sqrt{2^2 \times 5} = \sqrt{4 \times 5} = \sqrt{20}$

$\sqrt{3} = \sqrt{3}$

$5\sqrt{2} = \sqrt{5^2 \times 2} = \sqrt{25 \times 2} = \sqrt{50}$

Since, $3 < 20 < 50$, we have $\sqrt{3} < \sqrt{20} < \sqrt{50}$.

Hence, $\sqrt{3} < 2\sqrt{5} < 5\sqrt{2}$.

Answer 7B.

$$\text{Since } 2\sqrt[3]{3} = \sqrt[3]{2^3 \times 3} = \sqrt[3]{8 \times 3} = \sqrt[3]{24}$$
$$4\sqrt[3]{3} = \sqrt[3]{4^3 \times 3} = \sqrt[3]{64 \times 3} = \sqrt[3]{192}$$
$$3\sqrt[3]{3} = \sqrt[3]{3^3 \times 3} = \sqrt[3]{27 \times 3} = \sqrt[3]{81}$$

Since, $24 < 81 < 192$, we have $\sqrt[3]{24} < \sqrt[3]{81} < \sqrt[3]{192}$.

Hence, $2\sqrt[3]{3} < 3\sqrt[3]{3} < 4\sqrt[3]{3}$.

Answer 7C.

$$5\sqrt{7} = \sqrt{5^2 \times 7} = \sqrt{25 \times 7} = \sqrt{175}$$
$$7\sqrt{5} = \sqrt{7^2 \times 5} = \sqrt{49 \times 5} = \sqrt{245}$$
$$6\sqrt{2} = \sqrt{6^2 \times 2} = \sqrt{36 \times 2} = \sqrt{72}$$

Since, $72 < 175 < 245$, we have $\sqrt{72} < \sqrt{175} < \sqrt{245}$.

Hence, $6\sqrt{2} < 5\sqrt{7} < 7\sqrt{5}$.

Answer 7D.

$$\text{Since } 7\sqrt[3]{5} = \sqrt[3]{7^3 \times 5} = \sqrt[3]{343 \times 5} = \sqrt[3]{1715}$$
$$6\sqrt[3]{4} = \sqrt[3]{6^3 \times 4} = \sqrt[3]{216 \times 4} = \sqrt[3]{864}$$
$$5\sqrt[3]{6} = \sqrt[3]{5^3 \times 6} = \sqrt[3]{125 \times 6} = \sqrt[3]{750}$$

Since, $750 < 864 < 1715$, we have $\sqrt[3]{750} < \sqrt[3]{864} < \sqrt[3]{1715}$.

Hence, $5\sqrt[3]{6} < 6\sqrt[3]{4} < 7\sqrt[3]{5}$.

Answer 8A.

Since $\sqrt{2} = 2^{\frac{1}{2}}$ has power $\frac{1}{2}$,

$\sqrt[3]{5} = 5^{\frac{1}{3}}$ has power $\frac{1}{3}$

$\sqrt[4]{10} = 10^{\frac{1}{4}}$ has power $\frac{1}{4}$

Now, L.C.M. of 2, 3 and 4 = 12

$\therefore \sqrt{2} = 2^{\frac{1}{2}} = 2^{\frac{6}{12}} = \left(2^6\right)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$

$\sqrt[3]{5} = 5^{\frac{1}{3}} = 5^{\frac{4}{12}} = \left(5^4\right)^{\frac{1}{12}} = (625)^{\frac{1}{12}}$

$\sqrt[4]{10} = 10^{\frac{1}{4}} = 10^{\frac{3}{12}} = \left(10^3\right)^{\frac{1}{12}} = (1000)^{\frac{1}{12}}$

Since, $1000 > 625 > 64$, we have $(1000)^{\frac{1}{12}} > (625)^{\frac{1}{12}} > (64)^{\frac{1}{12}}$.

Hence, $\sqrt[4]{10} > \sqrt[3]{5} > \sqrt{2}$.

Answer 8B.

Since $5\sqrt{3} = \sqrt{5^2 \times 3} = \sqrt{25 \times 3} = \sqrt{75}$

$\sqrt{15} = \sqrt{15}$

$3\sqrt{5} = \sqrt{3^2 \times 5} = \sqrt{9 \times 5} = \sqrt{45}$

Since, $75 > 45 > 15$, we have $\sqrt{75} > \sqrt{45} > \sqrt{15}$.

Hence, $5\sqrt{3} > 3\sqrt{5} > \sqrt{15}$.

Answer 8C.

Since $\sqrt{6} = 6^{\frac{1}{2}}$ has power $\frac{1}{2}$,

$\sqrt[3]{8} = 2$

$\sqrt[4]{3} = 3^{\frac{1}{4}}$ has power $\frac{1}{4}$

Now, L.C.M. of 2, 1 and 4 = 4

$\therefore \sqrt{6} = 6^{\frac{1}{2}} = 6^{\frac{2}{4}} = \left(6^2\right)^{\frac{1}{4}} = (36)^{\frac{1}{4}}$

$\sqrt[3]{8} = 2 = 2^{\frac{4}{4}} = \left(2^4\right)^{\frac{1}{4}} = (16)^{\frac{1}{4}}$

$\sqrt[4]{3} = 3^{\frac{1}{4}} = \left(3^1\right)^{\frac{1}{4}} = (3)^{\frac{1}{12}}$

Since, $36 > 16 > 3$, we have $(36)^{\frac{1}{4}} > (16)^{\frac{1}{4}} > (3)^{\frac{1}{12}}$.

Hence, $\sqrt{6} > \sqrt[3]{8} > \sqrt[4]{3}$.

Answer 9.

Since 3 and 4 are rational numbers and $3 \times 4 = 12$ is not a perfect square.

$\therefore$ One irrational number between 3 and 4 $= \sqrt{3 \times 4} = \sqrt{12}$

And, an irrational number between 3 and $\sqrt{12} = \sqrt{3 \times \sqrt{12}} = \sqrt{3\sqrt{12}}$

$\therefore$ Required irrational numbers between 3 and 4 are: $\sqrt{12}$ and $\sqrt{3\sqrt{12}}$

Answer 10.

We know that $2\sqrt{3} = \sqrt{4 \times 3} = \sqrt{12}$ and $3\sqrt{5} = \sqrt{9 \times 5} = \sqrt{45}$.

Thus, we have $\sqrt{12} < \sqrt{13} < \sqrt{14} < \sqrt{17} < \ldots\ldots < \sqrt{43} < \sqrt{44} < \sqrt{45}$

So, any five irrational numbers between $2\sqrt{3}$ and $3\sqrt{5}$ are:

$\sqrt{13}, \sqrt{14}, \sqrt{23}, \sqrt{37}, \sqrt{41}$

Answer 11.

Since squares of $\sqrt{3}$ and $\sqrt{7}$ are 3 and 7 respectively.

Now, find two rational numbers between 3 and 7 such that each of them is a perfect square.

Let the numbers be 4 and 5.76,

where,

$\sqrt{4} = 2$

$\sqrt{5.76} = 2.4$

Hence, required rational numbers between $\sqrt{3}$ and $\sqrt{7}$ are 2 and 2.4.

Answer 12.

Since squares of $\sqrt{2}$ and $\sqrt{3}$ are 2 and 3 respectively.

Now, find four rational numbers between 2 and 3 such that each of them is a perfect square.

Let the numbers be 2.25, 2.4025, 2.56, 2.89,

where,

$\sqrt{2.25} = 1.5$

$\sqrt{2.4025} = 1.55$

$\sqrt{2.56} = 1.6$

$\sqrt{2.89} = 1.7$

Hence, required rational numbers between $\sqrt{2}$ and $\sqrt{3}$ are 1.5, 1.55, 1.6 and 1.7.

Answer 13A.

$\sqrt{150} = \sqrt{25 \times 6} = 5\sqrt{6}$, which is an irrational number.

Hence, $\sqrt{150}$ is a surd.

Answer 13B.

$\sqrt[3]{4}$ is an irrational number.

Hence, $\sqrt[3]{4}$ is a surd.

Answer 13C.

$\sqrt[3]{50}\,.\sqrt[3]{20} = \sqrt[3]{50 \times 20} = \sqrt[3]{1000} = 10$, which is a rational number.

Hence, $\sqrt[3]{50}\,.\sqrt[3]{20}$ is not a surd.

Answer 13D.

$\sqrt[3]{-27} = -3$, which is a rational number.

Hence, $\sqrt[3]{-27}$ is not a surd.

Answer 13E.

$\sqrt{2+\sqrt{3}}$ is an irrational number.

Hence, $\sqrt{2+\sqrt{3}}$ is a surd

Answer 13F.

$\sqrt[12]{8} \div \sqrt[6]{6}$

$= \dfrac{\sqrt[12]{8}}{\sqrt[6]{6}}$

Numerator and Denominator, both are irrational numbers.

Hence, $\sqrt[12]{8} \div \sqrt[6]{6}$ is a surd.

Answer 14.

Let us find $\sqrt{5}$.

Draw a number line.

Mark a point O representing zero.

Take point A on numberline such that OA = 2

Construct $AB \perp OA$ such that AB = 1 unit.

$\therefore \Delta OAB$ is a right triangle.

In ΔOAB, $(OB)^2 = (OA)^2 + (AB)^2$ (Pythagoras' Theorem)

$\therefore (OB)^2 = 2^2 + 1^2$

$\therefore (OB)^2 = 5 \Rightarrow OB = \sqrt{5}$

Now, let us find $\sqrt{6}$.

Construct $BC \perp OB$, such that BC = 1 unit.

$\therefore \Delta OBC$ is a right triangle.

In ΔOBC, $OC^2 = OB^2 + BC^2$ (Pythagoras' Theorem)

$\therefore OC^2 = \left(\sqrt{5}\right)^2 + 1^2$

$\therefore OC^2 = 6 \Rightarrow OC = \sqrt{6}$

Now, let us find $\sqrt{7}$.

Construct $CD \perp OC$, such that CD = 1 unit.

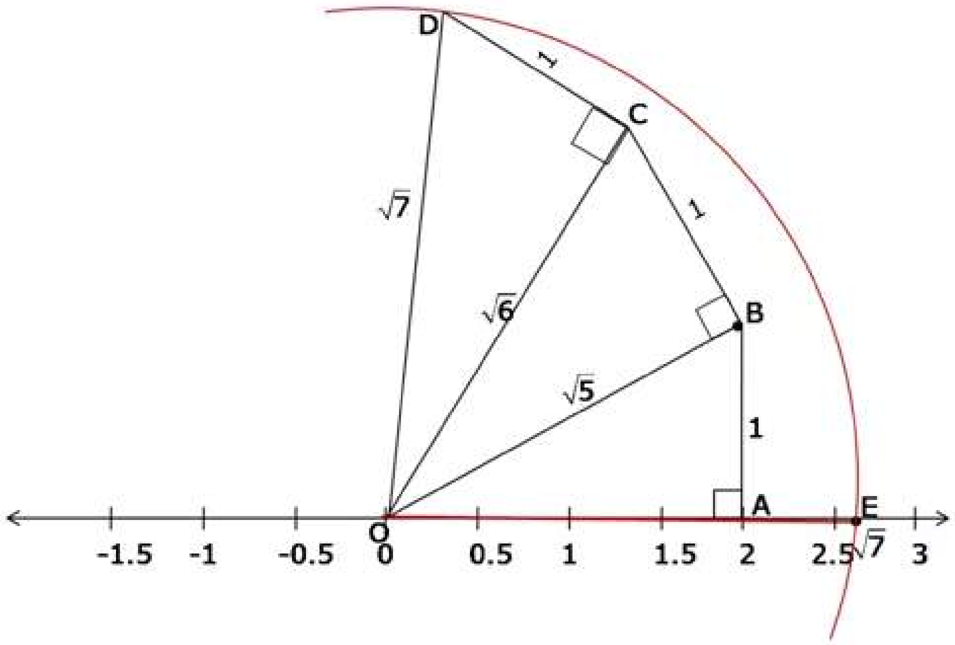
In ΔOCD, $OD^2 = OC^2 + CD^2$ (Pythagoras' Theorem)

$\therefore OD^2 = \left(\sqrt{6}\right)^2 + 1^2$

$\therefore OD^2 = 7 \Rightarrow \sqrt{7}$

Draw an arc of radius OD and centre O and let it intersect the number line at point E.

$\therefore \sqrt{7}$ is thus marked at point E on the number line.



Ex 1.3

Answer 1A.

$$\frac{3\sqrt{2}}{\sqrt{5}}$$
$$= \frac{3\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{3\sqrt{2} \times \sqrt{5}}{\left(\sqrt{5}\right)^2}$$
$$= \frac{3\sqrt{10}}{5}$$

Answer 1B.

$$\frac{1}{5+\sqrt{2}}$$
$$= \frac{1}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}}$$
$$= \frac{5-\sqrt{2}}{(5)^2-\left(\sqrt{2}\right)^2}$$
$$= \frac{5-\sqrt{2}}{25-2}$$
$$= \frac{5-\sqrt{2}}{23}$$

Answer 1C.

$$\frac{1}{\sqrt{3}+\sqrt{2}}$$
$$= \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$
$$= \frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$
$$= \frac{\sqrt{3}-\sqrt{2}}{3-2}$$
$$= \frac{\sqrt{3}-\sqrt{2}}{1}$$
$$= \sqrt{3}-\sqrt{2}$$

Answer 1D.

$$\frac{2}{3+\sqrt{7}}$$

$$= \frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}}$$

$$= \frac{2\left(3-\sqrt{7}\right)}{(3)^2-\left(\sqrt{7}\right)^2}$$

$$= \frac{2\left(3-\sqrt{7}\right)}{9-7}$$

$$= \frac{2\left(3-\sqrt{7}\right)}{2}$$

$$= 3-\sqrt{7}$$

Answer 1E.

$$\frac{5}{\sqrt{7}-\sqrt{2}}$$

$$= \frac{5}{\sqrt{7}-\sqrt{2}} \times \frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}$$

$$= \frac{5\left(\sqrt{7}+\sqrt{2}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{5\left(\sqrt{7}+\sqrt{2}\right)}{7-2}$$

$$= \frac{5\left(\sqrt{7}+\sqrt{2}\right)}{5}$$

$$= \sqrt{7}+\sqrt{2}$$

Answer 1F.

$$\frac{42}{2\sqrt{3}+3\sqrt{2}}$$

$$=\frac{42}{2\sqrt{3}+3\sqrt{2}}\times\frac{2\sqrt{3}-3\sqrt{2}}{2\sqrt{3}-3\sqrt{2}}$$

$$=\frac{42\left(2\sqrt{3}-3\sqrt{2}\right)}{\left(2\sqrt{3}\right)^2-\left(3\sqrt{2}\right)^2}$$

$$=\frac{84\sqrt{3}-126\sqrt{2}}{12-18}$$

$$=\frac{84\sqrt{3}-126\sqrt{2}}{-6}$$

$$=-14\sqrt{3}+21\sqrt{2}$$

$$=21\sqrt{2}-14\sqrt{3}$$

$$=7\left(3\sqrt{2}-2\sqrt{3}\right)$$

Answer 1G.

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$=\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$=\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{\left(\sqrt{3}\right)^2+2\times\sqrt{3}\times1+(1)^2}{3-1}$$

$$=\frac{3+2\sqrt{3}+1}{2}$$

$$=\frac{4+2\sqrt{3}}{2}$$

$$=2+\sqrt{3}$$

Answer 1H.

$$\frac{\sqrt{5}-\sqrt{7}}{\sqrt{3}}$$

$$=\frac{\sqrt{5}-\sqrt{7}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{\sqrt{5}\times\sqrt{3}-\sqrt{7}\times\sqrt{3}}{\left(\sqrt{3}\right)^2}$$

$$=\frac{\sqrt{15}-\sqrt{21}}{3}$$

Answer 1I.

$$\frac{3-\sqrt{3}}{2+\sqrt{2}}$$

$$=\frac{3-\sqrt{3}}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}}$$

$$=\frac{3\left(2-\sqrt{2}\right)-\sqrt{3}\left(2-\sqrt{2}\right)}{(2)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{6-3\sqrt{2}-2\sqrt{3}+\sqrt{6}}{4-2}$$

$$=\frac{6-3\sqrt{2}-2\sqrt{3}+\sqrt{6}}{2}$$

Answer 2.

(i) $\frac{5+\sqrt{6}}{5-\sqrt{6}}$

$$\frac{5+\sqrt{6}}{5-\sqrt{6}}$$

$$=\frac{5+\sqrt{6}}{5-\sqrt{6}}\times\frac{5+\sqrt{6}}{5+\sqrt{6}}$$

$$=\frac{(5+\sqrt{6})^2}{(5)^2-(\sqrt{6})^2}=\frac{25+6+10\sqrt{6}}{25-6}$$

$$=\frac{31+10\sqrt{6}}{19}$$

(ii) $\dfrac{4+\sqrt{8}}{4-\sqrt{8}}$

$$\frac{4+\sqrt{8}}{4-\sqrt{8}}$$

$$= \frac{4+\sqrt{8}}{4-\sqrt{8}} \times \frac{4+\sqrt{8}}{4+\sqrt{8}}$$

$$= \frac{(4+\sqrt{8})^2}{(4)^2-(\sqrt{8})^2} = \frac{16+8+8\sqrt{8}}{16-8}$$

$$= \frac{24+8\sqrt{8}}{8} = 3+\sqrt{8}$$

(iii) $\dfrac{\sqrt{15}+3}{\sqrt{15}-3}$

$$\frac{\sqrt{15}+3}{\sqrt{15}-3}$$

$$= \frac{\sqrt{15}+3}{\sqrt{15}-3} \times \frac{\sqrt{15}+3}{\sqrt{15}+3}$$

$$= \frac{(\sqrt{15}+3)^2}{(\sqrt{15})^2-(3)^2} = \frac{15+9+6\sqrt{15}}{15-9}$$

$$= \frac{24+6\sqrt{15}}{6} = 4+\sqrt{15}$$

(iv) $\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

$$\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$$

$$= \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

$$= \frac{(\sqrt{7}-\sqrt{5})^2}{(\sqrt{7})^2-\left(\sqrt{5}\right)^2} = \frac{7+5-2\sqrt{35}}{7-5} = \frac{12-2\sqrt{35}}{2}$$

$$= 6-\sqrt{35}$$

(v) $\dfrac{3\sqrt{5}+\sqrt{7}}{3\sqrt{5}-\sqrt{7}}$

$$\frac{3\sqrt{5}+\sqrt{7}}{3\sqrt{5}-\sqrt{7}}$$

$$= \frac{3\sqrt{5}+\sqrt{7}}{3\sqrt{5}-\sqrt{7}} \times \frac{3\sqrt{5}+\sqrt{7}}{3\sqrt{5}+\sqrt{7}}$$

$$= \frac{(3\sqrt{5}+\sqrt{7})^2}{(3\sqrt{5})^2-(\sqrt{7})^2} = \frac{45+7+6\sqrt{35}}{45-7}$$

$$= \frac{52+6\sqrt{35}}{38} = \frac{26+3\sqrt{35}}{19}$$

(vi) $\dfrac{2\sqrt{3}-\sqrt{6}}{2\sqrt{3}+\sqrt{6}}$

$$\frac{2\sqrt{3}-\sqrt{6}}{2\sqrt{3}+\sqrt{6}}$$

$$= \frac{2\sqrt{3}-\sqrt{6}}{2\sqrt{3}+\sqrt{6}} \times \frac{2\sqrt{3}-\sqrt{6}}{2\sqrt{3}-\sqrt{6}}$$

$$= \frac{(2\sqrt{3}-\sqrt{6})^2}{(2\sqrt{3})^2-(\sqrt{6})^2} = \frac{12+6-4\sqrt{18}}{12-6}$$

$$= \frac{18-4\sqrt{18}}{6} = \frac{9-2\sqrt{18}}{3} = \frac{9-6\sqrt{2}}{3} = 3-2\sqrt{2}$$

(vii) $\dfrac{5\sqrt{3}-\sqrt{15}}{5\sqrt{3}+\sqrt{15}}$

$$\frac{5\sqrt{3}-\sqrt{15}}{5\sqrt{3}+\sqrt{15}}$$

$$= \frac{5\sqrt{3}-\sqrt{15}}{5\sqrt{3}+\sqrt{15}} \times \frac{5\sqrt{3}-\sqrt{15}}{5\sqrt{3}-\sqrt{15}}$$

$$= \frac{(5\sqrt{3}-\sqrt{15})^2}{(5\sqrt{3})^2-(\sqrt{15})^2} = \frac{75+15-10\sqrt{45}}{75-15}$$

$$=$$

$$= \frac{90-10\sqrt{45}}{60} = \frac{9-1\sqrt{45}}{6} = \frac{9-3\sqrt{5}}{6}$$

$$= \frac{3-\sqrt{5}}{2}$$

(viii) $\dfrac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

$$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$$

$$=\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}\times\frac{3\sqrt{5}+2\sqrt{6}}{3\sqrt{5}+2\sqrt{6}}$$
$$=\frac{6\sqrt{30}+24-15-2\sqrt{30}}{(3\sqrt{5})^2-(2\sqrt{6})^2}$$
$$=\frac{6\sqrt{30}+9-2\sqrt{30}}{45-24}=\frac{4\sqrt{30}+9}{21}$$

(ix) $\dfrac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

$$\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

$$=\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}\times\frac{\sqrt{48}-\sqrt{18}}{\sqrt{48}-\sqrt{18}}$$
$$=\frac{7\sqrt{144}-7\sqrt{54}-5\sqrt{96}+5\sqrt{36}}{(\sqrt{48})^2-(\sqrt{18})^2}$$
$$=\frac{84-21\sqrt{6}-20\sqrt{6}+30}{48-18}$$
$$=\frac{114-41\sqrt{6}}{30}$$

(x) $\dfrac{\sqrt{12}+\sqrt{18}}{\sqrt{75}-\sqrt{50}}$

$$\frac{\sqrt{12}+\sqrt{18}}{\sqrt{75}-\sqrt{50}}$$

$$=\frac{\sqrt{12}+\sqrt{18}}{\sqrt{75}-\sqrt{50}}\times\frac{\sqrt{75}+\sqrt{50}}{\sqrt{75}+\sqrt{50}}$$
$$=\frac{(2\sqrt{3}+3\sqrt{2})(5\sqrt{3}+5\sqrt{2})}{(\sqrt{75})^2-(\sqrt{50})^2}$$
$$=\frac{30+10\sqrt{6}+15\sqrt{6}+30}{75-50}$$
$$=\frac{60+25\sqrt{6}}{25}=\frac{12+5\sqrt{6}}{5}$$

Answer 3.

(i) $\frac{3}{5-\sqrt{3}}+\frac{2}{5+\sqrt{3}}$

$\frac{3}{5-\sqrt{3}}+\frac{2}{5+\sqrt{3}}$

$=\frac{3(5+\sqrt{3})+2(5-\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})}$

$=\frac{15+3\sqrt{3}+10-2\sqrt{3}}{(5)^2-(\sqrt{3})^2}$

$=\frac{25+\sqrt{3}}{25-3}=\frac{25+\sqrt{3}}{22}$

(ii) $\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$

$\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}$

$=\frac{(4+\sqrt{5})^2+(4-\sqrt{5})^2}{(4-\sqrt{5})(4+\sqrt{5})}$

$=\frac{16+5+8\sqrt{5}+16+5-8\sqrt{5}}{16-5}$

$=\frac{42}{11}$

(iii) $\frac{\sqrt{5}-2}{\sqrt{5}+2}-\frac{\sqrt{5}+2}{\sqrt{5}-2}$

$\frac{\sqrt{5}-2}{\sqrt{5}+2}-\frac{\sqrt{5}+2}{\sqrt{5}-2}$

$=\frac{(\sqrt{5}-2)^2-(\sqrt{5}+2)^2}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$=\frac{5+4-4\sqrt{5}-5-4-4\sqrt{5}}{(\sqrt{5})^2-(2)^2}$

$=\frac{-8\sqrt{5}}{5-4}=-8\sqrt{5}$

(iv) $\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}-\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$$\frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}+\sqrt{3}}-\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$$
$$=\frac{(\sqrt{7}-\sqrt{3})^2-(\sqrt{7}+\sqrt{3})^2}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})}$$
$$=\frac{7+3-2\sqrt{21}-7-3-2\sqrt{21}}{(\sqrt{7})^2-(\sqrt{3})^2}$$
$$=\frac{-4\sqrt{21}}{7-3}=-\sqrt{21}$$

(v) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$
$$=\frac{(\sqrt{5}+\sqrt{3})^2+(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$$
$$=\frac{5+3+\sqrt{15}+5+3-\sqrt{15}}{5-3}$$
$$=\frac{16}{2}=8$$

Answer 4.

(i) $\dfrac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}+\dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

$$\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}+\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$$

Rationalizing the denominator of each term, we have

$$=\frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}+\frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})}-\frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$$
$$=\frac{\sqrt{12}-\sqrt{18}}{2-3}+\frac{3\sqrt{12}-3\sqrt{6}}{6-3}-\frac{4\sqrt{18}-4\sqrt{6}}{6-2}$$
$$=\frac{\sqrt{12}-\sqrt{18}}{-1}+\frac{3\sqrt{12}-3\sqrt{6}}{3}-\frac{4\sqrt{18}-4\sqrt{6}}{4}$$
$$=\sqrt{18}-\sqrt{12}+\sqrt{12}-\sqrt{6}-\sqrt{18}+\sqrt{6}$$
$$=0$$

(ii) $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}+\frac{2\sqrt{3}}{\sqrt{6}+2}$

$$\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}+\frac{2\sqrt{3}}{\sqrt{6}+2}$$

Rationalizing the denominator of each term, we have

$$=\frac{3\sqrt{2}(\sqrt{6}+\sqrt{3})}{(\sqrt{6}-\sqrt{3})(\sqrt{6}+\sqrt{3})}-\frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}+\frac{2\sqrt{3}(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)}$$
$$=\frac{3\sqrt{12}+3\sqrt{6}}{6-3}-\frac{4\sqrt{18}+4\sqrt{6}}{6-2}+\frac{2\sqrt{18}-4\sqrt{3}}{6-4}$$
$$=\frac{3\sqrt{12}+3\sqrt{6}}{3}-\frac{4\sqrt{18}+4\sqrt{6}}{4}+\frac{2\sqrt{18}-4\sqrt{3}}{2}$$
$$=\sqrt{12}+\sqrt{6}-\sqrt{18}-\sqrt{6}+\sqrt{18}-2\sqrt{3}$$
$$=\sqrt{12}-2\sqrt{3}$$
$$=2\sqrt{3}-2\sqrt{3}$$
$$=0$$

(iii) $\frac{6}{2\sqrt{3}-\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

$$\frac{6}{2\sqrt{3}-\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$$

Rationalizing the denominator of each term, we have

$$=\frac{6(2\sqrt{3}+\sqrt{6})}{(2\sqrt{3}-\sqrt{6})(2\sqrt{3}+\sqrt{6})}+\frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}-\frac{4\sqrt{3}(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}$$
$$=\frac{12\sqrt{3}+6\sqrt{6}}{12-6}+\frac{\sqrt{18}-\sqrt{12}}{3-2}-\frac{4\sqrt{18}+4\sqrt{6}}{6-2}$$
$$=\frac{12\sqrt{3}+6\sqrt{6}}{6}+\frac{\sqrt{18}-\sqrt{12}}{1}-\frac{4\sqrt{18}+4\sqrt{6}}{4}$$
$$=2\sqrt{3}+\sqrt{6}+\sqrt{18}-\sqrt{12}-\sqrt{18}-\sqrt{6}$$
$$=2\sqrt{3}-\sqrt{12}=2\sqrt{3}-2\sqrt{3}$$
$$=0$$

(iv) $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

Rationalizing the denominator of each term, we have

$$\begin{aligned}
&=\frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{(\sqrt{10}+\sqrt{3})(\sqrt{10}-\sqrt{3})}-\frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{(\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5})}-\frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{(\sqrt{15}+3\sqrt{2})(\sqrt{15}-3\sqrt{2})}\\
&=\frac{7\sqrt{30}-21}{10-3}-\frac{2\sqrt{30}-10}{6-5}-\frac{3\sqrt{30}-18}{15-18}\\
&=\frac{7\sqrt{30}-21}{7}-\frac{2\sqrt{30}-10}{1}-\frac{3\sqrt{30}-18}{-3}\\
&=\frac{7\sqrt{30}-21}{7}-\frac{2\sqrt{30}-10}{1}+\frac{3\sqrt{30}-18}{3}\\
&=\sqrt{30}-3-2\sqrt{30}+10+\sqrt{30}-6\\
&=-1
\end{aligned}$$

(v) $\frac{4\sqrt{3}}{\left(2-\sqrt{2}\right)}-\frac{30}{\left(4\sqrt{3}-3\sqrt{2}\right)}-\frac{3\sqrt{2}}{\left(3+2\sqrt{3}\right)}$

$$\frac{4\sqrt{3}}{\left(2-\sqrt{2}\right)}-\frac{30}{\left(4\sqrt{3}-3\sqrt{2}\right)}-\frac{3\sqrt{2}}{\left(3+2\sqrt{3}\right)}$$

Rationalizing the denominator of each term, we have

$$\begin{aligned}
&=\frac{4\sqrt{3}\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)}-\frac{30\left(4\sqrt{3}+3\sqrt{2}\right)}{\left(4\sqrt{3}-3\sqrt{2}\right)\left(4\sqrt{3}+3\sqrt{2}\right)}-\frac{3\sqrt{2}\left(3-2\sqrt{3}\right)}{\left(3+2\sqrt{3}\right)\left(3-2\sqrt{3}\right)}\\
&=\frac{8\sqrt{3}+4\sqrt{6}}{4-2}-\frac{120\sqrt{3}+90\sqrt{2}}{48-18}-\frac{9\sqrt{2}-6\sqrt{6}}{9-12}\\
&=\frac{8\sqrt{3}+4\sqrt{6}}{2}-\frac{120\sqrt{3}+90\sqrt{2}}{30}-\frac{9\sqrt{2}-6\sqrt{6}}{-3}\\
&=\frac{8\sqrt{3}+4\sqrt{6}}{2}-\frac{120\sqrt{3}+90\sqrt{2}}{30}+\frac{9\sqrt{2}-6\sqrt{6}}{3}\\
&=4\sqrt{3}+2\sqrt{6}-4\sqrt{3}-3\sqrt{2}+3\sqrt{2}-2\sqrt{6}\\
&=0
\end{aligned}$$

Answer 5.

$$\frac{\sqrt{2.5}-\sqrt{0.75}}{\sqrt{2.5}+\sqrt{0.75}}$$

$$= \frac{\sqrt{2.5}-\sqrt{0.75}}{\sqrt{2.5}+\sqrt{0.75}} \times \frac{\sqrt{2.5}-\sqrt{0.75}}{\sqrt{2.5}-\sqrt{0.75}}$$

$$= \frac{\left(\sqrt{2.5}-\sqrt{0.75}\right)^2}{\left(\sqrt{2.5}\right)^2-\left(\sqrt{0.75}\right)^2}$$

$$= \frac{2.5-2\times\sqrt{2.5}\times\sqrt{0.75}+0.75}{2.5-0.75}$$

$$= \frac{3.25-2\times\sqrt{0.25\times 10}\times\sqrt{0.25\times 3}}{1.75}$$

$$= \frac{3.25-2\times 0.25\sqrt{30}}{1.75}$$

$$= \frac{3.25-0.5\sqrt{30}}{1.75}$$

$$= \frac{3.25}{1.75}-\frac{0.5}{1.75}\sqrt{30}$$

$$= \frac{325}{175}-\frac{50}{175}\sqrt{30}$$

$$= \frac{13}{7}-\frac{2}{7}\sqrt{30}$$

$$= \frac{13}{7}+\left(-\frac{2}{7}\right)\sqrt{30}$$

$$= p+q\sqrt{30}$$

Hence, $p = \frac{13}{7}$ and $q = -\frac{2}{7}$.

Answer 6A.

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$= \frac{3-2\times\sqrt{3}\times 1+1}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3}$$

$$= 2+(-1)\sqrt{3}$$

$$= a+b\sqrt{3}$$

Hence, $a = 2$ and $b = -1$.

Answer 6B.

$$\frac{3+\sqrt{7}}{3-\sqrt{7}}$$

$$= \frac{3+\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$$

$$= \frac{\left(3+\sqrt{7}\right)^2}{(3)^2-\left(\sqrt{7}\right)^2}$$

$$= \frac{9+6\sqrt{7}+7}{9-7}$$

$$= \frac{16+6\sqrt{7}}{2}$$

$$= 8+3\sqrt{7}$$

$$= a+b\sqrt{7}$$

Hence, $a = 8$ and $b = 3$.

Answer 6C.

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$$

$$= \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{5\left(7-4\sqrt{3}\right)+2\sqrt{3}\left(7-4\sqrt{3}\right)}{(7)^2-\left(4\sqrt{3}\right)^2}$$

$$= \frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48}$$

$$= \frac{11-6\sqrt{3}}{1}$$

$$= 11+(-6)\sqrt{3}$$

$$= a+b\sqrt{3}$$

Hence, $a = 11$ and $b = -6$

Answer 6D.

$$\frac{1}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{5-3}$$

$$= \frac{\sqrt{5}+\sqrt{3}}{2}$$

$$= \frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{3}$$

$$= \frac{1}{2}\sqrt{5}-\left(-\frac{1}{2}\right)\sqrt{3}$$

$$= a\sqrt{5}-b\sqrt{3}$$

Hence, $a = \frac{1}{2}$ and $b = -\frac{1}{2}$.

Answer 6E.

$$\frac{\sqrt{3}-2}{\sqrt{3}+2}$$

$$= \frac{\sqrt{3}-2}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$= \frac{\sqrt{3}\left(\sqrt{3}-2\right)-2\left(\sqrt{3}-2\right)}{\left(\sqrt{3}\right)^2-(2)^2}$$

$$= \frac{3-2\sqrt{3}-2\sqrt{3}+4}{3-4}$$

$$= \frac{7-4\sqrt{3}}{-1}$$

$$= -\left(7-4\sqrt{3}\right)$$

$$= -7+4\sqrt{3}$$

$$= 4\sqrt{3}-7$$

$$= 4\sqrt{3}+(-7)$$

$$= a\sqrt{3}+b$$

Hence, $a = 4$ and $b = -7$.

Answer 6F.

$$\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}$$

$$= \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} \times \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}}$$

$$= \frac{\left(\sqrt{11}-\sqrt{7}\right)^2}{\left(\sqrt{11}\right)^2-\left(\sqrt{7}\right)^2}$$

$$= \frac{\left(\sqrt{11}\right)^2+\left(\sqrt{7}\right)^2-2\times\sqrt{11}\times\sqrt{7}}{11-7}$$

$$= \frac{11+7-2\sqrt{77}}{4}$$

$$= \frac{18-2\sqrt{77}}{4}$$

$$= \frac{18}{4}-\frac{2}{4}\sqrt{77}$$

$$= \frac{9}{2}-\frac{1}{2}\sqrt{77}$$

$$= a-b\sqrt{77}$$

Hence, $a = \frac{9}{2}$ and $b = \frac{1}{2}$.

Answer 6G.

$$\frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}$$

$$=\frac{7\sqrt{3}-5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\times\frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$$

$$=\frac{7\sqrt{3}\left(4\sqrt{3}-3\sqrt{2}\right)-5\sqrt{2}\left(4\sqrt{3}-3\sqrt{2}\right)}{\left(4\sqrt{3}\right)^2-\left(3\sqrt{2}\right)^2}$$

$$=\frac{84-21\sqrt{6}-20\sqrt{6}+30}{48-18}$$

$$=\frac{110-41\sqrt{6}}{30}$$

$$=\frac{110}{30}-\frac{41\sqrt{6}}{30}$$

$$=\frac{11}{3}-\frac{41}{30}\sqrt{6}$$

$$=a-b\sqrt{6}$$

Hence, $a=\frac{11}{3}$ and $b=\frac{41}{30}$.

Answer 6H.

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$$

$$=\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$$

$$=\frac{\left(\sqrt{2}+\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}$$

$$=\frac{\sqrt{2}\left(3\sqrt{2}+2\sqrt{3}\right)+\sqrt{3}\left(3\sqrt{2}+2\sqrt{3}\right)}{(9\times2)-(4\times3)}$$

$$=\frac{\left(3\times2+2\sqrt{6}\right)+\left(3\sqrt{6}+2\times3\right)}{18-12}$$

$$=\frac{6+2\sqrt{6}+3\sqrt{6}+6}{6}$$

$$=\frac{12+5\sqrt{6}}{6}$$

$$=2+\frac{5\sqrt{6}}{6}$$

$$=2-\left(-\frac{5}{6}\right)\sqrt{6}$$

$$=a-b\sqrt{6}$$

Hence, $a=2$ and $b=-\frac{5}{6}$.

Answer 6I.

$$\frac{7+\sqrt{5}}{7-\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}}$$

$$=\frac{7+\sqrt{5}}{7-\sqrt{5}}\times\frac{7+\sqrt{5}}{7+\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}}\times\frac{7-\sqrt{5}}{7-\sqrt{5}}$$

$$=\frac{\left(7+\sqrt{5}\right)^2}{7^2-\left(\sqrt{5}\right)^2}-\frac{\left(7-\sqrt{5}\right)^2}{7^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{7^2+2\times7\times\sqrt{5}+\left(\sqrt{5}\right)^2}{49-5}-\frac{7^2-2\times7\times\sqrt{5}+\left(\sqrt{5}\right)^2}{49-5}$$

$$=\frac{49+14\sqrt{5}+5}{44}-\frac{49-14\sqrt{5}+5}{44}$$

$$=\frac{54+14\sqrt{5}}{44}-\frac{54-14\sqrt{5}}{44}$$

$$=\frac{2\left(27+7\sqrt{5}\right)}{44}-\frac{2\left(22-7\sqrt{5}\right)}{44}$$

$$=\frac{27+7\sqrt{5}}{22}-\frac{27-7\sqrt{5}}{22}$$

$$=\frac{27}{22}+\frac{7\sqrt{5}}{22}-\frac{27}{22}+\frac{7\sqrt{5}}{22}$$

$$=\frac{14\sqrt{5}}{22}$$

$$=\frac{7\sqrt{5}}{11}$$

$$=0+\frac{7\sqrt{5}}{11}$$

$$=a+b\sqrt{5}$$

Hence, $a=0$ and $b=\frac{7}{11}$.

Answer 6J.

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$=\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2-1}+\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}\right)^2-1}$$

$$=\frac{\left(\sqrt{3}\right)^2-2\times\sqrt{3}\times1+1^2}{3-1}+\frac{\left(\sqrt{3}\right)^2+2\times\sqrt{3}\times1+1^2}{3-1}$$

$$=\frac{3-2\sqrt{3}+1}{2}+\frac{3+2\sqrt{3}+1}{2}$$

$$=\frac{4-2\sqrt{3}}{2}+\frac{4+2\sqrt{3}}{2}$$

$$=\frac{2\left(2-\sqrt{3}\right)}{2}+\frac{2\left(2+\sqrt{3}\right)}{2}$$

$$=2-\sqrt{3}+2+\sqrt{3}$$

$$=4+0$$

Hence, $a=4$ and $b=0$

Answer 7.

(i) $\sqrt{x}+\frac{1}{\sqrt{x}}$

Squaring Both sides we get

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=x+\frac{1}{x}+2 \quad ----(1)$$

we will first find out $x+\frac{1}{x}$

$$x+\frac{1}{x}=\left(7+4\sqrt{3}\right)+\frac{1}{\left(7+4\sqrt{3}\right)}$$

$$=\frac{\left(7+4\sqrt{3}\right)^2+1}{\left(7+4\sqrt{3}\right)}$$

$$=\frac{49+48+56\sqrt{3}+1}{\left(7+4\sqrt{3}\right)}$$

$$=\frac{98+56\sqrt{3}}{\left(7+4\sqrt{3}\right)}=\frac{14\left(7+4\sqrt{3}\right)}{\left(7+4\sqrt{3}\right)}=14$$

substituting in (1)

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=x+\frac{1}{x}+2=14+2=16$$

$$\therefore \sqrt{x}+\frac{1}{\sqrt{x}}=4$$

(ii) $x^2+\frac{1}{x^2}$

$$\left(x^2+\frac{1}{x^2}\right)=\left(x+\frac{1}{x}\right)^2-2 \quad ----(1)$$

we will first find out $x+\frac{1}{x}$

$$x+\frac{1}{x}=\left(7+4\sqrt{3}\right)+\frac{1}{\left(7+4\sqrt{3}\right)}$$

$$=\frac{\left(7+4\sqrt{3}\right)^2+1}{\left(7+4\sqrt{3}\right)}$$

$$=\frac{49+48+56\sqrt{3}+1}{\left(7+4\sqrt{3}\right)}$$

$$=\frac{98+56\sqrt{3}}{\left(7+4\sqrt{3}\right)}=\frac{14\left(7+4\sqrt{3}\right)}{\left(7+4\sqrt{3}\right)}=14$$

substitutingin(1)

$$\left(x^2+\frac{1}{x^2}\right)=\left(x+\frac{1}{x}\right)^2-2=196-2=194$$

$$\therefore \left(x^2+\frac{1}{x^2}\right)=194$$

(iii) $x^3+\frac{1}{x^3}$

$$\left(x^3+\frac{1}{x^3}\right)=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right) \quad ----(1)$$

we will first find out $x+\frac{1}{x}$

$$x+\frac{1}{x}=\left(7+4\sqrt{3}\right)+\frac{1}{\left(7+4\sqrt{3}\right)}$$

$$=\frac{\left(7+4\sqrt{3}\right)^2+1}{\left(7+4\sqrt{3}\right)}$$

$$=\frac{49+48+56\sqrt{3}+1}{\left(7+4\sqrt{3}\right)}$$

$$=\frac{98+56\sqrt{3}}{\left(7+4\sqrt{3}\right)}=\frac{14\left(7+4\sqrt{3}\right)}{\left(7+4\sqrt{3}\right)}=14$$

substitutingin(1)

$$\left(x^3+\frac{1}{x^3}\right)=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=(14)^3-3\times 14=2744-42$$

$$=2702$$

$$\therefore \left(x^3+\frac{1}{x^3}\right)=2702$$

(iv)

$$x = 7 + 4\sqrt{3}$$
$$\therefore \frac{1}{x} = \frac{1}{7 + 4\sqrt{3}}$$
$$= \frac{1}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}}$$
$$= \frac{7 - 4\sqrt{3}}{7^2 - \left(4\sqrt{3}\right)^2}$$
$$= \frac{7 - 4\sqrt{3}}{49 - 48}$$
$$= \frac{7 - 4\sqrt{3}}{1}$$
$$= 7 - 4\sqrt{3}$$
$$\therefore x + \frac{1}{x} = \left(7 + 4\sqrt{3}\right) + \left(7 - 4\sqrt{3}\right) = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} = 14$$
$$\text{Hence, } \left(x + \frac{1}{x}\right)^2 = (14)^2 = 196$$

Answer 8.

(i) $\frac{1}{x}$

$$\frac{1}{x} = \frac{1}{\left(4 - \sqrt{15}\right)}$$
$$= \frac{1}{\left(4 - \sqrt{15}\right)} \times \frac{\left(4 + \sqrt{15}\right)}{\left(4 + \sqrt{15}\right)}$$
$$= \frac{\left(4 + \sqrt{15}\right)}{16 - 15} = \left(4 + \sqrt{15}\right)$$

(ii) $x + \frac{1}{x}$

$$x + \frac{1}{x} = \left(4 - \sqrt{15}\right) + \frac{1}{\left(4 - \sqrt{15}\right)}$$
$$= \frac{\left(4 - \sqrt{15}\right)^2 + 1}{\left(4 - \sqrt{15}\right)}$$
$$= \frac{16 + 15 - 8\sqrt{15} + 1}{\left(4 - \sqrt{15}\right)}$$
$$= \frac{8\left(4 - \sqrt{15}\right)}{\left(4 - \sqrt{15}\right)} = 8$$

(iii) $x^2+\frac{1}{x^2}$

$$\left(x^2+\frac{1}{x^2}\right)=\left(x+\frac{1}{x}\right)^2-2 \quad ------(1)$$

we will first find the value of $x+\frac{1}{x}$

$$x+\frac{1}{x}=\left(4-\sqrt{15}\right)+\frac{1}{\left(4-\sqrt{15}\right)}$$

$$=\frac{\left(4-\sqrt{15}\right)^2+1}{\left(4-\sqrt{15}\right)}$$

$$=\frac{16+15-8\sqrt{15}+1}{\left(4-\sqrt{15}\right)}$$

$$=\frac{8\left(4-\sqrt{15}\right)}{\left(4-\sqrt{15}\right)}=8$$

substituting the values in (1)

$$\left(x^2+\frac{1}{x^2}\right)=\left(x+\frac{1}{x}\right)^2-2=8^2-2=64-2=62$$

$$\left(x^2+\frac{1}{x^2}\right)=62$$

(iv) $x^3+\frac{1}{x^3}$

$$\left(x^3+\frac{1}{x^3}\right)=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right) \quad ------(1)$$

we will first find the value of $x+\frac{1}{x}$

$$x+\frac{1}{x}=\left(4-\sqrt{15}\right)+\frac{1}{\left(4-\sqrt{15}\right)}$$

$$=\frac{\left(4-\sqrt{15}\right)^2+1}{\left(4-\sqrt{15}\right)}$$

$$=\frac{16+15-8\sqrt{15}+1}{\left(4-\sqrt{15}\right)}$$

$$=\frac{8\left(4-\sqrt{15}\right)}{\left(4-\sqrt{15}\right)}=8$$

substituting the values in (1)

$$\left(x^3+\frac{1}{x^3}\right)=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=8^3-24=488$$

$$\left(x^3+\frac{1}{x^3}\right)=488$$

(v)

$$x = 4 - \sqrt{15}$$
$$\therefore \frac{1}{x} = \frac{1}{4-\sqrt{15}}$$
$$= \frac{1}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}}$$
$$= \frac{4+\sqrt{15}}{4^2 - \left(\sqrt{15}\right)^2}$$
$$= \frac{4+\sqrt{15}}{16-15}$$
$$= \frac{4+\sqrt{15}}{1}$$
$$= 4+\sqrt{15}$$
$$\therefore x + \frac{1}{x} = \left(4-\sqrt{15}\right) + \left(4+\sqrt{15}\right) = 4-\sqrt{15} + 4 + \sqrt{15} = 8$$
$$\text{Hence, } \left(x + \frac{1}{x}\right)^2 = (8)^2 = 64$$

Answer 9.

$$x = \frac{\left(2+\sqrt{5}\right)}{\left(2-\sqrt{5}\right)}$$
$$= \frac{\left(2+\sqrt{5}\right)}{\left(2-\sqrt{5}\right)} \times \frac{\left(2+\sqrt{5}\right)}{\left(2+\sqrt{5}\right)}$$
$$= \frac{\left(2+\sqrt{5}\right)^2}{4-5} = -\left(4+5+4\sqrt{5}\right)$$
$$= -9-4\sqrt{5}$$
$$y = \frac{\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)}$$
$$= \frac{\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)} \times \frac{\left(2-\sqrt{5}\right)}{\left(2-\sqrt{5}\right)}$$
$$= \frac{\left(2-\sqrt{5}\right)^2}{4-5} = -\left(4+5-4\sqrt{5}\right)$$
$$= -9+4\sqrt{5}$$
$$\therefore x^2 - y^2 = (x+y)(x-y)$$
$$= (-9-4\sqrt{5}-9+4\sqrt{5})(-9-4\sqrt{5}+9-4\sqrt{5})$$
$$= (-18)(-8\sqrt{5}) = 144\sqrt{5}$$

Answer 10.

(i) x^2+y^2

$$x^2+y^2=(x+y)^2-2xy ----(1)$$

$$\therefore (x+y)=\frac{(\sqrt{3}+1)}{(\sqrt{3}-1)}+\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$$

$$=\frac{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2}{3-1}$$

$$=\frac{3+1+2\sqrt{3}+3+1-2\sqrt{3}}{2}$$

$$=\frac{8}{2}=4$$

$$\text{and } xy=\frac{(\sqrt{3}+1)}{(\sqrt{3}-1)}\times\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$$

$$=1$$

substituting in(1), we get

$$x^2+y^2=(x+y)^2-2xy=16-2=14$$

(ii) x^3+y^3

$$x^3+y^3=(x+y)^3-3xy(x+y) ----(1)$$

$$\therefore (x+y)=\frac{(\sqrt{3}+1)}{(\sqrt{3}-1)}+\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$$

$$=\frac{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2}{3-1}$$

$$=\frac{3+1+2\sqrt{3}+3+1-2\sqrt{3}}{2}$$

$$=\frac{8}{2}=4$$

$$\text{and } xy=\frac{(\sqrt{3}+1)}{(\sqrt{3}-1)}\times\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$$

$$=\frac{3-1}{3-1}=1$$

substitutingin(1), we get

$$x^3+y^3=(x+y)^3-3xy(x+y)$$

$$=64-3\times 4=64-12=52$$

(iii) $x^2 - y^2 + xy$

$$x^2 - y^2 + xy = (x+y)(x-y) + xy \quad ----(1)$$

$$\therefore (x+y) = \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} + \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$$

$$= \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{3-1}$$

$$= \frac{3+1+2\sqrt{3}+3+1-2\sqrt{3}}{2}$$

$$= \frac{8}{2} = 4$$

$$(x-y) = \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} - \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$$

$$= \frac{(\sqrt{3}+1)^2 - (\sqrt{3}-1)^2}{3-1}$$

$$= \frac{3+1+2\sqrt{3}-3-1+2\sqrt{3}}{2}$$

$$= 2\sqrt{3}$$

$$\text{and } xy = \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}$$

$$= \frac{3-1}{3-1} = 1$$

substituting in (1), we get

$$x^2 - y^2 + xy = (x+y)(x-y) + xy$$

$$= 4 \times 2\sqrt{3} + 1 = 8\sqrt{3} + 1$$

Answer 11.

(i) x^2+y^2

$$\left(x^2+y^2\right)=(x+y)^2-2xy \quad ----(1)$$

$$\text{Now, } x+y=\frac{1}{\left(3-2\sqrt{2}\right)}+\frac{1}{\left(3+2\sqrt{2}\right)}$$

$$=\frac{\left(3+2\sqrt{2}\right)+\left(3-2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$$

$$=\frac{6}{9-8}=6$$

$$\text{and } xy=\frac{1}{\left(3-2\sqrt{2}\right)}\times\frac{1}{\left(3+2\sqrt{2}\right)}$$

$$=\frac{1}{9-8}=1$$

sustituting the values in (1), we get

$$\left(x^2+y^2\right)=(x+y)^2-2xy=36-2=34$$

$$\left(x^2+y^2\right)=34$$

(ii) x^3+y^3

$$\left(x^3+y^3\right)=(x+y)^3-3xy(x+y) \quad ----(1)$$

$$\text{Now, } x+y=\frac{1}{\left(3-2\sqrt{2}\right)}+\frac{1}{\left(3+2\sqrt{2}\right)}$$

$$=\frac{\left(3+2\sqrt{2}\right)+\left(3-2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$$

$$=\frac{6}{9-8}=6$$

$$\text{and } xy=\frac{1}{\left(3-2\sqrt{2}\right)}\times\frac{1}{\left(3+2\sqrt{2}\right)}$$

$$=\frac{1}{9-8}=1$$

sustituting the values in (1), we get

$$\left(x^3+y^3\right)=(x+y)^3-3xy(x+y)$$

$$=216-3\times 6=198$$